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A

MATHEMATICAL AND PHILOSOPHICAL

DICTIONARY.

VOL. I.

A MATHEMATICAL AND PHILOSOPHICAL DICTIONARY:

CONTAINING

AN EXPLANATION OF THE TERMS, AND AN ACCOUNT OF THE SEVERAL SUBJECTS,

COMPRIZED UNDER THE HEADS

MATHEMATICS, ASTRONOMY, AND PHILOSOPHY

BOTH NATURAL AND EXPERIMENTAL:

WITH AN

HISTORICAL ACCOUNT OF THE RISE, PROGRESS, AND PRESENT STATE OF THESE SCIENCES:

ALSO

MEMOIRS OF THE LIVES AND WRITINGS OF THE MOST EMINENT AUTHORS,

BOTH ANCIENT AND MODERN,

WHO BY THEIR DISCOVERIES OR IMPROVEMENTS HAVE CONTRIBUTED TO THE ADVANCEMENT OF THEM.

IN TWO VOLUMES.

WITH MANY CUTS AND COPPER-PLATES.

By CHARLES HUTTON, LL.D.

F. R. SS. OF LONDON AND EDINBURGH, AND OF THE PHILOSOPHICAL SOCIETIES OF HAARLEM AND AMERICA;
AND PROFESSOR OF MATHEMATICS IN THE ROYAL MILITARY ACADEMY, WOOLWICH.

VOL. I.

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PREFACE.

AMONG the Dictionaries of Arts and Sciences which have been publiſhed, of late years, in various parts of Europe, it is matter of ſurpriſe that Philoſophy and Mathematics ſhould have been ſo far overlooked as not to be thought worthy of a ſeparate Treatiſe, in this form. Theſe Sciences conſtitute a large portion of the preſent ſtock of human knowledge, and have been uſually conſidered as poſſeſſing a degree of importance to which few others are entitled; and yet we have hitherto had no diſtinct Lexicon, in which their conſtituent parts and technical terms have been explained, with that amplitude and preciſion, which the great improvements of the Moderns, as well as the riſing dignity of the Subject, ſeem to demand.

THE only works of this kind in the Engliſh language, deſerving of notice, are Harris's Lexicon Technicum, and Stone's Mathematical Dictionary; the former of which, though a valuable performance at the time it was written, is now become too dry and obſolete to be referred to with pleaſure or ſatisfaction: and the latter, conſiſting only of one volume in 8vo, muſt be regarded merely as an unfiniſhed ſketch, or brief compendium, extremely limited in its plan, and neceſſarily deficient in uſeful information.

IT became, therefore, the only reſource of the Reader, in many caſes where explanation was wanted, to have recourſe to Chambers's Dictionary, in four large Volumes folio, or to the Encyclopædia Britannica, now in eighteen large volumes 4to, or the ſtill more ſtupendous performance of the French Encyclopediſts; and even here his expectations might be frequently diſappointed. Theſe great and uſeful works, aiming at a general comprehenſion of the whole circle of the Sciences, are ſometimes very deficient in their deſcriptions of particular branches; it being almoſt impoſſible, in ſuch extenſive undertakings, to appreciate, with exactneſs, the due value of every article: They are, beſides, ſo voluminous and heterogeneous in their nature, as to render a frequent reference to them extremely inconvenient; and even if this were not the caſe, their high price puts them out of the reach of the generality of readers.

WITH a view to obviate theſe defects, the Public are here preſented with a Dictionary of a moderate ſize and price, which is devoted ſolely to Philoſophical and Mathematical ſubjects. It is a work for which materials have been collecting through a courſe of many years; and is the reſult of great labour and reading. Not only moſt of the Encyclopedias already extant, and the various publications of the Learned Societies throughout Europe, have been carefully conſulted, but alſo all the original works, of any reputation, which have hitherto appeared upon theſe ſubjects, from the earlieſt writers down to the preſent times.

FROM the latter of theſe ſources, in particular, a conſiderable portion of information has been obtained, which the curious reader will find, in many caſes, to be highly intereſting and important. The Hiſtory of Algebra, for inſtance, which is detailed at conſiderable length in the Firſt Volume, under the head of that Article, will afford ſufficient evidence to ſhew in what a ſuperficial and partial way the inquiry has been hitherto inveſtigated, even by profeſſed writers on the ſubject; the principal of whom are M. Montucla, our countryman the celebrated Dr. Wallis, and the Abbé De Gua, a late French author, who has pretended to correct the Doctor's errors and miſrepreſentations.

REGULAR hiſtorical details are in like manner given of the origin and progreſs of each of theſe Sciences, as well as of the inventions and improvements by which they have been gradually brought from their firſt rude beginnings to their preſent advanced ſtate.

IT is alſo to be obſerved, that beſides the articles common to the generality of Dictionaries of this kind, an intereſting Biographical Account is here introduced of the moſt celebrated Philoſophers and Mathematicians, both ancient and modern; among which will be found the Lives of many eminent characters, who have hitherto been either wholly overlooked, or very imperfectly recorded. Complete liſts of their works are alſo ſubjoined to each Article, where they could be procured; which cannot but prove highly acceptable to that claſs of readers, who are deſirous of obtaining the moſt ſatisfactory information upon the ſubjects of their particular enquiries and purſuits. On the head of Biography however the Author has ſtill to lament the want of many other reſpectable names which he was deſirous to have added to his liſt of authors, not having been able to procure any circumſtantial accounts of their lives. He could have wiſhed to have compriſed in his liſt, the lives of all ſuch public literary characters as the Univerſity Profeſſors of Aſtronomy, Philoſophy and Mathematics, as well as thoſe of the other more reſpectable claſs of Authors on thoſe Sciences. He will therefore thankfully receive the communication of any ſuch memoirs from the hands of gentlemen poſſeſſed of them; as well as hints and information on ſuch uſeful improvements in the ſciences as may have been overlooked in this Dictionary, or any articles that may here have been imperfectly or incorrectly treated; that he may at ſome future time, by adding them to this work, render it ſtill more complete and deſerving the public notice.

As this work is an attempt to ſeparate the words in the ſciences of Aſtronomy, Mathematics, and Philoſophy, from thoſe of other arts or ſciences, in ſeveral of which there are already ſeparate Dictionaries; as in Chemiſtry, Geography, Muſic, Marine and Naval affairs, &c; words ſometimes occurred which it was rather doubtful whether they could be conſidered as properly belonging to the preſent work or not; in which caſe many of ſuch words have been here inſerted. But ſuch as appeared clearly and peculiarly to belong to any of thoſe other ſubjects, have been either wholly omitted, or elſe have had a very ſhort account only given of them. The readers of this work therefore, recollecting that it is not a General Dictionary of all the Arts and Sciences, will not expect to find all ſorts of words and ſubjects here treated of; but ſuch only as peculiarly appertain to the proper matter of the work. And therefore, although ſome few words may inadvertently have been omitted; yet when the Reader does not immediately find every word which he wiſhes to conſult, he will not always conſider them as omiſſions of the Author, but for the moſt part as relating to ſome other ſcience foreign to this Dictionary.

In all caſes where it could be conveniently done, the neceſſary figures and diagrams are inſerted in the ſame page with the ſubjects which they are deſigned to elucidate; a method which will be found much more commodious than that of putting them in ſeparate plates at the end of each volume, but, which has added very conſiderably to the expence of the undertaking: where the ſubjects are of ſuch a nature that they could not be otherwiſe well repreſented, they are engraved on Copperplates.

As the whole of this work was written before it was put to the preſs, the Reader will find it of an equal and uniform nature and conſtruction throughout; in which reſpect many publications of this kind are very defective, from the ſubjects being diffuſely treated under the firſt letters of the alphabet, while articles of equal importance in the latter part are ſo much abridged as to be rendered almoſt uſeleſs, in order that the whole might be compriſed in a limited number of ſheets, according to propoſals made before the works were compoſed. The preſent Dictionary having been completed without any of theſe unfavourable circumſtances, will be found in moſt caſes equally inſtructive and uſeful, and may be conſulted with no leſs advantage by the Man of Science than the Student.

A
PHILOSOPHICAL and MATHEMATICAL
DICTIONARY.

A.

ABACIST, an Arithmetician. In this ſenſe we find the word uſed by William of Malmeſbury, in his Hiſtory *de Geſtis Anglorum*, written about the year 1150; where he ſhews that one Gerbert, a learned monk of France, who was afterwards made pope of Rome in the year 998 or 999, by the name of Silveſter the 2d, was the firſt who got from the Saracens the abacus, and that he taught ſuch rules concerning it, as the Abaciſts themſelves could hardly underſtand.

ABACUS, *in Arithmetic*, an ancient inſtrument uſed by moſt nations for caſting up accounts, or performing arithmetical calculations: it is by ſome derived from the Greek αϐαξ, which ſignifies a cupboard or beaufet, perhaps from the ſimilarity of the form of this inſtrument; and by others it is derived from the Phœnician *abak*, which ſignifies duſt or powder, becauſe it was ſaid that this inſtrument was ſometimes made of a ſquare board or tablet, which was powdered over with fine ſand or duſt, in which were traced the figures or characters uſed in making calculations, which could thence be eaſily defaced, and the abacus refitted for uſe. But Lucas Paciolus, in the firſt part of his ſecond diſtinction, thinks it is a corruption of Arabicus, by which he meant their Algoriſm, or the method of numeral computation received from them.

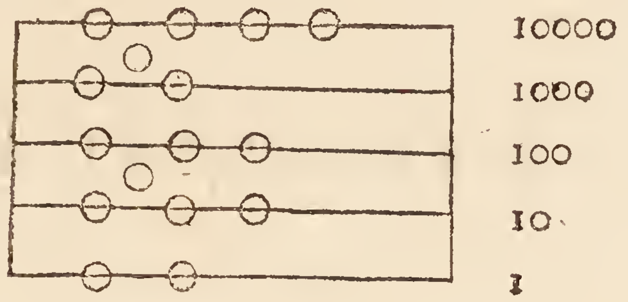
We find this inſtrument for computation in uſe, under ſome variations, with moſt nations, as the Greeks, Romans, Germans, French, Chineſe, &c.

The Grecian abacus was an oblong frame, over which were ſtretched ſeveral braſs wires, ſtrung with little ivory balls, like the beads of a necklace; by the various arrangements of which all kinds of computations were eaſily made. Mahudel, in Hiſt. Acad. R. Inſcr. t. 3. p. 390.

The Roman Abacus was a little varied from the Grecian, having pins ſliding in grooves, inſtead of ſtrings or wires and beads. Philoſ. Tranſ. No. 180.

The Chineſe Abacus, or Shwan-pan, like the Grecian, conſiſts of ſeveral ſeries of beads ſtrung on braſs wires, ſtretched from the top to the bottom of the inſtrument, and divided in the middle by a croſs piece from ſide to ſide. In the upper ſpace every ſtring has two beads, which are each counted for 5; and in the lower ſpace every ſtring has five beads, of different values, the firſt being counted as 1, the ſecond as 10, the third as 100, and ſo on, as with us. See SHWAN-PAN.

The Abacus chiefly uſed in European countries, is nearly upon the ſame principles, though the uſe of it is here more limited, becauſe of the arbitrary and unequal diviſions of money, weights, and meaſures, which, in China, are all divided in a tenfold proportion, like our ſcale of common numbers. This is made by drawing any number of parallel lines, like paper ruled for muſic, at ſuch a diſtance as may be at leaſt equal to twice the diameter of a calculus, or counter. Then the value of theſe lines, and of the ſpaces between them, increaſes, from the loweſt to the higheſt, in a tenfold proportion. Thus, counters placed upon the firſt line, ſignify ſo many units or ones; on the ſecond line 10's, on the third line 100's, on the fourth line 1000's, and ſo on: in like manner a counter placed in the firſt ſpace, between the firſt and ſecond line, denotes 5, in the ſecond ſpace 50, in the third ſpace 500, in the fourth ſpace 5000, and ſo on. So that there are never more than four counters placed on any line, nor more than one placed in any ſpace, this being of the ſame value as five counters on the next line below. So the counters on the Abacus, in the figure here below, expreſs the number or ſum 47382.



Beſides the above inſtruments of computation, there have been ſeveral others invented by different perſons; as *Napier's rods* or *bones*, deſcribed in his Rabdologia, which ſee under the word Napier; alſo the *Abacus Rhabdologicus*, a variation of Napier's, which is deſcribed in the firſt vol. of *Machines et Inventions approuvées par l'Academie Royale des Sciences*. An ingenious and general one was alſo invented by Mr. Gamaliel Smethurſt, and is deſcribed in the Philoſophical Tranſactions, vol. 46; where the inventor remarks that computations by it are much quicker and eaſier than by the pen, are leſs burthenſome to the memory, and can be performed by blind perſons, or in the dark as well as in the light. A very comprehenſive inſtrument of this kind was alſo contrived by the late learned Dr. Nicholas Saunderſon, by which he performed very intricate calculations: an account of it is prefixed to the firſt volume of his Algebra, and it is there by the editor called *Palpable Arithmetic*: which ſee.

Abacus, *Pythagorean*, ſo denominated from its inventor, Pythagoras; a table of numbers, contrived for readily learning the principles of arithmetic; and was probably what we now call the multiplication-table.

Abacus, or Abaciscus, in *Architecture*, the upper part or member of the capital of a column; ſerving as a crowning both to the capital and to the whole column. Vitruvius informs us that the *Abacus* was originally intended to repreſent a ſquare flat tile laid over an urn, or a baſket; and the invention is aſcribed to Calimachus, an ingenious ſtatuary of Athens, who, it is ſaid, adopted it on obſerving a ſmall baſket, covered with a tile, over the root of an Acanthus plant, which grew on the grave of a young lady; the plant ſhooting up, encompaſſed the baſket all around, till meeting with the tile, it curled back in the form of ſcrolls: Calimachus paſſing by, took the hint, and immediately executed a capital on this plan; repreſenting the tile by the Abacus, the leaves of the acanthus by the volutes or ſcrolls, and the baſket by the vaſe or body of the capital. See Acanthus.

Abacus is alſo uſed by Scamozzi for a concave moulding in the capital of the Tuſcan pedeſtal. And the word is uſed by Palladio for other members which he deſcribes. Alſo, in the ancient architecture, the ſame term is uſed to denote certain compartments in the incruſtation or lining of the walls of ſtate-rooms, moſaic-pavements, and the like. There were *Abaci* of marble, porphyry, jaſper, alabaſter, and even glaſs; variouſly ſhaped, as ſquare, triangular, and ſuch-like.

Abacus *Logiſticus* is a right angled triangle, whoſe ſides, about the right angle, contain all the numbers from 1 to 60; and its area the products of each two of the oppoſite numbers. This is alſo called a *canon of ſexageſimals*, and is no other than a multiplication-table carried to 60 both ways.

Abacus & *Palmulæ*, in the Ancient Muſic, denote the machinery by which the ſtrings of the polyplectra, or inſtruments of many ſtrings, were ſtruck, with a plectrum made of quills.

Abacus *Harmonicus* is uſed by Kircher for the ſtructure and diſpoſition of the keys of a muſical inſtrument, either to be touched with the hands or feet.

Abacus, in *Geometry*, a table or ſlate upon which ſchemes or diagrams are drawn.

ABAS, a weight uſed in Perſia for weighing pearls; and is an eighth part lighter than the European carat.

ABASSI, a ſilver coin current in Perſia, deriving its name from Schaw Abbas II. King of Perſia, and is worth near eighteen pence Engliſh money.

ABATIS, or Abattis, from the French *abattre*, to throw down, or beat down, in the Military Art, denotes a kind of retrenchment made by a quantity of whole trees cut down, and laid lengthways beſide each other, the cloſer the better, having all their branches pointed towards the enemy, which prevents his approach, at the ſame time that the trunks ſerve as a breaſt-work before the men. The Abattis is a very uſeful work on moſt occaſions, eſpecially on ſudden emergencies, when trees are near at hand; and has always been practiſed with conſiderable ſucceſs, by the ableſt commanders in all ages and nations.

ABBREVIATE; to abbreviate fractions in arithmetic and algebra, is to leſſen proportionally their terms, or the numerator and denominator; which is performed by dividing thoſe terms by any number or quantity, which will divide them without leaving a remainder. And when the terms cannot be any farther ſo divided, the fraction is ſaid to be in its leaſt terms.

So $\frac{16}{24} = \frac{8}{12} = \frac{4}{6} = \frac{2}{3}$,
by dividing the terms continually by 2.

And $\frac{294}{504} = \frac{147}{252} = \frac{49}{84} = \frac{7}{12}$,
by dividing by 2, 3, and 7.

Alſo $\frac{3\times8\times5}{6} = \frac{8\times5}{2} = 4\times5 = 20$,
by dividing by 3 and by 2.

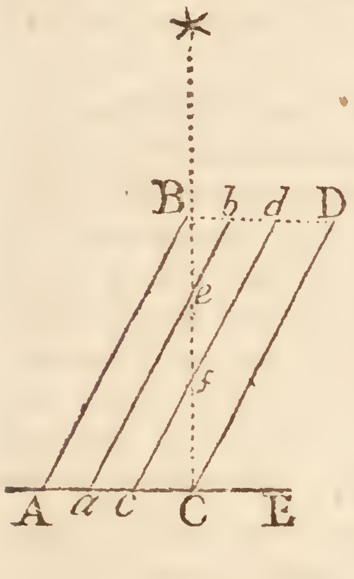
And $\frac{12abx^2}{4acx} = \frac{3bx}{c}$, by dividing by $4ax$.

And $\frac{ab^2+b^2x}{ax+x^2} = \frac{b^2}{x}$, by dividing by $a+x$.

ABBREVIATION, of fractions, in Arithmetic and Algebra, the reducing them to lower terms.

ABERRATION, in *Aſtronomy*, an apparent motion of the celeſtial bodies, occaſioned by the progreſſive motion of light, and the earth's annual motion in her orbit.

This effect may be explained and familiarized by the motion of a line parallel to itſelf, much after the manner that the compoſition and reſolution of forces are explained. If light have a progreſſive motion, let the proportion of its velocity to that of the earth in her orbit, be as the line B C to the line A C; then, by the compoſition of theſe two motions, the particle of light will ſeem to deſcribe the line B A or D C, inſtead of its real courſe B C; and will appear in the direction A B or C D, inſtead of its true direction C B. So that if A B repreſent a tube, carried with a parallel motion by an obſerver along the line A C, in the time that a particle of light would move over the ſpace B C, the different places of the tube being A B, *ab*, *cd*, C D; and when the eye, or end of the tube, is at A, let a particle of light enter the other end at B; then when the tube is at *a b*, the particle of light will be at *e*, exactly in the axis of the tube; and when the tube is at *c d*, the particle of light will arrive at *f*, ſtill in the axis of the tube; and laſtly, when the tube arrives at C D, the particle of light will arrive at the eye or point C, and conſequently will appear to come in the direction D C of the tube, inſtead of the true direction B C. And ſo on, one particle ſucceeding another, and forming a continued ſtream or ray of light in the apparent direction D C. So that the apparent angle made by the ray of light with the line A E, is the angle D C E, inſtead of the true angle B C E; and the difference, B C D or A B C, is the quantity of the aberration.



M. de Maupertuis, in his Elements of Geography, gives alſo a familiar and ingenious idea of the aberration, in this manner: "It is thus," ſays he, "concerning the direction in which a gun muſt be pointed to ſtrike a bird in its flight; inſtead of pointing it ſtraight to the bird, the fowler will point a little before it, in the path of its flight, and that ſo much the more as the flight of the bird is more rapid, with reſpect to the flight of the ſhot." In this way of conſidering the matter, the flight of the bird repreſents the motion of the earth, or the line A C, in our ſcheme above, and the flight of the ſhot repreſents the motion of the ray of light, or the line B C.

Mr. Clairaut too, in the Memoires of the Academy of Sciences for the year 1746, illuſtrates this effect in a familiar way, by ſuppoſing drops of rain to fall rapidly and quickly after each other from a cloud, under which a perſon moves with a very narrow tube; in which caſe it is evident that the tube muſt have a certain inclination, in order that a drop which enters at the top, may fall freely through the axis of the tube, without touching the ſides of it; which inclination muſt be more or leſs according to the velocity of the drops in reſpect to that of the tube: then the angle made by the direction of the tube and of the falling drops, is the aberration ariſing from the combination of thoſe two motions.

This diſcovery, which is one of the brighteſt that have been made in the preſent age, we owe to the accuracy and ingenuity of the late Dr. Bradley, Aſtronomer Royal; to which he was occaſionally led by the reſult of ſome accurate obſervations which he had made with another view, namely, to determine the annual parallax of the fixed ſtars, or that which ariſes from the motion of the earth in its annual orbit about the ſun.

The annual motion of the earth about the ſun had been much doubted, and warmly conteſted. The defenders of that motion, among other proofs of the reality of it, conceived the idea of adducing an inconteſtable one from the annual parallax of the fixed ſtars, if the ſtars ſhould be within ſuch a diſtance, or if inſtruments and obſervations could be made with ſuch accuracy, as to render that parallax ſenſible. And with this view various attempts have been made. Before the obſervations of M. Picard, made in 1672, it was the general opinion, that the ſtars did not change their poſition during the courſe of a year. Tycho Brahe and Ricciolus fancied that they had aſſured themſelves of it from their obſervations; and from thence they concluded that the earth did not move round the ſun, and that there was no annual parallax in the fixed ſtars. M. Picard, in the account of his *Voyage d'Uranibourg*, made in 1672, ſays that the pole ſtar, at different times of the year, has certain variations which he had obſerved for about 10 years, and which amounted to about 40″ a year: from whence ſome who favoured the annual motion of the earth were led to conclude that theſe variations were the effect of the parallax of the earth's orbit. But it was impoſſible to explain it by that parallax; becauſe this motion was in a manner contrary to what ought to follow only from the motion of the earth in her orbit.

In 1674 Dr. Hook publiſhed an account of obſervations which he ſaid he had made in 1669, and by which he had found that the ſtar γ Draconis was 23″ more northerly in July than in October: obſervations which, for the preſent, ſeemed to favour the opinion of the earth's motion, although it be now known that there could not be any truth or accuracy in them.

Flamſteed having obſerved the pole ſtar with his mural quadrant, in 1689 and the following years, found that its declination was 40″ leſs in July than in December; which obſervations, although very juſt, were yet however improper for proving the annual parallax; and he recommended the making of an inſtrument of 15 or 20 feet radius, to be firmly fixed on a ſtrong foundation, for deciding a doubt which was otherwiſe not ſoon likely to be brought to a concluſion.

In this ſtate of uncertainty and doubt, then, Dr. Bradley, in conjunction with Mr. Samuel Molineux, in the year 1725, formed the project of verifying, by a ſeries of new obſervations, thoſe which Dr. Hook had communicated to the public almoſt 50 years before. And as it was his attempt that chiefly gave riſe to this, ſo it was his method in making the obſervations, in ſome meaſure, that they followed; for they made choice of the ſame ſtar, and their inſtrument was conſtructed upon nearly the ſame principles: but had it not greatly exceeded the former in exactneſs, they might ſtill have continued in great uncertainty as to the parallax of the fixed ſtars. And this was chiefly owing to the accuracy of the ingenious Mr. George Graham, to whom the lovers of aſtronomy are alſo indebted for ſeveral other exact and convenient inſtruments.

The ſucceſs then of the intended experiment, evidently depending very much on the accuracy of the inſtrument, that leading object was firſt to be well ſecured. Mr. Molineux's apparatus then having been completed, and fitted for obſerving, about the end of November 1725, on the third day of December following, the bright ſtar in the head of Draco, marked γ by Bayer, was for the firſt time obſerved, as it paſſed near the zenith, and its ſituation carefully taken with the inſtrument. The like obſervations were made on the fifth, eleventh, and twelfth days of the ſame month; and there appearing no material difference in the place of the ſtar, a farther repetition of them, at that ſeaſon, ſeemed needleſs, it being a time of the year in which no ſenſible alteration of parallax, in this ſtar, could ſoon be expected. It was therefore curioſity that chiefly urged Dr. Bradley, being then at Kew, where the inſtrument was fixed, to prepare for obſerving the ſtar again on the 17th of the ſame month; when, having adjuſted the inſtrument as uſual, he perceived that it paſſed a little more ſoutherly this day than it had done before. Not ſuſpecting any other cauſe of this appearance, they firſt concluded that it was owing to the uncertainty of the obſervations, and that either this, or the foregoing, was not ſo exact as they had before ſuppoſed. For which reaſon they propoſed to repeat the obſervation again, to determine from what cauſe this difference might proceed: and upon doing it, on the 20th of December, the doctor found that the ſtar paſſed ſtill more ſoutherly than at the preceding obſervation. This ſenſible alteration ſurpriſed them the more, as it was the contrary way from what it would have been, had it proceeded from an annual parallax of the ſtar. But being now pretty well ſatisfied, that it could not be entirely owing to the want of exactneſs in the obſervations, and having no notion of any thing elſe that could cauſe ſuch an apparent motion as this in the ſtar; they began to ſuſpect that ſome change in the materials, or fabric of the inſtrument itſelf, might have occaſioned it. Under theſe uncertainties they remained for ſome time; but being at length fully convinced, by ſeveral trials, of the great exactneſs of the inſtrument; and finding, by the gradual increaſe of the ſtar's diſtance from the pole, that there muſt be ſome regular cauſe that produced it; they took care to examine very nicely, at the time of each obſervation, how much the variation was; till about the beginning of March 1726, the ſtar was found to be 20″ more ſoutherly than at the time of the firſt obſervation: it now indeed ſeemed to have arrived at its utmoſt limit ſouthward, as in ſeveral trials, made about this time, no ſenſible difference was obſerved in its ſituation. By the middle of April it appeared to be returning back again towards the north; and about the beginning of June, it paſſed at the ſame diſtance from the zenith, as it had done in December, when it was firſt obſerved.

From the quick alteration in the declination of the ſtar about this time, increaſing about one ſecond in three days, it was conjectured that it would now proceed northward, as it had before gone ſouthward, of its preſent ſituation; and it happened accordingly; for the ſtar continued to move northward till September following, when it became ſtationary again; being then near 20″ more northerly than in June, and upwards of 39″ more northerly than it had been in March. From September the ſtar again returned towards the ſouth, till, in December, it arrived at the ſame ſituation in which it had been obſerved twelve months before, allowing for the difference of declination on account of the preceſſion of the equinox.

This was a ſufficient proof that the inſtrument had not been the cauſe of this apparent motion of the ſtar; and yet it ſeemed difficult to deviſe one that ſhould be adequate to ſuch an unuſual effect. A nutation of the earth's axis was one of the firſt things that offered itſelf on this occaſion; but it was ſoon found to be inſufficient; for though it might have accounted for the change of declination in γ Draconis, yet it would not at the ſame time accord with the phenomena obſerved in the other ſtars, particularly in a ſmall one almoſt oppoſite in right aſcenſion to γ Draconis, and at about the ſame diſtance from the north pole of the equator: for though this ſtar ſeemed to move the ſame way, as a nutation of the earth's axis would have made it; yet changing its declination but about half as much as γ Draconis in the ſame time, as appeared on comparing the obſervations of both made on the ſame days, at different ſeaſons of the year, this plainly proved that the apparent motion of the ſtar was not occaſioned by a real nutation; ſince, had that been the caſe, the alteration in both ſtars would have been nearly equal.

The great regularity of the obſervations left no room to doubt, but that there was ſome uniform cauſe by which this unexpected motion was produced, and which did not depend on the uncertainty or variety of the ſeaſons of the year. Upon comparing the obſervations with each other, it was diſcovered that, in both the ſtars above mentioned, the apparent difference of declination from the *maxima*, was always nearly proportional to the verſed ſine of the ſun's diſtance from the equinoctial points. This was an inducement to think that the cauſe, whatever it was, had ſome relation to the ſun's ſituation with reſpect to thoſe points. But not being able to frame any hypotheſis, ſufficient to account for all the phenomena, and being very deſirous to ſearch a little farther into this matter, Dr. Bradley began to think of erecting an inſtrument for himſelf at Wanſtead; that, having it always at hand, he might with the more eaſe and certainty enquire into the laws of this new motion. The conſideration likewiſe of being able, by another inſtrument, to confirm the truth of the obſervations hitherto made with that of Mr. Molineux, was no ſmall inducement to the undertaking; but the chief of all was, the opportunity he ſhould thereby have of trying in what manner other ſtars ſhould be affected by the ſame cauſe, whatever it might be. For Mr. Molineux's inſtrument being originally deſigned for obſerving γ Draconis, to try whether it had any ſenſible parallax, it was ſo contrived, as to be capable of but little alteration in its direction; not above ſeven or eight minutes of a degree: and there being but few ſtars, within half that diſtance from the zenith of Kew, bright enough to be well obſerved, he could not, with his inſtrument, thoroughly examine how this cauſe affected ſtars that were differently ſituated, with reſpect to the equinoctial and ſolſticial points of the ecliptic.

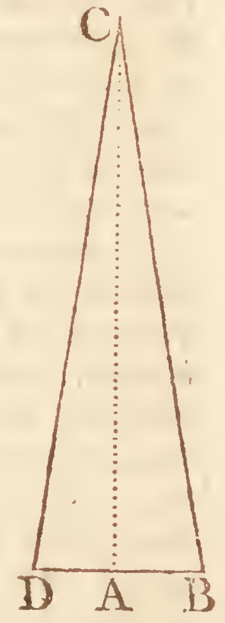
Theſe conſiderations determined him; and by the contrivance and direction of the ſame ingenious perſon, Mr. Graham, his inſtrument was fixed up the 19th of Auguſt 1727. As he had no convenient place where he could make uſe of ſo long a teleſcope as Mr. Molineux's, he contented himſelf with one of but little more than half the length, namely of 12 feet and a half, the other being 24 feet and a half long, judging from the experience he had already had, that this radius would be long enough to adjuſt the inſtrument to a ſufficient degree of exactneſs: and he had no reaſon afterwards to change his opinion; for by all his trials he was very well ſatisfied, that when it was carefully rectified, its ſituation might be ſecurely depended on to half a ſecond. As the place where his inſtrument was hung, in ſome meaſure determined its radius; ſo did it alſo the length of the arc or limb, on which the diviſions were made, to adjuſt it: for the arc could not conveniently be extended farther, than to reach to about $6\frac{1}{4}$ degrees on each ſide of his zenith. This however was ſufficient, as it gave him an opportunity of making choice of ſeveral ſtars, very different both in magnitude and ſituation: there being more than two hundred, inſerted in the Britiſh Catalogue, that might be obſerved with it. He needed not indeed to have extended the limb ſo far, but that he was willing to take in *Capella*, the only ſtar of the firſt magnitude that came ſo near his zenith.

His inſtrument being fixed, he immediately began to obſerve ſuch ſtars as he judged moſt proper to give him any light into the cauſe of the motion already mentioned. There was a ſufficient variety of ſmall ones, and not leſs than twelve that he could obſerve through all ſeaſons of the year, as they were bright enough to be ſeen in the day-time, when neareſt the ſun. He had not been long obſerving, before he perceived that the notion they had before entertained, that the ſtars were fartheſt north and ſouth when the ſun was near the equinoxes, was only true of thoſe ſtars which are near the ſolſticial colure. And after continuing his obſervations a few months, he diſcovered what he then apprehended to be a general law obſerved by all the ſtars, namely, that each of them became ſtationary, or was fartheſt north or ſouth, when it paſſed over his zenith at ſix of the clock, either in the evening or morning. He perceived alſo that whatever ſituation the ſtars were in, with reſpect to the cardinal points of the ecliptic, the apparent motion of every one of them tended the ſame way, when they paſſed his inſtrument about the ſame hour of the day or night; for they all moved ſouthward when they paſſed in the day, and northward when in the night; ſo that each of them was fartheſt north, when it came in the evening about ſix of the clock, and fartheſt ſouth when it came about ſix in the morning.

Though he afterwards diſcovered that the maxima, in moſt of theſe ſtars, do not happen exactly when they paſs at thoſe hours; yet, not being able at that time to prove the contrary, and ſuppoſing that they did, he endeavoured to find out what proportion the greateſt alterations of declination, in different ſtars, bore to each other; it being very evident that they did not all change their declination equally. It has been before noticed, that it appeared from Mr. Molineux's obſervations, that γ *Draconis* changed its declination above twice as much as the before-mentioned ſmall ſtar that was nearly oppoſite to it; but examining the matter more nicely, he found that the greateſt change in the declination of theſe ſtars, was as the ſine of the latitude of each ſtar reſpectively. This led him to ſuſpect that there might be the like proportion between the *maxima* of other ſtars; but finding that the obſervations of ſome of them would not perfectly correſpond with ſuch an hypotheſis, and not knowing whether the ſmall difference he met with might not be owing to the uncertainty and error of the obſervations, he deferred the farther examination into the truth of this hypotheſis, till he ſhould be farther furniſhed with a ſeries of obſervations made in all parts of the year; which would enable him not only to determine what errors the obſervations might be liable to, or how far they might ſafely be depended on; but alſo to judge, whether there had been any ſenſible change in the parts of the inſtrument itſelf.

When the year was completed, he began to examine and compare his obſervations; and having pretty well ſatisfied himſelf as to the general laws of the phenomena, he then endeavoured to find out the cauſe of them. He was already convinced that the apparent motion of the ſtars was not owing to a nutation of the earth's axis. The next that occurred to him, was an alteration in the direction of the plumb-line, by which the inſtrument was conſtantly adjuſted; but this, upon trial, proved inſufficient. Then he conſidered what refraction might do; but here alſo he met with no ſatisfaction. At laſt, through an amazing ſagacity, he conjectured that all the phenomena hitherto mentioned, proceeded from the progreſſive motion of light, and the earth's annual motion in her orbit: for he perceived, that if light were propagated in time, the apparent place of a fixed object would not be the ſame when the eye is at reſt, as when it is moving in any other direction but that of the line paſſing through the object and the eye; and that when the eye is moving in different directions, the apparent place of the object would be different.

He conſidered this matter in the following manner. He imagined C A to be a ray of light, falling perpendicularly upon the line B D: then, if the eye be at reſt at A, the object muſt appear in the direction A C, whether light be propagated in time, or in an inſtant. But if the eye be moving from B towards A, and light be propagated in time, with a velocity that is to the velocity of the eye, as A C to A B; then, light moving from C to A, whilſt the eye moves from B to A, that particle of it by which the object will be diſcerned, when the eye in its motion comes to A, is at C when the eye is at B. Joining the points B, C, he ſuppoſed the line B C to be a tube, inclined to the line B D in the angle D B C, and of ſuch a diameter as to admit of but one particle of light: then it was eaſy to conceive, that the particle of light at C, by which the object muſt be ſeen when the eye arrives at A, would paſs through the tube B C, ſo inclined to the line B D, and accompanying the eye in its motion from B to A; and that it would not come



to the eye, placed behind ſuch a tube, if it had any other inclination to the line B D. If, inſtead of ſuppoſing B C ſo ſmall a tube, we conceive it to be the axis of a larger; then, for the ſame reaſon, the particle of light at C cannot paſs through that axis, unleſs it be inclined to B D in the ſame angle D B C.

In the like manner, if the eye move the contrary way, from D towards A, with the ſame velocity; then the tube muſt be inclined in the angle B D C. Although therefore the true or real place of an object, be perpendicular to the line in which the eye is moving, yet the viſible place will not be ſo; ſince that muſt doubtleſs be in the direction of the tube. But the difference between the true and apparent place, will be, *cæteris paribus*, greater or leſs, according to the different proportions between the velocity of light and that of the eye: ſo that if we could ſuppoſe light to be propagated in an inſtant, then there would be no difference between the real and viſible place of an object, although the eye were in motion; for in that caſe, A C being infinite with reſpect to A B, the angle A C B, which is the difference between the true and viſible place, vaniſhes. But if light be propagated in time, which was then allowed by moſt philoſophers, then it is evident from the foregoing conſiderations, that there will always be a difference between the true and viſible place of an object, except when the eye is moving either directly towards, or from the object. And in all caſes, the ſine of the difference between the true and viſible place of the object, will be to the ſine of the viſible inclination of the object to the line in which the eye is moving, as the velocity of the eye, is to the velocity of light.

If light moved only 1000 times faſter than the eye, and an object, ſuppoſed to be at an infinite diſtance, were really placed perpendicularly over the plane in which the eye is moving; it follows, from what has been ſaid, that the apparent place of ſuch object will always be inclined to that plane, in an angle of 89° 56′½; ſo that it will conſtantly appear 3′½ from its true place, and will ſeem ſo much leſs inclined to the plane, that way towards which the eye tends. That is, if A C be to A B or A D, as 1000 to 1, the angle A B C will be 89° 56′½, and the angle A C B 3′½, and B C D or 2 A C B will be 7′, if the direction of the motion of the eye be contrary at one time to what it is at another.

If the earth revolve about the ſun annually, and the velocity of light were to the velocity of the earth's motion in its orbit, as 1000 is to 1; then it is eaſy to conceive, that a ſtar really placed in the pole of the ecliptic, would to an eye carried along with the earth, ſeem to change its place continually; and, neglecting the ſmall difference on account of the earth's diurnal revolution on its axis, it would ſeem to deſcribe a circle about that pole, every where diſtant from it by 3′½. So that its longitude would be varied through all the points of the ecliptic every year, but its latitude would always remain the ſame. Its right aſcenſion would alſo change, and its declination, according to the different ſituation of the ſun in reſpect of the equinoctial points; and its apparent diſtance from the north pole of the equator, would be 7′ leſs at the autumnal, than at the vernal equinox.

The greateſt alteration of the place of a ſtar, in the pole of the ecliptic, or, which in effect amounts to the ſame, the proportion between the velocity of light and the earth's motion in its orbit, being known, it will not be difficult to find what would be the difference, on this account, between the true and apparent place of any other ſtar at any time; and, on the contrary, the difference between the true and apparent place being given, the proportion between the velocity of light, and the earth's motion in her orbit, may be found.

After the hiſtory of this curious diſcovery, related by the author nearly in the terms above, he gives the reſults of a multitude of accurate obſervations, made on a great number of ſtars, at all ſeaſons of the year. From all which obſervations, and the theory as related above, he found that every ſtar, in conſequence of the earth's motion in her orbit and the progreſſive motion of light, appears to deſcribe a ſmall ellipſe in the heavens, the tranſverſe axis of which is equal to the ſame quantity for every ſtar, namely 40″ nearly; and that the conjugate axis of the ellipſe, for different ſtars, varies in this proportion, namely, as the right ſine of the ſtar's latitude; that is, radius is to the ſine of the ſtar's latitude, as the tranſverſe axis to the conjugate axis: and conſequently a ſtar in the pole of the ecliptic, its latitude being there 90°, whoſe ſine is equal to the radius, will appear to deſcribe a ſmall circle about that pole as a centre, whoſe radius is equal to 20″. He alſo gives the following law of the variation of the ſtar's declination: if A denote the angle of poſition, or the angle at the ſtar made by two great circles drawn from it through the poles of the ecliptic and equator, and B another angle, whoſe tangent is to the tangent of A, as radius is to the ſine of the ſtar's latitude; then B will be equal to the difference of longitude between the ſun and the ſtar, when the true and apparent declination of the ſtar are the ſame. And if the ſun's longitude in the ecliptic be reckoned from that point in which it is when this happens; then the difference between the true and apparent declination of the ſtar, will be always as the ſine of the ſun's longitude from that point. It will alſo be found that the greateſt difference of declination that can be between the true and apparent place of the ſtar, will be to 20″, the ſemitranſverſe axis of the ellipſe, as the ſine of A to the ſine of B.

The author then ſhews, by the compariſon of a number of obſervations made on different ſtars, that they exactly agree with the theory deduced from the progreſſive motion of light, and that conſequently it is highly probable that ſuch motion is the cauſe of thoſe variations in the ſituation of the ſtars. From which he infers, that the parallax of the fixed ſtars is much ſmaller, than hath been hitherto ſuppoſed by thoſe, who have pretended to deduce it from their obſervations. He thinks he may venture to ſay, that in the ſtars he had obſerved, the parallax does not amount to 2″; nay, that if it had amounted to 1″, he ſhould certainly have perceived it, in the great number of obſervations that he made, eſpecially of γ Draconis; which agreeing with the hypotheſis, without allowing any thing for parallax, nearly as well when the ſun was in conjunction with, as in oppoſition to, this ſtar, it ſeems very probable that the parallax of it is not ſo much as one ſingle ſecond; and conſequently that it is above 400000 times farther from us than the ſun.

From the greateſt variation in the place of the ſtars, namely 40″, Dr. Bradley deduces the ratio of the velocity of light in compariſon with that of the earth in her orbit. In the preceding figure, A C is to A B, as the velocity of light to that of the earth in her orbit, the angle A C B being equal to 20″; ſo that the ratio of thoſe velocities is that of radius to the tangent of 20″, or of radius to 20″, ſince the tangent has no ſenſible difference from ſo ſmall an arc: but the radius of a circle is equal to the arc of $57^{\circ}\frac{3}{10}$ nearly, or equal to 206260″; therefore the velocity of light is to the velocity of the earth, as 206260 to 20, or as 10313 to 1.

And hence alſo the time in which light paſſes over the ſpace from the ſun to the earth, is eaſily deduced; for this time will be to one year, as A B or 20″ to 360° or the whole circle; that is, 360° : 20″ : : $365\frac{1}{4}$ days : $8^{m}\ 7^{s}$, namely, light will paſs from the ſun to the earth in the time of 8 minutes, 7 ſeconds; and this will be the ſame, whatever the diſtance of the ſun is.

Dr. Bradley having annexed to his theory the rules or formulæ for computing the aberration of the fixed ſtars in declination and right aſcenſion; theſe rules have been variouſly demonſtrated, and reduced to other practical forms, by Mr. Clairaut in the Memoirs of the Academy of Sciences for 1737; by Mr. Simpſon in his Eſſays in 1740; by M. Fontaine des Crutes in 1744; and ſeveral other perſons. The reſults of theſe rules are as follow: Every ſtar appears to deſcribe in the courſe of a year, by means of the aberration, a ſmall ellipſe, whoſe greater axis is 40″, and the leſs axis, perpendicular to the ecliptic, is equal to 40″ multiplied by the ſine of the ſtar's latitude, the radius being 1. The eaſtern extremity of the longer axis, marks the apparent place of the ſtar, the day of the oppoſition; and the extremity of the leſs axe, which is fartheſt from the ecliptic, marks its ſituation three months after.

The greateſt aberration in longitude, is equal to 20″ divided by the coſine of its latitude. And the aberration for any time, is equal to 20″ multiplied by the coſine of the elongation of the ſtar found for the ſame time, and divided by the coſine of its latitude. This aberration is ſubtractive in the firſt and laſt quadrant of the argument, or of the difference between the longitudes of the ſun and ſtar; and additive in the ſecond and third quadrants. The greateſt aberration in latitude, is equal to 20″ multiplied by the ſine of the ſtar's latitude. And the aberration in latitude for any time, is equal to 20″ multiplied by the ſine of the ſtar's latitude, and multiplied alſo by the ſine of the elongation. The aberration is ſubtractive before the oppoſition, and additive after it.

The greateſt aberration in declination, is equal to 20″ multiplied by the ſine of the angle of poſition A, and divided by the ſine of B the difference of longitude between the ſun and ſtar when the aberration in declination is nothing. And the aberration in declination at any other time, will be equal to the greateſt aberration multiplied by the ſine of the difference between the ſun's place at the given time and his place when the aberration is nothing. Alſo the ſine of the latitude of the ſtar is to radius, as the tangent of A the angle of poſition at the ſtar, is to the tangent of B, the difference of longitude between the ſun and ſtar when the aberration in declination is nothing. The greateſt aberration in right-aſcenſion, is equal to 20″ multiplied by the coſine of A the angle of poſition, and divided by the ſine of C the difference in longitude between the ſun and ſtar when the aberration in right aſcenſion is nothing. And the aberration in right-aſcenſion at any other time, is equal to the greateſt aberration multiplied by the ſine of the difference between the ſun's place at the given time, and his place when the aberration is nothing. Alſo the ſine of the latitude of the ſtar is to radius, as the cotangent of A the angle of poſition, to the tangent of C.

ABERRATION *of the Planets*, is equal to the geocentric motion of the planet, the ſpace it appears to move as ſeen from the earth, during the time that light employs in paſſing from the planet to the earth. Thus, in the ſun, the aberration in longitude is conſtantly 20″, that being the ſpace moved by the ſun, or, which is the ſame thing, by the earth, in the time of $8^{m}\ 7^{s}$, which is the time in which light paſſes from the ſun to the earth, as we have ſeen in the foregoing article. In like manner, knowing the diſtance of any planet from the earth, by proportion it will be, as the diſtance of the ſun is to the diſtance of the planet, ſo is $8^{m}\ 7^{s}$ to the time of light paſſing from the planet to the earth: then computing the planet's geocentric motion in this time, that will be the aberration of the planet, whether it be in longitude, latitude, right-aſcenſion, or declination.

It is evident that the aberration will be greateſt in the longitude, and very ſmall in latitude, becauſe the planets deviate very little from the plane of the ecliptic, or path of the earth; ſo that the aberration in the latitudes of the planets, is commonly neglected, as inſenſible; the greateſt in Mercury being only $4''\frac{1}{3}$, and much leſs in the other planets. As to the aberrations in declination and right-aſcenſion, they muſt depend on the ſituation of the planet in the zodiac. The aberration in longitude, being equal to the geocentric motion, will be more or leſs according as that motion is; it will therefore be leaſt, or nothing at all, when the planet is ſtationary; and greateſt in the ſuperior planets Mars, Jupiter, Saturn, &c, when they are in oppoſition to the ſun; but in the inferior planets Venus and Mercury, the aberration is greateſt at the time of their ſuperior conjunction. Theſe maxima of aberration for the ſeveral planets, when their diſtance from the ſun is leaſt, are as below: viz, for

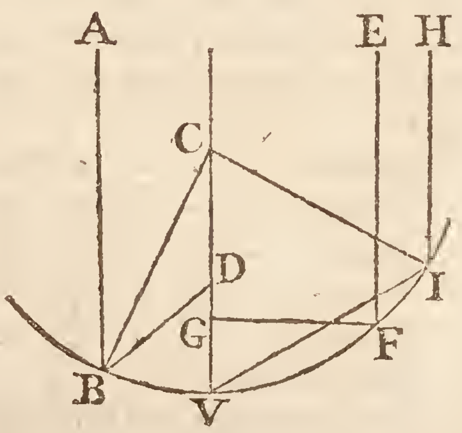
Saturn - - -	27″·0
Jupiter - - -	29 ·8
Mars - - -	37 ·8
Venus - - -	43 ·2
Mercury - -	59 ·0
The Moon - -	$\frac{2}{3}$

And between theſe numbers and nothing the aberrations of the planets, in longitude, vary according to their ſituations. But that of the ſun varies not, being conſtantly 20″, as has been before obſerved. And this may alter his declination by a quantity, which varies from 0 to near 8″; being greateſt or 8″ about the equinoxes, and vaniſhing in the ſolſtices.

The methods of computing theſe, and the formulas for all caſes, are given by M. Clairaut in the Memoirs of the Academy of Sciences for the year 1746, and by M. Euler in the Berlin Memoirs, vol. 2, for 1746.

Optic Aberration, the deviation or diſperſion of the rays of light, when reflected by a ſpeculum, or refracted by a lens, by which they are prevented from meeting or uniting in the ſame point, called the geometrical focus, but are ſpread over a ſmall ſpace, and produce a confuſion of images. Aberration is either lateral or longitudinal: the lateral aberration is meaſured by a perpendicular to the axis of the ſpeculum or lens, drawn from the focus to meet the refracted or reflected ray: the longitudinal aberration is the diſtance, on the axis, between the focus and the point where the ray meets the axis. The aberrations are very amply treated in Smith's Complete Syſtem of Optics, in 2 volumes 4to.

There are two ſpecies of aberration, diſtinguiſhed according to their different cauſes: the one ariſes from the figure of the ſpeculum or lens, producing a geometrical diſperſion of the rays, when theſe are perfectly equal in all reſpects; the other ariſes from the unequal refrangibility of the rays of light themſelves; a diſcovery that was made by Sir Iſaac Newton, and for this reaſon it is often called the Newtonian aberration. As to the former ſpecies of aberration, or that ariſing from the figure, it is well known that if rays iſſue from a point at a given diſtance; then they will be reflected into the other focus of an ellipſe having the given luminous point for one focus, or directly from the other focus of an hyperbola; and will be variouſly diſperſed by all other figures. But if the luminous point be infinitely diſtant, or, which is the ſame, the incident rays be parallel, then they will be reflected by a parabola into its focus, and variouſly diſperſed by all other figures. But thoſe figures are very difficult to make, and therefore curved ſpecula are commonly made ſpherical, the figure of which is generated by the revolution of a circular arc, which produces an aberration of all rays, whether they are parallel or not, and therefore it has no accurate geometrical focus which is common to all the rays. Let B V F repreſent a concave ſpherical ſpeculum, whoſe centre is C; and let A B, E F be incident rays parallel to the axis CV. Becauſe the angle of incidence is equal to the angle of reflection in all caſes, therefore if the radii C B, C F be drawn to the points of incidence, and thence B D making the angle C B D equal to the angle C B A, and F G making the angle C F G equal to the angle C F E; then B D, FG will be the reflected rays, and D, G, the points where they meet the axis. Hence it appears that the point of coincidence with the axis is equally diſtant from the point of incidence and the centre: for becauſe the angle C B D is equal to the angle C B A, which is equal to the alternate angle B C D, therefore their oppoſite ſides C D, D B are equal: and in like manner, in any other, G F is equal to G C. And hence it is evident that when B is indefinitely near the vertex V, then D is in the middle of the radius C V; and the nearer the incident ray is to the axis C V, the nearer will the reflected ray come to the middle point D; and the contrary. So that the aberration D G of any ray E F G, is always more and more, as the incident ray is farther from the axis, or the incident point F from the vertex V; till when the diſtance V I is 60 degrees, then the reflected ray falls in the vertex V, making the aberration equal to the whole length D V. And this ſhews the reaſon why ſpecula are made of a very ſmall ſegment of a ſphere, namely, that all their reflected rays may arrive very near the middle point or focus D, to produce an image the moſt diſtinct, by the leaſt aberration of the rays. And in like manner for rays refracted through lenſes.



In ſpherical lenſes, Mr. Huygens has demonſtrated that the aberration from the figure, in different lenſes, is as follows.

1. In all plano-convex lenſes, having their plane ſurface expoſed to parallel rays, the longitudinal aberration of the extreme ray, or that remoteſt from the axis, is equal to $\frac{9}{2}$ of the thickneſs of the lens.

2. In all plano-convex lenſes, having their convex ſurface expoſed to parallel rays, the longitudinal aberration of the extreme ray, is equal to $\frac{7}{6}$ of the thickneſs of the lens. So that in this poſition of the ſame plano-convex lens, the aberration is but about one-fourth of that in the former; being to it only as 7 to 27.

3. In all double convex lenſes of equal ſpheres, the aberration of the extreme ray, is equal to $\frac{5}{3}$ of the thickneſs of the lens.

4. In a double convex lens, the radii of whoſe ſpheres are as 1 to 6, if the more convex ſurface be expoſed to parallel rays, the aberration from the figure is leſs than in any other ſpherical lens; being no more than $\frac{15}{14}$ of its thickneſs.

But the foregoing ſpecies of aberration, ariſing from the figure, is very ſmall, and eaſily remedied, in compariſon with the other, ariſing from the unequal refrangibility of the rays of light; inſomuch that Sir Iſaac Newton ſhews in his Optics, pa. 84 of the 8vo. edition, that if the object-glaſs of a teleſcope be plano-convex, the plane ſide being turned towards the object, and the diameter of the ſphere, to which the convex ſide is ground, be 100 feet, the diameter of the aperture being 4 inches, and the ratio of the ſine of incidence out of glaſs into air, be to that of refraction, as 20 to 31; then the diameter of the circle of aberrations will in this caſe be only $\frac{961}{72000000}$ parts of an inch: while the diameter of the little circle, through which the ſame rays are ſcattered by unequal refrangibility, will be about the 55th part of the aperture of the object-glaſs, which here is 4 inches. And therefore the error ariſing from the ſpherical figure of the glaſs, is to the error ariſing from the different refrangibility of the rays, as $\frac{961}{72000000}$ to $\frac{4}{55}$, that is as 1 to 5449.

So that it may ſeem ſtrange that objects appear through teleſcopes ſo diſtinct as they do, conſidering that the error ariſing from the different refrangibility, is almoſt incomparably larger than that of the figure. Newton however ſolves the difficulty by obſerving that the rays, under their various aberrations, are not ſcattered uniformly over all the circular ſpace, but collected infinitely more denſe in the centre than in any other part of the circle; and that, in the way from the centre to the circumference, they grow more and more rare, ſo as at the circumference to become infinitely rare; and, by reaſon of their rarity, they are not ſtrong enough to be viſible, unleſs in the centre, and very near it.

In conſequence of the diſcovery of the unequal refrangibility of light, and the apprehenſion that equal refractions muſt produce equal divergencies in every ſort of medium, it was ſuppoſed that all ſpherical object-glaſſes of teleſcopes would be equally affected by the different refrangibility of light, in proportion to their aperture, of whatever materials they might be conſtructed: and therefore that the only improvement that could be made in refracting teleſcopes, was that of increaſing their length. So that Sir Iſaac Newton, and other perſons after him, deſpairing of ſucceſs in the uſe and fabric of lenſes, directed their chief attention to the conſtruction of reflecting teleſcopes.

However, about the year 1747, M. Euler applied himſelf to the ſubject of refraction; and purſued a hint ſuggeſted by Newton, for the deſign of making object-glaſſes with two lenſes of glaſs incloſing water between them; hoping that, by conſtructing them of different materials, the refractions would balance one another, and ſo the uſual aberration be prevented. Mr. John Dollond, an ingenious optician in London, minutely examined this ſcheme, and found that Mr. Euler's principles were not ſatisfactory. M. Clairaut likewiſe, whoſe attention had been excited to the ſame ſubject, concurred in opinion that Euler's ſpeculations were more ingenious than uſeful. This controverſy, which ſeemed to be of great importance in the ſcience of optics, engaged alſo the attention of M. Klingenſtierna of Sweden, who was led to make a careful examination of the 8th experiment in the ſecond part of Newton's Optics, with the concluſions there drawn from it. The conſequence was, that he found that the rays of light, in the circumſtances there mentioned, did not loſe their colour, as Sir Iſaac had imagined. This hint of the Swediſh philoſopher led Mr. Dollond to re-examine the ſame experiment: and after ſeveral trials it appeared, that different ſubſtances cauſed the light to diverge very differently, in proportion to their general refractive powers. In the year 1757 therefore he procured wedges of different kinds of glaſs, and applied them together ſo that the refractions might be made in contrary directions, that he might diſcover whether the refraction and divergency of colour would vaniſh together. The reſult of his firſt trials encouraged him to perſevere; for he diſcovered a difference far beyond his hopes in the qualities of different kinds of glaſs, with reſpect to their divergency of colours. The Venice glaſs and Engliſh crown glaſs were found to be nearly allied in this reſpect: the common Engliſh plate glaſs made the rays diverge more; and the Engliſh flint glaſs moſt of all. But without enquiring into the cauſe of this difference, he proceeded to adapt wedges of crown glaſs, and of white flint glaſs, ground to different angles, to each other, ſo as to refract in different directions; till the refracted light was entirely free from colours. Having meaſured the refractions of each wedge, he found that the refraction of the white glaſs was to that of the crown glaſs, nearly as 2 to 3: and he hence concluded in general, that any two wedges made in this proportion, and applied together ſo as to refract in contrary directions, would refract the light without any aberration of the rays.

Mr. Dollond's next object was to make ſimilar trials with ſpherical glaſſes of different materials, with the view of applying his diſcovery to the improvement of teleſcopes: and here he perceived that, to obtain a refraction of light in contrary directions, the one glaſs muſt be concave, and the other convex; and the latter, which was to refract the moſt, that the rays might converge to a real focus, he made of crown glaſs, the other of white flint glaſs. And as the refractions of ſpherical glaſſes are inverſely as their focal diſtances, it was neceſſary that the focal diſtances of the two glaſſes ſhould be inverſely as the ratios of the refractions of the wedges; becauſe that, being thus proportioned, every ray of light that paſſes through this compound glaſs, at any diſtance from its axis, will conſtantly be refracted, by the difference between two contrary refractions, in the proportion required; and therefore the different refrangibility of the light will be entirely removed.

But in the applications of this ingenious diſcovery to practice, Mr. Dollond met with many and great difficulties. At length, however, after many repeated trials, by a reſolute perſeverance, he ſucceeded ſo far as to conſtruct refracting teleſcopes much ſuperior to any that had hitherto been made; repreſenting objects with great diſtinctneſs, and in their true colours.

Mr. Clairaut, who had intereſted himſelf from the beginning in this diſcovery, now endeavoured to aſcertain the principles of Mr. Dollond's theory, and to lay down rules to facilitate the conſtruction of theſe new teleſcopes. With this view he made ſeveral experiments, to determine the refractive power of different kinds of glaſs, and the proportions in which they ſeparated the rays of light: and from theſe experiments he deduced ſeveral theorems of general uſe. M. D'Alembert made likewiſe a great variety of calculations to the ſame purpoſe; and he ſhewed how to correct the errors to which theſe teleſcopes are ſubject, ſometimes by placing the object-glaſſes at a ſmall diſtance from each other, and ſometimes by uſing eye-glaſſes of different refractive powers. But though foreigners were hereby ſupplied with the moſt accurate calculations, they were very defective in practice. And the Engliſh teleſcopes, made, as they imagined, without any preciſe rule, were greatly ſuperior to the beſt of their conſtruction.

M. Euler, whoſe ſpeculations had firſt given occaſion to this important and uſeful enquiry, was very reluctant in admitting Mr. Dollond's improvements, becauſe they militated againſt a pre-conceived theory of his own. At laſt however, after ſeveral altercations, being convinced of their reality and importance by M. Clairaut, ant, he aſſented ; and he ſoon after received farther ſatisfaction from the experiments of M. Zeiher, of Peterſburgh.

M. Zeiher ſhewed by experiments that it is the lead, in the compoſition of glaſs, which gives it this remarkable property, namely, that while the refraction of the mean rays is nearly the ſame, that of the extreme rays conſiderably differs. And, by increaſing the lead, he produced a kind of glaſs, which occaſioned a much greater ſeparation of the extreme rays than that of the flint glaſs uſed by Mr. Dollond, and at the ſame time conſiderably increaſed the mean refraction. M. Zeiher, in the courſe of his experiments, made glaſs of minium and lead, with a mixture alſo of alkaline ſalts ; and he found that this mixture greatly diminiſhed the mean refraction, and yet made hardly any change in the diſperſion : and he at length obtained a kind of glaſs greatly ſuperior to the flint glaſs of Mr. Dollond for the conſtruction of teleſcopes ; as it occaſioned three times as great a diſperſion of the rays as the common glaſs, whilſt the mean refraction was only as 1·61 to 1.

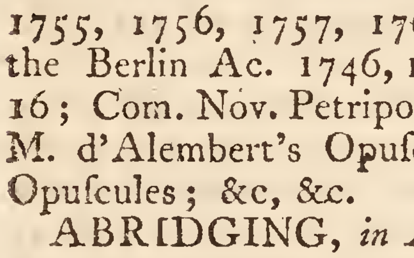
Other improvements were alſo made on the new or achromatic teleſcopes by the inventor Mr. John Dollond, and by his ſon Peter Dollond ; which may be ſeen under the proper words. For various diſſertations on the ſubject of the aberration of light, colours, and the figure of the glaſs, ſee Philoſ. Tranſ. vols. 35, 48, 50, 51, 52, 55, 60 ; Memoirs of the Academy of Sciences of Paris, for the years 1737, 1746, 1752, 1755, 1756, 1757, 1762, 1764, 1765, 1767, 1770 ; the Berlin Ac. 1746, 1762, 1766 ; Swed. Mem. vol. 16 ; Com. Nov. Petripol. 1762 ; M. Euler's Dioptrics ; M. d'Alembert's Opuſcules Math. ; M. de Rochon Opuſcules ; &c, &c.

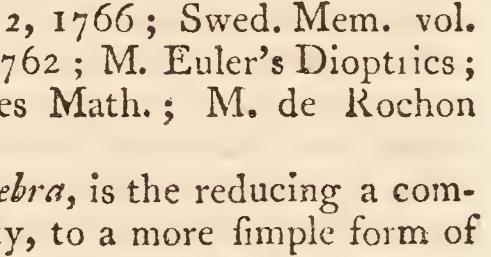
ABRIDGING, *in Algebra*, is the reducing a compound equation, or quantity, to a more ſimple form of expreſſion. This is done either to ſave room, or the trouble of writing a number of ſymbols ; or to ſimplify the expreſſion, either to eaſe the memory, or to render the formula more eaſy and general.

So the equation $x^3 - ax^2 + abx - abc = 0$, by putting $p = a$, $q = ab$, and $r = abc$, becomes $x^3 - px^2 + qx - r = 0$.

And the equation $x^2 + (a+b)x - \frac{ab}{c} = 0$, by putting $p = a + b$, and $q = \frac{ab}{c}$, becomes $x^2 + px - q = 0$.

ABSCISS, Abſciſſe, or Abſciſſa, is a part or ſegment cut off a line, terminated at ſome certain point, by an ordinate to a curve ; as A P or B P.

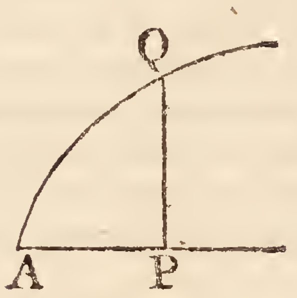


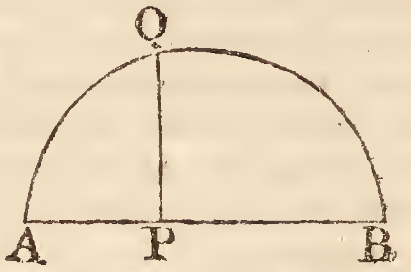


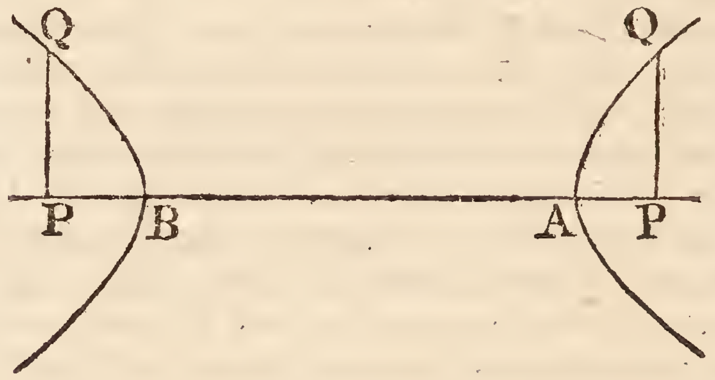
The abſciſs may either commence at the vertex of the curve, or at any other fixed point. And it may be taken either upon the axis or diameter of the curve, or upon any other line drawn in a given poſition.

Hence there are an infinite number of variable abſciſſes, terminated at the ſame fixed point at one end, the other end of them being at any point of the given line or diameter.

In the common parabola, each ordinate P Q has but one abſciſs A P ; in the ellipſe or circle, the ordinate has two abſciſſes A P, B P lying on the oppoſite ſides of it ; and in the hyperbola the ordinate P Q has alſo two abſciſſes, but they lie both on the ſame ſide of it. That is, in general, a line of the ſecond kind, or a curve of the firſt kind, may have two abſciſſes to each ordinate. But a line of the third order may have three abſciſſes to each ordinate ; a line of the fourth order may have four ; and ſo on.







The uſe of the abſciſſes is, in conjunction with the ordinates, to expreſs the nature of the curves, either by ſome proportion or equation including the abſciſs and its ordinate, with ſome other fixed invariable line or lines. Every different curve has its own peculiar equation or property by which it is expreſſed, and different from all others : and that equation or expreſſion is the ſame for every ordinate and its abſciſſes, whatever point of the curve be taken. So, in the circle, the ſquare of any ordinate is equal to the rectangle of its two abſciſſes, or $AP \cdot PB = PQ^2$; in the parabola, the ſquare of the ordinate is equal to the rectangle of the abſciſs and a certain given line called the parameter ; in the ellipſe and hyperbola, the ſquare of the ordinate is always in a certain conſtant proportion to the rectangle of the two abſciſſes, namely, as the ſquare of the conjugate to the ſquare of the tranſverſe, or as the parameter is to the tranſverſe axis ; and ſo other properties in other curves.

When the natures or properties of curves are expreſſed by algebraic equations, any general abſciſs, as A P, is commonly denoted by the letter x, and the ordinate P Q by the letter y ; the other or conſtant lines being repreſented by other letters. Then the equations expreſſing the nature of theſe curves are as follow ; namely, for the

circle - - $dx - x^2 = y^2$, where d is the diameter A B ;
parabola - $px = y^2$, where p is the parameter ;
ellipſe - $t^2 : c^2 :: tx - x^2 : y^2$, { where t is the tranſverſe,
hyperbola $t^2 : c^2 :: tx + x^2 : y^2$, { & c the conjugate axis.

ABSIS, ABSIDES. See Apsis, Apsides.

ABSOLUTE Equation, *in Aſtronomy*, is the ſum of the optic and excentric equations. The apparent inequality of a planet's motion, ariſing from its not being equally diſtant from the earth at all times, is called its optic equation; and this would ſubſiſt even if the planet's real motion were uniform. The excentric inequality is cauſed by the planet's motion being not uniform. To illuſtrate this, conceive the ſun to move, or to appear to move, in the circumference of a circle, in whoſe centre the earth is placed. It is manifeſt, that if the ſun move uniformly in this circle, then he muſt appear to move uniformly to a ſpectator at the earth; and in this caſe there will be no optic nor excentric equation. But ſuppoſe the earth to be placed out of the centre of the circle; and then, though the ſun's motion ſhould be really uniform, it would not appear to be ſo, being ſeen from the earth; and in this caſe there would be an optic equation, without an excentric one. Imagine farther, the ſun's orbit to be, not circular, but elliptical, and the earth in its focus: it will be full as evident that the ſun cannot appear to have an uniform motion in ſuch ellipſe; ſo that his motion will then be ſubject to two equations; that is, the optic equation, and the excentric equation. *See* Equation, and Optic Inequality.

Absolute *Number*, in Algebra, is that term or member of an equation that is completely known, and which is equal to all the other, or unknown terms, taken together; and is the ſame as what Vieta calls the *homogeneum comparationis*. So, of the equation $x^2 + 16x - 36 = 0$, or $x^2 + 16x = 36$, the abſolute number, or known term, is 36.

Absolute *Gravity*, *Motion*, *Space*, *Time*, &c. See the reſpective ſubſtantives.

ABSTRACT Mathematics, otherwiſe called pure mathematics, is that which treats of the properties of magnitude, figure, or quantity, abſolutely and generally conſidered, without reſtriction to any ſpecies in particular: ſuch as Arithmetic and Geometry. In this ſenſe, abſtract or pure mathematics, is oppoſed to mixed mathematics, in which ſimple and abſtract properties, and the relations of quantities, primitively conſidered in pure mathematics, are applied to ſenſible objects; as in aſtronomy, hydroſtatics, optics, &c.

Abstract *Number*, is a number, or collection of units, conſidered in itſelf, without being applied to denote a collection of any particular and determinate things. So, for example, 3 is an abſtract number, ſo far as it is not applied to ſomething: but when we ſay 3 feet, or 3 perſons, the 3 is no longer an abſtract, but a concrete number.

ABSURD, or Absurdum, a term commonly uſed in demonſtrating converſe propoſitions; a mode of demonſtration, in which the propoſition intended is not proved in a direct manner, by principles before laid down; but it proves that the contrary is abſurd or impoſſible; and ſo indirectly as it were proves the propoſition itſelf. The 4th propoſition in the firſt book of Euclid, is the firſt in which he makes uſe of this mode of proof; where he ſhews that if the extremities of two lines coincide, thoſe lines will coincide in all their parts, otherwiſe they would incloſe a ſpace, which is abſurd or contrary to the 10th axiom. Moſt converſe propoſitions are proved in this way, which mode of proof is called *reductio ad abſurdum*.

ABUNDANT Number, in *Arithmetic*, is a number whoſe aliquot parts, added all together, make a ſum which is greater than the number itſelf. Thus 12 is an abundant number, becauſe its aliquot parts, namely 1, 2, 3, 4, 6, when added together, make 16, which is greater than the number 12 itſelf.

An abundant number is oppoſed to a deficient one, which is leſs than the ſum of its aliquot parts taken together, as the number 14, whoſe aliquot parts 1, 2, 7, make no more than 10; and to a perfect number, which is exactly equal to the ſum of all its aliquot parts, as the number 6, which is equal to the ſum of 1, 2, 3, which are its aliquot parts.

ACADEMICIAN, a member of a ſociety called an academy, inſtituted for the promotion of arts, ſciences, or natural knowledge in general.

ACADEMICS, an ancient ſect of philoſophers, who followed the doctrine of Socrates and Plato, as to the uncertainty of knowledge, and the incomprehenſibility of truth.

Academic, in this ſenſe, amounts to much the ſame with Platoniſt; the difference between them being only in point of time. Thoſe who embraced the ſyſtem of Plato, among the ancients, were called *academici*, academician or academic; whereas thoſe who did the ſame ſince the reſtoration of learning, have aſſumed the denomination of Platoniſts.

We uſually reckon three ſects of academics; though ſome make five. The ancient academy was that of which Plato was the chief.

Arceſſilas, one of Plato's ſucceſſors, introducing ſome alterations into the philoſophy of this ſect, founded what they call the ſecond academy.

The eſtabliſhment of the third, called alſo the new academy, is attributed to Lacydes, or rather to Carneades.

Some authors add a fourth, founded by Philo; and a fifth, by Antiochus, called the Antiochan, which tempered the ancient academy with Stoiciſm.

The ancient academy doubted of every thing; and carried this principle ſo far as to make it a doubt, whether or no they ought to doubt. It was a kind of a principle with them, never to be certain or ſatisfied of any thing; never to affirm or to deny any thing, either for true or falſe.

The new academy was ſomewhat more reaſonable; they acknowledged ſeveral things for truths, but without attaching themſelves to any with entire aſſurance. Theſe philoſophers had found that the ordinary commerce of life and ſociety was inconſiſtent with the abſolute and univerſal doubtfulneſs of the ancient academy: and yet it is evident that they looked upon things rather as probable, than as true and certain: by this amendment thinking to ſecure themſelves from thoſe abſurdities into which the ancient academy had fallen.

ACADEMIST, the ſame as Academician.

ACADEMY, Academia, in *Antiquity*, a fine villa or pleaſure houſe, in one of the ſuburbs of Athens, about a mile from the city; where Plato, and the wiſe men who followed him, held aſſemblies for diſputes and philoſophical conference; which gave the name to the ſect of Academics.

The houſe took its name, *Academy*, from one Academus, or Ecademus, a citizen of Athens, to whom it originally belonged: he lived in the time of Theſeus; and here he uſed to have gymnaſtic ſports or exerciſes.

The academy was farther improved by Cimon, and adorned with fountains, trees, ſhady walks, &c, for the convenience of the philoſophers and men of learning, who here met to confer and diſpute for their mutual improvement. It was ſurrounded with a wall by Hipparchus, the ſon of Piſiſtratus; and it was alſo uſed as the burying-place for illuſtrious perſons, who had deſerved well of the republic.

It was here that Plato taught his philoſophy; and hence it was that all public places, deſtined for the aſſemblies of the learned and ingenious, have been ſince called *Academies*.

Sylla ſacrificed the delicious walks and groves of the academy, which had been planted by Cimon, to the ravages of war; and employed thoſe very trees in conſtructing machines to batter the walls of the city which they had adorned.

Cicero too had a villa, or country retirement, near Puzzuoli, which he called by the ſame name, *Academia*. Here he uſed to entertain his philoſophical friends; and here it was that he compoſed his Academical Queſtions, and his books *De Naturâ Deorum*.

ACADEMY, among the moderns, denotes a regular ſociety or company of learned perſons, inſtituted under the protection of ſome prince, or other public authority, for the cultivation and improvement of arts or ſciences.

Some authors confound Academy with Univerſity; but though much the ſame in Latin, they are very different things in Engliſh. An univerſity is properly a body compoſed of graduates in the ſeveral faculties; of profeſſors, who teach in the public ſchools; of regents or tutors, and ſtudents who learn under them, and aſpire likewiſe to degrees. Whereas an academy is not intended to teach, or profeſs any art or ſcience, but to improve it: it is not for novices to be inſtructed in, but for thoſe that are more knowing; for perſons of learning to confer in, and communicate their lights and diſcoveries to each other, for their mutual benefit and improvement.

The firſt modern academy we read of, was eſtabliſhed by Charlemagne, by the advice of Alcuin, an Engliſh monk: it was compoſed of the chief geniuſes of the court, the emperor himſelf being a member. In their academical conferences, every perſon was to give ſome account of the ancient authors he had read; and each one aſſumed the name of ſome ancient author, that pleaſed him moſt, or ſome celebrated perſon of antiquity. Alcuin, from whoſe letters we learn theſe particulars, took that of Flaccus, the ſurname of Horace; a young lord, named Augilbert, took that of Homer; Adelard, biſhop of Corbie, was called Auguſtin; Recluſe, biſhop of Mentz, was Dametas; and the king himſelf, David

Since the revival of learning in Europe, academies have multiplied greatly, moſt nations being furniſhed with ſeveral, and from their communications the chief improvements have been made in the arts and ſciences, and in cultivating natural knowledge. There are now academies for almoſt every art, or ſpecies of knowledge; but I ſhall give a ſhort account only of thoſe inſtitutions of this kind, which regard the cultivation of ſubjects mathematical or philoſophical, which are the proper and peculiar objects of our undertaking.

Italy abounds more in academies than all the world beſides; there being enumerated by Jarckius not leſs than five hundred and fifty in all; and even to the amount of twenty-five in Milan itſelf. Theſe are however moſtly of a private and inferior nature; the conſequence of their too great number.

The firſt academy of a philoſophical kind was eſtabliſhed at Naples, in the houſe of Baptiſta Porta, about the year 1560, under the name of *Academy Secretorum Naturæ*; being formed for the improvement of natural and mathematical knowledge. This was ſucceeded by the

ACADEMY *of Lyncei*, founded at Rome by prince Frederick Ceſi, towards the end of the ſame century. It was rendered famous by the notable diſcoveries made by ſeveral of its members; among whom was the celebrated Galileo Galilei.

Several other academies contributed alſo to the advancement of the ſciences; but it was by ſpeculations rather than by repeated experiments on the phenomena of nature: ſuch were the academy of Beſſarian at Rome, and that of Laurence de Medicis at Florence, in the 15th century; and in the 16th were that of Infiammati at Padua, of Vegna Juoli at Rome, of Ortolani at Placentia, and of Umidi at Florence. The firſt of theſe ſtudied fire and pyrotechnia, the ſecond wine and vineyards, the third pot-herbs and gardens, the fourth water and hydraulics. To theſe may be added that of Venice, called La Veneta, and founded by Frederick Badoara, a noble Venetian; another in the ſame city, of which Campegio, biſhop of Feltro, appears to have been the chief; alſo that of Coſenza, or La Conſentina, of which Bernadin Teleſio, Sertorio Quatromanni, Paulus Aquinas, Julio Cavalcanti, and Fabio Cicali, celebrated philoſophers, were the chief members. The compoſitions of all theſe academies, of the 16th century, were good in their kind; but none of them comparable to thoſe of the Lyncei.

ACADEMY *del Cimento*, that is, of Experiments, aroſe at Florence, ſome years after the death of Torricelli, namely in the year 1657, under the protection of prince Leopold of Tuſcany, afterwards cardinal de Medicis, and brother to the Grand Duke Ferdinand the Second. Galileo, Toricelli, Aggiunti, and Viviani had prepared the way for it: and ſome of its chief members were Paul del Buono, who in 1657 invented the inſtrument for trying the incompreſſibility of water, namely a thick globular ſhell of gold, having its cavity filled with water; then the globe being compreſſed by a ſtrong ſcrew, the water came through the pores of the gold rather than yield to the compreſſion: alſo, Alphonſus Borelli, well known for his ingenious treatiſe *De Motu Animalium*, and other works; Candide del Buono, brother of Paul; Alexander Marſili, Vincent Viviani, Francis Rhedi, and the Count Laurence Magalotti, ſecretary of this academy, who publiſhed a volume of their curious experiments in 1667, under the title of *Saggi di Naturali Eſperienze;* a copy of which being preſented to the Royal Society, it was tranſlated into Engliſh by Mr. Waller, and publiſhed at London, in 4to, 1684: A curious collection of tracts, containing ingenious experiments on the preſſure of the air, on the compreſſing of water, on cold, heat, ice, magnets, electricity, odours, the motion of ſound, projectiles, light, &c, &c. But we have heard little or nothing more of the academy ſince that time. It may not be improper to obſerve here, that the Grand Duke Ferdinand, above mentioned, was no mean philoſopher and chemiſt, and that he invented thermometers, of which the conſtruction and uſe may be ſeen in the collection of the academy del Cimento.

ACADEMY *degl' Inquieti* at Bologna, incorporated afterwards into that *della traccia* in the ſame city, followed the example of that *del Cimento.* The members met at the houſe of the abbot Antonio Sampieri; and here Geminiano Montanari, one of the chief members, made excellent diſcourſes on mathematical and philoſophical ſubjects, ſome parts of which were publiſhed in 1667, under the title of Penſieri Fiſico-Mathematici. This academy afterwards met in an apartment of Euſtachio Manfredi; and then in that of Jacob Sandri; but it arrived at its chief luſtre while its aſſemblies were held in the palace Marſilli.

ACADEMY *of Roſſano,* in the kingdom of Naples, called *La Societa Scientifica Roſſaneſe degl' Incurioſi,* was founded about the year 1540, under the name of *Naviganti;* and was renewed under that of *Spenſierati* by Camillo Tuſcano, about the year 1600. It was then an academy of belles-lettres, but was afterwards transformed into an academy of ſciences, on the ſolicitation of the learned abbot Don Giacinto Gimma; who, being made preſident under the title of promoter-general, in 1695, gave it a new ſet of regulations. He divided the academiſts into ſeveral claſſes, namely, grammarians, rhetoricians, poets, hiſtorians, philoſophers, phyſicians, mathematicians, lawyers, and divines; with a ſeparate claſs for cardinals and perſons of quality. To be admitted a member, it was neceſſary that the candidate have degrees in ſome faculty. Members, in the beginning of their books, are not allowed to take the title of *academiſt* without a written permiſſion from the preſident, which is not granted till the work has been examined by the cenſors of the academy. This permiſſion is the higheſt honour the academy can confer; ſince they hereby, as it were, adopt the work, and engage to anſwer for it againſt any criticiſms that may be made upon it. The preſident himſelf is not exempt from this law: and it is not permitted that any academiſt publiſh any thing againſt the writings of another, without leave obtained from the ſociety.

There have been ſeveral other academies of ſciences in Italy, but which have not ſubſiſted long, for want of being ſupported by the princes. Such were at Naples that of the *Inveſtiganti,* founded about the year 1679, by the marquis d'Arena, Don Andrea Concubletto; and that which, about the year 1698, met in the palace of Don Lewis della Cerda, the duke de Medina, and viceroy of Naples: at Rome, that of *Fiſico-Matematici,* which in 1686 met in the houſe of Signior Ciampini: at Verona, that of *Aletofili,* founded the ſame year by Signior Joſeph Gazola, and which met in the houſe of the count Serenghi della Cucca: at Breſcia, that of *Fileſotici,* founded the ſame year for the cultivation of philoſophy and mathematics, and terminated the year following: that of F. Franciſco Lana, a jeſuit of great ſkill in theſe ſciences: and laſtly that of Fiſico-Critici at Sienna, founded in 1691, by Signior Peter Maria Gabrielli.

Some other academies, ſtill ſubſiſting in Italy, repair with advantage the loſs of the former. One of the principal is the academy of Filarmonici at Verona, ſupported by the marquis Scipio Maffei, one of the moſt learned men in Italy; the members of which academy, though they cultivate the belles lettres, do not neglect the ſciences. The academy of *Ricovrati* at Padua ſtill ſubſiſts with reputation; in which, from time to time, learned diſcourſes are held on philoſophical ſubjects. The like may be ſaid of the academy of the *Muti di Reggio,* at Modena. At Bologna is an academy of ſciences, in a flouriſhing condition, known by the name of *The Inſtitute of Bologna;* which was founded in 1712 by count Marſigli, for cultivating phyſics, mathematics, medicine, chemiſtry, and natural hiſtory. The hiſtory of it is written by M. de Limiers, from memoirs furniſhed by the founder himſelf. Among the new academies, the firſt place, after the Inſtitute of Bologna, is given to that of the Counteſs Donna Clelio Grillo Boromeo, one of the moſt learned ladies of the age, to whom Signior Gimma dedicates his literary hiſtory of Italy. She had lately eſtabliſhed an academy of experimental philoſophy in her palace at Milan; of which Signior Valliſnieri was nominated preſident, and had already drawn up the regulations for it, though we do not find it has yet taken place. In the number of theſe academies may alſo be ranked the aſſembly of the learned, who of late years met at Venice in the houſe of Signior Criſtino Martinelli, a noble Venetian, and a great patron of learning.

ACADEMIA *Coſmografica,* or that of the Argonauts, was inſtituted at Venice, at the inſtance of F. Coronelli, for the improvement of geography; the deſign being to procure exact maps, geographical, topographical, hydrographical, and ichnographical, of the celeſtial as well as terreſtrial globe, and their ſeveral regions or parts, together with geographical, hiſtorical, and aſtronomical deſcriptions accommodated to them: to promote which purpoſes, the ſeveral members oblige themſelves, by their ſubſcription, to take one copy or more of each piece publiſhed under the direction of the academy; and to advance the money, or part of it, to defray the charge of publication. To this end there are three ſocieties ſettled, namely at Venice, Paris, and Rome; the firſt under F. Moro, provincial of the Minorites of Hungary; the ſecond under the abbot Laurence au Rue Payenne au Marais; the third under F. Ant. Baldigiani, jeſuit, profeſſor of mathematics in the Roman college; to whom thoſe addreſs themſelves who are willing to engage in this deſign. The Argonauts number near 200 members in the different countries of Europe; and their device is the terraqueous globe, with the motto *Plus ultra.* All the globes, maps, and geographical writings of F. Coronelli have been publiſhed at the expence of this academy.

The Academy *of Apatiſts*, or Impartial Academy, deſerves to be mentioned on account of the extent of its plan, including univerſally all arts and ſciences. It holds from time to time public meetings at Florence, where any perſon, whether academiſt or not, may read his works, in whatever form, language, or ſubject; the academy receiving all with the greateſt impartiality.

In France there are many academies for the improvement of arts and ſciences. F. Merſenne, it is ſaid, gave the firſt idea of a philoſophical academy in France, about the beginning of the ſeventeenth century, by the conferences of mathematicians and naturaliſts, held occaſionally at his lodgings; at which Des Cartes, Gaſſendus, Hobbes, Roberval, Paſcal, Blondel, and others, aſſiſted. F. Merſenne propoſed to each of them certain problems to examine, or certain experiments to be made. Theſe private aſſemblies were ſucceeded by more public ones, formed by M. Monmort, and M. Thevenot, the celebrated traveller. The French example animated ſeveral Engliſhmen of rank and learning to erect a kind of philoſophical academy at Oxford, towards the cloſe of Cromwell's adminiſtration; which after the reſtoration was erected, by public authority, into a Royal Society: an account of which ſee under the word. The Engliſh example, in its turn, animated the French. In 1666 Louis XIV, aſſiſted by the counſels of M. Colbert, founded an academy of ſciences at Paris, called the

Academie *Royale des Sciences, or Royal Academy of Sciences*, for the improvement of philoſophy, mathematics, chemiſtry, medicine, belles-lettres, &c. Among the principal members, at the commencement in 1666, were the reſpectable names of Carcavi, Huygens, Roberval, Frenicle, Auzout, Picard, Buot, Du Hamel the Secretary, and Mariotte. There was a perfect equality among all the members, and many of them received ſalaries from the king, as at preſent. By the rules of the academy, every claſs was to meet twice a week; the philoſophers and geometricians were to meet, ſeparately, every Wedneſday, and then both together on the Saturday, in a room of the king's library, where the philoſophical and mathematical books were kept: the hiſtory claſs was to meet on the Monday and Thurſday in the room of the hiſtorical books: and the claſs of belles-lettres on the Tueſday and Friday: and on the firſt Thurſday of every month all the claſſes met together, and by their ſecretaries made a mutual report of what had been tranſacted by each claſs during the preceding month.

In 1699, on the application of the preſident, the abbé Bignon, the academy received, under royal authority and protection, a new form and conſtitution; by the articles of which, the academy was to conſiſt of four ſorts of members, namely honorary, penſionary, aſſociates, and eleves. The honorary claſs to conſiſt of ten perſons, and the other three claſſes of twenty perſons each. The preſident to be choſen annually out of the honorary claſs, and the ſecretary and treaſurer to be perpetual, and of the penſionary claſs. The meetings to be twice a week, on the Wedneſday and Saturday; beſides two public meetings in the year.

Of the penſionaries, or thoſe who receive ſalaries, three to be geometricians, three aſtronomers, three mechaniſts, three anatomiſts, three botaniſts, and three chemiſts, the other two being the ſecretary and treaſurer. Of the twenty aſſociates, of which twelve to be French, and eight might be foreigners, two were to cultivate geometry, two aſtronomy, two mechanics, two anatomy, two botany, and two chemiſtry. Of the twenty eleves, one to be attached to each penſionary, and to cultivate his peculiar branch of ſcience. The penſionaries and their eleves to reſide at Paris. No regulars nor religious to be admitted, except into the honorary claſs: nor any perſon to be admitted a penſioner who was not known by ſome conſiderable work, or ſome remarkable diſcovery.

In 1716 the Duke of Orleans, then regent of France, by the king's authority made ſome alteration in their conſtitution. The claſs of eleves was ſuppreſſed; and inſtead of them were inſtituted twelve adjuncts, two to each of the ſix claſſes of penſioners. The honorary members were increaſed to twelve: and a claſs of ſix free aſſociates was made, who were not under the obligation of cultivating any particular branch of ſcience, and in this claſs only could the regulars or religious be admitted. A preſident and vice-preſident to be appointed annually from the honorary claſs, and a director and ſub-director annually from that of the penſioners. And no perſon to be allowed to make uſe of his quality of academician, in the title of any of his books that he publiſhed, unleſs ſuch book were firſt approved by the academy.

The academy has for a device or motto, *Invenit & perficit*. And the meetings, which were formerly held in the king's library, have ſince the year 1699 been held in a fine hall of the old Louvre.

Finally, in the year 1785 the king confirmed, by letters patent, dated April 23, the eſtabliſhment of the academy of ſciences, making the following alterations, and adding claſſes of agriculture, natural hiſtory, mineralogy, and phyſics; incorporating the aſſociates and adjuncts, and limiting to ſix the members of each claſs, namely three penſioners and three aſſociates; by which the former receive an increaſe of ſalary, and the latter approach nearer to becoming penſioners.

By the articles of this inſtrument it is ordained, that the academy ſhall conſiſt of eight claſſes, namely, that of geometry, 2d aſtronomy, 3d mechanics, 4th general phyſics, 5th anatomy, 6th chemiſtry and metallurgy, 7th botany and agriculture, and 8th natural hiſtory and mineralogy. That each claſs ſhall remain irrevocably fixed at ſix members; namely, three penſioners and three aſſociates, independent however of a perpetual ſecretary and treaſurer, of twelve free-aſſociates and of eight aſſociate-ſtrangers or foreigners, the ſame as before, except that the adjunct-geographer for the future be called the aſſociate-geographer.

The claſſes at firſt to be filled by the following perſons, namely, that of geometry by Meſſieurs de Borda, Jeaurat, Vandermonde, as penſioners; and Meſſieurs Couſin, Meuſnier, and Charles, as aſſociates: that of aſtronomy by Meſſieurs le Monnier, de la Lande, and le Gentil, as penſioners; and Meſſieurs Meſſier, de Caſſini, and Dagelat, as aſſociates: that of mechanics by Meſſieurs l'abbe Boſſut, l'abbe Rochon, and de la Place, as penſioners; and Meſſieurs Coulomb, le Gendre, and Perrier, as aſſociates: that of general phyſics by Meſſieurs Leroy, Briſſon, and

Bailly, as penſioners; and Meſſieurs Monge, Mechain, and Quatremere, as aſſociates: that of anatomy by Meſſieurs Daubluton, Tenon, and Portal, as penſioners; and Meſſieurs Sabatier, Vicq-d'azir, and Brouſſonet, as aſſociates: that of chemiſtry and metallurgy by Meſſieurs Cadet, Lavoiſier, and Beaume, as penſioners; and Meſſieurs Cornette, Bertholet, and Fourcroy, as aſſociates: that of botany and agriculture by Meſſieurs Guettard, Fougeroux, and Adanſon, as penſioners; and Meſſieurs de Juſſieu, de la Marck, and Desfontaines, as aſſociates: and that of natural hiſtory and mineralogy by Meſſieurs Deſmaretz, Saye, and l'abbe de Gua, as penſioners; and Meſſieurs Darcet, l'abbe Haui, and l'abbe Teſſier, as aſſociates. All names reſpectable in the common-wealth of letters; and from whom the world might expect ſtill farther improvements in the ſeveral branches of ſcience.

The late M. Rouille de Meſlay, counſellor of the parliament of Paris, founded two prizes, the one of 2500 livres, the other of 2000 livres, which the academy diſtributed alternately every year: the ſubjects of the former prize reſpecting phyſical aſtronomy, and of the latter, navigation and commerce.

The world is highly indebted to this academy for the many valuable works they have executed, or publiſhed, both individually and as a body collectively, eſpecially by their memoirs, making upwards of a hundred volumes in 4to, with the machines, indexes, &c. in which may be found moſt excellent compoſitions in every branch of ſcience. They publiſh a volume of theſe memoirs every year, with the hiſtory of the academy, and eloges of remarkable men lately deceaſed: alſo a general index to the volumes every ten years. An alteration was introduced into the volume for 1783, which it ſeems is to be continued in future, by omitting, in the hiſtory, the minutes or extracts from the regiſters, containing ſome preliminary account of the ſubjects of the memoires; but ſtill however retaining the eloges of diſtinguiſhed men, lately deceaſed.

M. l'abbe Rozier alſo has publiſhed in four 4to volumes, an excellent index of the contents of all the volumes, and the writings of all the members, from the beginning of their publications to the year 1770; with convenient blank ſpaces for continuing the articles in writing.

Their hiſtory alſo, to the year 1697, was written by M. Du Hamel; and after that time continued from year to year by M. Fontenelle, under the following titles, Du Hamel Hiſtoriæ Regiæ Academiæ Scientiarum, Paris, 4to. Hiſtoire de l'Academie Royale des Sciences, avec les Memoires de Mathematique & de Phyſique, tirez des Regiſtres de l'Academie, Paris, 4to. Hiſtoire de l'Academie Royale des Sciences depuis ſon etabliſſement en 1666, juſqu'en 1699, en 13 tomes, 4to. A new hiſtory, from the inſtitution of the academy, to the period from whence M. de Fontenelle commences, has been formed; with a ſeries of the works publiſhed under the name of this academy, during the firſt interval.

Since the foregoing account was written, it is ſaid the Academy has been ſuppreſſed and aboliſhed, by the preſent convention of France.

Beſides the academies in the capital, there are a great many in other parts of France. The

ACADEMIE *Royale*, at Caen, was eſtabliſhed by letters patent in the year 1705; but it had its riſe fifty years earlier in private conferences, held firſt in the houſe of M. de Brieux. M. de Segrais retiring to this city, to ſpend the reſt of his days, reſtored and gave new luſtre to their meetings. In 1707 M. Foucault, intendant of the generality of Caen, procured the king's letters patent for erecting them into a perpetual academy, of which M. Foucault was to be protector for the time, and the choice afterwards left to the members, the number of whom was fixed to thirty, choſen for this time by M. Foucault. But beſides the thirty original members, leave was given to add ſix ſupernumerary members, from the eccleſiaſtical communities in that city.

At Toulouſe is the *Academie des jeux floraux*, compoſed of forty perſons, the oldeſt of the kingdom: beſides an academy of ſciences and belles-lettres, founded in 1750.

At Montpelier is the royal ſociety of ſciences, which ſince 1708 makes but one body with the royal academy of ſciences at Paris.

There are alſo other academies at Bourdeaux, founded in 1703, at Soiſſons in 1674, at Marſeilles in 1726, at Lyons in 1700, at Pau in Bearn in 1721, at Montauban in 1744, at Angers in 1685, at Amiens in 1750, at Villefranche in 1679, at Dijon in 1740, at Nimes in 1682, at Beſançon in 1752, at Chalons in 1775, at Rochelle in 1734, at Beziers in 1723, at Rouen in 1744, at Metz in 1760, at Arras in 1773, &c. The number of theſe academies is continually augmenting; and even in ſuch towns as have no academies, the literati form themſelves into literary ſocieties, having nearly the ſame objects and purſuits.

In Germany and other parts of Europe, there are various academies of ſciences, &c. The

ACADEMIE *Royale des Sciences & des Belles-Lettres* of Pruſſia, was founded at Berlin, in the year 1700, by Frederic I. king of Pruſſia, of which the famous M. Leibnitz was the firſt preſident, and its great promoter. The academy received a new form, and a new ſet of ſtatutes in 1710; by which it was ordained, that the preſident ſhall be one of the counſellors of ſtate; and that the members be divided into four claſſes; the firſt to cultivate phyſics, medicine, and chemiſtry; the ſecond, mathematics, aſtronomy, and mechanics; the third, the German language, and the hiſtory of the country; and the fourth, oriental learning, particularly as it may concern the propagation of the goſpel among infidels. That each claſs elect a director for themſelves, who ſhall hold his poſt for life. That they meet in the caſtle called the New Marſhal, the claſſes to meet in their turns, one each week. And that the members of any of the claſſes have free acceſs into the aſſemblies of the reſt. Several volumes of their tranſactions have been publiſhed in Latin, from time to time, under the title of Miſcellanea Berolinenſia.

In 1743 the late famous Frederic II. king of Pruſſia, made great alterations and improvements in the academy. Inſtead of a great lord or miniſter of ſtate, who had uſually preſided over the academy, he wiſely judged that office would be better filled by a man of letters; and he honoured the French academy of ſciences by fixing upon one of its members for a preſident, namely M. Maupertuis, a diſtinguiſhed character in the literary world, and whoſe conduct in improving the academy was a proof of the ſound judgment of the king, who at the ſame time made new regulations for the academy, and took the title of its Protector. From that time the tranſactions of the academy have been publiſhed, under the title of Hiſtoire de l'Academie Royale des Sciences et Belles Lettres à Berlin, much in the manner of thoſe of the French academy of ſciences, and in the French language; and the volumes are now commonly publiſhed annually. Beſides the ordinary private meetings of the academy, it has two public ones in the year, in January and May, at the latter of which is given a prize gold medal, of the value of 50 ducats, or about 20 guineas. The ſubject of the prize is ſucceſſively phyſics, mathematics, metaphyſics, and general literature. For the academy has this peculiar circumſtance, that it embraces alſo metaphyſics, logic, and morality; having one claſs particularly appropriated to theſe objects, called the claſs of Speculative Philoſophy.

Imperial ACADEMY *of Peterſburgh*. This academy was projected by the Czar Peter I, commonly called Peter the Great, who in ſo many other inſtances alſo was inſtrumental in raiſing Ruſſia from the ſtate of barbarity in which it had been immerged for ſo many ages. Having viſited France in 1717, and among other things informed himſelf of the advantages of an academy of arts and ſciences, he reſolved to eſtabliſh one in his new capital, whither he had drawn by noble encouragements ſeveral learned ſtrangers, and made other preparations, when his death prevented him from fully accompliſhing that object, in the beginning of the year 1725. Thoſe preparations and intentions however were carried into execution the ſame year, by the eſtabliſhment of the academy, by his conſort the czarina Catherine, who ſucceeded him. And ſoon after the academy compoſed the firſt volume of their works, publiſhed in 1728, under the title of Commentarii Academiæ Scientiarum Imperialis Petropolitanæ; which they continued almoſt annually till 1746, the whole amounting to 14 volumes, which were publiſhed in Latin, and the ſubjects divided and claſſed under the following heads, namely mathematics, phyſics, hiſtory, and aſtronomy. Their device a tree bearing fruit not ripe, with the modeſt motto *paullatim*.

Moſt part of the ſtrangers who compoſed this academy being dead, or having retired, it was rather in a languiſhing ſtate at the beginning of the reign of the empreſs Elizabeth, when the count Raſomowſki was happily appointed preſident, who was inſtrumental in recovering its vigour and labours. This empreſs renewed and altered its conſtitution, by letters patent dated July 24, 1747, giving it a new form and regulations. It conſiſts of two chief parts, an academy, and a univerſity, having regular profeſſors in the ſeveral faculties, who read lectures as in our colleges. The ordinary aſſemblies are held twice a week, and public or ſolemn ones thrice in the year; in which an account is given of what has been done in the private ones. The academy has a noble building for their meetings, &c, with a good apparatus of inſtruments, a fine library, obſervatory, &c. Their firſt volume, after this renovation, was publiſhed for the years 1747 and 1748, and they have been ſince continued from year to year, now to the amount of near thirty volumes, under the title of Novi Commentarii Academiæ Scientiarum Imperialis Petropolitanæ. They are printed in the Latin language, and contain many excellent compoſitions in all the ſciences, eſpecially the mathematical papers of the late excellent M. L. Euler, which always made a conſiderable portion of every volume. The ſubjects are claſſed under heads in the following order, mathematics, phyſico-mathematics, phyſics, which include botany, anatomy, &c, and aſtronomy; the whole prefaced by hiſtorical extracts, or minutes, relating to each paper or memoir, after the manner of the volumes of the French academy; but wanting however the eloges of deceaſed eminent men. Their device is a heap of ripe fruits piled on a table, with the motto *En addit fructus ætate recentes*.

Imperial and Royal ACADEMY *of Sciences and Belles Lettres, at Bruſſels*. This academy was founded in the year 1773; and ſeveral volumes of their memoirs have been publiſhed.

Royal ACADEMY *of Sciences*, at Stockholm, was inſtituted in 1739, and ſince that time it has publiſhed about ſixty volumes of tranſactions, quarterly, in 8vo, in the Swediſh language.

For an account of the Royal Society of London, and ſeveral other ſimilar inſtitutions, ſee the words Journal, Society, &c.

American ACADEMY *of Arts and Sciences*, was eſtabliſhed in 1780 by the council and houſe of repreſentatives in the province of Maſſachuſet's Bay, for promoting the knowledge of the antiquities of America, and of the natural hiſtory of the country; for determining the uſes to which its various natural productions may be applied; for encouraging medicinal diſcoveries, mathematical diſquiſitions, philoſophical enquiries and experiments, aſtronomical, meteorological, and geographical obſervations, and improvements in agriculture, manufactures, and commerce; and, in ſhort, for cultivating every art and ſcience, which may tend to advance the intereſt, honour, dignity, and happineſs, of a free, independent, and virtuous people. The members of this academy are never to be leſs than forty, nor more than two hundred.

ACADEMY is alſo uſed among us for a kind of collegiate ſchool, or ſeminary; where youth are inſtructed in the liberal arts and ſciences in a private way: now indeed it is uſed for all kinds of ſchools.

Frederic I, king of Pruſſia, eſtabliſhed an academy at Berlin in 1703, for educating the young nobility of the court, ſuitable to their extraction. The expence of the ſtudents was very moderate, the king having undertaken to pay the extraordinaries. This illuſtrious ſchool, which was then called the academy of princes, has now loſt much of its firſt ſplendour.

The Romans had a kind of military academies eſtabliſhed in all the cities of Italy, under the name of *Campi Martis*. Here the youth were admitted to be trained for war at the public expence. And the Greeks, beſides academies of this kind, had military profeſſors, called *Tactici*, who taught all the higher offices of war, &c.

We have two royal academies of this kind in England, the expences of which are defrayed by the government; the one at Woolwich, for the artillery and military engineers; and the other at Portſmouth, for the navy. The former was eſtabliſhed by his late majeſty king George II, by warrants dated April the 30th and November the 18th, 1741, for inſtructing perſons belonging to the military part of the ordnance, in the ſeveral branches of mathematics, fortification, &c, proper to qualify them for the ſervice of artillery and the office of engineers. This inſtitution is under the direction of the maſter-general and board of ordnance for the time being: and at firſt the lectures of the maſters in the academy were attended by the practitioner-engineers, with the officers, ſerjeants, corporals, and private men of the artillery, beſides the cadets. At preſent however none are educated there but the gentlemen cadets, to the number of 90 or 100, where they receive an education perhaps not to be obtained or purchaſed for money in any part of the world. The maſter-general of the ordnance is always captain of the cadets' company, and governor of the academy; under him are a lieutenant-governor, and an inſpector of ſtudies. The maſters have been gradually increaſed, from two or three at firſt, now to the number of twelve, namely, a profeſſor of mathematics, and two other mathematical maſters, a profeſſor of fortification, and an aſſiſtant, two drawing maſters, two French maſters, with maſters for fencing, dancing, and chemiſtry. This inſtitution is of the greateſt conſequence to the ſtate, and it is hardly credible that ſo important an object ſhould be accompliſhed at ſo trifling an expence. It is to be lamented however that the academy is fixed in ſo unhealthy a ſituation; that the lecture rooms and cadets' barracks are ſo ſmall as to be inſufficient for the purpoſes of the inſtitution; and that the ſalaries of the profeſſors and maſters ſhould be ſo inadequate to their labours, and the benefit of their ſervices.

The Royal Naval Academy at Portſmouth was founded by George I, in 1722, for inſtructing young gentlemen in the ſciences uſeful for navigation, to breed officers for the royal navy. The eſtabliſhment is under the direction of the board of admiralty, who give ſalaries to two maſters, by one of whom the ſtudents are boarded and lodged, the expence of which is defrayed by their own friends, nothing being ſupplied by the government but their education.

ACANTHUS, *in Architecture*, the leaves of a plant which forms the ornament of the capital of the Corinthian order. Vitruvius aſcribes the uſe of it to the following accident. A young girl dying, her nurſe was deſirous of conſecrating to her manes certain toys which ſhe was fond of in her life-time; which the good woman carried in a little baſket, covered with a ſquare tile, and placed it among ſome green plants which grew on her grave. One of theſe, which happened to be the acanthus, as it grew up, invironed and in a manner embraced the baſket; which Callimachus, a noted Greek ſculptor, caſting his eyes upon, from thence took the hint of this elegant ornament. See ABACUS.

ACCELERATED *Motion*, is that which receives freſh acceſſions of velocity; and the acceleration may be either equably or unequably: if the acceſſions of velocity be always equal in equal times, the motion is ſaid to be equably or uniformly accelerated; but if the acceſſions, in equal times, either increaſe or decreaſe, then the motion is unequably or variably accelerated.

Acceleration is directly oppoſite to retardation, which denotes a diminution of velocity.

ACCELERATION comes chiefly under conſideration in phyſics, in the deſcent of heavy bodies, tending or falling towards the centre of the earth, by the force of gravity.

That bodies are accelerated in their natural deſcent, is evident both to the ſight, and from obſerving that the greater height they fall from, the greater force they ſtrike with, and the deeper impreſſions they make in ſoft ſubſtances.

The acceleration of falling bodies has been aſcribed to various cauſes, by different philoſophers. Some have attributed it to the preſſure of the air downwards: the more a body deſcends, the longer and heavier, ſay they, muſt be the column of atmoſphere incumbent upon it; to which they add, that the whole maſs of fluid preſſing by an infinity of right-lines all ultimately meeting in the earth's centre, ſuch central point muſt ſupport, as it were, the preſſure of the whole maſs; and that conſequently the nearer a body approaches to it, the more muſt it receive of the preſſure of a multitude of lines tending to unite in the central point.

Mr. Hobbes endeavours to account for this acceleration from a new impreſſion of the cauſe which makes bodies fall; in which he is ſo far right. But then he as far miſtakes, as to the cauſe of the fall, which he thinks is the air: at the ſame time, ſays he, that one particle of air aſcends, another deſcends; for in conſequence of the earth's motion being two-fold, that is circular and progreſſive, the air muſt at once both aſcend and circulate; whence it follows, that a body falling in this medium, and receiving a new preſſure every inſtant, muſt have its motion accelerated.

But to both theſe ſyſtems it may be anſwered, that the air is quite out of the queſtion; for it is very evident that bodies fall, and in falling have their motion accelerated, in vacuo, as in open air, and even more than in the air, in as much as this oppoſes and ſomewhat retards their fall.

The Gaſſendiſts aſſign another reaſon for the acceleration: they pretend that there are continually iſſuing out of the earth certain attractive corpuſcles, directed in an infinite number of rays; thoſe, ſay they, aſcend and then deſcend, in ſuch ſort that the nearer a body approaches to the earth's centre, the more of theſe attractive rays preſs upon it, in conſequence of which its motion becomes more accelerated.

The peripatetics endeavour to explain the matter thus: the motion of heavy bodies downward, ariſes, ſay they, out of an intrinſic principle that cauſes a tendency in them to the centre, as the place appropriated to their element; where, when they can once arrive, they will be at perfect reſt; and therefore, continue they, the nearer bodies approach to it, the more the velocity of their motion is increaſed: a notion too idle to merit confutation.

The Carteſians account for acceleration, by reiterated impulſes of their *materia ſubtilis*, acting continually on falling bodies, and propelling them downwards: a conceit equally unintelligible and abſurd with the former.

But, leaving all ſuch viſionary cauſes of acceleration, and only admitting the exiſtence of ſuch a force as gravity, ſo evidently inherent in all bodies, without regard to what may be the cauſe of it, the whole myſtery of tion will be cleared up. Conſider gravity then, alileo, only as a cauſe or force which acts cony on heavy bodies; and it will be eaſy to con that the principle of gravitation, which deteres bodies to deſcend, muſt by a neceſſary conſeuence accelerate them in falling.

A body then having once begun to deſcend, through the impulſe of gravity; the ſtate of deſcending is now, by Newton's firſt law of nature, become as it were natural to it; inſomuch that, were it left to itſelf, it would for ever continue to deſcend, even though the firſt cauſe of its deſcent ſhould ceaſe. But beſides this determination to deſcend, impreſſed upon it by the firſt cauſe of motion, which would be ſufficient to continue to infinity the degree of motion already begun, new impulſes are continually ſuperadded by the ſame cauſe; which continues to act upon the body already in motion, in the ſame manner as if it had remained at reſt. There being then two cauſes of motion, acting both in the ſame direction; it neceſſarily follows, that the motion which they unitedly produce, muſt be more conſiderable than what either could produce ſeparately. And as long as the velocity is thus continued, the ſame cauſe ſtill ſubſiſting to increaſe it more, the deſcent muſt of neceſſity be continually accelerated.

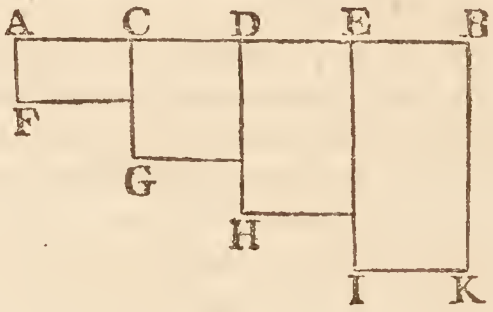
Suppoſing then that gravity, from whatever principle it ariſes, acts uniformly upon all bodies at the ſame diſtance from the centre of the earth: dividing the time which the heavy body takes up in falling to the earth, into indefinitely ſmall equal parts; gravity will impel the body toward the centre of the earth, in the firſt indefinitely ſhort inſtant of the deſcent. If after this we ſuppoſe the action of gravity to ceaſe, the body will continue perpetually to advance uniformly toward the earth's centre, with an indefinitely ſmall velocity, equal to that which reſulted from the firſt impulſe.

But then if we ſuppoſe that the action of gravity ſtill continues the ſame after the firſt impulſe; in the ſecond inſtant, the body will receive a new impulſe toward the earth, equal to that which it received in the firſt inſtant; and conſequently its velocity will be doubled; in the third inſtant, it will be tripled; in the fourth, quadrupled; in the fifth, quintupled; and ſo on continually: for the impulſe made in any preceding inſtant, is no ways altered by that which is made in the following one; but they are, on the contrary, always accumulated on each other.

So that the inſtants of time being ſuppoſed indefinitely ſmall, and all equal, the velocity acquired by the falling body, will be, in every inſtant, proportional to the times from the beginning of the deſcent; and conſequently the velocity will be proportional to the time in which it is produced. So that if a body, by this conſtant force, acquire a velocity of $16\frac{1}{12}$ feet ſuppoſe in one ſecond of time; it will acquire a velocity of $32\frac{1}{6}$ feet in two ſeconds, $48\frac{1}{4}$ feet in 3 ſeconds, $64\frac{1}{3}$ in 4 ſeconds, and ſo on. Nor ought it to ſeem ſtrange that all bodies, ſmall or large, acquire, by the force of gravity, the ſame velocity in the ſame time. For every equal particle of matter being endued with an equal impelling force, namely its gravity or weight, the ſum of all the forces, in any compound maſs of matter, will be proportional to the ſum of all the weights, or quantities of matter to be moved; conſequently, the forces and maſſes moved, being thus conſtantly increaſed in the ſame proportion, the velocities generated will be the ſame in all bodies, great or ſmall. That is, a double force moves a double maſs of matter, with the ſame velocity that the ſingle force moves the ſingle maſs; and ſo on. Or otherwiſe, the whole compound maſs falls all together with the ſame velocity, and in the ſame manner, as if its particles were not united, but as if each fell by itſelf, ſeparated all from one another. And thus all being let go at once, they would fall together, juſt as if they were united into one maſs.

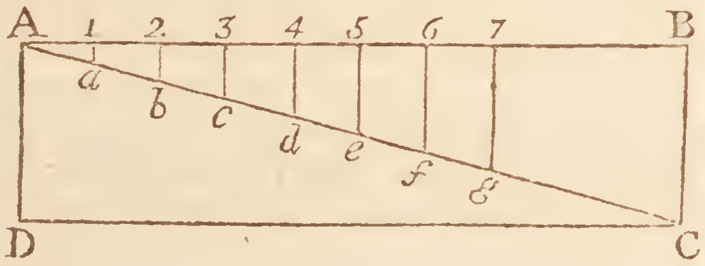
The foregoing law of the deſcent of falling bodies, namely that the velocities are always proportional to the times of deſcent, as well as the following laws concerning the ſpaces paſſed over, &c, were firſt diſcovered and taught by the great Galileo, and that nearly in the following manner.

Becauſe the conſtant velocity with which any body moves, or the ſpace it paſſes over in a given time, as ſuppoſe one ſecond, being multiplied by the time, or number of ſeconds it is in motion, expreſſes the ſpace paſſed over in that time; and the area or ſpace of a rectangular figure being denoted by the length multiplied by the breadth; therefore the ſpace ſo run over, may be conſidered as a rectangle compounded of the time and velocity, that is a rectangle of which the time denotes the length, and the velocity the breadth. Suppoſe then A to be the heavy body which deſcends, and A B to denote the whole time of any deſcent; which let be divided into a certain number of equal parts, denoting intervals or portions of the given time, as A C, C D, D E, &c. Imagine the body to deſcend, during the time expreſſed by the firſt of the diviſions A C, with a certain uniform velocity ariſing from the force of gravity acting on it, which let be denoted by A F, the breadth of the rectangle C F; then the ſpace run through during the time denoted by A C, with the velocity denoted by A F, will be expreſſed by the rectangular ſpace C F.



Now the action of gravity having produced, in the firſt moment, the velocity A F, in the body, before at reſt; in the firſt two moments it will produce the velocity C G, the double of the former; in the third moment, to the velocity C G will be added one degree more, by which means will be produced the velocity D H, triple of the firſt; and ſo of the reſt; ſo that during the whole time A B, the body will have acquired the velocity B K. Hence, taking the diviſions of the line A B at pleaſure; for example, the diviſions A C, C D, &c, for the times; the ſpaces run through during thoſe times, will be as the areas or rectangles C F, D G, &c; and ſo the ſpace deſcribed by the moving body during the whole time A B, will be equal to all the rectangles, that is, equal to the whole indented ſpace A B K I H G F. And thus it will happen if the increments of velocity be produced, as we may ſay, all at once, at the end of certain portions of finite time; for inſtance at C, at D, &c; ſo that the degree of motion remains the ſame to the inſtant that a new acceleration takes place.

By conceiving the diviſions of time to be ſhorter, for example but half as long as the former, the indentures of the figure will be proportionably more contracted, and it will approach nearer to a triangle; and ſo much the nearer as the diviſions of time are ſhorter: and if theſe be ſuppoſed infinitely ſmall; that is, if increments of the velocity be ſuppoſed to be acquired continually, and at each indiviſible particle of time, which is really the caſe, the rectangles ſo ſucceſſively produced, will form a true triangle, as A B C; the whole time A B conſiſting of minute portions A 1, 12, 23, &c; and the area of the triangle A B C, of all the minute ſurfaces, or minute trapeziums, which anſwer to the diviſions of the times; the area of the whole triangle A B C, denoting the ſpace run through during the whole time A B; and the area of any ſmaller triangle A 7 *g*, denoting the ſpace run through during the correſponding time A 7. But the triangles A 1 *a*, A 7 *g*, &c, being ſimilar, have their areas to each other as the ſquares of their like ſides A 1, A 7, &c; and conſequently the ſpaces gone through, in any times counted from the beginning, are to each other as the ſquares of the times.



Hence, in any right-angled triangle, as A B C, the one ſide A B repreſents the time, the other ſide B C the velocity acquired in that time, and the area of the triangle the ſpace deſcribed by the falling body.

From the preceding demonſtration is alſo drawn this other general theorem in motions that are uniformly accelerated; namely, that a body deſcending with a uniformly accelerated motion, deſcribes in the whole time of its deſcent, a ſpace, which is exactly the half of that which it would deſcribe uniformly in the ſame time, with the velocity it has acquired at the end of its accelerated fall. For it has been ſhewn that the whole ſpace which the falling body has run through in the time A B, is repreſented by the triangle A B C, the laſt velocity being B C; and the ſpace which the body would run through uniformly in the ſame time A B, conſtantly with the ſaid greateſt velocity B C, is repreſented by the rectangle A B C D: but it is well known that the rectangle A B C D is double the triangle A B C; and therefore the latter ſpace run through, is double the former; that is, the ſpace run through by the accelerated motion, is juſt half of that which the body would deſcribe in the ſame time, moving uniformly with the velocity acquired at the end of its accelerated fall.

Hence then, from the foregoing conſiderations are deduced the following general laws of uniformly accelerated motions, namely,

1ſt. That the velocities acquired, are conſtantly proportional to the times; in a double time a double velocity, &c.

2d. That the ſpaces deſcribed in the whole times, each counted from the commencement of the motion, are proportional to the ſquares of the times, or to the ſquares of the velocities; that is, in twice the time, the body will deſcribe 4 times the ſpace; in thrice the time, it will deſcribe 9 times the ſpace; in quadruple the time, 16 times the ſpace; and ſo on. In ſhort, if the times are proportional

to the numbers - 1, 2, 3, 4, 5, &c,
the ſpaces will be as 1, 4, 9, 16, 25, &c,

which are the ſquares of the former. So that if a body, by the natural force of gravity, fall through the ſpace of $16\frac{1}{12}$ feet in the firſt ſecond of time; then in the firſt two ſeconds of time it will fall through four times as much, or $64\frac{1}{3}$ feet; in the firſt three ſeconds it will fall nine times as much, or $144\frac{3}{4}$ feet; and ſo on. And as the ſpaces fallen through are as the ſquares of the times, or of the velocities; therefore the times, or the velocities, are proportional to the ſquare roots of the ſpaces.

3d. The ſpaces deſcribed by falling bodies, in a ſeries of equal inſtants or intervals of time, will be as

the odd numbers - 1, 3, 5, 7, 9, &c,
which are the differences of the ſquares or whole ſpaces } 1, 4, 9, 16, 25, &c,

that is, the body which has run through $16\frac{1}{12}$ feet in the firſt ſecond, will in the next ſecond run through $48\frac{1}{3}$ feet, in the third ſecond $80\frac{5}{12}$, and ſo on.

4th. If the body fall through any ſpace in any time, it acquires a velocity equal to double that ſpace; that is, in an equal time, with the laſt velocity acquired, if uniformly continued, it would paſs over juſt double the ſpace. So if a body fall through $16\frac{1}{12}$ feet in the firſt ſecond of time, then it has acquired a velocity of $32\frac{1}{6}$ feet in a ſecond; that is, if the body move uniformly for one ſecond, with the velocity acquired, it will paſs over $32\frac{1}{6}$ feet in this one ſecond: and if in any time the body fall through 100 feet; then in another equal time, if it move uniformly with the velocity laſt acquired, it will paſs over 200 feet. And ſo on.

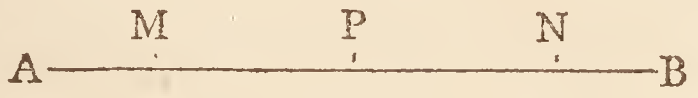
But, as the method of demonſtration uſed by Galileo, by means of infinitely ſmall parts forming a regular triangle, is not approved of by many perſons, the ſame laws may be otherwiſe demonſtrated thus: let the whole time of a body's free deſcent be divided into any number of parts, calling each of theſe parts 1; and let *a* denote the velocity acquired at the end of the firſt part of time; then will $2a$, $3a$, $4a$, &c, repreſent the velocities at the end of the 2d, 3d, 4th, &c, part of time, becauſe the velocities are as the times; and for the ſame reaſon $\frac{1}{2}a$, $\frac{3}{2}a$, $\frac{5}{2}a$, &c, will be the velocities at the middle point of the firſt, ſecond, third, &c, part of time. But now as the velocities increaſe uniformly, the ſpace deſcribed in any one of theſe parts of time, may be conſidered as uniformly deſcribed with its middle velocity, or the velocity in the middle of that part of time; and therefore multiplying thoſe mean velocities each by their common time 1, we have the ſame fractions $\frac{1}{2}a$, $\frac{3}{2}a$, $\frac{5}{2}a$, &c, for the ſpaces paſſed over in the ſucceſſive parts of the time; that is, the ſpace $\frac{1}{2}a$ in the firſt time, $\frac{3}{2}a$ in the ſecond, $\frac{5}{2}a$ in the third, and ſo on: then add theſe ſpaces ſucceſſively to one another, and we obtain $\frac{1}{2}a$, $\frac{4}{2}a$, $\frac{9}{2}a$, $\frac{16}{2}a$, &c, for the whole ſpaces deſcribed from the beginning of the motion to the end of the firſt, ſecond, third, &c, portion of time; namely $\frac{1}{2}a$ ſpace in one time, $\frac{4}{2}a$ in 2 times, $\frac{9}{2}a$ in 3 times, and ſo on: and it is evident that theſe ſpaces are as the numbers 1, 4, 9, 16, &c, which are as the ſquares of the times.

And from this mode of demonſtration, all the properties above-mentioned evidently flow; ſuch as that
the whole ſpaces - - $\frac{1}{2}a$, $\frac{4}{2}a$, $\frac{9}{2}a$, &c,
are as the ſquares of the times - 1, 2, 3, &c,
that the ſeparate ſpaces - $\frac{1}{2}a$, $\frac{3}{2}a$, $\frac{5}{2}a$, &c,
deſcribed in the ſucceſſive times, are as the odd numbers - } 1, 3, 5, &c,
and that the velocity a acquired in any time 1, is double the ſpace - - $\frac{1}{2}a$ deſcribed in the ſame time.

As the laws of acceleration are very important, I ſhall here inſert the two following propoſitions, ſent me by my learned friend Mr. Abram Robertſon, of Chriſt Church College Oxford, in which thoſe laws are demonſtrated in a manner ſomewhat different.

" Proposition I.

If from the point P in the ſtraight line A B, the points M, N begin to move at the ſame time, namely, M towards A with a motion uniformly retarded, and N from reſt towards B with a motion uniformly accelerated; and if the velocity of M decreaſes as much as the velocity of N increaſes in the ſame time; then the ſpace M N is generated by an uniform motion, equal to the velocity with which M begins to move.

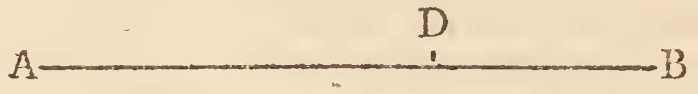


For, by hypotheſis, whatever is loſt in the velocity of M by retardation, is added to the velocity of N by acceleration: the joint velocities, therefore, of M and N muſt always be equal. But it is by the joint velocities of M and N that the ſpace M N is generated. Conſequently M N is generated by an uniform motion, which is evidently equal to the velocity with which M begins to move.

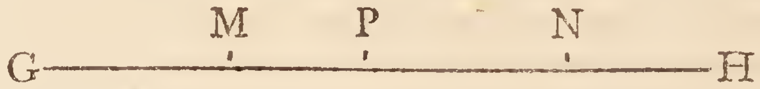
" Proposition II.

If a point begins to move in the direction of a ſtraight line, and continues to move in the ſame direction with a velocity uniformly accelerated; the ſpace paſſed over in any given time, will be equal to half the ſpace paſſed over in the ſame time with the velocity with which the acceleration ends.

Let the point D begin to move from A towards B, along the ſtraight line A B, with a motion uniformly accelerated; the ſpace A D paſſed over, is equal to half the ſpace which the point would paſs over, in the ſame time with the acquired velocity at D.



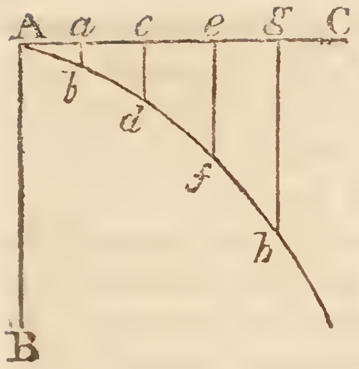
Let the points M, N begin to move in the ſtraight line G H, at the ſame time, with equal velocities uniformly accelerated; M beginning to move from G, and N from P; and at the ſame time that M comes to the point P, let N come to H. Then as M and N



move with equal velocities, uniformly accelerated, it is evident that the ſpaces, which they paſs over in the ſame time, are equal to one another; conſequently the ſpace G P is equal to the ſpace P H. Now as M begins to move from G with a velocity uniformly accelerated, it will arrive at P with an acquired velocity. Hence it is evident, if it be ſuppoſed to begin to move from P with this acquired velocity, and proceed toward G with a velocity uniformly retarded in the ſame degree that it was accelerated when it began to move from G, that it will paſs over the ſame ſpace G P in the ſame time. Wherefore, ſuppoſing the two points M, N to begin to move from P at the ſame time, namely the point M beginning to move with the acquired velocity mentioned above, and proceeding towards G with the velocity uniformly retarded, deſcribed above; and the point N as before with the velocity uniformly accelerated: then as the acceleration and retardation are uniform, they will be equal in equal ſpaces of time. Again, as M is retarded in the ſame degree that it was accelerated when it began to move from G, that is, in the ſame degree that N is accelerated, by the former prop. M N is generated by an uniform velocity. But when the point M arrives at G, its velocity becomes equal to 0 or nothing; and at the time that M arrives at G, N arrives at H with the acquired velocity. Wherefore, as the velocities of M and N taken jointly are equal, and conſequently uniform, the ſpace G H is paſſed over with the velocity of N at H, in the ſame time that P H is paſſed over by N beginning to move from P with a velocity uniformly accelerated to H. But P H is half of G H. " Hence the prop. is manifeſt."

And hence the other laws of the ſpaces, before-mentioned, eaſily follow.

Since the ſpaces deſcended are as the ſquares of the times, and the abſciſſes of a parabola are as the ſquares of the ordinates, therefore the relation of the times and ſpaces deſcended may be very well repreſented by the ordinates and abſciſſes of that figure. Thus if A B be the axis of the parabola A $bdfh$, and A C a tangent at



at the vertex perpendicular to the axis, divided into any number of equal parts A *a*, *a c*, *c e*, &c, for the times; and if there be drawn *a b*, *c d*, *e f*, &c, parallel to the axis: hence if *a b* be the ſpace deſcended in the time A *a*, then *c d* will be the ſpace deſcended in the time A *c*, and *e f* the ſpace deſcended in the time A *e*, and ſo on continually.

From the properties above-demonſtrated, are derived the following practical formulas or theorems for uſe. Namely, if g denote the ſpace paſſed over in the firſt ſecond of time, by a body urged by any conſtant force, denoted by 1, and t denote the time or number of ſeconds in which the body paſſes over any other ſpace s, and v the velocity acquired at the end of that time; then from the foregoing laws we have $v = 2gt$, and $s = gt^2$; and from theſe two equations reſult the following general formulas:

$$t = \frac{v}{2g} = \frac{2s}{v} = \sqrt{\frac{s}{g}},$$

$$v = 2gt = 2\sqrt{gs} = \frac{2s}{t},$$

$$s = gt^2 = \frac{vv}{4g} = \frac{tv}{2},$$

$$g = \frac{s}{tt} = \frac{v}{2t} = \frac{vv}{4s}.$$

And here, when the conſtant force 1, is the natural force of gravity, then the diſtance g deſcended in the firſt ſecond, in the latitude of London, is $16\frac{1}{12}$ feet: but if it be any other conſtant force, the value of g will be different, in proportion as the force is more or leſs.

The motion of an aſcending body, or of one that is impelled upwards, is diminiſhed or retarded by the ſame principle of gravity, acting in a contrary direction, after the ſame manner that a falling body is accelerated.

A body projected upwards, aſcends until it has loſt all its motion; which it does in the ſame ſpace of time, that the body would have taken up in acquiring, by falling, a velocity equal to that with which the falling body began to be projected upwards. And conſequently the heights to which bodies aſcend, when projected upwards with different velocities, are to each other as the ſquares of thoſe velocities.

ACCELERATED *Motion of Bodies on Inclined Planes.* The ſame general laws obtain here, as in bodies falling freely, or perpendicularly; namely, that the velocities are as the times, and the ſpaces deſcended down the planes as the ſquares of the times, or of the velocities. But thoſe velocities are leſs, according to the ſine of the plane's inclination; and the ſpaces leſs, according to the ſquare of the ſine. See INCLINED *Plane.*

ACCELERATED *Motion of Pendulums.* See PENDULUM.

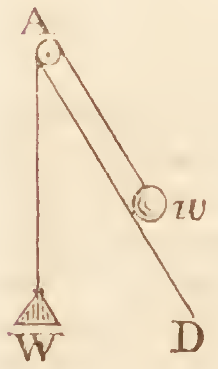
ACCELERATED *Motion of Projectiles.* See PROJECTILE.

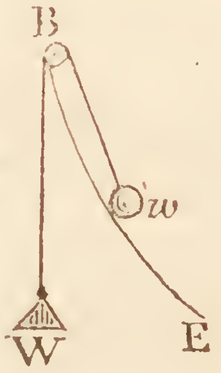
ACCELERATED *Motion of Compreſſed Bodies,* in expanding or reſtoring themſelves. See DILATATION, COMPRESSION, and ELASTICITY.

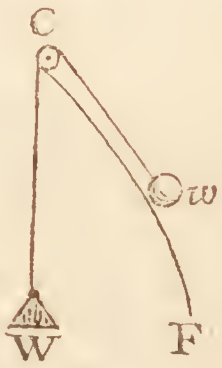
ACCELERATING FORCE, in Phyſics, is the force that accelerates the motion or velocity of bodies; and it is equal to, or expreſſed by, the quotient ariſing from the motive or abſolute force, divided by the maſs or weight of the body that is moved. In treating of phyſical conſiderations reſpecting forces, velocities, times, and ſpaces gone over, the firſt inquiry is the accelerating or accelerative force. This force is greater or leſs in proportion to the velocity it generates in the ſame time, and by this velocity it is meaſured. All accelerating forces are equal, and generate equal velocities, that have the motive forces directly proportional to the quantities of matter: ſo a double motive force will move a double quantity of matter with the ſame velocity, as alſo a triple motive force a triple quantity, a quadruple force a quadruple quantity, &c, all with the ſame velocity. And this is the reaſon why all bodies fall equally ſwift by the force of gravity; for the motive force is exactly proportional to their weight or maſs. In general, the accelerating force is in the direct ratio of the motive force, and inverſe ratio of the quantity of matter. When a body is let fall freely, to deſcend by the force of its natural gravity, it has been found by experiment that it falls through $16\frac{1}{12}$ feet in one ſecond of time, and requires a velocity of $32\frac{1}{6}$ feet in that time: but if the quantity of matter be doubled, and the motive force remain the ſame as before, by connecting the falling body to another of equal weight by means of a thread, this other body being laid on a horizontal plane, and the falling body hanging down off the plane, and drawing the other equal body along the plane after it; then the accelerating force will be only half of what it was before, and the ſpace fallen in one ſecond will be only $8\frac{1}{24}$ feet, and the velocity acquired $16\frac{1}{12}$: and if the quantity of matter be tripled, or the body drawn along the plane doubled; then the accelerating force will be only one-third of what it was at firſt, and the ſpace deſcended in one ſecond, and velocity acquired, each one-third of the firſt: and ſo on.

But accelerating forces are ſometimes variable, as well as ſometimes conſtant; and the variation may be either increaſing or decreaſing.

The nature of conſtant and variable accelerating forces, may be illuſtrated in the following manner. Let two weights W, *w*, be connected by a thread paſſing over a pully at A, B, or C; and let the weight W deſcend perpendicularly down, while it draws the ſmaller weight *w* up the line A D, or B E, or C F, the firſt

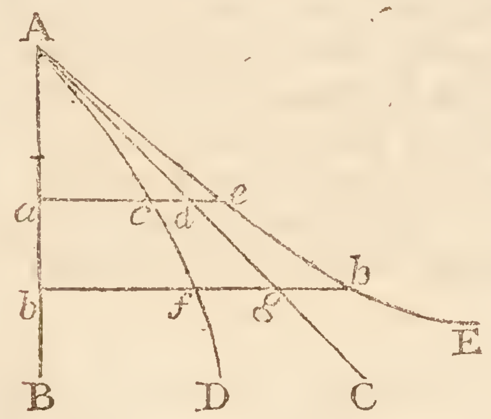






firſt being a ſtraight inclined plane, and the other two curves, the one convex and the other concave to the perpendicular. Then the ſmall weight *w* will always make ſome certain reſiſtance to the free deſcent of the large weight W, and that reſiſtance will be conſtantly the ſame in every part of the plane A D, the difficulty to draw it up being the ſame in every point of it, becauſe every part of it has the ſame inclination to the horizon, or to the perpendicular; and conſequently the acceſſions to the velocity of the deſcending weight W, will be always equal in equal times; that is, in this caſe W deſcends by a uniformly accelerating force. But in the two curves B E, C F, the reſiſtance or oppoſition of the ſmall weight *w* will be conſtantly altering as it is drawn up the curves, becauſe every part of them has a different inclination to the horizon, or to the perpendicular: in the former curve, the direction becomes more and more upright, or nearer perpendicular, as the ſmall weight *w* aſcends, and the oppoſition it makes to the deſcent of W, becomes more and more; and conſequently the acceſſions to the velocity of W will be always leſs and leſs in equal times; that is, W deſcends by a decreaſing accelerating force: but in the latter curve C F, as *w* aſcends, the direction of the curve becomes leſs and leſs upright, and the oppoſition it makes to the deſcent of W, becomes always leſs and leſs; and conſequently the acceſſions to the velocity of W will be always more and more in equal times; that is, W deſcends by an increaſing accelerating force. So that although the velocity continually increaſes in all theſe caſes, yet whilſt it increaſes in a conſtant ratio to the times of motion, in the plane A D; the velocity increaſes in a leſs ratio than the time it aſcended up B E, and in a greater ratio than the time increaſes in the other curve C F.

Now the relations between the times and velocities in all theſe caſes, may be very well repreſented by the relations between the abſciſſes and ordinates of certain lines. Thus let A B and A C be two ſtraight lines, making any angle B A C; and A D, A E two curves, the former concave, and the latter convex towards A B: divide A B into any parts A *a*, A *b*, &c, repreſenting the times of motion; and draw the perpendiculars *a c d e*, *b f g h*, &c, repreſenting the velocities. Then in the right line A C, the ordinates *a d*, *b g*, being as the abſciſſes A *a*, A *b*, this repreſents the caſe of uniformly accelerated motion, in which the velocities are always as the times: but in the curve A D, the ordinates *a c*, *b f* increaſe in a leſs ratio than the abſciſſes A *a*, A *b*; and therefore this repreſents the caſe of decreaſing acceleration, in which the velocities increaſe in a leſs ratio than the times: and in the other curve A E, the ordinates *a e*, *b h* increaſe in a greater ratio than the abſciſſes; and therefore this repreſents the caſe of increaſing acceleration, in which the velocities increaſe in a greater ratio than the times.



The ſeveral algebraic formulas or theorems, reſpecting the time, velocity, ſpace, for conſtant accelerating forces, are delivered above, at the article *Accelerated Motion*, where the value of each circumſtance is expreſſed in finite determinate quantities. But in the caſes of variably accelerated motions, the formulas will require the help of the method of fluxions to expreſs, not thoſe general relations themſelves, but the fluxions of them; and conſequently, taking the fluents of thoſe expreſſions, in particular caſes, the relations of time, ſpace, velocity, &c, are obtained.

Now if t denote the time in motion,
v the velocity generated by any force,
s the ſpace paſſed over,
and $2g$ the variable force at any part of the motion, or the velocity the force would generate in one ſecond of time, if it ſhould continue invariable, like the force of gravity, during that one ſecond; and therefore the value of this velocity $2g$, will be in proportion to $32\frac{1}{6}$ feet, as that variable force, is to 1 the force of gravity. Then becauſe the force may be ſuppoſed conſtant during the indefinitely ſmall time $\dot{t}$, and that in uniform motions the ſpaces and velocities are proportional to the times, we from thence obtain theſe two general fundamental porportions,

$$v : \dot{s} :: 1'' : \dot{t}, \text{ or } \dot{s} = \dot{t} v;$$

$$2g : \dot{v} :: 1'' : \dot{t}, \text{ or } \dot{v} = 2g\dot{t}.$$

From which are derived the four formulas below, in which the value of each quantity is expreſſed in terms of the reſt.

$$1. \;\text{- -}\; \dot{t} = \frac{\dot{s}}{v} = \frac{\dot{v}}{2g}.$$

$$2. \;\text{- -}\; \dot{v} = 2g\dot{t} = \frac{2g\dot{s}}{v}.$$

$$3. \;\text{- -}\; s = v\dot{t} = \frac{v\dot{v}}{2g}.$$

$$4. \;\text{-}\; 2g = \frac{\dot{v}}{t} = \frac{v\dot{v}}{s}.$$

And theſe theorems equally hold good for the deſtruction of motion and velocity, by means of retarding forces, as for the generation of the ſame by means of accelerating forces.

ACCELERATION, in *Mechanics*, the increaſe of velocity in a moving body.

ACCELERATION. *Aſtron.* The Diurnal Acceleration of the fixed ſtars, is the time which the ſtars, in one diurnal revolution, anticipate the mean diurnal revolution of the ſun; which is $3^m\ 55^s\frac{9}{10}$ of mean time, or nearly $3^m\ 56^s$: that is, a ſtar riſes, or ſets, or paſſes the meridian, about $3^m\ 56^s$ ſooner each day. This acceleration of the ſtars, which is only apparent in them, ariſes from the real retardation of the ſun, owing to his apparent motion in his orbit towards the eaſt, which is about $59'\ 8''\frac{2}{10}$ of a degree every day. So that the ſtar which paſſed the meridian yeſterday at the ſame moment with the ſun, is to-day about $59'\ 8''$ paſt the meridian to the weſt, when the ſun arrives at it; which will take him up about $3^m\ 56^s$ of time to paſs over; and therefore the ſtar paſſes by $3^m\ 56^s$ ſooner than the ſun each day, or anticipates his motion at that rate. The true quantity of this anticipation, or acceleration, is found by this proportion, $360^\circ\ 59'\ 8''\frac{1}{5} : 59'\ 8''\frac{1}{5} :: 24$ hours : $3^m\ 55^s\frac{9}{10}$, the fourth term of which is the acceleration.

The diurnal acceleration ſerves to regulate the lengths or vibration of pendulums. If I obſerve a fixed ſtar ſet or paſs behind a hill, ſteeple, or ſuch like, when the pendulum marks for inſtance $8^h\ 10^m$; and the next day, the eye being in the ſame place as before, the paſſage be at $8^h\ 6^m\ 4^s$; I thence conclude that the pendulum is well regulated, or truly meaſures mean time.

Acceleration *of a Planet*. A planet is ſaid to be accelerated in its motion, when its real diurnal motion exceeds its mean diurnal motion. And, on the other hand, the planet is ſaid to be retarded in its motion, when the mean exceeds the real diurnal motion. This inequality ariſes from the change in the diſtance of the planet from the ſun, which is continually varying; the planet moving always quicker in its orbit when nearer the ſun, and ſlower when farther off.

Acceleration *of the Moon*, is a term uſed to expreſs the increaſe of the moon's mean motion from the ſun, compared with the diurnal motion of the earth; by which it appears that, from ſome uncertain cauſe, it is now a little quicker than it was formerly. Dr. Halley was led to the diſcovery, or ſuſpicion, of this acceleration, by comparing the ancient eclipſes obſerved at Babylon, &c, and thoſe obſerved by Albategnius in the ninth century, with ſome of his own time; as may be ſeen in N° 218 of the Philoſophical Tranſactions. He could not however aſcertain the quantity of the acceleration, becauſe the longitudes of Bagdat, Alexandria, and Aleppo, where the obſervations were made, had not been accurately determined. But ſince his time the longitude of Alexandria has been aſcertained by Chazelles; and Babylon, according to Ptolemy's account, lies 50′ eaſt of Alexandria. From theſe data, Mr. Dunthorne, vol. 46 Philoſ. Tranſactions, compared the recorded times of ſeveral ancient and modern eclipſes, with the calculations of them by his own tables, and thereby verified the ſuſpicion that had been ſtarted by Dr. Halley; for he found that the ſame tables gave the moon's place more backward than her true place in ancient eclipſes, and more forward than her true place in later eclipſes; and thence he juſtly inferred that her motion in ancient times was ſlower, and in later times quicker, than the tables give it.

Not content however with barely aſcertaining the fact, he proceeded to determine, as well as the obſervations would allow, the quantity of the acceleration; and by means of the moſt authentic eclipſe, of which any good account remains, obſerved at Babylon in the year 721 before Chriſt, he found that the obſerved beginning of this eclipſe was about an hour and three quarters ſooner than the beginning by the tables; and that therefore the moon's true place preceded her place by computation by about 50′ of a degree at that time. Then admitting the acceleration to be uniform, and the aggregate of it as the ſquare of the time, it will be at the rate of about 10″ in 100 years.

Dr. Long, vol. ii. p. 436 of his Aſtronomy, enumerates the following cauſes from ſome one or more of which the acceleration may ariſe. Either 1ſt, the annual and diurnal motion of the earth continuing the ſame, the moon is really carried about the earth with a greater velocity than formerly: or, 2dly, the diurnal motion of the earth, and the periodical revolution of the moon, continuing the ſame, the annual motion of the earth about the ſun is retarded; which makes the ſun's apparent motion in the ecliptic a little ſlower than formerly; and conſequently the moon, in paſſing from any conjunction with the ſun, takes up a leſs time before ſhe again overtakes the ſun, and forms a ſubſequent conjunction: in both theſe caſes, the motion of the moon from the ſun is really accelerated, and the ſynodical month actually ſhortened: or, 3dly, the annual motion of the earth, and the periodical revolution of the moon, continuing the ſame, the rotation of the earth upon its axis is a little retarded; in this caſe, days, hours, minutes, &c, by which all periods of time muſt be meaſured, appear of a longer duration; and conſequently the ſynodical month will appear to be ſhortened, though it really contain the ſame quantity of abſolute time as it always did. If the quantity of matter in the body of the ſun be leſſened, by the particles of light continually ſtreaming from it, the motion of the earth about the ſun may become ſlower: if the earth increaſes in bulk, the motion of the moon about the earth may thereby be quickened.

ACCELERATIVE Force, &c, the ſame as Accelerating.

ACCESSIBLE, ſomething that may be approached, or to which we can come. In Surveying, it is ſuch a place as will admit of having a diſtance or length of ground meaſured from it; or ſuch a height or depth as can be meaſured by actually applying a proper inſtrument to it. For the means of doing which, ſee Altimetry, Longimetry, or Heights-and-Distances.

ACCIDENS, Accident, *Philoſ.*

Per Accidens is a term often uſed among philoſophers, to denote what does not follow from the nature of a thing, but from ſome accidental quality of it: in this ſenſe it ſtands oppoſed to *per ſe*, which denotes the nature and eſſence of a thing. Thus, fire is ſaid to burn *per ſe*, or conſidered as fire, and not *per accidens*; but a piece of iron, though red-hot, only burns *per accidens*, by a quality accidental to it, and not conſidered as iron.

Accidents, in *Aſtrology*, denote the moſt extraordinary occurrences in the courſe of a perſon's life, either good or bad: ſuch as a remarkable inſtance of good fortune, a ſignal deliverance, a great ſickneſs, &c.

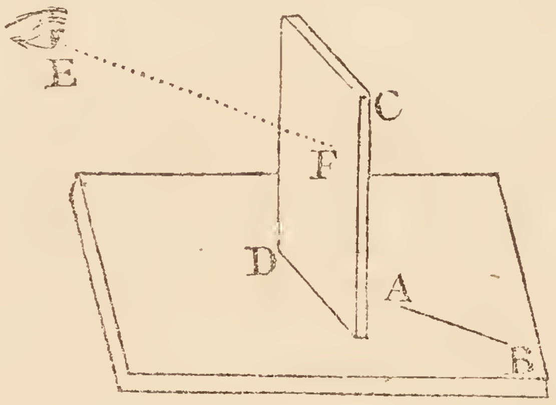
ACCIDENTAL, ſomething that partakes of the nature of an accident; or that is indifferent, or not eſſential to its ſubject.—Thus whiteneſs is accidental to marble, and ſenſible heat to iron.

Accidental *Colours*, ſo called by M. Buffon, are thoſe which depend on the affections of the eye, in contradiſtinction to ſuch as belong to light itſelf.

The impreſſions made upon the eye, by looking ſtedfaſtly on objects of a particular colour, are various according to the ſingle colour, or aſſemblage of colours, in the object; and they continue for ſome time after the eye is withdrawn, and give a falſe colouring to other objects that are viewed during their continuance. M. Buffon has endeavoured to trace the connection between theſe accidental colours, and thoſe that are natural, in a variety of inſtances. M. d'Arcy contrived a machine for meaſuring the duration of thoſe impreſſions on the eye; and from the reſult of ſeveral trials he inferred, that the effect of the action of light on the eye continued about eight thirds of a minute.

The ſubject has alſo been conſidered by M. de la Hire, and M. Aepinus, &c. See Mem. Acad. Paris 1743, and 1765; Nov. Com. Petrop. vol. 10; alſo Dr. Prieſtley's Hiſt. of Diſcoveries relating to Viſion, pa. 631.

Accidental *Point*, in Perſpective, is the point in which a right line drawn from the eye, parallel to another right line, cuts the picture or perſpective plane.



Let A B be the line given to be put into perſpective, C F D the picture or perſpective plane, and E the eye: draw E F parallel to A B; ſo ſhall F be the accidental point of the line A B, and indeed of all lines parallel to it, ſince only one parallel to them, namely E F, can be drawn from the ſame point E: and in the accidental point concur or meet the repreſentations of all the parallels to A B, when produced.

It is called the accidental point, to diſtinguiſh it from the principal point, or point of view, where a line drawn from the eye perpendicular to the perſpective plane, meets this plane, and which is the accidental point to all lines that are perpendicular to the ſame plane.

Accidental *Dignities*, and *Debilities*, in *Aſtrology*, are certain caſual diſpoſitions, and affections, of the planets, by which they are ſuppoſed to be either ſtrengthened, or weakened, by being in ſuch a houſe of the figure.

ACCLIVITY, the ſlope or ſteepneſs of a line or plane inclined to the horizon, taken upwards; in contradiſtinction to *declivity*, which is taken downwards. So the aſcent of a hill, is an *acclivity*: the deſcent of the ſame, a *declivity*.

Some writers on fortification uſe acclivity for *talus*: though more commonly the word talus is uſed to denote the ſlope, whether in aſcending or deſcending.

ACCOMPANYMENT, in *Muſic*, denotes either the different parts of a piece of muſic for the different inſtruments, or the inſtruments themſelves which accompany a voice, to ſuſtain it, as well as to make the muſic more full.

The Accompanyment is uſed in recitative, as well as in ſong; on the ſtage, as well as in the choir, &c.

The ancients had likewiſe their accompanyments on the theatre; and they had even different kinds of inſtruments to accompany the chorus, from thoſe which accompanied the actors in the recitation.

The accompanyment among the moderns, is often a different part, or melody, from the ſong it accompanies. But it is diſputed whether it was ſo among the ancients.

Organiſts ſometimes apply the word to ſeveral pipes which they occaſionally touch to accompany the treble; as the drone, the flute, &c.

ACCOMPT. See Account.

ACCORD, according to the modern French muſic, is the union of two or more ſounds heard at the ſame time, and forming together a regular harmony.

They divide Accords into *perfect* and *imperfect*; and again into *conſonances* and *diſſonances*.

Accord is more commonly called Concord, which ſee.

Accord is alſo ſpoken of the ſtate of an inſtrument, when its fixed ſounds have among themſelves all the juſtneſs that they ought to have.

ACCOUNT, or Accompt, in *Arithmetic*, &c, a calculation or computation of the number or order of certain things; as the computation of time, &c.

There are various ways of accounting; as, by enumeration, or telling one by one; or by the rules of arithmetic, addition, ſubtraction, &c.

Account, in *Chronology*, is nearly ſynonymous with ſtyle. Thus, we ſay the Engliſh, the foreign, the Julian, the Gregorian, the Old, or the New account, or ſtyle.

We account time by years, months, &c; the Greeks accounted it by olympiads; the Romans, by indictions, luſtres, &c.

Acherner, or Acharner, in *Aſtronomy*, a ſtar of the firſt magnitude in the ſouthern extremity of the conſtellation Eridanus, marked α by Bayer. Its longitude for 1761, ♓ 11° 55′ 1″; and latitude ſouth 59° 22′ 4″.

ACHILLES, a name given by the ſchools to the principal argument alleged by each ſect of philoſophers in behalf of their ſyſtem. In this ſenſe we ſay this is his Achilles; that is, his maſter-proof: alluding to the ſtrength and importance of the hero Achilles among the Greeks.

Zeno's argument againſt motion is peculiarly termed Achilles. That philoſopher made a compariſon between the ſwiftneſs of Achilles, and the ſlowneſs of a tortoiſe, pretending that a very ſwift animal could never overtake a ſlow one that was before it, and that therefore there is no ſuch thing as motion: for, ſaid he, if the tortoiſe were one mile before Achilles, and the motion of Achilles 100 times ſwifter than that of the tortoiſe, yet he would never overtake it; and for this reaſon, namely, that while Achilles runs over the mile, the tortoiſe will creep over one hundredth part of a mile, and will be ſo much the foremoſt; again while Achilles runs over this $\frac{1}{100}$th part, the tortoiſe will creep over the 100th part of that $\frac{1}{100}$th part, and will ſtill be this laſt part the foremoſt; and ſo on continually, according to an infinite ſeries of 100th parts: from which he concluded that the ſwifter could never overtake the ſlower in any finite time, but that they muſt go on approaching to infinity. But this ſophiſm lay in their conſidering as an infinite time, the ſum of the infinite ſeries of ſmall times in which Achilles could run over the infinite ſeries of ſpaces, $1 + \frac{1}{100} + \frac{1}{10000} + \frac{1}{1000000}$ &c, not knowing that the ſum of this infinite ſeries is equal to the quantity $1\frac{1}{99}$ of a mile, and that therefore Achilles will overtake the tortoiſe when the latter has crawled over $\frac{1}{99}$th of a mile.

ACHROMATIC, in *Optics*, without colour; a term which, it ſeems, was firſt uſed by M. de la Lande, in his aſtronomy, to denote teleſcopes of a new invention, contrived to remedy aberrations and colours. See *Aberration* and *Teleſcope*.

ACHROMATIC TELESCOPE, a ſingular ſpecies of refracting teleſcope, ſaid to be invented by the late Mr. John Dollond, optician to the king, and ſince improved by his ſon Mr. Peter Dollond, and others.

Every ray of light paſſing obliquely from a rarer into a denſer medium, changes its direction towards the perpendicular; and every ray paſſing obliquely from a denſer into a rarer medium, changes its direction from the perpendicular. This bending of the ray, cauſed by the change of its direction, is called its refraction; and the quality of light which ſubjects it to this refraction, is called its refrangibility. Every ray of light, before it is refracted, is white, though it conſiſts of a number of component rays, each of which is of a different colour. As ſoon as it is refracted, it is ſeparated into its component rays, which, from that time, proceed diverging from each other, like rays from a centre: and this divergency is cauſed by the different refrangibility of the component rays, in ſuch ſort, that the more the original or component ray is refracted, the more will the compound rays diverge when the light is refracted by one given medium only.

From hence it has been concluded, that any two different mediums that can be made to produce equal refractions, will neceſſarily produce equal divergencies: whence it ſhould alſo follow, that equal and contrary refractions ſhould not only deſtroy each other, but that the divergency of the colours cauſed by one refraction, ſhould be corrected by the other; and that to produce refraction that would not be affected by the different refrangibility of light, is impoſſible.

But Mr. Dollond has proved, by many experiments, that theſe concluſions are not well founded; from which experiments it appeared, that a ray of light, after equal and contrary refractions, was ſtill ſpread into component rays differently coloured: in other words, that two different mediums may cauſe equal refraction, but different divergency; and equal divergency, with different refraction. It follows therefore that refraction may be produced, which is not affected by the different refrangibility of light. In other words, that, if the mediums be different, different refractions may be produced, though at the ſame time the divergency cauſed by one refraction ſhall be exactly counteracted by the divergency cauſed by the other; and ſo an object may be ſeen through mediums which, together, cauſe the rays to converge, without appearing of different colours.

This is the foundation of Mr. Dollond's improvement of refracting teleſcopes. By ſubſequent experiments he found, that different ſorts of glaſs differed greatly in their refractive qualities, with reſpect to the divergency of colours. He found that crown glaſs cauſes the leaſt divergency, and white flint the moſt, when they are wrought into forms that produce equal refractions. He ground a piece of white flint glaſs into a wedge, whoſe angle was about 25 degrees; and a piece of crown glaſs to another, whoſe angle was about 29 degrees; and theſe he found refracted nearly alike, but that their divergency of colours was very different.

He then ground ſeveral other pieces of crown glaſs to wedges of different angles, till he got one that was equal, in the divergency it produced, to that of a wedge of flint glaſs of 25 degrees; ſo that when they were put together, in ſuch a manner as to refract in contrary directions, the refracted light was perfectly free from colour. Then meaſuring the fractions of each wedge, he found that that of the white flint glaſs, was to that of the crown glaſs, nearly as two to three. And hence any two wedges, made of theſe two ſubſtances, and in this proportion, would, when applied together ſo as to refract in contrary directions, refract the light without any effect ariſing from the different refrangibility of the component rays.

Therefore, to make two ſpherical glaſſes that refract the light in contrary directions, one muſt be concave, and the other convex; and as the rays, after paſſing through both, muſt meet in a focus, the exceſs of the refraction muſt be in the convex one: and as the convex is to refract moſt, it appears from the experiment that it muſt be made of crown glaſs; and as the concave is to refract leaſt, it muſt be made of white flint.

And farther, as the refractions of ſpherical glaſſes are in an inverſe ratio of their focal diſtances, it follows that the focal diſtances of the two glaſſes ſhould be in the ratio of the refractions of the wedges; for, being thus proportioned, every ray of light that paſſes through this combined glaſs, at whatever diſtance from its axis, will conſtantly be refracted by the difference between two contrary refractions, in the proportion required; and therefore the effect of the different refrangibility of light will be prevented.

The removal of this impediment, however, produced another: for the two glaſſes, which were thus combined, being ſegments of very deep ſpheres, the aberrations from the ſpherical ſurfaces became ſo conſiderable, as greatly to diſturb the diſtinctneſs of the image. Yet conſidering that the ſurfaces of ſpherical glaſſes admit of great variations, though the focal diſtance be limited, and that by theſe variations their aberration might be made more or leſs at pleaſure; Mr. Dollond plainly ſaw that it was poſſible to make the aberrating of any two glaſſes equal; and that, as in this caſe the refractions of the two glaſſes were contrary to each other, and their aberrations being equal, theſe would deſtroy each other.

Thus he obtained a perfect theory of making object glaſſes, to the apertures of which he could hardly perceive any limits: for if the practice could come up to the theory, they muſt admit of apertures of great extent, and conſequently bear great magnifying powers.

The difficulties of the practice are, however, ſtill very conſiderable. For firſt, the focal diſtances, as well as the particular ſurfaces, muſt be proportioned with the utmoſt accuracy to the denſities and refracting powers of the glaſſes, which vary even in the ſame ſort of glaſs, when made at different times. Secondly, there are four ſurfaces to be wrought perfectly ſpherical.

However, Mr. Dollond could conſtruct refracting teleſcopes upon theſe principles, with ſuch apertures and magnifying powers, under limited lengths, as greatly exceed any that were produced before, in forming the images of objects bright, diſtinct, and uninfected with colours about the edges, through the whole extent of a very large field or compaſs of view; of which he has given abundant and undeniable teſtimony. See Teleſcope.

There has lately appeared in the Gentleman's Magazine (1790, pa. 890) a paper on the refracting teleſcope, by an author who ſigns *Veritus*, in which the invention is aſcribed to another perſon, not heretofore mentioned; in theſe words: "As the invention has been claimed by M. Euler, M. Klingenſtierna, and ſome other foreigners, we ought, for the honour of England, to aſſert our right, and give the merit of the diſcovery to whom it is due; and therefore, without farther preface, I ſhall obſerve, that the inventor was Cheſter More Hall, Eſq. of More-hall, in Eſſex, who, about 1729, as appears by his papers, conſidering the different humours of the eye, imagined they were placed ſo as to correct the different refrangibility of light. He then conceived, that if he could find ſubſtances having ſuch properties as he ſuppoſed theſe humours might poſſeſs, he ſhould be enabled to conſtruct an object glaſs that would ſhew objects colourleſs. After many experiments he had the good fortune to find thoſe properties in two different ſorts of glaſs, and making them diſperſe the rays of light in different directions, he ſucceeded. About 1733 he completed ſeveral achromatic object glaſſes (though he did not give them this name), that bore an aperture of more than $2\frac{1}{2}$ inches, though the focal length did not exceed 20 inches; one of which is now in the poſſeſſion of the Rev. Mr. Smith, of Charlotte Street, Rathbone Place.

This glaſs has been examined by ſeveral gentlemen of eminence and ſcientific abilities, and found to poſſeſs the properties of the preſent achromatic glaſſes.

Mr. Hall uſed to employ the working opticians to grind his lenſes; at the ſame time he finiſhed them with the radii of the ſurfaces, not only to correct the different refrangibility of rays, but alſo the aberration ariſing from the ſpherical figure of the lenſes. Old Mr. Baſs, who at that time lived in Bridewell precinct, was one of theſe working opticians, from whom Mr. Hall's invention ſeems to have been obtained.

In the trial at Weſtminſter-hall about the patent for making achromatic teleſcopes, Mr. Hall was allowed to be the inventor; but Lord Mansfield obſerved, that "It was not the perſon that locked up his invention in his ſcrutoire that ought to profit by a patent for ſuch an invention, but he who brought it forth for the benefit of the public." This, perhaps, might be ſaid with ſome degree of juſtice, as Mr. Hall was a gentleman of property, and did not look to any pecuniary advantage from his diſcovery; and, conſequently, it is very probable that he might not have an intention to make it generally known at that time.

That Mr. Ayſcough, optician on Ludgate Hill, was in poſſeſſion of one of Mr. Hall's achromatic teleſcopes in 1754, is a fact which at this time will not be diſputed."

ACHRONICAL, or *Achronycal.* See Acronychal.

ACOUSTICS. This term, in phyſico-mathematical meaning, ſignifies the doctrine of hearing, and the art of aſſiſting that ſenſe by means of ſpeaking trumpets, hearing trumpets, whiſpering galleries, and ſuch like. See Stentrophonic Tube.

Sturmius, in his Elements of Univerſal Mechanics, treating of Acouſtics, after examining into the nature of ſounds, deſcribes the ſeveral parts of the external and internal ear, and their ſeveral uſes and connexions with each other; and from thence deduces the mechaniſm of hearing: and laſtly, he treats of the means of adding an intenſity of force to the voice and other ſounds; and explains the nature of echoes, otacouſtic tubes, and ſpeaking trumpets. See Sound, Ear, Voice, and Echo.

Dr. Hook, in the preface to his Micrography, aſſerts that the loweſt whiſper, by certain means, may be heard at the diſtance of a furlong; and that he knew a way by which it is eaſy to hear any one ſpeak through a wall of three feet thick; alſo that by means of an extended wire, ſound may be conveyed to a very great diſtance, almoſt in an inſtant.

ACRE, from the Saxon *æcre*, or German *acker*, a *field*, of the Latin *ager*. It is a meaſure of land, containing, by the ordinance for meaſuring land, made in the 33d and 34th of Edward I, 160 perches or ſquare poles of land; that is, 16 in length and 10 in breadth, or in that proportion: and as the ſtatute length of a pole is $5\frac{1}{2}$ yards, or $16\frac{1}{2}$ feet, therefore the acre will contain 4840 ſquare yards, or 43560 ſquare feet. The chain with which land is commonly meaſured, and which was invented by Gunter, is 4 poles or 22 yards in length; and therefore the acre is juſt 10 ſquare chains; that is, 10 chains in length and one in breadth, or in that proportion. Farther, as a mile contains 1760 yards, or 80 chains in length, therefore the ſquare mile contains 640 acres.

The acre, in ſurveying, is divided into 4 roods, and the rood is 40 perches.

The French acre, *arpent*, is equal to $1\frac{1}{4}$ Engliſh acre;
The Straſburg contains about $\frac{1}{2}$ an Engliſh acre;
The Welch acre contains about 2 Engliſh acres;
The Iriſh acre contains 1 ac. 2 r. $19\frac{27}{121}$ p. Engliſh.

Sir William Petty, in his Political Arithmetic, reckons that England contains 39 million acres: but Dr. Greve ſhews, in the Philoſ. Tranſ. N° 330, that England contains not leſs than 46 million acres. Whence he infers that England is above 46 times as large as the province of Holland, which it is ſaid contains but about one million of acres.

By a ſtatute of the 31ſt of Elizabeth, it is ordained, that if any man erect a cottage, he ſhall annex four acres of land to it.

ACRONYCHAL, or Acronycal, in *Aſtronomy*, is ſaid of a ſtar or planet, when it is oppoſite to the ſun. It is from the Greek ακρονυχος, the point or extremity of night, becauſe the ſtar roſe at ſun-ſet, or the beginning of night, and ſet at ſun-riſe, or the end of night; and ſo it ſhone all the night.

The acronychal is one of the three Greek poetic riſings and ſettings of the ſtars; and ſtands diſtinguiſhed from Coſmical and Heliacal. And by means of which, for want of accurate inſtruments, and other obſervations, they might regulate the length of their year.

ACROTERIA, or Acroters, in *Architecture*, ſmall pedeſtals, uſually without baſes, placed on pediments, and ſerving to ſupport ſtatues.

Thoſe at the extremities ought to be half the height of the tympanum; and that in the middle, according to Vitruvius, one eighth part more.

Acroteria alſo are ſometimes uſed to ſignify figures, whether of ſtone or metal, placed as ornaments or crownings, on the tops of temples, or other buildings.

It is alſo ſometimes uſed to denote thoſe ſharp pinacles or ſpiry battlements, that ſtand in ranges about flat buildings, with rails and baluſtres.

ACTION, in *Mechanics* or *Phyſics*, a term uſed to denote, ſometimes the effort which ſome body or power exerts againſt another body or power, and ſometimes it denotes the effects reſulting from ſuch effort.

The Carteſians reſolve all phyſical action into metaphyſical. Bodies, according to them, do not act on one another; the action comes all immediately from the Deity; the motions of bodies, which ſeem to be the cauſe, being only the occaſions of it.

It is one of the laws of nature, that action and reaction are always equal, and contrary to each other in their directions.

Action is either inſtantaneous or continued; that is, either by collition or percuſſion, or by preſſure. Theſe two ſorts of action are heterogeneous quantities, and are not comparable, the ſmalleſt action by percuſſion exceeding the greateſt action of preſſure, as the ſmalleſt ſurface exceeds the longeſt line, or as the ſmalleſt ſolid exceeds the largeſt ſurface: thus, a man by a ſmall blow with a hammer, will drive a wedge below the greateſt ſhip on the ſtocks, or under any other weight; that is, the ſmalleſt percuſſion overcomes the preſſure of the greateſt weight. Theſe actions then cannot be meaſured the one by the other, but each muſt have a meaſure of its own kind, like as ſolids muſt be meaſured by ſolids, and ſurfaces by ſurfaces: time being concerned in the one, but not in the other.

If a body be urged at the ſame time by equal and contrary actions, it will remain at reſt. But if one of theſe actions be greater than its oppoſite, motion will enſue towards the part leaſt urged.

The actions of bodies upon each other, in a ſpace that is carried uniformly forward, are the ſame as if the ſpace were at reſt; and any powers or forces that act upon all bodies, ſo as to produce equal velocities in them in the ſame, or in parallel right lines, have no effect on their mutual actions, or relative motions. Thus the motion of bodies on board of a ſhip that is carried uniformly forward, are performed in the ſame manner as if the ſhip was at reſt. And the motion of the earth about its axis has no effect on the actions of bodies and agents at its ſurface, except in ſo far as it is not uniform and rectilineal. In general, the actions of bodies upon each other, depend not on their abſolute, but relative motion.

Quantity of Action, in Mechanics, a name given by M. de Maupertuis, in the Memoirs of the Academy of Sciences of Paris for 1744, and in thoſe of Berlin for 1746, to the continual product of the maſs of a body, by the ſpace which it runs through, and by its celerity. He lays it down as a general law, that in the changes made in the ſtate of a body, the quantity of action neceſſary to produce ſuch change is the leaſt poſſible. This principle he applies to the inveſtigation of the laws of refraction, and even the laws of reſt, as he calls them; that is, of the equilibrium or equipollency of preſſures; and even to the modes of acting of the Supreme Being. In this way Maupertuis attempts to connect the metaphyſics of final cauſes with the fundamental truths of mechanics; to ſhew the dependence of the colliſion of both elaſtic and hard bodies, upon one and the ſame law, which before had always been referred to ſeparate laws; and to reduce the laws of motion, and thoſe of equilibrium, to one and the ſame principle.

But this quantity of motion, of Maupertuis, which is defined to be the product of the maſs, the ſpace paſſed over, and the celerity, comes to the ſame thing as the maſs multiplied by the ſquare of the velocity, when the ſpace paſſed over is equal to that by which the velocity is meaſured; and ſo the quantity of force will be proportional to the maſs multiplied by the ſquare of the velocity; ſince the ſpace is meaſured by the velocity continued for a certain time.

In the ſame year that Maupertuis communicated the idea of his principle, profeſſor Euler, in the ſupplement to his book, intitled *Methodus inveniendi lineas curvas maximi vel minimi proprietate gaudentes*, demonſtrates, that in the trajectories which bodies deſcribe by central forces, the velocity multiplied by what the foreign mathematicians call the element of the curve, always makes a minimum: which Maupertuis conſidered as an application of his principle to the motion of the planets.

It appears from Maupertuis's Memoir of 1744, that it was his reflections on the laws of refractions, that led him to the theorem above mentioned. The principle which Fermat, and after him Leibnitz, made uſe of, in accounting for the laws of refraction, is ſufficiently known. Thoſe mathematicians pretended, that a particle of light, in its paſſage from one point to another, through two mediums, in each of which it moves with a different velocity, muſt do it in the ſhorteſt time poſſible: and from this principle they have demonſtrated geometrically, that the particle cannot go from the one point to the other in a right line; but being arrived at the ſurface that ſeparates the two mediums, it muſt alter its direction in ſuch a manner, that the ſine of its incidence ſhall be to the ſine of its refraction, as its velocity in the firſt medium is to its velocity in the ſecond: whence they deduced the well known law of the conſtant ratio of thoſe ſines.

This explanation, though very ingenious, is liable to this preſſing difficulty, namely, that the particle muſt approach towards the perpendicular, in that medium where its velocity is the leaſt, and which conſequently reſiſts it the moſt: which ſeems contrary to all the mechanical explanations of the refraction of bodies, that have hitherto been advanced, and of the refraction of light in particular.

Sir Iſaac Newton's way of accounting for it, is the moſt ſatisfactory of any that has hitherto been offered, and gives a clear reaſon for the conſtant ratio of the ſines, by aſcribing the refraction to the attractive force of the mediums; from which it follows, that the denſeſt mediums, whoſe attraction is the ſtrongeſt, ſhould cauſe the ray to approach the perpendicular; a fact confirmed by experiment. But the attraction of the medium could not cauſe the ray to approach towards the perpendicular, without increaſing its velocity; as may eaſily be demonſtrated. Thus then, according to Newton, the refraction muſt be towards the perpendicular, when the velocity is increaſed: contrary to the law of Fermat and Leibnitz.

Maupertuis has attempted to reconcile Newton's explanation with metaphyſical principles. Inſtead of ſuppoſing, as the aforeſaid gentlemen do, that a particle of light proceeds from one point to another in the ſhorteſt time poſſible; he contends that a particle of light paſſes from one point to another in ſuch a manner, that the quantity of action ſhall be the leaſt poſſible. This quantity of action, ſays he, is a real expence, in which nature is always frugal. In virtue of this philoſophical principle he diſcovers, that not only the ſines are in a conſtant ratio, but alſo that they are in the inverſe ratio of the velocities, according to Newton's explanation, and not in the direct ratio, as had been pretended by Fermat and Leibnitz.

It is remarkable that, of the many philoſophers who have written on refraction, none ſhould have fallen upon ſo ſimple a manner of reconciling metaphyſics with mechanics; ſince no more is neceſſary to that, than making a ſmall alteration in the calculus founded upon Fermat's principle. Now according to that principle, the time, that is, the ſpace divided by the velocity, ſhould be a minimum; ſo that calling the ſpace run through in the firſt medium S, with the velocity V, and the ſpace run through in the ſecond medium s, with the velocity v, we ſhall have $\frac{S}{V} + \frac{s}{v} = a$ minimum; that is to ſay, $\frac{\dot{S}}{V} + \frac{\dot{s}}{v} = 0$, or $\frac{\dot{S}}{V} = -\frac{\dot{s}}{v}$. Now it is eaſy to perceive, that the ſines of incidence and refraction are to each other, as $\dot{S}$ to $-\dot{s}$; whence it follows, that thoſe ſines are in the direct ratio of the velocities V, v; which is exactly what Fermat makes it to be. But in order to have thoſe ſines to be in the inverſe ratio of the velocities, it is only ſuppoſing $V\dot{S} + v\dot{s} = 0$; which gives $SV + sv$ or $S \times V + s \times v$ a minimum: which is Maupertuis's principle.

In the Memoirs of the Academy of Berlin, above cited, may be ſeen all the other applications which Maupertuis has made of this principle. And whatever may be determined as to his metaphyſical baſis of it, as alſo to the idea he has annexed to the quantity of action, it will ſtill hold good, that the product of the ſpace by the velocity is a minimum in ſome of the moſt general laws of nature.

ACTIVE, the quality of an agent, or of communicating motion or action to ſome body. In this ſenſe the word ſtands oppoſed to paſſive: thus we ſay an active cauſe, active principle, &c.

Sir Iſaac Newton ſhews that the quantity of motion in the world muſt be always decreaſing, in conſequence of the vis inertiæ, &c. So that there is a neceſſity for certain active principles to recruit it: ſuch he takes the cauſe of gravity to be, and the cauſe of fermentation; adding, that we ſee but little motion in the univerſe, except what is owing to theſe active principles.

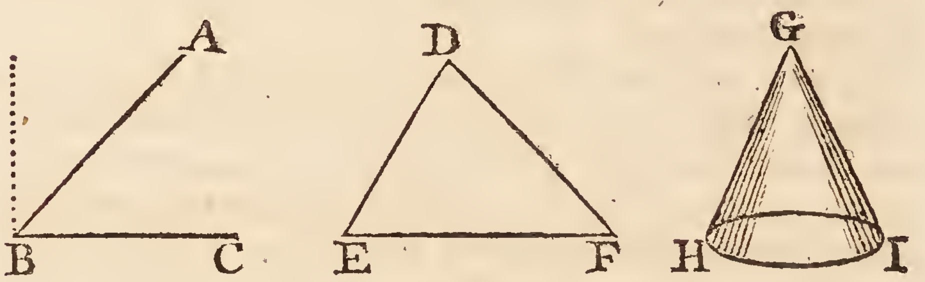
ACTIVITY, the virtue or faculty of acting. As the activity of an acid, a poiſon, &c: the activity of fire exceeds all imagination.

According to Sir Iſaac Newton, bodies derive their activity from the principle of attraction.

Sphere of ACTIVITY, is the ſpace which ſurrounds a body, as far as its efficacy or virtue extends to produce any ſenſible effect. Thus we ſay, the ſphere of activity of a loadſtone, of an electric body, &c.

ACUBENE, in *Aſtronomy*, the Arabic name of a ſtar of the fourth magnitude, in the ſouthern claw of Cancer, marked α by Bayer. Its longitude for 1761, ♌ 10° 18′ 9″, ſouth latitude 5° 5′ 56″.

ACUTE, or ſharp; a term oppoſed to obtuſe. Thus, ACUTE *Angle*, in *Geometry*, is that which is leſs than a right angle; and is meaſured by leſs than 90°, or by leſs than a quadrant of a circle. As the angle A B C.



ACUTE *angled Triangle*, is that whoſe three angles are all acute; and is otherwiſe called an oxygenous triangle. As the triangle D E F.

ACUTE-*angled Cone*, is that whoſe oppoſite ſides make an acute angle at the vertex, or whoſe axis, in a right cone, makes leſs than half a right angle with the ſide As the cone G H I.

Pappus, in his Mathematical Collections, ſays, this name was given to ſuch a cone by Euclid and the ancients, before the time of Apollonius. And they called an

ACUTE-*angled Section of a Cone*, an Ellipſis, which was made by a plane cutting both ſides of an acute-angled cone: not knowing that ſuch a ſection could be generated from any cone whatever, till it was ſhewn by Apollonius.

ACUTE, in *Muſic*, is underſtood of a tone, or ſound, which is high, ſharp, or ſhrill, in reſpect of ſome other: in which ſenſe the word ſtands oppoſed to grave. And both theſe ſounds are independent of loudneſs or force: ſo that the tone may be acute or high, without being loud; and loud without being high or acute. For both the affections of acute and grave, depend intirely on the quickneſs or ſlowneſs of the vibrations by which they are produced.

Sounds conſidered as grave and acute, that is, in the relation of gravity and acuteneſs, conſtitute what is called tune, the foundation of all harmony.

ADAGIO, in *Muſic*, one of the terms uſed by the Italians to expreſs a degree or diſtinction of time.

Adagio denotes the ſloweſt time except grave.

Sometimes the word is repeated, as *adagio, adagio*, to denote a ſtill ſlower time than the former.

Adagio alſo ſignifies a ſlow movement, when uſed ſubſtantively.

ADAMAS, in *Aſtrology*, a name given to the moon.

ADAR, in the Hebrew *Chronology*, is the 6th month of their civil year, but the 12th of their eccleſiaſtical year. It contains only 29 days; and it anſwers to our February; but ſometimes entering into the month of March, according to the courſe of the moon.

ADDITION, the uniting or joining of two or more things together; or the finding of one quantity equal to two or more others taken together.

ADDITION, in *Arithmetic*, is the firſt of the four fundamental rules or operations of that ſcience; and it conſiſts in finding a number equal to ſeveral others taken together, or in finding the moſt ſimple expreſſion of a number according to the eſtabliſhed notation. The quantity ſo found equal to ſeveral others taken together, is named their ſum.

The ſign or character of addition is +, and is called *plus*. This character is ſet between the quantities to be added, to denote their ſum: thus, $3 + 6 = 9$, that is, 3 plus 6 are equal to 9; and $2 + 4 + 6 = 12$, that is, 2 plus 4 plus 6 are equal to 12.

Simple numbers are either added as above; or elſe by placing them under one another, as in the margin, and adding them together, one after another, beginning at the bottom: thus 2 and 4 make 6, and 6 make 12.

$$\begin{array}{r} 6\\ 4\\ 2\\ \hline 12\\ \hline \end{array}$$

Compound numbers, or numbers conſiſting of more figures than one, are added, by firſt ranging the numbers in columns under each other, placing always the numbers of the ſame denomination under each other, that is, units under units, tens under tens, and ſo on; and then adding up each column ſeparately, beginning at the right hand, ſetting down the ſum of each column below it, unleſs it amount to ten or ſome number of tens, and in that caſe ſetting down only the overplus, and carrying one for each ten to the next column. Thus, to add 451 and 326,

$$\left.\begin{array}{r} 451\\ 326 \end{array}\right\}\text{ that is }\left\{\begin{array}{l} 400 + 50 + 1\\ 300 + 20 + 6 \end{array}\right.$$

$$\text{Sum } 777 \quad = \quad 700 + 70 + 7$$

Alſo to add the numbers $329 + 1562 + 20347 + 712048$; ſet them down as in the margin, and beginning at the loweſt number on the right hand, ſay 8 and 7 make 15, and 2 make 17, and 9 make 26; ſet down 6, and carry 2 to the next column, ſaying 2 and 4 make 6, and 4 make 10, and 6 make 16, and 2 make 18; ſet down 8, and carry 1, ſaying 1 and 3 make 4, and 5 make 9, and 3 make 12; ſet down 2, and carry 1, ſaying 1 and 2 make 3, and 1 make 4, which ſet down; then 1 and 2 make 3; and 7 is 7 to ſet down: ſo the ſum of all together is 734286. Or it is the ſame as the ſums of the columns ſet under one another, as in the margin, and then theſe added up in the ſame manner.

$$\begin{array}{r} 329\\ 1562\\ 20347\\ 712048\\ \hline 734286\\ \hline 26\\ 16\\ 11\\ 3\\ 3\\ 7\\ \hline 734286\\ \hline \end{array}$$

When a great number of ſeparate ſums or numbers are to be added, as in long accounts, it is eaſier to break or ſeparate them into two or more parcels, which are added up ſeverally, and then their ſums added together for the total ſum. And thus alſo the truth of the addition may be proved, by dividing the numbers into parcels different ways, as the totals muſt be the ſame in both caſes when the operation is right.

Another method of proving addition was given by Dr. Wallis, in his Arithmetic, publiſhed 1657, by caſting out the nines, which method of proof extends alſo to the other rules of arithmetic. The method is this: add the figures of each line of numbers together ſeverally, caſting out always 9 from the ſums as they ariſe in ſo adding, adding the overplus to the next figure, and ſetting at the end of each line what is over the nine or nines; then do the ſame by the ſum-total, as alſo by the former exceſſes of 9, ſo ſhall the laſt exceſſes be equal when the work is right. So the former example will be proved as below:

$$\begin{array}{r|c|c} 329 & & 5\\ 1562 & & 5\\ 20347 & \text{Exceſs of 9's} & 7\\ 712048 & & 4\\ \cline{1-1}\cline{3-3} 734286 & & 3\\ \hline \end{array}$$

When the numbers are of different denominations; as pounds, ſhillings, and pence; or yards, feet, and inches; place the numbers of the ſame kind under one another, as pence under pence, ſhillings under ſhillings, &c; then add each column ſeparately, and carry the overplus as before, from one column to another. As in the following examples:

l.	s.	d.		Yards.	Feet.	Inches.
271	12	3		271	10	3
94	14	7		36	2	7
42	5	10		14	2	5
408	12	8	ſums	323	0	3

ADDITION *of Decimals*, is performed in the ſame manner as that of whole numbers, placing the numbers of the ſame denomination under each other, in which caſe the decimal ſeparating points will range ſtraight in one column; as in this example, to add together theſe numbers $371{\cdot}0496 + 25{\cdot}213 + 1{\cdot}704 + 924{\cdot}61 + {\cdot}0962$.

$$\begin{array}{r@{\cdot}l} 371 & 0496\\ 25 & 213\\ 1 & 704\\ 924 & 61\\ & 0962\\ \hline \text{The ſum } 1322 & 6728\\ \hline \end{array}$$

ADDITION *of Vulgar Fractions*, is performed by bringing all the propoſed fractions to a common denominator, if they have different ones, which is an indiſpenſable preparation; then adding all the numerators together, and placing their ſum over the common denominator for the ſum total required.

So $\frac{3}{7} + \frac{2}{7} = \frac{5}{7}$. And $\frac{1}{5} + \frac{2}{5} + \frac{3}{5} = \frac{6}{5} = 1\frac{1}{5}$.
And $\frac{1}{2} + \frac{2}{3} = \frac{3}{6} + \frac{4}{6} = \frac{7}{6} = 1\frac{1}{6}$.
And $\frac{5}{6} + \frac{2}{3}$ of $\frac{3}{4} = \frac{5}{6} + \frac{3}{6} = \frac{8}{6} = \frac{4}{3} = 1\frac{1}{3}$.
And $\frac{1}{2}$ of $\frac{3}{4} + \frac{2}{3}$ of $5 = \frac{3}{8} + \frac{10}{3} = \frac{9}{24} + \frac{80}{24} = \frac{89}{24} = 3\frac{17}{24}$.

ADDITION *in Algebra*, or the addition of indeterminate quantities, denoted by letters of the alphabet, is performed by connecting the quantities together by their proper ſigns, and uniting or reducing ſuch as are ſuſceptible of it, namely ſimilar quantities, by adding their co-efficients together if the ſigns are the ſame, but ſubtracting them when different. Thus the quantity a added to the quantity b, makes $a + b$; and a joined with $-b$, makes $a - b$; alſo $-a$ and $-b$ make $-a-b$; and $3a$ and $5a$ make $3a + 5a$ or $8a$, by uniting the ſimilar numbers 3 and 5 to make 8.

$$\begin{array}{lccccccc} \text{Thus alſo} & 3a & + & 2bc & - & 4a^2b & + & c^3 \\ \text{added to} & 2a & - & 2bc & + & 3a^2b & + & 2c^3 \\ \hline \text{makes} & 5a & & 0 & - & a^2b & + & 3c^3 \\ \hline \end{array}$$

Alſo $\frac{4a}{3c} + \frac{2a}{3c} = \frac{6a}{3c} = \frac{2a}{c}$;

and $3\sqrt{ac} + 5\sqrt{ac} - 1\frac{1}{2}\sqrt{ac} = 6\frac{1}{2}\sqrt{ac}$;

and $a\sqrt{ac} + b\sqrt{ac} = \overline{a+b}\sqrt{ac}$ or $(a+b)\sqrt{ac}$;

and $5\sqrt{a^2-x^2} + 7\sqrt{a^2-x^2} = 12\sqrt{a^2-x^2}$;

and $6\sqrt{3} + 4\sqrt{3} - 2\sqrt{3} = 8\sqrt{3}$;

and $\frac{\overline{3a+7}\sqrt{2x}}{a+x} + \frac{\overline{7-3a}\sqrt{2x}}{a+x} = \frac{14\sqrt{2x}}{a+x}$;

and $\frac{8a\sqrt{ax}}{a+2c} + \frac{16c\sqrt{ax}}{a+2c} = \frac{\overline{8a+16c}\sqrt{ax}}{a+2c} = 8\sqrt{ax}$;

and $-\frac{3ax}{2c} - \frac{5ax}{2c} = -\frac{8ax}{2c} = -\frac{4ax}{c}$;

and $-a\sqrt{x} - b\sqrt{x} = -(a+b)\sqrt{x}$;

and $+3 - 2 = 1$; and $-3 + 2 = -1$;

and $\frac{7ax}{b} - \frac{4ax}{c} = \frac{7acx}{bc} - \frac{4abx}{bc} = (7c-4b)\frac{ax}{bc}$.

In the addition of ſurd or irrational quantities, they muſt be reduced to the ſame denomination, or to the ſame radical, if that can be done; then add or unite the rational parts, and ſubjoin the common ſurd. Otherwiſe connect them with their own ſigns.

So $\sqrt{8} + \sqrt{18} = \sqrt{4 \times 2} + \sqrt{9 \times 2} = 2\sqrt{2} + 3\sqrt{2} = 5\sqrt{2} = \sqrt{50}$;

and $\sqrt{12} + \sqrt{17} = \sqrt{4 \times 3} + \sqrt{9 \times 3} = 2\sqrt{3} + 3\sqrt{3} = 5\sqrt{3} = \sqrt{75}$;

and $\sqrt{50} + \sqrt{75} = \sqrt{25 \times 2} + \sqrt{25 \times 3} = 5\sqrt{2} + 5\sqrt{3} = (\sqrt{2} + \sqrt{3})\,5$;

but of $\sqrt{5}$ and $\sqrt{6}$ the ſum is ſet down $\sqrt{5} + \sqrt{6}$, becauſe the terms are incommenſurable, and not reducible to a common ſurd.

ADDITION *of Logarithms*. See Logarithms.

ADDITION *of Ratios*, the ſame as compoſition of ratios; which ſee.

ADDITIVE, denotes ſomething to be added to another, in contradiſtinction to ſomething to be taken away or ſubtracted. So aſtronomers ſpeak of additive equations, and geometricians of additive ratios.

ADELARD, or ATHELARD, was a learned monk of Bath, in England, who flouriſhed about the year 1130, as appears by ſome manuſcripts of his in Corpus Chriſti, and Trinity Colleges, Oxford. Voſſius ſays he was univerſally learned in all the ſciences of his time; and that, to acquire all ſorts of knowledge, he travelled into France, Germany, Italy, Spain, Egypt, and Arabia. He wrote many books himſelf, and tranſlated others from different languages: thus, he tranſlated, from Arabic into Latin, Euclid's Elements, at a time before any Greek copies had been diſcovered; alſo Erichiafarim, upon the ſeven planets. He wrote a book on the ſeven liberal arts, another on the aſtrolabe, another on the cauſes of natural compoſitions, beſides ſeveral on phyſics and on medicine.

Although Voſſius refers to Oxford for ſome of theſe manuſcripts, it would yet ſeem they were not to be found there in Wallis's time; for the Doctor, ſpeaking of this author, and other Engliſh authors and travellers about the ſame age, ſays, "A particular account of theſe travels of Shelley and Morley was a while ſince to be ſeen in two prefaces to two manuſcript books of theirs in the library of Corpus-Chriſti College in Oxford, but hath lately (by ſome unknown hand) been cut out, and carried away; which prefaces (one or both of them) did alſo make mention of the travels of Athelardus Bathonienſis, and are, to that purpoſe, cited by Voſſius out of that manuſcript copy. Whoever hath them, would do a kindneſs (by ſome way or other) to reſtore them, or at leaſt a copy of them." Wallis's Algebra, pa. 6.

ADELM, ALDHELMUS, or ALTHELMUS, a learned Engliſhman, who flouriſhed about the year 680. He was firſt abbot of Malmſbury, and afterward biſhop of Shirburn. He died in the year 709, in the monaſtery of Malmſbury.

Adelm was the ſon of Kenred or Kenten, who was the brother of Ina, king of the Weſt Saxons in England. Beſide certain books in theology, he compoſed ſeveral on the mathematical ſciences &c; as Arithmetic, and Aſtrology, and librum de philoſophorum diſciplinis. See Bede's Hiſtory, lib. 5. cap. 19. He is alſo mentioned by Bale and William of Malmſbury.

ADERAIMIN, or ALDERAIMIN, the Arabic name of a ſtar of the third magnitude, in the left ſhoulder of Cepheus, marked α by Bayer. Its longitude for 1761, ♈ 9° 30′ 8″ north latitude 68° 56′ 20″.

ADFECTED, ſee AFFECTED.

ADHESION, ADHERENCE, in *Phyſics*, is the ſtate of two bodies, joined or faſtened together, whether by mutual attraction, the interpoſition of their own parts, or the impulſe or preſſure of external bodies. See COHESION.

Thus two hollow hemiſpheres, exhauſted of air, are made to adhere firmly together by the preſſure of the atmoſphere on their convex or external ſurfaces; for if they are introduced into an exhauſted receiver, they preſently fall aſunder. Alſo two very well poliſhed planes adhere firmly together, partly by the external preſſure of the atmoſphere, and partly by the attraction of their parts.

In No. 389 of the Philoſ. Tranſ. Dr. Deſaguliers has given experiments of the adheſion of leaden bullets to each other: the cauſe of which he reſolves into the principle of attraction.

M. Muſſchenbroeck, in his Eſſai de Phyſique, has given a great many remarks on the adheſion of bodies, and relates various experiments which he had made upon this matter, but chiefly relative to the reſiſtance made by bodies to fracture, in virtue of the adheſion of their parts; which adheſion he aſcribes principally to their mutual attraction. Common experiments prove the mutual adheſion of the parts of water to each other, as well as to the bodies they touch. The ſame may be ſaid of the particles of air, on which M. Petit has a memoir among thoſe of the Paris Academy of Sciences for the year 1731.

Some authors however are not willing to admit that the adheſion of the parts of water, or indeed of bodies in general, is to be attributed to the attraction of their parts; and they reaſon thus: ſuppoſe, ſay they, that attraction acts at any ſmall diſtance, as for example to the diſtance of one-tenth of an inch, from a particle of water: and about this particle deſcribe a circle whoſe radius is one-tenth of an inch: then the particle of water will be attracted only by the particles included within the circle; but as theſe particles act in contrary directions, their mutual effects muſt deſtroy one another, and there can be no attraction of the particle, ſince it will have no more tendency one way than another.

ADHIL, in *Aſtronomy*, a ſtar, of the ſixth magnitude, upon the garment of Andromeda, under the laſt ſtar in her foot.

ADJACENT, whatever lies immediately by the ſide of another.

ADJACENT *Angle*, in *Geometry*, is ſaid of an angle when it is immediately contiguous to another, ſo that they have both one common ſide. And the term is more particularly uſed when the two angles have not only one common ſide, but alſo when the other two ſides form one continued right line.

ADJACENT *bodies*, *in Phyſics*, are underſtood of thoſe that are near, or next to, ſome other body.

ADJUTAGE, or rather AJUTAGE; which ſee.

ADSCRIPTS, in *Trigonometry*, is uſed by ſome mathematicians, for the tangents of arcs. Vieta calls them alſo proſines.

ADVANCE-FOSSE, in Fortification, a ditch thrown round the eſplanade or glacis of a place, to prevent its being ſurpriſed by the beſiegers.

The ditch ſometimes made in that part of the lines or retrenchments neareſt the enemy, to prevent him from attacking them, is alſo called the advance-foſſe.

The advance-foſſe ſhould always be full of water, otherwiſe it will ſerve to cover the enemy from the fire of the place, if he ſhould become maſter of the foſſe. Beyond the advance-foſſe it is uſual to conſtruct lunettes, redouts, &c.

ADVENT, *Adventus*, in the Calendar, the time immediately preceding Chriſtmas; and was anciently employed in pious preparation for the *adventus*, or coming on, of the feaſt of the Nativity. Advent includes four Sundays, or weeks; commencing either with the Sunday which falls on St. Andrew's day, namely the 30th day of November, or the neareſt Sunday to that day, either before or after.

ÆOLIPILE, *Æolipile*, in Hydraulics, a hollow ball of metal, with a very ſmall hole or opening; chiefly uſed to ſhew the convertibility of water into elaſtic ſteam. The beſt way of fitting up this inſtrument, is with a very ſlender neck or pipe, to ſcrew on and off, for the convenience of introducing the water into the inſide; for by unſcrewing the pipe, and immerging the ball in water, it readily fills, the hole being pretty large; and then the pipe is ſcrewed on. But if the pipe do not ſcrew off, its orifice is too ſmall to force its way in againſt the included air; and therefore to expel moſt of the air, the ball is heated red hot, and ſuddenly plunged with its orifice into water, which will then ruſh in till the ball is about two-thirds filled with the water. The water having been introduced, the ball is ſet upon the fire, which gradually heats the contained water, and converts it into elaſtic ſteam, which ruſhes out by the pipe with great violence and noiſe; and thus continues till all the water is ſo diſcharged; though not with a conſtant and uniform blaſt, but by fits: and the ſtronger the fire is, the more elaſtic will the ſteam be, and the force of the blaſt. Care ſhould be taken that the ball be not ſet upon a violent fire with very little water in it, and that the ſmall pipe be not ſtopped with any thing; for in ſuch caſe, the included elaſtic ſteam will ſuddenly burſt the ball with a very dangerous exploſion.

This inſtrument was known to the ancients, being mentioned by Vitruvius, lib. 1. cap. 6. It is alſo treated of, or mentioned, by ſeveral modern authors, as Deſcartes, in his Meteor. cap. 4; and Father Merſennus, in prop. 29 Phædom. Pneumat. uſes it to weigh the air, by firſt weighing the inſtrument when red hot, and having no water in it; and afterwards weighing the ſame when it becomes cold. But the concluſion gained by this means, cannot be quite accurate, as there is ſuppoſed to be no air in the ball when it is red hot; whereas it is ſhewn by Varenius, in his Geography, cap. 19, ſect. 6, prop. 10, that the air is rarefied but about 70 times; and conſequently the weight obtained by the above proceſs, will be about one-70th too ſmall, or more or leſs according to the intenſity of the heat.

In Italy it is ſaid that the Æolipile is often uſed to cure ſmoaky chimneys: for being hung over the fire, the blaſt ariſing from it carries up the loitering ſmoke along with it.

And ſome have imagined that the æolipile might be employed as bellows to blow up a fire, having the blaſt from the pipe directed into the fire: but experience would ſoon convince them of their miſtake; for it would rather blow the fire *out* than *up*, as it is not air, but rarefied water, that is thus violently blown through the pipe.

ÆOLUS, in Mechanics, a ſmall portable machine, not long ſince invented by Mr. Tidd, for refreſhing and changing the air in rooms which are made too cloſe.

The machine is adapted to ſupply the place of a ſquare of glaſs in a ſaſh-window, where it works with little or no noiſe, on the principle of the ſails of a mill, or a ſmoke-jack; and thus admitting an agreeable quantity of air, at a convenient part of the room.

ÆOLUS's *Harp*, or *Æolian Harp*, an inſtrument ſo named, from its producing an agreeable melody, merely by the action of the wind.

Neither the age nor inventor of this inſtrument are very well known. It is not mentioned by Merſennus in his Harmonics, where he deſcribes moſt ſorts of muſical inſtruments: and yet the deſcription and uſe of it was given ſoon after, by Kircher, in his book, Magia Phenotactica & Phonurgia.

The conſtruction of this inſtrument is thus; let a box be made of as thin deal as poſſible, its length anſwering exactly to the width of the window in which it is to be placed; five or ſix inches deep, and ſeven or eight inches wide. Acroſs the top, and near each end, glue on a bit of wainſcot, about half an inch high, and a quarter of an inch thick, to ſerve as two bridges for the ſtrings to be ſtretched over, by means of pins inſerted into holes a little behind the bridges, nearer the ends, half the number being at one end, and half at the other end: theſe pins are like thoſe of a harpſichord; and for their better ſupport in the thin deal, a piece of beech of about an inch ſquare, and length equal to the breadth of the box, is glewed on the inſide of the lid, immediately under the place of the pins, the holes for receiving them being bored through this piece. It is ſtrung with ſmall catgut, or blue firſt fiddle ſtrings, more or leſs at pleaſure, on the outſide and lengthways of the lid, fixing one end to one of the ſmall pins, and twiſting the other end about the oppoſite or ſtretching pin. A couple of ſound-holes are cut in the lid; and the thinner this is, the better will be the performance.

When the ſtrings are tuned uniſon, and the inſtrument placed, with the top or ſtringed ſide outwards, in the window to which it is fitted, the air blowing upon that window, the inſtrument will give a ſound like a diſtant choir, increaſing or decreaſing according to the ſtrength of the wind.

ÆRA, in Chronology, is the ſame as epoch, or epocha, and means a fixed point of time, from which to begin a computation of the years enſuing.

The word is ſometimes alſo written *era* in ancient authors Its origin is conteſted, though it is generally ſuppoſed that it had its riſe in Spain. Some imagine that it is formed from *a. er. a.* the abbreviations of the words, *annus erat Auguſti*, or from *a. e. r. a.* the initials of the words *annus erat regni Auguſti*, becauſe the Spaniards began their computation from the time that their country came under the dominion of Auguſtus. Others derive it from *æs*, *braſs*, the tribute money with which Auguſtus taxed the world. It is alſo ſaid that *æra* originally ſignified a number ſtamped on money to determine its current value. And that the ancients uſed *æs* or *æra* as an article, as we do the word *item*, to each particular of an account; and hence it came to ſtand for a ſum or number itſelf.

ÆRA alſo means the way or mode of accounting time. Thus we ſay ſuch a year of the Chriſtian æra, &c.

Spaniſh ÆRA, otherwiſe called the year of Cæſar, was introduced after the ſecond diviſion of the Roman provinces, between Auguſtus, Anthony, and Lepidus, in the 714th year of Rome, the 4676th year of the Julian period, and the 38th year before Chriſt. In the 447th year of this æra, the Alani, the Vandals, Suevi, &c, entered Spain. It is frequently mentioned in the Spaniſh affairs; their councils, and other public acts, being all dated according to it. Some ſay it was aboliſhed under Peter IV, king of Arragon, in the year of Chriſt 1358, and the Chriſtian æra introduced inſtead of it. But Mariana obſerves that it ceaſed in the year of Chriſt 1383, under John I, king of Caſtile. The like was afterwards done in Portugal.

Chriſtian ÆRA. It is generally allowed by Chronologers, that the computation of time from the birth of Chriſt, was only introduced in the ſixth century in the reign of Juſtinian; and it is commonly aſcribed to Dionyſius Exiguus. This æra came then into uſe in deeds, and ſuch like; before which time either the olympiads, the year of Rome, or that of the reign of the emperors, was uſed for ſuch purpoſes.

See an account of the other principal æras under the word Epoch.

AERIAL *Perſpective*, is that which repreſents bodies diminiſhed and weakened, in proportion to their diſtance from the eye.

Aërial Perſpective chiefly reſpects the colours of objects, whoſe force and luſtre it diminiſhes more or leſs, to make them appear as if more or leſs remote.

It is founded upon this, that the longer the column of air an object is ſeen through, the more feebly do the viſual rays emitted from it affect the eye.

AEROGRAPHY, a deſcription of the air, or atmoſphere, its limits, dimenſions, properties, &c.

AEROLOGY, the doctrine or ſcience of the air, and its phænomena, its properties, good and bad qualities, &c. It is much the ſame with the foregoing word, Aerography.

AEROMETRY, *Aerometria*, the ſcience of meaſuring the air, its powers and properties; comprehending not only the quantity of the air itſelf, as a fluid body, but alſo its preſſure or weight, its elaſticity, rarefaction, condenſation, &c.

The term is not much uſed at preſent; this branch of natural philoſophy being uſually called pneumatics, which ſee. Wolfius, late profeſſor of mathematics at Hall, having reduced ſeveral properties of the air to geometrical demonſtrations, firſt publiſhed at Leipſic his Elements of Aerometry, in the German language, and afterwards more enlarged in Latin, which have ſince been inſerted in his *Curſus Mathematicus*, in five volumes in 4to.

AERONAUTICA, the pretended art of ſailing through the air, or atmoſphere, in a veſſel, ſuſtained as a ſhip in the ſea.

AEROSTATICA, is properly the doctrine of the weight, preſſure, and balance of the air and atmoſphere.

AEROSTATION, in its proper and primary ſenſe, denotes the ſcience of weights ſuſpended in the air; but in the modern application of the term, it ſignifies the art of navigating or floating in the air, both as to the practice and principles of it. Hence alſo the machines which are employed for this purpoſe, are called *aeroſtats*, or *aeroſtatic* machines; and which, on account of their round and bell-like ſhape, are otherwiſe called *air balloons.* Alſo *aeronaut* is the name given to the perſon who navigates or floats in the air by means of ſuch machines.

Principles of Aerostation. The fundamental principles of this art have been long and generally known, as well as ſpeculations on the theory of it; but the ſucceſsful application of them to practice ſeems to be altogether a modern diſcovery. Theſe principles chiefly reſpect the weight or preſſure, and elaſticity of the air, with its ſpecific gravity, and that of the other bodies to be raiſed or floated in it: the particular detail of which principles may be ſeen under the reſpective words in this dictionary. Suffice it therefore in this place to obſerve, that any body which is ſpecifically, or bulk for bulk, lighter than the atmoſphere, or air encompaſſing the earth, will be buoyed up by it, and aſcend, like as wood, or a cork, or a blown bladder, aſcends in water. And thus the body would continue to aſcend to the top of the atmoſphere, if the air were every where of the ſame denſity as at the ſurface of the earth. But as the air is compreſſible and elaſtic, its denſity decreaſes continually in aſcending, on account of the diminiſhed preſſure of the ſuperincumbent air, at the higher elevations above the earth; and therefore the body will aſcend only to ſuch height where the air is of the ſame ſpecific gravity with itſelf; where the body will float, and move along with the wind or current of air, which it may meet with at that height. This body then is an aeroſtatic machine, of whatever form or nature it may be. And an air-balloon is a body of this kind, the whole maſs of which, including its covering and contents, and the weights annexed to it, is of leſs weight than the ſame bulk of air in which it riſes.

We know of no ſolid bodies however that are light enough thus to aſcend and float in the atmoſphere; and therefore recourſe muſt be had to ſome fluid or aeriform ſubſtance.

Among theſe, that which is called inflammable air is the moſt proper of any that have hitherto been diſcovered. It is very elaſtic, and from ſix to ten or eleven times lighter than common atmoſpheric air at the ſurface of the earth, according to the different methods of preparing it. If therefore a ſufficient quantity of this kind of air be incloſed in any thin bag or covering, the weight of the two together will be leſs than the weight of the ſame bulk of common air; and conſequently this compound maſs will riſe in the atmoſphere, and continue to aſcend till it attain a height at which the atmoſphere is of the ſame ſpecific gravity as itſelf; where it will remain or float with the current of air, as long as the inflammable air does not eſcape through the pores of its covering. And this is an inflammable air-balloon.

Another way is to make uſe of common air, rendered lighter by warming it, inſtead of the inflammable air. Heat, it is well known, rarefies and expands common air, and conſequently leſſens its ſpecific gravity; and the diminution of its weight is proportional to the heat applied. If therefore the air, incloſed in any kind of a bag or covering, be heated, and conſequently dilated, to ſuch a degree, that the exceſs of the weight of an equal bulk of common air, above the weight of the heated air, be greater than the weight of the covering and its appendages, the whole compound maſs will aſcend in the atmoſphere, till, by the diminiſhed denſity of the ſurrounding air, the whole become of the ſame ſpecific gravity with the air in which it floats; where it will remain till, by the cooling and condenſation of the included air, it ſhall gradually contract and deſcend again, unleſs the heat is renewed or kept up. And ſuch is a heated air-balloon, otherwiſe called a Montgolfier, from its inventor.

Now it has been diſcovered, by various experiments, that one degree of heat, according to the ſcale of Fahrenheit's thermometer, expands the air about one five-hundredth part; and therefore that it will require about 500 degrees, or nearer 484 degrees of heat, to expand the air to juſt double its bulk. Which is a degree of heat far above what it is practicable to give it on ſuch occaſions. And therefore, in this reſpect, common air heated, is much inferior to inflammable air, in point of levity and uſefulneſs for aeroſtatic machines.

Upon ſuch principles then depends the conſtruction of the two ſorts of air-balloons. But before treating of this branch more particularly, it will be proper to give a ſhort hiſtorical account of this late-diſcovered art.

Hiſtory of Aerostation. Various ſchemes for riſing in the air, and paſſing through it, have been deviſed and attempted, both by the ancients and moderns, and that upon different principles, and with various ſucceſs. Of theſe, ſome attempts have been upon mechanical principles, or by virtue of the powers of mechaniſm: and ſuch are conceived to be the inſtances related of the flying pigeon made by Archytas, the flying eagle and fly by Regiomontanus, and various others. Again, other projects have been formed for attaching wings to ſome part of the body, which were to be moved either by the hands or feet, by the help of mechanical powers; ſo that ſtriking the air with them, after the manner of the wings of a bird, the perſon might raiſe himſelf in the air, and tranſport himſelf through it, in imitation of that animal. But of theſe and various other devices of the like nature, a particular account will be given under the article *artificial flying*, as belonging rather to that ſpecies or principle of motion, than to our preſent ſubject of aeroſtation, which is properly the ſailing or floating in the air by means of a machine rendered ſpecifically lighter than that element, in imitation of aqueous navigation, or the ſailing upon the water in a ſhip, or veſſel, which is ſpecifically lighter than the water.

The firſt rational account that we have upon record, for this ſort of ſailing, is perhaps that of our countryman Roger Bacon, who died in the year 1292. He not only affirms that the art is feaſible, but aſſures us that he himſelf knew how to make an engine, in which a man ſitting might be able to carry himſelf through the air like a bird; and he farther affirms that there was another perſon who had tried it with ſucceſs. And the ſecret it ſeems conſiſted in a couple of large thin ſhells, or hollow globes, of copper, exhauſted of air; ſo that the whole being thus rendered lighter than air, they would ſupport a chair, on which a perſon might ſit.

Biſhop Wilkins too, who died in 1672, in ſeveral of his works, makes mention of ſimilar ideas being entertained by divers perſons. “ It is a pretty notion to this purpoſe, ſays he (in his Diſcovery of a New World, prop. 14), mentioned by Albertus de Saxonia, and out of him by Francis Mendoza, that the air is in ſome part of it navigable. And that upon this ſtatick principle, any braſs or iron veſſel (ſuppoſe a kettle), whoſe ſubſtance is much heavier than that of the water; yet being filled with the lighter air, it will ſwim upon it, and not ſink.” And again, in his Dedalus, chap. 6, “ Scaliger conceives the framing of ſuch volant automata to be very eaſy. *Volantis columbæ machinulam, cujus autorem Archytam tradunt*, vel facillime *profiteri audeo.* Thoſe ancient motions were thought to be contrived by the force of ſome included air: So *Gellius*, *Ita erat ſcilicet libramentis ſuſpenſum, & aura ſpiritus incluſa, atque occulta conſitum, &c.* As if there had been ſome lamp, or other fire within it, which might produce ſuch a forcible rarefaction, as ſhould give a motion to the whole frame.” From which it would ſeem that Biſhop Wilkins had ſome confuſed notion of ſuch a thing as a heated air-balloon.

Again F. Franciſco Lana, in his Prodroma, printed in 1670, propoſes the ſame method with that of Roger Bacon, as his own thought.

He conſidered that a hollow veſſel, exhauſted of air, would weigh leſs than when filled with that fluid; he alſo reaſoned that, as the capacity of ſpherical veſſels increaſes much faſter than their ſurface, if there were two ſpherical veſſels, of which the diameter of one is double the diameter of the other; then the capacity of the former will be equal to 8 times the capacity of the latter, but the ſurface of that only equal to 4 times the ſurface of this: and the one ſphere have its diameter equal to triple the diameter of the other; then the capacity of the greater will be equal to 27 times the capacity of the leſs, while its ſurface is only 9 times greater: and ſo on, the capacities increaſing as the cubes of the diameters, while the ſurfaces increaſe only as the ſquares of the ſame diameters. And from this mathematical principle, father Lana deduces, that it is poſſible to make a ſpherical veſſel of any given matter, and thickneſs, and of ſuch a ſize as, when emptied of air, it will be lighter than an equal bulk of that air, and conſequently that it will aſcend in that element, together with ſome additional weight attached to it. After ſtating theſe principles, father Lana computes that a round veſſel of plate braſs, 14 feet in diameter, weighing 3 ounces the ſquare foot, will only weigh 1848 ounces; whereas a quantity of air of the ſame bulk will weigh $2155\frac{2}{3}$ ounces, allowing only one ounce to the cubic foot; ſo that the globe will not only be ſuſtained in the air, but will alſo carry up a weight of $307\frac{2}{3}$ ounces: and by increaſing the bulk of the globe, without increaſing the thickneſs of the metal, he adds, a veſſel might be made to carry a much greater weight.

Such then were the ingenious ſpeculations of learned men, and the gradual approaches towards this art. But one thing more was yet wanting: although acquainted in ſome degree with the weight of any quantity of air, conſidered as a detached ſubſtance, it ſeems they were not aware of its great elaſticity, and the univerſal preſſure of the atmoſphere; by which preſſure, a globe of the dimenſions above-deſcribed, and exhauſted of its air, would immediately be cruſhed inwards, for want of the equivalent internal counter preſſure, to be ſought for in ſome element, much lighter than common air, and yet nearly of equal preſſure or elaſticity with it; a property or circumſtance attending common air when conſiderably heated. It is evident then that the ſchemes of ingenious men hitherto muſt have terminated in mere ſpeculation; otherwiſe they could never have recorded ſchemes, which, on the firſt attempt to put in practice, muſt have manifeſted their own inſufficiency, by an immediate failure of ſucceſs: For inſtead of exhauſting the veſſel of air, it muſt either be filled with common air heated, or with ſome other equally elaſtic and lighter air. So that upon the whole it appears, that the art of traverſing the air, is an invention of our own time; and the whole hiſtory of it is comprehended within a very ſhort period.

The rarefaction and expanſion of air by heat, is a property of it that has long been known, not only to philoſophers, but even to the vulgar: by this means it is, that the ſmoke is continually carried up our chimneys; and the effect of heat upon air, is made very ſenſible by bringing a bladder, only partly full of air, near a fire; when the air preſently expands with the heat, and diſtends the bladder ſo as almoſt to burſt it: and ſo well are the common people acquainted with this effect, that it is the common practice of thoſe who kick blown bladders about for foot-balls, to bring them from time to time to the fire, to reſtore the ſpring of the air, and diſtenſion of the ball, loſt by the continual waſte of that fluid through the ſides of it.

But the great levity of inflammable air, is a very modern diſcovery. As to the inflammable property of this air itſelf, it had been long known to miners, and eſpecially in coal mines, by the dreadful effects it ſometimes produces by its exploſions. Among them it is ſometimes vulgarly called ſulphur, but more properly the fire damp, or inflammable damp, to diſtinguiſh it from the choak damp, and other damps, a ſpecies of air ſometimes found in deep wells and mines, and which does not explode nor take fire, but preſently extinguiſhes candles, and ſuffocates the perſons who may happen to go into it. But it ſeems that it was Mr. Cavendiſh who firſt diſcovered with exactneſs the ſpecific gravity of inflammable air; and his experiments and obſervations upon it, are publiſhed in the 56th volume of the Philoſophical Tranſactions for the year 1766. Soon after this diſcovery of Mr. Cavendiſh, it occurred to the ingenious Dr. Black of Edinburgh, that if a bladder, or other veſſel, ſufficiently light and thin, were filled with this air, it would form altogether a maſs lighter than the ſame bulk of atmoſpheric air, and conſequently that it would aſcend in it.

This idea he mentioned in his chemical lectures in the year 1767 or 1768; and he farther propoſed to exhibit the experiment, by filling the allantois of a calf with ſuch air. The allantois however was not prepared juſt at the time when he was at that part of his lectures, and other avocations afterwards prevented his deſign: ſo that, conſidering it only as an amuſing experiment, and being fully ſatisfied of the truth of ſo evident an effect, he contented himſelf with barely mentioning the experiment from time to time in his lectures. About the year 1777 or 1778 too it occurred to Mr. Cavallo, that it might be poſſible to conſtruct a veſſel, which, when filled with inflammable air, would aſcend in the atmoſphere: and there is no doubt but that ſimilar ideas would occur to many other perſons, of ſo evident a conſequence of Mr. Cavendiſh's diſcovery.

But it ſeems to have been Mr. Cavallo who firſt actually attempted the experiment, in which however he ſucceeded no farther than in being able to raiſe ſoap bubbles of two or three inches diameter: a thing which had been done by children for their amuſement time immemorial. Theſe experiments Mr. Cavallo made in the beginning of the year 1782, and an account of them was read at a public meeting of the Royal Society on the 20th day of June of that year. From which it appears that he tried bladders and paper of various ſorts. But the bladders, however thin they were made by ſcraping, &c, were ſtill found too heavy to aſcend in the atmoſphere, when fully inflated with the inflammable air: and in uſing China paper, he found that this air paſſed through its pores, like water through a ſieve. And having failed of ſucceſs by blowing the ſame air into a thick ſolution of gum, thick varniſhes, and oil paint, he was obliged to reſt ſatisfied with ſoap-balls or bubbles, which, being filled with inflammable air, by dipping the end of a ſmall glaſs tube, connected with a bladder containing the air, into a thick ſolution of ſoap, and gently compreſſing the bladder, aſcended rapidly in the atmoſphere, and broke againſt the ceiling of the room.

Here however it ſeems the matter might have reſted, had it not been for experiments made in France ſoon after, by the two brothers Stephen and Joſeph Montgolfier, upon principles ſuggeſted, not by the levity of inflammable air, which probably they had never heard of, but by that of ſmoke and clouds aſcending in the atmoſphere. Theſe two brothers it ſeems were natives of Annonay, a town in the Vivarais, about 36 miles diſtant from Lyons; and that in their youth, Stephen, the elder, had aſſiduouſly ſtudied the mathematics, but the other had applied himſelf more particularly to natural philoſophy and chemiſtry. They were not intended for any particular way of buſineſs, but the death of a brother obliged them to put themſelves at the head of a conſiderable paper manufactory at Annonay. In the intervals of time allowed by their buſineſs they amuſed themſelves in ſeveral philoſophical purſuits, and particularly with the experiments in aeroſtation, of which we are now to give ſome account. It would be perhaps impoſſible to know all the particular ſteps and ideas which finally produced this diſcovery: but it has been ſaid that the real principle, upon which the effect of the aeroſtatic machine depends, was unknown even for a conſiderable time after its diſcovery: that M. Montgolfier attributed the effect of the machine, not to the rarefaction of the air, which is the true cauſe, but to a certain gas, ſpecifically lighter than common air, which was ſuppoſed to be developed from burning ſubſtances, and which was commonly called Montgolfier's gas. Be this however as it may, it is well known that the two brothers began to think of the experiment of the aeroſtatic machine about the middle or the latter part of the year 1782. The natural aſcenſion of the ſmoke and the clouds in the atmoſphere, ſuggeſted the firſt idea; and to imitate thoſe bodies, or to incloſe a cloud in a bag, and let the latter be lifted up by the buoyancy of the former, was the firſt project of thoſe celebrated gentlemen.

Accordingly the firſt experiment was made at Avignon by Stephen, the elder brother, about the middle of November 1782. Having prepared a bag of fine ſilk, in the ſhape of a parallelopipedon, and of about 40 cubic feet in capacity, he applied burning paper to an aperture in the bottom, which rarefied the air, and thus formed a kind of cloud in the bag; and when it became ſufficiently expanded, it aſcended rapidly to the ceiling.

Soon afterwards the experiment was repeated with the ſame machine at Annonay, by the two brothers, in the open air; when the bag aſcended to the height of about 70 feet. Encouraged by this ſucceſs, they conſtructed another machine, of about 650 cubic feet capacity; which, when inflated as before, broke the cords which confined it, and after aſcending rapidly to the height of about 600 feet, deſcended and fell on the adjoining ground. With another larger machine, of 37 feet diameter, they repeated the experiment on the 25th day of April, which anſwered exceedingly well: the machine had ſuch force of aſcenſion, that, breaking abruptly from its confinement of ropes, it roſe to the height of more than 1000 feet, and then, being carried by the wind, deſcended and fell at a place about three quarters of a mile from the place of its aſcenſion. The capacity of this machine was equal to above 23 thouſand cubic feet, and, being nearly globular, when inflated, it meaſured 117 Engliſh feet in circumference. The covering was formed of linen, lined with paper; and its aperture at the bottom was fixed to a wooden frame, of about 4 feet ſquare, or 16 feet in ſurface. When filled with vapour, which it was conjectured might be about half as heavy as common air, it was capable of lifting up about 490 pounds, beſides its own weight, which, together with that of the wooden frame, was equal to about 500 pounds. With this ſame machine the next experiment was publicly performed at Annonay, on the 5th of June 1783, before a great multitude of ſpectators. The flaccid bag was ſuſpended on a pole 35 feet high; ſtraw and chopped wool were burned under the opening at the bottom; the vapour, or rather ſmoke, ſoon inflated the bag, ſo as to diſtend it in all its parts; and this enormous maſs aſcended in the air with ſuch velocity, that in leſs than ten minutes it reached the height of above 6 thouſand feet, when a breeze carried it in an horizontal direction to the diſtance of 7668 feet, or near a mile and a half, where it deſcended gently to the ground.

As ſoon as the news of this experiment reached Paris, the philoſophers of that city, conceiving that a new ſpecies of gas, of about half the weight of common air, had been diſcovered by Meſſrs. Montgolfier; and knowing that the weight of inflammable air was but about the eighth or tenth part of the weight of common air, they juſtly concluded that inflammable air would anſwer the purpoſe of this experiment better than the gas of Montgolfier, and accordingly they reſolved to make trial of it.

A ſubſcription was opened by M. Faujas de St. Fond, towards defraying the expence of the experiment. A ſufficient ſum of money having ſoon been raiſed, Meſſrs. Roberts were appointed to conſtruct the machine, and M. Charles, profeſſor of experimental philoſophy, to ſuperintend the work. After a conſiderable time ſpent, and ſurmounting many difficulties in obtaining a ſufficient quantity of inflammable air, and ſearching out a ſubſtance light enough for the covering, they at length conſtructed a globe of the ſilk called luteſtring, which was rendered impervious to the incloſed air by a varniſh of elaſtic gum or caeutchouc, diſſolved in ſome kind of ſpirit or eſſential oil. The diameter of this globe was about 13 feet; and it had only one aperture, like a bladder, to which a ſtop-cock was adapted: and the weight of this covering, when empty, together with that of the ſtop-cock, was 25 pounds.

On the 23d of Auguſt 1783, they began to fill the globe with inflammable air; but this, being their firſt attempt, was attended with many obſtructions and diſappointments, which took up two or three days to overcome.

At length however it was prepared for exhibition, and on the 27th it was carried from the Place des Victoires, where it had been prepared, to the Champs de Mars, a ſpacious open ground in the front of the Military School, where, after introducing ſome more inflammable air, and diſengaging it from the cords by which it was held down, it roſe, in leſs than two minutes, to the height of 3123 feet: the ſpecific gravity of the balloon, when it went up, being 35 pounds leſs than that of common air. At that height the balloon entered a cloud, but ſoon appeared again; and at laſt it was loſt among other clouds. After floating about in the air for about three quarters of an hour, it fell in a field about 15 miles from the place of its aſcent; where, as we may eaſily imagine, it occaſioned great amazement to the peaſants who found it. Its fall was owing to a rent in the covering, probably occaſioned by the ſuperior elaſticity of the inflammable air, over that of the rare part of the atmoſphere to which it had aſcended.

In conſequence of this brilliant experiment, numberleſs ſmall balloons were made, moſtly of goldbeater's ſkin, from 6 and 9 to 18 or 20 inches diameter; their cheapneſs putting it in the power of almoſt every family to ſatisfy its curioſity relative to the new experiment; and in a few days time balloons were ſeen flying all about Paris, from whence they were ſoon after ſent abroad.

Mr. Joſeph Montgolfier repeted an experiment with a machine of his conſtruction before the commiſſaries of the Academy of Sciences, on the 11th and 12th of September. The machine was about 74 feet high, and 43 feet in diameter; it was made of canvaſs, covered with paper both within and without, and weighed 1000 pounds. It was filled with rarefied air in 9 minutes, and in one trial the weight of eight men was not ſufficient to keep it down. It was not ſuffered to go up, as it had been intended for exhibition before the Royal Family, a few days after. By the violence of the rain, however, which fell about this time, it was ſo much ſpoiled, that he thought proper to conſtruct another for that purpoſe, in which he uſed ſo great diſpatch, that it was completed in the ſhort ſpace of four days time. This machine was conſtructed of cloth made of linnen and cotton thread, and painted with water colours both within and without. Its height was 60 feet, and diameter 43 feet. Having made the neceſſary preparation for inflating it, the operation was begun about one o'clock on the 19th of the ſame month, before the king and queen, the court, and the inhabitants of the place, as well as all the Pariſians who could procure a conveyance to Verſailles. The balloon was ſoon filled, and in eleven minutes after the commencement of the operation, the ropes being cut, it aſcended, bearing up with it a wicker cage, containing a cock, a duck, and a ſheep, the firſt animals that ever aſcended into the atmoſphere with an aeroſtatic machine. Its power of aſcenſion, or the weight by which it was lighter than an equal bulk of common air, allowing for the animals and their cage, was 696 pounds. The balloon roſe to the height of 1440 feet; and being driven by the wind for the ſpace of eight minutes, it gradually deſcended in conſequence of two large rents made in the covering by the wind, and fell in a wood at the diſtance of 10,200 feet, or about two miles from Verſailles. The animals landed again as ſafe as when they went up, and the ſheep was found feeding.

The ſucceſs of this experiment induced M. Pilatre de Rozier, with a philoſophical intrepidity which will be recorded with applauſe in the hiſtory of aeroſtation, to offer himſelf as the firſt adventurer in this aerial navigation. For this purpoſe M. Montgolfier conſtructed a new machine, of an oval ſhape, in a garden of the fauxbourg St. Antoine; its diameter being about 48 feet, and height 74 feet. To the aperture in the lower part was annexed a wicker gallery about three feet broad, with a balluſtrade of three feet high. From the middle of the aperture an iron grate, or brazier, was ſuſpended by chains, deſcending from the ſides of the machine, in which a fire was lighted for inflating the machine; and towards the aperture port-holes were opened in the gallery, through which any perſon, who might venture to aſcend, might feed the fire on the grate with fuel, and regulate at pleaſure the dilatation of the air incloſed in the machine: the weight of the whole being upwards of 1600 pounds. On the 15th of October 1783, the fire being lighted, and the balloon inflated, M. P. de Rozier placed himſelf in the gallery, and, to the aſtoniſhment of a multitude of ſpectators, aſcended as high as the length of the reſtraining cords would permit, which was about 84 feet from the ground, and there kept the machine afloat about four minutes and a half, by repeatedly throwing ſtraw and wool upon the fire: the machine then deſcended gradually and gently, through a medium of increaſing denſity, to the ground; and the intrepid adventurer aſſured the admiring ſpectators that he had not experienced the leaſt inconvenience in this aerial excurſion. This experiment was repeted on the 17th with nearly the ſame ſucceſs; and again ſeveral times on the 19th, when M. P. de Rozier, by a partial aſcent and deſcent, ſeveral times repeted, evinced to the multitude of obſervers that the machine may be made to aſcend and deſcend at the pleaſure of the aeronaut, by merely increaſing or diminiſhing the fire in the grate. The balloon having been hauled down, by the ropes which always confined it, M. Gironde de Villette placed himſelf in the gallery oppoſite to M. de Rozier, and the machine being ſuffered to aſcend, it hovered for about 9 minutes over Paris, in the ſight of all its inhabitants, at the height of 330 feet. And on their deſcending, the marquis of Arlandes aſcended with M. de Rozier much in the ſame manner.

In conſequence of the report of theſe experiments, ſigned by the commiſſaries of the Academy of Sciences, it was ordered that the annual prize of 600 livres ſhould be given to Meſſrs. Montgolfier for the year 1783.

In the experiments above-recited, the machine was always ſecured by long ropes, to prevent its entire eſcape: but they were ſoon ſucceeded by unconfined aerial navigation. For this purpoſe the ſame balloon of 74 feet in height was conveyed to La Muette, a royal palace in the Bois de Boulogne: and all things being got ready, on the 21ſt of November 1783, M. P. de Rozier and the marquis d'Arlandes took their poſt in oppoſite ſides of the gallery, and at 54 minutes after one the machine was abſolutely abandoned to the element, and it aſcended calmly and majeſtically in the atmoſphere. On reaching the height of about 280 feet the intrepid aeronauts waved their hats to the aſtoniſhed multitude: but they ſoon after roſe too high to be diſtinguiſhed, and it is ſuppoſed they roſe to more than 3000 feet in height. At firſt they were driven, by a north-weſt wind, horizontally over the river Seine and part of Paris, taking care to clear the ſteeples and high buildings by increaſing the fire; and in riſing they met with a current of air which carried them ſouthward. Having thus paſſed the Boulevard, and finally deſiſting from ſupplying the fire with fuel, they deſcended very gently in a field beyond the new Boulevard, about 9000 yards, or a little more than 5 miles diſtant from the palace de La Muette, having been between 20 and 25 minutes in the air. The weight of the whole apparatus including that of the two travellers, was between 1600 and 1700 pounds.

Notwithſtanding the rapid progreſs of aeroſtation in France, it is remarkable that we have no authentic account of any experiments of this kind being attempted in other countries. Even in our own iſland, where all arts and ſciences find an indulgent nurſery, and many their birth, no aeroſtatic machine was ſeen before the month of November 1783. Various ſpeculations have been made on the reaſons of this ſtrange neglect of ſo novel and brilliant an experiment. But none ſeemed to carry any ſhew of probability except that it was ſaid to be diſcouraged by the leader of a philoſophical ſociety, expreſsly inſtituted for the improvement of natural knowledge, for the reaſon, as it was ſaid, that it was the diſcovery of a neighbouring nation. Be this however as it may, it is a fact that the firſt aeroſtatic experiment was exhibited in England by a foreigner unconnected and unſupported. This was a count Zambeccari, an ingenious Italian, who happened to be in London about that time. He made a balloon of oiled-ſilk, 10 feet in diameter, weighing only 11 pounds: it was gilt, both for ornament, and to render it more impermeable to the inflammable air with which it was to be filled. The balloon, after being publicly ſhewn for ſeveral days in London, was carried to the Artillery Ground, and there being filled about three-quarters with inflammable air, and having a direction incloſed in a tin box for any perſon by whom it ſhould afterwards be found, it was launched about one o'clock on the 25th of November 1783. At half paſt three it was taken up near Petworth in Suſſex, 48 miles diſtant from London; ſo that it travelled at the rate of near 20 miles an hour. Its deſcent was occaſioned by a rent in the ſilk, which muſt have been the effect of the rarefaction of the inflammable air when the balloon aſcended to a rarer part of the atmoſphere.

The French philoſophers having executed the firſt aerial voyage with a balloon inflated by heated air, reſolved to attempt a ſimilar voyage with a balloon filled with inflammable air, which ſeemed to be preferable to dilated air in every reſpect, the expence of preparing it only excepted. A ſubſcription was opened however to defray that expence, which was eſtimated at about ten thouſand livres; and the balloon was conſtructed by Meſſrs. Roberts, of gores of ſilk, varniſhed with a ſolution of elaſtic gum. Its form was ſpherical, and it meaſured $27\frac{1}{2}$ feet in diameter. The upper hemiſphere was covered by a net, which was faſtened to a hoop encircling its middle, and called its equator. To this equator was ſuſpended by ropes a car or boat, covered with painted linen, and beautifully ornamented, which ſwung a few feet below the balloon. To prevent the burſting of the machine by the expanſion of the inflammable air in a rarer medium, or to cauſe the balloon to deſcend, it was furniſhed with a valve, which might be opened by means of a ſtring deſcending from it, for diſcharging a part of the internal air, without admitting the external to enter: And the car was ballaſted with bags of ſand, for the purpoſe of lightening it occaſionally, and cauſing it to aſcend: ſo that by letting ſome of the air eſcape through the valve, they might deſcend; and by diſcharging ſome of their ſand ballaſt, aſcend. To this balloon was likewiſe annexed a long pipe by which it was filled. The apparatus for filling it conſiſted of ſeveral caſks placed round a large tub of water, each having a long tin tube, that terminated under a veſſel or funnel which was inverted into the water of the tub, and communicated with the long pipe annexed to the lower part of the balloon. Iron filings and diluted vitriolic acid being put into the caſks, the inflammable air which was produced from theſe materials, paſſed through the tin tubes, thence through the water of the tubs to the inverted funnel, and ſo through the pipe into the balloon. When inflated, the weight of the common air which was equal in bulk to the balloon, was $771\frac{1}{2}$ pounds; alſo the power of aſcenſion, or weight juſt neceſſary to keep it from aſcending, was 20 pounds, and the weight of the balloon, with its car, paſſengers, and all its appendages, was $604\frac{1}{2}$ pounds, which two together make $624\frac{1}{2}$ pounds: and this taken from $771\frac{1}{2}$ pounds, the weight of common air diſplaced, leaves 147 pounds for the weight of the inflammable air contained in the balloon, and which is to $771\frac{1}{2}$ pounds, the weight of the ſame bulk of common air, nearly as 1 to $5\frac{1}{4}$; that is, the inflammable air uſed in this experiment was $5\frac{1}{4}$ times lighter than common air.

The firſt of December was fixed on for the diſplay of this grand experiment; and every preparation was made for conducting it with advantage. The garden of the Thuilleries at Paris was the ſcene of operation; which was ſoon crowded and encompaſſed with a prodigious multitude of obſervers. Signals were given, from time to time, by the firing of cannon, waving of flags, &c: and a ſmall montgolfier was launched, for ſhewing the direction of the wind, and for the amuſement of the people previous to the general diſplay. At three quarters after one o'clock, M. Charles and one of the Roberts, having ſeated themſelves in the boat attached to the balloon, and being furniſhed with proper inſtruments, cloathing, and proviſions, left the ground, and aſcended with a moderately accelerated velocity to the height of about 600 yards; the ſurrounding multitude ſtanding ſilent with fear and amazement; while the aerial navigators at this height made ſignals of their ſafety. When they left the ground, the thermometer, according to Fahrenheit's ſcale, ſtood at 59 degrees; and the barometer, at 30·18 inches: and at the utmoſt height to which they aſcended, the barometer fell to 27 inches; from which they deduced their height as above to be 600 yards, or one third part of a mile. During the reſt of the voyage the quickſilver in the barometer was generally between 27 and 27·65 inches, riſing and falling, as part of the ballaſt was thrown out, or ſome of the inflammable air eſcaped from the balloon. The thermometer generally ſtood between 53 and 57 degrees. Soon after their aſcent, they remained ſtationary for ſome time: they then moved horizontally in the direction north-north-weſt: and having croſſed the Seine, and paſſed over ſeveral towns and villages, to the great amazement of the inhabitants, they deſcended in a field, about 27 miles diſtant from Paris, at three-quarters paſt 3 o'clock; ſo that they had travelled at the rate of near 15 miles an hour, without feeling the leaſt inconvenience.

The balloon ſtill containing a conſiderable quantity of inflammable air, M. Charles re-aſcended alone, and it was computed he went to the height of 3100 yards, or almoſt 2 miles, the barometer being then at 20 Engliſh inches: having amuſed himſelf in the air about 33 minutes, he pulled the ſtring of the valve, and deſcended at 3 miles diſtance from the place of his aſcent. All the inconvenience he experienced in his great elevation, was a dry ſharp cold, with a pain in one of his ears and a part of his face, which he aſcribed to the dilatation of the internal air: a circumſtance that uſually happens to perſons who ſuddenly change the denſity of their atmoſphere, either by aſcending into a rarer, or deſcending into a denſer one. The ſmall balloon, launched at the beginning by M. Montgolfier, was found to have moved in a direction oppoſite to that of the aeronauts; from which it is inferred that there were two currents of air at different heights above the earth.

In the month of December this year, ſeveral experiments were made at Philadelphia in America with air balloons, by Meſſrs. Rittenhouſe and Hopkins. They conſtructed and filled a great many ſmall balloons, and connected them together; in which a man went up ſeveral times, and was drawn down again; and finally, the ropes being cut, he aſcended to the height of 100 feet, and floated to a conſiderable diſtance; but, being afraid, he cut open the balloons with a knife, and ſo deſcended.

About the cloſe of this year ſmall balloons were ſent up in many places, and were become very common in ſome parts of France and England. And in the beginning of the year following, their number and magnitude increaſed conſiderably; and ſome of the more remarkable ones were as follow:—On the 19th of January M. Joſeph Montgolfier, accompanied by ſix other perſons, aſcended from Lyons with a rarefied-air balloon, to the height of 1000 yards. This was the largeſt machine that had been hitherto made, being 131 feet high, and 104 feet in diameter: it was formed of a double covering of linen, with three layers of paper between them; and it weighed, when it went up, 1600 pounds, including the gallery, paſſengers, &c. It was at firſt intended for ſix paſſengers; but before it went up, it was not judged ſafe to freight it with more than three: however no authority nor ſolicitations could prevail upon any of the ſix to quit their place, nor even to caſt lots which three ſhould reſign their pretenſions: ſo that the ſpectators ſaw them all aſcend with terror and anxiety; and to add to their diſtreſs, when the ropes were cut, and the machine had aſcended a foot or two from the ground, a ſeventh perſon ſuddenly leaped into the gallery, and the fire being increaſed, the whole aſcended together. To add to the terror of the ſcene, after being in the air about 15 minutes, a large rent of about 50 feet in length was made by the balloon taking fire, in conſequence of which it deſcended very rapidly to the ground, though fortunately without injury to any of the aeronauts.

On the 22d of February an inflammable air balloon was launched from Sandwich in Kent. It was but a ſmall one, being only 5 feet in diameter; but it was rendered remarkable by being the firſt machine that croſſed the ſea from England to France. It was found in a field at Warneton, about 9 miles from Liſle in French Flanders, two hours and a half after it left Sandwich, the diſtance being about 74 miles; ſo that it floated at the rate of about 30 miles an hour.

The chevalier Paul Andreani, of Milan, was the firſt aerial traveller in Italy. The chevalier was at the ſole expence of this machine, but was aſſiſted in the conſtruction by two brothers of the name of Gerli. They all three aſcended together near Milan on the 25th of February, and remained in the atmoſphere about 20 minutes, when they deſcended, all their fuel being exhauſted. This machine was a montgolfier, of a ſpherical ſhape, and about 68 feet in diameter. From calculations made on the power of this, and other machines of the ſame ſort, it appears that the included air is rarefied commonly but about one-third, or that the included warm air weighs about two-thirds of the ſame bulk of the external or common air.

The next aerial voyage was performed on the 2d of March 1784, by M. Jean Pierre Blanchard, a man who has ſince that time made more voyages than any other perſon, and who has rendered himſelf famous by being the firſt who has floated in the air over the channel from England to France. M. Blanchard it ſeems had for many years been in purſuit of mechanical means for flying through the air; but on hearing of the late invented air balloons, he dropped his former purſuits, and turned his attention to them. He accordingly conſtructed one of 27 feet diameter, to which a boat was ſuſpended with two wings, and a rudder to ſteer it by, as alſo a large parachute ſpread horizontally between the boat and the balloon, deſigned to check the fall in caſe the balloon ſhould burſt. The machine being filled with inflammable air, he aſcended, from the Champs de Mars at Paris, to the height of near ten thouſand feet, or almoſt 2 miles; and after floating in the air for an hour and a quarter, he deſcended at Billancourt near Seve, having experienced by turns heat, cold, hunger, and an exceſſive drowſineſs. It appears from his own account, and as might have been expected, that the wings and rudder of his boat had little or no power in turning the balloon from the direction of the wind.

In the courſe of this year, 1784, aeroſtatic experiments and aerial voyages became ſo frequent, that the limits of this article will not allow of any thing farther than mentioning thoſe which were attended with any remarkable circumſtances. On the 25th of April, Meſſrs. de Morveau and Bertrand aſcended from Dijon, with an inflammable-air balloon, to the height of thirteen thouſand feet, or near 2 miles and a half, where the thermometer marked 25 degrees. They were in the air 1 hour and 25 minutes, in which time they floated 18 miles.

On the 20th of May, four ladies and a gentleman aſcended from Paris, in a large montgolfier, above the higheſt buildings, and remained ſuſpended there a conſiderable time, the balloon being confined by ropes from flying away.

On the 23d of May, M. Blanchard, with the ſame balloon as before, aſcended from Rouen, to ſuch height that the mercury in the barometer ſtood at 20·57 inches, which on the earth had been at 30·16. It was obſerved that in this voyage M. Blanchard's wings or oars could not turn him aſide from the direction of the wind.

On the 4th of June M. Fleurant and Madame Thible, the firſt lady who made an aerial voyage, aſcended at Lyons in a machine of 70 feet diameter. They went to the height of 8500 feet, and floated about 2 miles in 45 minutes.

On the 14th of June, M. Couſtard de Maſſi and M. Mouchet aſcended at Nantes to a great height, with a balloon of 32½ feet diameter, filled with inflammable air extracted from zink; and they floated to the diſtance of 27 miles in 58 minutes.

On the 23d of June, the firſt aerial traveller M. Pilatre de Rozier, accompanied with M. Prouts, aſcended at Verſailles, in the preſence of the royal family and the king of Sweden, with a large montgolfier, whoſe diameter was 79 feet, and its height 91 feet and a half. They floated to the diſtance of 36 miles in three-quarters of an hour, when they deſcended, which is at the rate of 48 miles an hour. In conſequence of this experiment the king granted to M. de Rozier a penſion of 2000 livres.

On the 15th of July the duke of Chartres, the two brothers Roberts, and a fourth perſon aſcended from the park of St. Cloud, with an inflammable-air machine, of an oblong form, its diameter being 34 feet, and its length, which went in a direction parallel to the horizon, was 55½ feet; and they remained in the atmoſphere for 45 minutes in the greateſt fear and danger. The machine contained an interior ſmall balloon, filled with common air, by means of which it was propoſed to cauſe the machine to aſcend or deſcend without the loſs of any inflammable air or ballaſt; and the boat was furniſhed with a helm and oars, which were intended to guide it. Three minutes after aſcending, the machine was loſt in the clouds, and involved in a denſe vapour. A violent agitation of the air, reſembling a whirlwind, greatly alarmed the aeronauts, turned the machine three times round in a moment, and gave it ſuch ſhocks as prevented them from uſing any of their inſtruments for managing the machine. After many ſtruggles, with great difficulty they tore about 7 or 8 feet of the lower part of the covering, by which the inflamable air eſcaped, and they deſcended to the ground with great rapidity, though without any hurt. At the place of departure the barometer ſtood at 30·12 inches, and at their greateſt elevation it ſtood at 24·36 inches; ſo that their aſcent was about 5100 feet, or near one mile.

On the 18th of July, M. Blanchard, with a Mr. Boley, made his third voyage with the ſame balloon as he had before, and roſe ſo high as to ſink the barometer from 30·1 to 25·34 inches, anſwering to a height of about 4600 feet. In 2 hours and a quarter they floated 45 miles, which is at the rate of 20 miles an hour. In this voyage M. Blanchard pretended that he was able to turn the machine with his wings, and to make it aſcend and deſcend at pleaſure. After deſcending, it is ſaid the balloon remained all the night at anchor full of air; and that the next day ſeveral ladies amuſed themſelves by aſcending ſucceſſively to the height of 80 feet, the length of the ropes by which it was anchored.

In the courſe of this ſummer two perſons had nearly loſt their lives by aſcending with machines of warmed air. The one in Spain, on the 5th of June, by the machine taking fire, was much burnt, and ſo hurt by the fall that his life was long deſpaired of. The latter having aſcended a few feet, the machinery entangled under the eves of a houſe, which broke the ropes, and the man fell about twenty feet: the machine preſently took fire, and was conſumed. Other montgolfiers were alſo burned about London, by taking fire, through the defects of their conſtruction.

The firſt aerial voyage performed in England was by one Vincent Lunardi, a native of Italy, who aſcended from the Artillery Ground, London, with an inflammable-air balloon on the 15th of September. His machine was made of oiled ſilk, painted in alternate ſtripes of blue and red; and its diameter was 33 feet. From a net, which covered about two-thirds of the balloon, 45 cords deſcended to a hoop hanging below the balloon, to which the gallery was attached. The machine had no valve; and its neck, which terminated in the form of a pear, was the aperture through which the inflammable air was introduced, and through which it might be let out. The balloon was filled with air produced from zink by means of diluted vitriolic acid. And when the aeronaut departed, at 2 o'clock, he took up with him a dog, a cat, and a pigeon. After throwing out ſome ſand to clear the houſes, he aſcended to a conſiderable height; and the direction of his motion at firſt was north-weſt by weſt;

but as the balloon roſe higher, it came into another current of air, which carried it nearly north. In the courſe of his voyage the thermometer was as low as 29 degrees, and the drops of water which had collected round the balloon were frozen. About half after three he deſcended very near the ground, and landed the cat, which was almoſt dead with cold: then riſing, he proſecuted his voyage, till at 10 minutes paſt 4 o'clock he landed near Ware in Hertfordſhire. He pretends that he deſcended by means of his oars or wings; but other circumſtances related by him, ſtrongly contradict the fact.

The longeſt and moſt intereſting voyage performed about this time, was that of Meſſrs. Roberts and M. Colin Hullin, who aſcended at Paris, at noon on the 19th of September, with an aeroſtat, filled with inflammable air, which was 27^{3} feet in diameter, and $46\frac{3}{4}$ feet long, the machine being made to float with its longeſt part parallel to the horizon, and having a boat of near 17 feet long attached to it. The boat was fitted up with ſeveral wings or oars, ſhaped like an umbrella, and they aſcended at 12 o'clock with 450 pounds of ſand ballaſt, and after various manœuvres finally deſcended, at 40 minutes paſt 6 o'clock, near Arras in Artois, 150 miles from Paris, having ſtill 200 pounds of ballaſt remaining in the boat. In one part they found the current of air uniform from 600 to 4200 feet high, which it ſeems was their greateſt height, and the fall of the barometer had been near 5·6 degrees. They found that by means of their oars they could accelerate their courſe a little in the direction of the wind, when it moved ſlowly, which may be true; but there is great reaſon to doubt of the accuracy of their experiments by which they pretended to cauſe their path to deviate about 22 degrees from the wind, going with a conſiderable velocity.

The ſecond aerial voyage in England, was performed by Mr. Blanchard, and Mr. Sheldon profeſſor of anatomy to the Royal Academy, being the firſt Engliſhman who aſcended with an aeroſtatic machine. They aſcended at Chelſea the 16th of October, at 9 minutes paſt 12 o'clock. Mr. Blanchard having landed Mr. Sheldon at about 14 miles from Chelſea, re-aſcended alone, and finally landed near Rumſey in Hampſhire, about 75 miles diſtant from London, having gone nearly at the rate of 20 miles an hour. The wings uſed on this occaſion it ſeems produced no deviation from the direction of the wind. Mr. Blanchard ſaid that he aſcended ſo high as to feel a great difficulty of breathing: and that a pigeon, which flew away from the boat, laboured for ſome time to ſuſtain itſelf with its wings in the rarefied air, but after wandering a good while, returned, and reſted on the ſide of the boat.

On the 4th of October, Mr. Sadler, an ingenious tradeſman at Oxford, aſcended at that place with an inflammable-air balloon of his own conſtruction and filling. And again on the 12th of the ſame month he aſcended at Oxford, and floated to the diſtance of 14 miles in 17 minutes, which is at the rate of near 50 miles an hour.

The 30th of November this year Mr. Blanchard's fifth aerial voyage, ſtill with his old machine, was performed in company with Dr. J. Jeffries, a native of America. Their voyage was about 21 miles; and it does not appear that the greateſt action of their oars produced any effect in directing the courſe of the balloon.

On the 4th of January, 1785, a Mr. Harper aſcended at Birmingham with an inflammable-air balloon, and went to the diſtance of 50 miles in an hour and a quarter, and ſuffered no other inconvenience than a temporary deafneſs, and what might be expected from the changes of wet and cold. The thermometer deſcended from 40 to 28 degrees.

On the 7th of January, Mr. Blanchard, accompanied with Dr. Jeffries, performed his ſixth aerial voyage, by actually croſſing the Britiſh channel from Dover to Calais, with the ſame balloon which had five times before carried him ſucceſsfully through the air. They aſcended with only 30 pounds of ſand ballaſt, beſides their proviſions, ſome books, inſtruments, and other neceſſaries. The machine parted with the gas very rapidly, and their ballaſt was ſoon all exhauſted; after which, from time to time they threw out every thing elſe in the boat, to prevent themſelves from dropping into the ſea. In this way they diſpoſed of all their proviſions, their books and inſtruments, and finally the moſt part of their very clothes themſelves. This however bringing them near the French coaſt, they gradually aſcended, cleared the cliffs and houſes, and landed in the foreſt of Guiennes. It is remarkable that a bottle, being thrown out when they were in danger of falling into the ſea, ſtruck the water with ſuch force, that they heard and felt the ſhock very ſenſibly on the car and balloon. In conſequence of this voyage the king of France preſented M. Blanchard with a gift of 12000 livres, and granted him a penſion of 1200 livres a year.

On the 19th of January, Mr. Croſbie aſcended at Dublin in Ireland, with an inflammable-air balloon to a great height. He roſe ſo rapidly that he was out of ſight in 3 minutes and a half. By ſuddenly opening the valve he deſcended juſt at the edge of the ſea, as he was driving towards the channel, being unprovided for properly paſſing over to England.

On the 23d of March, Count Zambeccari and Admiral Sir Edward Vernon aſcended at London, and ſailed to Horſham in Suſſex, at the diſtance of 35 miles in leſs than an hour. The voyage proved very dangerous, owing to ſome of the machinery about the valve being damaged, which obliged them to cut open ſome part of the balloon when they were about two miles perpendicular height above the earth, the barometer having fallen from 30·4 to 20·8 inches. In deſcending they paſſed through a denſe cloud, which felt very cold, and covered them with ſnow. The obſervations they made were, that the balloon kept perpetually turning round its vertical axis, ſometimes ſo rapidly as to make each revolution in 4 or 5 ſeconds; that a peculiar noiſe, like ruſtling, was heard among the clouds, and that the balloon was greatly agitated in the deſcent.

On May the 5th, Mr. Sadler, and William Windham, eſq. member of parliament for Norwich, aſcended at Moulſey-hurſt. The machine took a ſouth-eaſt courſe, and the current of air was ſo ſtrong that they were in great danger of being driven to ſea. They had the good fortune however to deſcend near the con-flux of the Thames and Medway, not a mile from the water's edge. By an accident they loſt their balloon: for while the aeronauts were buſied in ſecuring their inſtruments, the country people, whom they had employed in holding down the machine, ſuddenly let go the cords, when the balloon inſtantly aſcended, and was driven many miles out to ſea, where it fell, and was taken up by a trading veſſel. It was afterwards reſtored again, and another voyage made with it from Mancheſter to Pontefract, in which Mr. Sadler was ſtill more unfortunate; for no perſon being near when it deſcended, and not being able to confine it by his own ſtrength, he was dragged by it over trees and hedges; and at laſt was forced to quit it at the utmoſt peril of his life; after which it roſe, and was out of ſight in a few minutes. It was afterwards found near Gainſborough.

On the 12th of May Mr. Croſbie aſcended, at Dublin, as high as the tops of the houſes; but ſoon deſcended again with a velocity that alarmed all the ſpectators for his ſafety. On his ſtepping out of the car, in an inſtant Mr. M‘Guire, a college youth, ſprung into it, and the balloon aſcended with him to the aſtoniſhment of the beholders, and preſently he was carried with great velocity towards the channel in the direction of Holyhead. This being obſerved, a crowd of horſemen purſued full ſpeed the courſe he ſeemed to take, and could plainly perceive the balloon deſcending into the ſea. Lord H. Fitzgerald, who was among the foremoſt, inſtantly diſpatched a ſwift-ſailing veſſel mounted with oars, and all the boats that could be got, to the relief of the gallant youth; whom they found almoſt ſpent with ſwimming, juſt time enough to ſave his life.

The fate of M. Pilatre de Rozier, the firſt aerial traveller, and his companion M. Romain, has been much lamented. They aſcended at Boulogne the 15th of June, with intent to croſs the channel to England: for the firſt 20 minutes they ſeemed to take the proper direction; when preſently the whole apparatus was ſeen in flames, and the unfortunate adventurers fell to the ground from the height of more than a thouſand yards, and were killed on the ſpot, their bones being broken, and their bodies cruſhed in a ſhocking manner. The machine in which they aſcended, conſiſted of a ſpherical balloon, 37 feet in diameter, filled with inflammable air; and under this balloon was ſuſpended a ſmall montgolfier, or fire balloon, of 10 feet diameter; the gallery which ſuſpended the aeronauts, was attached to the net of the upper balloon by cords, which were faſtened to a hoop rather larger than the montgolfier, and deſcended perpendicularly to the gallery. The montgolfier was intended to promote and prolong the aſcenſion, by rarefying the atmoſpheric air, and by that means gaining levity. It is not certainly known whether the balloon was actually ſet on fire by the montgolfier, or, being over-rarefied by the heat beneath, burſt, and by that means the inflammable air was ſet in a blaze.

On the 19th of July, at 20 minutes paſt 2 o'clock, Mr. Croſbie aſcended at Dublin, with intent to croſs the channel to Holyhead in England. The uſual form of the boat had been changed, for a capacious wicker baſket, of a circular form, round the upper edges of which were faſtened a great many bladders, which were intended to render his gallery buoyant, in caſe of a diſaſter at ſea. About 300 pounds of ballaſt were put into the baſket, but the aeronaut diſcharged half a hundred on his firſt riſe. At firſt the current of air carried him due weſt; but it ſoon changed his courſe to nearly north-eaſt, pointing nearly towards Whitehaven. At upwards of 40 miles from the Iriſh ſhore, he found himſelf within clear ſight of both lands, and he ſaid it was impoſſible to give any adequate idea of the unſpeakable beauties which the ſcenery of the ſea, bounded by both lands, preſented. He roſe at one time ſo high, that by the intenſe cold his ink was frozen, and the mercury ſunk quite into the ball of the thermometer. He was ſick, and felt a ſtrong prepulſion on the tympanum of the ears. At his utmoſt height he thought himſelf ſtationary; but on liberating ſome gas, he deſcended to a current of air blowing north, and extremely rough. He now entered a thick cloud, and encountered ſtrong blaſts of wind, with thunder and lightning, which brought him rapidly towards the ſurface of the water. Here the balloon made a circuit, but falling lower, the water entered his car, and he loſt his notes of obſervation. All his endeavours to throw out ballaſt were of no avail; the force of the wind plunged him into the ocean; and with much difficulty he put on his cork jacket. The propriety of his idea was now very manifeſt in the conſtruction of his boat; as by the admiſſion of the water into the lower part of it, and the ſuſpenſion of his bladders, which were arranged at the top, the water, added to his own weight, became proper ballaſt; and the balloon maintaining its poiſe, it became a powerful ſail, by means of which, and a ſnatch block to his car, he went before the wind as regularly as a ſailing veſſel. In this ſituation he found himſelf inclined to eat, and he took a little fowl. At the diſtance of a league he diſcovered ſome veſſels crowding after him; but as his progreſs outſtripped all their endeavours, he lengthened the ſpace of the balloon from the car, which gave a check to the rapidity of his ſailing, and he was at length overtaken and ſaved by the Dunleary barge, which took him on board, and ſteered to Dunleary, towing the balloon after them.

A ſimilar accident happened to Major Money, who aſcended at Norwich, on the 22d of July, at 20 minutes paſt 4 in the afternoon; when meeting with an improper current, and not being able to let himſelf down, on account of the ſmallneſs of the valve, he was driven out to ſea, where, after blowing about for near two hours, he dropped into the water. Here the ſtruggles were aſtoniſhing which he made to keep the balloon up, which was torn, and hung only like an umbrella over his head. A ſhip was once within a mile, but, he adds, whether from want of humanity, or by miſtaking the balloon for a ſea monſter, they ſheered off, and left him to his fate: but a boat chaſed him for two hours, till juſt dark, and then bore away. He now gave up all hopes, and began to wiſh that providence had given him the fate of Pilatre de Rozier, rather than ſuch a lingering death. Exerting himſelf however to preſerve life as long as poſſible, by keeping the balloon floating over his head, to keep himſelf out of the water, into which nevertheleſs he ſunk gradually inch by inch, as it loſt its power, till he was at length breaſt deep in water, when he was providentially taken up by a revenue cutter, at half paſt eleven at night, but ſo weak that he was obliged to be lifted out of the car into the ſhip.

About the latter end of Auguſt, the longeſt aerial voyage hitherto made, was performed by Mr. Blanchard, who aſcended at Liſle, accompanied by the Chevalier de L'Epinard, and travelled 300 miles in their balloon before it deſcended. On this occaſion, as on ſome former ones, Mr. Blanchard made trial of a parachute, like a large umbrella, invented to break the fall in caſe of an accident happening the balloon: with this machine he dropped a dog from the car ſoon after his aſcenſion, which deſcended gently and unhurt.

On September the 8th, Thomas Baldwin, Eſq. aſcended from the city of Cheſter, at 40 minutes paſt one o'clock, and deſcended at Rixton-Moſs, at 25 miles diſtance, after a voyage of 2 hours and a quarter. The greateſt perpendicular altitude aſcended was about a mile and a half, and the aeronaut computed that in ſome parts of the voyage he moved at the rate of 30 miles an hour. Mr. Baldwin publiſhed a very circumſtantial account of his voyage, with many ingenious philoſophical remarks relating to aeroſtation, of which ſubject his book may be conſidered as one of the beſt treatiſes yet given to the public.

October the 5th, Mr. Lunardi made the firſt aerial voyage in Scotland. He aſcended at Edinburgh, and after various turnings, landed near Cupar in Fife, having deſcribed a track of 40 miles over the ſea, and 10 over the land, in an hour and a half. He ſaid the mercury in the barometer ſunk as low as 18·3 inches at his greateſt elevation.

November the 19th the celebrated Blanchard aſcended at Ghent to a great height, and after many dangers deſcended at Delft without his car, which he cut away to lighten the machine when he was deſcending too rapidly, and flung himſelf by the cords to the balloon, which ſerved him then in the nature of a parachute. On his firſt aſcent, when he was almoſt out of ſight, he let down a dog, by means of a parachute, which came eaſily to the ground.

November the 25th Mr. Lunardi aſcended at Glaſgow, and in two hours it is ſaid he deſcribed a track of 125 miles. It is further remarkable that, being overcome with drowſineſs, he ſays he ſlept for about 20 minutes in the bottom of the car, during this voyage.

Many other voyages were made in different countries, and with various ſucceſs. But ſince the year 1785, the rage for balloons has conſiderably abated, and we have gradually had leſs and leſs of theſe aerial excurſions, ſo that it is now become rather an uncommon thing to hear of one of them performed in any country whatever: which ſpeedy decline in this new art is perhaps to be aſcribed chiefly to the following cauſes; namely, a leſs degree of eagerneſs in people to purſue ſuch experiments, from their curioſity having been ſatisfied; ſecondly, the trouble, danger, and great expence, attending them; and laſtly, the want of the means of conducting them, and the ſmall degree of utility to which they have hitherto been applied. The failure in the many attempts that have been made to direct balloons at pleaſure through the air, cannot but be felt as a very diſcouraging circumſtance: and it is to be feared that it will ever be felt as ſuch, notwithſtanding the pretenſions of ſome perſons on this head; for they never have cauſed, nor is it to be expected they ever can cauſe the machine to deviate ſenſibly from the courſe of the wind, except only in the caſe when this moves with a very ſmall celerity. For when the current blows only at the rate of 10 miles an hour, which is but a very gentle wind, it may be ſhewn that a balloon of 50 feet in diameter will require a force equal to the preſſure of 72 pounds weight, to cauſe it to deviate 30 degrees from the courſe of the wind; and a force equal to double or triple that weight, when the wind blows with a double or triple velocity, that is, at the rate of 20 or 30 miles an hour; and ſo in proportion. To obviate the danger of a fall, ariſing from any accident happening to the balloon, ſome experiments have been made with a parachute, chiefly by Mr. Blanchard, whoſe endeavours and perſeverance it ſeems have continued longer than in any other perſon: we ſtill hear of his excurſions in different parts of Europe, and improvements of the parachute, wings, &c; and have juſt read accounts of two voyages lately performed by him; with which, being very curious, we ſhall conclude our narration of theſe aerial excurſions. They will be beſt related in Mr. Blanchard's own words, taken from his letter, dated Leipſick, October the 9th, 1787, to the editors of the Paris Journal. "I did not mention," ſays he, "in your intereſting paper, my aſcenſion at Straſburg on the 26th of laſt Auguſt: the weather was ſo horrible that I mounted only for the ſake of contenting the aſtoniſhing crowd of ſtrangers aſſembled there from all parts of the country. Every body ſeemed ſatiſfied at the attempt, but I aſſure you, gentlemen, that I was far from being pleaſed with ſo common an experiment. The only remarkable thing that occurred at that time, was the following circumſtance: At the height of about 2000 yards, or a mile and half a quarter, I let down a dog tied to the parachute, who, inſtead of deſcending gently, was forcibly carried, by a whirlwind, above the clouds. I met him ſoon after, bending his courſe directly downwards, and, as on recollecting his maſter, he began to bark a little, I was going to take hold of the parachute, when another whirlwind lifted him again to a great height. I loſt him for the ſpace of ſix minutes, and perceived him afterwards, with my teleſcope, as if ſleeping in the cradle or baſket belonging to the machine. Continually agitated, and impetuouſly toſſed through every point of the compaſs, by the violence of the different currents of air, I determined to end my voyage on the other ſide of the Rhine, after having paſſed vertically over Zell. I deſcended at a ſmall village, with an intention to be aſſiſted a little, and about thirty men ſoon came within reach of the balloon very a-propos, and fixed me to the ground. The wind was ſo violent that anchors or ropes would have been of no ſervice. I had however added to the large aeroſtatic globe a ſmaller one, of 60 pounds aſcenſional force, which would have contributed to fix me, when once I let it looſe; but notwithſtanding this precaution, the men's aſſiſtance was very neceſſary to me. The parachute was ſtill wavering in the air, and did not come down till 12 minutes after."

" I performed my 27th aſcenſion at Leipſick the 29th of September, in the midſt of an incredible number of ſpectators, forming one of the moſt brilliant aſſemblies I ever beheld. The ſky was as clear and ſerene as poſſible, and the air ſo calm that many of my friends, and multitudes of others, could follow me on horſeback, and even on foot. I was ſometimes ſo near them that they thought they could reach me, but I could ſoon find the means of riſing; and once, when they had actually taken hold of the cords, to ſee me float with the ſtrings in their hands, I ſuddenly cut them, and mounted again in the air. All theſe amuſing evolutions were in ſight of the town and its environs. At length I yielded to the earneſt ſolicitations of the company, and entered the town triumphantly in my car, followed by a concourſe of people tranſported with joy, and amidſt the acclamations of thouſands. The next day I emptied the inflammable air into another globe, with which I intended to try ſome experiments; and I let it off with a cradle, in which a dog was fixed. The balloon, having reached a conſiderable height, made an exploſion in its under part, as I had imagined it would, having previouſly diſpoſed it in a proper manner for that purpoſe; by which means the little animal fell gently to the ground."

" The day before yeſterday having repeated this experiment, at the town's requeſt, I prepared the globe in ſuch a manner as to cauſe an exploſion in its upper part, and added a parachute with two ſmall dogs fixed to it. They went ſo high that, notwithſtanding the ſerenity of the ſky, the balloon was loſt in its immenſe expanſe. Teleſcopes of the beſt ſort became uſeleſs, and I began to be apprehenſive for the death of the little animals, on account of the ſeverity of the cold. They deſcended however about two hours after, quite ſafe and well, in the town of Delitzſch, three miles from Leipſick. I went yeſterday to claim them, and found them again over the town in the air with the parachute. Such experiments had been already tried many times in the courſe of the day, and ſome officers had thrown them from the top of a ſteeple, in the ſight of all the inhabitants of Delitzſch, from whence they deſcended ſafe."

We have lately heard of Mr. Blanchard's 32d aſcenſion at Brunſwick in the month of Auguſt 1788, in which he much aſſiſted his aſcent by means of his wings.

For ſeveral figures of balloons, ſee plate 1.

Practice of AEROSTATION. The firſt conſideration in the practice of aeroſtation, is the form and the ſize of the machine. Various ſhapes have been tried and propoſed, but the globular, or the egg-like figure, is the moſt proper and convenient, for all purpoſes; and this form alſo will require leſs cloth or ſilk than any other ſhape of the ſame capacity; ſo that it will both come cheaper, and have a greater power of aſcenſion. The bag or cover of an inflammable-air balloon, is beſt made of the ſilk ſtuff called luſtring, varniſhed over. But for a montgolfier, or heated-air balloon, on account of its great ſize, linen cloth has been uſed, lined within or without with paper, and varniſhed. Small balloons are made either of varniſhed paper, or ſimply of paper unvarniſhed, or of gold beater's ſkin, or ſuch-like light ſubſtances.

With reſpect to the form of a balloon, it will be neceſſary that the operator remember the common proportions between the diameters, circumferences, ſurfaces, and ſolidities of ſpheres; for inſtance, that of different ſpheres, the circumferences are as the diameters; that the ſurfaces are as the ſquares of the diameters; and the ſolidities as the cubes of the ſame diameters: that any diameter is to its circumference as 7 to 22, or as 1 to $3\frac{1}{7}$; and therefore 3 times and $\frac{1}{7}$ of any diameter will be its circumference; ſo that if the diameter of a balloon be 35 feet, its circumference will be 110 feet. And if the diameter be multiplied by the circumference, the product will be the ſurface of the ſphere; thus 35 multiplied by 110 gives 3850, which is the ſurface of the ſame ſphere in ſquare feet: and if this ſurface be divided by the breadth of the ſtuff, in feet, which the balloon is to be made of, the quotient will be the number of feet in length neceſſary to conſtruct the balloon; ſo if the ſtuff be 3 feet wide, then 3850 divided by 3, gives $1283\frac{1}{3}$ feet, or 428 yards nearly, the requiſite quantity of ſtuff of 3 feet or one yard wide, to form the balloon of 35 feet diameter. Hence alſo, by knowing the weight of a given piece of the ſtuff, as of a ſquare foot, or ſquare yard, it is eaſy to find the weight of the whole bag, namely by multiplying the ſurface, in ſquare feet or yards, by the weight of a ſquare foot or yard: ſo if each ſquare yard weigh 16 ounces or 1 pound, then the whole bag will weigh 428 pounds. Again, the capacity, or ſolid contents, of the ſphere, will be found by multiplying $\frac{1}{6}$ of the ſurface by the diameter, or by taking $\frac{11}{21}$ of the cube of the diameter; which gives 22458 cubic feet for the capacity of the ſaid balloon, that is, it will contain, or diſplace, 22458 cubic feet of air. From the content and ſurface of the balloon, ſo found, is to be derived its power or levity, thus: on an average, a cubic foot of common air weighs $1\frac{1}{5}$ ounce, and therefore to the number 22458, which is the content of our balloon, adding its $\frac{1}{5}$th part, we have 26950 ounces, or 1684 pounds, for the weight of the common air diſplaced or occupied by the balloon. From this weight muſt be deducted the weight of the bag, namely 428 pounds, and then there remains 1256 pounds levity of the balloon, without however conſidering the contained air, whether it be heated air, or of the inflammable kind. If inflammable air be uſed, as it is of different weights, from $\frac{1}{4}$ to $\frac{1}{10}$ or $\frac{1}{12}$ the weight of common air, according to the modes of preparing it, let us ſuppoſe for inſtance that it is $\frac{1}{6}$ of the weight of common air; then $\frac{1}{6}$ of 1684 is 261 pounds, which is the weight of the bag full of that air; which being taken from 1256, leaves 995 pounds for the levity of the balloon when ſo filled with that inflammable air, or the weight which it will carry up, conſiſting of the car, the ropes, the paſſengers, the neceſſaries, and ballaſt. But if heated air be uſed; then as it is known from experiment that, by heating, the contained air is diminiſhed in denſity about one-third only, therefore from 1684, take $\frac{1}{3}$ of itſelf, and there remains 1123 for the weight of the contained warm air; and this being ſubtracted from 1256, leaves only 133 pounds for the levity of the balloon in this caſe; which being too ſmall to carry up the car, paſſengers, &c, it ſhews that for thoſe purpoſes a larger balloon is neceſſary, on

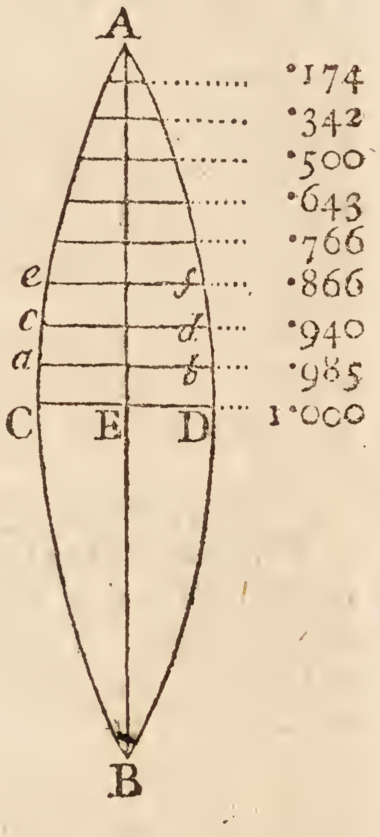
Montgolfier's principles. But if now, from the preceding computation, it be required to find how much the ſize of the balloon muſt be increaſed, that its levity, or power of aſcenſion, may be equal to any given weight, as ſuppoſe 1000 pounds; then becauſe the levities are nearly as the cubes of the diameters, therefore the diameters will be nearly as the cube roots of the levities; but the levities 133 and 1000 are nearly as 1 to 8, the cube roots of which are as 1 to 2, and conſequently 1 : 2 : : 35 : 70 feet, the diameter of a montgolfier, made of the ſame thickneſs of ſtuff as the former, capable of lifting 1000 pounds.

On the ſame principles we can eaſily find the ſize of a balloon that ſhall juſt float in air when made of ſtuff of a given thickneſs or weight, and filled with air of a given denſity; the rule for which is this: from the weight of a cubic foot of common air, ſubtract that of a cubic foot of the lighter or contained air; then divide 6 times the weight of a ſquare foot of the ſtuff, by the remainder; and the quotient will be the diameter, in feet, of the balloon that will juſt float at the ſurface of the earth. Suppoſe, for inſtance, that the materials are as before, namely, the ſtuff 1 pound to the ſquare yard, or $\frac{16}{9}$ ounces to the ſquare foot, which taken 6 times is $\frac{32}{3}$; then the cubic foot of common air weighing $1\frac{1}{5}$ ounce, and of heated air $\frac{2}{3}$ of the ſame, whoſe difference is $\frac{2}{5}$; therefore $\frac{32}{3}$ divided by $\frac{2}{5}$, gives $26\frac{2}{3}$ feet, which is the diameter of a montgolfier that will juſt float: but if inflammable air be uſed of $\frac{1}{6}$ the weight of common air, the difference between $1\frac{1}{5}$ and $\frac{1}{6}$ of it, is 1; by which dividing $\frac{32}{3}$ or $10\frac{2}{3}$, the quotient is the ſame $10\frac{2}{3}$ feet, which therefore is the diameter of an inflammable-air balloon that will juſt float. And if the diameter be more than theſe dimenſions, the balloons will riſe up into the atmoſphere.

The height nearly to which a given balloon will riſe in the atmoſphere, may be thus found, having given only the diameter of the balloon, and the weight which juſt balances it, or that is juſt neceſſary to keep it from riſing: compute the capacity or content of the globe in cubic feet, and divide its reſtraining weight in ounces by that content, and the quotient will be the difference between the denſity or ſpecific gravity of the atmoſphere at the earth's ſurface and that at the height to which the balloon will riſe; therefore ſubtract that difference or quotient from $1\frac{1}{5}$ or 1·2, the denſity at the earth, and the remainder will be the denſity at that height: then the height anſwering to that denſity will be found ſufficiently near in the annexed table. Thus, in the foregoing examples, in which the diameter of the balloon is 35 feet, its capacity 22458, and the levity of the firſt one 995 pounds, or 15920 ounces, the quotient of the latter number divided by the former, is ·709, which is the denſity at the utmoſt height, and to which in the table anſwers a little more than $2\frac{1}{2}$ miles, or $2\frac{5}{8}$ miles nearly, which therefore is the height to which the balloon will aſcend. And when the ſame balloon was filled with heated air, its levity was found equal to only 133 pounds, or 2128 ounces, then dividing this by 22458 the capacity, the quotient ·095 taken from 1·200, leaves 1·105 for the denſity; to which in the table correſponds almoſt half a mile, or nearer $\frac{3}{8}$ of a mile. And ſo high nearly would theſe balloons aſcend, if they keep the ſame figure, and loſe none of the contained air: or rather, thoſe are the heights they would ſettle at; for their acquired velocity would firſt carry them above that height, ſo far as till all their motion ſhould be deſtroyed; then they would deſcend and paſs below that height, but not ſo much as they had gone above; after which they would re-aſcend, and paſs that height again, but not ſo far as they had gone below it; and ſo on for many times, vibrating alternately above and below that point, but always leſs and leſs every time. The foregoing rule, for finding the height to which the balloon will aſcend, is independent of the different ſtates of the thermometer at that higheſt point, and at the ſurface of the earth; but for greater accuracy, including the allowances depending on the different ſtates of the thermometer, ſee under the word Atmosphere, where the more accurate rules are given at large.

Height in miles.	Denſity.
0	1·200
$\frac{1}{4}$	1·141
$\frac{1}{2}$	1 085
$\frac{3}{4}$	1·031
1	0·980
$1\frac{1}{4}$	0·932
$1\frac{1}{2}$	0·886
$1\frac{3}{4}$	0·842
2	0·800
$2\frac{1}{4}$	0·761
$2\frac{1}{2}$	0·723
$2\frac{3}{4}$	0·687
3	0·653

The beſt way to make up the whole coating of the balloon, is by different pieces or ſlips joined lengthways from end to end, like the pieces compoſing the ſurface of a geographical globe, and contained between one meridian and another, or like the ſlices into which a melon is uſually cut, and ſuppoſed to be ſpread flat out. Now the edges of ſuch pieces cannot be exactly deſcribed by a pair of compaſſes, not being circular, but flatter or leſs round than circular arches; but if the ſlips are ſufficiently narrow, or numerous, they will differ the leſs from circles, and may be deſcribed as ſuch. But more accurately, the breadths of the ſlip, at the ſeveral diſtances from the point to the middle, where it is broadeſt, are directly as the ſines of thoſe diſtances, radius being the ſine of the half length of the ſlip, or of the diſtance of either point from the middle of the ſlip: that is, If ACBD be one of the ſlips, AB being half the circumference, or AE a quadrant conceived to be equal to AC or AD; then will CD be to *a b*, as radius or the ſine of AC, to the ſine of A*a*. So that if the quadrant AE or AC be divided into any number of equal parts, as here ſuppoſe 9, then divide the quadrant or 90 degrees by the number of parts 9, and the quotient 10 is the number of degrees in each part; and hence the arcs AC, A*a*, A*c*, &c, will be reſpectively 90°, 80°, 70°, &c; and CD being radius, the ſeveral breadths *ab*, *cd*, *ef*, &c, will be reſpectively the ſines of 80°, 70°, 60°, &c, which are here placed oppoſite them, the radius being 1. Therefore when it is propoſed to cut out ſlips for a globe of a given diameter; compute the circumference, and make AE or AC a quarter of that circumference, and CD of any breadth, as 3 feet, or 2 feet, or any other quantity;



quantity; then multiply each of the decimal numbers, ſet oppoſite the figure, by that quantity, or breadth of CD, ſo ſhall the products be the ſeveral breadths *ab*, *cd*, *ef*, &c.

Various ſchemes have been deviſed for conducting balloons in any direction, whether vertical or ſideways. As to the vertical directions, namely upwards or downwards, the means are obvious, viz. in order to aſcend, the aeronaut throws out ſome ballaſt; and that he may deſcend, he opens a valve in the top of his machine by means of a ſtring, to let ſome of the gas eſcape; or if it be a montgolfier, he increaſes or diminiſhes the fire, as he would aſcend or deſcend. But to direct the machine in a ſide or horizontal courſe, is a very difficult operation, and what has hitherto not been accompliſhed, except in a ſmall degree, and when the current of air is very gentle indeed. The difficulty of moving the balloon ſideways, ariſes from the want of wind blowing upon it; for as it floats along with the current of air, it is relatively in a calm, and the aeronaut feels no more wind than if the machine were at reſt in a perfect calm. For this reaſon, any thing in the nature of ſails can be of no uſe; and all that can be hoped for, is to be attempted by means of oars; and how ſmall the effect of theſe muſt be, may eaſily be conceived from the rarity of the medium againſt which they muſt act, and the great magnitude of the machine to be forced through it. We can eaſily aſſign what force is neceſſary to move a given machine in the air with any propoſed velocity. From very accurate experiments I have determined, that a globe of $6\frac{3}{8}$ inches in diameter, and moving with a velocity of 20 feet per ſecond of time, ſuffers a reſiſtance from the air which is juſt equal to the weight or preſſure of one ounce Averdupois; and farther that with different ſurfaces, and the ſame velocity, the reſiſtances are directly proportional to the ſurface nearly, a double ſurface having a double reſiſtance, a triple ſurface a triple reſiſtance, and ſo on; and alſo that with different velocities, the reſiſtances are proportional to the ſquares of the velocities nearly, ſo that a double velocity produced a quadruple reſiſtance, and three times the velocity nine times the reſiſtance, and ſo on. And hence we can aſſign the reſiſtance to move a given balloon, with any velocity. Thus, take the balloon as before of 35 feet diameter; then by compariſon as above it is found that this globe, if moved with the velocity of 20 feet per ſecond, or almoſt 14 miles per hour, will ſuffer a reſiſtance equal to 271 pounds; to move it at the rate of 7 miles an hour, the reſiſtance will be 68 pounds; and to move it $3\frac{1}{2}$ miles an hour, the reſiſtance will be 17 pounds; and ſo on: and with ſuch force muſt the aeronauts act on the air in a contrary direction, to communicate ſuch a motion to the machine. And if the balloon move through a rarer part of the atmoſphere, than that at the ſurface of the earth, as $\frac{1}{3}$, or $\frac{1}{4}$, &c, rarer, and conſequently the reſiſtance be leſs in the ſame proportion; yet the force of the oars will be diminiſhed as much; and therefore the ſame difficulty ſtill remains. In general, the aeronaut muſt ſtrike the air, by means of his oars, with a force juſt equal to the reſiſtance of the air on the balloon, and therefore he muſt ſtrike that air with a velocity which muſt be greater as the ſurface of the oar is leſs than the reſiſted ſurface of the globe, but not in the ſame proportion, becauſe the force is as the ſquare of the velocity.

Now ſuppoſe the aeronaut act with an oar equal to 100 ſquare feet of ſurface, to move the balloon above mentioned at the rate of 20 feet per ſecond, or 14 miles an hour; then muſt he move this oar with the great velocity of 62 feet per ſecond, or near 43 miles per hour: and ſo in proportion for other velocities of the balloon. From whence it is highly probable, that it will never be in the power of man to guide ſuch machine with any tolerable degree of ſucceſs, eſpecially when any conſiderable wind blows, which is almoſt always the caſe.

As ſome aeronauts have thought of uſing parachutes, made ſomething like umbrellas, to break their fall, in caſe of any accident happening to the balloon, we ſhall here conſider the principles and power of ſuch a machine. Let us ſuppoſe a perſon wants to know what the ſize of a parachute muſt be, that he may deſcend with it at the uniform rate of 10 feet in a ſecond, which is nearly equal to the velocity he acquires by falling or leaping from the height only of 17 inches, and which it is preſumed he may do with ſafety. Now in order to deſcend with any uniform velocity, the reſiſtance of the air muſt be equal to the whole weight that deſcends: then ſuppoſe the weight of the aeronaut to be 150 pounds, and that the parachute is flat, and circular, and made of ſuch materials as that every ſquare foot of its ſurface weighs 2 ounces, and farther that the weight increaſes in the ſame proportion as the ſurface; then the diameter of the parachute neceſſary to deſcend with the moderate velocity of 10 feet per ſecond, muſt be upwards of 78 feet in diameter: but if the parachute be not a flat ſurface, but concave on the lower ſide, its power will be rather the greater, and the diameter may be ſomewhat leſs. If it be required to know the power of a flat circular parachute, or what reſiſtance it meets with from air of a mean denſity, when deſcending with a given velocity; ſay as the number 800 is to the ſquare of the velocity in feet, ſo is the ſquare of the diameter in feet, to a fourth number, which will be the reſiſtance in pounds. And hence, if it be required to know with what velocity a parachute will deſcend with a given weight; ſay as the given diameter is to the ſquare root of the weight, ſo is the number $28\frac{1}{3}$ to a fourth term, which will be the velocity when the deſcent is in air of a mean denſity. So if the diameter of a balloon be 50, and its weight together with that of a man be 530 pounds, the ſquare root of which is 23 very nearly; then as $50 : 23 :: 28\frac{1}{3} : 13$, ſo that the man and parachute will deſcend with the velocity of 13 feet per ſecond; which it is preſumed he may ſafely do, as he would meet with a ſhock only equal to that of leaping freely from a height only of 2 feet 2 inches.

The methods of extracting inflammable air from various ſubſtances, for filling balloons, and for other purpoſes, may be ſeen under the words Air and Gas. And as to the methods of filling and conſtructing balloons, being matters merely mechanical, they are omitted in this place.

Ample information however on theſe, and many other particulars, may be met with in ſeveral books expreſsly written on the ſubject; as in Cavallo's Hiſtory and Practice of Aeroſtation, 8vo, 1785; in Baldwin's Airopaidia, 8vo, 1786; &c.

It has often been diſcuſſed, ſays the former of theſe gentlemen, whether the preference ſhould be given to machines raiſed by inflammable air, or to thoſe raiſed by heated air. Each of them has its peculiar advantages and diſadvantages; a juſt conſideration of which ſeems to decide in favour of thoſe made with inflammable air. The principal comparative advantages of the other ſort are, that they do not require to be made of ſo expenſive materials; that they are filled with little or no expence; and that the combuſtibles neceſſary to fill them are found almoſt every where; ſo that when the ſtock of fuel is exhauſted, the aeronaut may deſcend and recruit it again, in order to proceed on his voyage. But then this ſort of machines muſt be made larger than the other, to take up the ſame weight; and the preſence of a fire is a continual trouble, and a continual danger: in fact, among the many aerial voyages that have been made and attempted with ſuch machines, very few have ſucceeded without an inconvenience, or an accident; and ſome indeed have been attended with dangerous and even fatal conſequences; from which the other ſort is in a great meaſure exempt. But, on the other hand, the inflammable air balloon muſt be made of a ſubſtance impermeable to the ſubtle gas: the gas itſelf cannot be produced without a conſiderable expence; and it is not eaſy to find the materials and apparatus neceſſary for the production of it in every place. However, it has been found that an inflammable-air balloon, of 30 feet in diameter, may be made ſo cloſe as to ſuſtain two perſons, and a conſiderable quantity of ballaſt, in the air for more than 24 hours, when properly managed; and poſſibly one man might be ſupported by the ſame machine for three days: and it is probable that the ſtuff for theſe balloons may be ſo far improved, as to be quite impermeable to the gas, or very nearly ſo; in which caſe, the machine, once filled, would continue to float for a long ſpace of time. At Paris they have already attained to a great degree of perfection in this point; and ſmall balloons have been kept floating in a room for many weeks, without loſing any conſiderable quantity of their levity: but the difficulty lies in the large machines: for in theſe, the weight of the ſtuff itſelf, with the weight and ſtreſs of the ropes and boat, and the folding them up, may eaſily crack and rub off the varniſh, and make them leaky.

In regard to philoſophical obſervations, derived from the new ſubject of aeroſtation, there have been very few made; the novelty of the diſcovery, and of the proſpect enjoyed from the car of an aeroſtatic machine, have commonly diſtracted the attention of the aeronauts; not to remark that many of the adventurers were inadequate to the purpoſe of making improvements in philoſophy, being moſtly influenced either by pecuniary motives, or the vanity of adding their names to the liſt of aerial travellers.—The agreeable ſtillneſs and tranquillity experienced aloft in the atmoſphere, have been matter of general obſervation. Some machines have aſcended to a great height, as far, it has been ſaid, as two miles; and they have commonly paſſed through fogs and clouds, above which they have enjoyed the clear light and heat of the ſun, whilſt the earth beneath was actually covered by denſe clouds, which poured down abundance of rain. In aſcending very high, the aeronauts have often experienced a pain in their ears, ariſing, it is ſuppoſed, from the internal air being not of the ſame denſity as the air without; but the pain uſually went off in a ſhort time: and it ſeems that this effect is ſimilar to what is experienced by perſons who deſcend by a diving-bell to conſiderable depths in the ſea: I remember often to have heard the late unfortunate Mr. Spalding, the celebrated diver, ſpeak of this effect, with a marked and philoſophical accuracy: after deſcending two or three fathoms below the ſurface, he began to feel a pain in his ears, which gradually increaſed to a very great degree if the deſcent was too quick; his method was therefore to deſcend ſlowly, and to make a ſtop for ſome minutes at the depth of 5 fathom, which is equal nearly to the preſſure of the atmoſphere, and where conſequently the air in his bell was of double the denſity of common air at the ſurface; after reſting here awhile, his ears, as he expreſſed it, gave a crack, and he was ſuddenly relieved of the pain. He then deſcended 5 fathoms more, with the ſame ſymptoms, and the ſame effect: and ſo on continually, from one five fathoms to another, deſcending leiſurely, and ſtopping a little at each ſtage, to give time for his conſtitution to adapt itſelf to the degree of condenſation of the air; after which he felt no more inconvenience, till he came to aſcend again, which was performed with the ſame caution and circumſtances. One experiment is recorded, in which the air of a high region, being brought down, and examined by means of nitrous air, was found to be purer than the air below. The temperature of the upper regions too, it has been found, is much colder than that of the air near the earth; the thermometer, in ſome aeroſtatic machines, having deſcended many degrees below the freezing point of water, while it was conſiderably higher than that degree at the earth's ſurface.

ÆSTIVAL, ſee Estival.

ÆSTUARY, or Estuary, in *Geography*, an arm of the ſea, running up a good way into the land. Such as Briſtol channel, many of the friths in Scotland, and ſuch like.

ÆTHER, ſee Ether.

AFFECTED, or Adfected, Equation, in *Algebra*, is an equation in which the unknown quantity riſes to two or more ſeveral powers or degrees. Such, for example, is the equation $x^3 - px^2 + qx = r$, in which there are three different powers of x, namely, x^3, x^2, and x.

The term, affected, is alſo uſed ſometimes in algebra, when ſpeaking of quantities that have co-efficients:

Thus in the quantity $2a$, a is ſaid to be affected with the co-efficient 2.

It is alſo ſaid, that an algebraic quantity is affected with the ſign + or −, or with a radical ſign; meaning no more than that it has the ſign + or −, or that it includes a radical ſign.

The term adfected, or affected, I think, was introduced by Vieta.

AFFECTION, in *Geometry*, a term uſed by ſome ancient writers, ſignifying the ſame as property.

Affection. *Phyſ.* The affections of a body are certain modifications occaſioned or induced by motion ; in virtue of which the body is diſpoſed after ſuch, or ſuch a manner.

The affections of bodies, are ſometimes divided into *primary* and *ſecondary*.

Primary Affections, are thoſe which ariſe either out of the idea of matter, as magnitude, quantity, and figure ; or out of the idea of form, as quality and power ; or out of both, as motion, place, and time.

Secondary, or *derivative Affections*, are ſuch as ariſe out of primary ones, as diviſibility, continuity, contiguity, &c, which ariſe out of quantity ; regularity, irregularity, &c, which ariſe out of figure, &c.

AFFIRMATIVE QUANTITY, or POSITIVE QUANTITY, one which is to be added, or taken effectively ; in contradiſtinction to one that is to be ſubtracted, or taken defectively.—The term affirmative was introduced by Vieta.

AFFIRMATIVE SIGN, or POSITIVE SIGN, in *Algebra*, the ſign of addition, thus marked +, and is called *plus*, or *more*, or *added to*. When ſet before any ſingle quantity, it ſerves to denote that it is an affirmative or a poſitive quantity ; when ſet between two or more quantities, it denotes their ſum, ſhewing that the latter are to be added to the former. So $+ 6$, and $+ a$, and $+ AB$, are affirmative quantities ; alſo $+ 6 + 8 + 10$ denote the ſum of 6, 8, and 10, which is 24, and are read thus, 6 plus 8 plus 10. Alſo $a + b + c$ denote the ſum of the quantities repreſented by a, b and c, when added together. It ſeems now not eaſy to aſcertain with certainty, when, or by whom, this ſign was firſt introduced ; but it was probably by the Germans, as I find it firſt uſed by Stifelius in his Arithmetic, printed in 1544.

The early writers on Algebra uſed the word *plus* in Latin, or *piu* in Italian, for addition, and afterwards the initial *p* only, as a contraction ; like as they uſed *minus*, or *meno*, or the initial *m* only, for ſubtraction : and thus theſe operations were denoted in Italy by Lucas de Burgo, Tartalea, and Cardan, while the ſigns + and − were employed much about the ſame time in Germany by Stifelius, Scheubelius, and others, for the ſame operations.

AGE, in *Chronology*, is uſed for a century, being a ſyſtem or a period of a hundred years.

Chronologiſts alſo divide the time ſince the creation of the world into three ages: The firſt, from Adam till Moſes, which they call the age of nature ; the ſecond from Moſes to Jeſus Chriſt, called the age of the law ; and the third, or age of grace, from Jeſus Chriſt till the end of the world.

AGE *of the Moon*, in Aſtronomy, is the number of days elapſed ſince the laſt new moon. To find the moon's age, for any time nearly, for ordinary uſes ; add together the epact, the day of the current month, and the number of months from March to the preſent month incluſive ; the ſum is the moon's age : but if the ſum exceed 29, deduct 29 from it in months that have 30 days, or 30 in thoſe that have 31 ; and the remainder will be the age.—At the end of 19 years the moon's age returns upon the ſame day of the month, but falls a little ſhort of the ſame hour of the day.

AGENT, *Agens*, in *Phyſics*, that by which a thing is done or effected ; or any thing having a power by which it acts on another, called the patient, or by its action induces ſome change in it.

AGGREGATE, the ſum or reſult of ſeveral things added together. See *Sum*.

AGITATION, in *Phyſics*, a briſk inteſtine motion, excited among the particles of a body. Thus fire agitates the ſubtleſt particles of bodies. Fermentations, and efferveſcences, are produced by a briſk agitation of the particles of the fermenting body.

AGUILON (FRANCIS), or AQUILON, was a jeſuit of Bruſſels, and profeſſor of philoſophy at Doway, and of theology at Antwerp. He was one of the firſt that introduced mathematical ſtudies into Flanders. He wrote a large work on *Optics*, in 6 books, which was publiſhed in folio, at Antwerp, in 1613 ; and a treatiſe of *Projections of the Sphere*. He promiſed alſo to treat upon *Catoptrics* and *Dioptrics*, but this was prevented by his death, which happened at Seville, in the year 1617.

AIR, in *Phyſics*, a thin, fluid, tranſparent, elaſtic, compreſſible, and dilatable body, which ſurrounds this terraqueous globe, and covers it to a conſiderable height.

Some of the ancients conſidered air as an element, namely, one of the four elements, air, earth, water, and fire, of which they conceived all bodies to be compoſed ; and though it be certain that air, taken in the common acceptation, be far from the ſimplicity of an elementary ſubſtance, yet ſome of its parts may properly be ſo called. So that air may be diſtinguiſhed into proper or elementary, and vulgar or heterogeneous.

Elementary AIR, or *Air* properly ſo called, is a ſubtle, homogeneous, elaſtic fluid ; being the baſis, or fundamental ingredient, of the whole air of the atmoſphere, from which it takes its name. And in this ſenſe Dr. Hales, and other modern philoſophers, conſider it as entering into the compoſition of moſt, or perhaps all bodies ; exiſting in them in a ſolid ſtate, devoid of its elaſticity, and moſt of its diſtinguiſhing properties, and ſerving as their cement ; but, by certain proceſſes, capable of being diſengaged from them, recovering its elaſticity, and reſembling the air of our atmoſphere.

The particular nature of this aerial matter we know but little about : what authors have ſaid concerning it being chiefly conjectural. There is no way of examining air pure and defecated from the ſeveral matters with which it is mixed ; and conſequently we cannot pronounce what are its peculiar properties, abſtractedly from other bodies.

Dr. Hook, and ſome others, maintain that it is the ſame with the *ether*, or that imaginary fine, fluid, active matter, conceived to be diffuſed through the whole expanſe of the celeſtial regions : which comes to much the ſame thing as Newton's ſubtle medium, or ſpirit. In this ſenſe it is ſuppoſed to be a body *ſui generis*, incorruptible, immutable, incapable of being generated, but preſent in all places, and in all bodies.

Other philoſophers place its eſſence in elaſticity, making that its diſtinctive character. Theſe ſuppoſe that it may be generated, and that it is nothing elſe but the matter of other bodies, rendered by the changes it has undergone, ſuſceptible of a permanent elaſticity. Mr. Boyle produces a number of experiments, which he made on the production of air, that is, according to him, the extraction of a ſenſible quantity of air from a body in which there appeared to be little or none at all, by whatever means this may be effected. He obſerves that among the different methods for this purpoſe, the chief are fermentation, corroſion, diſſolution, decompoſition, ebullition of water and other fluids, the reciprocal action of bodies, eſpecially ſaline ones, upon one another; he adds, that different ſolid and mineral bodies, in the parts of which no elaſticity could be conceived to exiſt, being plunged into corroſive mediums, which alſo are quite unelaſtic, will, by the attenuation of their parts from their mutual colliſion, produce a conſiderable quantity of elaſtic air.

Sir Iſaac Newton is of the ſame opinion, according to whom the particles of a denſe compact fixed ſubſtance, adhering to each other by a powerful attractive force, cannot be ſeparated but by a violent heat, and perhaps never without fermentation; and theſe bodies, rarefied by heat and fermentation, are finally transformed into a truly permanent elaſtic air. On theſe principles, he adds, gunpowder produces air on exploſion. Optics, Qu. 31, &c.

Common, or *heterogeneous* Air, is an aſſemblage of corpuſcles of various kinds, which together conſtitute one fluid maſs, in which we live and move, and which we conſtantly breathe; which compound maſs altogether, is called the atmoſphere.

In this popular and extenſive meaning of the term, Mr. Boyle acknowledges that air is the moſt heterogeneous body in the univerſe; and Boerhaave proves that it is an univerſal chaos, a mere jumble of all ſpecies of created things. Beſides the matter of light or fire, which continually flows into it from the celeſtial bodies, and perhaps the magnetic effluvia of the earth, whatever fire can volatilize muſt be found in the air.

Hence, for inſtance, 1. All ſorts of vegetable matter muſt be contained in the air; being either exhaled from plants growing all over the face of the earth, or rendered volatile by putrefaction, not excepting even the more ſolid and vaſcular parts of them.

2. It is no leſs certain that the air muſt contain particles of every ſubſtance belonging to the animal kingdom. For the copious emanations which are perpetually iſſuing from the bodies of animals, in the perſpiration conſtantly kept up by the vital heat, are abſorbed by the air; and in ſuch quantities too, during the courſe of an animal life, that, could they be recollected, they would be ſufficient to compoſe a good round number of the like animals. And beſides, when a dead animal continues expoſed to the air, all its particles evaporate, and are quickly diſſipated; ſo that the ſubſtance which compoſed the animal, is almoſt wholly incorporated with the air.

3. The whole foſſil kingdom muſt neceſſarily be found in the atmoſphere; for all of that kind, as ſalts, ſulphurs, ſtones, metals, &c. are convertible into fume, and muſt conſequently take place among aerial ſubſtances. Gold itſelf, the moſt fixed of all natural bodies, is found among ores, cloſely adhering to ſulphurs in mines, and ſo is raiſed along with the mineral.

Of all the emanations which float in the vaſt ocean of the atmoſphere, perhaps the principal are ſuch as conſiſt of ſaline particles. Many writers ſuppoſe that they are of a nitrous kind; but it is probable that they are of all ſorts, as vitriol, alum, marine ſalt, and many others. And Mr. Boyle thinks that there may be great quantities of compound ſalts, not to be met with on or in the bowels of the earth, formed by the fortuitous concourſe and mixture of different ſaline ſpirits.

We often find the window-glaſs of old buildings corroded, as if eaten by worms; though we know of no particular ſalt that is capable of producing ſuch an effect.

Sulphurs too muſt make a conſiderable portion of this compound maſs, on account of the many volcanos, grotts, caverns, and mines, diſperſed over the face of the globe.

Finally, the various attritions, ſeparations, diſſolutions, and other mutual operations of matter of different ſorts upon one another, may be regarded as the ſources of many other neutral, or anonymous bodies, unknown to us, which riſe and float in the air.

Air, taken in this extenſive ſenſe, is one of the moſt general and conſiderable agents in nature; being concerned in the preſervation of animal and vegetable life, and in the production of moſt of the phenomena that take place in the material world.

Its properties and effects, having been the principal objects of the reſearches and diſcoveries of modern philoſophers, have been reduced to preciſe laws and demonſtrations, forming no inconſiderable branch of mixed mathematics, under the titles of Pneumatics, Aerometry, &c.

Mechanical Properties and Effects of Air. Of theſe the moſt conſiderable are its fluidity, its weight, and its elaſticity.

1. *Its Fluidity.*—The great fluidity of the air is manifeſt from the great facility with which bodies traverſe it; as in the propagation of, and eaſy conveyance it affords to, ſounds, odours and other effluvia and emanations that eſcape from bodies: for theſe effects prove that it is a body whoſe parts give way to any force, and in yielding are eaſily moved amongſt themſelves; which is the definition of a fluid. That the air is a fluid is alſo proved from this circumſtance, that it is found to exert an equal preſſure in all directions; an effect which could not take place otherwiſe than from its extreme fluidity. Neither has it been found that the air can be deprived of this property, whether it be kept for many years together confined in glaſs veſſels, or be expoſed to the greateſt natural or artificial cold, or condenſed by the moſt powerful preſſure; for in none of theſe circumſtances has it ever been reduced to a ſolid ſtate. It is true indeed that real permanent air may be extracted from ſolid bodies, and may alſo be abſorbed by them; and we alſo know that in this caſe it muſt be exceedingly condenſed, and reduced to a bulk many hundred times leſs than in its natural ſtate: but in what form it exiſts in thoſe bodies, or how their particles are combined together, is a myſtery which remains hitherto inexplicable.

Thoſe philoſophers who, with the Carteſians, make fluidity to conſiſt in a perpetual inteſtine motion of the parts, think they can prove that this character belongs to air: thus, in a darkened room, where the repreſentations of external objects are introduced by a ſingle ray, the corpuſcles with which the air is replete, are ſeen to be in a continual fluctuation. Some moderns attribute the fluidity of the air, to the fire which is intermixed with it; without which, ſay they, the whole atmoſphere would harden into a ſolid impenetrable maſs: and indeed it muſt be allowed that the more fire it contains, the greater will its fluidity, mobility, and permeability be; and according as the different poſitions of the ſun augment or diminiſh the degree of fire, the air always receives a proportional temperature, and is kept in a continual reciprocation.

2. *Its Weight or Gravity.*—The weight or gravity of the air, is a property belonging to it as a body; for gravity is a property eſſential to matter, or at leaſt a property found in all bodies. But independent of this, we have many direct proofs of its gravity from ſenſe and experiment: thus, the hand laid cloſe upon the end of a veſſel, out of which the air is drawn at the other end, ſoon feels the load of the incumbent atmoſphere: thus alſo, thin glaſs veſſels, exhauſted of their air, are eaſily cruſhed to pieces by the weight of the external air: and ſo two hollow ſegments of a ſphere, 4 inches in diameter, exactly fitting each other, being emptied of air, are, by the weight of the ambient air, preſſed together with a force which requires the weight of 188 pounds to ſeparate them; and that they are thus forcibly held together by the preſſure of the air, is made evident by ſuſpending them in an exhauſted receiver, for then they quickly ſeparate of themſelves, and fall aſunder. Again, if a tube, cloſe at one end, be filled with quickſilver, and the open end be immerged in a baſon of the ſame fluid, and ſo held upright, the quickſilver in the tube will be kept raiſed up in it to the height of about 30 inches above the ſurface of that in the baſon, being ſupported and balanced by the preſſure of the external air upon that ſurface: and that this is the cauſe of the ſuſpenſion of the quickſilver in the tube, is made evident by placing the whole apparatus under the receiver of an air-pump; for then the fluid will deſcend in the tube in proportion as the receiver is exhauſted of its air; and then on gradually letting in the air again, the quickſilver reaſcends to its former height in the tube: and this is what is called, from its inventor, the Terricellian experiment. Nay farther, air can actually be weighed like any other body: for a rigid veſſel, full even of common air, by a nice balance is found to weigh more than when the air is exhauſted from it; and the effect is proportionally more ſenſible, if the veſſel be weighed full of condenſed air, and more ſtill if it be weighed in a receiver void of air.

But although we have innumerable proofs of the gravitating property of the air, yet the full diſcovery of the laws and circumſtances of it are certainly due to the moderns. It cannot indeed be denied, that ſeveral of the ancients had ſome confuſed notions about this property: thus Ariſtotle ſays that all the elements have gravity, and even air itſelf; and as a proof of it, ſays that a bladder inflated with air, weighs more than the ſame when empty; and Plutarch and Stobæus quote him as teaching that the air in its weight is between that of fire and of earth; and farther, he himſelf, treating of reſpiration, reports it as the opinion of Empedocles, that he aſcribes the cauſe of it to the weight of the air, which by its preſſure forces itſelf into the lungs; and much in the ſame way are the ſentiments of Aſclepiades expreſſed by Plutarch, who repreſents him as ſaying, among other things, that the external air, by its weight, forcibly opened its way into the breaſt. But nevertheleſs it is certain, however unreaſonable it may ſeem, that Ariſtotle's followers departed in this inſtance from their maſter, by aſſerting the contrary for many ages together. Indeed many of the phenomena ariſing from this property, have been remarked from the higheſt antiquity. Many centuries ſince, it was known that by ſucking the air from an open pipe, having its extremity immerſed in water, this fluid riſes above its level, and occupies the place of the air. In conſequence of ſuch obſervations, ſucking pumps were contrived, and various other hydraulic machines; as Heron's ſyphons, deſcribed in his Spiritalia or Pneumatics, and the watering-pots known in Ariſtotle's time under the name of *clepſydræ*, which alternately ſtop or run as the finger cloſes or opens their upper orifice. Indeed the reaſon aſſigned, by philoſophers many ages after, for this phenomenon, was a pretended horror that nature conceives for a vacuum, which, rather then endure it, makes a body aſcend contrary to the powerful ſolicitation of its gravity. Even Galileo, with all his ſagacity, could not for ſome time hit upon any thing more ſatisfactory; for he only aſſigned a limit to this dread of vacuity: having obſerved that ſucking pumps would not raiſe water higher than 16 braſſes, or 34 Engliſh feet, he limited this abhorring force of nature, to one that was equivalent to the weight of a column of water 34 feet high, on the ſame baſe as the void ſpace. Conſequently he pointed out a way of making a vacuum, by means of a hollow cylinder, whoſe piſton is charged with a weight ſufficient to detach it from the cloſe bottom turned upwards: this effort he called the meaſure of the force of vacuity, and made uſe of it for explaining the coheſion of the parts of bodies.

Galileo however was well appriſed of the weight of the air as a body: in his Dialogues he ſhews two ways of demonſtrating it, by weighing it in bottles: the tranſition was eaſy from one diſcovery to another: yet ſtill Galileo's knowledge of the matter was imperfect, that is, as to the particular inſtance of the ſuſpenſion of a fluid above its level, by the preſſure of the external air.

At length Torricelli fell upon the lucky gueſs, that the counterpoiſe which keeps fluids above their level, when nothing preſſes upon their internal ſurface, is the maſs of air reſting upon the external one. He diſcovered it in the following manner: In the year 1643 this diſciple of Galileo, on occaſion of executing an experiment on the vacuum formed in pumps, above the column of water, when it exceeds 34 feet, thought of uſing ſome heavier fluid, ſuch as quickſilver. He conceived that whatever might be the cauſe by which a column of water of 34 feet high is ſuſtained above its level, the ſame force would ſuſtain a column of any other fluid, which weighed as much as that column of water, on the ſame baſe; whence he concluded that quickſilver, being about 14 times as heavy as water, would not be ſuſtained higher than 29 or 30 inches.

He therefore took a glaſs tube of ſeveral feet in length, ſealed it hermetically at one end, and filled it with quickſilver; then inverting it, and holding it upright, by preſſing his finger againſt the lower or open orifice, he immerſed that end in a veſſel of quickſilver; then removing his finger, and ſuffering the fluid to run out, the event verified his conjecture; the quickſilver, faithful to the laws of hydroſtatics, deſcended till the column of it was about 30 inches high above the ſurface of that in the veſſel below. And hence Torricelli concluded that it was no other than the weight of the air incumbent on the ſurface of the external quickſilver, which counterbalanced the fluid contained in the tube.

By this experiment Torricelli not only proved, what Galileo had done before, that the air had weight, but alſo that it was its weight which kept water and quickſilver raiſed in pumps and tubes, and that the weight of the whole column of it was equal to that of a like column of quickſilver of 30 inches high, or of water 34 or 35 feet high; but he did not aſcertain the weight of any particular quantity of it, as a gallon, or a cubic foot of it, nor its ſpecific gravity to water, which had been done by Galileo, though to be ſure with no great accuracy, for he only proved that water was more than 400 times heavier than air.

Torricelli's experiment became famous in a ſhort time. Father Merſenne, who kept up a correſpondence with moſt of the literati in Italy, was informed of it in 1644, and communicated it to thoſe of France, who preſently repeated the experiment: Meſſrs. Paſcal and Petit made it firſt, and varied it ſeveral ways; which gave occaſion to the ingenious treatiſe which Paſcal publiſhed at 23 years of age, intitled *Experiences Nouvelles touchant la Vuide*. In this treatiſe indeed he makes uſe of the old principle of *fuga vacui;* but afterwards getting ſome notion of the weight of the air, he ſoon adopted Torricelli's idea, and deviſed ſeveral experiments to confirm it. One of theſe was to procure a vacuum above the reſervoir of quickſilver; in which caſe he found the column ſink down to the common level: but this appearing to him not ſufficiently powerful to diſſipate the prejudices of the ancient philoſophy, he prevailed on M. Perier, his brother-in-law, to execute the famous experiment of Puy-de-Domme, who found that the height of the quickſilver half-way up the mountain was leſs, by ſome inches, than at the foot of it, and ſtill leſs at the top: ſo that it was now put out of doubt that it was the weight of the atmoſphere which counterpoiſed the quickſilver.

Des Cartes too had a right notion of this effect of the air, to ſuſtain fluids above their level, as appears by ſome of his letters about this time, and ſome years before; and in one of thoſe he lays claim to the idea of the Puy-de-Domme experiment: After having deſired M. de Carcavi to inform him of the ſucceſs of that experiment, which public rumour had advertiſed him had been made by M. Paſcal himſelf, he adds, "I had reaſon to expect this from him, rather than from you, becauſe I firſt propoſed it to him two years ſince, aſſuring him at the ſame time, that although I had not tried it, yet I could not doubt of the conſequence; but as he is a friend of M. Roberval, who profeſſes himſelf no friend to me, I ſuppoſe he is guided by that gentleman's paſſions." See more of this hiſtory under BAROMETER.

As to the actual weight of any given portion of common air, it ſeems that Galileo was the firſt who determined it experimentally; and he gives two different methods, in his Dialogues, for weighing it in bottles: he did not however perform the experiment very accurately, as he ſtated from the reſult that the gravity of water was to that of air rather above 400 to 1.

A quantity of air was next weighed by Merſenne in a very ingenious manner. His idea was to weigh a veſſel both when full of air, and when emptied of it: to make the vacuum for this purpoſe, he knew no better way than by expelling the air out of an eolipile by heating it red hot: by weighing it both when cold and hot, he found a certain difference; which however was not the exact weight of that capacity of air, becauſe the vacuum was not perfect. But by plunging the eolipile, when red hot, into water, juſt ſo much water entered as was equal in bulk to the air that had been expelled; then he took it out and weighed it with the water, which gave the weight of the ſame bulk of water; and on comparing this with the former difference, or weight of air expelled, he found their proportion to be as 1300 to 1. Which is as wide of the truth as Galileo's proportion, namely 400 to 1, but the contrary way. And it is remarkable that the mean between the two, namely 850 to 1, is very near the true proportion as ſettled by other more accurate experiments.

Mr. Boyle, by a more accurate experiment, found the proportion to be that of 938 to 1. And Mr. Hauksbee found it as 850 to 1, proceeding on the ſame principles as Merſenne, with a three-gallon glaſs bottle, but extracting the air out of it with the air pump, inſtead of expelling it by fire; the height of the barometer being at that time 29·7 inches. Alſo by other accurate experiments made before the Royal Society by Mr. Hauksbee, Dr. Halley, Mr. Cotes, and others, the proportion was always between 800 and 900 to 1, but rather nearer the latter, namely, being firſt found as 840 to 1, then as 852 to 1, and a third time as 860 to 1; the barometer then ſtanding at $29\frac{3}{4}$ inches, and the weather warm. Mr. Cavendiſh determines the ratio 800 to 1, the barometer being $29\frac{3}{4}$, and the thermometer at 50°; and Sir George Shuckburgh, by a very accurate experiment, finds it 836 to 1, the barometer being at that time at 29·27, and the thermometer at 51°. And the medium of all theſe is about 832 or or 833 to 1, when reduced to the preſſure of 30 inches of the barometer, and the mean temperature 55° of the thermometer. Upon the whole therefore it may be ſafely concluded that, when the barometer is at 30 inches, and the thermometer at the mean temperature 55°, the denſity or gravity of water is to that of air, as $833\frac{1}{3}$ to 1, that is as $\frac{2500}{3}$ to 1, or as 2500 to 3; and that for any changes in the height of the barometer, the ratio varies proportionally; and alſo that the denſity of the air is altered by the $\frac{1}{440}$th part for every degree of the thermometer above or below temperate.

This number, which is a very good medium among them all, I have choſen with the fraction $\frac{1}{3}$, becauſe it gives exactly $1\frac{1}{5}$ ounce for the mean weight of a cubic foot of air, the weight of the cubic foot of water being juſt 1000 ounces averdupois, and that of quickſilver equal to 13600 ounces.

Air, then, having been ſhewn to be a heavy fluid ſubſtance, the laws of its gravitation and preſſure muſt be the ſame as thoſe of water and other fluids; and conſequently its preſſure muſt be proportional to its perpendicular altitude. Which is exactly conformable to experiment; for on removing the Terricellian tube to different heights, where the column of air is ſhorter, the column of quickſilver which it ſuſtains is ſhorter alſo, and that nearly at the rate of 100 feet for $\frac{1}{10}$ of an inch of quickſilver. And on theſe principles depend the ſtructure and uſe of the barometer.

And from the ſame principle it likewiſe follows that air, like other fluids, preſſes equally in all directions. And hence it happens that ſoft bodies endure this preſſure without change of figure, and hard or brittle bodies without breaking; being equally preſſed on all parts; but if the preſſure be taken off, or diminiſhed, on one ſide, the effect of it is immediately perceived on the other. See ATMOSPHERE, for the total quantity of effects and preſſure, and the laws of different altitudes, &c.

From the weight and fluidity of the air, jointly conſidered, many effects and uſes of it may eaſily be deduced. By the combination of theſe two qualities, it cloſely inveſts the earth, with all the bodies upon it, conſtringing and binding them down with a great force, namely a preſſure equal to about 15 pounds upon every ſquare inch. Hence, for example, it prevents the arterial veſſels of plants and animals from being too much diſtended by the impetus of the circulating juices, or by the elaſtic force of the air ſo copiouſly abounding in them. For hence it happens, that on a diminution of the preſſure of the air, in the operation of cupping, we ſee the parts of the body grow tumid, which cauſes an alteration in the circulation of the fluids in the capillary veſſels. And the ſame cauſe hinders the fluids from tranſpiring through the pores of their containing veſſels, which would otherwiſe cauſe the greateſt debility, and often deſtroy the animal. To the ſame two qualities of the air, weight and fluidity, is owing the mixture of bodies contiguous to one another, eſpecially fluids; for ſeveral liquids, as oils and ſalts, which readily mix of themſelves in air, will not mix at all in vacuo. With many other natural phenomena.

3. *Elaſticity.* Another quality of the air, from whence ariſe a multitude of effects, is its elaſticity; a quality by which it yields to the preſſion of any other bodies, by contracting its volume; and dilates and expands itſelf again on the removal or diminution of the preſſure. This quality is the chief diſtinctive property of air, the other two being common to other fluids alſo.

Of this property we have innumerable inſtances. Thus, for example, a blown bladder being ſqueezed in the hand, we find a ſenſible reſiſtance from the included air; and upon taking off the preſſure, the compreſſed parts immediately reſtore themſelves to their former round figure. And on this property of elaſticity depend the ſtructure and uſes of the air-pump.

Every particle of air makes a continual effort to dilate itſelf, and ſo it acts forcibly againſt all the neighbouring particles, which alſo exert the like force in return; but if their reſiſtance happen to ceaſe, or be weakened, the particle immediately expands to an immenſe extent. Hence it is that thin glaſs bubbles, or bladders, filled with air, and placed under the receiver of an air-pump, do, upon pumping out the air, burſt aſunder by the force of the air which they contain. So likewiſe a cloſe flaccid bladder, containing only a ſmall quantity of air, being put under the receiver, ſwells as the receiver is exhauſted, and at length appears quite full. And the ſame thing happens by carrying the flaccid bladder to the top of a very high mountain.

The ſame experiment ſhews that this elaſtic property of the air is very different from the elaſticity of ſolid bodies, and that theſe are dilated after a different manner from the air. For when air ceaſes to be compreſſed, it not only dilates, but then occupies a far greater ſpace, and exiſts under a volume immenſely greater than before; whereas ſolid elaſtic bodies only reſume the figure they had before they were compreſſed.

It is plain that the weight or preſſure of the air does not at all depend on its elaſticity, and that it is neither more nor leſs heavy than if it were not at all elaſtic. But from its being elaſtic, it follows that it is ſuſceptible of a preſſure, which reduces it to ſuch a ſpace, that the force of its elaſticity, which re-acts againſt the preſſing weight, is exactly equal to that weight. Now the law of the elaſticity is ſuch, that it increaſes in proportion to the denſity of the air, and that its denſity increaſes in proportion to the forces or weights which compreſs it. But there is a neceſſary equality between action and re-action; that is, the gravity of the air, which effects its compreſſion, and the elaſticity of it, which gives it its tendency to expanſion, are equal.

So that, the elaſticity increaſing or diminiſhing, in the ſame proportion as the denſity increaſes or diminiſhes, that is, as the diſtance between its particles decreaſe or increaſe; it is no matter whether the air be compreſſed, and retained in any ſpace, by the weight of the atmoſphere, or by any other cauſe; as in either caſe it muſt endeavour to expand with the ſame force. And therefore, if ſuch air as is near the earth be incloſed in a veſſel, ſo as to have no communication with the external air, the preſſure of ſuch incloſed air will be exactly equal to that of the whole external atmoſphere. And accordingly we find that quickſilver is ſuſtained to the ſame height, by the elaſtic force of air incloſed in a glaſs veſſel, as by the whole preſſure of the atmoſphere.—And on this principle of the condenſation and elaſticity of the air, depends the ſtructure and uſe of the air-gun.

That the denſity of the air is always directly proportional to the force or weight which compreſſes it, was proved by Boyle and Mariotte, at leaſt as far as their experiments go on this head: and Mr. Mariotte has ſhewn that the ſame rule takes place in condenſed air. However, this rule is not to be admitted as ſcrupulouſly exact; for when air is very forcibly compreſſed, ſo as to be reduced to $\frac{1}{4}$th of its ordinary bulk, the effect does not anſwer preciſely to the rule; for in this caſe the air begins to make a greater reſiſtance, and requires a ſtronger compreſſion, than according to the rule. And hence it would ſeem, that the particles of air cannot, by means of any poſſible weight or preſſure, how

how great ſoever, be brought into perfect contact, or that it cannot thus be reduced to a ſolid maſs; and conſequently that there muſt be a limit to which this condenſation of the air can never arrive. And the ſame remark is true with regard to the rarefaction of air, namely, that in very high degrees of rarefaction, the elaſticity is decreaſed rather more than in proportion to the weight or denſity of the air: and hence there muſt alſo be a limit to the rarefaction and expanſion of the air, by which it is prevented from expanding to infinity.

We know not however how to aſſign thoſe limits to the elaſticity of the air, nor to deſtroy or alter it, without changing the very nature of air, which is effected by chemical proceſſes. To what degree air is ſuſceptible of condenſation, by compreſſion, is not certainly known. Mr. Boyle condenſed it 13 times more than in its natural ſtate, by this means: others have compreſſed it into $\frac{1}{60}$th part of its ordinary volume; Dr. Hales made it 38 times more denſe, by means of a preſs; but by freezing water in a hollow caſt-iron ball or ſhell, he reduced it to 1838 times leſs ſpace than it naturally occupies; in which ſtate it muſt have been of more than twice the denſity or ſpecific gravity of water: And as water is not compreſſible, except in a very ſmall degree, it follows from this experiment, that the particles of air muſt be of a nature very different from thoſe of water; ſince it would otherwiſe be impoſſible to reduce air to a volume above 800 times leſs than in its common ſtate; an inference however which militates directly againſt an aſſertion made by Dr. Halley, from ſome experiments performed in London, and others at Florence by the Academy del Cimento, namely, that it may be ſafely concluded that no force whatever is capable to reduce air into a ſpace 800 times leſs than that which it naturally occupies near the ſurface of the earth.

The elaſticity of the air exerts its force equally in all directions; and when it is at liberty, and freed from the cauſe which compreſſed it, it expands equally in all directions, and in conſequence always aſſumes a ſpherical figure in the interſtices of the fluids in which it is lodged. This is evident in liquors placed in the receiver of an air pump, by exhauſting the air; at firſt there appears a multitude of exceeding ſmall bubbles, like grains of fine ſand, diſperſed through the fluid maſs, and riſing upwards; and as more air is pumped out, they enlarge in ſize; but ſtill they continue round. Alſo if a plate of metal be immerged in the liquor, on pumping, its ſurface will be ſeen covered over with ſmall round bubbles, compoſed of the air which adhered to it, now expanding itſelf. And for the ſame reaſon it is that large glaſs globes are always blown up of a ſpherical ſhape, by blowing air through an iron tube into a piece of melted glaſs at the end of the pipe.

The expanſion of the air, by virtue of its elaſtic property, when only the compreſſing force is taken off, or diminiſhed, is found to be ſurpriſingly great; and yet we are far from knowing the utmoſt dilatation of which it is capable. In ſeveral experiments made by Mr. Boyle, it expanded firſt into 9 times its former ſpace; then into 31 times; then into 60, and then into 150 times. Afterwards, it was brought to dilate into 8000 times its firſt ſpace; then into 10000, and at laſt even into 13679 times its ſpace; and this ſolely by its own natural expanſive force, by only removing the preſſure, but without the help of fire. And on this principle depends the conſtruction and uſe of the MANOMETER.

The elaſticity of the air, under one and the ſame preſſure, is ſtill farther increaſed by heat, and diminiſhed by cold, and that, by ſome late accurate experiments made by Sir George Shuckburgh, at the rate of the 440th part of its volume nearly, for each degree of the variation of heat, from that of temperate, in Fahrenheit's thermometer.

Mr. Hauksbee obſerved that a portion of air incloſed in a glaſs tube, when the temperature was at the freezing point, formed a volume which was to that of the ſame quantity of air in the greateſt heat of ſummer here in England, as 6 to 7. And it has been found by ſeveral experiments, that air is expanded $\frac{1}{3}$ of its natural bulk by applying the heat of boiling water to it.

Dr. Hales found that the air in a retort, when the bottom of the veſſel juſt became red hot, was dilated into twice its former ſpace; and that in a white, or almoſt melting heat, it filled thrice its former ſpace: but Mr. Robins found that air was expanded, by means of the white or fuſing heat of iron, to 4 times its former bulk.

See ſeveral ingenious experiments on the elaſticity of the air, in the Philoſ. Tranſ. for the year 1777, by Sir George Shuckburgh and Colonel Roy.

This properly explains the common effect obſerved on bringing a cloſe flaccid bladder near the fire to warm it; when it is preſently found to ſwell as if more air were blown into it. And upon this principle depends the ſtructure and office of the thermometer; as alſo the air balloons, lately invented by Mr. Montgolfier, for floating in the atmoſphere.

M. Amontons firſt diſcovered that, with the ſame degree of heat, air will expand in a degree proportioned to its denſity. And on this foundation the ingenious author has formed a diſcourſe, to prove "that the ſpring and weight of the air, with a moderate degree of warmth, may enable it to produce even earthquakes, and others of the moſt vehement commotions of nature." He computes that at the depth of the 74th part of the earth's radius below the ſurface, the natural preſſure of the air would reduce to the denſity of gold; and thence infers that all matter below that depth, is probably heavier than the heavieſt metal that we know of. And hence again, as it is proved that the more the air is compreſſed, the more does the ſame degree of fire increaſe the force of its elaſticity; we may infer that a degree of heat, which in our orb can produce only a moderate effect, may have a very violent one in ſuch lower orb; and that, as there are many degrees of heat in nature, beyond that of boiling water, it is probable there may be ſome whoſe violence, thus aſſiſted by the weight of the air, may be ſufficiently powerful to tear aſunder the ſolid globe. Mem. de l'Acad. 1703.

Many philoſophers have ſuppoſed that the elaſtic property of the air depends on the figure of its corpuſcles, which they take to be ramous: ſome maintain that they are ſo many minute *flocculi*, reſembling fleeces of wool: others conceive them rolled up like hoops, and curled like wires, or ſhavings of wood, or coiled like the ſprings of watches, and endeavouring to expand themſelves by virtue of their texture.

But Sir Iſaac Newton (Optics, Qu. 31, &c.) explains the matter in a different way; ſuch a contexture of parts he thinks by no means ſufficient to account for that amazing power of elaſticity obſerved in air, which is capable of dilating itſelf into above a million of times more ſpace than it occupied before: but, he obſerves, as it is known that all bodies have an attractive and a repelling power; and as both theſe are ſtronger in bodies, the denſer, more compact, and ſolid they are; hence it follows that when, by heat, or any other powerful agent, the attractive force is overcome, and the particles of the body ſeparated ſo far as to be out of the ſphere of attraction; the repelling power, then commencing, makes them recede from each other with a ſtrong force, proportionable to that with which they before cohered; and thus they become permanent air.

And hence, he ſays, it is, that as the particles of air are groſſer, and riſe from denſer bodies, than thoſe of tranſient air, or vapour, true air is more ponderous than vapour, and a moiſt atmoſphere lighter than a dry one.

And M. Amontons makes the elaſticity of air to ariſe from the fire it contains; ſo that by augmenting the degree of heat, the rarefaction will be increaſed to a far greater degree than by a mere ſpontaneous dilatation.

The elaſtic power of the air becomes the ſecond great ſource of the remarkable effects of this important fluid. By this property it inſinuates itſelf into the pores of bodies, where, by means of this virtue of expanding, which is ſo eaſily excited, it muſt put the particles of thoſe bodies into perpetual vibrations, and maintain a continual motion of dilatation and contraction in all bodies, by the inceſſant changes in its gravity and denſity, and conſequently its elaſticity and expanſion.

This reciprocation is obſervable in ſeveral inſtances, particularly in plants, in which the tracheæ or air-veſſels perform the office of lungs; for as the heat increaſes or diminiſhes, the air alternately dilates and contracts, and ſo by turns compreſſes the veſſels, and eaſes them again; thus promoting a circulation of their juices. And hence it is found that no vegetation or germination is carried on in vacuo.

It is from the ſame cauſe too, that ice is burſt by the continual action of the air contained in its bubbles. Thus, too, glaſſes and other veſſels are frequently cracked, when their contained liquors are frozen; and thus alſo large blocks of ſtone, and entire columns of marble, ſometimes ſplit in the winter ſeaſon, from ſome little bubble of included air acquiring an increaſed elaſticity: and for the ſame reaſon it is that ſo few ſtones will bear to be heated by a fire, without cracking into many pieces, by the increaſed expanſive force of ſome air confined within their pores. From the ſame ſource ariſe alſo all putrefaction and fermentation; neither of which can be carried on in vacuo, even in the beſt diſpoſed ſubjects. And even reſpiration, and animal life itſelf, are ſuppoſed, by many authors, to be conducted, in a great meaſure, by the ſame principle of the air. And as we find ſuch great quantities of air generated by the ſolution of animal and vegetable ſubſtances, a good deal muſt conſtantly be raiſed from the diſſolution of theſe elements in the ſtomach and bowels.

In fact, all natural corruption and alteration ſeem to depend on air; and even metals, particularly gold, only ſeem to be durable and incorruptible, in ſo far as they are impervious to air.

As to the different kinds of air, with its generation, and the effects of different ingredients of it, &c, they are omitted here, as properly belonging to a Chemical Dictionary, or to a General Dictionary of Arts, &c.

For the reſiſtance of the air, ſee RESISTANCE.

AIR-GUN, in PNEUMATICS, is a machine for propelling bullets with great violence, by the ſole means of condenſed air.

The firſt account we meet with of an air-gun, is in the *Elemens d'Artillerie* of David Rivaut, who was preceptor to Louis XIII. of France. He aſcribes the invention to one Marin, a burgher of Liſieux, who preſented one to Henry IV.

To conſtruct a machine of this kind, it is only neceſſary to take a ſtrong veſſel of any ſort, into which the air is to be thrown or condenſed by means of a ſyringe, or otherwiſe, the more the better; then a valve is ſuddenly opened, which lets the air eſcape by a ſmall tube in which a bullet is placed, and which is thus violently forced out before the air.

It is evident then that the effect is produced by virtue of the elaſtic property of the air; the force of which, as has been ſhewn in the laſt article, is directly proportional to its condenſation; and therefore the greater quantity that can be forced into the engine, the greater will be the effect. Now this effect will be exactly ſimilar to that of a gun charged with powder, and therefore we can eaſily form a compariſon between them: for inflamed gun-powder is nothing more than very condenſed elaſtic air; ſo that the two forces are exactly ſimilar. Now it is ſhewn by Mr. Robins, in his New Principles of Gunnery, that the fluid of inflamed gun-powder, has, at the firſt moment, a force of elaſticity equal to about a 1000 times that of common air; and therefore it is neceſſary that air ſhould be condenſed a 1000 times more than in its natural ſtate, to produce the ſame effect as gun-powder. But then it is to be conſidered, that the velocities with which equal balls are impelled, are directly proportional to the ſquare roots of the forces; ſo that if the air in an air-gun be condenſed only 10 times, then the velocity it will project a ball with, will be, by that rule, $\frac{1}{10}$th of that ariſing from gun-powder; and if the air were condenſed 20 times, it would communicate a velocity of $\frac{1}{7}$ of that of gun-powder. But in reality the air-gun ſhoots its ball with a much greater proportion of velocity than as above, and for this reaſon, namely, that as the reſervoir, or magazine of condenſed air, is commonly very large in proportion to the tube which contains the ball, its denſity is very little altered by expanding through that narrow tube, and conſequently the ball is urged all the way by nearly the ſame uniform force as at the firſt inſtant; whereas the elaſtic fluid ariſing from inflamed gun-powder is but very ſmall in proportion to the tube or barrel of the gun, occupying at firſt indeed but a very ſmall portion of it next the but-end : and therefore by dilating into a comparatively large ſpace, as it urges the ball along the barrel, its elaſtic force is proportionally weakened, and it acts always leſs and leſs on the ball in the tube. From which cauſe it happens, that air condenſed into a good large machine only 10 times, will ſhoot its ball with a velocity but little inferior to that given by the gunpowder. And if the valve of communication be ſuddenly ſhut again by a ſpring, after opening it to let ſome air eſcape, then the ſame collection of it may ſerve to impel many balls, one after another.

In all caſes in which a conſiderable force is required, and conſequently a great condenſation of air, it will be requiſite to have the condenſing ſyringe of a ſmall bore, perhaps not more than half an inch in diameter : otherwiſe the force to produce the compreſſion will become ſo great, that the operator cannot work the machine: for, as the preſſure againſt every ſquare inch is about 15 pounds, and againſt every circular inch about 12 pounds, if the ſyringe be one inch in diameter, when one atmoſphere is injected, there will be a reſiſtance of 12 pounds againſt the piſton ; when 2, of 24 pounds ; and when 10 are injected, there will be a force of 120 pounds to overcome ; whereas 10 atmoſpheres act againſt the half-inch piſton, whoſe area is but $\frac{1}{4}$ of the former, with $\frac{1}{4}$ of the force only, namely, 30 pounds ; and 40 atmoſpheres may be injected with ſuch a ſyringe, as well as ten with the larger.

There are air-guns of various conſtructions ; an eaſy and portable one is repreſented in Plate II, fig. 1. which is a ſection lengthways through the axis, to ſhew the inſide. It is made of braſs, and has two barrels ; the inner barrel D A of a ſmall bore, from which the bullets are ſhot ; and a larger barrel E S C D R, on the outſide of it. In the ſtock of the gun there is a ſyringe M N P S, whoſe rod M draws out to take in air ; and by puſhing it in again, the piſton S N drives the air before it, through the valve P E into the cavity between the two barrels. The ball K is put down into its place in the ſmall barrel, with the rammer, as in another gun. There is another valve at S L, which, being opened by the trigger O, permits the air to come behind the ball, ſo as to drive it out with great force. If this valve be opened and ſhut ſuddenly, one charge of condenſed air may make ſeveral diſcharges of bullets ; becauſe only part of the injected air will then go out at a time, and another bullet may be put into the place K : but if the whole air be diſcharged on a ſingle bullet, it will impel it more forcibly. This diſcharge is effected by means of a lock (fig. 2) when fixed to its place as uſual in other guns ; for the trigger being pulled, the cock will go down and drive a lever which opens the valve.

Dr. Macbride (Exper. Eſſ. p. 81) mentions an improvement of the air-gun, made by Dr. Ellis ; in which the chamber for containing the condenſed air is not in the ſtock, which renders the machine heavy and unweildy, but has five or ſix hollow ſpheres belonging to it, of about 3 inches diameter, fitted to a ſcrew on the lock of the gun. Theſe ſpheres are contrived with valves, to confine the air which is forced into their cavities, ſo that a ſervant may carry them ready charged with condenſed air : and thus the gun of this conſtruction is rendered as light and portable as one of the ſmalleſt fowling-pieces.

Fig. 3 repreſents one made by the late Mr. B. Martin of London, and now by ſeveral of the mathematical inſtrument and gun-makers of the metropolis ; which, for ſimplicity and perfection, perhaps exceeds any other that has been contrived. A is the gun-barrel, of the ſize and weight of a common fowling-piece, with the lock, ſtock, and ramrod. Under the lock, at *b*, is a round ſteel-tube, having a ſmall moveable pin in the inſide, which is puſhed out when the trigger *a* is pulled, by the ſpringwork within the lock ; to this tube *b* is ſcrewed a hollow copper ball, perfectly air-tight. This copper ball is fully charged with condenſed air by means of a ſyringe, previous to its being applied to the tube *b*. Hence, if a bullet be rammed down in the barrel, the copper ball ſcrewed faſt at *b*, and the trigger *a* be pulled ; then the pin in *b* will forcibly puſh open a valve within the copper ball, and let out a portion of the condenſed air ; which air will ruſh up through the aperture of the lock, and forcibly act againſt the bullet, driving it to the diſtance of 60 or 70 yards, or farther. If the air be ſtrongly condenſed at every diſcharge, only a portion of the air eſcapes from the ball ; therefore, by re-cocking the piece, another diſcharge may be made ; and this repeated 15 or 16 times. An additional barrel is ſometimes made, and applied for the diſcharge of ſhot, inſtead of the ball above deſcribed.

Sometimes the ſyringe is applied to the end of the barrel C (fig. 4) ; the lock and trigger ſhut up in a braſs caſe *d* ; and the trigger pulled, or the diſcharge made, by pulling the chain *b*. In this contrivance there is a round chamber for the condenſed air at the end of the ſpring at *e*, and it has a valve acting in a ſimilar manner to that of the copper ball. When this inſtrument is not in uſe, the braſs caſe *d* is made to ſlide off, and the inſtrument then becomes a walking ſtick : from which circumſtance, and the barrel being made of cane, or braſs, &c, it has been called the *Air-cane*. The head of the cane unſcrews and takes off at *a*, where the extremity of the piſton-rod in the barrel is ſhewn. An iron rod is placed in a ring at the end of this, and the air is condenſed in the barrel in a manner ſimilar to that of the gun as above ; but its force and action is not near ſo ſtrong as in the gun.

MAGAZINE AIR-*Gun*. This is an improvement of the common air-gun, made by an ingenious artiſt, called L. Colbe. By his contrivance, ten bullets are ſo lodged in a cavity, near the place of diſcharge, that they may be ſucceſſively drawn into the barrel, and ſhot ſo quickly as to be nearly of the ſame uſe as ſo many different guns ; the only motion required, after the air has been injected, being that of ſhutting and opening the hammer, and cocking and pulling the trigger. Fig. 3 is a longitudinal ſection of this gun, as large in every part as the gun itſelf ; and as much of its length is ſhewn as is peculiar to this conſtruction ; the reſt of it being like the ordinary air-gun. E E is part of the ſtock ; G is the end of the injecting ſyringe, with its valve H, opening into the cavity F F F F between the barrels. K K is the ſmall or ſhooting barrel, which receives the bullets, one at a time, from the magazine D E, being a ſerpentine cavity, in which the bullets *b*, *b*, *b*, &c, are lodged, and cloſed at the end D; from whence, by one motion of the hammer, they are brought into the barrel at I, and thence are ſhot out by the opening of the valve V, which lets in the condenſed air from the cavity F F F into the channel V K I, and ſo along the inner barrel K K K, whence the bullet is diſcharged. *s* I *s i* M *k* is the key of a cock, having a hole through it; which hole, in the preſent ſituation, makes part of the barrel K K, being juſt of the ſame bore: ſo that the air, which is let in at every opening of the valve V, comes behind this cock, and taking the ball out of it, carries it forward, and ſo out of the mouth of the piece.

To bring in another bullet to ſucceed I, which is done in an inſtant, bring the cylindrical cavity of the key of the cock, which made part of the barrel K K K, into the ſituation *i k*, ſo that the part I may be at K; then turning the gun upſide-down, one bullet next the cock will fall into it out of the magazine, but will go no farther into this cylindrical cavity, than the two little pieces *s s* will permit it; by which means only one bullet at a time will be taken in to the place I, to be diſcharged again as before.

A more particular deſcription of the ſeveral parts may be ſeen in Deſaguliers' Exper. Philoſ. vol. ii. pa. 399 et ſeq.

Air-Pump, in *Pneumatics*, is a machine for exhauſting the air out of a proper veſſel, and ſo to make what is commonly called a vacuum; though in reality the air in the receiver is only rarefied to a great degree, ſo as to take off the ordinary effects of the atmoſphere. So that by this machine we learn, in ſome meaſure, what our earth would be without air; and how much all vital, generative, nutritive, and alterative powers depend upon it.

The principle on which the air-pump is conſtructed, is the ſpring or elaſticity of the air; as that on which the common, or water-pump is formed, is the gravity of the ſame air: the one gradually exhauſting the air from a veſſel by means of a piſton, with a proper valve, working in a cylindrical barrel or tube; and the other exhauſting water in a ſimilar manner.

The air-pump has proved one of the principal means of performing philoſophical diſcoveries, that has been invented by the moderns. The idea of ſuch a machine occurred to ſeveral perſons, nearly about the ſame time. But the firſt it ſeems was completed by Otto Guericke, the celebrated conſul of Magdeburg, who exhibited his firſt public experiments with it, before the emperor and the ſtates of Germany, at the breaking up of the imperial diet at Ratiſbon, in the year 1654. But it was not till the year 1672 that Guericke publiſhed a deſcription of the inſtrument, with an account of his experiments, in his Experimenta Nova Magdeburgica de Vacuo Spacia: though an account of them had been publiſhed by Schottus in 1657, in his Mechanica Hydraulico Pneumatica.

Dr. Hook and M. Duhamel aſcribe the invention of the air-pump to Mr. Boyle. But that great man frankly confeſſes that Guericke was beforehand with him in the execution. Some attempts, he aſſures us, he had indeed made upon the ſame foundation, before he knew any thing of what had been done abroad: but the information he afterwards received from the account given by Schottus, enabled him, with the aſſiſtance of Dr. Hook, after two or three unſucceſsful trials, to bring his deſign to maturity. The product of their labours was a new air-pump, much more eaſy, convenient, and manageable, than the German one. And hence, or rather from the great variety of experiments to which this illuſtrious author applied the machine, it was afterwards called *Machina Boyliana*, and the vacuum produced by it, *Vacuum Boylianum*.

Structure of the Air-Pump. Moſt of the air-pumps that were firſt made, conſiſted of only one barrel, or hollow cylinder of braſs, with a valve at the bottom, opening inwards; and a moveable embolus or piſton, having likewiſe a valve opening upwards, and ſo exactly fitted to the barrel, that when it is drawn up from the bottom, by means of an indented iron rod or rack, and a handle turning a ſmall indented wheel, playing in the teeth of that rod, all the air will be drawn up from the cavity of the barrel: there is alſo a ſmall pipe opening into the bottom of the barrel, by means of which it communicates with any proper veſſel to be exhauſted of air, which is called a receiver, from its office in receiving the ſubjects upon which experiments are to be made in vacuo: the whole being fixed in a convenient frame of wood-work, where the end of the pipe turns up into a horizontal plate, upon which the receiver is placed, juſt over that end of the pipe.

The other parts of the machine, being only accidental circumſtances, chiefly reſpecting conveniency, have been diverſified and improved from time to time, according to the addreſs and ſeveral views of the makers. That of Otto Guericke was very rude and inconvenient, requiring the labour of two ſtrong men, for more than two hours, to extract the air from a glaſs, which was alſo placed under water; and yet allowed of no change of ſubjects for experiments.

Mr. Boyle, from time to time, removed ſeveral of theſe inconveniences, and leſſened others: but ſtill the working of his pump, which had but one barrel, was laborious, by reaſon of the preſſure of the atmoſphere, a great part of which was to be removed at every lift of the piſton, when the exhauſtion was nearly completed. Various improvements were ſucceſſively made in the machine by the philoſophers about that time, and ſoon after, who cultivated this new and important branch of pneumatics; as Papin, Merſenne, Mariotte, and others; but ſtill they laboured under a difficulty of working them, from the circumſtance of the ſingle barrel, till Papin, in his farther improvements of the air-pump, removed that inconvenience, by the uſe of a ſecond barrel and piſton, contrived to riſe, as the other fell, and to fall as that roſe; by which, and the great improvements made by Mr. Haukſbee, the preſſure of the atmoſphere on the deſcending piſton, always nearly balanced that of the aſcending one; ſo that the winch, which worked them up and down, was eaſily moved by a very gentle force with one hand: and beſides, the exhauſtion was hereby made in leſs than half the time.

Some of the Germans, and others likewiſe, made improvements in the air-pump, and contrived it to perform the counter office of a condenſer, in order to examine the properties of the air depending on its condenſation.

Mr. Boyle contrived a mercurial gauge or index to the air-pump, which is deſcribed in his firſt and ſecond Phyſico-Mechanical Continuations, for meaſuring the degrees of the air's rarefaction in the receiver. This gauge is ſimilar to the barometer, being a long glaſs tube, having its lower end immerſed in an open baſon of quickſilver, but its other end, which was open alſo, communicating with the receiver: which being exhauſted, this tube is equally exhauſted of air at the ſame time, and the external air preſſes the quickſilver up into the tube, to a height proportioned to the degree of exhauſtion.

Mr. Vream, an ingenious pneumatic operator, made an improvement in Haukſbee's air-pump, by reducing the alternate up-and-down motion of the hand and winch to a circular one. In his method, the winch is turned quite round, and yet the piſtons are alternately raiſed and depreſſed: by which the trouble of ſhifting the hand backwards and forwards, as well as the loſs of time, and the ſhaking of the pump, are prevented.

The air-pump, thus improved, is repreſented in plate III. fig. 1; where *o o* is the receiver to be exhauſted, ground truly level at the bottom, ſet over a hole in the plate, from which deſcends the bent pipe *h h* to the ciſtern *d d*, with which the two barrels *a a* communicate, in which the piſtons are worked by a toothed wheel, by turning the handle *b b*; by which the racks *c c*, with the piſtons, are worked alternately up and down. *l l* is the gauge tube, immerſed in a baſon of quickſilver *m* at bottom, and communicating with the receiver at top; from which however it may be occaſionally diſengaged, by turning a cock. And *n* is another cock, by turning of which, the air is again let in to the exhauſted receiver; into which it is heard to ruſh with a conſiderable hiſſing noiſe.

Notwithſtanding the great excellency of Mr. Haukſbee's air-pump, it was ſtill ſubject to inconveniences, from which it was in a great meaſure relieved by ſome contrivances of Mr. Smeaton, which are deſcribed at large in the Philoſ. Tranſ. for the year 1752. The principal improvements ſuggeſted by Mr. Smeaton, relate to the gauge, the valves of the piſton, and the piſton going cloſer down to the bottom of the barrel; for his pump has only one. By the laſt of theſe, the air was extracted more perfectly at each ſtroke. By the ſecond, he remedied an inconvenience ariſing from the valve hole of the piſton being too wide properly to ſupport the bladder valve which covered it: inſtead of the uſual circular orifice, Mr. Smeaton perforated the piſton with ſeven ſmall and equal hexagonal holes, one in the centre, and the other ſix around, forming together the appearance of a tranſverſe ſection of a honeycomb; the bars or diviſions between which, ſerved to ſupport the preſſure of the air on the valve. His gage conſiſts of a bulb of glaſs, of a pear-like ſhape, and capable of holding about half a pound of quickſilver: it is open at the lower end, the other terminating in a tube hermetically ſealed; and it has annexed to it a ſcale, divided into parts of about $\frac{1}{10}$ of an inch, and anſwering to the 1000th part of the whole capacity. During the exhauſtion of the receiver, the gage is ſuſpended in it by a wire; but when the pump has been worked as much as neceſſary, the gage is puſhed down, till the open end be immerſed in a baſon of quickſilver placed underneath. The air is then let into the receiver again, and the quickſilver driven by it from the baſon, up into the gauge, till the air remaining in it become of the ſame denſity as the air without; and as the air always takes the higheſt place, the tube being uppermoſt, the expanſion will be determined by the number of diviſions occupied by the air at the top. This air-pump is made to act alſo as a condenſing engine, as ſome German machines had done before, by the very ſimple apparatus of turning a cock.

By means of this gauge, Mr. Smeaton judged that his machine was incomparably better than any former ones, as it ſeemed to rarefy the air in the receiver 1000, or even 2000 times, while the beſt of the former conſtruction only rarefied about 140 times: and ſo the caſe has ſince been always underſtood, an implicit confidence being placed in Mr. Smeaton's accuracy, till the fallacy was accidentally detected in the manner related at large by Mr. Nairne in the Philoſ. Tranſ. for the year 1777. This accurate and ingenious artiſt wanting to make trial of Mr. Smeaton's pear-gauge, executed an air-pump of his improved conſtruction, in the beſt manner poſſible; which, in various experiments made with it, appeared, by the pear-gauge, to rarefy the air to an amazing degree indeed, being at times from 4000 to 10000, or 50000, or even 100000 times rarefied. But upon meaſuring the ſame expanſion by the uſual long and ſhort tube gauges, which both accurately agreed together, he found that theſe never ſhewed a rarefaction of more than 600 times: widely different from the ſame as meaſured by the pear or internal gauge, by experiments often repeated. ‘ Finding, ſays Mr. Nairne, ſtill this diſagreement between the pear-gauge and the other gauges, I tried a variety of experiments; but none of them appeared to me ſatisfactory, till one day in April 1776, ſhewing an experiment with one of theſe pumps to the honourable Henry Cavendiſh, Mr. Smeaton, and ſeveral other gentlemen of the Royal Society, when the two gauges differed ſome thouſand times from one another, Mr. Cavendiſh accounted for it in the following manner. “ It appeared, he ſaid, from ſome experiments of his father's, Lord Cavendiſh, that water, whenever the preſſure of the atmoſphere on it is diminiſhed to a certain degree, is immediately turned into vapour, and is as immediately turned back again into water on reſtoring the preſſure. This degree of preſſure is different according to the heat of the water: when the heat is 72° of Fahrenheit's ſcale, it turns into vapour as ſoon as the preſſure is no greater than that of three quarters of an inch of quickſilver, or about 1-40th of the uſual preſſure of the atmoſphere; but when the heat is only 41°, the preſſure muſt be reduced to that of a quarter of an inch of quickſilver before the water turns into vapour. It is true, that water expoſed to the open air, will evaporate at any heat, and with any preſſure of the atmoſphere; but that evaporation is intirely owing to the action of the air upon it; whereas the evaporation here ſpoken of, is performed without any aſſiſtance from the air. Hence it follows, that when the receiver is exhauſted to the above-mentioned degree, the moiſture adhering to the different parts of the machine will turn into vapour, and ſupply the place of the air, which is continually drawn away by the working of the pump; ſo that the fluid in the pear-gauge, as well as that in the receiver, will conſiſt in a good meaſure of vapour. Now letting the air into the receiver, all the vapour within the pear-gauge will be reduced to water, and only the real air will remain uncondenſed; conſequently the pear-gauge ſhews only how much real air is left in the receiver, and not how much the preſſure or ſpring of the included fluid is diminiſhed; whereas the common gauges ſhew how much the preſſure of the included fluid is diminiſhed, and that equally, whether it conſiſt of air or of vapour." Mr. Cavendiſh having explained ſo ſatisfactorily the cauſe of the diſagreement between the two gauges, Mr. Nairne conſidered that, if he were to avoid moiſture as much as poſſible, the two gauges ſhould nearly agree. And in fact they were found ſo to do, each ſhewing a rarefaction of about 600, when all moiſture was perfectly cleared away from the pump, and the plate and the edges of the receiver were ſecured by a cement inſtead of ſetting it upon a ſoaked leather, as in the uſual way. But by future experiments, Mr. Nairne found that the ſame excellent machine would not exhauſt more than 50 or 60 times, when the receiver was ſet upon leather ſoaked in water, the heat of the room being about 57°. And from the whole, Mr. Nairne concludes that the air-pump of Otto Guericke, and thoſe contrived by Mr. Gratorix, and Dr. Hook, and the improved one by Mr. Papin, both uſed by Mr. Boyle, as alſo Haukſbee's, s'Graveſande's, Muſchenbroeck's, and thoſe of all who have uſed water in the barrels of their pumps, could never have exhauſted to more than between 40 and 50, if the heat of the place was about 57; and although Mr. Smeaton, with his pump, where no water was in the barrel, but where leather ſoaked in a mixture of water and ſpirit of wine was uſed on the pump-plate, to ſet the receiver upon, may have exhauſted all but a thouſandth, or even a ten-thouſandth part of the common air, according to the teſtimony of his pear-gauge; yet ſo much vapour muſt have ariſen from the wet leather, that the contents of the receiver could never be leſs than a 70th or 80th part of the denſity of the atmoſphere. But when nothing of moiſture is uſed about this machine, it will, when in its greateſt perfection, rarefy its contents of air about 600 times.

It is evident that by means of theſe two gauges we can aſcertain the ſeveral quantities of vapour and permanent air which make up the contents of the receiver, after the exhauſtion is made as perfect as can be; for the uſual external gauge determines the whole contents, made up of the vapour and air, whilſt the pear-gauge ſhews the quantity of real permanent air; conſequently the difference is the quantity of vapour.

The principal cauſe which prevents this pump from exhauſting beyond the limit above-mentioned, is the weakened elaſticity of the air within the receiver, which, decreaſing in proportion as the quantity of the air within is diminiſhed, becomes at laſt incapable of lifting up the valve of communication between the receiver and the barrel; and conſequently no more air can then paſs from the former to the latter.

Several ingenious perſons have uſed their endeavours to remove this imperfection in the beſt air-pumps. Amongſt theſe it ſeems that one Mr. Haas has ſucceeded tolerably well; having, by means of a contrivance to open the communication valve in the bottom of the barrel, made his machine ſo perfect, that when every thing is in the greateſt perfection, it rarefies the contents of the receiver as far as 1000 times, even when meaſured by the exterior gauge. The deſcription of this machine, and an account of ſome experiments performed with it, are given by Mr. Cavallo in the Philoſ. Tranſ. for the year 1783.

But the imperfections it ſeems have more recently been removed by an ingenious contrivance of Mr. Cuthbertſon, a mathematical inſtrument maker at Amſterdam, now of London, whoſe air-pump has neither cocks nor valves, and is ſo conſtructed, that what ſupplies their place has the advantages of both, without the inconveniences of either. He has alſo made improvements in the gauges, by means of which he determines the height of the mercury in the tube, by which the degree of exhauſtion is indicated, to the hundredth part of an inch. And to obviate the inconvenience of the elaſtic vapour ariſing from the wet leather, upon which the receiver is placed, for common experiments, he recommends the uſe of leather dreſſed with allum, and ſoaked in hog's lard, which he found to yield very little of this vapour; but when the utmoſt degree of exhauſtion is required, his advice is, to dry the receiver well, and ſet it upon the plate without any leather, only ſmearing its outer edges with hog's lard, or with a mixture of three parts of hog's lard and one of oil. But the uſe of the leather has long been laid aſide by our Engliſh inſtrument-makers, a circumſtance which probably had not come to Mr. Cuthbertſon's knowledge. An account of this inſtrument, and of ſome experiments performed with it, was publiſhed at Amſterdam in the year 1787; from which experiments it appears that, by a coincidence of the ſeveral gauges, a rarefaction of 1200 times was ſhewn; but when the atmoſphere was very dry, the exhauſtion has been ſo complete, that the gauges have ſhewn the air in the receiver to be rarefied above 2400 times.

There are made alſo by different perſons, portable, or ſmall air-pumps, of various conſtructions, to ſet upon a table, to perform experiments with. In theſe, the gauge is varied according to the fancy of the maker, but commonly it conſiſts of a bent glaſs tube, like a ſyphon, open only at one end. The gauge is placed under a ſmall receiver communicating, by a pipe, with the principal pipe leading from the general receiver to the barrels. The cloſe end of the gauge, of 3 or 4 inches long, before the exhauſtion, has the quickſilver forced cloſe up to the top by the preſſure of the air on the open end; but when the exhauſtion is conſiderably advanced, it begins to deſcend, and then the difference of the heights of the quickſilver in the two legs, compared with the height in the barometrical tube, determines the degree of exhauſtion: ſo if the difference between the two be one inch, when the barometer ſtands at 30, the air is rarefied 30 times; but if the difference be only half an inch, the rarefaction is 60 times, and ſo on. See Plate III. fig. 2.

The Uſe of the Air-Pump. In whatever manner or form this machine be made, the uſe and operation of it are always the ſame. The handle, which works the piſton, is moved up and down in the barrel, by which means a barrel of the contained air is drawn out at every ſtroke of the piſton, in the following manner: by puſhing the piſton down to the bottom of the barrel, where the air is prevented from eſcaping downwards, by its elaſticity it opens the valve of the piſton, and eſcapes upwards above it into the open air; then raiſing the piſton up, the external atmoſphere ſhuts down its valve, and a vacuum would be made below it, but for the air in the receiver, pipe, &c, which now raiſes the valve in the bottom of the barrel, and ruſhes in and fills it again, till the whole air in the receiver and barrel be of one uniform denſity, but leſs than it was before the ſtroke, in proportion as the ſum of all the capacities of the receiver, pipe, and barrel together, is to the ſame ſum wanting the barrel. And thus is the air in the receiver diminiſhed at each ſtroke of the piſton, by the quantity of the barrel or cylinder full, and therefore always in the ſame proportion: ſo that by thus repeating the operation again and again, the air is rarefied to any propoſed degree, or till it has not elaſticity enough to open the valve of the piſton or of the barrel, after which the exhauſtion cannot be any farther carried on: the gauge, in compariſon with the barometer, ſhewing at any time what the degree of exhauſtion is, according to the particular nature and conſtruction of it.

But, ſuppoſing no vapour from moiſture, &c, to riſe in the receiver, the degree of exhauſtion, after any number of ſtrokes of the piſton, may be determined by knowing the reſpective capacities of the barrel and the receiver, including the pipe, &c. For as we have ſeen above that every ſtroke diminiſhes the denſity in a conſtant proportion, namely as much as the whole content exceeds that of the cylinder or barrel; and conſequently the ſum of as many diminutions as there are ſtrokes of the piſton, will ſhew the whole diminution by all the ſtrokes. So, if the capacity of the barrel be equal to that of the receiver, in which the communication pipe is always to be included; then, the barrel being half the ſum of the whole contents, half the air will be drawn out at one ſtroke; and conſequently the remaining half, being dilated through the whole or firſt capacity, will be of only half the denſity of the firſt: in like manner, after the ſecond ſtroke, the denſity of the remaining contents will be only half of that after the firſt ſtroke, that is only $\frac{1}{4}$ of the original denſity: continuing this operation, it follows that the denſity of the remaining air will be $\frac{1}{8}$ after 3 ſtrokes of the piſton, $\frac{1}{16}$ after 4 ſtrokes, $\frac{1}{32}$ after 5 ſtrokes, and ſo on, according to the powers of the ratio $\frac{1}{2}$; that is, ſuch power of the ratio as is denoted by the number of the ſtrokes. In like manner, if the barrel be $\frac{1}{3}$ of the whole contents, that is, the receiver double of the barrel, or $\frac{2}{3}$ of the whole contents; then the ratio of diminution of denſity being $\frac{2}{3}$, the denſity of the contents, after any number of ſtrokes of the piſton, will be denoted by ſuch power of $\frac{2}{3}$ whoſe exponent is that number; namely, the denſity will be $\frac{2}{3}$ after one ſtroke, $(\frac{2}{3})^2$ or $\frac{4}{9}$ after two ſtrokes, $(\frac{2}{3})^3$ or $\frac{8}{27}$ after 3 ſtrokes, and in general it will be $(\frac{2}{3})^n$ after n ſtrokes: the original denſity of the air being 1. Hence then, univerſally, if s denote the ſum of the contents of the receiver and barrel, and r that of the receiver only without the barrel, and n any number of ſtrokes of the piſton; then, the original denſity of the air being 1, the denſity after n ſtrokes will be $(\frac{r}{s})^n$ o $\frac{r^n}{s^n}$, namely the n power of the ratio $\frac{r}{s}$. So, for example, if the capacity of the receiver be equal to 4 times that of the barrel; then their ſum s is 5, and r is 4; and the denſity of the contents after 30 ſtrokes, will be $(\frac{4}{5})^{30}$, or the 30th power of $\frac{4}{5}$, which is $\frac{1}{808}$ nearly; ſo that the air in the receiver is rarefied 808 times.

See alſo the Memoires de l'Acad. Royale des Sciences for the years 1693 and 1705.

From the ſame formula, namely $(\frac{r}{s})^n = d$ the denſity, we eaſily derive a rule for finding the number of ſtrokes of the piſton, neceſſary to rarefy the air any number of times, or to reduce it to a given denſity d, that of the natural air being 1. For ſince $(\frac{r}{s})^n = d$, by taking the logarithm of this equation, it is $n \times \log. \frac{r}{s} = \log.$ of d; and hence $n = \frac{\log. d}{\log. \frac{r}{s}} = \frac{\log. d}{\log. r - \log. s}$; that is, divide the log. of the propoſed denſity by the log. of the ratio of the receiver to the ſum of the receiver and barrel together, and the quotient will ſhew the number of ſtrokes of the piſton requiſite to produce the degree of exhauſtion required. So, for example, if the receiver be equal to 5 times the barrel, and it be propoſed to find how many ſtrokes of the piſton will rarefy the air 100 times; then $r = 5$, $s = 6$, $d = \frac{1}{100}$, whoſe log. is -2, and $\frac{r}{s} = \frac{5}{6}$, whoſe log. is $-\cdot07918$; therefore $\frac{2}{\cdot07918} = 25\frac{1}{4}$ nearly, which is the number of ſtrokes required.

And, farther, the ſame formula reduced, would give us the proportion between the receiver and barrel, when the air is rarefied to any degree by an aſſigned number of ſtrokes of the piſton. For ſince $(\frac{r}{s})^n = d$ the denſity, therefore, extracting the n root of both ſides, it is $\frac{r}{s} = \sqrt[n]{d}$: that is, the n root of the denſity is equal to the ratio of the receiver to the ſum of the receiver and barrel. So, if the denſity d be $\frac{1}{128}$, and the number of ſtrokes $n = 7$; then the 7th root of $\frac{1}{128}$ is $\frac{1}{2}$; which ſhews that the receiver is equal to half the receiver and barrel together, or that the capacity of the barrel is juſt equal to that of the receiver.

Some of the principal effects and phenomena of the air-pump, are the following: That, in the exhauſted receiver, heavy and light bodies fall equally ſwift; ſo, a guinea and feather fall from the top of a tall receiver to the bottom exactly together. That moſt animals die in a minute or two: but however, That vipers and frogs, though they ſwell much, live an hour or two; and after being ſeemingly quite dead, come to life again in the open air: That ſnails ſurvive about ten hours; efts, or ſlow-worms, two or three days; and leeches five or ſix. That oyſters live for 24 hours. That the heart of an eel taken out of the body, continues to beat for good part of an hour, and that more briſkly than in the air. That warm blood, milk, gall, &c, undergo a conſiderable intumeſcence and ebullition. That a mouſe or other animal may be brought, by degrees, to ſurvive longer in a rarefied air, than naturally it does. That air may retain its uſual preſſure, after it is become unfit for reſpiration. That the eggs of ſilk-worms hatch in vacuo. That vegetation ſtops. That fire extinguiſhes ; the flame of a candle uſually going out in one minute ; and a charcoal in about five minutes. That red-hot iron, however, ſeems not to be affected ; and yet ſulphur or gun-powder are not lighted by it, but only fuſed. That a match, after lying ſeemingly extinct a long time, revives again on re-admitting the air. That a flint and ſteel ſtrike ſparks of fire as copiouſly, and in all directions, as in air. That magnets, and magnetic needles, act the ſame as in air. That the ſmoke of an extinguiſhed luminary gradually ſettles to the bottom in a darkiſh body, leaving the upper part of the receiver clear and tranſparent ; and that on inclining the veſſel ſometimes to one ſide, and ſometimes to another, the fume preſerves its ſurface horizontal, after the nature of other fluids. That heat may be produced by attrition. That camphire will not take fire ; and that gun-powder, though ſome of the grains of a heap of it be kindled by a burning glaſs, will not give fire to the contiguous grains. That glow-worms loſe their light in proportion as the air is exhauſted, and at length become totally obſcure ; but on re-admitting the air, they preſently recover it all. That a bell, on being ſtruck, is not heard to ring, or very faintly. That water freezes. But that a ſyphon will not run. That electricity appears like the aurora borealis. With multitudes of other curious and important particulars, to be met with in the numerous writings on this machine, namely, beſides the Philoſ. Tranſactions of moſt academies and ſocieties, in the writings of Torricelli, Paſcal, Merſenne, Guericke, Schottus, Boyle, Hook, Duhamel, Mariotte, Haukſbee, Hales, Muſchenbroeck, Graveſande, Deſaguliers, Franklin, Cotes, Helſham, and a great many other authors.

AIR-VESSEL, in *Hydraulics*, is a veſſel of air within ſome water engines, which being compreſſed, by forcing in a conſiderable quantity of water, by its uniform ſpring, forces it out at the pipe in a conſtant uninterrupted ſtream, to a great height.

Air-veſſel too, in the improved fire engines, is a metallic cylinder, placed between the two forcing pumps, by the action of whoſe piſtons the water is forced into this veſſel, through two pipes, with valves ; then the air, previouſly contained in it, is compreſſed by the water, in proportion to the quantity admitted, and this air, by its ſpring, forces the water through a pipe by a conſtant and equal ſtream ; whereas in the common ſquirting engine, the ſtream is diſcontinued between the ſeveral ſtrokes.

AIRY TRIPLICITY, in *Aſtrology*, the ſigns of Gemini, Libra, and Aquarius.

AJUTAGE, or ADJUTAGE, in *Hydraulics*, part of the apparatus of a *jet d'eau*, or artificial fountain ; being a kind of tube fitted to the aperture or mouth of the ciſtern, or the pipe ; through which the water is to be played in any direction, and in any ſhape or figure.

It is chiefly the diverſity in the ajutage, that makes the different kinds of fountains. So that, by having ſeveral ajutages, to be applied occaſionally, one fountain is made to have the effect of many.

Mariotte, Graveſande, and Deſaguliers have written pretty fully on the nature of ajutages, or ſpouts for jets d'eau, and eſpecially the former. He affirms, from experiment, that an even poliſhed round hole, made in the thin end of a pipe, gives a higher jet than either a cylindrical or a conical ajutage ; but that, of theſe two latter however, the conical is better than the cylindrical figure. See his Traite du Mouvement des Eaux, part 4.

The quantity of water diſcharged by ajutages of equal area, but of different figures, is the ſame. And for like figures, but of different ſizes, the quantity diſcharged, is directly proportional to the area of the ajutage, or to the ſquare of its diameter, or of any ſide or other linear dimenſion : ſo, an ajutage of a double diameter, or ſide, will diſcharge 4 times the quantity of water ; of a triple diameter, 9 times the quantity ; and ſo on ; ſuppoſing them at an equal depth below the ſurface or head of water. But if the ajutage be at different depths below the head, then the celerity with which the water iſſues, and conſequently the quantity of it run out in any given time, is directly proportional to the ſquare-root of the altitude of the head, or depth of the hole : ſo at 4 times the depth, the celerity and quantity is double ; at 9 times the depth, triple ; and ſo on.

It has been found that jets do not riſe quite ſo high as the head of water ; owing chiefly to the reſiſtance of the air againſt it, and the preſſure of the upper parts of the jet upon the lower : and for this reaſon it is, that if the direction of the ajutage be turned a very little from the perpendicular, it is found to ſpout rather higher than when the jet is exactly upright.

It is found by experiment too, that the jet is higher or lower, according to the ſize of the ajutage : that a circular hole of about an inch and a quarter in diameter, jets higheſt ; and that the farther from that ſize, the worſe. Experience alſo ſhews that the pipe leading to the ajutage, ſhould be much larger than it ; and if the pipe be a long one, that it ſhould be wider the farther it is from the ajutage.

For the other circumſtances relating to jets and the iſſuing of water under various circumſtances, ſee EXHAUSTION, FLUX, FOUNTAIN, JET D'EAU, &c, to which they more properly belong.

ALBATEGNI, an Arabic prince of Batan in Meſopotamia, who was a celebrated aſtronomer, about the year of Chriſt 880, as appears by his obſervations. He is alſo called *Muhammed ben Geber Albatani*, *Mahomet the ſon of Geber*, and *Muhamedes Aractenſis*. He made aſtronomical obſervations at Antioch, and at Racah or Aracta, a town of Chaldea, which ſome authors call a town of Syria or of Meſopotamia. He is highly ſpoken of by Dr. Halley, as a *vir admirandi acuminis, ac in adminiſtrandis obſervationibus exercitatiſſimus*.

Finding that the tables of Ptolomy were imperfect, he computed new ones, which were long uſed as the beſt among the Arabs : theſe were adapted to the meridian of Aracta or Racah. Albategni compoſed in

Arabic a work under the title of *The Science of the Stars*, compriſing all parts of aſtronomy, according to his own obſervations and thoſe of Ptolomy. This work, tranſlated into Latin by Plato of Tibur, was publiſhed at Nuremberg in 1537, with ſome additions and demonſtrations of Regiomontanus; and the ſame was reprinted at Bologna in 1645, with this author's notes. Dr. Halley detected many faults in theſe editions: Philoſ. Tranſ. for 1693, N° 204.

In this work, Albategni gives the motion of the ſun's apogee ſince Ptolomy's time, as well as the motion of the ſtars, which he makes 1 degree in 70 years. He made the longitude of the firſt ſtar of Aries to be 18°·2′; and the obliquity of the ecliptic 23° 35′. And upon Albategni's obſervations were founded the Alphonſine tables of the moon's motions; as is obſerved by Nic. Muler, in the Tab. Friſicæ, pa. 248.

ALBERTUS MAGNUS, a very learned man in the 13th century, who, among a multitude of books, wrote ſeveral upon the various mathematical ſciences, as Arithmetic, Geometry, Perſpective or Optics, Muſic, Aſtrology and Aſtronomy, particularly under the titles, *de ſphæra, de aſtris, de aſtronomia, item ſpeculum aſtronomicum*.

Albertus Magnus was born at Lawingen on the Danube, in Suabia, in 1205, or according to ſome in 1193; and he died at a great age, at Cologn, November 15, 1280. Voſſius and other authors ſpeak of him as a great genius, and deeply ſkilled in all the learning of the age. His writings were ſo numerous, that they make 21 volumes in folio, in the Lyons edition of 1615. He has paſſed alſo for the author of ſome writings relating to midwifery, &c, under the title of *De natura rerum*, and *De ſecretis mulierum*, in which there are many phraſes and expreſſions unavoidable on ſuch a ſubject, which gave great offence, and raiſed a clamour againſt him as the ſuppoſed author, and inconſiſtent with his character, being a Dominican friar, and ſometime biſhop of Ratiſbon; which dignity however he ſoon reſigned, through his love for ſolitude, to enter again into the monaſtic life. But the advocates of Albert aſſert, that he was not the author of either of theſe two works. It muſt be acknowledged however, that there are, in his Comment upon the Maſter of Sentences, ſome queſtions concerning the practice of conjugal duty, in which he has uſed ſome words rather too groſs for chaſte and delicate ears: but they allege what he himſelf uſed to ſay in his own vindication, that he came to the knowledge of ſo many monſtrous things at confeſſion, that it was impoſſible to avoid touching upon ſuch queſtions. Albert was certainly a man of a moſt curious and inquiſitive turn of mind, which gave riſe to other accuſations againſt him; ſuch as, that he laboured to find out the philoſopher's ſtone; that he was a magician; and that he made a machine in the ſhape of a man, which was an oracle to him, and explained all the difficulties he propoſed: the common cant accuſations of thoſe times of ignorance and ſuperſtition. But having great knowledge in the mathematics and mechanics, by his ſkill in theſe ſciences he probably formed a head, with ſprings capable of articulate ſounds; like the machines of Boetius and others.

John Matthæus de Luna, in his treatiſe De Rerum Inventoribus, has attributed the invention of fire-arms to Albert; but in this he is refuted by Naude, in his Apologie des grands hommes.

ALBUMAZAR, otherwiſe called ABUASSAR, and JAPHAR, was a celebrated Arabian philoſopher and aſtrologer, of the 9th or 10th century, or according to ſome authors much earlier. Blancanus, Voſſius, &c, ſpeak of him as one of the moſt learned aſtronomers of his time, or aſtrologers, which was then the ſame thing. He wrote a work *De Magnis Conjunctionibus Annorum Revolutionibus, ac eorum Perfectionibus*, printed at Venice in 1515, at the expence of Melchior Seſſa, a work chiefly aſtrological.

He wrote alſo *Introductio in Aſtronomiam*, printed in the year 1489. And it is reported that he obſerved a comet in his time, above the orb of Venus.

ALCOHOL, in the Arabian Aſtrology, is when a heavy ſlow-moving planet receives another lighter one within its orb, ſo as to come in conjunction with it.

ALDEBARAN, an Arabian name of a fixed ſtar, of the firſt magnitude, juſt in the eye of the ſign or conſtellation Taurus, or the bull, and hence it is popularly called the bull's eye. For the beginning of the year 1800, its

Right Aſcenſion is	—	66°	6′	51″·10
Annual variation in AR		0	0	51 ·31
Declination	—	16	5	52 ·00 N.
And Annual variat. in Decl.		0	0	8 ·30

ALDERAIMIN, a ſtar of the third magnitude in the right ſhoulder of the conſtellation Cepheus.

ALDHAFERA, or Aldhaphra, in the Arabian Aſtronomy, denotes a fixed ſtar of the third magnitude, in the mane of the ſign or conſtellation Leo, the lion.

ALEMBERT (JOHN LE ROND D'), an eminent French mathematician and philoſopher, and one of the brighteſt ornaments of the 18th century. He was perpetual ſecretary to the French Academy of Sciences, and a member of moſt of the philoſophical academies and ſocieties of Europe.

D'Alembert was born at Paris, the 16th of November 1717. He derived the name of John le Rond from that of the church near which, after his birth, he was expoſed as a foundling. But his father, informed of this circumſtance, liſtening to the voice of nature and duty, took meaſures for the proper education of his child, and for his future ſubſiſtence in a ſtate of eaſe and independence. His mother, it is ſaid, was a lady of of rank, the celebrated Mademoiſelle Tencin, ſiſter to cardinal Tencin, archbiſhop of Lyons.

He received his firſt education among the Janſeniſts, in the College of the Four Nations, where he gave early ſigns of genius and capacity. In the firſt year of his philoſophical ſtudies, he compoſed a Commentary on the Epiſtle of St. Paul to the Romans. The Janſeniſts conſidered this production as an omen, that portended to the party of Port-Royal a reſtoration to ſome part of their former ſplendor, and hoped to find one day in d'Alembert a ſecond Paſcal. To render this reſemblance more complete, they engaged their pupil in the ſtudy of the mathematics; but they ſoon perceived that his growing attachment to this ſcience was likely to diſappoint the hopes they had formed with reſpect to his future deſtination: they therefore endeavoured to divert him from this line; but their endeavours were fruitleſs.

On his quitting the college, finding himſelf alone, and unconnected in the world, he ſought an aſylum in the houſe of his nurſe. He hoped that his fortune, though not ample, would enlarge the ſubſiſtence, and better the condition of her family, which was the only one that he could conſider as his own. It was here therefore that he fixed his reſidence, reſolving to apply himſelf entirely to the ſtudy of geometry.—And here he lived, during the ſpace of 40 years, with the greateſt ſimplicity, diſcovering the augmentation of his means only by increaſing diſplays of his beneficence, concealing his growing reputation and celebrity from theſe honeſt people, and making their plain and uncouth manners the ſubject of good-natured pleaſantry and philoſophical obſervation. His good nurſe perceived his ardent activity; heard him mentioned as the writer of many books; but never took it into her head that he was a great man, and rather beheld him with a kind of compaſſion. "You will never, ſaid ſhe to him one day, be any thing but a philoſopher—and what is a philoſopher?—a fool, who toils and plagues himſelf all his life, that people may talk of him when he is dead."

As d'Alembert's fortune did not far exceed the demands of neceſſity, his friends adviſed him to think of ſome profeſſion that might enable him to increaſe it. He accordingly turned his views to the law, and took his degrees in that faculty: but ſoon after, abandoning this line, he applied himſelf to the ſtudy of medicine. Geometry however was always drawing him back to his former purſuits; ſo that after many ineffectual ſtruggles to reſiſt its attractions, he renounced all views of a lucrative profeſſion, and gave himſelf up entirely to mathematics and poverty.

In the year 1741 he was admitted a member of the Academy of Sciences; for which diſtinguiſhed literary promotion, at ſo early an age (24), he had prepared the way by correcting the errors of a celebrated work (The *Analyſe Demontrée* of Reyneau), which was eſteemed claſſical in France in the line of analytics. He afterwards ſet himſelf to examine, with cloſe attention and aſſiduity, what muſt be the motion and path of a body, which paſſes from one fluid into another denſer fluid, in a direction oblique to the ſurface between the two fluids. Every one knows the phenomenon which happens in this caſe, and amuſes children, under the denomination of *Ducks and Drakes*; but it was d'Alembert who firſt explained it in a ſatisfactory and philoſophical manner.

Two years after his election to a place in the academy, he publiſhed his *Treatiſe on Dynamics*. The new principle developed in this treatiſe, conſiſted in eſtabliſhing an equality, at each inſtant, between the changes that the motion of a body has undergone, and the forces or powers which have been employed to produce them: or, to expreſs the ſame thing otherwiſe, in ſeparating into *two parts* the action of the moving powers, and conſidering the *one* as producing alone the motion of the body, in the ſecond inſtant, and the *other* as employed to deſtroy that which it had in the firſt.

So early as the year 1744, d'Alembert had applied this principle to the theory of the equilibrium, and the motion of fluids: and all the problems before reſolved in phyſics, became in ſome meaſure its corollaries. The diſcovery of this new principle was followed by that of a new calculus, the firſt eſſays of which were publiſhed in a *Diſcourſe on the General Theory of the Winds*, to which the prize-medal was adjudged by the Academy of Berlin in the year 1746, which proved a new and brilliant addition to the fame of d'Alembert. This new calculus of *Partial Differences* he applied, the year following, to the problem of vibrating chords, the reſolution of which, as well as the theory of the oſcillations of the air and the propagation of ſound, had been but imperfectly given by the mathematicians who preceded him; and theſe were his maſters or his rivals.

In the year 1749 he furniſhed a method of applying his principle to the motion of any body of a given figure. He alſo reſolved the problem of the preceſſion of the equinoxes; determining its quantity, and explaining the phenomenon of the nutation of the terreſtrial axis diſcovered by Dr. Bradley.

In 1752, d'Alembert publiſhed a treatiſe on the *Reſiſtance of Fluids*, to which he gave the modeſt title of an *Eſſay*; though it contains a multitude of original ideas and new obſervations. About the ſame time he publiſhed, in the Memoirs of the Academy of Berlin, *Reſearches concerning the Integral Calculus*, which is greatly indebted to him for the rapid progreſs it has made in the preſent century.

While the ſtudies of d'Alembert were confined to mere mathematics, he was little known or celebrated in his native country. His connections were limited to a ſmall ſociety of ſelect friends. But his cheerful converſation, his ſmart and lively ſallies, a happy knack at telling a ſtory, a ſingular mixture of malice of ſpeech with goodneſs of heart, and of delicacy of wit with ſimplicity of manners, rendering him a pleaſing and intereſting companion, his company began to be much ſought after in the faſhionable circles. His reputation at length made its way to the throne, and rendered him the object of royal attention and beneficence. The conſequence was a penſion from government, which he owed to the friendſhip of count d'Argenſon.

But the tranquillity of d'Alembert was abated when his fame grew more extenſive, and when it was known beyond the circle of his friends, that a fine and enlightened taſte for literature and philoſophy accompanied his mathematical genius. Our author's eulogiſt aſcribes to envy, detraction, &c, all the oppoſition and cenſure that d'Alembert met with on account of the famous Encyclopédie, or Dictionary of Arts and Sciences, in conjunction with Diderot. None ſurely will refuſe the well-deſerved tribute of applauſe to the eminent diſplays of genius, judgment, and true literary taſte, with which d'Alembert has enriched that great work. Among others, the Preliminary Diſcourſe he has prefixed to it, concerning the riſe, progreſs, connections, and affinities of all the branches of human knowledge, is perhaps one of the moſt capital productions the philoſophy of the age can boaſt of.

Some time after this, d'Alembert publiſhed his *Philoſophical, Hiſtorical*, and *Philological Miſcellanies*. Theſe were followed by the *Memoirs of Chriſtina queen of Sweden*; in which d'Alembert ſhewed that he was acquainted with the natural rights of mankind, and was bold enough to aſſert them. His *Eſſay on the Intercourſe of Men of Letters with Perſons high in Rank and Office*, wounded the former to the quick, as it expoſed to the eyes of the public the ignominy of thoſe ſervile chains, which they feared to ſhake off, or were proud to wear. A lady of the court hearing one day the author accuſed of having exaggerated the deſpotiſm of the great, and the ſubmiſſion they require, anſwered ſlyly, "If he had conſulted me, I would have told him ſtill more of the matter."

D'Alembert gave elegant ſpecimens of his literary abilities in his tranſlations of ſome ſelect pieces of Tacitus. But theſe occupations did not divert him from his mathematical ſtudies: for about the ſame time he enriched the Encyclopédie with a multitude of excellent articles in that line, and compoſed his *Reſearches on ſeveral Important Points of the Syſtem of the World*, in which he carried to a higher degree of perfection the ſolution of the problem concerning the perturbations of the planets, that had ſeveral years before been preſented to the Academy.

In 1759 he publiſhed his *Elements of Philoſophy*: a work much extolled as remarkable for its preciſion and perſpicuity.

The reſentment that was kindled (and the diſputes that followed it) by the article *Geneva*, inſerted in the Encyclopédie, are well known. D'Alembert did not leave this field of controverſy with flying colours. Voltaire was an auxiliary in the conteſt: but as he had no reputation to loſe, in point of candour and decency; and as he weakened the blows of his enemies, by throwing both them and the ſpectators into fits of laughter, the iſſue of the war gave him little uneaſineſs. It fell more heavily on d'Alembert; and expoſed him, even at home, to much contradiction and oppoſition.

It was on this occaſion that the late king of Pruſſia offered him an honourable aſylum at his court, and the office of preſident of his academy: and the king was not offended at d'Alembert's refuſal of theſe diſtinctions, but cultivated an intimate friendſhip with him during the reſt of his life. He had refuſed, ſome time before this, a propoſal made by the empreſs of Ruſſia to entruſt him with the education of the Grand Duke; —a propoſal accompanied with all the flattering offers that could tempt a man, ambitious of titles, or deſirous of making an ample fortune: but the objects of his ambition were tranquillity and ſtudy.

In the year 1765, he publiſhed his *Diſſertation on the Deſtruction of the Jeſuits*. This piece drew upon him a ſwarm of adverſaries, who only confirmed the merit and credit of his work by their manner of attacking it.

Beſide the works already mentioned, he publiſhed nine volumes of memoirs and treatiſes, under the title of *Opuſcules*; in which he has reſolved a multitude of problems relating to aſtronomy, mathematics, and natural philoſophy; of which his panegyriſt, Condorcet, gives a particular account, more eſpecially of thoſe which exhibit new ſubjects, or new methods of inveſtigation.

He publiſhed alſo *Elements of Muſic*; and rendered, at length, the ſyſtem of Rameau intelligible: but he did not think the mathematical theory of the ſonorous body ſufficient to account for the rules of that art.

In the year 1772 he was choſen ſecretary to the French Academy of Sciences. He formed, ſoon after this preferment, the deſign of writing the lives of all the deceaſed academicians, from 1700 to 1772; and in the ſpace of three years he executed this deſign, by compoſing 70 eulogies.

D'Alembert died on the 29th of October 1783, being nearly 66 years of age. In his moral character there were many amiable lines of candour, modeſty, diſintereſtedneſs, and beneficence; which are deſcribed, with a diffuſive detail, in his eulogium, by Condorcet, in the *Hiſt. de l'Acad. Royale des Sciences*, 1783.

As it may be curious and uſeful to have in one view an entire liſt of d'Alembert's writings, I ſhall here inſert a catalogue of them, from Rozier's *Nouvelle Table des Articles contenus dans les volumes de l'Academie Royale des Sciences de Paris*, &c, as follows:

Traité de Dynamique, in 4to, Paris, 1743. The 2d ed. in 1758.

Traité de l'Equilibre et du Mouvement des Fluides. Paris, 1744; and the 2d edition in 1770.

Reflexions ſur la Cauſe Générale des Vents; which gained the prize at Berlin in 1746; and was printed at Paris in 1747, in 4to.

Recherches ſur la Préceſſion des Équinoxes, & ſur la Nutation de l'Axe de la Terre dans le Syſtème Newtonien. Paris, 1749, in 4to.

Eſſais d'une Nouvelle Théorie du Mouvement des Fluides. Paris, 1752, in 4to.

Recherches ſur differens Points importans du Syſtème du Monde. Paris, 1754 and 1756, 3 vol. in 4to.

Elemens de Philoſophie, 1759.

Opuſcules Mathématiques, ou Memoires ſur différens Sujets de Géométrie, de Méchaniques, d'Optiques, d'Aſtronomie. Paris, 9 vol. in 4to; 1761 to 1773.

Elémens de Muſique, théorique & pratique, ſuivant les Principes de M. Rameau, eclairés, développés, & ſimplifiés. 1 vol. in 8vo. à Lyon.

De la Deſtruction des Jeſuites, 1765.

In the Memoirs of the Academy of Paris are the following pieces, by d'Alembert: viz,

Précis de Dynamique, 1743, Hiſt. 164.

Précis de l'Equilibre & de Mouvement des Fluides, 1744, Hiſt. 55.

Methode générale pour déterminer les Orbites & les Mouvements de toutes les Planètes, en ayant égard à leur action mutuelle, 1745, p. 365.

Précis des Réflexions ſur la Cauſe Générale des Vents, 1750, Hiſt 41.

Précis des Recherches ſur la Préceſſion des Équinoxes, et ſur la Nutation de l'Axe de la Terre dans le Syſtème Newtonien, 1750, Hiſt. 134.

Eſſai d'une Nouvelle Théorie ſur la Réſiſtance des Fluides, 1752, Hiſt. 116.

Précis des Eſſais d'une Nouvelle Théorie de la Réſiſtance des Fluides, 1753, Hiſt. 289.

Précis des Recherches ſur les differens Points importans du Syſtème du Monde, 1754, Hiſt. 125.

Recherches ſur la Préceſſion des Equinoxes, & ſur la Nutation de l'Axe de la Terre, dans l'Hypotheſe de la Diſſimilitude des Méridiens, 1754, p. 413, Hiſt. 116.

Reponſe à un Article du Mémoire de M. l'Abbé de la Caille, ſur la Théorie du Soleil, 1757, p. 145, Hiſt. 118.

Addition à ce Mémoire, 1757, p. 567, Hiſt. 118.

Précis des Opuſcules Mathématiques, 1761, Hiſt. 86.

Précis du troiſième volume des Opuſcules Mathématiques, 1764, Hiſt. 92.

Nouvelles Recherches ſur les Verres Optiques, pour ſervir de ſuite à la théorie qui en à été donnée dans le volume 3^e des Opuſcules Mathématiques. Premier Mémoire, 1764, p. 75, Hiſt. 175.

Nouvelles Recherches ſur les Verres Optiques, pour ſervir de ſuite à la théorie qui en a été donnée dans le troiſième volume des Opuſcules Mathématiques. Second Mémoire, 1765, p. 53.

Obſervations ſur les Lunettes Achromatiques, 1765, p. 53, Hiſt. 119.

Suite des Recherches ſur les Verres Optiques. Troiſième Mémoire, 1767, p. 43, Hiſt. 153.

Recherches ſur le Calcul Intégral, 1767, p. 573.

Accident arrivé par l'Exploſion d'une Meule d'Emouleur, 1768, Hiſt. 31.

Précis des Opuſcules de Mathématiques, 4^e & 5^e volumes. Leur Analyſe, 1768, Hiſt. 83.

Recherches ſur les Mouvemens de l'Axe d'une Planete quelconque dans l'hypotheſe de la Diſſimilitude des Méridienes, 1768 p. 1, Hiſt. 95.

Suite des Recherches ſur les Mouvemens, &c, 1768 p. 332, Hiſt. 95.

Recherches ſur le Calcul Intégral, 1769, p. 73.

Mémoire ſur les Principes de la Mech. 1769, p. 278.

And in the Memoirs of the Academy of Berlin, are the following pieces, by our author: viz,

Recherches ſur le Calcul Integral, premiere partie, 1746.

Solution de quelques problemes d'aſtronomie, 1747.

Recherches ſur la courbe que forme une Corde Tendue, miſe en Vibration, 1747.

Suite des recherches ſur le Calcul Intégral, 1748.

Lettre à M. de Maupertuis, 1749.

Addition aux recherches ſur la courbe que forme une Corde Tendue miſe en Vibration, 1750.

Addition aux recherches ſur le Calcul Intégral, 1750.

Lettre à M. le profeſſeur Formey, 1755.

Extr. de differ. lettres à M. de la Grange, 1763.

Sur les Tautochrones, 1765.

Extr. de differ. lettres à M. de la Grange, 1769.

Alſo in the Memoirs of Turin are,

Differentes Lettres à M. de la Grange, en 1764 & 1765, tom. 3 of theſe Memoirs.

Recherches ſur differens ſujets de Math. t. 4.

ALFECCA, or Alfeta, a name given to the ſtar commonly called Lucida Coronæ.

ALFRAGAN, ALFERGANI, or FARGANI, a celebrated Arabic aſtronomer, who flouriſhed about the year 800. He was ſo called from the place of his nativity, Fergan, in Sogdiana, now called Maracanda, or Samarcand, anciently a part of Bactria. He is alſo called Ahmed (or Muhammed) ben-Cothair, or Katir. He wrote the Elements of Aſtronomy, in 30 chapters or ſections. In this work the author chiefly follows Ptolomy, uſing the ſame hypotheſes, and the ſame terms, and frequently citing him.

There are three Latin tranſlations of Alfragan's work. The firſt was made in the 12th century, by Joannes Hiſpalenſis; and was publiſhed at Ferrara in 1493, and at Nuremberg in 1537, with a preface by Melancthon. The ſecond was by James Chriſtman, from the Hebrew verſion of James Antoli, and appeared at Frankfort in 1590. Chriſtman added to the firſt chapter of the work an ample commentary, in which he compares together the calendars of the Romans, the Egyptians, the Arabians, the Perſians, the Syrians, and the Hebrews, and ſhews the correſpondence of their years.

The third and beſt tranſlation was made by Golius, profeſſor of mathematics and Oriental languages at Leyden: this work, which came out in 1669, after the death of Golius, is accompanied with the Arabic text, and many learned notes upon the firſt nine chapters; for this author was not ſpared to carry them farther.

ALGAROTI, commonly called *Count Algaroti*, a celebrated Italian of the preſent century, well ſkilled in Architecture and the Newtonian philoſophy, &c. Algaroti was born at Padua, but in what year has not been mentioned. Led by curioſity, as well as a deſire of improvement, he travelled early into foreign countries; and was very young when he arived in France in 1736. It was here that he compoſed his *Newtonian Philoſophy for the Ladies*, as Fontenelle had done his Carteſian Aſtronomy, in the work intitled *The Plurality of Worlds*. He was much noticed by the king of Pruſſia, who conferred on him many marks of his eſteem. He died at Piſa the 23d of May, 1764, and gave orders for his own mauſoleum, with this inſcription upon it; *Hic jacet Algarotus, ſed non omnis.* He was eſteemed to be well ſkilled in painting, ſculpture, and architecture. His works, which are numerous, and upon a variety of ſubjects, abound with vivacity, elegance, and wit: a collection of them has lately been made, and printed at Leghorn; but that for which he is chiefly intitled to a place in this work is his *Newtonian Philoſophy for the Ladies*, a ſprightly, ingenious, and popular work.

ALGEBRA, a general method of reſolving mathematical problems by means of equations. Or, it is a method of performing the calculations of all ſorts of quantities by means of general ſigns or characters. At firſt, numbers and things were expreſſed by their names at full length; but afterwards theſe were abridged, and the initials of the words uſed inſtead of them; and, as the art advanced farther, the letters of the alphabet came to be employed as general repreſentations of all ſorts of quantities; and other marks were gradually introduced, to expreſs all ſorts of operations and combinations; ſo as to entitle it to different appellations—univerſal arithmetic, and literal arithmetic, and the arithmetic of ſigns.

The etymology of the name, *Algebra*, is given in various ways. It is pretty certain, however, that the word is Arabian, and that from thoſe people we had the name, as well as the art itſelf, as is teſtified by Lucas le Burgo, the firſt European author whoſe treatiſe was printed on this art, and who alſo refers to former authors and maſters, from whoſe writings he had learned it. The Arabic name he gives it, is *Alghebra e Almucabala*, which is explained to ſignify the art of *reſtitution and compariſon*, or *oppoſition and compariſon*, or *reſolution and equation*, all which agree well enough with the nature of this art. Some however derive it from various other arabic words; as from Geber, a celebrated philoſopher, chemiſt, and mathematician, to whom alſo they aſcribe the invention of this ſcience: ſome likewiſe derive it from the word Geber, which with the particle *al*, makes Algeber, which is purely Arabic, and ſignifies the reduction of broken numbers or fractions to integers.

But Peter Ramus, in the beginning of his Algebra, ſays "the name Algebra is Syriac, ſignifying the art and doctrine of an excellent man. For *Geber*, in Syriac, is a name applied to men, and is ſometimes a term of honour, as maſter or doctor among us. That there was a certain learned mathematician, who ſent his Algebra, written in the Syriac language, to Alexander the Great, and he named it *Almucabala*, that is, the book of dark or myſterious things, which others would rather call the doctrine of Algebra. And to this day the ſame book is in great eſtimation among the learned in the oriental nations, and by the Indians who cultivate this art it is called *Aljabra*, and *Alboret*; though the name of the author himſelf is not known." But Ramus gives no authority for this ſingular paragraph. It has however on various occaſions been diſtinguiſhed by other names. Lucas Paciolus, or de Burgo, in Italy, called it *l'Arte Magiore: ditta dal vulgo la Regola de la Coſa over Alghebra e Almucabala*; calling it *l'Arte Magiore*, or the greater art, to diſtinguiſh it from common arithmetic, which is called *l'Arte Minore*, or the leſſer art. It ſeems too that it had been long and commonly known in his country by the name *Regola de la Coſa*, or *Rule of the Thing*; from whence came our rule of coſs, coſic numbers, and ſuch like terms. Some of his countrymen followed his denomination of the art; but other Italian and Latin writers called it *Regula rei & cenſus*, the rule of the thing and the product, or the root and the ſquare, as the unknown quantity in their equations commonly aſcended no higher than the ſquare or ſecond power. From this Italian word *cenſus*, pronounced *chenſus*, came the barbarous word *zenzus*, uſed by the Germans and others, for quadratics; with the ſeveral zenzic or ſquare roots. And hence ♃, ʒ, ȼ, which are derived from the letters *r*, *z*, *c*, the initials of *res*, *zenzus*, *cubus*, or root, ſquare, cube, came to be the ſigns or characters of theſe words: like as ℞ and √, derived from the letters *R*, *r*, became the ſigns of radicality.

Later authors, and other nations, uſed ſome the one of thoſe names, and ſome another. It was alſo called *Specious Arithmetic* by Vieta, on account of the ſpecies, or letters of the alphabet, which he brought into general uſe; and by Newton it was called *Univerſal Arithmetic*, from the manner in which it performs all arithmetical operations by general ſymbols, or indeterminate quantities.

Some authors define algebra to be *the art of reſolving mathematical problems*: but this is the idea of analyſis, or the analytic art in general, rather than of algebra, which is only one particular ſpecies of it.

Indeed algebra properly conſiſts of two parts: firſt, the method of calculating magnitudes or quantities, as repreſented by letters or other characters: and ſecondly the manner of applying theſe calculations in the ſolution of problems.

In algebra, as applied to the reſolution of problems, the firſt buſineſs is to tranſlate the problem out of the common into the algebraic language, by expreſſing all the conditions and quantities, both known and unknown, by their proper characters, arranged in an equation, or ſeveral equations if neceſſary, and treating the unknown quantity, whether it be number, or line, or any other thing, in the ſame way as if it were a known one: this forms the compoſition. Then the reſolution, or analytic part, is the diſentangling the unknown quantity from the ſeveral others with which it is connected, ſo as to retain it alone on one ſide of the equation, while all the other, or known, quantities, are collected on the other ſide, and ſo giving the value of the unknown one. And as this diſentangling of the quantity ſought, is performed by the converſe of the operations by which it is connected with the others, taking them always backwards in the contrary order, it hence becomes a ſpecies of the analytic art, and is called the modern analyſis, in contradiſtinction to the ancient analyſis, which chiefly reſpected geometry, and its applications.

There have ariſen great controverſies and ſharp diſputes among authors, concerning the hiſtory of the progreſs and improvements of Algebra; ariſing partly from the partiality and prejudices which are natural to all nations, and partly from the want of a cloſer examination of the works of the older authors on this ſubject. From theſe cauſes it has happened, that the improvements made by the writers of one nation, have been aſcribed to thoſe of another; and the diſcoveries of an earlier author, to ſome one of much later date. Add to this alſo, that the peculiar methods of many authors have been deſcribed ſo little in detail, that our information derived from ſuch hiſtories, is but very imperfect, and amounting only to ſome general and vague ideas of the true ſtate of the arts. To remedy this inconvenience therefore, and to reform this article, I have taken the pains carefully to read over in ſucceſſion all the older authors on this ſubject, which I have been able to meet with, and to write down diſtinctly a particular account and deſcription of their ſeveral compoſitions, as to their contents, notation, improvements, and peculiarities; from the compariſon of all which, I have acquired an idea more preciſe and accurate than it was poſſible to obtain from other hiſtories, and in a great many inſtances very different from them. The full detail of theſe deſcriptions would employ a volume of itſelf, and would be far too extenſive for this place: I muſt therefore limit this article to a very brief abridgment of my notes, remarking only the moſt material circumſtances in each author; from which a general idea of the chain of improvements may be perceived, from the firſt rude beginnings, down to the more perfect ſtate; from which it will appear that the diſcoveries and improvements made by any one ſingle author, are ſcarcely ever either very great or numerous; but that, on the contrary, the improvements are almoſt always very ſlow and gradual, from former writers, ſucceſſively made, not by great leaps, and after long intervals of time, but by gradations which, viewed in ſucceſſion, become almoſt imperceptible.

As to the origin of the analytic art, of which Algebra is a ſpecies, it is doubtleſs as old as any ſcience in the world, being the natural method by which the mind inveſtigates truths, cauſes, and theories, from their obſerved effects and properties. Accordingly, traces of it are obſervable in the works of the earlieſt philoſophers and mathematicians, the ſubject of whoſe enquiries moſt of any require the aid of ſuch an art. And this proceſs conſtituted their Analytics. Of that part of analytics however which is properly called Algebra, the oldeſt treatiſe which has come down to us, is that of Diophantus of Alexandria, who flouriſhed about the year 350 after Chriſt, and who wrote, in the Greek language, 13 books of Algebra or Arithmetic, as mentioned by himſelf at the end of his addreſs to Dionyſius, though only 6 of them have hitherto been printed; and an imperfect book on multangular numbers, namely in a Latin tranſlation only, by Xilander, in the year 1575, and afterwards in 1621 and 1670 in Greek and Latin by Gaſpar Bachet. Theſe books however do not contain a treatiſe on the elementary parts of Algebra, but only collections of difficult queſtions relating to ſquare and cube numbers, and other curious properties of numbers, with their ſolutions. And Diophantus only prefaces the books by an addreſs to one Dionyſius, for whoſe uſe it was probably written, in which he juſt mentions certain precognita, as it were to prepare him for the problems themſelves. In theſe remarks he ſhews the names and generation of the powers, the ſquare, cube, 4th, 5th, 6th, &c, which he calls dynamis, cubus, dynamodinamis, dynamocubus, cubocubus, according to the ſum of the indices of the powers; and he marks theſe powers with the initials thus $\delta^{\bar{\upsilon}}$, $\kappa^{\bar{\upsilon}}$, $\delta\delta^{\bar{\upsilon}}$, $\delta\kappa^{\bar{\upsilon}}$, $\kappa\kappa^{\bar{\upsilon}}$, &c: the unknown quantity he calls ſimply αριθμος, *numerus*, the number; and in the ſolutions he commonly marks it by the final thus $\tilde{\varsigma}$; alſo he denotes the monades, or indefinite unit, by $\overline{\mu^{o}}$. Diophantus there remarks on the multiplication and diviſion of ſimple ſpecies together, ſhewing what powers or ſpecies they produce; declares that minus (λειψις) multiplied by minus produces plus (υπαρξιν); but that minus multiplied by plus, produces minus; and that the mark uſed for minus is $\pitchfork$, namely the ψ inverted and curtailed, but he uſes no mark for plus, but a word or conjunction copulative. As to the operations, viz. of addition, ſubtraction, multiplication, and diviſion of compound ſpecies, or thoſe connected by plus and minus, Diophantus does not teach, but ſuppoſes his reader to know them. He then remarks on the preparation or ſimplifying of the equations that are derived from the queſtions, which we call reduction of equations, by collecting like quantities together, adding quantities that are minus, and ſubtracting ſuch as are plus, called by the moderns Tranſpoſition, ſo as to bring the equation to ſimple terms, and then depreſſing it to a lower degree by equal diviſion when the powers of the unknown quantity are in every term: which preparation, or reduction of the complex equation, being now made, or reduced to what we call a final equation, Diophantus goes no farther, but barely ſays what the root or *res ignota* is, without giving any rules for finding it, or for the reſolution of equations; thereby intimating that ſuch rules were to be found in ſome other work, done either by himſelf or others. Of the body of the work, *Lib.* 1 contains 43 queſtions, concerning one, two, three, or four unknown numbers, having certain relations to each other, viz. concerning their ſums, differences, ratios, products, ſquares, ſums and differences of ſquares, &c, &c; but none of them concerning either ſquare or cubic numbers. *Lib.* 2 contains 36 queſtions. The firſt five queſtions are concerning two numbers, though only one condition is given in each queſtion; but he ſupplies another by aſſuming the numbers in a given ratio, viz. as 2 to 1. The 6th and 7th contain each two conditions: then in the 8th queſtion he firſt comes to treat of ſquare numbers, which is this, to divide a given ſquare number into two other ſquares; and the 9th is the ſame, but performed in a different way: the reſt, to the end, are, almoſt all, about one, two, or three ſquares. *Lib.* 3 contains 24 queſtions concerning ſquares, chiefly including three or four numbers. *Lib.* 4 begins with cubes; the firſt of which is this, to divide a given number into two cubes whoſe ſides ſhall have a given ſum: here he has occaſion to cube the two binomials $5 + n$ and $5 - n$; the manner of doing which ſhews that he knew the compoſition of the cube of a binomial; and many other places manifeſt the ſame thing. Only part of the queſtions in this book are concerning cubes; the reſt are relating to ſquares. Two or three queſtions in this book have general ſolutions, and the theorems deduced are general, and for any numbers indefinitely; but all the other queſtions, in all the four books, find only particular numbers. *Lib.* 5 is alſo concerning ſquare and cube numbers, but of a more difficult kind, beginning with ſome that relate to numbers in geometrical progreſſion. *Lib.* 6 contains 26 propoſitions, concerning right-angled triangles; ſuch as to make their ſides, areas, perimeters, &c, &c, ſquares or cubes, or rational, &c. In ſome parts of this book it appears, that he was acquainted with the compoſition of the 4th power of the binomial root, as he ſets down all the terms of it; and, from his great ſkill in ſuch matters, it ſeems probable that he was acquainted with the compoſition of other higher powers, and with other parts of Algebra, beſides what are here treated of. At the end is part of a book, in 10 propoſitions, concerning arithmetical progreſſions, and multangular or polygonal numbers. Diophantus once mentions a compound quadratic equation; but the reſolution of his queſtions is by ſimple equations, and by means of only one unknown letter or character, which he chooſes ſo ingeniouſly, that all the other unknown quantities in the queſtion are eaſily expreſſed by it, and the final equation reduced to the ſimpleſt form which it ſeems the queſtion can admit of. Sometimes he ſubſtitutes for a number ſought immediately, and then expreſſes the other numbers or conditions by it: at other times he ſubſtitutes for the ſum or difference, &c, and thence derives the reſt, ſo as always to obtain the expreſſions in the ſimpleſt form. Thus, if the ſum of two numbers be given, he ſubſtitutes for their difference; and if the difference be given, he ſubſtitutes for their ſum: and in both caſes he has the two numbers eaſily expreſſed by adding and ſubtracting the half ſum and half difference; and ſo in other caſes he uſes other ſimilar ingenious notations. In ſhort, the chief excellence in this collection of queſtions, which ſeems to be only a ſet of exerciſes to ſome rules which had been given elſewhere, is the neat mode of ſubſtitution or notation; which being once made, the reduction

tion to the final equation is eaſy and evident: and there he leaves the ſolution, only mentioning that the root or αριθμος is ſo much. Upon the whole, this work is treated in a very able and maſterly manner, manifeſting the utmoſt addreſs and knowledge in the ſolutions, and forcing a perſuaſion that the author was deeply ſkilled in the ſcience of Algebra, to ſome of the moſt abſtruſe parts of which theſe queſtions or exerciſes relate. However, as he contrives his aſſumptions and notations ſo as to reduce all his conditions to a ſimple equation, or at leaſt a ſimple quadratic, it does not appear what his knowledge was in the reſolution of compound or affected equations.

But although Diophantus was the firſt author on Algebra that we now know of, it was not from him, but from the Moors or Arabians that we received the knowledge of Algebra in Europe, as well as that of moſt other ſciences. And it is matter of diſpute who were the firſt inventors of it; ſome aſcribing the invention to the Greeks, while others ſay that the Arabians had it from the Perſians, and theſe from the Indians, as well as the arithmetical method of computing by ten characters, or digits; but the Arabians themſelves ſay it was invented amongſt them by one *Mahomet ben Muſa*, or ſon of Moſes, who it ſeems flouriſhed about the 8th or 9th century. It is more probable, however, that Mahomet was not the inventor, but only a perſon well ſkilled in the art; and it is farther probable, that the Arabians drew their firſt knowledge of it from Diophantus or other Greek writers, as they did that of Geometry and other ſciences, which they improved and tranſlated into their own language; and from them it was that we received theſe ſciences, before the Greek authors were known to us, after the Moors ſettled in Spain, and after the Europeans began to hold communications with them, and that our countrymen began to travel amongſt them to learn the ſciences. And according to the teſtimony of Abulphavagius, the Arithmetic of Diophantus was tranſlated in Arabic by Mahomet ben-yahya Baziani. But whoever were the inventors and firſt cultivators of Algebra, it is certain that the Europeans firſt received the knowledge, as well as the name, from the Arabians or Moors, in conſequence of the cloſe intercourſe which ſubſiſted between them for ſeveral centuries. And it appears that the art was pretty generally known, and much cultivated, at leaſt in Italy, if not in other parts of Europe alſo, long before the invention of printing, as many writers upon the art are ſtill extant in the libraries of manuſcripts; and the firſt authors, preſently after the invention of printing, ſpeak of many former writers on this ſubject, from whom they learned the art.

It was chiefly among the Italians that this art was firſt cultivated in Europe. And the firſt author whoſe works we have in print, was Lucas Paciolus, or Lucas de Burgo, a Cordelier, or Minorite Friar. He wrote ſeveral treatiſes of Arithmetic, Algebra, and Geometry, which were printed in the years 1470, 1476, 1481, 1487, and in 1494 his principal work, intitled *Summa de Arithmetica, Geometria, Proportioni, et Proportionalita*, is a very maſterly and complete treatiſe on thoſe ſciences, as they then ſtood. In this work he mentions various former writers, as Euclid, St. Auguſtine, Sacroboſco or Halifax, Boetius, Prodocimo, Giordano, Biagio da Parma, and Leonardus Piſanus, from whom he learned thoſe ſciences. The order of the work is, 1ſt Arithmetic, 2d Algebra, and 3d Geometry. Of the Arithmetic the contents, and the order of them, are nearly as follow. Firſt, of numbers figurate, odd and even, perfect, prime and compoſite, and many others. Then of Common Arithmetic in 7 parts, namely numeration or notation, addition, ſubtraction, multiplication, diviſion, progreſſion, and extraction of roots. Before him, he ſays, duplation and mediation, or doubling and halving, were accounted two rules in Arithmetic; but that he omits them, as being included in multiplication and diviſion. He aſcribes the preſent notation and method of Arithmetic to the Arabs; and ſays that according to ſome the word *Abaco* is a corruption of *Modo Arabico*, but that according to others it was from a Greek word. All thoſe primary operations he both performs and demonſtrates in various ways, many of which are not in uſe at preſent, proving them not only by what is called caſting out the nines, but alſo by caſting out the ſevens, and otherwiſe. In the extraction of roots he uſes the initial ℞ for a root; and when the roots can be extracted, he calls them diſcrete or rational; otherwiſe ſurd, or indiſcrete, or irrational. The ſquare root is extracted much the ſame way as at preſent, namely, dividing always the laſt remainder by double the root found; and ſo he continues the ſurd roots continually nearer and nearer in vulgar fractions. Thus, for the root of 6, he firſts finds the neareſt whole number 2, and the remainder 2 alſo; then $\frac{2}{4}$ or $\frac{1}{2}$ is the firſt correction, and $2\frac{1}{2}$ the ſecond root: its ſquare is $6\frac{1}{4}$, therefore $\frac{1}{4}$ divided by 5, or $\frac{1}{20}$ is the next correction, and $2\frac{1}{2}$ minus $\frac{1}{20}$, or $2\,\frac{9}{20}$ is the 3d root: its ſquare is $6\frac{1}{400}$, therefore $\frac{1}{400}$ divided by $4\frac{9}{10}$, or $\frac{1}{1960}$, is the 3d correction, which gives $2\frac{881}{1960}$ for the 4th root, whoſe ſquare exceeds 6 by only $\frac{1}{3841600}$: and ſo on continually: and this proceſs he calls approximation. He obſerves that fractions, which he ſets down the ſame way as we do at preſent, are extracted, by taking the root of the denominator, and of the denominated, for ſo he calls the numerator: and when mixed numbers occur, he directs to reduce the whole to a fraction, and then extract the roots of its two terms as above: as if it be $12\frac{1}{4}$; this he reduces to $\frac{49}{4}$, and then the roots give $\frac{7}{2}$ or $3\frac{1}{2}$: in like manner he finds that $4\frac{1}{2}$ is the root of $20\frac{1}{4}$; $5\frac{1}{2}$ the root of $30\frac{1}{4}$; "and ſo on (he adds) in infinitum;" which ſhews that he knew how to form the ſeries of ſquares by addition. He then extracts the cube root, by a rule much the ſame as that which is uſed at preſent; from which it appears that he was well acquainted with the co-efficients of the binomial cubed, namely 1, 3, 3, 1; and he directs how the operation may be continued "in infinitum" in fractions, like as in the ſquare root. After this, he deſcribes geometrical methods for extracting the ſquare and cube roots inſtrumentally: he then treats profeſſedly of vulgar fractions, their reductions, addition, ſubtraction, and other operations, much the ſame as at preſent: then of the rule-of-three, gain-and-loſs, and other rules uſed by merchants.

Paciolus next enters on the algebraical part of this work, which he calls "*L'Arte Magiore; ditta dal vulgo la Regola de la Coſa, over Alghebra e Almucabala:*" which laſt name he explains by *reſtauratio & oppoſitio*, and aſſigns as a reaſon for the firſt name, becauſe it treats of things above the common affairs in buſineſs, which make the *Arte Minore.* Here he aſcribes the invention of Algebra to the Arabians, and denominates the ſeries of powers, with their marks or abbreviations, as *n*°. or *numero*, the abſolute or known number; *co.* or *coſa*, the thing or 1ſt power of the unknown quantity; *ce.* or *cenſo*, the product or ſquare; *cu.* or *cubo*, the cube, or 3d power; *ce. ce.* or *cenſo de cenſo*, the ſquare-ſquared, or 4th power; *p*°. *r*°. or *primo relato*, or 5th power; *ce. cu.* or *cenſo de cubo*, the ſquare of the cube, or 6th power; and ſo on, compounding the names or indices according to the multiplication of the numbers 2, 3, &c, and not according to their ſum or addition, as uſed by Diophantus. He deſcribes alſo the other characters made uſe of in this part, which are for the moſt part no more than the initials or other abbreviations of the words themſelves; as R̸ for *radici*, the root; R̸. R̸. *radici de radici*, the root of the root; R̸ u. *radici univerſale*, or *radici legata*, or *radici unita*; R̸ *cu. radici cuba*; and $\overline{\mathrm{q}3}^{a}$ *quantita*, quantity; *p* for *piu* or plus, and *m* for *meno* or minus; and he remarks that the neceſſity and uſe of theſe two laſt characters are for connecting, by addition or ſubtraction, different powers together; as 3 *co.* p. 4 *ce.* m. 5 *cu.* p. 2 *ce. ce.* m. 6 *n*ⁱ. that is, 3 *coſa* piu 4 *cenſa* meno 5 *cubo* piu 2 *cenſa-cenſa* meno 6 *numeri*, or, as we now write the ſame thing, $3x + 4x^2 - 5x^3 + 2x^4 - 6$. He firſt treats very fully of proportions and proportionalities, both arithmetical and geometrical, accompanied with a large collection of queſtions concerning numbers in continued proportion, reſolved by a kind of Algebra. He then treats of *el Cataym*, which he ſays, according to ſome, is an Arabic or Phenician word, and ſignifies the Double Rule of Falſe Poſition: but he here treats of both ſingle and double poſition, as we do at preſent, dividing the *el Cataym* into ſingle and double. He gives alſo a geometrical demonſtration of both the caſes of the errors in the double rule, namely when the errors are both plus or both minus, and when the one error is plus and the other minus; and adds a large collection of queſtions, as uſual. He then goes through the common operations of Algebra, with all the variety of ſigns, as to plus and minus; proving that, in multiplication and diviſion, like ſigns give plus, and unlike ſigns give minus. He next treats of different roots *in infinitum*, and the extraction of roots; giving alſo a copious treatiſe on radicals or ſurds, as to their addition, ſubtraction, multiplication and diviſion, and that both in ſquare roots and cube roots, and in the two together, much the ſame as at preſent. He makes here a digreſſion concerning the 15 lines in the 10th book of Euclid, treating them as ſurd numbers, and teaching the extraction of the roots of the ſame, or of compound ſurds or binomials, ſuch as of 23 *p* R̸ 448, or of R̸ 18 *p* R̸ 10; and gives this rule, among ſeveral others, namely: Divide the firſt term of the binomial into two ſuch parts that their product may be $\frac{1}{4}$ of the number in the ſecond term; then the roots of thoſe two parts, connected by their proper ſign *p* or *m*, is the root of the binomial; as in this 23 *p* R̸ 448, the two parts of 23 are 7 and 16, whoſe product, 112, is $\frac{1}{4}$ of 448, therefore their roots give 4 *p* R̸ 7 for the root R̸ u. 23 *p* R̸ 448. He next treats of equations both ſimple and quadratic, or ſimple and compound, as he calls it; and this latter he performs by completing the ſquare, and then extracting the root, juſt as we do at preſent. He alſo reſolves equations of the ſimple 4th power, and of the 4th combined with the 2d power, which he treats the ſame way as quadratics; expreſſing his rules in a kind of bad verſe, and giving geometrical demonſtrations of all the caſes. He uſes both the roots or values of the unknown quantity, in that caſe of the quadratics which has two poſitive roots; but he takes no notice of the negative roots in the other two caſes. But as to any other compound equations, ſuch as the cube and any other power, or the 4th and 1ſt, or 4th and 3d, &c, he gives them up as impoſſible, or at leaſt ſays that no general rule has yet been found for them, any more, he adds, than for the quadrature of the circle. —The remainder of this part is employed on rules in trade and merchandiſe, ſuch as Fellowſhip, Barter, Exchange, Intereſt, Compoſition or Alligation, with various other caſes in trade. And in the third part of the work, he treats of Geometry, both theoretical and practical.

From this account of Lucas de Burgo's book, we may perceive what was the ſtate of Algebra about the year 1500, in Europe; and probably it was much the ſame in Africa and Aſia, from whence the Europeans had it. It appears that their knowledge extended only to quadratic equations, of which they uſed only the poſitive roots; that they uſed only one unknown quantity; that they had no marks or ſigns for either quantities or operations, excepting only ſome few abbreviations of the words or names themſelves; and that the art was only employed in reſolving certain numeral problems. So that either the Africans had not carried Algebra beyond quadratic equations, or elſe the Europeans had not learned the whole of the art, as it was then known to the former. And indeed it is not improbable but this might be the caſe: for whether the art was brought to us by an European, who, travelling in Africa, there learned it; or whether it was brought to us by an African; in either caſe we might receive the art only in an imperfect ſtate, and perhaps far ſhort of the degree of perfection to which it had been carried by their beſt authors. And this ſuſpicion is rendered rather probable by the circumſtance of an Arabic manuſcript, ſaid to be on cubic equations, depoſited in the Library of the univerſity of Leyden by the celebrated Warner, bearing a title which in Latin ſignifies *Omar Ben Ibrahim al'Ghajamæi Algebra cubicarum æquationum, ſive de problematum ſolidorum reſolutione*; and of which book I am in ſome hopes of procuring either a copy or a tranſlation, by means of my worthy friend Dr. Damen, the learned Profeſſor of Mathematics in that univerſity, and by that means to throw ſome light on this doubtful ſubject.

Since this was written, death has prematurely put an end to the uſeful labours of this ingenious and worthy ſucceſſor of Graveſande.

After the publication of the books of Lucas de Burgo, the ſcience of Algebra became more generally known, and improved, eſpecially by many perſons in Italy; and about this time, or ſoon after, namely about the year 1505, the firſt rule was there found out by Scipio Ferreus, for reſolving one caſe of a compound cubic equation. But this ſcience, as well as other branches of

Mathematics, was moſt of all cultivated and improved there by Hieronymus Cardan of Bononia, a very learned man, whoſe arithmetical writings were the next that appeared in print, namely in the year 1539, in 9 books, in the Latin language, at Milan, where he practiſed phyſic, and read public lectures on Mathematics; and in the year 1545 came out a 10th book, containing the whole doctrine of cubic equations, which had been in part revealed to him about the time of the publication of his firſt 9 books. And as it is only this 10th book which contains the new diſcoveries in Algebra, I ſhall here confine myſelf to it alone, as it will alſo afford ſufficient occaſion to ſpeak of his manner of treating Algebra in general. This book is divided into 40 chapters, in which the whole ſcience of cubic equations is moſt amply and ably treated. Chap. 1 treats of the nature, number and properties of the roots of equations, and particularly of ſingle equations that have double roots. He begins with a few remarks on the invention and name of the art: calls it *Ars Magna*, or *Coſa*, or *Rules of Algebra*, after Lucas de Burgo and others: ſays it was invented by Mahomet, the ſon of one Moſes an Arabian, as is teſtified by Leonardus Piſanus; and that he left four rules or caſes, which perhaps only included quadratic equations: that afterwards three derivatives were added by an unknown author, though ſome think by Lucas Paciolus; and after that again three other derivatives, for the cube and 6th power, by another unknown author; all which were reſolved like quadratics: that then Scipio Ferreus, Profeſſor of Mathematics at Bononia, about 1505, found out the rule for the caſe *cubum & rerum numero æqualium*, or, as we now write it, $x^3 + bx = c$, which he ſpeaks of as a thing admirable: that the ſame thing was next afterwards found out, in 1535, by Tartalea, who revealed it to him, Cardan, after the moſt earneſt intreaties: that, finally, by himſelf and his quondam pupil Lewis Ferrari, the caſes are greatly augmented and extended, namely, by all that is not here expreſsly aſcribed to others; and that all the demonſtrations of the rules are his own, except only three adopted from Mahomet for the quadratics, and two of Ferrari for cubics.

He then delivers ſome remarks, ſhewing that all ſquare numbers have two roots, the one poſitive, and the other negative, or, as he calls them, *vera & ficta*, true and fictitious or falſe; ſo the *æſtimatio rei*, or root, of 9, is either 3 or -3; of 16 it is 4 or -4; the 4th root of 81 is 3 or -3; and ſo on for all even *denominations* or powers. And the ſame is remarked on compound caſes of even powers that are added together; as if $x^4 + 3x^2 = 28$, then the æſtimatio x is $= 2$ or -2; but that the form $x^4 + 12 = 7x^2$ has four anſwers or roots, in real numbers, two plus and two minus, viz. 2 or -2, and $\surd 3$ or $-\surd 3$; while the caſe $x^4 + 12 = 6x^2$ has no real roots; and the caſe $x^4 = 2x^2 + 8$ has two, namely 2 and -2: and in like manner for other even powers. So that he includes both the poſitive and negative roots; but rejects what we now call imaginary ones. I here expreſs the caſes in our modern notation, for brevity ſake, as he commonly expreſſes the terms by words at full length, calling the abſolute or known term the *numero*, the 1ſt power the *res*, the 2d the *quadratum*, the 3d the *cubum*, and ſo on, uſing no mark for the unknown quantity, and only the initials *p* and *m* for plus and minus, and ℞ for radix or root. The *res* he ſometimes calls *poſitio*, and *quantitas ignota*; and in ſtating or ſetting down his equations, he, as well as Lucas de Burgo before him, ſets down the terms on that ſide where they will be plus, and not minus.

On the other hand, he remarks that the odd denominations, or powers, have only one æſtimatio, or root, and that true or poſitive, but none fictitious or negative, and for this reaſon, that no negative number raiſed to an odd power, will give a poſitive number; ſo of $2x = 16$, the root is 8 only; and if $2x^3 = 16$, the root is 2 only: and if there be ever ſo many odd denominations, added together, equal to a number, there will be only one æſtimatio or root; as if $x^3 + 6x = 20$, the only root is 2. But that when the ſigns of ſome of the terms are different as to plus and minus, they may have more roots; and he ſhews certain relations of the co-efficients, when they have two or more roots: ſo the equation $x^3 + 16 = 12x$ has two æſtimatios, the one true or 2, and the other fictitious or -4, which he obſerves is the ſame as the true æſtimatio of the caſe $x^3 = 12x + 16$, having only the ſign of the abſolute number changed from the former, the 3d root 2 being the ſame as the firſt, which therefore he does not count. He next ſhews what are the relations of the co-efficients when a cubic equation has three roots, of which two are true, and the 3d fictitious, which is always equal to the ſum of the other two, and alſo equal to the true root of the ſame equation with the ſign of the abſolute number changed: thus, in the equation $x^3 + 9 = 12x$, the two true roots are 3 and $\surd 5\frac{1}{4} - 1\frac{1}{2}$, and the fictitious one is $-\surd 5\frac{1}{4} - 1\frac{1}{2}$, which laſt is the ſame as the true root of $x^3 = 12x + 9$, viz. $\surd 5\frac{1}{4} + 1\frac{1}{2}$; and he here infers generally that the fictitious æſtimatio of the caſe - - - - - - $x^3 + c = bx$, always anſwers to the true root of $x^3 = bx + c$. Cardan alſo ſhews what the relation of the co-efficients is, when the caſe has no true roots, but only one fictitious root, which is the ſame as the true root of the reciprocal caſe, formed by changing the ſign of the abſolute number. Thus, the caſe $x^3 + 21 = 2x$ has no true root, and only one falſe root, viz. -3, which is the ſame as the true root of $x^3 = 2x + 21$: and he ſhews in general, that changing the ſign of the abſolute number in ſuch caſes as want the 2d term, or changing the ſigns of the even terms when it is not wanting, changes the ſigns of all the three roots, which he alſo illuſtrates by many examples; thus, the roots of $x^3 + 11x^2 = 72$, are $+\surd 40 - 4$, and -3, and $-\surd 40 - 4$; and the roots of $x^3 + 72 = 11x^2$, are $-\surd 40 + 4$, and $+3$, and $+\surd 40 + 4$.

And he further obſerves, that the ſum of the three roots, or the difference between the true and fictitious roots, is equal to 11, the co-efficient of the 2d term. He alſo ſhews how certain cubic caſes have one, or two, or three roots, according to circumſtances: that the caſe $x^4 + d = bx^2$ has ſometimes four roots, and ſometimes none at all, that is, no real ones: that the caſe $x^3 + bx = ax^2 + c$ may have three true æquatios, or poſitive roots, but no fictitious or negative ones; and for this reaſon, that the odd powers of minus being minus, and the even powers plus, the two terms $x^3 + bx$ would be negative, and equal to a poſitive ſum $ax^2 + c$, which is abſurd: and farther, that the caſe $x^3 + ax^2 + bx = c$ has three roots, one plus and two minus, which are the ſame, with the ſigns changed, as the roots of the caſe $x^3 + bx + c = ax^2$. He alſo ſhews the relation of the co-efficients when the equation has only one real root, in a variety of caſes: but that the caſe $x^3 + c = ax^2 + bx$ has always one negative root, and either two poſitive roots, or none at all; the number of roots failing by pairs, or the impoſſible roots, as we now call them, being always in pairs. Of all theſe circumſtances Cardan gives a great many particular examples in numeral co-efficients, and ſubjoins geometrical demonſtrations of the properties here enumerated; ſuch as, that the two correſponding or reciprocal caſes have the ſame root or roots, but with different ſigns or affections; and how many true or poſitive roots each caſe has.

Upon the whole, it appears from this ſhort chapter, that Cardan had diſcovered moſt of the principal properties of the roots of equations, and could point out the number and nature of the roots, partly from the ſigns of the terms, and partly from the magnitude and relations of the co-efficients. He ſhews in effect, that when the caſe has all its roots, or when none are impoſſible, the number of its poſitive roots is the ſame as the number of changes in the ſigns of the terms, when they are all brought to one ſide: that the co-efficient of the 2d term is equal to the ſum of all the roots poſitive and negative collected together, and conſequently that when the 2d term is wanting, the poſitive roots are equal to the negative ones: and that the ſigns of all the roots are changed, by changing only the ſigns of the even terms: with many other remarks concerning the nature of equations.

In chap. 2, Cardan enumerates all the caſes of compound equations of the 2d and 3d order, namely, 3 quadratics, and 19 cubics; with 44 derivatives of theſe two, that is, of the ſame kind, with higher denominations.

In chap. 3 are treated the roots of ſimple caſes, or ſimple equations, or at leaſt that will reduce to ſuch, having only two terms, the one equal to the other. He directs to depreſs the denominations equally, as much as they will, according to the height of the leaſt; then divide by the number or co-efficient of the greateſt; and laſtly extract the root on both ſides. So if $20x^3 = 180x^5$, then $20 = 180x^2$, and $\frac{1}{9} = x^2$, and $x = \frac{1}{3}$.

Chap. 4 treats of both general and particular roots, and contains various definitions and obſervations concerning them. It is here ſhewn that the ſeveral caſes of quadratics and cubics have their roots of the following forms or kinds, namely that the caſe

$x^2 = ax + b$ has its root of this kind $\sqrt{19} + 3$,
$x^2 + ax = b$ has its root of this kind $\sqrt{19} - 3$,
$x^2 + b = ax$ has two roots like $3 + \sqrt{2}$ and $3 - \sqrt{2}$,
$x^3 = bx + c$ has its root of this kind $\sqrt[3]{4} + \sqrt[3]{2}$.
$x^3 + bx = c$ has its root of this kind $\sqrt[3]{4} - \sqrt[3]{2}$,
$x^3 = ax^2 + c$ has this kind of root $\sqrt[3]{16} + 2 + \sqrt[3]{4}$
$x^3 + ax^2 = c$ has this kind of root $\sqrt[3]{16} - 2 + \sqrt[3]{4}$,
where the three parts $\sqrt[3]{16}$, 2, $\sqrt[3]{4}$, are in continual proportion.

Chap. 5 treats of the æſtimatio of the loweſt degree of compound caſes, that is, affected quadratic equations; giving the rule for each of the three caſes, which conſiſts in completing the ſquare, &c, as at preſent, and which it ſeems was the method given by the Arabians; and proving them by geometrical demonſtrations from Eucl. I. 43, and II. 4 and 5, in which he makes ſome improvement of the demonſtrations of Mahomet. And hence it appears that the work of this Arabian author was in being, and well known in Cardan's time.

Chap. 6, on the methods of finding new rules, contains ſome curious ſpeculations concerning the ſquares and cubes of binomial and reſidual quantities, and the proportions of the terms of which they conſiſt, ſhewn from geometrical demonſtrations, with many curious remarks and properties, forming a foundation of principles for inveſtigating the rules for cubic equations.

Chap. 7 is on the tranſmutation of equations, ſhewing how to change them from one form to another, by taking away certain terms out of them; as $x^3 + ax^2 = c$, to $y^3 = by + d$, &c. The rules are demonſtrated geometrically; and a table is added, of the forms into which any given caſes will reduce; which transformations are extended to equations of the 4th and 5th order. And hence it appears that Cardan knew how to take away any term out of an equation.

Chap. 8 ſhews generally how to find the root of any ſuch equation as this $ax^m = x^n + b$, where m and n are any exponents whatever, but n the greater; and the rule is, to ſeparate or divide the co-efficient a into two ſuch parts z and $a - z$, as that the abſolute number b ſhall be equal to $\overline{a - z} \cdot z^{\frac{m}{n-m}}$, the product of the one part $a - z$, and the $\frac{m}{n-m}$ power of the other part: then the root x is $= z^{\frac{1}{n-m}}$. The rule is general for quadratics, cubics, and all the higher powers; and could not have been formed without the knowledge of the compoſition of the terms from the roots of the equation.

Chap. 9 and 10 contain the reſolution of various queſtions producing equations not higher than quadratics.

Chap. 11 is of the caſe or form $x^3 + 3bx = 2c$. Cardan now comes to the actual reſolution of the firſt caſe of cubic equations. He begins with relating a ſhort hiſtory of the invention of it, obſerving that it was firſt found out, about 30 years before, by Scipio Ferrens of Bononia, and by him taught to Antonio Maria Florido of Venice, who having a conteſt afterwards with Nicolas Tartalea of Breſcia, it gave occaſion to Tartalea to find it out himſelf, who after great entreaties taught it to Cardan, but ſuppreſſed the demonſtration. By help of the rule alone, however, Cardan of himſelf diſcovered the ſource or geometrical inveſtigation, which he gives here at large, from Eucl. II. 4. In this proceſs he makes uſe of the Greek letters α, β, γ, o, &c, to denote certain indefinite numbers or quantities, to render the inveſtigation general; which may be conſidered as the firſt inſtance of ſuch literal notation in Algebra. He then gives the rule in words at length, which comes to this,

$$x = \sqrt[3]{\sqrt{c^2 + b^3} + c} - \sqrt[3]{\sqrt{c^2 + b^3} - c};$$

illuſtrating it in a variety of examples; in the reſolution of which, he extracts the cubic roots of ſuch of the binomials as will admit of it, by ſome rule which he had for that purpoſe; ſuch as $x^3 + 6x = 20$, which $x = \sqrt[3]{\sqrt{108} + 10} - \sqrt[3]{\sqrt{108} - 10} - \sqrt{3} - 1 = 2 = \overline{\sqrt{3} + 1}$.

Chap. 12, of the caſe $x^3 = 3bx + 2c$. This he treats exactly as the laſt, and finds the rule

$$x = \sqrt[3]{c + \sqrt{c^2 - b^3}} + \sqrt[3]{c - \sqrt{c^2 - b^3}};$$

which he illuſtrates by many examples, as uſual. But when b^3 exceeds c^2, which has ſince been called the irreducible caſe, he refers to another following book, called *Aliza*, for other rules of ſolution, to overcome this difficulty, about which he took infinite pains.

Chap. 13, of the caſe $x^3 + 2c = 3bx$. This caſe, by a geometrical proceſs, he reduces to the caſe in the laſt chapter: thus, find the æſtimatio y of the caſe $y^3 = 3by + 2c$, having the ſame co-efficients as the given caſe $x^3 + 2c = 3bx$; then is $x = \frac{1}{2}y \pm \frac{1}{2}\sqrt{12b - 3y^2}$, giving two roots. He ſhews alſo how to find the ſecond root, when the firſt is known, independent of the foregoing caſe. From this relation of theſe two caſes he deduces ſeveral corollaries, one of which is, that the æſtimatio or root of the caſe $y^3 = 3by + 2c$, is equal to the ſum of the roots of the caſe $x^3 + 2c = 3bx$. As in the example $y^3 = 16y + 21$, whoſe æſtimatio is $\sqrt{9\frac{1}{4}} + 1\frac{1}{2}$, which is equal to the ſum of 3 and $\sqrt{9\frac{1}{4}} - 1\frac{1}{2}$, the two roots of the caſe $x^3 + 21 = 16x$.

In chapters 14, 15, and 16, he treats of the three caſes which contain the 2d and 3d powers, but wanting the firſt power, according to all the varieties of the ſigns; which he performs by exterminating the 2d term, or that which contains the 2d power of the unknown quantity x, by ſubſtituting $y \pm \frac{1}{3}$ the co-efficient of that term for x, and ſo reducing theſe caſes to one of the former. In theſe chapters Cardan ſometimes alſo gives other rules; thus, for the caſe $x^3 + 3ax^2 = 2c$, find firſt the æſtimatio y of the caſe $y^3 = 3ay + \sqrt{2c}$, then is $x = y^2 - 3a$: alſo for the caſe $x^3 + 2c = 3ax^2$, firſt find the two roots of $y^3 + 2c = 3ay\sqrt[3]{2c}$, then is $x = \frac{\sqrt[3]{4c^2}}{y}$ the two values of x according to the two values of y. He here alſo gives another rule, by which a ſecond æſtimatio or root is found, when the firſt is known, namely, if e be the firſt eſtimatio or value of x in the caſe $x^3 + 2c = 3ax^2$, then is the other value of

$$x = \tfrac{1}{2}\sqrt{\overline{3a - e} \cdot \overline{3a + 3e}} + \tfrac{1}{2} \cdot \overline{3a - e}.$$

In chapters 17, 18, 19, 20, 21, 22, 23, Cardan treats of the caſes in which all the four terms of the equation are preſent; and this he always effects by taking away the 2d term out of the equation, and ſo reducing it to one of the foregoing caſes which want that term, giving always geometrical inveſtigations, and adding a great many examples of every caſe of the equations.

Chap. 24, of the 44 derivative caſes; which are only higher powers of the forms of quadratics and cubics.

Chap. 25, of imperfect and ſpecial caſes: containing many particular examples when the co-efficients have certain relations amongſt them, with eaſy rules for finding the roots; alſo 8 other rules for the irreducible caſe $x^3 = bx + c$.

Chap. 26, in like manner, contains eaſy rules for biquadratics, when the co-efficients have certain ſpecial relations.

Then the following chapters, from chap. 27 to chap. 38, contain a great number of queſtions and applications of various kinds, the titles of which are theſe: *De tranſitu capituli ſpecialis in capitulum ſpeciale; De operationibus radicum pronicarum ſeu mixtarum & Allellarum; De regula modi; De regula Aurea; De regula Magna*, or the method of finding out ſolutions to certain queſtions; *De regula æqualis poſitionis*, being a method of ſubſtituting for the half ſum and half difference of two quantities, inſtead of the quantities themſelves; *De regula inæqualiter ponendi, ſeu proportionis; De regula medii; De regula aggregati; De regula liberæ poſitionis; De regula falſum ponendi*, in which ſome quantities come out negative; *Quomodo excidant partes & denominationes multiplicando.* Among the foregoing collection of queſtions, which are chiefly about numbers, there are ſome geometrical ones, being the application of Algebra to Geometry, ſuch as, In a right-angled triangle, given the ſum of each leg and the adjacent ſegment of the hypotenuſe, made by a perpendicular from the right angle, to determine the area &c; with other ſuch geometrical queſtions, reſolved algebraically.

Chap. 39, *De regula qua pluribus poſitionibus invenimus ignotam quantitatem*; which is employed on biquadratic equations. After ſome examples of his own, Cardan gives a rule of Lewis Ferrari's, for reſolving all biquadratics, namely by means of a cubic equation, which Ferrari inveſtigated at his requeſt, and which Cardan here demonſtrates, and applies in all its caſes. The method is very general, and conſiſts in forming three ſquares, thus: firſt, complete one ſide of the equation up to a ſquare, by adding or ſubtracting ſome multiples or parts of ſome of its own terms on both ſides, which it is always eaſy to do: 2d, ſuppoſing now the three terms of this ſquare to be but one quantity, viz, the firſt term of another ſquare to which this ſame ſide is to be completed, by annexing the ſquare of a new and aſſumed indeterminate quantity, with double the product of the roots of both; which evidently forms the ſquare of a binomial, conſiſting of the aſſumed indeterminate quantity and the root of the firſt ſquare: 3d, the other ſide of the equation is then made to become the ſquare of a binomial alſo, by ſuppoſing the product of its 1ſt and 3d terms to be equal to the ſquare of half its 2d term; for it conſiſts of only three terms, or three different denominations of the original unknown quantity: then this equality will determine the value of the aſſumed indeterminate quantity, by means of a cubic equation, and from it, that of the original ignota, by the equal roots of the 2d and 3d ſquares. Here we have a notable example of the uſe of aſſuming a new indeterminate quantity to introduce into an equation, long before Des Cartes was born, who made uſe of a like aſſumption for a ſimilar purpoſe. And this method is very general, and is here applied to all forms of biquadratics, either having all their terms, or wanting ſome of them. To illuſtrate this rule I ſhall here ſet down the proceſs of one of his examples, which is this, $x^4 + 4x + 8 = 10x^2$. Now firſt ſubtract $2x^2 + 4x + 7$ from both ſides, then the firſt becomes a ſquare, viz, $x^4 - 2x^2 + 1$ or $\overline{x^2 - 1}^2 = 8x^2 - 4x - 7$. Next aſſume the indeterminate y, and ſubtract $2y\,(x^2 - 1) - y^2$ from both ſides, making the firſt ſide again a ſquare, viz, $\overline{x - 1}^2 - 2y \cdot \overline{x - 1} + y^2$ or $\overline{x^2 - 1 - y}^2 = \overline{8 - 2y} \cdot x^2 - 4x + y^2 + 2y - 7$. Of this latter ſide, make the product of the 1ſt and 3d terms equal to the ſquare of half the 2d term, that is, $\overline{8 - 2y} \cdot y^2 + 2y - 7 = 2^2$, which reduces to $y^3 + 30 = 2y^2 + 15y$; the poſitive roots of which are $y = 2$ or $\surd 15$; and hence, uſing 2 for y, the equation of equal ſquares becomes $\overline{x^2 - 1 - y}^2$ or $\overline{x^2 - 3}^2 = 4x^2 - 4x + 1$, the roots of which give $x^2 - 3 = 2x \backsim 1$; and hence $x^2 = 2x + 2$, or $x^2 + 2x = 4$; the two poſitive roots of which are $\surd 3 + 1$ and $\surd 5 - 1$, which are two of the values of x in the given equation $x^4 + 4x + 8 = 10x^2$. The other roots he leaves to be tried by the reader.

The 40th, or laſt, chap. is entitled, Of modes of general ſuppoſition relating to this art; with ſome rules of an unuſual kind; and æſtimatios or roots of a nature different from the foregoing ones. Some of theſe are as follow: If $x^3 = ax^2 + c$, and $x - a = y$, and $x : y :: c : d$; then is $y^3 + ay^2 = d$.

Secondly, if $x^3 + ax^2 = c$,
and $y^3 = ay^2 + c$,
then is $x + a : y - a :: y^2 : x^2$.

Thirdly, when $x^3 + c = ax^2$, the ſquare will be taken away, by putting $x = y + \frac{1}{3}a$; and then the equation becomes $y^3 + c - 2(\frac{1}{3}a)^3 = \frac{1}{3}a^2y$.

Cardan adds ſome other remarks concerning the ſolutions of certain caſes and queſtions, all evincing the accuracy of his ſkill, and the extent of his practice; and then he concludes the book with a remark concerning a certain transformation of equations, which quite aſtoniſhes us to find that the ſame perſon who, through the whole work, has ſhewn ſuch a profound and critical ſkill in the nature of equations, and the ſolution of problems, ſhould yet be ignorant of one of the moſt obvious tranſmutations attending them, namely increaſing or diminiſhing the roots in any proportion. Cardan having obſerved that the form $x^3 = bx + c$ may be changed into another ſimilar one, viz, $y^3 = \frac{b}{c}y + \surd\frac{1}{c}$, of which the co-efficient of the term y is the quotient ariſing from the co-efficient of x divided by the abſolute number of the firſt equation: and that the abſolute number of the 2d equation is the root of the quotient of 1 divided by the ſaid abſolute number of the firſt; he then adds, that finding the æſtimatio or root of the one equation from that of the other is very difficult, *valde difficilis*.

It is matter of wonder that Cardan, among ſo many tranſmutations, ſhould never think of ſubſtituting inſtead of x in ſuch equations, another poſitio or root, greater or leſs than the former in any indefinite proportion, that is, multiplied or divided by a given number; for this would have led him immediately to the ſame transformation as he makes above, and that by a way which would have ſhewn the conſtant proportion between the two roots. Thus, inſtead of x in the given form - - - - - - $x^3 = bx + c$, ſubſtitute dy, and it becomes - $d^3y^3 = bdy + c$; and this divided by d^3 becomes - - $y^3 = \frac{b}{d^2}y + \frac{c}{d^3}$; and here if d be taken $= \surd c$, it becomes $y^3 = \frac{b}{c}y + \surd\frac{1}{c}$; which is the transformation in queſtion, and in which it is evident that x is $= y\surd c$, and $y = \frac{x}{\surd c}$. Inſtead of this, Cardan gives the following ſtrange way of finding the one root x from the other y, when this latter is by any means known; viz, Multiply the firſt given equation by $y^2x + 1$, then add $\frac{x^2}{4y^2}$ to both ſides, and laſtly extract the roots of both, which can always be done, as they will always be both of them ſquares; and the roots will give the value of x by a quadratic equation.

Thus, $x^3 = bx + c$ multiplied by $y^2x + 1$ gives $y^2x^4 + x^3 = by^2x^2 + \overline{cy^2 + b} \cdot x + c$; add $\frac{x^2}{4y^2}$, then $y^2x^4 + x^3 + \frac{x^2}{4y^2} = \overline{by^2 + \frac{1}{4y^2}} \cdot x^2 + \overline{b + cy^2} \cdot x + c$; and the roots are $yx^2 + \frac{x}{2y} = \surd(by^2 + \frac{1}{4y^3})x^2 + (b + cy^2)x + c$; and this 2d ſide of the equation he ſays will always have a root alſo. It is indeed true that it will have an exact root; but the reaſon of it is not obvious, which is, becauſe y is the root of the equation $y^3 = \frac{b}{c}y + \surd\frac{1}{c}$. Cardan has not ſhewn the reaſon why this happens; but I apprehend he made it out in this manner, viz, ſimilar to the way in which he forms the laſt ſquare in the caſe of biquadratic equations, namely, by making the product of the 1ſt and 3d terms equal to the ſquare of half the 2d term: thus, in the preſent caſe, it is $4c(by^2 + \frac{1}{4y^2}) = (b + cy^2)^2$, which reduces to $y^3 = \frac{b}{c}y + \surd\frac{1}{c}$ the equation in queſtion. Therefore taking y the root of the equation $y^3 = \frac{b}{c}y + \surd\frac{1}{c}$, and ſubſtituting its value in the quantity $(by^2 + \frac{1}{4y^2})x^2 + (b + cy^2)\,x + c$, this will become a complete ſquare.

Of Cardan's *Libellus de Aliza Regula.*

Subjoined to the above Treatiſe on cubic equations, is this *Libellus de Aliza regula*, or the algebraic logiſtics, in which the author treats of ſome of the abſtruſer parts of Arithmetic and Algebra, eſpecially cubic equations, with many more attempts on the irreducible caſe $x^3 = bx + c$. This book is divided into 60 chapters;

but I ſhall only ſet down the titles of ſome few of them, whoſe contents require more particular notice.

Chap. 4. *De modo redigendi quantitates omnes, quæ dicuntur latera prima ex decimo Euclidis in compendium.* He treats here of all Euclid's irrational lines, as ſurd numbers, and performs various operations with them.

Chap. 5. *De conſideratione binomiorum & reciſorum, &c; ubi de æſtimatione capitulorum.* Contains various operations of multiplying compound numbers and ſurds.

Chap. 6. *De operationibus* p: & m: (i. e. + and −) *ſecundum communem uſum.* Here it is ſhewn that, in multiplication and diviſion, *plus* always gives the ſame ſigns, and *minus* gives the contrary ſigns. So alſo in addition, every quantity retains its own ſign; but in ſubtraction they change the ſigns. That the $\sqrt{}+$, or the ſquare root of plus, is +; but the $\sqrt{}-$, or the ſquare root of minus, is nothing as to common uſe: (but of this below.) That $\sqrt[3]{}-$ is −; as $\sqrt{}-8$ is -2. That a reſidual, compoſed of + and − may have a root alſo compoſed of + and −: So $\sqrt{5-\sqrt{24}}$ is $=\sqrt{3}-\sqrt{2}$. The rules for the ſigns in multiplication and diviſion are illuſtrated by this example; to divide 8 by $2+\sqrt{6}$ or $\sqrt{6}+2$. Take the two correſponding reſiduals $2-\sqrt{6}$ and $\sqrt{6}-2$, and by theſe multiply both the diviſor and dividend; then the products are + and − reſpectively, and the quotients ſtill both alike. Thus,

Divid.	Diviſ.	Divid.	Diviſ.
8	$\sqrt{6}+2$	8	$2+\sqrt{6}$
$\sqrt{6}-2$	$\sqrt{6}-2$	$2-\sqrt{6}$	$2-\sqrt{6}$
$\sqrt{384}-16$	divide $+4$	$16-\sqrt{384}$	div. -2
Quot. $\sqrt{96}-8$.		Quot. $\sqrt{96}-8$.	

And this method of performing diviſion of compound ſurds, was fully taught before him, by Lucas de Burgo, namely, reducing the compound diviſor to a ſimple quantity, by multiplying by the correſponding quantity, having the ſign changed.

In chap. 11 and 18, and elſewhere, Cardan makes a general notation of a, b, c, d, e, f, for any indefinite quantities, and treats of them in a general way.

Cap. 2. *De contemplatione* p: & m: (or + and −), & *quod* m: *in* m: *facit* p: & *de cauſis horum juxta veritatem.* Cardan here demonſtrates geometrically that, in multiplication and diviſion, like ſigns give plus, and unlike ſigns give minus. And he illuſtrates this numerically, by ſquaring the quantity 8, or $6+2$, or $10-2$, which muſt all produce the ſame thing, namely 64.

Among many of the chapters which treat of the irreducible caſe $x^3=bx+c$, there is a peculiar kind of way given in chap. 31, which is entitled *De æſtimatione generali* $x^3=bx+c$ *ſolida vocata, & operationibus ejus;* in which he ſhews how to approximate to the root of that caſe, in a manner ſimilar to approximating the ſquare root and cube root of a number. The rule he uſes for this purpoſe, is the 3d in chap. 25 of the laſt book, and it is this: Divide b into two parts, ſuch that the ſum of the products of each, multiplied by the ſquare of the other, may be equal to $\frac{1}{2}c$; then the ſum of the roots of theſe parts is the æſtimatio or value of x required. So, of this equation $x^3=10x+24$; the two parts are 9 and 1, and their roots 3 and 1, and their ſum $4=x$, as in the margin.

9 1
×
3 1
$3+9$ or $12=\frac{1}{2}c$.
$x=3+1=4$

Again, take $x^3=6x+1$. Here he invents a new notation to expreſs the root or *radix*, which he calls *ſolida*, viz, $x=\sqrt{}$ *ſolida* 6 *in* $\frac{1}{2}$, that is, the roots of the two parts of 6, ſo that each part multiplied by the root of the other, the two products may be $\frac{1}{2}$ or $\frac{1}{2}c$. Then to free this from fractions, and make the operation eaſier, multiply that root by ſome number as ſuppoſe 4, that is the ſquare part 6 by the ſquare of 4, and the ſolid part $\frac{1}{2}$ by the cube of 4; then $x=\frac{1}{4}\sqrt{}$ *ſolida* 96 *in* 32. Now, by a few trials, it is found that the parts are
nearly $95\frac{8}{9}$ and $\frac{1}{9}$, which give too much,
or $95\frac{9}{10}$ and $\frac{1}{10}$, which give too little,
and thereof $95\frac{17}{19}$ and $\frac{2}{19}$ are ſtill nearer. Divide both by 4^2 or 16, then $5\frac{151}{152}$ and $\frac{1}{152}$ are the quot. And the ſum of their roots, or $x=\sqrt{5\frac{151}{152}}+\sqrt{\frac{1}{152}}$ is nearly the value of the root x.

Cap. 42. *De duplici æquatione comparanda in capitulo cubi & numeri æqualium rebus.* Treats of the two poſitive roots of that caſe, neglecting the negative one; and ſhewing, not only that that caſe has two ſuch roots, but that the ſame number may be the common root of innumerable equations.

Cap. 57. *De tractatione æſtimationis generalis capituli* $x^3=bx+c$. Cardan here again reſumes the conſideration of the irreducible caſe, making ingenious obſervations upon it, but ſtill without obtaining the root by a general rule. In this place alſo, as well as elſewhere, he ſhews how to form an equation in this caſe, that ſhall have a given binomial root, as ſuppoſe $\sqrt{m}+n$, where the equation will be $x^3=(m+3n^2)x+2n(m-n^2)$, having $\sqrt{m}+n$ for one root, namely the poſitive root. From which it appears that he was well acquainted with the compoſition of cubic equations from given roots.

Cap. 59. *De ordine & exemplis in binomiis ſecundo & quinto.* Contains a great many numeral forms of the ſame irreducible caſe $x^3=bx+c$, with their roots; from which are derived theſe following caſes, with many curious remarks. When

$$x^3=(c+1)\,x+c,\ \text{then}\ x=\sqrt{c+\tfrac{1}{4}}+\tfrac{1}{2}.$$

$$x^3=(\tfrac{1}{2}c+4)\,x+c,\ \text{then}\ x=\sqrt{\tfrac{1}{2}c+\tfrac{4}{4}}+\tfrac{2}{2}.$$

$$x^3=(\tfrac{1}{3}c+9)\,x+c,\ \text{then}\ x=\sqrt{\tfrac{1}{3}c+\tfrac{9}{4}}+\tfrac{3}{2}.$$

$$x^3=(\tfrac{1}{4}c+16)\,x+c,\ \text{then}\ x=\sqrt{\tfrac{1}{4}c+\tfrac{16}{4}}+\tfrac{4}{2}.$$

$$x^3=(\frac{1}{n}c+n^2)\,x+c,\ \text{then}\ x=\sqrt{\frac{1}{n}c+\frac{n^2}{4}}+\frac{n}{2}.$$

Cap. 60. *Demonſtratio generalis capituli cubi æqualis rebus & numero.* This demonſtration of the irreducible caſe is geometrical, like all the reſt. Some more ingenious remarks are again added, as if he reluctantly finiſhed the book without perfectly overcoming the difficulty of the irreducible caſe. Cardan here alſo uſes the letters a and b for any two indefinite numbers, in order to ſhew the form and manner of the arithmetical operations: thus $\dfrac{a}{b}$ is the fraction for their quotient; alſo

$\sqrt{\dfrac{a}{b}}$ or $\dfrac{\sqrt{a}}{\sqrt{b}}$ the ſquare root of that quotient, and

$\sqrt[3]{\dfrac{a}{b}}$ or $\dfrac{\sqrt[3]{a}}{\sqrt[3]{b}}$ the cube root of it, &c.

Having conſidered the chief contents of Cardan's algebra, it will now be proper to ſum them up, and ſet down a liſt of the improvements made by him, as collected from his writings:

And 1ſt, Tartalea having only communicated to him the rules for reſolving theſe three caſes of cubic equations, viz,

$$\left.\begin{array}{l} x^3 + bx = c, \\ x^3 = bx + c, \\ x^3 + c = bx, \end{array}\right\}$$

he from thence raiſed a very large and complete work, laying down rules for all forms and varieties of cubic equations, having all their terms, or wanting any of them, and having all poſſible varieties of ſigns; demonſtrating all theſe rules geometrically; and treating very fully of almoſt all ſorts of tranſformations of equations, in a manner heretofore unknown.

2nd, It appears that he was well acquainted with all the roots of equations that are real, both poſitive and negative; or, as he calls them, true and fictitious; and that he made uſe of them both occaſionally. He alſo ſhewed, that the even roots of poſitive quantities, are either poſitive or negative; that the odd roots of negative quantities, are real and negative; but that the even roots of them are impoſſible, or nothing as to common uſe. He was alſo acquainted with,

3d, The number and nature of the roots of an equation, and that partly from the ſigns of the terms, and partly from the magnitude and relation of the coefficients. He alſo knew,

4th, That the number of poſitive roots is equal to the number of changes of the ſigns of the terms.

5th, That the coefficient of the ſecond term of the equation, is the difference between the poſitive and negative roots.

6th, That when the ſecond term is wanting, the ſum of the negative roots is equal to the ſum of the poſitive roots.

7th, How to compoſe equations that ſhall have given roots.

8th, That, changing the ſigns of the even terms, changes the ſigns of all the roots.

9th, That the number of roots failed in pairs; or what we now call impoſſible roots were always in pairs.

10th, To change the equation from one form to another, by taking away any term out of it.

11th, To increaſe or diminiſh the roots by a given quantity. It appears alſo,

12th, That he had a rule for extracting the cube root of ſuch binomials as admit of extraction.

13th, That he often uſed the literal notation $a, b, c, d,$ &c.

14th, That he gave a rule for biquadratic equations, ſuiting all their caſes; and that, in the inveſtigation of that rule, he made uſe of an aſſumed indeterminate quantity, and afterwards found its value by the arbitrary aſſumption of a relation between the terms.

15th, That he applied Algebra to the reſolution of geometrical problems. And

16th, That he was well acquainted with the difficulty of what is called the irreducible caſe, viz, $x^3 = bx + c$, upon which he ſpent a great deal of time, in attempting to overcome it. And though he did not fully ſucceed in this caſe, any more than other perſons have done ſince, he nevertheleſs made many ingenious obſervations about it, laying down rules for many particular forms of it, and ſhewing how to approximate very nearly to the root in all caſes whatever.

OF TARTALEA.

Nicholas Tartalea, or Tartaglia, of Breſcia, was contemporary with Cardan, and was probably older than he was, but I do not know of any book of Algebra publiſhed by him till the year 1546, the year after the date of Cardan's work on Cubic Equations, when he printed his *Queſiti & Inventioni diverſe*, at Venice, where he reſided as a public lecturer on mathematics. This work is dedicated to our king Henry the VIIIth of England, and conſiſts of 9 books, containing anſwers to various queſtions which had been propoſed to him at different times, concerning mechanics, ſtatics, hydroſtatics, &c.; but it is only the 9th, or laſt, that we ſhall have occaſion to take notice of in this place, as it contains all thoſe queſtions which relate to arithmetic and algebra. Theſe are all ſet down in chronological order, forming a pretty collection of queſtions and ſolutions on thoſe ſubjects, with a ſhort account of the occaſion of each of them. Among theſe, the correſpondence between him and Cardan forms a remarkable part, as we have here the hiſtory of the invention of the rules for cubic equations, which he communicated to Cardan, under the promiſe, and indeed oath, to keep them ſecret, on the 25th of March 1539. But, notwithſtanding his oath, finding that Cardan publiſhed them in 1545, as above related, it ſeems Tartalea publiſhed the correſpondence between them in revenge for his breach of faith; and it elſewhere appears, that many other ſharp bickerings paſſed between them on the ſame account, which only ended with the death of Tartalea, in the year 1557. It ſeems it was a common practice among the mathematicians, and others, of that time, to ſend to each other nice and difficult queſtions, as trials of ſkill, and to this cauſe it is that we owe the principal queſtions and diſcoveries in this collection, as well as many of the beſt diſcoveries of other authors. The collection now before us contains queſtions and ſolutions, with their dates, in a regular order, from the year 1521, and ending in 1541, in 42 dialogues, the laſt of which is with an Engliſh gentleman, namely, Mr. Richard Wentworth, who it ſeems was no mean mathematician, and who learned ſome algebra, &c, of Tartalea, while he reſided at Venice. The queſtions at firſt are moſtly very eaſy ones in arithmetic, but gradually become more difficult, and exerciſing ſimple and quadratic equations, with complex calculations of radical quantities: all ſhewing that he was well ſkilled in the art of Algebra as it then ſtood, and that he was very ingenious in applying it to the ſolutions of queſtions. Tartalea made no alteration in the notation or forms of expreſſion uſed by Lucas de Burgo, calling the firſt power of the unknown quantity, in his language, *coſa*, the ſecond power *cenſu*, the third *cubo*, &c, and writing the names of all the operations in words at length, without uſing any contractions, except the initial ℞ for root or radicality. So that the only thing remarkable in this collection, is the diſcovery of the rules for cubic equations, with the curious circumſtances attending the ſame.

The firſt two of theſe were diſcovered by Tartalea in the year 1530, namely for the two caſes $x^3 + ax^2 = c$, and $x^3 = ax^2 + c$, as appears by Queſt. 14 and 25 of this collection, on occaſion of a queſtion then propoſed to him by one Zuanni de Tonini da Coi or Colle, John Hill, who kept a ſchool at Breſcia. And from the 25th letter we learn, that he diſcovered the rules for the other two caſes $x^3 + bx = c$, and $x^3 = bx + c$, on the 12th and 13th of February 1535, at Venice, where he had come to reſide the year before. And the occaſion of it was this: There was then at Venice one Antonio Maria Fiore or Florido, who, by his own account, had received from his preceptor Scipio Farreo, about thirty years before, a general rule for reſolving the caſe $x^3 + bx = c$. Being a captious man, and preſuming on this diſcovery, Florido uſed to brave his contemporaries, and by his inſults provoked Tartalea to enter into a wager with him, that each ſhould propoſe to the other thirty different queſtions; and that he who ſooneſt reſolved thoſe of his adverſary, ſhould win from him as many treats for himſelf and friends. Theſe queſtions were to be propoſed on a certain day at ſome weeks diſtance; and Tartalea made ſuch good uſe of his time, that eight days before the time appointed for delivering the propoſitions, he diſcovered the rules both for the caſe $x^3 + bx = c$, and the caſe $x^3 = bx + c$. He therefore propoſed ſeveral of his queſtions ſo as to fall either on this latter caſe, or on the caſes of the cube and ſquare, expecting that his adverſary would propoſe his in the former. And what he ſuſpected fell out accordingly; the conſequence of which was, that on the day of meeting Tartalea reſolved all his adverſary's queſtions in the ſpace of two hours, without receiving one anſwer from Florido in return; to whom, however, Tartalea generouſly remitted the forfeit of the thirty treats won of him.

Queſtion 31 firſt brings us acquainted with the correſpondence between Tartalea and Cardan. This correſpondence is very curious, and would well deſerve to be given at full length in their own words, if it were not too long for this place. I may enlarge farther upon it under the article *Cubic Equations*; but muſt here be content with a brief abſtract only. Cardan was then a reſpectable phyſician, and lecturer in mathematics at Milan; and having nearly finiſhed the printing of a large work on Arithmetic, Algebra, and Geometry, and having heard of Tartalea's diſcoveries in cubic equations, he was very deſirous of drawing thoſe rules from him, that he might add them to his book before it was finiſhed. For this purpoſe he firſt applied to Tartalea, by means of a third perſon, a bookſeller, whom he ſent to him, in the beginning of the year 1539, with many flattering compliments, and offers of his ſervices and friendſhip, &c, accompanied with ſome critical queſtions for him to reſolve, according to the cuſtom of the times. Tartalea however refuſed to diſcloſe his rules to any one, as the knowledge of them gained him great reputation among all people, and gave him a great advantage over his competitors for fame, who were commonly afraid of him on account of thoſe very rules. He only ſent Cardan therefore, at his requeſt, a copy of the thirty queſtions which had been propoſed to him in the conteſt with Florido. Not to be rebuffed ſo eaſily, Cardan next applied, in the moſt urgent manner, by letter to Tartalea; which however procured from him only the ſolution of ſome other queſtions propoſed by Cardan, with a few of the queſtions that had been propoſed to Florido, but none of their ſolutions. Finding he could not thus prevail, with all his fair promiſes, Cardan then fell upon another ſcheme. There was at Milan a certain Marquis dal Vaſto, a great patron of Cardan, and, it was ſaid, of learned men in general. Cardan conceived the idea of making uſe of the influence of this nobleman to draw Tartalea to Milan, hoping that then, by perſonal intreaties, he ſhould ſucceed in drawing the long-concealed rules from him. Accordingly he wrote a ſecond letter to Tartalea, much in the ſame ſtrain with the former, ſtrongly inviting him to come and ſpend a few days in his houſe at Milan, and repreſenting that, having often commended him in the higheſt terms to the marquis, this nobleman deſired much to ſee him; for which reaſon Cardan adviſed him, as a friend, to come to viſit them at Milan, as it might be greatly to his intereſt, the marquis being very liberal and bountiful; and he beſides gave Tartalea to underſtand, that it might be dangerous to offend ſuch a man by refuſing to come, who might, in that caſe, take offence, and do him ſome injury. This manœuvre had the deſired effect: Tartalea on this occaſion laments to himſelf in theſe words, "By this I am reduced to a great dilemma; for if I go not to Milan, the marquis may take it amiſs, and ſome evil may befal me on that account; I ſhall therefore go, although very unwillingly." When he arrived at Milan however, the marquis was gone to Vigeveno, and Tartalea was prevailed on to ſtay three days with Cardan, in expectation of the marquis returning, at the end of which he ſet out from Milan, with a letter from Cardan, to go to Vigeveno to that nobleman. While Tartalea was at Milan the three days, Cardan plied him by all poſſible means to draw from him the rules for the cubic equations; and at length, juſt as Tartalea was about to depart from Milan, on the 25th of March 1539, he was overcome by the moſt ſolemn proteſtations of ſecrecy that could be made. Cardan ſays, "I ſhall ſwear to you on the holy evangeliſts, and by the honour of a gentleman, not only never to publiſh your inventions, if you reveal them to me; but I alſo promiſe to you, and pledge my faith as a true chriſtian, to note them down in cyphers, ſo that after my death no other perſon may be able to underſtand them." To this Tartalea replies, "If I refuſe to give credit to theſe aſſurances, I ſhould deſervedly be accounted utterly void of belief. But as I intend to ride to Vigeveno, to ſee his excellency the marquis, as I have been here now theſe three days, and am weary of waiting ſo long; whenever I return therefore, I promiſe to ſhew you the whole." Cardan anſwers, "Since you determine at any rate to go to Vigeveno, to the marquis, I ſhall give you a letter for his excellency, that he may know who you are. But now before you depart, I intreat you to ſhew me the rule for the equations, as you have promiſed." "I am content," ſays Tartalea: "But you muſt know, that to be able on all occaſions to remember ſuch operations, I have brought the rule into rhyme; for if I had not uſed that precaution, I ſhould often have forgot it; and although my rhymes are not very good, I do not value that, as it is ſufficient that they ſerve to bring the rule to mind as often as I repeat them. I ſhall here write the rule with my own hand, that you may be ſure I give you the diſcovery exactly." Theſe rude verſes contain, in rather dark and enigmatical language, the rule for theſe three caſes, viz.

$$\left.\begin{array}{l} x^3 + bx = c, \\ x^3 = bx + c, \\ x^3 + c = bx, \end{array}\right\}$$

which differ however only in the ſign of one quantity, and the rule amounts to this: Find two numbers, z and y, ſuch that their difference in the firſt caſe, and their ſums in the 2d and 3d, may be equal to c the abſolute number, and their product equal to the cube of $\frac{1}{3}$ of b the coefficient of the leſs power; then the difference of their cube roots will be equal to x in the firſt caſe, and the ſum of their cube roots equal to x in the 2d and 3d caſes: that is, taking $z-y=c$ in the 1ſt caſe, or $z+y=c$ in the 2d and 3d, and $zy = (\frac{1}{3}b)^3$; then $x = \sqrt[3]{z} - \sqrt[3]{y}$ in the firſt caſe, and $x = \sqrt[3]{z} + \sqrt[3]{y}$ in the other two. At parting, T. fails not again to remind C. of his obligation: "Now your excellency will remember not to break your promiſed faith, for if unhappily you ſhould inſert theſe rules either in the work you are now printing, or in any other, although you ſhould even give them under my name, and as of my invention, I promiſe and ſwear that I ſhall immediately print another work that will not be very pleaſing to you." "Doubt not, ſays C. but that I ſhall obſerve what I have promiſed: Go, and reſt ſecure as to that point: and give this letter of mine to the marquis." It ſhould ſeem however that T. was much diſpleaſed at having ſuffered himſelf to be worried as it were out of his rules, for as ſoon as he quitted Milan, inſtead of going to wait upon the marquis, he turned his horſe's head, and rode ſtraight home to Venice, ſaying to himſelf, "By my faith I ſhall not go to Vigeveno, but ſhall return to Venice, come of it what will."

After T's departure it ſeems C. applied himſelf immediately to reſolving ſome examples in the cubic equations by the new rules, but not ſucceeding in them, for indeed he had miſtaken the words, as it was very eaſy to do in ſuch bad verſes, having miſtaken $(\frac{1}{3}b)^3$ for $\frac{1}{3}b^3$, or the cube of $\frac{1}{3}$ of the coefficient, for $\frac{1}{3}$ of the cube of the coefficient; accordingly we find him writing to T. in fourteen days after the above, blaming him much for his abrupt departure without ſeeing the marquis, who was ſo liberal a prince he ſaid, and requeſting T. to reſolve him the example $x^3 + 3x = 10$. This T. did to his ſatisfaction, rightly gueſſing at the nature of his miſtake; and concludes his anſwer with theſe emphatical words, "Remember your promiſe." On the 12th of May following C. returns him a letter of thanks, together with a copy of his book, ſaying, "As to my work, juſt finiſhed, to remove your ſuſpicion, I ſend you a copy, but unbound, as it is yet too freſh to be beaten. But as to the doubt you expreſs leſt I may print your inventions, my faith which I gave you with an oath ſhould ſatisfy you; for as to the finiſhing of my book, that could be no ſecurity, as I could always add to it whenever I pleaſe. But on account of the dignity of the thing, I excuſe you for not relying on that which you ought to have done, namely on the faith of a gentleman, inſtead of the finiſhing of a book, which might at any time be enlarged by the addition of new chapters; and there are beſides a thouſand other ways. But the ſecurity conſiſts in this, that there is no greater treachery than to break one's faith, and to aggrieve thoſe who have given us pleaſure. And when you ſhall try me, you will find whether I be your friend or not, and whether I ſhall make an ungrateful return for your friendſhip, and the ſatisfaction you have given me."

It was within leſs than two months after this, however, that T. received the alarming news of Cardan's ſhewing ſome ſymptoms of breaking the faith he had ſo lately pledged to him; this was in a letter from a quondam pupil of his, in which he writes, "A friend of mine at Milan has written to me, that Dr. Cardano is compoſing another algebraical work, concerning ſome lately-diſcovered rules; hence I imagine they may be thoſe ſame rules which you told me you had taught him; ſo that I fear he will deceive you." To which T. replies, "I am heartily grieved at the news you inform me of, concerning Dr. Cardano of Milan; for if it be true, they can be no other rules but thoſe I gave him; and therefore the proverb truly ſays, 'That which you wiſh not to be known, tell to nobody.' Pray endeavour to learn more of this matter, and inform me of it."

Tartalea, after this, kept on the reſerve with Cardan, not anſwering ſeveral letters he ſent him, till one written on the 4th of Auguſt the ſame year, 1539, complaining greatly of T's neglect of him, and farther requeſting his aſſiſtance to clear up the difficulty of the irreducible caſe $x^3 = bx + c$, which C. had thus early been embarraſſed with: he ſays that when $(\frac{1}{3}b)^3$ exceeds $(\frac{1}{2}c)^2$, the rule cannot be applied to the equation in hand, becauſe of the ſquare root of the negative quantities. On this occaſion T. turns the tables on C. and plays his own game back upon him; for being aware of the above difficulty, and unable to overcome it himſelf, he wanted to try if C. could be encouraged to accompliſh it, by pretending that the caſe might be done, though in another way. He ſays thus to himſelf, "I have a good mind to give no anſwer to this letter, no more than to the other two. However I will anſwer it, if it be but to let him know what I have been told of him. And as I perceive that a ſuſpicion has ariſen concerning the difficulty or obſtacle in the rule for the caſe $x^3 = bx + c$, I have a mind to try if he can alter the data in hand, ſo as to remove the ſaid obſtacle, and to change the rule into another form, although I believe indeed that it cannot be done; however there is no harm in trying."—"M. Hieronime, I have received your letter, in which you write that you underſtand the rule for the caſe $x^3 = bx + c$, but that when $(\frac{1}{3}b)^3$ exceeds $(\frac{1}{2}c)^2$, you cannot reſolve the equation by following the rule, and therefore you requeſt me to give you the ſolution of this equation $x^3 = 9x + 10$. To which I reply, that you have not uſed a good method in that caſe, and that your whole proceſs is intirely falſe. And as to reſolving you the equation you have ſent, I muſt ſay that I am very ſorry that I have already given you ſo much as I have done, for I have been informed, by a credible perſon, that you are about to publiſh another Algebraical work, and that you have been boaſting through Milan of having diſcovered ſome new rules in algebra. But, take notice, that if you break your faith with me, I ſhall certainly keep my word with you, nay, I even aſſure you to do more than I promiſed." In Cardan's anſwer to this he ſays, "You have been miſinformed as to my intention to publiſh more on Algebra. But I ſuppoſe you have heard ſomething about my work *de myſteriis æternitatis*, which you take for ſome Algebra I intend to publiſh. As to your repenting of having given me your rules, I am not to be moved from the faith I promiſed you for any thing you ſay." To this, and many other things contained in the ſame letter, T. returned no anſwer, being ſtill ſuſpicious of Cardan's intentions, and declining any more correſpondence with him. This however did not diſcourage C. for we find him writing again to T. on the 5th of January, 1540, to clear up another difficulty which had occurred in this buſineſs, namely to extract the cube root of the binomials, of which the two parts of the rule always conſiſted, and for which purpoſe it ſeems C. had not yet found out a rule. On this occaſion he informs T. that his quondam competitor Zuanne Colle had come to Milan, where, in ſome conteſts between them, Colle gave Cardan to underſtand that he had found out the rules for the two caſes $x^3 + bx = c$, and $x^3 = bx + c$, and farther that he had diſcovered a general rule for extracting the cube roots of all ſuch binomials as can be extracted; and that, in particular, the cube root of $\sqrt{108} + 10$ is $\sqrt{3} + 1$, and that of $\sqrt{108} - 10$ is $\sqrt{3} - 1$, and conſequently that $\sqrt[3]{\sqrt{108} + 10} - \sqrt[3]{\sqrt{108} - 10}$ is $= \overline{\sqrt{3} + 1} - \overline{\sqrt{3} - 1} = 2$. He then earneſtly entreats T. to try to find out the rule, and the ſolution of certain other queſtions which had been propoſed to him by Colle. By this letter T. is ſtill more confirmed in his reſolution of ſilence; ſo that, without returning any anſwer, he only ſets down among his own memorandums ſome curious remarks on the contents of the letter, and then concludes to himſelf, "Wherefore I do not chooſe to anſwer him again, as I have no more affection for him than for M. Zuanne, and therefore I ſhall leave the matter between them." Among thoſe remarks he ſets down a rule for extracting the cube root of ſuch binomials as can be extracted, and that is done from either member of the binomial alone, thus: Take either term of the binomial, and divide it into two ſuch parts that one of them may be a complete cube, and the other part exactly diviſible by 3; then the cube root of the ſaid cubic part will be one term of the required root, and the ſquare root of the quotient ariſing from the diviſion of $\frac{1}{3}$ of the 2d part by the cube root of the firſt, will be the other member of the root ſought. This rule will be better underſtood in characters thus: let m be one member of the given binomial, whoſe cube root is ſought, and let it be divided into the two parts a^3 and $3b$, ſo that $a^3 + 3b$ be $= m$; then is $a + \sqrt{\frac{b}{a}}$ the cube root required, if it have one. Thus in the quantity $\sqrt{108} + 10$, taking the term 10 for m, then 10 divides into 1 and 9, where $a^3 = 1$ or $a = 1$, and $3b = 9$ or $b = 3$: therefore $a + \sqrt{\frac{b}{a}}$ becomes $1 + \sqrt{3}$ for the cube root of $\sqrt{108} + 10$. And taking the other member $\sqrt{108}$, this divides into the two equal parts $\sqrt{27}$ and $\sqrt{27}$, making $a^3 = \sqrt{27}$, and $3b = \sqrt{27}$; hence $a = \sqrt{3}$, and $b = \sqrt{3}$ alſo; conſequently $a + \sqrt{\frac{b}{a}}$ is $= \sqrt{3} + \sqrt{\frac{3}{3}}$ or $\sqrt{3} + 1$ for the cube root of the binomial ſought, the ſame as before. "And thus, he adds, we may know whether any propoſed binomial or reſidual be a cube or a noncube; for if it be a cube, the ſame two terms for the root muſt ariſe from both the given terms ſeparately; and if the two terms of the root cannot thus be brought to agree both ways, ſuch binomial or reſidual will not be a cube." And thus ends the correſpondence between them, at leaſt for this time. But it ſeems they had ſtill more violent diſputes when C. in violation of his faith ſo often pledged to the contrary, publiſhed his work on cubic equations 4 years afterwards, viz, in the year 1545, of which we have before given an account, which diſputes, it is ſaid, continued till the death of Tartalea in the year 1557.

The laſt article in the volume contains a dialogue on ſome other forms of the cubic equations, in the year 1541, between T. and a Mr. Richard Wentworth, an Engliſh gentleman, who it ſeems had reſided ſome time at Venice, on ſome public ſervice from England, as T. in the dedication of the volume to Henry VIII. king of England, makes mention of him as "a gentleman of his ſacred majeſty." Mr. Wentworth had learned ſome mathematics of T. and being about to depart for England, requeſts T. to ſhew him his newly diſcovered rules for cubic equations, as a farewell-leſſon; and it is worth while to note a few particulars in this conference, as they ſhew pretty nicely the limited knowledge of T. at that time, as to the nature and roots of ſuch equations. T. had before, it ſeems, ſhewed Mr. W. the rules for the caſes of the 3d and 1ſt powers, and now the latter deſires him to do the ſame as to the three caſes in which the 3d and 2d powers only are concerned. On this T. profeſſes great gratitude to Mr. W. for many obligations, but deſires to be excuſed from giving him the rules for theſe, becauſe he ſays he intends ſoon to compoſe a new work on Arithmetic, Geometry, and Algebra, which he intends to dedicate to him, and in which he means to inſert all his new diſcoveries. On Mr. W. urging him further, T. gives him the roots of ſome equations of that kind, as for inſtance:

If $x^3 + 6x^2 = 100$, then

$$x = \sqrt[3]{42 + \sqrt{17000}} + \sqrt[3]{42 - \sqrt{17000}} - 2,$$

If $x^3 + 9x^2 = 100$, then $x = \sqrt{24} - 2$,
If $x^3 + 3x^2 = 2$, then $x = \sqrt{3} - 1$,
If $x^3 + 4 = 5x^2$, then $x = \sqrt{8} + 2$,
If $x^3 + 6 = 7x^2$, then $x = \sqrt{15} + 3$;
but not the rules for finding them.

In the courſe of the converſation T. tells him that "all ſuch equations admit of two different anſwers, and perhaps more; and hence it follows that they have, or admit of, two different rules, and perhaps more, the one more difficult than the other." And on Mr. W. expreſſing his wonder at this circumſtance of a plurality of roots, T. replies, "It is however very true, though hardly to be believed, and indeed if experience had not confirmed it, I ſhould ſcarcely have believed it myſelf." He then commits a ſtrange blunder in an example which he takes to illuſtrate this by, namely the equation $x^3 + 3x = 14$, which, he ſays, it is evident has the number 2 for one of its roots; and yet, he adds, "whoever ſhall reſolve the ſame equation by my rule, will find the value of x to be $\sqrt[3]{7 + \sqrt{50}} + \sqrt[3]{7 - \sqrt{50}}$, which is proved to be a true root by ſubſtituting it in the equation for x. And therefore, continues he, it is manifeſt that the caſe $x^3 + bx = c$ admits of two rules, namely, one (as in the above example) which ought to give the value of x rational, viz 2, and the other is my rule, which gives the value of x irrational, as appears above; and there is reaſon to think that there may be ſuch a rule as will give the value of $x = 2$, although our anceſtors may not have found it out."——"And theſe two different anſwers will be found not only in every equation of this form $x^3 + bx = c$, when the value of x happens to be rational, as in the example $x^3 + 3x = 14$ above, but the ſame will alſo happen in all the other five forms of cubic equations: and therefore there is reaſon to think that they alſo admit of two different rules; and by certain circumſtances attending ſome of them, I am almoſt certain that they admit of more than two rules, as, God willing, I ſhall ſoon demonſtrate." Now all this diſcourſe ſhews a ſtrange mixture of knowledge and ignorance: it is very probable that he had met with ſome equations which admit of a plurality of roots; indeed it was hardly poſſible for him to avoid it; but it ſeems he had no ſuſpicion what the number of roots might be, nor that his reaſoning in this inſtance was founded on an error of his own, miſtaking the root $x = \sqrt[3]{7 + \sqrt{50}} + \sqrt[3]{7 - \sqrt{50}}$, of the equation $x^3 + 3x = 14$, for a different root from the number or root 2, when in reality it is the very ſame, as he might eaſily have found, if he had extracted the cube roots of the binomials by the rule which he himſelf had juſt given above for that purpoſe: for by that rule he would have found $\sqrt[3]{7 + \sqrt{50}} = 1 + \sqrt{2}$, and $\sqrt[3]{7 - \sqrt{50}} = 1 - \sqrt{2}$, and therefore their ſum is $2 = x$, the ſame root as the other, which T. thought had been different. And beſides this root 2, the equation in hand, $x^3 + 3x = 14$, admits of no other real roots. Nor does any equation of the ſame form, $x^3 + bx = c$, admit of more than one real root.

It ſeems alſo they had not yet diſcovered that all caſes belong to the rules and forms for quadratic equations, which have only two powers in them, in which the exponent of the one is juſt double of the exponent of the other, as $x^{2n} + bx^n = c$; but ſome particular caſes only of this ſort they had as yet ventured to refer to quadratics, as the caſe $x^4 + bx^2 = c$. But in the concluſion of this dialogue T. informs W. of another caſe of this ſort which he had accompliſhed, as a notable diſcovery, in theſe words: "I well remember, ſays he, that in the year 1536, on the night of St. Martin, which was on a Saturday, meditating in bed when I could not ſleep, I diſcovered the general rule for the caſe $x^6 + bx^3 = c$, and alſo for the other two, its accompanying caſes, in the ſame night." And then he directs that they are to be reſolved like quadratics, by completing the ſquare, &c. And in theſe reſolutions it is remarkable that he uſes only the poſitive roots, without taking any notice of the negative ones.

Tartalea alſo publiſhed at Venice, in 1556, &c, a very large work, in folio, on Arithmetic, Geometry, and Algebra. This is a very complete and curious work upon the firſt two branches; but that of Algebra is carried no farther than quadratic equations, called *book the firſt*, with which the work terminates. It is evidently incomplete, owing to the death of the author, which happened before this latter part of the work was printed, as appears by the dates, and by the prefaces. It appears alſo, from ſeveral parts of this work, that the author had many ſevere conflicts with Cardan and his friend Lewis Ferrari: and particularly, there was a public trial of ſkill between them, in the year 1547; in which it would ſeem that Tartalea had greatly the advantage, his queſtions moſtly remaining unanſwered by his antagoniſts.

OF MICHAEL STIFELIUS.

After the foregoing analyſis of the works of the firſt algebraic writers in Italy, it will now be proper to conſider thoſe of their contemporaries in Germany; where, excepting for the diſcoveries in cubic equations, the art was in a more advanced ſtate, and of a form approaching nearer to that of our modern Algebra; the ſtate and circumſtances indeed being ſo different, that one would almoſt be led to ſuppoſe they had derived their knowledge of it from a different origin.

Here Stifelius and Scheubelius were writers of the ſame time with Cardan and Tartalea, and even before their diſcoveries, or publication, concerning the rules for cubic equations, Stifelius's *Arithmetica Integra* was publiſhed at Norimberg in 1544, being the year before Cardan's work on cubic equations, and is an excellent treatiſe, both on Arithmetic and Algebra. The work is divided into three books, and is prefaced with an Introduction by the famous Melanchthon. The firſt book contains a complete and ample Treatiſe on Arithmetic, the ſecond an Expoſition of the 10th book of Euclid's Elements, and the third a Treatiſe of Algebra, and it is therefore properly the part with which we are at preſent concerned. In the dedication of this part, he aſcribes the invention of Algebra to Geber, an Arabic Aſtronomer; and mentions beſides, the authors Campanus, Chriſt. Rudolph, and Adam Ris, Riſen, or Gigas, whoſe rules and examples he has chiefly given. In other parts of the book he ſpeaks, and makes uſe alſo, of the works of Bretius, Campanus, Cardan (i. e. his Arithmetic publiſhed in 1539, before the work on cubic equations appeared), de Cuſa, Euclid, Jordan, Milichius, Schonerus, and Stapulenſis.

Chap. 1. *Of the Rule of Algebra, and its parts.* Stifelius here deſcribes the notation and marks of powers, or denominations as he calls them, which marks for the ſeveral powers are thus:

1ſt,	2d,	3d,	4th,	5th,	6th,	&c.
♃,	ʒ,	ϛꝭ,	ʒʒ,	ſs,	ʒϛꝭ,	&c.

which are formed from the initials of the barbarous way in which the Germans pronounced and wrote the Latin and Italic names of the powers, namely, res or coſa, zenſus, cubo, zenſi-zenſus, ſurſolid, zenſi-cubo, &c. And the coſs or firſt power ♃, he calls the radix or root, which is the firſt time that we meet with this word in the printed authors. He alſo here uſes the ſigns or characters, + and −, for addition and ſubtraction, and the firſt of any that I know of: for in Italy they uſed none of theſe characters for a long time after. He has no mark however for equality, but makes uſe of the word itſelf.

Chap. 2. Of the Parts of the Rule of Geber or Algebra: teaching the various reductions by addition, ſubtraction, multiplication, diviſion, involution, and evolution, &c.

Chap. 3. Of the Algorithm of Coſſic Numbers: teaching the uſual operations of addition, ſubtraction, multiplication, diviſion, involution, and extraction of roots, much the ſame as they are at preſent. Single terms, or powers, he calls ſimple quantities; but ſuch as 1ȝ + 1ꝝ a compoſite or compound, and 2ꝝ − 8 a defective one. In multiplication and diviſion, he proves that like ſigns give +, and unlike ſigns −. He ſhews that the powers 1, ꝝ, ȝ, ꞓ, &c, form a geometrical progreſſion from unity; and that the natural ſeries of numbers 0, 1, 2, 3, &c, from 0, are the exponents of the coſſic powers; and he, for the firſt time, expreſsly calls them exponents: thus,

Exponents, 0, 1, 2, 3, 4, 5, 6, &c.
Powers, 1, ꝝ, ȝ, ꞓ, ȝȝ, ſs, ȝꞓ, &c.

And he ſhews the uſe of the exponents, in multiplication, diviſion, powers, and roots, as we do at preſent; viz, adding the exponents in multiplication, and ſubtracting them in diviſion, &c. And theſe operations he demonſtrates from the nature of arithmetical and geometrical progreſſions. It is remarkable that theſe compound denominations of the powers are formed from the ſimple ones according to the *products* of the exponents, while thoſe of Diophantus are formed according to the *ſums* of them; thus the 6th power here is ȝꞓ or quadrato-cubi, but with Diophantus it is cubo-cubi; and ſo of others. Which is preſumptive evidence that the Europeans had not taken their Algebra immediately from him, independent of other proofs.

Chap. 4. Of the extraction of the roots of coſſic numbers. He here treats of quadratic equations, which he reſolves by completing the ſquare, from Euclid II. 4 &c. Alſo quadratics of the higher orders, ſhewing how to reſolve them in all caſes, whatever the height may be, provided the exponents be but in arithmetical progreſſion, as

$$\left.\begin{array}{l} 2,\ 1,\ 0 \\ 4,\ 2,\ 0 \\ 6,\ 3,\ 0 \\ 8,\ 4,\ 0 \end{array}\right\}$$

&c; where it is plain that he always counts 0 for the exponent of the unknown quantity in the abſolute term.

Chap. 5. Of irrational coſſic numbers, and of ſurd or negative numbers. In this treatiſe of radicals, or irrationals, he firſt uſes the character √ to denote a root, and ſets after it the mark of the power whoſe root is intended; as √ȝ 20 for the ſquare root of 20, and √ꞓ 20 for the cube root of the ſame, and ſo on. He treats here alſo of negative numbers, or what he calls ſurd or fictitious, or numbers leſs than 0. On which he takes occaſion to obſerve, that when a geometrical progreſſion is continued downwards below 1, then the exponents of the terms, or the arithmetical progreſſion, will go below 0 into negative numbers, and will yet be the true exponents of the former; as in theſe,

Expon.	−3	−2	−1	0	1	2	3
Pow.	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4	8

And he gives examples to ſhew that theſe negative exponents perform their office the ſame as the poſitive ones, in all the operations.

Chap. 6. Of the perfection of the Rule of Algebra, and of Secondary Roots. In the reduction of equations he uſes a more general rule than thoſe who had preceded him, who detailed the rule in a multitude of caſes; inſtead of which, he directs to multiply or divide the two ſides equally, to tranſpoſe the terms with + or −, and laſtly to extract ſuch root as may be denoted by the exponent of the higheſt power.

As to ſecondary roots, Cardan treated of a 2d *ignota* or unknown, which he called *quantitas*, and denoted it by the initial *q*, to diſtinguiſh it from the firſt. But here Stifelius, for diſtinction ſake, and to prevent one root from being miſtaken for others, aſſigns literal marks to all of them, as A, B, C, D, &c, and then performs all the uſual operations with them, joining them together as we do now, except that he ſubjoins the initial of the power, inſtead of its numeral exponent:

thus, 3A into 9B makes 27AB,
ȝȝ into 4B makes 12ȝB,
2ꞓ into 4Aȝ makes 8ꞓAȝ,
1A ſquared makes 1Aȝ,
6 into 3C makes 18C,
2Aȝ into 5Aꞓ makes 10Aſs, &c, &c.
8ꞓAȝ divided by 4ꞓ makes 2Aȝ, &c.
The ſquare root of 25Aȝ is 5A, &c.
Alſo 2A added to 2ꝝ makes 2ꝝ + 2A,
and 2A ſubtr. from 2ꝝ makes 2ꝝ − 2A.

And he ſhews how to uſe the ſame, in queſtions concerning ſeveral unknown numbers; where he puts a different character for each of them, as ꝝ, A, B, C, &c; he then makes out, from the conditions of the queſtion, as many equations as there are characters; from theſe he finds the value of each letter, in terms of ſome one of the reſt; and ſo, expelling them all but that one, reduces the whole to a final equation, as we do at preſent.

The remainder of the book is employed with the ſolutions of a great number of queſtions to exerciſe all the rules and methods; ſome of which are geometrical ones.

From this account of the ſtate of Algebra in Stifelius, it appears that the improvements made by himſelf, or other Germans, beyond thoſe of the Italians, as contained in Cardan's book of 1539, were as follow:

1ſt. He introduced the characters +, −, √, for plus, minus, and root, or *radix*, as he calls it.

2d. The initials ꝝ, ȝ, ꞓ, &c. for the powers.

3d. He treated all the higher orders of quadratics by the ſame general rule.

4th. He introduced the numeral exponents of the powers, −3, −2, −1, 0, 1, 2, 3, &c, both poſitive and negative, ſo far as integral numbers, but not fractional ones; calling them by the name *exponens*, exponent: and he taught the general uſes of the exponents, in the ſeveral operations of powers, as we now uſe them, or the logarithms.

5th. And laſtly, he uſed the general literal notation A, B, C, D, &c, for ſo many different unknown or general quantities.

OF SCHEUBELIUS.

John Scheubelius publiſhed ſeveral books upon Arithmetic and Algebra. The one now before me, is intitled *Algebræ Compendioſa Faciliſque Deſcriptio, quâ depromuntur*

muntur magna Arithmetices miracula. Authore Johanne Scheubelio Mathematicarum Profeſſore in Academia Tubingenſi. Pariſiis 1552. But at the end of the book it is dated 1551. The work is moſt beautifully printed, and is a very clear and ſuccinct treatiſe; and both in the form and matter much reſembles a modern printed book. He ſays that the writers aſcribe this art to Diophantus, which is the firſt time that I find this Greek author mentioned by the modern algebraiſts: he farther obſerves, that the Latins call it *Regula Rei & Cenſus*, the rule of the thing and the ſquare (or of the 1ſt and 2d power); and the Arabs, Algebra. His characters and operations are much the ſame as thoſe of Stifelius, uſing the ſigns and characters +, −, √, and the powers ȣ, ȥ, 3, ʒʒ, &c, where the character ȣ is uſed for 1 or unity, or a number, or the 0 power; prefixing alſo the numeral coefficients; thus 44ʃ3 + 113 + 31ȣ − 53ȥ. He uſes alſo the exponents 0, 1, 2, 3, &c, of the powers, the ſame way as Stifelius, before him. He performs the algebraical calculations, firſt in integers, and then in fractions, much the ſame as we do at preſent. Then of equations, which he ſays may be of infinite degrees, though he treats only of two, namely the firſt and ſecond orders, or what we call ſimple and quadratic equations, in the uſual way, taking however only the poſitive roots of theſe; and adverting to all the higher orders of quadratics, namely, x^4, ax^2, b;

$$x^6,\ ax^3,\ b;$$
$$x^8,\ ax^4,\ b;\ \&c.$$

Next follows a tract on ſurds, both ſimple and compound, quadratic, cubic, binomial, and reſidual. Here he firſt marks the notation, obſerving that the root is either denoted by the initial of the word, or, after ſome authors, by the mark √:, viz. the ſq. root √:, the cube root w√:, and the 4th root, or root of the root thus v√:, which latter method he moſtly uſes. He then gives the Arithmetic of ſurds, in multiplication, diviſion, addition, and ſubtraction. In theſe laſt two rules he ſquares the ſum or difference of the ſurds, and then ſets the root to the whole compound, which he calls *radix collecti*, what Cardan calls *radix univerſalis*. Thus $\surd 12 \pm \surd 20$ is ra. col. $32 \pm \surd 960$. But when the terms will reduce to a common ſurd, he then unites them into one number; as $\surd 27 + \surd 12$ is equal $\surd 75$. Alſo of cubic ſurds, and 4th roots. In binomial and reſidual ſurds, he remarks the different kinds of them which anſwer to the ſeveral irrational lines in the 10th book of Euclid's elements; and then gives this general rule for extracting the root of any binomial or reſidual $a \pm b$, where one or both parts are ſurds, and a the greater quantity, namely, that the ſquare root of it is $\sqrt{\frac{a+\sqrt{a^2-b^2}}{2}} \pm \sqrt{\frac{a-\sqrt{a^2-b^2}}{2}}$, which he illuſtrates by many examples. This rule will only ſucceed however, ſo as to come out in ſimple terms, in certain caſes, namely, either when $a^2 - b^2$ is a ſquare, or when a and $\sqrt{a^2 - b^2}$ will reduce to a common ſurd, and unite: in all other caſes the root is in two compound ſurds, inſtead of one. He gives alſo another rule, which comes however to the ſame thing as the former, though by the words of them they ſeem to be different.

Scheubelius wrote much about the time of Cardan and Stifelius. And as he takes no notice of cubic equations, it is probable he had neither ſeen nor heard any thing about them; which might very well happen, the one living in Italy, and the other in Germany. And, beſides, I know not if this be the firſt edition of Scheubel's book: it is rather likely it is not, as it is printed at Paris, and he himſelf was profeſſor of mathematics at Tubingen in Germany.

ROBERT RECORDE.

The firſt part of his Arithmetic was publiſhed in 1552; and the ſecond part in 1557, under the title of, "The Whetſtone of Witte, which is the ſeconde parte of Arithmetike: containing the Extraction of Rootes: The Coſſike Practiſe, with the Rule of Equation: and the Workes of Surde Nombers." The work is in dialogue between the maſter and ſcholar; and is nearly after the manner of the Germans, Stifelius and Scheubelius, but eſpecially the latter, whom he often quotes, and takes examples from. The chief parts of the work are, 1ſt. The properties of abſtract and figurate numbers. 2nd. The extraction of the ſquare and cube roots, much the ſame as at preſent. Here, when the number is not an exact power, but having ſome remainder over, he either continues the root into decimals as far as he pleaſes, by adding to the remainders always periods of cyphers; or elſe makes a vulgar fraction for the remaining part of the root, by taking the remainder for the numerator, and double the root for the denominator, in the ſquare root; but in the cube root he takes for the nominator either the triple ſquare of the root, which is Cardan's rule, or the triple ſquare and triple root, with one more, which is Scheubel's rule. 3d. Of Algebra, or "Coſſike Nombers." He uſes the notation of powers with their exponents the ſame as Stifel, with all the operations in ſimple and compound quantities, or integers and fractions. And he gives alſo many examples of extracting the roots of compound algebraic quantities, even when the roots are from two to ſix terms, in imitation of the ſame proceſs in numbers, juſt as we do at preſent; which is the firſt inſtance of this kind that I have obſerved. As of this quantity:

$$253\,\text{ʒʒ} + \overset{\text{Square}}{80ʃ3} - 2633 - 633\ (5\,\text{ʒʒ} \overset{\text{Root.}}{+} 83 - 9\,\text{ȥ}.$$

4th. "The Rule of Equation, commonly called Algeber's Rule." He here, firſt of any, introduces the character =, for brevity ſake. His words are, "And to avoide the tediouſe repetition of theſe woordes: is equalle to: I will ſette as I doe often in woorke uſe, a paire of paralleles, or gemowe lines of one lengthe, thus: =, bicauſe noe 2 thynges can be moare equalle." He gives the rules for ſimple and quadratic equations, with many examples. He gives alſo ſome examples in higher compound equations, with a root for each of them, but gives no rule how to find it. 5th. "Of Surde Nombers." This is a very ample treatiſe on ſurds, both ſimple and compound, and ſurds of various degrees, as ſquare, cubic, and biquadratic, marking the roots in Scheubel's manner, thus: √, w√, v√. He here uſes the names bimedial, binomial, and reſidual; but ſays they have been uſed by others before him, though this is the firſt place where I have obſerved the two latter.— Hence it appears that the things which chiefly are new in this author, are theſe three, viz.

1. The extraction of the roots of compound algebraic quantities.
2. The uſe of the terms binomial and reſidual.
3. The uſe of the ſign of equality, or =.

OF PELETARIUS.

The firſt edition of this author's algebra was printed in 4to at Paris, in 1558, under this title, *Jacobi Peletarii Cenomani, de occulta parte Numerorum, quam Algebram vocant. Lib. duo.*

In the preface he ſpeaks of the ſuppoſed authors of Algebra, namely Geber, Mahomet the ſon of Moſes, an Arabian, and Diophantus. But he thinks the art older, and mentions ſome of his contemporary writers, or a very little before him, as Cardan, Stifel, Scheubel, Chr. Januarius; and a little earlier again, Lucas Paciolus of Florence, and Stephen Villafrancus a Gaul.

Of the two books, into which the work is divided, the firſt is on rational, and the ſecond on irrational or ſurd quantities; each being divided into many chapters. It will be ſufficient to mention only the principal articles.

He calls the ſeries of powers *numeri creati*, or derived numbers, or alſo radicals, becauſe they are all raiſed from one root or *radix*. He names them thus, radix, quadratus cubus, quadrato-quadratus, or biquadratus, ſuperſolidus, quadrato-cubus, &c; and marks them thus ℞, *q*, *cſ*, *qq*, *ſs*, *qcſ*, *bſs*, *&c.* Of theſe he gives the following ſeries in numbers, having the common ratio 2, with their marks ſet over them, and the exponents ſet over theſe again, in an arithmetical ſeries, beginning at 0, thus:

0	1	2	3	4	5	6	7	8	
1	℞	*q*	*cſ*	*qq*	*ſs*	*qcſ*	*bſs*	*qqq*	
1	2	4	8	16	32	64	128	256	&c.

And he ſhews the uſe of the exponents, the ſame as Stifelius and Scheubelius; like whom alſo he prefixes coefficients to quantities of all kinds, as alſo the radical √. But he does not follow them in the uſe of the ſigns + and —, but employs the initials *p* and *m* for the ſame purpoſe. After the operations of addition, &c, he performs involution and evolution alſo much the ſame way as at preſent: thus, in powers, raiſe the coefficient to the power required, and multiply the exponent, or ſign, as he calls it, by 2, or 3, or 4, &c, for the 2nd, 3d, 4th, &c, power; and the reverſe for extraction: and hence he obſerves, if the number or coefficient will not exactly extract, or the ſign do not exactly divide, the quantity is a ſurd.

After the operations of compound quantities, and fractions, and reduction of equations, namely, ſimple and quadratic equations, as uſual, in chap. 16, *De Inveniendis generatim Radicibus Denominatorum*, he gives a method of finding the roots of equations among the diviſors of the abſolute number, when the root is rational, whether it be integral or fractional; for then, he obſerves, the root always lies hid in that number, and is ſome one of its diviſors. This is exemplified in ſeveral inſtances, both of quadratic and cubic equations, and both for integral and fractional roots. And he here obſerves, that he knows not of any perſon who has yet given general rules for the ſolution of cubic equations; which ſhews that when he wrote this book, either Cardan's laſt book was not publiſhed, or elſe it had not yet come to his knowledge.

Chap. 17 contains, in a few words, directions for bringing queſtions to equations, and for reducing theſe. He here obſerves, that ſome authors call the unknown number *res*, and others the *poſitio*; but that he calls it *radix*, or root, and marks it thus ℞: hence the term, root of an equation. But it was before called radix by Stifelius.

Chap. 21 *&* *ſeq.* treat of ſecondary roots, or a plurality of roots, denoted by A, B, C, &c, after Stifelius.

The 2d book contains the like operations in ſurds, or irrational numbers, and is a very complete work on this ſubject indeed. He treats firſt of ſimple or ſingle ſurds, then of binomial ſurds, and laſtly of trinomial ſurds. He gives here the ſame rule for extracting the root of a binomial and reſidual as Scheubelius, viz, $\sqrt{a \pm b} = \sqrt{\frac{a + \sqrt{a^2 - b^2}}{2}} \pm \sqrt{\frac{a - \sqrt{a^2 - b^2}}{2}}$. Individing by a binomial or reſidual, he proceeds as all others before him had done, namely, reducing the diviſor to a ſimple quantity, by multiplying it by the ſame two terms with the ſign of one of them changed, that is by the binomial if it be a reſidual, or by the ſame reſidual if it be a binomial; and multiplying the dividend by the ſame thing: thus $\frac{3}{\sqrt{5} - 2} = \frac{3}{\sqrt{5} - 2} \times \frac{\sqrt{5} + 2}{\sqrt{5} + 2} = \frac{3\sqrt{5} + 6}{5 - 4} = 3\sqrt{5} + 6$.

And, in imitation of this method, in diviſion by trinomial ſurds, he directs to reduce the trinomial diviſor firſt to a binomial or reſidual, by multiplying it by the ſame trinomial with the ſign of one term changed, and then to reduce this binomial or reſidual to a ſimple nomial as above; obſerving to multiply the dividend by the ſame quantities as the diviſor. Thus, if the diviſor be $4 + \sqrt{2} - \sqrt{3}$; multiplying this by $4 + \sqrt{2} + \sqrt{3}$, the product is $15 + 8\sqrt{2}$; then this binomial multiplied by the reſidual $15 - 8\sqrt{2}$, gives $225 - 128$ or 97 for the ſimple diviſor: and the dividend, whatever it is, muſt alſo be multiplied by the two $4 + \sqrt{2} + \sqrt{3}$ and $15 - 8\sqrt{2}$. Or in general, if the diviſor be $a + \sqrt{b} - \sqrt{c}$; multiply it by $a + \sqrt{b} + \sqrt{c}$, which
gives $(a + \sqrt{b})^2 - c = a^2 + b - c + 2a\sqrt{b}$;
then multiply this by $a^2 + b - c - 2a\sqrt{b}$,
and it gives - $(a^2 + b - c)^2 - 4a^2 b$, which will be rational, and will all collect into one ſingle term. But Tartalea muſt have been in poſſeſſion of ſome ſuch rule as this, as one of the queſtions he propoſed to Florido was of this nature, namely to find ſuch a quantity as multiplied by a given trinomial ſurd, ſhall make it rational: and it appears, from what is done above, that, the given trinomial being $\overline{a + \sqrt{b} - \sqrt{c}}$, the anſwer will be $\overline{a + \sqrt{b} + \sqrt{c}} \times \overline{a^2 + b - c + 2\sqrt{b}}$.

Chap. 24 ſhews the compoſition of the cube of a binomial or reſidual, and thence remarks on the root of the caſe or equation 1 *cſ* *p* 3℞ eqnal to 10, which he ſeems to know ſomething about, though he had not Cardan's rules.

Chap. 30, which is the laſt, treats of certain precepts relating to ſquare and cubic numbers, with a table of ſuch ſquares and cubes for all numbers to 140; alſo ſhewing how to compute them both, by adding always their differences.

He then concludes with remarking that there are many curious properties of theſe numbers, one of which is this, that the ſum of any number of the cubes, taken from the beginning, always makes a ſquare number, the root of which is the ſum of the roots of the cubes; ſo that the ſeries of ſquares ſo formed, have for their roots — 1, 3, 6, 10, 15, 21, &c. whoſe diff. are the natural n^{os} 1, 2, 3, 4, 5, 6, &c. Namely, $1^3 = 1^2$; $1^3 + 2^3 = 3^2$; $1^3 + 2^3 + 3^3 = 6^2$, &c. Or in general, $1^3 + 2^3 + 3^3 \cdots n^3 = (1 + 2 + 3 \cdots n)^2 = \frac{1}{2}n . \overline{n + 1}$.

This work of Peletarius is a very ingenious and maſterly compoſition, treating in an able manner of the ſeveral parts of the ſubject then known, excepting the cubic equations. But his real diſcoveries, or improvements, may be reduced to theſe three, viz.

1ſt. That the root of an equation, is one of the diviſors of the abſolute term.

2d. He taught how to reduce trinomials to ſimple terms, by multiplying them by compound factors.

3d. He taught curious precepts and properties concerning ſquare and cube numbers, and the method of conſtructing a ſeries of each by addition only, namely by adding ſucceſſively their ſeveral orders of differences.

RAMUS.

Peter Ramus wrote his arithmetic and algebra about the year 1560. His notation of the powers is thus, *l*, *q*, *c*, *bq*, being the initials of latus, quadratus, cubus, biquadratus. He treats only of ſimple and quadratic equations. And the only thing remarkable in his work, is the firſt article, on the names and invention of Algebra, which we have noticed at the beginning of this hiſtory.

BOMBELLI.

Raphael Bombelli's Algebra was publiſhed at Bologna in the year 1579, in the Italian language. It ſeems however it was written ſome time before, as the dedication is dated 1572. In a ſhort, but neat, introduction, he firſt adverts, in a few words, to the great excellence and uſefulneſs of arithmetic and algebra. He then laments that it had hitherto been treated in ſo imperfect and irregular a way; and declares it is his intention to remedy all defects, and to make the ſcience and practice of it as eaſy and perfect as may be. And for this purpoſe he firſt reſolved to procure and ſtudy all the former authors. He then mentions ſeveral of theſe, with a ſhort hiſtory or character of them; as Mahomet the ſon of Moſes, an Arabian; Leonard Piſano; Lucas de Burgo, the firſt printed author in Europe; Oroncius; Seribelius; Boglione Franceſi; Stifelius in Germany; a certain Spaniard, doubtleſs meaning Nunez or Nonius; and laſtly Cardan, Ferrari, and Tartalea; with ſome others ſince, whoſe names he omits. He then adds a curious paragraph concerning Diophantus: he ſays that ſome years ſince there had been found, in the Vatican library, a Greek work on this art, compoſed by a certain Diophantus, of Alexandria, a Greek author, who lived in the time of Antoninus Pius; which work having been ſhewn to him by Mr. Antonio Maria Pazzi Reggiano, public lecturer on mathematics at Rome; and finding it to be a good work, theſe two formed the reſolution of giving it to the world, and he ſays that they had already tranſlated five books, of the ſix which were then extant, being as yet hindered by other avocations from completing the work. He then adds the following ſtrange circumſtance, viz. *that they had found that in the ſaid work the Indian authors are often cited; by which they learned that this ſcience was known among the Indians before the Arabians had it:* a paragraph the more remarkable as I have never underſtood that any other perſon could ever find, in Diophantus, any reference to Indian writers: and I have examined his work with ſome attention, for that purpoſe.

Bombelli's work is divided into three books. In the firſt, are laid down the definitions and operations of powers and roots, with various ſorts of radicals, ſimple and compound, binomial, reſidual, &c; moſtly after the rules and manner of former writers, excepting in ſome few inſtances, which I ſhall here take notice of. And firſt of his rule for the cube root of binomials or reſiduals, which for the ſake of brevity, may be expreſſed in modern notation as follows: let $\sqrt{b} + a$ be the binomial, the term $\sqrt{b}$ being greater than a; then the rule for the cube root of $\sqrt{b} + a$ comes to this, $P - Q + \sqrt{\overline{P - Q}^2 + \sqrt[3]{b - a^2}}$ where $P = \sqrt[3]{\sqrt{\frac{a^2}{64} + \frac{b - a^2}{64}} + \frac{a}{8}}$, and $Q = \sqrt[3]{\sqrt{\frac{a^2}{64} + \frac{b - a^2}{64}} - \frac{a}{8}}$. Which is a rule that can be of little or no uſe; for, in the firſt place, $\overline{P - Q}^2 + \sqrt[3]{b - a^2}$ is the ſame as $\overline{P + Q}^2$; and P or $\sqrt[3]{\sqrt{\frac{a^2}{64} + \frac{b - a^2}{64}} + \frac{a}{8}}$ is $=$

$$\sqrt[3]{\sqrt{\frac{b}{64}} + \frac{a}{8}} = \sqrt[3]{\frac{\sqrt{b} + a}{8}} = \frac{1}{2}\sqrt[3]{\sqrt{b} + a};$$

therefore the whole $P - Q + \sqrt{\overline{P - Q}^2 + \sqrt[3]{b - a^2}}$ reduces to $P - Q + P + Q = 2P = 2 \times \frac{1}{2}\sqrt[3]{\sqrt{b} + a} = \sqrt[3]{\sqrt{b} + a}$, the original quantity firſt propoſed. The next thing remarkable in this 1ſt book, is his method for the ſquare roots of negative quantities, and his rule for the cube roots of ſuch imaginary binomials as ariſe from the irreducible caſe in cubic equations. His words, tranſlated, are theſe: "I have found another ſort of cubic root, very different from the former, which ariſes from the caſe of the cube equal to the firſt power and a number, when the cube of the $\frac{1}{3}$d part of the (coef. of the) 1ſt power, is greater than the ſquare of half the abſolute number, which ſort of ſquare root hath in its algoriſm, names and operations different from the others; for in that caſe, the exceſs cannot be called either plus or minus; I therefore call it *plus of minus* when it is to be added, and *minus of minus* when it is to be ſubtracted." He then gives a ſet of rules for the ſigns when ſuch roots are multiplied, and illuſtrates them by a great many examples. His rule for the cube roots of ſuch binomials, viz. ſuch as $a + \sqrt{-b}$, is this: Firſt find $\sqrt[3]{a^2 + b}$; then, by trials ſearch out a number c, and a ſq. root $\sqrt{d}$, ſuch, that the ſum of their ſquares $c^2 + d$ may be $= \sqrt[3]{a^2 + b}$

and alſo $c^3 - 3cd = a$; then ſhall $c + \sqrt{-d}$ be $= \sqrt[3]{a + \sqrt{-b}}$ ſought. Thus, to extract the cube root of $2 + \sqrt{-121}$: here $\sqrt[3]{a^2 + b} = \sqrt[3]{125} = 5$; then taking $c = 2$, and $d = 1$, it is $c^2 + d = 5 = \sqrt[3]{a^2 + b}$, and $c^3 - 3cd = 8 - 6 = 2 = a$, as it ought; and therefore $2 + \sqrt{-1}$ is = the cube root of $2 + \sqrt{-121}$, as required.

The notation in this book, is the initial *R* for root, with *q* or *c* &c after it, for quadrate or cubic, &c root. Alſo *p* for *plus*, and *m* for *minus*.

In the 2d book, Bombelli treats of the algoriſm with unknown quantities, and the reſolution of equations. He firſt gives the definitions and characters of the unknown quantity and its powers, in which he deviates from the former authors, but profeſſes to imitate Diophantus. He calls the unknown quantity *tanto*, and marks it thus - - $\overset{1}{\smile}$,
Its ſquare or 2d power *potenza*, $\overset{2}{\smile}$,
Its cube - - *cubo*, $\overset{3}{\smile}$,
and the higher names are compounded of theſe, and marked $\overset{4}{\smile}$, $\overset{5}{\smile}$, $\overset{6}{\smile}$, $\overset{7}{\smile}$, &c, ſo that he denotes all the powers by their exponents ſet over the common character $\smile$. And all theſe powers he calls by the general name *dignita*, dignity. He then performs all the algoriſm of theſe powers, by means of their exponents, as we do at preſent, viz, adding them in multiplication, ſubtracting in diviſion, multiplying them by the index in involution, and dividing by the ſame in evolution.

In equations he goes regularly through all the caſes, and varieties of the ſigns and terms; firſt all the ſimple or ſingle powers, and then all the compound caſes; demonſtrating the rules geometrically, and illuſtrating them by many examples.

In compound quadratics, he gives two rules: the firſt is by freeing the *potenza* or ſquare from its coefficient by diviſion, and then completing the ſquare, &c, in the uſual way: and the 2d rule, when the firſt term has its coefficient, may be thus expreſſed; if $ax^2 + bx = c$, then $x = \frac{\sqrt{ac + \frac{1}{4}b^2} - \frac{1}{2}b}{a}$. He takes only the poſitive root or roots; and in the caſe $ax^2 + c = bx$, which has two, he obſerves that the nature of the problem muſt ſhew which of the two is the proper one.

In the cubic equations, he gives the rules and tranſformations, &c, after the manner of Cardan; remarking that ſome of the caſes have only one root, but others two or three, of which ſome are true, and others falſe or negative. And in one place he ſays that by means of the caſe $x^3 = bx + c$ he *triſects* or *divides an angle into three equal parts*.

When he arrives at biquadratic equations, and particularly to this caſe $x^4 + ax = b$, he ſays, "Since I have ſeen Diophantus's work, I have always been of opinion that his chief intention was to come to this equation, becauſe I obſerve he labours at finding always ſquare numbers, and ſuch, that adding ſome number to them, may make ſquares; and I believe that the ſix books, which are loſt, may treat of this equation, &c." —"But Lewis Ferrari," he adds, "of this city, alſo laboured in this way, and found out a rule for ſuch caſes, which was a very fine invention, and therefore I ſhall here treat of it the beſt I can." This he accordingly does, in all the caſes of biquadratics, both with reſpect to the number of terms in the equation, and the ſigns of the terms, except I think this moſt general caſe only $rx - qx^2 + px^3 - x^4 = s$; fully applying Ferrari's method in all caſes. Which concludes the 2d book.

The 3d book conſiſts only of the reſolution of near 300 practical queſtions, as exerciſes in all the rules and equations, ſome of which are taken from Diophantus and other authors.

Upon the whole it appears that this is a plain, explicit, and very orderly treatiſe on algebra, in which are very well explained the rules and methods of former writers. But Bombelli does not produce much of improvement or invention of his own, except his notation, which varies from others, and is by means of one general character, with the numeral indices of Stifelius. He alſo firſt remarks that angles are triſected by a cubic equation. But I know not how to account for his aſſertion, that Diophantus often cites the Indian authors; which I think muſt be a miſtake in Bombelli.

CLAVIUS.

Chriſtopher Clavius wrote his Algebra about the year 1580, though it was not publiſhed till 1608, at Orleans. He moſtly follows Stifelius and Scheubelius in his notation and method, &c, having ſcarcely any variations from them; nor does he treat of cubic equations. He mentions the names given to the art, and the opinions about its origin, in which he inclines to aſcribe it to Diophantus, from what Diophantus ſays in his preface to Dyoniſius.

STEVINUS.

The Arithmetic of Simon Stevin of Bruges, was publiſhed in 1585, and his Algebra a little afterwards. They were alſo printed in an edition of his works at Leyden in 1634, with ſome notes and additions of Albert Girard, who it ſeems died the year before, this edition being publiſhed for the benefit of Girrard's widow and children. The Algebra is an ingenious and original work. He denotes the *res*, or unknown quantity, in a way of his own, namely by a ſmall circle $\bigcirc$, within which he places the numeral exponent of the power, as ⓪, ①, ②, ③, &c, which are the 0, 1, 2, 3, &c power of the quantity $\bigcirc$; where ⓪, or the 0 power, is the beginning of quantity, or arithmetical unit. He alſo extends this notation to roots or fractional exponents, and even to radical ones.

Thus $\textcircled{\tfrac{1}{2}}$, $\textcircled{\tfrac{1}{3}}$, $\textcircled{\tfrac{1}{4}}$, &c, are the ſq. root, cube root, 4th root, &c;
and $\textcircled{\tfrac{2}{3}}$ is the cube root of the ſquare;
and $\textcircled{\tfrac{3}{2}}$ is the ſq. root of the cube. And ſo of others.

The firſt three powers, ①, ②, ③, he alſo calls *coſte* (ſide), *quarre* (ſquare), *cube* (cube); and the firſt of them, ①, the prime quantity, which he obſerves is alſo *metaphorically* called the racine or root, (the mark of which is alſo $\surd$), becauſe it repreſents the root or origin from whence all other quantities ſpring or ariſe, called the *potences* or powers of it. He condemns the terms ſurſolids, and numbers abſurd, irrational, irregular, inexplicable, or ſurd, and ſhews that all numbers are denoted the ſame way, and are all equally proper expreſſions of ſome length or magnitude, or ſome power of the ſame root. He alſo rejects all the compound expreſſions of ſquare-ſquared, cube-ſquared, cube-cubed, &c, and ſhews that it is beſt to name them all by their exponents, as the 1ſt, 2d, 3d, 4th, 5th, 6th, &c power or quantity in the ſeries. And on his extenſion of the new notation he juſtly obſerves that what was before obſcure, laborious, and tireſome, will by theſe marks be clear, eaſy, and pleaſant. He alſo makes the notation of algebraic quantities more general in their coefficients, including in them not only integers, as 3①, but alſo fractions and radicals, as $\frac{3}{4}$②, and $\surd 2$③, &c. He has various other peculiarities in his notations; all ſhewing an original and inventive mind. A quantity of ſeveral terms, *he* calls a multinomial, and alſo binomial, trinomial, &c, according to the number of the terms. He uſes the ſigns + and −, and ſometimes : for equality; alſo × for diviſion of fractions, or to multiply croſswiſe thus, $\frac{5}{7}\times\frac{2}{3}:\frac{15}{14}$.

He teaches the generation of powers by means of the annexed table of numbers, which are the coefficients of all the terms except the firſt and laſt. And he makes uſe of the ſame numbers alſo for extracting all roots whatever: both which things had firſt been done by Stifelius. In extracting the roots of non-quadrate or non-cubic numbers, he has the ſame approximations as at preſent, viz, either to continue the extraction indefinitely in decimals, by adding periods of ciphers, or by making a fraction of the remainder in this manner, viz, $\surd N = n + \frac{N - n^2}{2n + 1}$ nearly, and $\sqrt[3]{N} = n + \frac{N - n^3}{3n^2 + 3n + 1}$ nearly; where n is the neareſt exact root of N; which is Peletarius's rule, and which differs from Tartalea's rule, as this wants the 1 in the denominator. And in like manner he goes on to the roots of higher powers.

2
3 . 3
4 . 6 . 4
5 . 10 . 10 . 5
6 . 15 . 20 . 15 . 6
&c.

He then treats of equations, and their inventors, which according to him are thus:

Mahomet, ſon of Moſes, an Arabian, invented theſe -	① egale *à* ⓪, its derivatives, ② egale *à* ①, ⓪,
And ſome unknown author, the derivatives of this.	
Some unknown author invented theſe - - - -	③ egale *à* ① ⓪, ③ egale *à* ② ⓪,

But afterwards he mentions Ferreus, Tartalea, Cardan, &c, as being alſo concerned in the invention of them.

Lewis Ferrari invented ④ egale *à* ③ ② ① ⓪.

He ſays alſo that Diophantus once reſolves the caſe ② egale *à* ① ⓪. In his reduction of equations, which is full and maſterly, he always puts the higheſt power on one ſide alone, equal to all the other terms, ſet in their order, on the other ſide, whether they be + or −. And he demonſtrates all the rules both arithmetically and geometrically. In cubics, he gives up the irreducible caſe, as hopeleſs: but ſays that Bombelli reſolves it by *plus of minus*, and *minus of minus*; thus, if 1③ = 30① + 36, then 1② = $\sqrt[3]{18 + \text{of} - 26} + \sqrt[3]{18 - \text{of} - 26}$, that is, 1① = $\sqrt[3]{18 + 26\surd -1} + \sqrt[3]{18 - 26\surd -1}$. He reſolves biquadratics by means of cubics and quadratics. In quadratics, he takes both the two roots, but looks for no more than two in cubics or biquadratics. He gives alſo a general method of approaching indefinitely near, in decimals, to the root of any equation whatever: but it is very laborious, being little more than trying all numbers, one after another, finding thus the 1ſt figure, then the 2d, then the 3d, &c, among theſe ten characters 0, 1, 2, 3, 4, 5, 6, 7, 8, 9. And finally he applies the rules in the reſolution of a great many practical queſtions.

Although a general air of originality and improvement runs through the whole of Stevinus's work, yet his more remarkable or peculiar inventions, may be reduced to theſe few following: viz,

1ſt. He invented not only a new character for the unknown quantity, but greatly improved the notation of powers, by numeral indices, firſt given by Stifelius as to integral exponents; which Stevinus extended to fractional and all other ſorts of exponents, thereby denoting all ſorts of roots the ſame way as powers, by numeral exponents. A circumſtance hitherto thought to be of much later invention.

2d. He improved and extended the uſe and notation of coefficients, including in them fractions and radicals, and all ſorts of numbers in general.

3d. A quantity of ſeveral terms, he called generally a multinomial; and he denoted all nomials whatever by particular names expreſſing the number of their terms, binomial, trinomial, quadrinomial, &c.

4th. A numeral reſolution of all equations whatever by one general method.

Beſides which, he hints at ſome unknown author as the firſt inventor of the rules for cubic equations; by whom may probably be intended the author of the Arabic manuſcript treatiſe on cubic equations, given to the library at Leyden by the celebrated Warner.

VIETA.

Moſt of Vieta's algebraical works were written about or a little before, the year 1600, but ſome of them were not publiſhed till after his death, which happened in the year 1603. And his whole mathematical works were collected together by Francis Schooten, and elegantly printed in a folio volume in 1646. Of theſe, the algebraical parts are as follow:

1. Iſagoge in Artem Analyticam.
2. Ad Logiſticen Specioſam Notæ priores.
3. Zeteticorum libri quinque.
4. De Æquationum Recognitione, & Emendatione.
5. De Numeroſâ Poteſtatum ad Exegeſin Reſolutione.

Of all theſe I ſhall give a very minute account, eſpecially in ſuch parts as contain any diſcoveries, as we here meet with more improvements and inventions in the nature of equations, than in almoſt any former author. And firſt of the *Iſagoge* or Introduction to the Analytic Art. In this ſhort introduction Vieta lays down certain præcognita in this art, as definitions, axioms, notations, common precepts or operations of addition, ſubtraction, multiplication, and diviſion, with rules for queſtions, &c. From which we find,

1ſt. That the names of his powers are latus, quadratum, cubus, quadrato-quadratum, quadrato-cubus, cubo-cubus, &c; in which he follows the method of Diophantus, and not that derived from the Arabians.

2d. That he calls powers pure or adfected, and firſt here uſes the terms coefficient, affirmative, negative, ſpecious logiſtics or calculations, homogeneum comparationis, or the abſolute known term of an equation, homogeneum adfectionis, or the 2d or other term which makes the equation adfected, &c. 3d. That he uſes the capital letters to denote the known as well as unknown quantities, to render his rules and calculations general, namely, the vowels A, E, I, O, U, Y for the unknown quantities, and the conſonants B, C, D, &c, for the known ones. 4th. That he uſes the ſign $+$ between two terms for addition; $-$ for ſubtraction, placing the greater before the leſs; and when it is not known which term is the greater, he places $=$ between them for the difference, as we now uſe $\backsim$; thus $A = B$ is the ſame as $A \backsim B$; that he expreſſes diviſion by placing the terms like a fraction, as at preſent; though he was not firſt in this. But that he uſes no characters for multiplication or equality, but writes the words themſelves, as well as the names of all the powers, as he uſes no exponents, which cauſes much trouble and prolixity in the progreſs of his work; and the numeral coefficients ſet after the literal quantities, have a diſagreeable effect.

II. *Ad Logiſticen Specioſam, Notæ Priores.* Theſe conſiſt of various theorems concerning ſums, differences, products, powers, proportionals, &c, with the geneſis of powers from binomial and reſidual roots, and certain properties of rational right-angled triangles.

III. *Zeteticorum libri quinque.* The zetetics or queſtions in theſe 5 books are chiefly from Diophantus, but reſolved more generally by literal arithmetic. And in theſe queſtions are alſo inveſtigated rules for the reſolution of quadratic and cubic equations. In theſe alſo Vieta firſt uſes a line drawn over compound quantities, as a vinculum.

IV. *De Æquationum Recognitione, & Emendatione.* Theſe two books, which contain Vieta's chief improvements in Algebra, were not publiſhed till the year 1615, by Alexander Anderſon, a learned and ingenious Scotchman, with various corrections and additions. The 1ſt of theſe two books conſiſts of 20 chapters. In the firſt ſix chapters, rules are drawn from the zetetics for the reſolution of quadratic and cubic equations. Theſe rules are by means of certain quantities in continued proportion, but in the reſolution they come to the ſame thing as Cardan's rules. In the cubics, Vieta ſometimes changes the negative roots into affirmative, as Cardan had done, but he finds only the affirmative roots. And he here refers the irreducible caſe to angular ſections for a ſolution, a method which had been mentioned by Bombelli.

Chap. 7 treats of the general method of transforming equations, which is done either by changing the root in various ways, namely by ſubſtituting another inſtead of it which is either increaſed or diminiſhed, or multiplied or divided by ſome known number, or raiſed or depreſſed in ſome known proportion; or by retaining the ſame root, and equally multiplying all the terms. Which ſorts of transformation, it is evident, are intended to make the equation become ſimpler, or more convenient for ſolution. And all or moſt of theſe reductions and transformations were alſo practiſed by Cardan.

Chap. 8 ſhews what purpoſes are anſwered by the foregoing transformations; ſuch as taking away ſome of the terms out of an equation, and particularly the 2d term, which is done by increaſing or diminiſhing the root by the coefficient of the 2d term divided by the index of the firſt: by which means alſo the affected quadratic is reduced to a ſimple one. And various other effects are produced.

Chap. 9 ſhews how to deduce compound quadratic equations from pure ones, which is done by increaſing or diminiſhing the root by a given quantity, being one application of the foregoing reductions.

Chap. 10, the reduction of cubic equations affected with the 1ſt power, to ſuch as are affected with the 2d power; by the ſame means.

In *chap.* 11, by the ſame means alſo, the 2d term is reſtored to ſuch cubic equations as want it.

In *chap.* 12, quadratic and cubic equations are raiſed to higher degrees by ſubſtituting for the root, the ſquare or cube of another root divided by a given quantity.

In *chap.* 13 affected biquadratic equations are deduced from affected quadratics in this manner, when expreſſed in the modern notation:

If $A^2 + BA = Z$, then ſhall $A^4 + \overline{B^3 + 2BZ} \cdot A = Z^2 + B^2Z$.

For ſince $A^2 + BA = Z$, therefore $A^2 = Z - BA$, and its ſquare is
$A^4 = Z^2 - 2BAZ + B^2A^2$: but $B^2A^2 = B^2Z - B^3A$, therefore
$A^4 = Z^2 - 2BAZ + B^2Z - B^3A$, or $A^4 + \overline{B^3 + 2BZ} \cdot A = Z^2 + B^2Z$.

And in like manner for the biquadratic affected with its other terms. And in a ſimilar manner alſo, in chap. 14, affected cubic equations are deduced from the affected quadratics.

In *chap.* 15 it is ſhewn that the quadratic $BA - A^2 = Z$ has two values of the root A, or has ambiguous roots, as he calls them; and alſo that the cubics, biquadratics, &c, which are raiſed or deduced from that quadratic, have alſo double roots.

Having, in the foregoing chapters, ſhewn how the coefficients of equations of the 3d and 4th degree are formed from thoſe of the 2d degree, of the ſame root; and that certain quadratics, and others raiſed from them, have double roots; then in the 16th chap. Vieta ſhews what relation thoſe two roots bear to the coefficients of the two loweſt terms of an equation conſiſting of only three terms. Thus,

$$\left.\begin{array}{l} \text{If } BA - A^2 = Z \\[2ex] \text{and } BE - E^2 = Z \end{array}\right\} \begin{array}{l} \text{then } B = \dfrac{A^2 - E^2}{A - E} = A + E, \\[2ex] \text{and } Z = \dfrac{A^2E - AE^2}{A - E} = AE. \end{array}$$

$$\left.\begin{array}{l} \text{If } BA - A^3 = Z \\[2ex] \text{and } BE - E^3 = Z, \end{array}\right\} \begin{array}{l} \text{then } B = \dfrac{A^3 - E^3}{A - E} = A^2 + AE + E^2, \\[2ex] \text{and } Z = \dfrac{A^3E - AE^3}{A - E} = A^2E + AE^2. \end{array}$$

$$\left.\begin{array}{l} \text{If } A^m + BA^n = Z, \\ \text{and } E^m - BE^n = Z, \end{array}\right\} \text{then } B = \frac{E^m - A^m}{E^n + A^n}, \& \; Z = \frac{A^mE^n + A^nE^m}{E^n + A^n},$$

And ſo on for the ſame terms with their ſigns variouſly changed.

Chap. 17 contains ſeveral theorems concerning quantities in continued geometrical progreſſion. Which are preparatory to what follows, concerning the double roots of equations, the nature of which he expounds by means of ſuch properties of proportional quantities.

Chap. 18, *Æquationum ancipitum conſtitutiva*; treating of the nature of the double roots of equations. Thus, if a, b, c, d, &c, be quantities in continual progreſſion; then, 1ſt, of equations affected with the firſt power,

If $BA-A^2=Z$; then $B=a+b$, $Z=ab$, & $A=a$ or b.

If $BA-A^3=Z$; then $B=a^2+b^2+c^2$, $Z=a(b^2+c^2)$, & $A=a$ or c.

And in general, if $BA-A^{n+1}$; then $B=a^n+b^n+c^n \cdots k^n$, $Z=a(b^n+c^n+d^n \cdots k^n)$, and $A=a$ or k the firſt or laſt term. Where the number of terms a^n, b^n, &c, in B is $n+1$, and the number of terms in Z is n.

2d, For equations containing only the higheſt two powers.

If $BA^2-A^3=Z$; then $B=a+b+c$, $Z=a(b+c)^2$, or $=c(a+b)^2$, and $A=a+b$, or $b+c$.

If $BA^3-A^4=Z$; then $B=a+b+c+d$, $Z=a(b+c+d)^3$ or $=d(a+b+c)^3$, and $A=a+b+c$ or $=b+c+d$.

And, in general, if $BA^n-A^{n+1}=Z$;

then $B=a+b+c \cdots i+k$,

$Z=a(b+c \cdots k)$ or $=k(a+b+c \cdots i)$,

and $A=a+b+c \cdots i$ or $=b+c+d \cdots k$, the ſum of all except the laſt, or ſum of all except the firſt; where the number of terms in B is $n+1$, and the number of terms in Z is n.

3d. Of equations affected by the intermediate powers.

If $BA^2-A^4=Z$; then $B=a^2+b^2$, $Z=a^2b^2$, & $A^2=a^2$ or b^2.

If $BA^3-A^6=Z$; then $B=a^3+b^3$, $Z=a^3b^3$, & $A^3=a^3$ or b^3.

If $BA^4-A^6=Z$; then $B=a^2+b^2+c^2$, $Z=a(b^2+c^2)$, and $A^2=a+b$ or $b+c$.

4th. Of the remaining caſes.

If $BA^2-A^5=Z$;

then $B=(a+b)^3+(b+c)^3+a(b+c)^2$ or $+c(a+b)^2$,

and $Z=\overline{B-(a+b)^3}\,.\,(a+b)^2$

or $=\overline{(b+c)^3+c(a+b)^2}\cdot(a+b)^2$,

and $A=a+b$ or $b+c$.

If $BA^3-A^5=Z$;

then $B=(a+b+c)^2+(b+c+d)^2-c(a+b+c)$ or $-b(b+c+d$,

and $Z=\overline{B-(b+c+d)^2}\,.\,(b+c+d)^3$

or $=\overline{B-(a+b+c)^2}\cdot(a+b+c)^3$;

and $A=a+b+c$ or $b+c+d$.

Chap. 19. *Æqualitatum contradicentium conſtitutiva.* Of the relation of equations of like terms, but the ſign of one term different; containing theſe 5 theorems, viz,

$$1.\ \left.\begin{array}{l}\text{If } A^2+BA=Z,\\ \text{and } E^2-BE=Z;\end{array}\right\}\begin{array}{l}\text{then } B=b-a,\ Z=ab;\\ \text{and } A=a,\ E=b.\end{array}$$

$$2.\ \left.\begin{array}{l}\text{If } A^4+BA=Z,\\ \\ \text{and } E^4-BE=Z;\end{array}\right\}\begin{array}{l}\text{then } B=(b+d)^3-(a+c)^3,\\ \quad Z=a(d^3-b^3) \text{ or } =d(c^3-a^3);\\ \text{and } A=a,\ E=d.\end{array}$$

$$3.\ \left.\begin{array}{l}\text{If } A^6+BA=Z,\\ \\ \\ \text{and } E^6-BE=Z;\end{array}\right\}\begin{array}{l}\text{then } B=b^5+d^5+f^5-a^5-c^5-e^5,\\ \quad Z=a(b^5+d^5+f^5-c^5-e^5)\\ \quad \text{or } =f(a^5+c^5+e^5-b^5-d^5);\\ \text{and } A=a,\ E=f.\end{array}$$

$$4.\ \left.\begin{array}{l}\text{If } A^4+BA^3=Z,\\ \\ \text{and } E^4-BE^3=Z;\end{array}\right\}\begin{array}{l}\text{then } B=b+d-a-c,\ Z=\\ \quad a(d-b)^3 \text{ or } =d(c-a)^3;\\ \text{and } A=c-a,\ E=d-b.\end{array}$$

$$5.\ \left.\begin{array}{l}\text{If } A^6+BA^5=Z,\\ \\ \\ \text{and } E^6-BE^5=Z;\\ \\ \end{array}\right\}\begin{array}{l}\text{then } B=b+d+f-a-c-e,\\ \quad Z=a(b+d+f-c-e)^5\\ \quad \text{or } =f(a+c+e-b-d)^5;\\ \text{and } A=a+c+e-b-d,\\ \quad E=b+d+f-c-e.\end{array}$$

Chap. 20. *Æqualitatum inverſarum conſtitutiva.* Containing theſe ſix theorems, viz,

$$1.\ \left.\begin{array}{l}\text{If } BA-A^3=Z,\\ \\ \text{and } E^3-BE=Z;\end{array}\right\}\begin{array}{l}\text{then } B=c^2-a^2,\ Z=a(c^2-b^2)\\ \quad \text{or } =c(b^2-a^2);\\ \text{and } A=a,\ E=c.\end{array}$$

$$2.\ \left.\begin{array}{l}\text{If } BA-A^5=Z,\\ \\ \\ \text{and } E^5-BE=Z;\end{array}\right\}\begin{array}{l}\text{then } B=a^4+c^4+e^4-b^4-d^4,\\ \quad Z=a(c^4+e^4-b^4-d^4)\\ \quad \text{or } =e(b^4+d^4-a^4-c^4);\\ \text{and } A=a,\ E=e.\end{array}$$

$$3.\ \left.\begin{array}{l}\text{If } BA^2+A^3=Z,\\ \\ \text{and } BE^2-E^3=Z;\end{array}\right\}\begin{array}{l}\text{then } B=c-a,\ Z=a(c-b)^2\\ \quad \text{or } =c(b-a)^2;\\ \text{and } A=b-a,\ E=c-b.\end{array}$$

$$4.\ \left.\begin{array}{l}\text{If } BA^4+A^5=Z,\\ \\ \\ \text{and } BE^4-E^5=Z;\\ \\ \end{array}\right\}\begin{array}{l}\text{then } B=a+c+e-b-d,\\ \quad Z=a(c+e-b-d)^4\\ \quad \text{or } =e(b+d-a-c)^4;\\ \text{and } A=b+d-a-c,\\ \quad E=c+e-b-d.\end{array}$$

$$5.\ \left.\begin{array}{l}\text{If } BA^3+A^5=Z,\\ \\ \\ \text{and } BE^3-E^5=Z;\end{array}\right\}\begin{array}{l}\text{then } B=(a+f)\,.\,(d-a),\\ \quad Z=(d-a)^3B+(d-a)^5\\ \quad \text{or } =(e-b)^3B-(e-b)^5;\\ \text{and } A=d-a,\ E=e-b.\end{array}$$

$$6.\ \left.\begin{array}{l}\text{If } BA^2+A^5=Z,\\ \\ \\ \text{and } BE^2-E^5=Z;\end{array}\right\}\begin{array}{l}\text{then } B=(a+f)\cdot(c-a)^2,\\ \quad Z=\overline{B+(c-a)^3}\cdot(c-a)^2\\ \quad \text{or } =\overline{B-(d-b)^3}\cdot(d-b)^2\\ \text{and } A=c-a,\ E=d-b.\end{array}$$

Chap. 21. *Alia rurſus æqualitatum inverſarum conſtitutiva.* In theſe two theorems:

$$1.\ \left.\begin{array}{l}\text{If } B^2A-A^3=Z,\\ \\ \\ \text{and } E^3-B^2A=Z;\end{array}\right\}\begin{array}{l}\text{then } B=a^2+b^2+c^2,\\ \quad Z=(a+c)b^2 \text{ or } =a(b^2+c^2)\\ \quad \text{or } =c(a^2+b^2);\\ \text{and } A=a \text{ or } c,\ E=a+c.\end{array}$$

$$2.\ \left.\begin{array}{l}\text{If } BA^2-A^3=Z,\\ \\ \\ \text{and } BE^2+E^3=Z;\end{array}\right\}\begin{array}{l}\text{then } B=a+b+c,\\ \quad Z=(a+b+c)b^2 \text{ or } =\\ \quad a(b+c)^2 \text{ or } =c(a+b)^2;\\ \text{and } A=a+b \text{ or } b+c,\ E=b.\end{array}$$

Next follows the 2d of the pieces publiſhed by Alexander Anderſon, namely,

De Emendatione Æquationum, in 14 chapters.

Chap. 1. Of preparing equations for their reſolution in numbers, by taking away the 2d term; by which affected quadratics are reduced to pure ones, and cubic equations affected with the 2d term are reduced to ſuch as are affected with the 3d only. Several examples of both ſorts of equations are given. He here too remarks upon the method of taking away any other term out of an equation, when the higheſt power is combined with that other term only; and this Vieta effects by means of the coefficients, or, as he calls them, the unciæ of the power of a binomial. All which was alſo performed by Cardan for the ſame purpoſe.

Chap. 2. *De tranſmutatione* Πρῶτον—ἔσχατον, *quæ remedium eſt adverſus vitium negationis.* Concerning the transformations by changing the given root A for another root E, which is equal to the homogeneum comparationis divided by the firſt root A; by which means negative terms are changed to affirmative, and radicals are taken out of the equation when they are contained in the homogeneum comparationis.

Chap. 3, *De Anaſtrophe*, ſhewing the relation between the roots of correlate equations; from whence, having given the root of the one equation, that of the other becomes known; and it conſiſts of theſe following 8 theorems, moſtly deduced from the laſt 4 chapters of the foregoing *recognitio æquationum.*

$$1.\ \left.\begin{array}{l}\text{If } BA - A^3 = Z,\\ \text{and } E^3 - BE = Z;\end{array}\right\}\ \text{then } EA - A^2 = E^2 - B.$$

$$2.\ \left.\begin{array}{l}\text{If } BA^2 - A^3 = Z,\\ \text{and } BE^2 + E^3 = Z,\end{array}\right\}\ \text{then } (E+B)A - A^2 = BD + D^2.$$

$$3.\ \left.\begin{array}{l}\text{If } BA - A^5 = Z,\\ \text{and } E^5 - BE = Z;\end{array}\right\}\ \begin{array}{l}\text{then } E^3A - E^2A^2 + EA^3 - A^4\\ \quad = E^4 - B.\end{array}$$

$$4.\ \left.\begin{array}{l}\text{If } BA^4 - A^5 = Z,\\ \text{and } BE^4 + E^5 = Z;\end{array}\right\}\ \begin{array}{l}\text{then } (BE^2+E^3)A - (BE+E^2)A^2\\ \quad + (B+E)A^3 - A^4 = BE^3 + E^4.\end{array}$$

$$5.\ \left.\begin{array}{l}\text{If } A^3 - BA = Z,\\ \text{and } BE - E^3 = Z;\end{array}\right\}\ \text{then } A^2 - EA = B - E^2.$$

$$6.\ \left.\begin{array}{l}\text{If } BA^2 + A^3 = Z,\\ \text{and } BE^2 - E^3 = Z,\end{array}\right\}\ \text{then } A^2 + (B-E)A = BE - E^2.$$

$$7.\ \left.\begin{array}{l}\text{If } BA - A^3 = Z,\\ \text{and } BE - E^3 = Z;\end{array}\right\}\ \text{then } A^2 + EA = B - E^2.$$

$$8.\ \left.\begin{array}{l}\text{If } BA^2 - A^3 = Z,\\ \text{and } BE^2 - E^3 = Z;\end{array}\right\}\ \text{then } A^2 + (E-B)A = BE - E^2.$$

Chap. 4, *De Iſomæria, adverſus vitium fractionis.* To take away fractions out of an equation. Thus, if $A^3 + \frac{B}{D}A = Z$. Put $A = \frac{E}{D}$; then $E^3 + BDE = ZD^3$.

Chap. 5, *De Symmetrica Climactiſmo adverſus vitium aſymmetriæ.* To take away radicals or ſurds out of equations, by ſquaring &c the other ſide of the equation.

Chap. 6. To reduce biquadratic equations by means of cubics and quadratics, by methods which are ſmall variations from thoſe of Ferrari and Cardan.

Chap. 7. The reſolution of cubic equations by rules which are the ſame with Cardan's.

Chap. 8. *De Canonica æquationum tranſmutatione, ut coefficientes ſubgraduales ſint quæ præſcribuntur.* To tranſmute the equation ſo that the coefficient of the lower term, or power, may be any given number, he changes the root in the given proportion, thus: Let A be the root of the equation given, E that of the tranſmuted equation, B the given coefficient, and X the required one; then take $A = \frac{BE}{X}$, which ſubſtitute in the given equation, and it is done.—He commonly changes it ſo, that X may be 1; which he does, that the numeral root of the equation may be the eaſier found; and this he here performs by trials, by taking the neareſt root of the higheſt power alone; and if that does not turn out to be the root of the whole equation, he concludes that it has no rational root.

Chap. 9. To reduce certain peculiar forms of cubics to quadratics, or to ſimpler forms, much the ſame as Cardan had done. Thus,

1. If $A^3 - 2B^2A = B^3$; then is $A^2 - BA = B^2$.
2. If $2B^2A - A^3 = B^3$; then is $A^2 + BA = B^2$.
3. If $A^3 - 3B^2A = 2B^3$; then is $A = 2B$.
4. If $3B^2A - A^3 = 2B^3$; then is $A = B$.
5. If $A^3 - BA^2 + DA = BD$; then is $A = B$.
6. If $A^3 + BA^2 - D^2A = BD^2$; then is $A = D$.
7. If $BA^2 + D^2A - A^3 = BD^2$; then is $A = B$ or $= D$.
8. If $D^2A + BDA - A^3 = B^3$; then is $(D+B)A - A^2 = B^2$
9. If $A^3 - DA^2 + BDA = B^3$; then is $(D-B)A - A^2 = B^2$
10. If $A^3 - 3BA = B\sqrt{2B}$; then is $A = \sqrt{\tfrac{3}{2}B} - \sqrt{\tfrac{1}{2}B}$.
11. If $3BA - A^3 = B\sqrt{2B}$; then is $A = \sqrt{\tfrac{3}{2}B} - \sqrt{\tfrac{1}{2}B}$.

Chap. 10. *Similium reductionum continuatio.* Being ſome more ſimilar theorems, when the equation is affected with all the powers of the unknown quantity A.

Chap. 11, 12, 13 relate alſo to certain peculiar forms of equations, in which the root is one of the terms of a certain ſeries of continued proportionals.

Chap. 14, which is the laſt in this tract, contains, in four theorems, the general relation between the roots of an equation and the coefficients of its terms, when all its roots are poſitive. Namely,

1. If $\overline{B+D}\,.\,A - A^2 = BD$; then is $A = B$ or D.
2. If $A^3 \overline{-B-D-G}\cdot A^2 \overline{+BD+BG+DG}\cdot A = BDG$; then is $A = B$ or D or G.
3. If $\overline{BDG+BDH+BGH+DGH}\cdot A \overline{-BD-BG-BH-DG-DH-GH}\,.\,A^2 \overline{+B+D+G+H}\cdot A^3 - A^4 = BDGH$; then is $A = B$ or D or G or H.
4. If $A^5 \overline{-B-D-G-H-K}\,.\,A^4 \overline{+BD+BG+BH+BK+DG+DH+DK+GH+GK+HK}\cdot A^3 \overline{-BDG-BDH-BDK-BGH-BGK-BHK-DGH-DGK-DHK-GHK}\cdot A^2 \overline{+BDGH+BDGK+BDHK+BGHK+DGHK}\cdot A = BDGHK$; then is $A = B$ or D or G or H or K.

And from theſe laſt 4 theorems it appears that Vieta was acquainted with the compoſition of theſe equations, that is, when all their roots are poſitive, for he never adverts to negative roots; and from other parts of the work it appears that he was not aware that the ſame properties will obtain in all ſorts of roots whatever. But it is not certain in what manner he obtained theſe theorems, as he has not given any account of the inveſtigations, though that was uſually his way on other occaſions; but he here contents himſelf with barely announcing the theorems as above, and for this ſtrange reaſon, that he might at length bring his work to a concluſion.

To this piece is added, by Alexander Anderſon, an Appendix, containing the conſtruction of the cubic equations by the triſection of an angle, and a demonſtration of the property referred to by Vieta for this purpoſe.

De Numeroſa Poteſtatum Purarum Reſolutione. Vieta here gives ſome examples of extracting the roots of pure powers, in the way that had been long before practiſed, by pointing the number into periods of figures according to the index of the root to be extracted, and then proceeding from one period to another, in the uſual way.

De Numeroſa Poteſtatum adfectarum Reſolutione. And here, in cloſe imitation of the above method for the roots of pure powers, Vieta extracts thoſe of adfected ones; or finding the roots of affected equations, placing always the homogeneum comparationis, or abſolute term, on one ſide, and all the terms affected with the unknown quantity, and their proper ſigns, on the other ſide. The method is very laborious, and is but little more than what was before done by Stevinus on this ſubject, depending not a little upon trials. The examples he uſes are ſuch as have either one or two roots, and indeed ſuch as are affected commonly with only two powers of the unknown quantity, and which therefore admit only of thoſe two varieties as to the number of roots, namely according as the higher of the two powers is affirmative or negative, the homogeneum comparationis, on the other ſide of the equation, being always affirmative; and he remarks this general rule, if the higher power be negative, the equation has two roots; otherwiſe, only one; that is, affirmative roots; for as to negative and imaginary ones, Vieta knew nothing about them, or at leaſt he takes no notice of them. By the foregoing extraction, Vieta finds both the greater and leſs root of the two that are contained in the equation, and either of them that he pleaſes; having firſt, for this purpoſe, laid down ſome obſervations concerning the limits within which the two roots are contained. Alſo, having found one of the roots, he ſhews how the other root may be found by means of another equation, which is a degree lower than the given one; though not by depreſſing the given equation, by dividing it as is now done; but from the nature of proportionals, and the theorems relating to equations, as given in the former tracts, he finds the terms of another equation, different from that laſt mentioned, from the root &c of which, the 2d root of the original equation may be obtained.

In the courſe of this work, Vieta makes alſo ſome obſervations on equations that are ambiguous, or have three roots; namely, that the equation $1C - 6Q + 11N = 6$, or as we write it $x^3 - 6x^2 + 11x = 6$ is ambiguous, when the 2d term is negative, and the 3d term affirmative, and when $\frac{1}{3}$ of the ſquare of 6 the coefficient of the 2d term, exceeds 11, the coefficient of the 3d term, and has then three roots. Or in general, if $x^3 - ax^3 + bx = c$, and $\frac{1}{3}a^2 > b$, the equation is ambiguous, and has three roots. He ſhews alſo, from the relation of the coefficients, how to find whether the roots are in arithmetical progreſſion or not, and how far the middle root differs from the extremes, by means of a cubic equation of this form $x^3 - ba = c$. In all or moſt of which remarks he was preceded by Cardan.—Vieta alſo remarks that the caſe $x^3 - 9x^2 + 24x = 20$, has three roots by the ſame rule, viz, 2, 2, 5, but that two of them are equal. And farther, that when $\frac{1}{3}a^2$ is $= b$, then all the three roots are equal, as in the caſe $x^3 - 6x^2 + 12x = 8$, the three roots of which are 2, 2, 2. But when $\frac{1}{3}a^2$ is leſs than b, the caſe is not ambiguous, having but one root. And when $ab = c$, then $a = x$ is one root itſelf.

Many curious notes are added at the end, with remarks on the method of finding the approximate roots, when they are not rational, which is done in two ways, in imitation of the ſame thing in the extraction of pure powers, viz, the one by forming a fraction of the remainder after all the figures of the homogeneum comparationis are exhauſted; the other by increaſing the root of the equation in a 10 fold, or 100 fold, &c, proportion, and then dividing the root which reſults by 10, or 100, &c: and this is a decimal approximation. And Vieta obſerves that the roots will be increaſed 10 or 100 fold, &c, by adding the correſponding number of ciphers to the coefficient of the 2d term, double that number to the 3d, triple the ſame number to the 4th, and ſo on. So if the equation were

$$1C + 4Q + 6N = 8,$$

then $1C + 40Q + 600N = 8000$ will have its root 10 fold, and $1C + 400Q + 60000N = 8000000$ will have it 100 fold.

Beſides the foregoing algebraical works, Vieta gave various conſtructions of equations by means of circles and right lines, and angular ſections, which may be conſidered as an algebraical tract, or a method of exhibiting the roots of certain equations having all their roots affirmative, and by means of which he reſolved the celebrated equation of 45 powers, propoſed to all the world by Adrianus Romanus.

Having now delivered a particular analyſis of Vieta's algebraical writings, it will be proper, as with other authors, to collect into one view the particulars of his more remarkable peculiarities, inventions, and improvements.

And firſt it may be obſerved, that his writings ſhew great originality of genius and invention, and that he made alterations and improvements in moſt parts of algebra; though in other parts and reſpects his method is inferior to ſome of his predeceſſors; as, for inſtance, where he neglects to avail himſelf of the negative roots of Cardan; the numeral exponents of Stifelius, inſtead of which he uſes the names of the powers themſelves; or the fractional exponents of Stevinus; or the commodious way of prefixing the coefficient before the quantity or factor; and ſuch like circumſtances; the want of which gives his Algebra the appearance of an age much earlier than its own. But his real inventions of things before not known, may be reduced to the following particulars.

1ſt. Vieta introduced the general uſe of the letters of the alphabet to denote indefinite given quantities; which had only been done on ſome particular occaſions before his time. But the general uſe of letters for the unknown quantities was before pretty common with Stifelius and his ſucceſſors. Vieta uſes the vowels A, E, I, O, U, Y for the unknown quantities, and the conſonants B, C, D, &c, for known ones.

2d. He invented, and introduced many expreſſions or terms, ſeveral of which are in uſe to this day: ſuch as coefficient, affirmative and negative, pure and adfected or affected, unciæ, homogeneum adfectionis, homogeneum comparationis, the line or vinculum over compound quantities thus $\overline{A + B}$. And his method of ſetting down his equations, is to place the homogeneum comparationis, or abſolute known term, on the right-hand ſide alone, and on the other ſide all the terms which contain the unknown quantity, with their proper ſigns.

3d. In moſt of the rules and reductions for cubic and other equations, he made ſome improvements, and variations in the modes.

4th. He ſhewed how to change the root of an equation in a given proportion.

5. He derived or raiſed the cubic and biquadratic, &c equations, from quadratics; but not by compoſition in Harriot's way, but by ſquaring and otherwiſe multiplying certain parts of the quadratic. And as ſome quadratic equations have two roots, therefore the cubics and others raiſed from them, have alſo the ſame two roots, and no more. And hence he comes to know what relation theſe two roots bear to the coefficients of the two loweſt terms of cubic and other equations, when they have only 3 terms, namely, by comparing them with ſimilar equations ſo raiſed from quadratics. And, on the contrary, what the roots are, in terms of ſuch coefficients.

6. He made ſome obſervations on the limits of the two roots of certain equations.

7. He ſtated the general relation between the roots of certain equations and the coefficients of its terms, when the terms are alternately plus and minus, and none of them are wanting, or the roots all poſitive.

8. He extracted the roots of affected equations, by a method of approximation ſimilar to that for pure powers.

9. He gave the conſtruction of certain equations, and exhibited their roots by means of angular ſections; before adverted to by Bombelli.

OF ALBERT GIRARD.

Albert Girard was an ingenious Dutch or Flemiſh mathematician, who died about the year 1633. He publiſhed an edition of Stevinus's Arithmetic in 1625, augmented with many notes; and the year after his death was publiſhed by his widow, an edition of the whole works of Stevinus, in the ſame manner, which Girard had left ready for the preſs. But the work which entitles him to a particular notice in this hiſtory, is his "*Invention Nouvelle en l'Algebre, tant pour la ſolution des equations, que pour recognoiſtre le nombre des ſolutions qu'elles reçoivent, avec pluſieurs choſes qui ſont neceſſaires a la perfection de ceſte divine ſcience*:" which was printed at Amſterdam 1629, in ſmall quarto in 63 pages, viz, 49 pages on Arithmetic and Algebra, and the reſt on the meaſure of the ſuperficies of ſpherical triangles and polygons, by him then lately diſcovered.

In this work Girard firſt premiſes a ſhort tract on Arithmetic; in the notation of which he has ſomething peculiar, viz, dividing the numbers into the ranks of millions, billions, trillions, &c.

He next delivers the common rules of Algebra, both in integers, fractions and radicals; with the notation of the quantities and ſigns. In this part he uſes ſometimes the letters A, B, C, &c, after the manner of Vieta, but more commonly the characters of Stevinus, viz, ⓪, ①, ②, ③, &c, for the powers of the unknown quantity, with their roots $\textcircled{\tfrac{5}{2}}$, $\textcircled{\tfrac{1}{2}}$, $\textcircled{\tfrac{1}{4}}$, $\textcircled{\tfrac{2}{3}}$, $\textcircled{\tfrac{3}{4}}$, &c, uſed by Stevinus; and ſometimes the more uſual marks of the roots as, $\sqrt{}$ or $\sqrt[2]{}$, $\sqrt[3]{}$, $\sqrt[4]{}$, &c; prefixing the coefficients, as 6②, or $3\sqrt[5]{32}$, or $2\textcircled{\tfrac{1}{2}}$. In the ſigns he follows his predeceſſors ſo far as to have + for plus, − or ÷ for minus, = for general or indefinite difference, $A + B$ for the ſum, $A - B$ or $A = B$ for the difference, AB the product, and $\frac{A}{B}$ for the quotient of A and B. He uſes the parentheſes ( ) for the vinculum or bond of compound quantities, as is now commonly practiſed on the continent; as $A(AB + Bq)$, or $\sqrt[3]{}(A \text{ cub.} - 3AqB)$; and he introduces the new characters ff for *greater than*, and § for *leſs than*; but he uſes no character for equality, only the word itſelf.

Girard gives a new rule for extracting the cube root of binomials, which however is in a good meaſure tentative, and which he explains thus: To extract the cube root of $72 + \sqrt{5120}$.

$$\text{The ſquares of the terms } \left\{\begin{array}{l} 5184 \\ 5120 \\ \hline \end{array}\right.$$

their difference 64, and its cube root 4. Which ſhews that the difference between the ſquares of the terms required is 4; and the rational part 72 being the greater, the greater term of the root will be rational alſo; and farther, that the greater terms of the power and root are commenſurable, as alſo the two leſs terms. Then having made a table as in the margin, where the ſquare of the rational term always exceeds that of the other, by the number 4 above mentioned, one of theſe binomials muſt be the cubic root ſought, if the given quantity have ſuch a root, and it muſt be one of theſe four forms, for it is known to be carried far enough by obſerving that the cube root of 72 is leſs than 5, and the cube root of 5120 leſs than 21; indeed, this being the caſe, the laſt binomial is excluded, as evidently too great; and the firſt is excluded becauſe one of its terms is 0; therefore the root muſt be either $3 + \sqrt{5}$ or $4 + \sqrt{12}$. And to know whether of theſe two it muſt be, try which of them has its two terms exact diviſors of the correſponding terms of the given quantity; then it is found that 3 and 4 are both diviſors of 72, but that only 5, and not 12, is a diviſor of 5120; therefore $3 + \sqrt{5}$ is the root ſought, which upon trial is found to anſwer. It is remarkable here that Girard uſes $4 + \sqrt{20}$ inſtead of $4 + \sqrt{12}$, and $5 + \sqrt{29}$ inſtead of $5 + \sqrt{20}$, contrary to his own rule.

$2 + \sqrt{0}$
$3 + \sqrt{5}$
$4 + \sqrt{12}$
$5 + \sqrt{21}$

Girard then gives diſtinct and plain rules for bringing queſtions to equations, and for the reduction of thoſe equations to their ſimpleſt form, for ſolution, by the uſual modes, and alſo by the way called by Vieta *Iſomeria*, multiplying the terms of the equation by the terms of a geometrical progreſſion, by which means the roots are altered in the proportion of 1 to the ratio of the progreſſion. He then treats of the methods of finding the roots of the ſeveral ſorts of equations, quadratic, cubic, &c; and adds remarks on the proper number of conditions or equations for limiting queſtions. The quadratics are reſolved by completing the ſquare, and both the poſitive and negative roots are taken; and he obſerves that ſometimes the equation is impoſſible, as ② equ. 6① − 25, whoſe roots, he adds, are $3 + \sqrt{-16}$ and $3 - \sqrt{-16}$.

The cubic equations he reſolves by Cardan's rule, except the irreducible caſe, which he the firſt of any reſolves by a table of ſines; the other caſes alſo he reſolves by tables of ſines and tangents; and adds geometrical conſtructions by means of the hyperbola or the triſection of angles. He next adds a particular mode of reſolving all ſorts of equations, that have rational roots, upon the principle of the roots being diviſors of the laſt or abſolute term, as before mentioned by Peletarius; and then gives the method of approximating to other roots that are not rational, much the ſame way as Stevinus.

Having found one root of an equation, by any of the former methods, by means of it he depreſſes the equation one degree lower, then finds another root, and ſo on till they are all found; for he ſhews that every algebraic equation admits of as many ſolutions or roots, as there are units in the index of the higheſt power, which roots may be either poſitive or negative, or imaginary, or, as he calls them, greater than nothing, or leſs than nothing, or involved; ſo the roots of the equation 1 ③ equ. 7① − 6, are 2, 1, and − 3; and the roots of the equation 1④ equ. 4① − 3 are

$$\begin{array}{r} 1, \\ 1, \\ -1+\sqrt{-2}, \\ -1-\sqrt{-2}. \end{array}$$

In depreſſing an equation to lower degrees, he does not uſe the method of reſolution of Harriot, but that which is derived from the general relation of the roots and coefficients of the terms, which he here fully and univerſally ſtates, viz, that the coefficient of the 2d term is equal to the ſum of all the roots; that of the 3d term equal to the ſum of all the products of the roots, taken two by two; that of the 4th term, the ſum of the products, taken three by three; and ſo on, to the laſt or abſolute term, which is the continual product of all the roots; a property which was before ſtated by Vieta, as to the equations that have all their roots poſitive; and here extended by Girard to all ſorts of roots whatever: but how either Vieta or he came by this property, no where appears that I know of. From this general property, among other deductions, Girard ſhews how to find the ſums of the powers of the roots of an equation; thus, let A, B, C, D, &c, be the 1ſt, 2d, 3d, 4th, &c, coefficient, after the firſt term, or the ſums of the products taken one by one, two by two, three by three, &c; then, in all ſorts of equations,

$$\left.\begin{array}{l} A \\ Aq-2B \\ A\text{ cub.}-3AB+3C \\ Aqq-4AqB+4AC+2Bq-4D \end{array}\right\}\begin{array}{c}\text{will be the}\\ \text{ſum of the}\end{array}\left\{\begin{array}{l} \text{roots,} \\ \text{ſquares,} \\ \text{cubes,} \\ \text{biquadrates.} \end{array}\right.$$

Girard next explains the uſe of negative roots in Geometry, ſhewing that they repreſent lines only drawn in a direction contrary to thoſe repreſenting the poſitive roots; and he remarks that this is a thing hitherto unknown. He then terminates the Algebra by ſome queſtions having two or more unknown quantities; and ſubjoins to the whole a tract on the menſuration of the ſurfaces of ſpherical triangles and polygons, by him lately diſcovered.

From the foregoing account it appears that,

1ſt, He was the firſt perſon who underſtood the general doctrine of the formation of the coefficients of the powers, from the ſums of their roots, and their products, &c.

2d, He was the firſt who underſtood the uſe of negative roots in the ſolution of geometrical problems.

3d, He was the firſt who ſpoke of the imaginary roots, and underſtood that every equation might have as many roots real and imaginary, and no more, as there are units in the index of the higheſt power. And he was the firſt who gave the whimſical name of *quantities leſs than nothing* to the negative. And,

4th, He was the firſt who diſcovered the rules for ſumming the powers of the roots of any equation.

OF HARRIOT.

Thomas Harriot, a celebrated aſtronomer, philoſopher, and mathematician, flouriſhed about the year 1610, about which time it is probable he wrote his Algebra, as he was then, and had been for many years before, celebrated for his mathematical and aſtronomical labours. In that year he made obſervations on the ſpots in the ſun, and on Jupiter's ſatellites, the ſame year alſo in which Galileo firſt obſerved them: he left many other curious aſtronomical obſervations, and amongſt them, ſome on the remarkable comets of the years 1607 and 1618. His Algebra was left behind him unpubliſhed, as well as thoſe other papers, at his death, which happened in the year 1621, being then 60 years of age, and but ſix years after the firſt publication of the principal parts of Vieta's Algebra by Alexander Anderſon; ſo that it is probable that Harriot's Algebra was written before this time, and indeed that he had never ſeen theſe pieces. Harriot's Algebra was publiſhed by his friend Walter Warner, in the year 1631: and it would doubtleſs be highly grateful to the learned in theſe ſciences, if his other curious algebraical and aſtronomical works were publiſhed from his original papers in the poſſeſſion of the Earl of Egremont, to whom they have deſcended from Henry Percy, the Earl of Northumberland, that noble Mæcenas of his day. The book is in folio, and intitled *Artis Analyticæ Praxis, ad Æquationes Algebraicas nova, expedita, & generali methodo, reſolvendas*; a work in all parts of it ſhewing marks of great genius and originality, and is the firſt inſtance of the modern form of Algebra in which it has ever ſince appeared. It is prefaced by 18 definitions, which are theſe: 1ſt, Logiſtica Specioſa; 2d, Equation; 3d, Syntheſis; 4, Analyſis; 5, Compoſition and Reſolution; 6, Forming an Equation; 7, Reduction of an Equation; 8, Verification; 9, Numeroſa & Specioſa; 10, Excogitata; 11, Reſolution; 12, Roots; 13 and 14, The kinds and generation of equations by multiplication, from binomial roots or factors, called original equations,

$$\text{as }\begin{array}{c|l} a+b & =aa+ba \\ a-c & -ca-bc, \\ \hline \end{array}$$

$$\text{or }\begin{array}{c|l} a+b & =aaa+baa+bca \\ a+c & +caa-bda \\ a-d & -daa-cda-bcd, \\ \hline \end{array}$$

where he puts *a* for the unknown quantity, and the ſmall conſonants, *b*, *c*, *d*, &c, for its literal values or roots; 15, The firſt form of canonical equations, which are derived from the above originals, by tranſpoſing the homogeneum, or abſolute term,

$$\begin{array}{l} \text{thus } aa+ba \\ \qquad\qquad -ca=+bc, \text{ \&c}; \end{array}$$

16, The ſecondary canonicals, formed from the primary by expelling the 2d term,

$$\begin{array}{l} \text{thus } aa=+bb, \\ \text{or } aaa-bba \\ \qquad\quad -bca \\ \qquad\quad -cca=+bbc \\ \qquad\qquad\qquad\ +bcc; \end{array}$$

17, That theſe are called canonicals, becauſe they are adapted to canons or rules for finding the numeral roots, &c. 18, Reciprocal equations, in which the homogeneum is the product of the coefficients of the other terms, and the firſt term, or higheſt power of the root, is equal to the product of the powers in the other terms, as $aaa - caa + bba = + bbc$.

After theſe definitions, the work is divided into two principal parts; 1ſt, of various generations, reductions, and preparations of equations for their reſolution in the 2d part. The former is divided into 6 ſections as follows.

Sect. 1. *Logiſtices Speciosæ*, exemplified in the 4 operations of addition, ſubtraction, multiplication, and diviſion; as alſo the reduction of algebraic fractions, and the ordinary reduction of irregular equations to the form proper for the reſolution of them, namely, ſo that all the unknown terms be on one ſide of the equation, and the known term on the other, the powers in the terms ranged in order, the greateſt firſt, and the firſt or higheſt power made poſitive, and freed from its coefficient; as $aa + ba = cd$,

or $aaa + baa - cda = - c^2d$.

In this part he explains ſome unuſual characters which he introduces, namely

$=$ for equality, as $a = b$.

$>$ for majority, as $a > b$,

$<$ for minority, as $a < b$;

but the firſt had been before introduced by Robert Recorde.

Sect. 2. The generation of original equations from binomial factors or roots, and the deducing of canonicals from the originals. He ſuppoſes that every equation has as many roots as dimenſions in its higheſt power; then ſuppoſing the values of the unknown letter a in any equation to be b, c, d, f, &c, that is $a = b$, and $a = c$, and $a = d$, &c; by tranſpoſition, or equal ſubtraction, theſe become $a - b = 0$, and $a - c = 0$, and $a - d = 0$, &c, or the ſame letters with contrary ſigns, for negative values or roots; then two of theſe binomial factors multiplied together, gives a quadratic equation, three of them a cubic, four of them a biquadratic, and ſo on, with all the terms on one ſide of the equation, and 0 on the other ſide, ſince, every binomial factor being $= 0$, the continual product of all of them muſt alſo be $= 0$. Thus,

$$\left.\begin{array}{l} a+b \\ a+c \\ \underline{a-d} \end{array}\right| = \begin{array}{l} aaa + baa + bca \\ \quad + caa - bda \\ \quad - daa - cda - bcd = 0 \end{array}$$

an original equation,

and

$$\begin{array}{l} aaa + baa + bca \\ \quad + caa - bda \\ \quad - daa - cda = + bcd \end{array}$$

its canonical, deduced from it. And theſe operations are carried through all the caſes of the 2d, 3d and 4th powers, as to the varieties of the ſigns + and −, and the proportions of the roots as to equal and unequal, with the reciprocals, &c. From which are made evident, at one glance of the eye, all the relations and properties between the roots of equations, and the coefficients of the terms.

Sect. 3. *Æquationum canonicarum ſecundariarum a primariis reductio per gradus alicujus parodici ſublationem radice ſuppoſititia invariata manente.* Containing a great many examples of preparing equations by taking away the 2d, 3d, or any other of the intermediate terms, which is done by making the poſitive coefficients in that term, equal to the negative ones, by which means the whole term vaniſhes, or becomes equal to nothing.

They are extended as far as equations of the 5th degree; and at the end are collected, and placed in regular order, all the ſecondary canonicals, ſo reduced, ſo that by the uniform law which is viſible through them all, the ſeries may be continued to the higher degrees as far as we pleaſe.

Sect. 4. *Æquationum canonicarum tam primariarum, quam ſecundariarum, radicum deſignatio.* A great many literal equations are here ſet down, and their roots aſſigned from the form of the equation, that is all their poſitive roots; for their negative roots are not noticed here; and it is every where proved that they cannot have any more poſitive roots than theſe, and conſequently the reſt are negative. That thoſe are roots, he proves by ſubſtituting them inſtead of the unknown letter a in the equation, when they make all the terms on one ſide come to the ſame thing as the homogeneum on the other ſide.

Sect. 5, *In qua æquationum communium per canonicarum æquipollentiam, radicum numerus determinatur.* On the number of the roots of common equations, that is the poſitive roots. This Harriot determines by comparing them with the like caſes found among his canonical forms, which two equations, having the ſame number of terms with the ſame ſigns, and the relations of the coefficients and homogeneum correſpondent, he calls equipollents. And whatever was the number of poſitive roots uſed in the compoſition of the canonical, the ſame, he infers, is the number in the propoſed common equation. It is remarkable that in all the examples here uſed, the number of poſitive roots is juſt equal to the number of the changes in the ſigns from + to − and from − to +, which is a circumſtance, though not here expreſsly mentioned, that could not eſcape the obſervation, or the eye, of any one, much leſs of ſo clear and comprehenſive a ſight as that of Harriot. In this ſection are contained many ingenious diſquiſitions concerning the limits and magnitudes of quantities, with ſeveral curious lemmas laid down to demonſtrate the propoſitions by, which lemmas are themſelves demonſtrated in a pure mathematical way, from the magnitudes themſelves, independent of geometrical figures; ſuch as, 1, If a quantity be divided into any two unequal parts, the ſquare of half the line will be greater than the product of the two unequal parts. 2, In three continued proportionals, the ſum of the extremes is greater than double the mean. 3, In four continued proportionals, the ſum of the extremes is greater than the ſum of the two means. 4, In any two quantities, one-fourth the ſquare of the ſum of the cubes, is greater than the cube of the product of the two quantities. 5, Of any two quantities q and r, then $\frac{1}{27}(qq + qr + rr)^3 > \frac{1}{4}(qqr + qrr)^2$. 6, If any quantity be divided into three unequal parts, the ſquare of $\frac{1}{3}$ of the whole quantity is greater than $\frac{1}{3}$ of the ſum of the three products made of the three unequal parts. 7, Alſo the cube of the $\frac{1}{3}$ part of the whole, is greater than the ſolid or continual product of the three unequal parts.

Sect. 6. *Æquationum communium reductio per gradus alicujus parodici excluſionem & radicis ſuppoſititiæ mutationem.* Here are a great many examples of reducing and transforming equations of the 2d, 3d, and 4th degrees; chiefly either by multiplying the roots of equations in any proportion, as was done by Vieta, or increaſing or diminiſhing the root by a given quantity, after the manner of Cardan. The former of theſe reductions is performed by multiplying the terms of the equation by the correſponding terms of a geometrical progreſſion, the 1ſt term being 1, and the 2d term the quantity by which the root is to be multiplied. And the other reduction, or transforming to another root, which may be greater or leſs than the given root by a given quantity, is performed commonly by ſubſtituting $e +$ or $- b$ for the given root a, by which the equation is reduced to a ſimpler form. Other modes of ſubſtitution are alſo uſed; one of which is this, viz, ſubſtituting $\frac{ee \pm bb}{e}$ or $e \pm \frac{bb}{e}$ for the root a in the given equation $aaa \mp 3.bba = 2.ccc$, by which it reduces to this quadratic form $e^6 \mp 2c^3 e^3 = - b^6$, from whence Cardan's forms are immediately deduced; namely $e = \surd 3)\ c^3 - \surd c^6 \pm b^6$, and therefore a or $e \pm \frac{bb}{e} = \surd 3)\ c^3 + \surd c^6 \pm b^6 \pm \surd 3)\ c^3 - \surd c^6 \pm b^6$; where he denotes the cube or 3d root thus $\surd 3)$, but without any vinculum over the compound quantities.

In this ſection, Harriot makes various remarks as they occur: thus he remarks, and demonſtrates, that $eee - 3.bbe = -ccc$
$-2.bbb$ is an impoſſible equation, or has no affirmative root. He remarks alſo that the three caſes of the equation $aaa - 3.bba = + 2.ccc$ are ſimilar to the three conic ſections; namely to the hyperbola when $c > b$, to the parabola when $c = b$, or to the ellipſis when $c < b$, and for which reaſon this caſe is not generally reſoluble in ſpecies.

Having thus ſhewn how to ſimplify equations, and prepare them for ſolution, Harriot enters next upon the ſecond part of his work, being the

Exegetice Numeroſa,

or the numeral reſolution of all ſorts of equations by a general method, which is exemplified in a great number of equations, both ſimple and affected as far as the 5th power incluſive; and they are commonly prepared, by the foregoing parts, by freeing them from their 2d term, &c. Theſe extractions are explained and performed in a way different from that of Vieta; and the examples are firſt in perfect or terminate roots, and afterwards for irrational or interminate ones, to which Harriot approximates by adding always periods of ciphers to the given number or reſolvend, as far as neceſſary in decimals, which are continued and ſet down as ſuch, but with their proper denominator 10, or 100, or 1000, &c.

He then concludes the work with

Canones Directorii,

which form a collection of the caſes or theorems for making the foregoing numeral extractions, ready arranged for uſe, under the various forms of equations, with the factors neceſſary to form the ſeveral reſolvends and ſubtrahends.

And from a review of the whole work, it appears that Harriot's inventions, peculiarities, and improvements in algebra, may be comprehended in the following particulars.

1ſt. He introduced the uniform uſe of the ſmall letters a, b, c, d, &c, viz, the vowels a, e, &c for unknown quantities, and the conſonants b, c, d, f, &c for the known ones; which he joins together like the letters of a word, to repreſent the multiplication or product of any number of theſe literal quantities, and prefixing the numeral coefficient as we do at preſent, except only ſeparated by a point, thus $5.bbc$. For a root he ſet the index of the root after the mark $\surd$; as $\surd 3)$ for the cube root. He alſo introduced the characters $>$ and $<$ for greater and leſs; and in the reduction of equations, he arranged the operations in ſeparate ſteps or lines, ſetting the explanations in the margin on the left hand, for each line. By which, and other means, he may be conſidered as the introducer of the modern ſtate of Algebra, which quite changed its form under his hands.

2d. He ſhewed the univerſal generation of all the compound or affected equations, by the continual multiplication of ſo many ſimple ones, or binomial roots; thereby plainly exhibiting to the eye the whole circumſtances of the nature, myſtery and number of the roots of equations; with the compoſition and relations of the coefficients of the terms; and from which many of the moſt important properties have ſince been deduced.

3d. He greatly improved the numeral exegeſis, or extraction of the roots of all equations, by clear and explicit rules and methods, drawn from the foregoing generation or compoſition of affected equations of all degrees.

OF OUGHTRED'S CLAVIS.

Oughtred was contemporary with Harriot, but lived a long time after him. His Clavis was firſt publiſhed in 1631, the ſame year in which Harriot's Algebra was publiſhed by his friend Warner. In this work, Oughtred chiefly follows Vieta, in the notation by the capitals A, B, C, D, &c, in the deſignation of products, powers, and roots, though with ſome few variations. His work may be comprehended under the following particulars.

1. *Notation.* This extends to both Algebra and Arithmetic, vulgar and decimal. The Algebra chiefly after the manner of Vieta, as aboveſaid. And he ſeparates the decimals from the integers thus, 21|56, which is the firſt time I have obſerved ſuch a ſeparation, and the decimals ſet down without their denominator.

2. The common rules or operations of Arithmetic and Algebra. In algebraic multiplication, he either joins the letters together like a word, or connects them by the mark ×, which is the firſt introduction of this character of multiplication: thus $A \times A$ or AA or Aq. But omitting the vinculum over compound factors, uſed by Vieta. He introduces here many neat and uſeful contractions in multiplication and diviſion of decimals: as that common one of inverting the multiplier, to have fewer decimals, and abridge the work; that of omitting always one figure at a time, of the diviſor, for the ſame purpoſe; dividing by the component factors of a number inſtead of the number itſelf; as 4 and 6 for 24; and many other neat contractions. He

ſtates his proportions thus 7.9 : : 28.36, and denotes continued proportion thus ÷ ; which is the firſt time I have obſerved theſe characters.

3. Invents and deſcribes various ſymbolical marks or abbreviations, which are not now uſed.

4. *The geneſis and analyſis of powers.* Denotes powers like Vieta, and alſo roots, thus $\surd q6$, $\surd c20$, $\surd qq24$, &c ; and much in his manner too performs the numeral extraction of roots. He here gives a table of the powers of the binomial A + E as far as the 10th power, with all their terms and coefficients, or unciæ as he calls them, after Vieta.

5. *Equations.* He here gives expreſs and particular directions for the ſeveral ſorts of reductions, according as the form of the equation may require. And he uſes the letter *u* after $\surd$, for univerſal, inſtead of the vinculum of Vieta. And obſerves that the ſigns of all the terms of the powers of A + E are poſitive, but thoſe of A − E are alternately poſitive and negative.

6. Next follow many properties of triangles and other geometrical figures; and the firſt inſtance of applying Algebra to Geometry, ſo as to inveſtigate new geometrical properties ; and after the algebraical reſolution of each problem, he commonly deduces and gives a geometrical conſtruction adapted to it. He gives alſo a good tract on angular ſections.

7. The work concludes with the numeral reſolution of affected equations, in which he follows the manner of Vieta, but he is more explicit.

OF DESCARTES.

Deſcartes's Geometry was firſt publiſhed in 1637, being ſix years after the publication of Harriot's Algebra. That work was rather an application of Algebra to Geometry, than the ſcience either of Algebra or Geometry itſelf, purely and properly ſo called. And yet he made improvements in both. We muſt obſerve however, that all the properties of equations, &c, which he ſets down, are not to be conſidered as even meant by himſelf for new inventions or diſcoveries; but as ſtatements and enumerations of properties, before known and taught by other authors, which he is about to make ſome uſe or application of, and for which reaſon it is that he mentions thoſe properties.

Deſcartes's Geometry conſiſts of three books. The firſt of theſe is, *De Problematibus, quæ conſtrui poſſunt, adhibendo tantum rectas lineas & circulos.* He here accommodates or performs arithmetical operations by Geometry, ſuppoſing ſome line to repreſent unity, and then, by means of proportionals, ſhewing how to multiply, divide, and extract roots by lines. He next deſcribes the notation he uſes, but not becauſe it is a new one, for it is the ſame as had been uſed by former authors, viz, $a + b$ for the addition of a and b, alſo $a - b$ for their ſubtraction, ab multiplication, $\frac{a}{b}$ diviſion, aa or a^2 the ſquare of a, a^3 its cube, &c : alſo $\surd\overline{a^2 + b^2}$ for the ſquare root of $a^2 + b^2$, and $\surd\overline{\text{C}.a^3 - b^3 + abb}$ for the cube root, &c. He then obſerves, after Stifelius, that there muſt be as many equations as there are unknown lines or quantities ; and that they muſt be reduced all to one final equation, by exterminating all the unknown letters except one ; when the final equation will appear like theſe,

$z \propto b$, or
$z^2 \propto - az + b^2$, or
$z^3 \propto + az^2 + b^2z - c^3$, or
$z^4 \propto + az^3 + b^2z^2 - c^3z + d^4$, &c.

Where he uſes $\propto$ for = or equality, ſetting the higheſt term or power alone on one ſide of the equation, and all the other terms on the other ſide, with their proper ſigns.

Deſcartes next defines plane problems, namely, ſuch as can be reſolved by right lines and circles, deſcribed on a plane ſuperficies ; and then the final equation riſes only to the 2d power of the unknown letter. He then conſtructs ſuch equations, viz. quadratics, by the circle, thus finding geometrically the root or roots, that is, the poſitive ones. But when the lines, by which the roots are determined, neither cut nor touch, he obſerves that the equation has then no poſſible root, or that the problem is impoſſible. He then concludes this book with the algebraical ſolution of the celebrated problem, before treated of by the antients, namely, to find a point, or the locus of all the points, from whence a line being drawn to meet any number of given lines in given angles, the product of the ſegments of ſome of them ſhall have a given ratio to that of the reſt.

Lib. 2. *De Natura Linearum Curvarum.* This is a good algebraical treatiſe on curve lines in general, and the firſt of the kind that has been produced by the moderns. Here the nature of the curve is expreſſed by an equation containing two unknown or variable lines, and others that are known or conſtant, as $y^2 \propto cy - \frac{cxy}{b} + ay - ac$. But, not relating to pure Algebra, the particulars will be moſt properly placed under the article of curve lines, and other terms relating to them. Only one diſcovery, among many ingenious applications of Algebra to Geometry, may here be particularly noticed, as it may be conſidered as the firſt ſtep towards the arithmetic of infinites; and that is the method of tangents, here given, or, which comes to the ſame thing, of drawing a line perpendicular to a curve at any point, which is an ingenious application of the general form of an equation, generated in Harriot's way, that has two equal roots, to the equation of the curve. Of which a particular account will be given at the article TANGENTS.

Lib. 3. *De Conſtructione Problematum Solidorum, et Solida excedentium.* Deſcartes begins this book with remarks on the nature and roots of equations, obſerving that they have as many roots as dimenſions, which he ſhews, after Harriot, by multiplying a certain number of ſimple binomial equations together, as $x - 2 \propto 0$, and $x - 3 \propto 0$, and $x - 4 \propto 0$, producing $x^3 - 9xx + 26x - 24 \propto 0$. He here remarks that equations may ſometimes have their roots *falſe*, or what we call negative, which he oppoſes to thoſe that are poſitive, or as he calls them *true*, as Cardan had done before. As a natural deduction from the generation or compoſition of equations, by multiplication, he infers their reſolution, or depreſſion, or decompoſition, namely, dividing them by the binomial factors which were multiplied to produce the equation : and he obſerves that by this operation it is known that this diviſor is one of the binomial roots, and that there can be no more roots than dimenſions, or than thoſe which form with the unknown letter *x*, bino-

x, binomials that will exactly divide the equation, as Harriot had ſhewn before. Deſcartes adverts to ſeveral other properties, moſtly known before, which he has occaſion to make uſe of in the progreſs of his work; ſuch as, that equations may have as many true roots as the terms have changes of the ſigns + and −, and as many falſe ones as ſucceſſions of the ſame ſigns: which number and nature of the roots had before been partly ſhewn by Cardan and Vieta, from the relation of the coefficients, and their ſigns, and more fully by Harriot in his 5th ſection. And hence Deſcartes infers the method of changing the true roots to falſe, and the falſe to true, namely by changing the ſigns of the even terms only, as Cardan had taught before. Deſcartes then adverts to other reductions and tranſmutations which had been taught by Cardan, Vieta, and Harriot, ſuch as, To increaſe or diminiſh the roots by any quantity; To take away the 2d term: To alter the roots in any proportion, and thence to free the equation from fractions and radicals.

Deſcartes next remarks that the roots of equations, whether true or falſe, may be either real or imaginary; as in the equation $x^3 - 6xx + 13x - 10 \propto 0$, which has only one real root, namely 2. The imaginary roots were firſt noticed by Albert Girard, as before mentioned. He then treats of the depreſſion of a cubic equation to a quadratic, or plane problem, that it may be conſtructed by the circle, by dividing it by ſome one of the binomial factors, which, in Harriot's way, compoſe the equation. Peletarius having ſhewn that the ſimple root is one of the diviſors of the known term of the equation, and Harriot that that term is the continual product of all the roots, Deſcartes therefore tries all the ſimple diviſors of that term, till he finds one of them which, connected with the unknown letter x, by + or −, will exactly divide the equation. And the proceſs is the ſame for higher powers than the cube. But when a diviſor cannot be thus found, for depreſſing a biquadratic equation to a cubic, he gives another rule, which is a new one, for diſſolving it into two quadratics, by means of a cubic equation, in this manner:

Let the given biqu. be $+\ x^4 * \cdot pxx \cdot qx \cdot r \propto 0$;

$$\text{Which ſuppoſe equal to the product of theſe two}\left\{\begin{array}{l} +xx - yx + \frac{1}{2}yy \cdot \frac{1}{2}p \cdot \dfrac{q}{2y} \propto 0, \\ +xx + yx + \frac{1}{2}yy \cdot \frac{1}{2}p \cdot \dfrac{q}{2y} \propto 0; \end{array}\right.$$

where the ſign of $\frac{1}{2}p$ in the two quadratics muſt be the ſame as the ſign of p in the given equation, and in the 1ſt quadratic the ſign of $\dfrac{q}{2y}$ muſt be the ſame as the the ſign of q, but in the 2d quadratic the contrary. Then if there be found the root yy of this cubic equation $y^6 \cdot 2py^4 + \begin{smallmatrix} +pp \\ \cdot 4r \end{smallmatrix} yy - qq \propto 0$,

where the ſign of $2p$ is the ſame as of p in the given biquadratic, but the ſign of $4r$ contrary to that of r in the ſame: Then the value of y, hence deduced, being ſubſtituted for it in the two quadratic equations, and their two pairs of roots taken, they will be the four roots of the propoſed biquadratic. And thus alſo, he hints, may equations of the 6th power be reduced to thoſe of the 5th, and thoſe of the 8th power to thoſe of the 7th, and ſo on. Deſcartes does not give the inveſtigation of this rule; but it has evidently been done, by aſſuming indeterminate quantities, after the manner of Ferrari and Cardan, as coefficients of the terms of the two quadratic equations, and, after multiplying the two together, determining their values by comparing the reſulting terms with thoſe of the propoſed biquadratic equation.

After theſe reductions, which are only mentioned for the ſake of the geometrical conſtructions which follow, by ſimplifying and depreſſing the equations as much as they will admit, Deſcartes then gives the conſtruction of ſolid and other higher problems, or of cubic and higher equations, by means of parabolas and circles; where he obſerves that the falſe roots are denoted by the ordinates to the parabola lying on the contrary ſide of the axis to the true roots. Finally, theſe conſtructions are illuſtrated by various problems concerning the triſecting of an angle, and the finding of two or four mean proportionals; which concludes this ingenious work.

From the foregoing analyſis may eaſily be collected the real inventions and improvements made in algebra by Deſcartes. His work, as has been obſerved before, is not algebra itſelf, but the application of algebra to geometry, and the algebraical doctrine of curve lines, expreſſing and explaining their nature by algebraical equations, and on the contrary, conſtructing and explaining equations by means of the curve lines. What reſpects the geometrical parts of this tract we ſhall have occaſion to advert to elſewhere; and therefore ſhall here only enumerate the circumſtances which belong more peculiarly to the ſcience of Algebra, which I ſhall diſtinguiſh into the two heads of improvements and inventions. And

1ſt. Of his improvements. That he might fit equations the better for their application in the conſtruction of problems, Deſcartes mentions, as it were by-the-bye, many things concerning the nature and reduction of equations, without troubling himſelf about the firſt inventors of them, ſtating them in his own terms and manner, which is commonly more clear and explicit, and often with improvements of his own. And under this head we find that he chiefly followed Cardan, Vieta, and Harriot, but eſpecially the laſt, and explains ſome of their rules and diſcoveries more diſtinctly, and varies but a little in the notation, putting the firſt letters of the alphabet for the known, and the latter letters for the unknown quantities; alſo x^3 for aaa, &c; and $\propto$ for =. But Herigone uſed the numeral exponents in the ſame manner two years before. Deſcartes explained or improved moſt parts of the reductions of equations, in their various tranſmutations, the number and nature of their roots, true and falſe, real and what he calls imaginary, called involved by Girard; and the depreſſion of equations to lower degrees.

2d. As to his inventions and diſcoveries in algebra, they may be comprehended in theſe particulars, namely, the application of algebra to the geometry of curve lines, the conſtructing equations of the higher orders, and a rule for reſolving biquadratic equations by means of a cubic and two quadratics.

Having now traced the ſcience of Algebra from its origin and rude ſtate, down to its modern and more poliſhed form, in which it has ever ſince continued, with very little variation; having analyſed all or moſt of the principal authors, in a chronological order, and deduced the inventions and improvements made by each of them; from this time the authors both become too numerous, and their improvements too inconſiderable, to merit a detail in the ſame minute and circumſtantial way: and beſides, theſe will be better explained in a particular manner under the word or article to which each of them ſeverally belongs. It may therefore now ſuffice to enumerate, or announce only in a curſory manner, the chief improvements and authors on algebra down to the preſent time.

After the publication of the Geometry of Deſcartes, a great many other ingenious men followed the ſame courſe, applying themſelves to algebra and the new geometry, to the mutual improvement of them both; which was done chiefly by reaſoning on the nature and forms of equations, as generated and compoſed by Harriot. Before proceeding upon theſe however, it is but proper to take notice here of Fermat, a learned and ingenious mathematician, who was contemporary and a competitor of Deſcartes for his brighteſt diſcoveries, which he was in poſſeſſion of before the geometry of Deſcartes appeared. Namely, the application of algebra to curve lines, which he expreſſed by an algebraical equation, and by them conſtructing equations of the 3d and 4th orders; alſo a method of tangents, and a method de maximis et minimis, which approach very near to the method of Fluxions or Increments, which they ſtrikingly reſemble both in the manner of treating the problems, and in the algebraic notation and proceſs. The particulars of which, ſee under their proper heads. Beſides theſe, Fermat was deeply learned in the Diophantine problems, and the beſt edition of Diophantus's Arithmetic, is that which contains the notes of Fermat on that ingenious work.

But to return to the ſucceſſors of Deſcartes. His geometry having been publiſhed in Holland, ſeveral learned and ingenious mathematicians of that country, preſently applied themſelves to cultivate and improve it; as Schooten, Hudde, Van-Heuraet, De Witte, Sluſius, Huygens, &c; beſides M. de Beaune, and perhaps ſome others in France.

Francis Schooten, profeſſor of mathematics in the univerſity of Leyden, was one of the firſt cultivators of the new geometry. He tranſlated Deſcartes's Geometry out of French into Latin, and publiſhed it in 1649, with his commentary upon it, as alſo Brief Notes of M. de Beaune; both of them containing many ingenious and uſeful things. And in 1659 he gave a new edition of the ſame in two volumes, with the addition of ſeveral other ingenious pieces: as two poſthumous tracts of de Beaune, the one on the nature and conſtitution, the other on the limits of equations, ſhewing how to aſſign the limits between which are contained the greateſt and leaſt roots of equations, extended and completed by Eraſmus Bartholine: two letters of M. Hudde on the reduction of equations, and on the maxima and minima of quantities, containing many ingenious rules; among which are ſome concerning the drawing of tangents, and on the equal roots of equations, which he determines by multiplying the terms of the equation by the terms of any arithmetical progreſſion, 0 being one of the terms, the equation is commonly depreſſed one degree lower: alſo a tract of Van Heuraet on the rectification of curve lines; the elements of curves by De Witte; Schooten's principles of univerſal mathematics, or introduction to Deſcartes's geometry, which had before been publiſhed by itſelf in 1651; and to the end of the work is added a poſthumous piece of Schooten's (for he died while the 2d vol. was printing) intitled *Tractatus de concinnandis demonſtrationibus geometricis ex calculo algebraico.* Schooten alſo publiſhed, in 1657, *Exercitationes Mathematicæ*, in which are contained many curious algebraical and analytical pieces, amongſt others of a geometrical nature.

An elaborate commentary on Deſcartes's Geometry was alſo publiſhed by F. Rabuel, a Jeſuit; and James Bernoulli, enriched with notes, an edition of the ſame, printed at Baſil in 169—.

The celebrated Huygens alſo, among his great diſcoveries, very much cultivated the algebraical analyſis: and he is often cited by Schooten, who relates divers inventions of his, while he was his pupil.

Sluſius, a canon of Liege, publiſhed in 1659, *Meſolabum, ſeu duæ mediæ propor. per circulum & ellips. vel hyperb. infinitis modis exhibitæ*; by which, any ſolid problem may be conſtructed by infinite different ways. And in 1668 he gave a ſecond edition of the ſame, with the addition of the analyſis, and a miſcellaneous collection of curious and important problems, relating to ſpirals, centres of gravity, maxima and minima, points of inflexion, and ſome Diophantine problems; all ſhewing him deeply ſkilled in Algebra and Geometry.

There have been a great number of other writers and improvers of Algebra, of which it may ſuffice ſlightly to mention the chief part, as in the following catalogue.

Peter Nonius, or Nunez, a Spaniard, wrote about the time of Cardan, or ſoon after.

In 1619 ſeveral pieces of Van Collen, or Ceulen, were tranſlated out of Dutch into Latin, and publiſhed at Leyden by W. Snell; among which are contained a particular treatiſe on ſurds, and his proportion of the circumference of a circle, to its diameter.

In 1621 Bachet publiſhed, in Greek and Latin, an edition of Diophantus, with many notes. And another edition of the ſame was publiſhed in 1670, with additions by Fermat.

In 1624 Bachet's *Problemes Plaiſans et Delectables*, being curious problems in mathematical recreations.

In 1634 Herigone publiſhed, at Paris, the firſt courſe of mathematics, in 5 vols. 8vo; in the 2d of which is contained a good treatiſe on Algebra; in which he uſes the notation by ſmall letters, introduced by the Algebra of Harriot, which was publiſhed three years before, though the reſt of it does not reſemble that work, and one would ſuſpect that Herigone had not ſeen it. The whole of this piece bears evident marks of originality and ingenuity. Beſides + for *plus*, he uſes ∽ for *minus*, and | for *equality*, with ſeveral other uſeful abbrevations and marks of his own. In the notation of powers, he does not repeat the letters like Harriot, but ſubjoins the numeral exponents, to the letter, as Deſcartes did two years afterwards. And Herigone uſes the ſame numeral exponents for roots, as $\surd 3$ for the cube root.

In 1635 Cavalerius publiſhed his *Indiviſibles*; which proved a new æra in analytics, and gave riſe to other new modes of computation in analytics.

About 1640, et ſeq. Roberval made ſeveral notable improvements in analytics, which are publiſhed in the early volumes of the Memoirs of the Academy of Sciences; as, 1. A tract on the compoſition of motion, and a method of tangents. 2, *De recognitione æquationum.* 3, *De geometrica planarum & cubicarum æquationum reſolutione.* 4, A treatiſe on indiviſibles, &c.

In 1643 De Billy publiſhed *Nova Geometriæ Clavis Algebra.* And in 1670 *Diophantus Redivivus.* He was an author particularly well ſkilled in Diophantine problems.

In 1644 Renaldine publiſhed, in 4to, *Opus Algebraicum,* both ancient and modern, with mathematical reſolution and compoſition. And in 1665, in folio, the ſame, greatly enlarged, or rather a new work, which is very heavy and tedious. In this work Renaldine uſes the parentheſes $(a^2 + b^2)$ as a vinculum, inſtead of the line over, as $\overline{a^2 + b^2}$.

In 1655 was publiſhed Wallis's *Arithmetica Infinitorum,* being a new method of reaſoning on quantities, or a great improvement on the Indiviſibles of Cavalerius, and which in a great meaſure led the way to infinite ſeries, the binomial theorem, and the method of fluxions. Wallis here treats ingeniouſly of quadratures and many other problems, and gives the firſt expreſſion for the quadrature of the circle by an infinite ſeries. Another ſeries is here added for the ſame purpoſe, by the Lord Brouncker.

In 1659 was publiſhed *Algebra Rhonii Germanice;* which was in 1668, tranſlated into Engliſh by Mr. Thomas Brancker, with additions and alterations by Dr. John Pell.

In 1661 was publiſhed in Dutch, a neat piece of Algebra by Mr. Kinckhuyſen; which Sir I. Newton, while he was profeſſor of mathematics at Cambridge, made uſe of and improved, and he meant to republiſh it, with the addition of his method of fluxions and infinite ſeries; but he was prevented by the accidental burning of ſome of his papers.

In 1665 or 1666 Sir Iſaac Newton made ſeveral of his brighteſt diſcoveries, though they were not publiſhed till afterwards: ſuch as the binomial theorem; the method of fluxions and infinite ſeries; the quadrature, rectification, &c of curves; to find the roots of all ſorts of equations, both numeral and literal, in infinite converging ſeries; the reverſion of ſeries, &c. Of each of which a particular account may be ſeen in their proper articles.

In 1666 M. Frenicle gave ſeveral curious tracts concerning combinations, magic ſquares, triangular numbers, &c; which were printed in the early volumes of Memoirs of the Academy of Sciences.

In 1668 Thomas Brancker publiſhed a tranſlation of Rhonius's Algebra, with many additions by Dr. John Pell, who uſed a peculiar method of regiſtering the ſteps in any algebraical proceſs, by means of marks and abbreviations in a ſmall column drawn down the margin, by which each line, or ſtep, is clearly explained, as was before done by Harriot in words at length.

In 1668 Mercator publiſhed his *Logarithmotechnia,* or method of conſtructing logarithms; in which he gives the quadrature of the hyperbola, by means of an infinite ſeries of algebraical terms, found by dividing a ſimple algebraic quantity by a compound one, and for the firſt time that this operation was given to the public, though Newton had before that expanded all ſorts of compound algebraical quantities into infinite ſeries.

In the ſame year was publiſhed James Gregory's *Exercitationes Geometricæ,* containing, among other things, a demonſtration of Mercator's quadrature of the hyperbola, by the ſame ſeries.

And in the ſame year was publiſhed, in the Philoſophical Tranſactions, Lord Brouncker's quadrature of the hyperbola by another infinite ſeries of ſimple rational terms, which he had been in poſſeſſion of ſince the year 1657, when it was announced to the public by Dr. Wallis. Lord Brouncker's ſeries for the quadrature of the circle, had been publiſhed by Wallis in his Arithmetic of Infinites.

In 1669 Dr. Iſaac Barrow publiſhed his Optical and Geometrical Lectures, abounding with profound reſearches on the dimenſions and properties of curve lines; but particularly to be noticed here for his method of tangents, by a mode of calculation ſimilar to that of Fluxions, or Increments, from which theſe differ but little, except in the notation.

In 1673 was publiſhed, in 2 vols. folio, *Elements of Algebra,* by John Kerſey; a very ample and complete work, in which Diophantus's problems are fully explained.

In 1675 were publiſhed Nouveaux Elemens des Mathematiques, par J. Preſtet, prêtre: a prolix and tedious work, which he preſumptuouſly dedicated to God Almighty.

About 1677 Leibnitz diſcovered his *Methodus Differentialis,* or elſe made a variation in Newton's Fluxions, or an extenſion of Barrow's method, for it is not certain which. He gave the firſt inſtance of it in the Leipſic Acts for the year 1684. He alſo improved infinite ſeries, and gave a ſimple one for the quadrature of the circle, in the ſame acts for 1682.

In 1682 Iſmael Bulliald publiſhed, in folio, his *Opus Novum ad Arithmeticam Infinitorum,* being a large amplification of Wallis's Arithmetic of Infinites.

In 1683 Tſchirnauſen gave a memoir, in the Leipſic Acts, concerning the extraction of the roots of all equations in a general way; in which he promiſed too much, as the method did not ſucceed.

In 1684 came out, in Engliſh and Latin, 4to, Thomas Baker's *Geometrical Key, or Gate of Equations Unlock'd;* being an improvement of Deſcartes's conſtruction of all equations under the 5th degree, by means of a circle and only one and the ſame parabola for all equations, uſing any diameter inſtead of the axis of the parabola.

In 1685 was publiſhed, in folio, Wallis's *Treatiſe of Algebra, both Hiſtorical and Practical,* with the addition of ſeveral other pieces; ſhewing the origin, progreſs, and advancement of that ſcience, from time to time. It cannot be denied that, in this work, Wallis has ſhewn too much partiality to the Algebra of Harriot. Yet, on the other hand, it is as true, that M. de Gua, in his account of it, in the Memoirs of the Academy of Sciences for 1741, has run at leaſt as far into the ſame extreme on the contrary ſide, with reſpect to the diſcoveries of Vieta; and both theſe I believe from the ſame cauſe, namely, the want of examining the works of all former writers on Algebra, and ſpecifying their ſeveral diſcoveries; as has been done in the courſe of this article.

In 1687 Dr. Halley gave, in the Philoſ. Tranſ. the conſtruction of cubic and biquadratic equations, by a parabola and circle; with improvements on what had been done by Deſcartes, Baker, &c. Alſo, in the ſame Tranſactions, a memoir on the number of the roots of equations, with their limits and ſigns.

In 1690 was publiſhed, in 4to, by M. Rolle, *Traité d'Algèbre;* in 1699 *Une Methode pour la Reſolution des Problemes indeterminés;* and in 1704 *Memoires ſur l'inverſe des tangents;* and other pieces.

In 1690 Joſeph Raphſon publiſhed *Analyſis Æquationum Univerſalis;* being a general method of approximating to the roots of equations in numbers. And in 1715 he publiſhed the *Hiſtory of Fluxions*, both in Engliſh and Latin.

In 1690 was alſo publiſhed, in 4 vols 4to, Dechale's *Curſus ſeu mundus mathematicus;* in which is a piece of algebra, of a very old-faſhioned ſort, conſidering the time when it was written.

About 1692, and at different times afterwards, De Lagny publiſhed many pieces on the reſolution of equations in numbers, with many theorems and rules for that purpoſe.

In 1693 was publiſhed, in a neat little volume, *Synopſis Algebraica, opus poſthumum Johannis Alexandri.*

In 1694, Dr. Halley gave, in the Philoſ. Tranſ. an ingenious tract on the numeral extraction of all roots, without any previous reduction. And this tract is alſo added to ſome editions of Newton's Univerſal Arithmetic.

In 1695 Mr. John Ward, of Cheſter, publiſhed, in 8vo, *A Compendium of Algebra*, containing plain, eaſy, and conciſe rules, with examples in an eaſy and clear way. And in 1706 he publiſhed the firſt edition of his *Young Mathematician's Guide*, or a plain and eaſy introduction to the mathematics: a book which is ſtill in great requeſt, eſpecially with beginners, and which has been ever ſince the ordinary introduction of the greateſt part of the mathematicians of this country.

In 1696 the Marquis de l'Hôpital publiſhed his *Analyſe des infiniment petits*. And gave ſeveral papers to the Leipſic Acts and the Memoires of the Academy of Sciences. He left behind him alſo an ingenious treatiſe, which was publiſhed in 1707, intitled *Traité analytique des Sections Coniques, et de la conſtruction des lieux geometriques.*

In 1697, and ſeveral other years, Mr. Ab. Demoivre gave various papers, in the Philoſ. Tranſ. containing improvements in Algebra: viz. in 1697, A method of raiſing an infinite multinomial to any power, or extracting any root of the ſame. In 1698, The extraction of the root of an infinite equation. In 1707, Analytical ſolution of certain equations of the 3d, 5th, 7th, &c degree. In 1722, Of algebraic fractions and recurring ſeries. In 1738, The reduction of radicals into more ſimple forms. Alſo in 1730, he publiſhed *Miſcellanea analytica de ſeriebus & quadraturis*, containing great improvements in ſeries, &c.

In 169 , Mr. Richard Sault publiſhed, in 4to, *A New Treatiſe of Algebra, apply'd to numeral queſtions and geometry. With a converging ſeries for all manner of Adfected Equations.* The ſeries here alluded to, is Mr. Raphſon's method of approximation, which had been lately publiſhed.

In 1699 Hyac. Chriſtopher publiſhed at Naples, in 4to, *De conſtructione æquationum.*

In 1702 was publiſhed Ozanam's Algebra; which is chiefly remarkable for the Diophantine analyſis. He had publiſhed his mathematical dictionary in 1691, and in 1693 his courſe of mathematics, in 5 vols 8vo, containing alſo a piece of algebra.

In 1704, Dr. John Harris publiſhed his *Lexicon Technicum*, the firſt dictionary of arts and ſciences: a very plain and uſeful book, eſpecially in the mathematical articles. And in 1705 a neat little piece on algebra and fluxions.

In 1705 M. Guiſnée publiſhed, in 4to, his *Application de l'algebre a la geometrie:* a uſeful book.

In 1706 Mr. William Jones publiſhed his *Synopſis Palmariorum Matheſeos*, or a new introduction to the mathematics: a very uſeful compendium in the mathematical ſciences. And in 1711 he publiſhed, in 4to, a collection of Sir Iſaac Newton's papers, intitled *Analyſis per quantitatum ſeries, fluxiones, ac differentias: cum enumeratione linearum tertii ordinis.*

In 1707 was publiſhed by Mr. Whiſton, the firſt edition of Sir Iſaac Newton's *Arithmetica Univerſalis: ſive de compoſitione et reſolutione arithmetica liber:* and many editions have been publiſhed ſince. This work was the text book uſed by our great author in his lectures, while he was profeſſor of mathematics in the univerſity of Cambridge. And although it was never intended for publication, it contains many and great improvements in analytics; particularly in the nature and tranſmutation of equations; the limits of the roots of equations; the number of impoſſible roots; the invention of diviſors, both ſurd and rational; the reſolution of problems, arithmetical and geometrical; the linear conſtruction of equations; approximating to the roots of all equations, &c. To the later editions of the book is commonly ſubjoined Dr. Halley's method of finding the roots of equations. As the principal parts of this work are not adapted to the circumſtance of beginners, there have been publiſhed commentaries upon it by ſeveral perſons, as s'Graveſande, Caſtilion, Wilder, &c.

In 1708 M. Reyneau publiſhed his *Analyſe Demontrée*, in 2 vols 4to. And in 1714 *La Science du Calcul, &c.*

In 1709 was publiſhed an Engliſh tranſlation of Alexander's algebra. With an ingenious appendix by Humphry Ditton.

In 1715 Dr. Brooke Taylor publiſhed his *Methodus Incrementorum:* an ingenious and learned work. And in the Philoſ. Tranſ. for 1718, An improvement of the method of approximating to the roots of equations in numbers.

In 1717 M. Nicole gave, in the memoirs of the academy of ſciences, a tract on the calculation of finite differences. And in ſeveral following years, he gave various other tracts on the ſame ſubject, and on the reſolution of equations of the 3d degree, and particularly on the irreducible caſe in cubic equations.

Alſo in 1717 was publiſhed a treatiſe on Algebra by Philip Ronayne.

Alſo in 1717 Mr. James Sterling publiſhed *Lineæ tertii Ordinis*; an ingenious work, containing good improvements in analytics. Alſo in 1730 *Methodus Differentialis: ſive tractatus de ſummatione et interpolatione ſerierum infinitarum:* with great improvements on infinite ſeries.

In 1726 and 1729 Maclaurin gave, in the Philoſ. Tranſ. tracts on the imaginary roots of equations. And afterwards was publiſhed, from his poſthumous papers, his treatiſe on Algebra, with its application to curve lines.

In 1727 came out s'Graveſande's Algebra, with a ſpecimen of a commentary on Newton's univerſal arithmetic.

In 1728 Mr. Campbell gave, in the Philoſ. Tranſ. an ingenious paper on the number of impoſſible roots of equations.

In 1732 was publiſhed Wolfius's Algebra, in his courſe of mathematics, in 5 vols. 4to.

In 1735 Mr. John Kirkby publiſhed his arithmetic and algebra. And in 1748 his doctrine of ultimators.

In 1740 were publiſhed Mr. Thomas Simpſon's Eſſays; in 1743 his Diſſertations, and in 1757 his Tracts; in all which are contained ſeveral improvements in ſeries and other parts of Algebra. As alſo in his algebra, firſt printed in 1745, and in his Select Exerciſes, in 1752.

Alſo in 1740 was publiſhed profeſſor Saunderſon's Elements of Algebra, in 2 vols. 4to.

In 1741 M. de la Caille publiſhed *leçons de mathematiques; ou elemens d'algebre & de geometrie.*

Alſo in 1741, in the memoirs of the academy of ſciences, were given two articles by M. de Gua, on the number of poſitive, negative, and imaginary roots of equations. With an hiſtorical account of the improvements in Algebra; in which he ſeverely cenſures Wallis for his partiality; a circumſtance in which he himſelf is not leſs faulty.

In 1746 M. Clairaut publiſhed his Elemens d'algebre, in which are contained ſeveral improvements, eſpecially on the irreducible caſe in cubic equations. He has alſo ſeveral good papers on different parts of analytics, in the memoirs of the academy of ſciences.

In 1747 M. Fontaine gave, in the memoirs of the academy of ſciences, a paper on the reſolution of equations. Beſides ſome analytical papers in the memoirs of other years.

In 1761 M. Caſtillion publiſhed, in 2 vols 4to, Newton's univerſal arithmetic, with a large commentary.

In 1763 Mr. Emerſon publiſhed his Increments. In 1764 his Algebra, &c.

In 1764 Mr. Landen publiſhed his Reſidual Analyſis. In 1765 his Mathematical Lucubrations. And in 1780 his Mathematical Memoirs. All containing good improvements in infinite ſeries, &c.

In 1770 was publiſhed, in the German language, Elements of Algebra by M. Euler. And in 1774 a French tranſlation of the ſame. The memoirs of the Berlin and Peterſburgh academies alſo abound with various improvements on ſeries and other branches of analyſis by this great man.

In 1775 was publiſhed at Bologna, in 2 vols 4to, *Compendio d'Analiſi di* Girolamo Saladini.

Beſides the foregoing, there have been many other authors who have given treatiſes on Algebra, or who have made improvements on ſeries and other parts of Algebra; as Schonerus, Coignet, Salignac, Laloubere, Hemiſchius, Degraave, Meſcher, Heniſchius, Roberval, the Bernoullis, Malbranche, Agneſi, Wells, Dodſon, Manfredi, Regnault, Rowning, Maſeres, Waring, Lorgna, de la Grange, de la Place, Bertrand, Kuhnius, Hales, and many others.

Algebra, *numeral*, is that which is chiefly concerned in the ſolution of numeral problems, and in which all the given quantities are expreſſed by numbers only. As uſed by the more early authors, Diophantus, Paciolus, Stifelius, &c.

Algebra, *ſpecious*, or *literal*, is that commonly uſed by the moderns, in which all the quantities, both known and unknown, are repreſented or expreſſed by ſpecies or general characters, as the letters of the alphabet, &c; in conſequence of which general deſignation, all the concluſions become univerſal theorems for performing every operation of the like kind. There are ſpecimens of this method from Cardan and others about his time, but it was more generally employed and introduced by Vieta.

Algebraical, ſomething relating to *algebra*.

Thus we ſay *algebraical* ſolutions, curves, characters or ſymbols, &c.

Algebraical *Curve*, is a curve in which the general relation between the abſciſſes and ordinates can be expreſſed by a common algebraical equation.

Theſe are alſo called *geometrical lines* or *curves*, in contradiſtinction to *mechanical* or *tranſcendental* ones.

ALGEBRAIST, a perſon ſkilled in *algebra*.

ALGENEB, or Algenib, a fixed ſtar of the ſecond magnitude, on the right ſide of Perſeus.

ALGOL, or *Meduſa's Head*, a fixed ſtar of the third magnitude, in the conſtellation Perſeus.

ALGORAB, a fixed ſtar of the third magnitude, in the right wing of the conſtellation Corvus.

ALGORISM, or Algorithm, is ſimilar to logiſtics, ſignifying the art of computing in any particular way, or about ſome particular ſubject; or the common rules of computing in any art. As the *algorithm* of numbers, of algebra, of integers, of fractions, of ſurds, &c; meaning the common rules for performing the operations of arithmetic, or algebra, or fractions, &c.

ALHAZEN, Allacen, or Abdilazum, was a learned Arabian, who lived in Spain about the year 1100, according to his editor Riſner, and Weidler. He wrote upon Aſtrology; and his work upon Optics was printed, in Latin, at Baſil, in 1572, under the title of *Opticæ Theſaurus*, by Riſner. Alhazen was the firſt who ſhewed the importance of refractions in aſtronomy, ſo little known to the ancients. He is alſo the firſt author who has treated on the twilight, upon which he wrote a work, in which he alſo ſpeaks of the height of the clouds.

ALIDADE, an Arabic name for the label, index, or ruler, which is moveable about the centre of an aſtrolabe, quadrant, &c, and carrying the ſights or teleſcope, and by which are ſhewn the degrees cut off the limb or arch of the inſtrument.

ALIQUANT *part*, is that part which will not exactly meaſure or divide the whole, without leaving ſome remainder. Or the *aliquant* part is ſuch, as being taken or repeated any number of times, does not make up the whole exactly, but is either greater or leſs than it. Thus 4 is an aliquant part of 10; for 4 twice taken makes 8 which is leſs than 10, and three times taken makes 12 which is greater than 10.

ALIQUOT *part*, is ſuch a part of any whole, as will exactly meaſure it without any remainder. Or the *aliquot* part is ſuch, as being taken or repeated a certain number of times, exactly makes up, or is equal to the whole. So 1 is an aliquot part of 6, or of any other whole number; 2 is alſo an aliquot part of 6, being contained juſt 3 times in 6; and 3 is alſo an aliquot part of 6, being contained juſt 2 times: ſo that all the aliquot parts of 6 are 1, 2, 3.

All the *aliquot* parts of any number may be thus found: Divide the given number by its leaſt diviſor; then divide the quotient alſo by its leaſt diviſor; and ſo on always dividing the laſt quotient by its leaſt diviſor, till the quotient 1 is obtained; and all the diviſors, thus taken, are the prime aliquot parts of the given number. Then multiply continually together theſe prime diviſors, viz. every two of them, every three of them, every four of them, &c; and the products will be the other or compound *aliquot* parts of the given number. So if the *aliquot* parts of 60 be required; firſt divide it by 2, and the quotient is 30: then 30 divided by 2 alſo, gives 15, and 15 divided by 3 gives 5, and 5 divided by 5 gives 1: ſo that all the prime diviſors or aliquot parts are 1, 2, 2, 3, 5. Then the compound ones, by multiplying every two, are 4, 6, 10, 15; and every three 10, 20, 30. So that all the *aliquot* parts of the given number 60, are 1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30.—In like manner it will be found that all the *aliquot* parts of 360 are 1, 2, 3, 4, 5, 6, 8, 9, 10, 12, 15, 18, 20, 24, 30, 36, 40,
90 45, 60, 72, 180.

ALLEN (THOMAS) a celebrated mathematician of the 16th century. He was born at Uttoxeter in Staffordſhire, in 1542; was admitted a ſcholar of Trinity college, Oxford, in 1561; where he took his degree of maſter of arts in 1567. In 1570 he quitted his college and fellowſhip, and retired to Gloceſter hall, where he ſtudied very cloſely, and became famous for his knowledge in antiquity, philoſophy and mathematics. He received an invitation from Henry earl of Northumberland, a great friend and patron of the mathematicians, and he ſpent ſome time at the earl's houſe; where he became acquainted with thoſe celebrated mathematicians Thomas Harriot, John Dee, Walter Warner, and Nathaniel Torporley. Robert earl of Leiceſter, too, had a great eſteem for Allen, and would have conferred a biſhopric upon him; but his love for ſolitude and retirement made him decline the offer. His great ſkill in the mathematics gave occaſion to the ignorant and vulgar to look upon him as a magician or conjurer. Allen was very curious and indefatigable in collecting ſcattered manuſcripts relating to hiſtory, antiquity, aſtronomy, philoſophy, and mathematics: which collections have been quoted by ſeveral learned authors, and mentioned as in the Bibliotheca Alleniana. He publiſhed in Latin the ſecond and third books of Ptolemy, *Concerning the Judgment of the Stars*, or, as it is uſually called, of the *quadripartite conſtruction*, with an expoſition. He wrote alſo notes on many of Lilly's books, and ſome on John Bale's work, *De ſcriptoribus Maj. Britanniæ*. He died at Gloceſter hall in 1632, being 90 years of age.

Mr. Burton, the author of his funeral oration, calls him the very ſoul and ſun of all the mathematicians of his age. And Selden mentions him as a perſon of the moſt extenſive learning and conſummate judgment, the brighteſt ornament of the univerſity of Oxford. Alſo Camden ſays he was ſkilled in moſt of the beſt arts and ſciences. A. Wood has alſo tranſcribed part of his character from a manuſcript in the library of Trinity college, in theſe words: "He ſtudied polite literature with great application; he was ſtrictly tenacious of academic diſcipline, always highly eſteemed both by foreigners and thoſe of the univerſity, and by all of the higheſt ſtations in the church of England, and the univerſity of Oxford. He was a ſagacious obſerver, an agreeable companion, &c."

ALLIGATION, one of the rules in arithmetic, by which are reſolved queſtions which relate to the compounding or mixing together of divers ſimples or ingredients, being ſo called from *alligare*, to tie or connect together, probably from certain vincula, or crooked ligatures, commonly uſed to connect or bind the numbers together.

It is probable that this rule came to us from the Mooriſh or Arabic writers, as we find it, with all the other rules of arithmetic, in *Lucas de Burgo*, and the other early authors in Europe.

Alligation is of two kinds, *medial* and *alternate*.

ALLIGATION *medial* is the method of finding the rate or quality of the compoſition, from having given the rates and quantities of the ſimples or ingredients.

The rule of operation is this: multiply each quantity by its rate, and add all the products together; then divide the ſum of the products by the ſum of the quantities, or whole compound, and the quotient will be the rate ſought.

For example, Suppoſe it were required to mix together 6 gallons of wine, worth 5s. a gallon; 8 gallons, worth 6s. the gallon; and 4 gallons, worth 8s. the gallon; and to find the worth or value, per gallon, of the whole mixture.

	Gal.		s.		products.
Here	6	mult. by	5	gives	30
	8	- by	6	-	48
	4	- by	8	-	32
whole comp.	18				110 ſum of prod.

Then 18) 110 ($6\frac{2}{18}$ or $6\frac{1}{9}$s. is the rate ſought.
108
2

ALLIGATION *alternate* is the method of finding the quantities of ingredients or ſimples, neceſſary to form a compound of a given rate.

The *rule* of operation is this: 1ſt, Place the given rates of the ſimples in a column, under each other; noting which rates are leſs, and which are greater than the propoſed compound. 2d, Connect or link with a crooked line, each rate which is leſs than the propoſed compound rate, with one or any number of thoſe which are greater than the ſame; and every greater rate with one or any number of the leſs ones. 3d, Take the difference between the given compound rate and that of each ſimple rate, and ſet this difference oppoſite every rate with which that one is linked. 4th, Then if only one difference ſtand oppoſite any rate, it will be the quantity belonging to that rate; but when there are ſeveral differences to any one, take their ſum for its quantity.

For example, Suppoſe it were required to mix together gold of various degrees of fineneſs, viz. of 19, of 21, and of 23 caracts fine, ſo that the mixture ſhall be of 20 caracts fine. Hence,

$$\begin{array}{lllll} & \text{Rates} & \text{Diffs.} & & \text{Sum of Diffs.} \\ \text{Comp. rate } 20 & \left\{\begin{array}{l}21\\19\\23\end{array}\right) & \begin{array}{l}\text{- - - }1\\ \text{- - - }1\\ \text{- - - }1\end{array} & +3 & \left\{\begin{array}{l}\text{1 of 21 caracts fine,}\\ \text{4 of 19 caracts fine,}\\ \text{1 of 23 caracts fine.}\end{array}\right.\end{array}$$

That is, there muſt be an equal quantity of 21 and 23 caracts fine, and 4 times as much of 19 caracts fine.

Various limitations, both of the compound and the ingredients, may be conceived; and in ſuch caſes, the differences are to be altered proportionally.

Queſtions of this ſort are however commonly beſt and eaſieſt reſolved by common Algebra, of which they form a ſpecies of indeterminate problems, as they admit of many, or an indefinite number of anſwers.

There is recorded a remarkable inſtance of a diſcovery made by Archimedes, both by *alligation* and ſpecific gravity at the ſame time, namely, concerning the crown of Hiero, king of Syracuſe. The king had ordered a crown to be made of pure gold, but when brought to him, a ſuſpicion aroſe that it was mixed with alloy of either ſilver or copper, and the king recommended it to Archimedes to diſcover the cheat without defacing the crown. Archimedes, after long thinking on the matter, without lighting on the means of doing it, being one day in the bath, and obſerving how his body raiſed the water higher, conceived the idea that different metals of the ſame weight would occupy different ſpaces, and ſo raiſe or expel different quantities of water. Upon which he procured two other maſſes, each of the ſame weight with the crown, the one of pure gold, and the other of alloy; then immerſing them all three, ſeparately, in water, and obſerving the ſpace each occupied, by the quantity it raiſed the water, he from thence computed the quantities of gold and alloy contained in the crown.

ALLIOTH, a ſtar in the tail of the great bear. The word in Arabic denotes a horſe; and they gave this name to each of the three ſtars, in the tail of the great bear, as they are placed like three horſes, thus arranged for the purpoſe of drawing the waggon commonly called Charles's wain, repreſented by the four ſtars on the body of the ſame conſtellation.

ALMACANTAR. See ALMUCANTAR.

ALMAGEST, the name of a celebrated book compoſed by Ptolemy; being a collection of a great number of the obſervations and problems of the ancients, relating to geometry and aſtronomy, but eſpecially the latter. And being the firſt work of this kind which has come down to us, and containing a catalogue of the fixed ſtars, with their places, beſide numerous records and obſervations of eclipſes, the motions of the planets, &c, this work will ever be held dear and valuable to the cultivators of aſtronomy.

In the original Greek it is called συνταξις μεγιστη, the *great compoſition* or *collection*. And to the word μεγιστη, *megiſte*, the Arabians joined the particle *al*, and thence called it *Almagheſti*, or, as we call it, from them, the Almageſt.

Ptolemy was born about the year of Chriſt 69, and died in 147, and wrote this work, conſiſting of 13 books, at Alexandria in Egypt, where the Arabians found it on the capture of that kingdom. It was by them tranſlated out of Greek, into Arabic, by order of the caliph Almaimon, about the year 827; and firſt into Latin about 1230, by favour of the emperor Frederic II. The Greek text however was not known in Europe till about the beginning of the 15th century, when it was brought from Conſtantinople, then taken by the Turks, by *George*, a monk of Trabezond, who tranſlated it into Latin, which tranſlation has ſeveral times been publiſhed.

Riccioli, an Italian jeſuit, alſo publiſhed, in 1651, a body of Aſtronomy, which, in imitation of Ptolemy, he called *Almageſtum Novum*, the *New Almageſt*; being a large collection of ancient and modern obſervations and diſcoveries, in the ſcience of Aſtronomy.

ALMAMON, caliph of Bagdat, a philoſopher and aſtronomer in the beginning of the 9th century, he having aſcended the throne in the year 814. He was ſon of Harun Al-Raſhid, and grand-ſon of Almanſor. His name is otherwiſe written *Mamon*, *Almaon*, *Almamun*, *Alamoun*, or *Al-Maimon*. Having been educated with great care and with a love for the liberal ſciences, he applied himſelf to cultivate and encourage them in his own country. For this purpoſe he requeſted the Greek emperors to ſupply him with ſuch books on philoſophy as they had among them; and he collected ſkilful interpreters to tranſlate them into the Arabic language. He alſo encouraged his ſubjects to ſtudy them; frequenting the meetings of the learned, and aſſiſting at their exerciſes and deliberations. He cauſed Ptolemy's Almageſt to be tranſlated in 827, by Iſaac Ben-honain, and Thabet Ben-korah, according to Herbelot, but according to others by Sergius, and Alhazen, the ſon of Joſeph. In his reign, and doubtleſs by his encouragement, an aſtronomer of Bagdat, named Habaſh, compoſed three ſets of aſtronomical tables.

Almamon himſelf made many aſtronomical obſervations, and determined the obliquity of the ecliptic to be then 23° 35′ (or 23° 33′ in ſome manuſcripts), but Voſſius ſays 23° 51′ or 23° 34′. He alſo cauſed ſkilful obſervers to procure proper inſtruments to be made, and to exerciſe themſelves in aſtronomical obſervations; which they did accordingly at Shemaſi in the province of Bagdat, and upon Mount Caſius near Damas.

Under the auſpices of Mamon alſo a degree of the meridian was meaſured on the plains of Sinjar or Sindgiar (or according to ſome Fingar), upon the borders of the Red Sea; by which the degree was found to contain 56⅔ miles, of 4000 coudees each, the coudee being a foot and a half: but it is not known what foot is here meant, whether the Roman, the Alexandrian, or ſome other. Riccioli makes this meaſure of the degree amount to 81 ancient Roman miles, which value anſwers to 62046 French toiſes; a quantity more than the true value of the degree by almoſt one-third.

Finally, Mamon revived the ſciences in the Eaſt to ſuch a degree, that many learned men were found, not only in his own time, but after him, in a country where the ſtudy of the ſciences had been long forgotten. This learned king died near Tarſus in Cilicia, by having eaten too freely of ſome dates, on his return from a military expedition, in the year 833.

ALMANAC, a calendar or table, in which are ſet down and marked the days and feaſts of the year, the common eccleſiaſtical notes, the courſe and phaſes of the moon, &c, for each month: and anſwers to the *faſti* of the ancient Romans.

The etymology of the word is much controverted among grammarians.—Some derive it from the Arabic, viz, from the particle *al*, and *manah*, to count. While Scaliger, and others, derive it from the ſame *al*, and the Greek μανακος, the courſe of the months. But Golius controverts theſe opinions, and aſcribes the word to another origin, though he ſtill makes it of Arabic extract, which it more evidently is. He ſays that, in the Eaſt it is the cuſtom for the people, at the beginning of the year, to make preſents to their princes; and that, among the reſt, the aſtrologers preſent them with their almanacs, or ephemerides, for the year enſuing; whence theſe came to be called *almanha*, that is, new-year's gifts. But this derivation ſeems rather ſtrained and improbable; for, by the ſame rule, the gifts or productions of other artiſts, or claſſes of men, might alſo be called almanacs. There are other gueſſes at the etymology; and Verſtegan writes the word *almonac*, and makes it of German original. Our anceſtors, he obſerves, uſed to carve the courſes of the moon, for the whole year, upon a ſquare piece of wood, which they called *al-monaght*, which is as much as to ſay, in old Engliſh or Saxon, *all-moon-heed.*

Almanacs are of various kinds and compoſition, ſome books, others ſheets, &c, ſome annual, others perpetual. The eſſential part is the calendar of months, weeks, and days; the motions, changes, and phaſes of the moon; with the riſing and ſetting of the ſun and moon. To theſe are commonly added various matters, aſtronomical, aſtrological, chronological, meteorological, and even political, rural, medical, &c; as alſo eclipſes, ſolar ingreſſes, aſpects, and configurations of the heavenly bodies, lunations, heliocentric and geocentric motions of the planets, prognoſtications of the weather, and predictions of other events, the tides, twilight, equation of time, &c.

Till about the 4th century, almanacs bore the marks of heatheniſm only; from thence to the 7th century, they were a mixture of heatheniſm and chriſtianity; and ever ſince they have been altogether chriſtian: but at all times, aſtrological and other predictions have been conſidered as an eſſential part, and ſtill are ſo to this day with ſeveral of them, notwithſtanding that moſt people *affect* to diſbelieve in ſuch predictions.

Nautical ALMANAC, and *Aſtronomical Ephemeris*, is a kind of national almanac, chiefly for nautical purpoſes, which was begun in the year 1767 under the direction of the Board of Longitude, on the recommendation of the preſent worthy Aſtronomer Royal, who has the immediate conducting of it. It is ſtill publiſhed by anticipation for ſeveral years before hand, for the convenience of ſhips going out upon long voyages, for which it is highly uſeful, and was found eminently ſo in the courſe of the late voyages round the world for making diſcoveries. Beſides moſt things eſſential to general uſe, that are to be found in other almanacs, it contains many new and important particulars; more eſpecially, the diſtances of the moon from the ſun and fixed ſtars, which are computed for the meridian of the Royal Obſervatory of Greenwich, and ſet down to every three hours of time, expreſsly deſigned for computing the longitude at ſea, by comparing theſe with the like diſtances obſerved there.

ALMANAR, in the Arabian aſtrology, denotes the pre-eminence or prevalence of one planet over another.

ALMUCANTARS, ALMACANTARS, or ALMICANTARS, from the Arabic *almocantharat*, are circles parallel to the horizon, conceived to paſs through every degree of the meridian; ſerving to ſhew the height of the ſun, moon, or ſtars, &c; and are the ſame as the parallels of altitude.

ALMUCANTAR-*Staff*, was an inſtrument formerly uſed at ſea to obſerve the ſun's amplitude at riſing or ſetting, and thence to determine the variation of the compaſs, &c. The inſtrument had an arch of 15 degrees, made of ſome ſmooth wood.

ALPHONSINE *Tables*, are aſtronomical tables compiled by order of Alphonſus, king of Caſtile. In the compiling of theſe it is thought that prince himſelf aſſiſted. See *Aſtronomical tables.*

ALPHONSUS the 10th, king of Leon and Caſtile, who has been ſurnamed The Wiſe, on account of his attachment to literature, and is now more celebrated for having been an aſtronomer than a king. He was born in 1203; ſucceeded his father Ferdinand the 3d, in 1252; and died in 1284, conſequently at the age of 81.

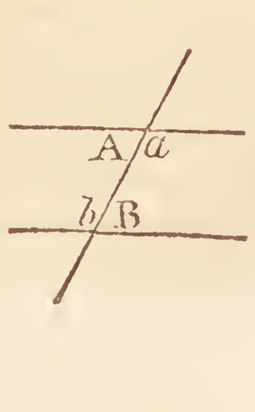
The affairs of the reign of Alphonſus were very extraordinary and unfortunate for him. But we ſhall here only conſider him in that part of his character, on account of which he has a place in this work, namely, as an aſtronomer and man of letters. He underſtood aſtronomy, philoſophy, and hiſtory, as if he had been only a man of letters; and compoſed books upon the motions of the heavens, and on the hiſtory of Spain, which are highly commended. "What can be more ſurpriſing," ſays Mariana, "than that a prince, educated in a camp, and handling arms from his childhood, ſhould have ſuch a knowledge of the ſtars, of philoſophy, and the tranſactions of the world, as men of leiſure can ſcarcely acquire in their retirements? There are extant ſome books of Alphonſus on the motions of the ſtars, and the hiſtory of Spain, written with great ſkill and incredible care." In his aſtronomioal purſuits he diſcovered that the tables of Ptolemy were full of errors; and thence he conceived the firſt of any the reſolution of correcting them. For this purpoſe, about the year 1240, and during the life of his father, he aſſembled at Toledo the moſt ſkilful aſtronomers of his time, Chriſtians, Moors, or Jews, when a plan was formed for conſtructing new tables. This taſk was accompliſhed about 1252, the firſt year of his reign; the tables being drawn up chiefly by the ſkill and pains of Rabbi Iſaac Hazan a learned Jew, and the work called the *Alphonſine Tables*, in honour of the prince, who was at vaſt expences concerning them. He fixed the epoch of the tables to the 30th of May 1252, being the day of his acceſſion to the throne. They were printed for the firſt time in 1483, at Venice, by Radtolt, who excelled in printing at that time; an edition extremely rare: there are others of 1492, 1521, 1545, &c. (Weidler, p. 280).

We muſt not omit a memorable ſaying of Alphonſus, which has been recorded for its boldneſs and pretended impiety; namely, "that if he had been of God's privy council when he made the world, he could have adviſed him better." Mariana however ſays only in general, that Alphonſus was ſo bold as to blame the works of

Providence, and the conſtruction of our bodies; and he ſays that this ſtory concerning him reſted only upon a vulgar tradition. The Jeſuit's words are curious: "Emanuel, the uncle of Sanchez (the ſon of Alphonſus), in his own name, and in the name of other nobles, deprived Alphonſus of his kingdom by a public ſentence: which that prince merited, for daring ſeverely and boldly to cenſure the works of divine Providence, and the conſtruction of the human body, as tradition ſays he did. Heaven moſt juſtly puniſhed the folly of his tongue." Though the ſilence of ſuch an hiſtorian as Mariana, in regard to Ptolemy's ſyſtem, ought to be of ſome weight, yet we cannot think it improbable, that if Alphonſus did paſs ſo bold a cenſure on any part of the univerſe, it was on the celeſtial ſphere, and meant to glance upon the contrivers and ſupporters of that ſyſtem. For, beſides that he ſtudied nothing more, it is certain that at that time aſtronomers explained the motions of the heavens by intricate and confuſed hypotheſes, which did no honour to God, nor anywiſe anſwered the idea of an able workman. So that, from conſidering the multitude of ſpheres compoſing the ſyſtem of Ptolemy, and thoſe numerous eccentric cycles and epicycles with which it is embarraſſed, if we ſuppoſe Alphonſus to have ſaid, "That if God had aſked his advice when he made the world, he would have given him better council," the boldneſs and impiety of the cenſure will be greatly diminiſhed.

ALSTED (JOHN-HENRY), a German proteſtant divine, and one of the moſt indefatigable writers of the 17th century. He was ſome time profeſſor of philoſophy and divinity at Herborn in the county of Naſſau: from thence he went into Tranſilvania, to be profeſſor at Alba Julia; where he continued till his death, which happened in 1638, being then 50 years of age. He applied himſelf chiefly to compoſe methods, and to reduce the ſeveral branches of arts and ſciences into ſyſtems. His *Encyclopædia* has been much eſteemed even by Roman Catholics; it was printed at Lyons, and ſold very well throughout all France. Voſſius mentions the Encyclopædia in general, but ſpeaks of his treatiſe of *Arithmetic* more particularly, and allows the author to have been a man of great reading and univerſal erudition. His *Theſaurus Chronologicus* is by ſome eſteemed one of his beſt works, and has gone through ſeveral editions, though others ſpeak of it with contempt. In his *Triumphus Biblicus* Alſted endeavours to prove that the materials and principles of all the arts and ſciences may be found in the ſcriptures; but he gained very few to his opinion. John Himmelius wrote a piece againſt his *Theologia Polemica*, which was one of Alſted's beſt performances. It ſeems he was a millenarian, having publiſhed, in 1672, a treatiſe *De Mille Annis*, in which he aſſerts that the faithful ſhall reign with Jeſus Chriſt upon earth a thouſand years; after which will be the general reſurrection, and the laſt judgment; and he pretended that this reign would commence in the year 1694.

ALTERNATE *angles*, are the internal angles, A and B, or *a* and *b*, made by a line cutting two parallel lines, and lying on oppoſite ſides of the cutting line. It is the property of theſe angles to be always equal to each other, namely the angle A = the angle B, and the angle *a* = the angle *b*. And the exterior alternate angles are alſo equal.



ALTERNATE *Ratio* or *Proportion*, is the ratio of the one antecedent to the other, or of one conſequent to the other, in any proportion, in which the quantities are of the ſame kind. So if A : B : : C : D, then alternately, or by alternation A : C : : B : D.

ALTERNATION, or *Permutation*, of quantities or things, is the varying or changing the order or poſition of them.

As ſuppoſe two things *a* and *b*; theſe may be placed either thus *ab* or *ba* that is two ways, or 1 × 2. If there be three things, *a*, *b*, *c*, then the 3d thing *c*, may be placed three different ways with reſpect to each of the two poſitions *ab* and *ba* of the other two things, it may ſtand either before them, or between them, or after them both, that is, it may ſtand either 1ſt, 2d, or 3d; and therefore with three things there will be three times as many changes as with two, that is 1 × 2 × 3 or ſix changes with three things. Again, if there be four things *a*, *b*, *c*, *d*; then the fourth thing *d* may be placed in four different ways with reſpect to each of the ſix poſitions of the other three; for it may be ſet either 1ſt or 2d or 3d or 4th in the order of each poſition; conſequently from four things there will be four times as many alternations as there are from three things; and therefore 1 × 2 × 3 × 4 = 24 is the number of changes with four things. And ſo on, always multiplying the laſt found number of alternations by the next number of things; or to find the number of changes for any number of things, as *n*, multiply the ſeries of natural numbers 1, 2, 3, 4, 5, &c, to *n*, continually together, and the laſt product will be the number of alternations ſought; ſo 1 × 2 × 3 × 4 × 5 - - - - *n* is the number of changes in *n* things.

So if, for example, it were required to find how many changes may be rung on 12 bells; it would be 1 × 2 × 3 × 4 × 5 × 6 × 7 × 8 × 9 × 10 × 11 × 12 = 479001600, the number of changes. Now ſuppoſing there might be rung 10 changes in one minute, that is 10 × 12 or 120 ſtrokes in a minute, or 2 ſtrokes in each ſecond of time; then, according to this rate, it would take upwards of 91 years to ring over all theſe changes on the 12 bells only. Alſo, if but two more bells were added, making 14 bells; then, at the ſame rate of ringing, it would require about 16575 years to ring all the changes on 14 bells but once over. And if the number of bells were 24, it would require more than 117000000000000000 years to ring all the different changes upon them!

ALTIMETRY, ALTIMETRIA, the art of taking or meaſuring altitudes or heights, whether acceſſible or inacceſſible. Or

Altimetria is the part of practical Geometry which reſpects the theory and practice of meaſuring both heights and depths, and both in reſpect of perpendicular and oblique lines.

ALTING (JAMES), was born at Heidelberg in 1618. He travelled into England in 1640, where he was ordained by the learned Dr. Prideaux, biſhop of Worceſter. He afterwards ſucceeded Gomarus in the profeſſorſhip of Groninghen. He died in 1697; and recommended the edition of his works to Menſo Alting (author of Notitia German. Infer. Antiquæ); but they were publiſhed in 5 vols folio, with his life, by

Bekker of Amſterdam. They contain various analytical, exegetical, practical, problematical, and philoſophical tracts, which ſhew his great induſtry and knowledge.

ALTITUDE, in Geometry is the third dimenſion of body, conſidered with reſpect to its elevation above the ground: and is otherwiſe called its height when meaſured from bottom to top, or its depth when meaſured from top to bottom.

ALTITUDE *of a figure*, is the diſtance of its vertex from the baſe, or the length of a perpendicular let fall from its vertex to the baſe. The altitudes of figures are uſeful in computing their areas or ſolidities.

ALTITUDE, or *Height* of any point of a terreſtrial object, is the perpendicular let fall from that point to the plane of the horizon. *Altitudes* are diſtinguiſhed into *acceſſible* and *inacceſſible*.

Acceſſible ALTITUDE of an object, is that whoſe baſe there is acceſs to, to meaſure the neareſt diſtance to it on the ground, from any place.

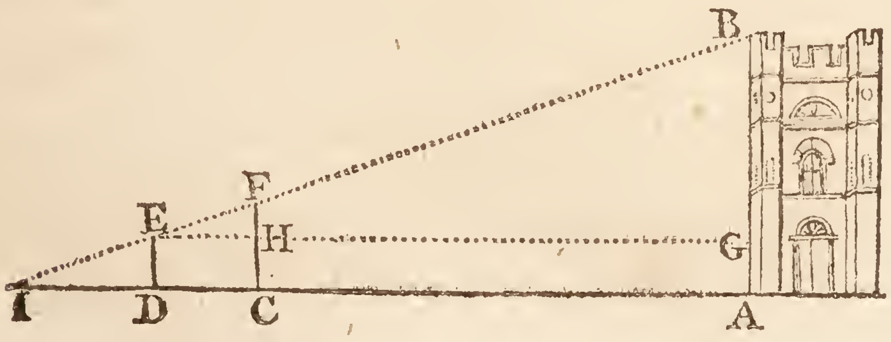
Inacceſſible ALTITUDE, of an object, is that whoſe baſe there is not free acceſs to, by which a diſtance may be meaſured to it, by reaſon of ſome impediment, ſuch as water, wood, or the like.

To meaſure or *take* ALTITUDES. If an altitude cannot be meaſured by ſtretching a ſtring from top to bottom, which is the direct and moſt accurate way, then ſome indirect way is uſed, by actually meaſuring ſome other line or diſtance which may ſerve as a baſis, in conjunction with ſome angles, or other proportional lines, either to compute, or geometrically determine, the altitude of the object ſought.

There are various ways of meaſuring altitudes, or depths, by means of different inſtruments, and by ſhadows or reflected images, on optical principles. There are alſo various ways of computing the altitude in numbers, from the meaſurements taken as above, either by geometrical conſtruction, or trigonometrical calculation, or by ſimple numeral computation from the property of parallel lines, &c.

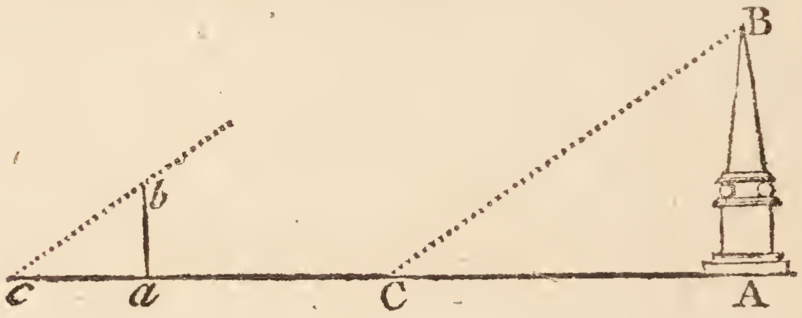
The inſtruments moſtly uſed in meaſuring altitudes, are the quadrant, theodolite, geometrical ſquare, line of ſhadows, &c; the deſcriptions of each of which may be ſeen under their reſpective names.

To meaſure an Acceſſible Altitude Geometrically. Thus, ſuppoſe the height of the acceſſible tower AB be required. Firſt, by means of two rods, the one longer than the other: plant the longer upright at C; then move the ſhorter back from it, till by trials you find ſuch a place, D, that the eye placed at the top of it at E, may ſee the top of the other, F, and the top of the object B ſtraight in a line: next meaſure the diſtances DA or EG and DC or EH, alſo HF the difference between the heights of the rods: then, by ſimilar triangles, as EH : EG : : HF : the 4th proportional GB; to which add AG or DE, and the ſum will be the whole altitude AB ſought.

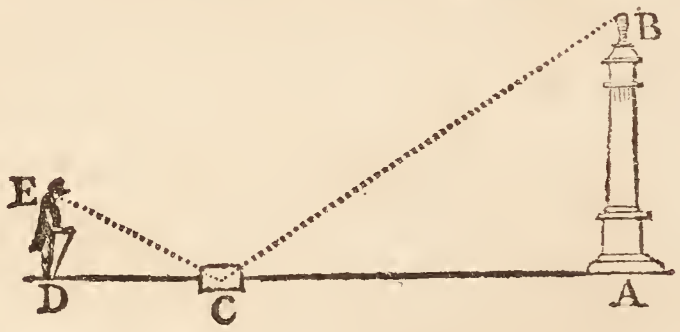


Or, with one rod CF only: plant it at ſuch a place C, that the eye at the ground, or near it, at I, may ſee the tops F and B in a right line: then, having meaſured IC, IA, CF, the 4th proportional to theſe will be the altitude AB ſought.

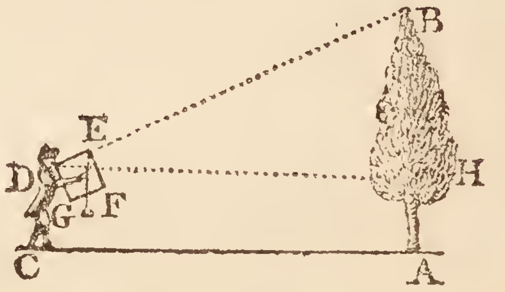
Or thus, by means of Shadows. Plant a rod *ab* at *a*, and meaſure its ſhadow *ac*, as alſo the ſhadow AC of the object AB; then the 4th proportional to *ac*, *ab*, AC will be the altitude AB ſought.



Or thus, by means of Optical Reflection. Place a veſſel of water, or a mirror or other reflecting ſurface, horizontal at C; and move off from it to ſuch a diſtance, D, that the eye E may ſee the image of the top of the object in the mirror at C: then, by ſimilar figures, CD : DE : : CA : AB the altitude ſought.

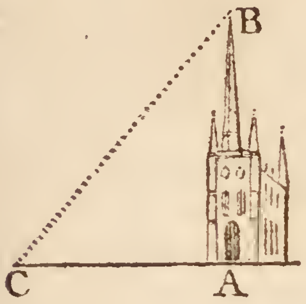


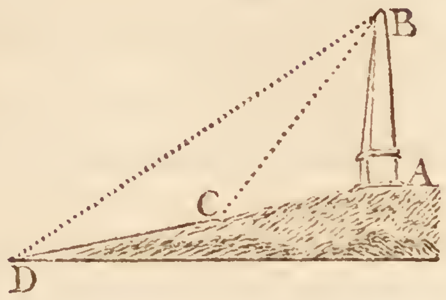
Or thus, by the Geometrical Square. At any place, C, fix the ſtand, and turn the ſquare about the centre of motion, D, till the eye there ſee the top of the object through the ſights or teleſcope on the ſide DE of the quadrant, and note the number of diviſions cut off the other ſide by the plumb line EG: then as EF : FG : : DH : HB; to which add AH or CD, for the whole height AB.



To meaſure an Acceſſible Altitude Trigonometrically. At any convenient ſtation, C, with a quadrant, theodolite, or other graduated inſtrument, obſerve the angle of elevation ACB above the horizontal line AC; and meaſure the diſtance AC. Then, A being a right angle, it will be, as radius is to the tangent of the angle A, ſo is AC to AB ſought.

If AC be not horizontal, but an inclined plane; then the angle above it muſt be obſerved at two ſtations C and D in a right line, and the diſtances AC, CD both meaſured. Then, from the angle C take the angle D, and there remains the angle CBD; hence in the triangle BCD, are given the angles and the ſide DC, to find the ſide CB; and then in the triangle ABC, are given the two ſides CA and CB, with the included angle C, to find the third ſide AB.

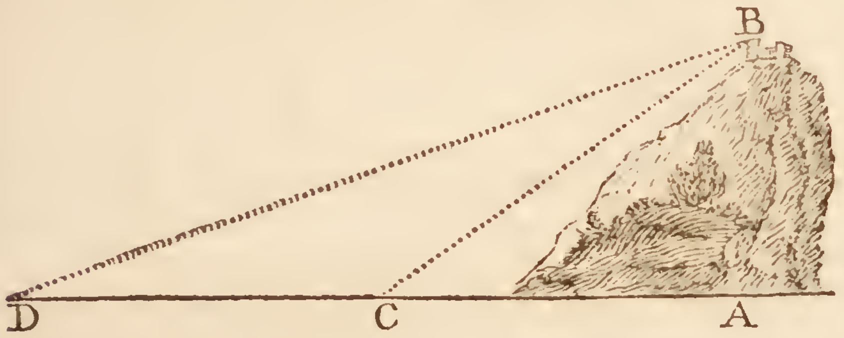




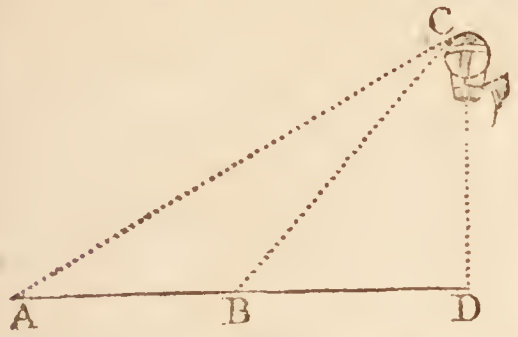
Or thus, meaſure only the diſtance AC, and the angles A and C: then, in the triangle ABC, are given all the angles and the ſide AC, to find the ſide AB.

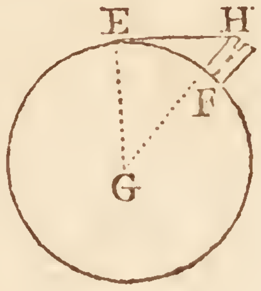
To meaſure an Inacceſſible Altitude, as a hill, cloud, or other object. This is commonly done, by obſerving the angle of its altitude at two ſtations, and meaſuring the diſtance between them. Thus, for the height AB of a hill, meaſure the diſtance CD at the foot of it, and obſerve the quantity of the two angles C and D. Then, from the angle C taking the angle D, leaves the angle CBD ; hence

As ſine $\angle$CBD : ſine $\angle$D : : CD : CB ; and
As rad. : ſine $\angle$ACB : : CB : AB the altitude.



And for a balloon, or cloud, or other moveable object C, let two obſervers at A and B, in a plane with C, take at the ſame time the angles A and B, and meaſure the diſtance between them AB ; then calculate the altitude CD exactly as in the laſt example.





To find the height of an object, by knowing the utmoſt diſtance at which its top can be juſt ſeen in the horizon. As ſuppoſe the top H of a tower FH can be juſt ſeen from E when the diſtance EF is 25 miles, ſuppoſing the circumference of the earth to be 25000 miles, or the radius 3979 miles or 21009120 feet. Firſt, as 25000 : 25 : : $360^\circ : 21' \, 36''$ equal to the angle G ; then as radius : ſec. $\angle$G : : EG : GH, which will be found to be 21009536 feet ; from which take EG or GF, and there remains 416 feet, for FH the height of the tower ſought.— Or rather thus, as 10000000 radius : 198 = ſec. $\angle$G — radius : : 21009120 = EG : 416 = FH, as before.

Or the ſame may be found eaſier thus : The horizon dips nearly 8 inches or $\frac{2}{3}$ of a foot at the diſtance of 1 mile, and according to the ſquare of the diſtance for other diſtances ; therefore as 1^2 or 1 : 25^2 or 625 : : $\frac{2}{3}$: $\frac{2}{3}$ of 625 or 416 feet, the ſame as before.

There is a very eaſy method of taking great terreſtrial altitudes, ſuch as mountains &c, by means of the difference between the heights of the barometer obſerved at the bottom and top of the ſame. Which ſee under the article BAROMETER.

ALTITUDE *of the Eye*, in *Perſpective*, is a right line let fall from the eye, perpendicular to the geometrical plane.

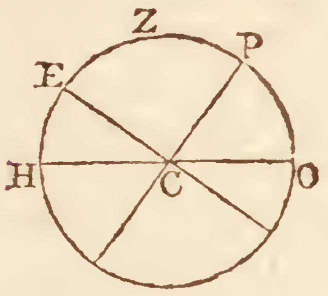
ALTITUDE, in *Aſtronomy*, is the arch of a vertical circle, meaſuring the height of the ſun, moon, ſtar, or other celeſtial object, above the horizon.

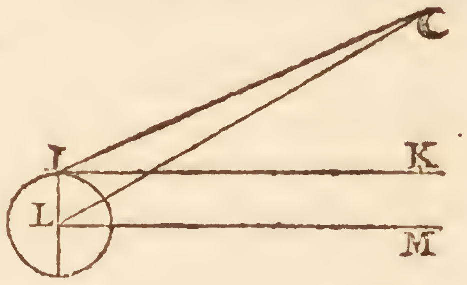
This altitude may be either *true* or *apparent*. The apparent altitude is that which appears by ſenſible obſervations made at any place on the ſurface of the earth. And the true altitude is that which reſults by correcting the apparent, on account of refraction and parallax.

The quantity of the refraction is different at different altitudes ; and the quantity of the parallax is different according to the diſtance of the different luminaries : in the fixed ſtars this is too ſmall to be obſerved ; in the ſun it is but about $8\frac{1}{4}$ ſeconds ; but in the moon it is about 52 minutes.

Altitudes are obſerved by a quadrant, or ſextant, or by the ſhadow of a gnomon or high pole, and by various other ways, as may be ſeen in moſt books of aſtronomy.

Meridian ALTITUDE, is an arch of the meridian intercepted between any point in it and the horizon. So if HO be the horizon, and HEZO the meridian ; then the arch HE, or the angle HCE, is the meridian altitude of an object in the meridian at the point E.





ALTITUDE, or *elevation*, *of the Pole*, is the angle OCP, or arch OP of the meridian, intercepted between the horizon and pole P.

This is equal to the latitude of the place ; and it may be found by obſerving the meridian altitude of the pole ſtar, when it is both above and below the pole, and taking half the ſum, when corrected on account of refraction. Or the ſame may be found by the declination and meridian altitude of the ſun.

ALTITUDE, or *elevation*, of the *equator*, is the angle HCE, or arch HE of the meridian, between the horizon and the equator at E ; and it is equal to ZP the colatitude of the place.

ALTITUDE *of the Tropics*, the ſame as what is otherwiſe called the *ſolſtitial altitude* of the ſun, or his meridian altitude when in the ſolſtitial points.

ALTITUDE, *or height*, *of the horizon*, or of ſtars &c ſeen in it, is the quantity by which it is raiſed by refraction.

Refraction of ALTITUDE, is an arch of a vertical circle, by which the true altitude of the moon, or a ſtar, or other object, is increaſed by means of the refraction ;

and is different at different altitudes, being nothing in the zenith, and greateſt at the horizon, where it is about 33′.

Parallax of Altitude, is an arch of a vertical circle, by which the true altitude, obſerved at the centre of the earth, exceeds that which is obſerved on the ſurface; or the difference between the angles ☾ LM and ☾ IK of altitude there; and is equal to the angle I ☾ L formed at the moon or other body, and ſubtended by the radius IL of the earth.

It is evident that this angle is leſs, as the luminary is farther diſtant from the earth; and alſo leſs, for any one luminary, as it is higher above the horizon; being greateſt there, and nothing in the zenith.

Altitude *of the Nonageſimal*, is the altitude of the 90th degree of the ecliptic, counted upon it from where it cuts the horizon, or of the middle or higheſt point of it which is above the horizon, at any time; and is equal to the angle made by the ecliptic and horizon where they interſect at that time.

Altitude *of the cone of the earth's or moon's ſhadow*, the height of the ſhadow of the body, made by the ſun, and meaſured from the centre of the body. To find it, ſay, As the tangent of the angle of the ſun's apparent ſemidiameter is to radius, ſo is 1 to a 4th proportional, which will be the height of the ſhadow, in ſemidiameters of the body.

So, the greateſt height of the earth's ſhadow, is 217·8 ſemidiameters of the earth, when the ſun is at his greateſt diſtance, or his ſemidiameter ſubtends an angle of about 15′ 47″; and the height of the ſame is 210·7 ſemidiameters of the earth, when the ſun is neareſt the earth, or when his ſemidiameter is about 16′ 19″: And proportionally between theſe limits for the intermediate diſtances or ſemidiameters of the ſun.

The altitudes of the ſhadows of the earth and moon, are nearly as 11 to 3, the proportion of their diameters.

Altitude, or *exaltation*, in aſtrology, denotes the ſecond of the five eſſential dignities, which the planets acquire by virtue of the ſigns they are found in.

Altitude *of motion*, is a term uſed by Dr. Wallis, for the meaſure of any motion, eſtimated in the line of direction of the moving force.

Altitude, in ſpeaking of fluids, is more frequently expreſſed by the term *depth*. The preſſure of fluids, in every direction, is in proportion to their altitude or depth.

Altitude *of the mercury*, in the *barometer* and *thermometer*, is marked by degrees, or equal diviſions, placed by the ſide of the tube of thoſe inſtruments.

The altitude of the barometer, or of the mercury in its tube, at London, is uſually compriſed between the limits of 28 and 31 inches; and the mean height, for every day in ſeveral years, is nearly 29·87 inches.

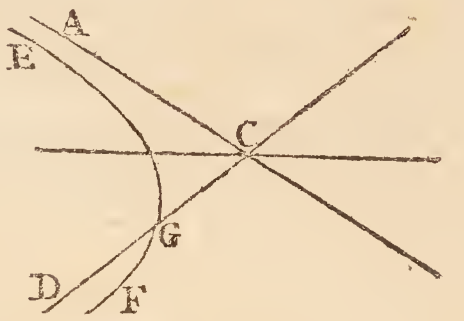
Altitude *of the pyramids in Egypt*, was meaſured ſo long ſince as the time of Thales, which he effected by means of their ſhadow, and that of a pole ſet upright beſide them, making the altitudes of the pole and pyramid proportional to the lengths of their ſhadows. Plutarch has given an account of the manner of this operation, which is one of the firſt geometrical obſervations we have an exact account of.

Altitude, *circles of*, *parallels of*, *quadrant of*, &c. See the reſpective words.

Equal Altitude *Inſtrument*, is an inſtrument uſed to obſerve a celeſtial object, when it has the ſame or an equal altitude, on both ſides of the meridian, or before and after it paſſes the meridian: an inſtrument very uſeful in adjuſting clocks &c, and for comparing equal and apparent time.

AMBIENT, encompaſſing round about; as the bodies which are placed about any other body, are called ambient bodies, and ſometimes *circum-ambient* bodies; and the whole maſs of the air or atmoſphere, becauſe it encompaſſes all things on the face of the earth, is called the *ambient* air.

AMBIGENAL *Hyperbola*, a name given by Newton, in his *Enumeratio linearum tertii ordinis*, to one of the triple hyperbolas EGF of the ſecond order, having one of its infinite legs, as EG, falling within the angle ACD, formed by the aſymptotes AC, CD, and the other leg GF falling without that angle.



AMBIT, *of a figure*, in Geometry, is the perimeter, or line, or ſum of the lines, by which the figure is bounded.

AMBLIGON, or Ambligonal, in Geometry, ſignifies obtuſe-angular, as a triangle which has one of its angles obtuſe, or conſiſting of more than 90 degrees.

AMICABLE *numbers*, denote pairs of numbers, of which each of them is mutually equal to the ſum of all the aliquot parts of the other. So the firſt or leaſt pair of amicable numbers are 220 and 284; all the aliquot parts of which, with their ſums, are as follow, viz,

of 220, they are 1, 2, 4, 5, 10, 11, 20, 22, 44, 55, 110, their ſum — — 284;
of 284, they are 1, 2, 4, 71, 142, and their ſum is 220.

The 2d pair of amicable numbers are 17296 and 18416, which have alſo the ſame property as above.

And the 3d pair of amicable numbers are 9363584 and 9437056.

Theſe three pairs of amicable numbers were found out by F. Schooten, ſect. 9 of his *Exercitationes Mathematicæ*, who I believe firſt gave the name of *amicable* to ſuch numbers, though ſuch properties of numbers it ſeems had before been treated of by Rudolphus, Deſcartes, and others.

To find the firſt pair, Schooten puts $4x$ and $4yz$, or a^2x and a^2yz for the two numbers where $a = 2$; then making each of theſe equal to the ſum of the aliquot parts of the other, gives two equations, from which are found the values of x and z, and conſequently, aſſuming a proper value for y, the two amicable numbers themſelves $4x$ and $4yz$.

In like manner for the other pairs of ſuch numbers; in which he finds it neceſſary to aſſume $16x$ and $16yz$ or a^4x and a^4yz for the 2d pair, and $128x$ and $128yz$ or a^7x and a^7yz for the 3d pair.

Schooten then gives this practical rule, from Deſcartes, for finding amicable numbers, viz; Aſſume the number 2, or ſome power of the number 2, ſuch that if unity or 1 be ſubtracted from each of theſe three following quantities, viz;
from 3 times the aſſumed number,
alſo from 6 times the aſſumed number,
and from 18 times the ſquare of the aſſumed number,
the three remainders may be all prime numbers; then the laſt prime number being multiplied by double the aſſumed number, the product will be one of the amicable numbers ſought, and the ſum of its aliquot parts will be the other.

That is, if a be put = the number 2, and n ſome integer number, ſuch that $3a^n - 1$, and $6a^n - 1$, and $18a^{2n} - 1$ be all three prime numbers; then is $\overline{18a^{2n} - 1} \times 2a^n$ one of the amicable numbers; and the ſum of its aliquot parts is the other.

AMONTONS (William), an ingenious French experimental philoſopher, was born in Normandy the 31ſt of Auguſt 1663. While at the grammar ſchool, he by ſickneſs contracted a deafneſs that almoſt excluded him from the converſation of mankind. In this ſituation he applied himſelf to the ſtudy of geometry and mechanics; with which he was ſo delighted that it is ſaid he refuſed to try any remedy for his diſorder, either becauſe he deemed it incurable, or becauſe it increaſed his attention to his ſtudies. Among other objects of his ſtudy, were the arts of drawing, of land-ſurveying, and of building; and ſhortly after he acquired ſome knowledge of thoſe more ſublime laws by which the univerſe is regulated. He ſtudied with great care the nature of barometers and thermometers; and wrote his treatiſe of *Obſervations and Experiments concerning a new Hour-glaſs, and concerning Barometers, Thermometers, and Hygroſcopes;* as alſo ſome pieces in the Journal des Savans. In 1687, he preſented a new hygroſcope to the Academy of Sciences, which was much approved. He found out a method of conveying intelligence to a great diſtance in a ſhort ſpace of time: this was by making ſignals from one perſon to another, placed at as great diſtances from each other as they could ſee the ſignals by means of teleſcopes. When the Royal Academy was new regulated in 1699, Amontons was choſen a member of it, as an eleve under the third Aſtronomer; and he read there his *New Theory of Friction*, in which he happily cleared up an important object in mechanics. In fact he had a particular genius for making experiments: his notions were juſt and delicate: and he knew how to prevent the inconveniences of his new inventions, and had a wonderful ſkill in executing them. He died of an inflammation in his bowels, the 11th of October 1705, being only 42 years of age.

The eloge of Amontons may be ſeen in the volume of the Memoirs of the Academy of Sciences for the year 1705, Hiſt. pa. 150. And his pieces contained in the different volumes of that work, which are pretty numerous, and upon various ſubjects, as the air, action of fire, barometers, thermometers, hygrometers, friction, machines, heat, cold, rarefactions, pumps, &c, may be ſeen in the volumes for the years 1696, 1699, 1702, 1703, 1704, and 1705.

AMPHISCII, or Amphiscians, are the people who inhabit the torrid zone; which are ſo called, becauſe they have their ſhadow at noon turned ſometimes one way, and ſometimes another, namely, at one time of the year towards the north, and at the other towards the ſouth.

AMPLITUDE, in *gunnery*, the range of the projectile, or the right line upon the ground ſubtending the curvilinear path in which it moves.

Amplitude, in *aſtronomy*, is an arch of the horizon, intercepted between the true eaſt or weſt point, and the centre of the ſun or a ſtar at its riſing or ſetting: ſo that the amplitude is of two kinds; *ortive* or eaſtern, and *occiduous* or weſtern. Each of theſe amplitudes is alſo either northern or ſouthern, according as the point of riſing or ſetting is in the northern or ſouthern part of the horizon: and the complement of the amplitude, or the arch of diſtance of the point of riſing or ſetting, from the north or ſouth point of the horizon, is the azimuth.

The amplitude is of uſe in navigation, to find the variation of the compaſs or magnetic needle. And the rule to find it is this: As the coſine of the latitude is to radius, ſo is the ſine of the ſun's or ſtar's declination, to the ſine of the amplitude. So in the latitude of London, viz, 51° 31′, when the ſun's declination is 23° 28′; then

cos. 51° 31′ the lat.	- - -	−	9·7939907
ſin. 23 28 the decl.	-	+	9·6001181
ſin. 39 47 the ampl.			9·8061274

That is, the ſun then riſes or ſets 39° 47′ from the eaſt or weſt point, to the north or ſouth according as the declination is north or ſouth.

Magnetical Amplitude, is an arch of the horizon, contained between the ſun or ſtar, at the riſing or ſetting, and the magnetical eaſt or weſt point of the horizon, pointed out by the magnetical compaſs, or the amplitude or azimuth compaſs. And the difference between this magnetical amplitude, ſo obſerved, and the true amplitude, as computed in the laſt article, is the variation of the compaſs.

So if, for inſtance, the magnetical amplitude be obſerved, by the compaſs, to be 61° 47′, at the time when it is computed to be - 39 47,
then the difference - - 22 0 is the variation weſt.

ANABIBAZON, a name ſometimes given to the dragon's tail, or northern node of the moon.

ANACAMPTICS, or the ſcience of the reflections of ſounds, frequently uſed in reference to echoes, which are ſaid to be ſounds produced *anacamptically*, or by reflection. And in this ſenſe it was uſed by the ancients for that part of optics which is otherwiſe called Catoptrics.

ANACHRONISM, in Chronology, an error in computation of time, by which an event is placed earlier than it really happened. Such is that of Virgil, who makes Dido to reign at Carthage in the time of Æneas, though, in reality, ſhe did not arrive in Africa till 300 years after the taking of Troy.

An error on the other ſide, by which a fact is placed later, or lower than it ſhould be, is called a *parachroniſm*. But in common uſe, this diſtinction, though proper, is not attended to; and the word *anachroniſm* is uſed indifferently for the miſtake on both ſides.

ANACLASTICS, *or* Anaclatics, an ancient name for that part of Optics which conſiders refracted light;

being the ſame as what is more uſually called *dioptrics.* See the Compendium of *Ambroſius Rhodius, lib.* 3. *Opticæ, pa.* 384 *& ſeq.*

Anaclastic *Curves*, a name given by M. de Mairan to certain apparent curves formed at the bottom of a veſſel full of water, to an eye placed in the air; or the vault of the heavens, ſeen by refraction through the atmoſphere.

M. de Mairan determines theſe curves by a principle not admitted by all authors; but Dr. Barrow, at the end of his Optics, determines the ſame curves by other principles.

ANALEMMA, a planiſphere, or projection of the ſphere, orthographically made on the plane of the meridian, by perpendiculars from every point of that plane, the eye ſuppoſed to be at an infinite diſtance, and in the eaſt or weſt point of the horizon. In this projection, the ſolſtitial colure, and all its parallels, are projected into concentric circles, equal to the real circles in the ſphere; and all circles whoſe planes paſs through the eye, as the horizon and its parallels, are projected into right lines equal to their diameters; but all oblique circles are projected into ellipſes, having the diameter of the circle for the tranſverſe axis.

This inſtrument, having the furniture drawn on a plate of wood or braſs, with an horizon fitted to it, is uſed for reſolving many aſtronomical problems; as the time of the ſun's riſing and ſetting, the length and hour of the day, &c. It is alſo uſeful in dialling, for laying down the ſigns of the zodiac, with the lengths of days, and other matters of furniture, upon dials.

The oldeſt treatiſe we have on the analemma, was written by Ptolemy, which was printed at Rome in 1562, with a commentary by F. Commandine. Pappus alſo treated of the ſame. Since that time, many other authors have treated very well of the analemma; as Aguilonius, Taquet, Dechales, Witty, &c.

ANALOGY, the ſame as proportion, or equality, or ſimilitude of ratios. Which ſee.

ANALYSIS, is, generally, the reſolution of any thing into its component parts, to diſcover the thing or the compoſition. And in mathematics it is properly the method of reſolving problems, by reducing them to equations. *Analyſis* may be diſtinguiſhed into the *ancient* and the *modern.*

The *ancient analyſis*, as deſcribed by Pappus, is the method of proceeding from the thing ſought as *taken* for granted, through its conſequences, to ſomething that is *really* granted or known; in which ſenſe it is the reverſe of ſyntheſis or compoſition, in which we lay that down *firſt* which was the *laſt* ſtep of the analyſis, and tracing the ſteps of the analyſis back, making that antecedent here which was conſequent there, till we arrive at the thing ſought, which was taken or aſſumed as granted in the firſt ſtep of the analyſis. This chiefly reſpected geometrical enquiries.

The principal authors on the ancient analyſis, as recounted by Pappus, in the 7th book of his *Mathematical Collections*, are Euclid in his *Data, Poriſmata, & de Locis ad Superficiem;* Apollonius *de Sectione Rationis, de Sectione Spatii, de Tactionibus, de Inclinationibus, de Locis Planis, & de Sectionibus Conicis;* Ariſtæus, *de Locis Solidis;* and Eratoſthenes, *de Mediis Proportionalibus:* from which Pappus gives many examples in the ſame book. To theſe authors we may add Pappus himſelf. The ſame ſort of analyſis has alſo been well cultivated by many of the moderns; as Fermat, Viviani, Getaldus, Snellius, Huygens, Simſon, Stewart, Lawſon, &c, and more eſpecially Hugo d'Omerique, in his *Analyſis Geometrica*, in which he has endeavoured to reſtore the Analyſis of the ancients. And, on this head, Dr. Pemberton tells us "that Sir Iſaac Newton uſed to cenſure himſelf for not following the ancients more cloſely than he did; and ſpoke with regret of his miſtake, at the beginning of his mathematical ſtudies, in applying himſelf to the works of Deſcartes, and other algebraical writers, before he had conſidered the Elements of Euclid with that attention ſo excellent a writer deſerves: that he highly approved the laudable attempt of Hugo d'Omerique to reſtore the ancient analyſis."

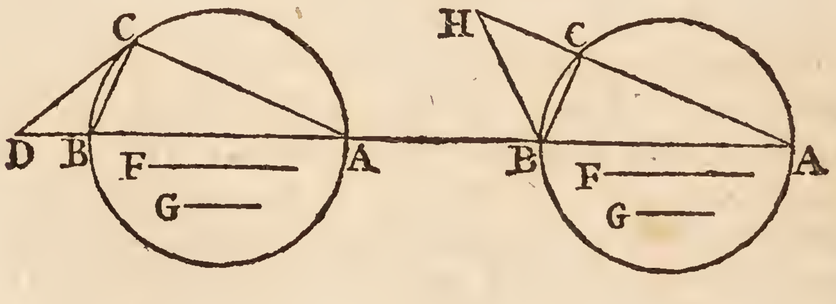
In the application of the ancient analyſis in geometrical problems, every thing cannot be brought within ſtrict rules; nor any invariable directions given, by which we may ſucceed in all caſes; but ſome previous preparation is neceſſary, a kind of mental contrivance and conſtruction, to form a connexion between the *data* and *quæſita*, which muſt be left to every one's fancy to find out; being various, according to the various nature of the problems propoſed: Right lines muſt be drawn in particular directions, or of particular magnitudes; biſecting perhaps a given angle, or perpendicular to a given line; or perhaps tangents muſt be drawn to a given curve, from a given point; or circles deſcribed from a given centre, with a given radius, or touching given lines, or other given circles; or ſuch-like other operations. Whoever is converſant with the works of Archimedes, Apollonius, or Pappus, well knows that they founded their analyſis upon ſome ſuch previous operations; and the great ſkill of the analyſt conſiſts in diſcovering the moſt proper affections on which to found his analyſis: for the ſame problem may often be effected in many different ways: of which it may be proper to give here an example or two. Let there be taken, for inſtance, this problem, which is the 155th prop. of the 7th book of Pappus.

From the extremities of the baſe A, B, of a given ſegment of a circle, it is required to draw two lines AC, BC, meeting at a point C in the circumference, ſo that they ſhall have a given ratio to each other, ſuppoſe that of F to G.

The ſolution of this problem, as given by Pappus, is thus.

Analysis.

Suppoſe the thing done, and that the point C is found: then ſuppoſe CD is drawn a tangent to the circle at C, and meeting the line AB produced in the point D. Now by the hypotheſis AC : BC : : F : G, and alſo AC^2 : BC^2 : : DA : DB, as may be thus proved.



Since DC touches the circle, and BC cuts it, the angle BCD is equal to BAC by Euc. iii. 32; alſo the angle D is

D is common to both the triangles DCA, DCB; theſe are therefore ſimilar, and ſo, by vi 4, DA : DC : : DC : DB, and hence $DA^2 : DC^2 :: DA : DB$ by cor. vi 20. But alſo, by vi 4, DA : AC : : DC : CB, and by permutation DA : DC : : AC : BC, or $DA^2 : DC^2 :: AC^2 : BC^2$; and hence, by equality, $AC^2 : BC^2 :: DA : DB$.

But the ratio of AC^2 to BC^2 is given by prop. LVII of Simſon's edition of the *Data*, becauſe the ratio of AC to BC is given, and conſequently that of DA to DB is given. Now ſince the ratio of DA to DB is given, therefore alſo, by Data vi, that of DA to AB, and hence, by Data ii, DA is given in magnitude.

And here the analyſis properly ends. For it having been ſhewn that DA is given, or that a point D may be found in AB produced, ſuch, that a tangent being drawn from it to the circumference, the point of contact will be the point ſought; we may now begin the compoſition, or ſynthetical demonſtration; which muſt be done by finding the point D, or laying down the line AD, which, it was affirmed, was given, in the laſt ſtep of the analyſis.

SYNTHESIS.

Conſtruction. Make as $F^3 : G^2 :: AD : DB$, (which may be done, ſince AB is given, by making it as $F^2 - G^2 : G^2 :: AB : DB$, and then by compoſition it will be as $F^2 : G^2 :: AD : DB$); and then from the point D, thus found, draw a tangent to the circle, and from the point of contact C drawing CA and CB, the thing is done.

Demonſtration. Since, by the conſtr. $F^2 : G^2 :: AD : DB$, and alſo $AD : DB :: AC^2 : BC^2$, which has been already demonſtrated in the analyſis, and might be here proved in the ſame manner. Therefore $F^2 : G^2 :: AC^2 : BC^2$, and conſequently F : G : : AC : BC. *Q. E. D.*

Here we ſee an inſtance of the method of *reſolution* and *compoſition*, as it was practiſed by the ancients, the ſolution here given being that of Pappus himſelf. But as the method of referring and reducing every thing to the *Data*, and conſtantly quoting the ſame, may appear now to be tedious and troubleſome: and indeed it is unneceſſary to thoſe who have already made themſelves maſters of the ſubſtance of that valuable book of Euclid, and have by practice and experience acquired a facility of reaſoning in ſuch matters: I ſhall therefore now ſhew how we may abate ſomething of the rigour and ſtrict form of the ancient method of ſolution, without diminiſhing any part of its admirable elegance and perſpicuity. And this may be done by the inſtance of another ſolution, of the many more which might be given, of the ſame problem, as follows.

ANALYSIS.

Let us again ſuppoſe that the thing is done, viz AC : BC : : F : G, and let there be drawn BH making the angle ABH equal to the angle ACB, and meeting AC produced in H. Then, the angle A being alſo common, the two triangles ABC and ABH are equiangular, and therefore, by vi 4, AC : BC : : AB : BH, in a given ratio; and, AB being given, therefore BH is given in poſition and magnitude.

SYNTHESIS.

Conſtruction. Draw BH making the angle ABH equal to that which may be contained in the given ſegment, and take AB to BH in the given ratio of F to G. Draw ACH, and BC.

Demonſtration. The triangles ABC, ABH are equiangular, therefore, vi 4, AC : CB : : AB : BH, which is the given ratio by conſtruction.

Modern ANALYSIS, conſiſts chiefly of algebra, arithmetic of infinites, infinite ſeries, increments, fluxions, &c; of each of which a particular account may be ſeen under their reſpective articles.

Theſe form a kind of arithmetical and ſymbolical analyſis, depending partly on modes of arithmetical computation, partly on rules peculiar to the ſymbols made uſe of, and partly on rules drawn from the nature and ſpecies of the quantities they repreſent, or from the modes of their exiſtence or generation.

The modern analyſis is a general inſtrument by which the fineſt inventions and the greateſt improvements have been made in mathematics and philoſophy, for near two centuries paſt. It furniſhes the moſt perfect examples of the manner in which the art of reaſoning ſhould be employed; it gives to the mind a wonderful ſkill for diſcovering things unknown, by means of a ſmall number that are given; and by employing ſhort and eaſy ſymbols for expreſſing ideas, it preſents to the underſtanding things which otherwiſe would ſeem to lie above its ſphere. By this means geometrical demonſtrations may be greatly abridged: a long train of arguments, in which the mind cannot, without the greateſt effort of attention, diſcover the connection of ideas, is converted into viſible ſymbols; and the various operations which they require, are ſimply effected by the combination of thoſe ſymbols. And, what is ſtill more extraordinary, by this artifice, a great number of truths are often expreſſed in one line only: inſtead of which, by following the ordinary way of explanation and demonſtration, the ſame truths would occupy whole pages or volumes. And thus, by the bare contemplation of one line of calculation, we may underſtand in a ſhort time whole ſciences, which otherwiſe could hardly be comprehended in ſeveral years.

It is true that Newton, who beſt knew all the advantages of analyſis in geometry and other ſciences, laments, in ſeveral parts of his works, that the ſtudy of the ancient geometry is abandoned or neglected. And indeed the method employed by the ancients in their geometrical writings, is commonly regarded as more rigorous, than that of the modern analyſis: and though it be greatly inferior to that of the moderns, in point of diſpatch and facility of invention; it is nevertheleſs highly uſeful in ſtrengthening the mind, improving the reaſoning faculties, and in accuſtoming the young mathematician to a pure, clear, and accurate mode of inveſtigation and demonſtration, though by a long and laboured proceſs, which he would with difficulty have ſubmitted to if his taſte had before been vitiated, as it were, by the more piquant ſweets of the modern analyſis. And it is principally on this that the complaints of Newton are founded, who feared leſt by the too early and frequent uſe of the modern analyſis, the ſcience of geometry ſhould loſe that rigour and purity which characteriſe its inveſtigations, and the mind become debilitated by the facility of our analyſis. This great man was therefore well founded, in recommending, to a certain extent, the ſtudy of the ancient geometricians: for, their demonſtrations being more difficult, give more exerciſe to the mind, accuſtom it to a cloſer application, give it a greater ſcope, and habituate it to patience and reſolution, ſo neceſſary for making diſcoveries. But this is the only or principal advantage from it; for if we ſhould look no farther than the method of the ancients, it is probable that, even with the beſt genius, we ſhould have made but few or ſmall diſcoveries, in compariſon of thoſe obtained by means of the modern analyſis. And even with regard to the advantage given to inveſtigations made in the manner of the ancients, namely of being more rigorous, it may perhaps be doubted whether this pretenſion be well founded. For to inſtance in thoſe of Newton himſelf, although his demonſtrations be managed in the manner of the ancients; yet at the ſame time it is evident that he inveſtigates his theorems by a method different from that employed in the demonſtrations, which are commonly analytical calculations, diſguiſed by ſubſtituting the name of lines for their algebraical value: and though it be true that his demonſtrations are rigorous, it is no leſs ſo that they would be the ſame when tranſlated and delivered in algebraic language; and what difference can it make in this reſpect, whether we call a line AB, or denote it by the algebraic character a? Indeed this laſt deſignation has this peculiarity, that when all the lines are denoted by algebraic characters, many operations can be performed upon them, without thinking of the lines or the figure. And this circumſtance proves of no ſmall advantage: the mind is relieved, and ſpared as much as poſſible, that its whole force may be employed in overcoming the natural difficulty of the problem alone.

Upon the whole therefore the ſtate of the compariſon ſeems to be this; That the method of the ancients is fitteſt to begin our ſtudies with, to form the mind and to eſtabliſh proper habits; and that of the moderns to ſucceed, by extending our views beyond the preſent limits, and enabling us to make new diſcoveries and improvements.

Analyſis is divided, with reſpect to its object, into that of *finites*, and that of *infinites*.

Analyſis of finite quantities, is what is otherwiſe called *algebra*, or *ſpecious arithmetic*.

Analyſis of infinites, called alſo the *new analyſis*, is that which is concerned in calculating the relations of quantities which are conſidered as infinite, or infinitely little; one of its chief branches being the *method of fluxions*, or the *differential calculus*. And the great advantage of the modern mathematicians over the ancients, ariſes chiefly from the uſe of this modern analyſis.

Analysis *of powers*, is the ſame as reſolving them into their roots, and is otherwiſe called *evolution*.

Analysis *of curve lines*, ſhews their conſtitution, nature and properties, their points of inflexion, ſtation, retrogradation, variation, &c.

ANALYST, a perſon who analyſes ſomething, or makes uſe of the analytical method. In mathematics, it is a perſon ſkilled in algebra, or in the mathematical analyſis in general.

Analyst, the title of an ingenious, though ſophiſtical book, written by the celebrated Dr. Berkeley, againſt the doctrine of fluxions.

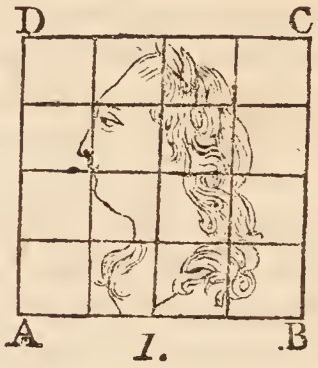
ANALYTIC, or **Analytical**, ſomething belonging to, or partaking of, the nature of analyſis; or performed by the method of analyſis.

Thus we ſay *analytical* demonſtration, *analytical* enquiry, *analytical* table or ſcheme, *analytical* method, &c. The *analytical* ſtands oppoſed to the *ſynthetical*, or that which proceeds by the way of *ſyntheſis*.

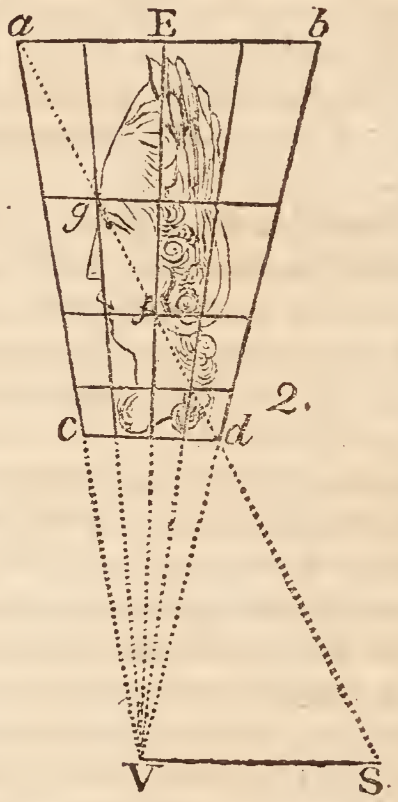
ANALYTICS, the ſcience, or doctrine, and uſe of analyſis.

ANAMORPHOSIS, in perſpective and painting, a monſtrous projection; or a repreſentation of ſome image, either on a plane or curve ſurface, deformed or diſtorted; but which in a certain point of view ſhall appear regular, and drawn in juſt proportion.

To conſtruct an Anamorphoſis, or monſtrous projection, on a plane.—Draw the ſquare ABCD (fig. 1), of any ſize at pleaſure, and divide it by croſſing lines into a number of areolæ or ſmaller ſquares: and then in this ſquare, or reticle, called alſo the *craticular prototype*, draw the regular image which is to be diſtorted.—Or, about any image, propoſed to be diſtorted, draw a reticle of ſmall ſquares.



1.



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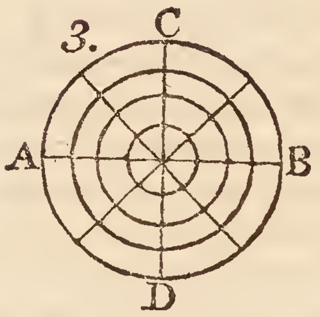


3.

Then draw the line ab (fig. 2.) equal to AB, dividing it into the ſame number of equal parts, as the ſide of the prototype AB; and on its middle point E erect the perpendicular EV, and alſo VS perpendicular to EV, making EV ſo much the longer, and VS ſo much the ſhorter, as it is intended the image ſhall be more diſtorted. From each of the points of diviſion draw right lines to the point V, and draw the right line aS. Laſtly through the points c, e, f, g, &c, draw lines parallel to ab: So ſhall $abcd$ be the ſpace upon which the monſtrous projection is to be drawn; and is called the *craticular ectype*.

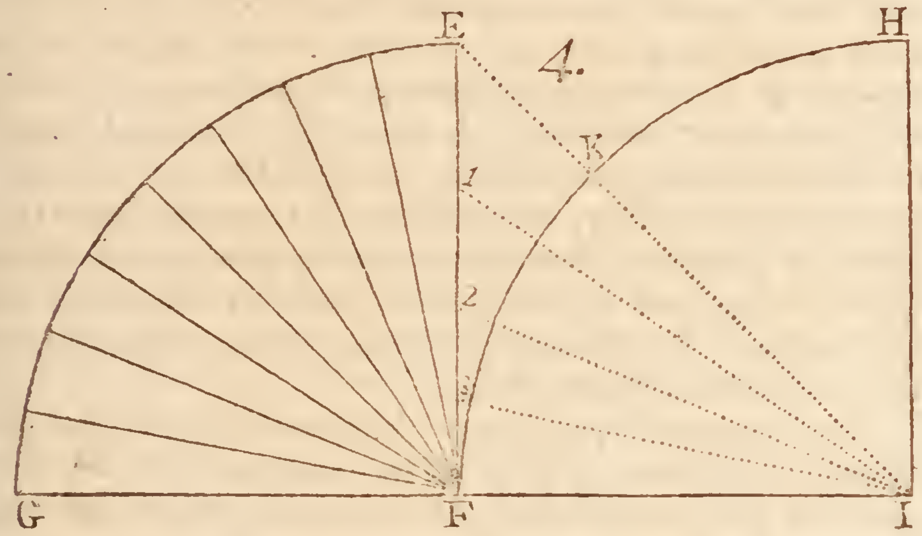
Then, in every areola, or ſmall trapezium, of the ſpace $abcd$, draw what appears contained in the correſponding areola of the original ſpace ABCD: ſo ſhall there be produced a deformed image in the ſpace $abcd$, which yet will appear in juſt proportion to an eye diſtant from it the length of EV, and raiſed above it by a height equal to VS.

It will be amuſing to contrive it ſo, that the deformed image may not repreſent a mere chaos, but ſome certain figure: thus, a river with ſoldiers, waggons, and other objects on the ſide of it, have been ſo drawn and diſtorted, that when viewed by an eye at S, it appeared like the face of a ſatyr.

An image may alſo be diſtorted mechanically, by perforating through in ſeveral places with a fine pin; then, placing it againſt a candle or lamp, obſerve where the rays, which paſs through theſe ſmall holes, fall on any plane or curve ſuperficies; for they will give the correſpondent points of the image deformed, and by means of which the deformation may be completed.

To draw an Anamorphoſis upon the convex ſurface of a cone. It appears from the conſtruction above, that we have only to make a craticular ectype upon the ſurface of the cone, which may appear equal to the craticular prototype, to an eye placed at a proper height above the vertex of the cone. Hence,

Let the baſe, or circumference, ABCD, of the cone (fig. 3) be divided by radii into any number of equal parts; and let ſome one radius be likewiſe divided into equal parts; then through each point of diviſion draw concentric circles: ſo ſhall the craticular prototype be formed.



With double the diameter AB, as a radius, deſcribe the quadrant EFG (fig. 4) ſo as the arch EG be equal to the whole periphery; then this quadrant, being plied or bent round, will form the ſuperficies of a cone, whoſe baſe is the circle.

Next divide the arch EG into the ſame number of equal parts as the craticular prototype is divided into; and draw radii from all the points of diviſion. Produce GF to I, ſo that FI be equal to FG; and from the centre I, with the radius IF, deſcribe the quadrant FKH; and draw the right line IE. Then divide the arch KF into the ſame number of equal parts as the radius of the craticular prototype is divided into; and from the centre I draw radii through all the points of diviſion, meeting EF in 1, 2, 3, &c. Laſtly, from the centre F, with the radii F1, F2, F3, &c, deſcribe concentric circles. So will the craticular ectype be formed, whoſe areolas will appear equal to each other.

Hence, what is delineated in every areola of the craticular prototype, being transferred into the areolas of the craticular ectype, the images will be diſtorted or deformed; and yet they will appear in juſt proportion to an eye elevated above the vertex at a height equal to the height of the cone itſelf.

If the chords of the quadrants be drawn in the craticular prototype, and chords of each of the 4th parts in the craticular ectype, every thing elſe remaining the ſame, there will be obtained the craticular ectype in a quadrangular pyramid.

And hence it will be eaſy to deform an image, in any other pyramid, whoſe baſe is any regular polygon.

Becauſe the illuſion is more perfect when the eye, by the contiguous objects, cannot eſtimate the diſtance of the parts of the deformed image, it is therefore proper to view it through a ſmall hole.

Anamorphoſes, or monſtrous images, may alſo be made to appear in their natural ſhape and juſt proportions, by means of mirrors of certain ſhapes, from which thoſe images are reflected again; and then they are ſaid to be reformed.

For farther particulars, ſee Wolfius's *Catoptrics and Dioptrics*, and ſome other optical authors.

ANAPHORA, in *Aſtrology*, the ſecond houſe, or that part of the heavens which is 30 degrees from the horoſcope.

The term *anaphora* is alſo ſometimes applied promiſcuouſly to ſome of the ſucceeding houſes, as the 5th, the 8th, and the 11th. In this ſenſe *anaphora* is the ſame as *epanaphora*, and ſtands oppoſed to *cataphora*.

ANASTROUS *ſigns*, in *Aſtronomy*, a name given to the *duodecatemoria*, or the 12 portions of the ecliptic, which the ſigns poſſeſſed anciently, but have ſince deſerted by the preceſſion of the equinox.

ANAXAGORAS, one of the moſt celebrated philoſophers among the ancients. He was born at Clazomene in Ionia, about the 70th Olympiad. He was a diſciple of Anaximenes; and he gave up his patrimony, to be more at leiſure for the ſtudy of philoſophy, giving lectures in that ſcience at Athens. Being perſecuted in this place, and at laſt baniſhed from it, he opened a ſchool at Lampſacum, where he was greatly honoured during his life, and ſtill more after his death, ſtatues having been erected to his memory. It is ſaid he made ſome predictions relative to the phenomena of nature, as earthquakes &c, upon which he wrote ſome treatiſes. His principal tenets may be reduced to the following:—All things were in the beginning confuſedly mixed together, without order and without motion. The principle of things is at the ſame time one and multiplex, which had the name of *homœmeries*, or ſimilar particles, deprived of life. But there is beſide this, from all eternity, another principle, an infinite and incorporeal ſpirit, who gave motion to theſe particles; in virtue of which, ſuch as are homogeneal united, and ſuch as were heterogeneal ſeparated according to their different kinds. All things being thus put into motion by the ſpirit, and every thing being united to ſuch as are ſimilar, thoſe that had a circular motion produced heavenly bodies, the lighter particles aſcending, while thoſe that were heavier deſcended. The rocks of the earth, being drawn up by the whirling force of the air, took fire, and became ſtars, beneath which the ſun and moon took their ſtations.—It was ſaid he alſo wrote upon the *Quadrature of the Circle*; the treatiſe upon which, Plutarch ſays, he compoſed during his impriſonment at Athens.

ANAXIMANDER, a very celebrated Greek philoſopher, was born at Miletus in the 42d olympiad; for, according to Apollodorus, he was 64 years of age in the 2d year of the 58th olympiad. He was one of the firſt who publicly taught philoſophy, and wrote upon philoſophical ſubjects. He was the kinſman, companion, and diſciple of Thales. He wrote alſo upon the ſphere and geometry, &c. And he carried his reſearches into nature very far, for the time in which he lived. It is ſaid that he diſcovered the obliquity of the zodiac; that he firſt publiſhed a geographical table: that he invented the gnomon, and ſet up the firſt ſun-dial in an open place at Lacedæmon. He taught, that infinity of things was the principal and univerſal element; that this infinite always preſerved its unity, but that its parts underwent changes; that all things came from it; and that all were about to return to it. By this obſcure and indeterminate principle he probably meant the chaos of other philoſophers. He held that the worlds are infinite; that the ſtars are compoſed of air and fire, which are carried about in their ſpheres, and that theſe ſpheres are gods; and that the earth is placed in the midſt of the univerſe, as in a common centre. Farther, that infinite worlds were the produce of infinity; and that corruption proceeded from ſeparation.

ANAXIMENES, an eminent Greek philoſopher, born at Miletus, the friend, ſcholar, and ſucceſſor of Anaximander. He diffuſed ſome degree of light upon the obſcurity of his maſter's ſyſtem. He made the firſt principle of things to conſiſt in the air, which he conſidered as infinite or immenſe, and to which he aſcribed a perpetual motion; that this air was the ſame as ſpirit or God, ſince the divine power reſided in it, and agitated it. The ſtars were as fiery nails in the heavens; the ſun a flat plate of fire; the earth an extended flat ſurface, &c.

ANDERSON (ALEXANDER), one of the brighteſt ornaments of the mathematical world, who flouriſhed about 200 years ago. He was born at Aberdeen in Scotland, it would ſeem towards the latter part of the 16th century, as he was profeſſor of mathematics at Paris in the early part of the 17th, where he publiſhed ſeveral ingenious works in geometry and algebra, both of his own, and of his friend Vieta's. Thus he publiſhed his "Supplementum Apollonii Redivivi; (of Ghetaldus) ſive analyſis problematis hactenus deſiderati ad Apollonii Pergæi doctrinam περι νευϛεων, a Marino Ghetaldo Patritio Raguſino hujuſque, non ita pridem reſtitutam. In qua exhibetur mechanice æqualitatum tertii gradus ſive ſolidarum, in quibus magnitudo omnino data, æquatur homogeneæ ſub altero tantum coëfficiente ignoto. Huic ſubnexa eſt variorum problematum practice." Paris, 1612, in 4to.

"Αιτιολογια: Pro Zetetico Apolloniani problematis a ſe jam pridem edito in ſupplemento Apollonii Redivivi. Ad clariſſimum & ornatiſſimum virum Marinum Ghetaldum Patritium Raguſinum. In qua ad ea quae obiter mihi perſtrinxit Ghetaldus reſpondetur, & analytices clarius detegitur." Paris, 1615, in 4to.

He publiſhed alſo,

"Franciſci Vietæ Fontenacenſis de Aequationum Recognitione & Emendatione Tractatus duo." Paris, 1615, in 4to; with a Dedication, Preface, and an Appendix, by Anderſon.

And Vieta's Angulares Sectiones, with the Demonſtrations by Anderſon.

Alexander was couſin german to a Mr. David Anderſon, of Finſhaugh, a gentleman who alſo poſſeſſed a ſingular turn for mathematical and mechanical knowledge. This mathematical genius was hereditary in the family of the Anderſons, and from them it ſeems to have been tranſmitted to their deſcendants of the name of Gregory in the ſame country: the daughter of the ſaid David Anderſon having been the mother of the celebrated mathematician James Gregory, and who herſelf firſt inſtructed her ſon James in the elements of the Mathematics, upon her obſerving in him, while yet a child, a ſtrong propenſity to thoſe ſciences.

The time either of the birth or death of our author Alexander, has not come to my knowledge.

ANDROGYNOUS, an appellation given, by aſtrologers, to ſuch of the planets as are ſometimes hot, and ſometimes cold; as mercury, which is accounted hot and dry when near the ſun, and cold and moiſt when near the moon.

ANDROMEDA, in *Aſtronomy*, a conſtellation of the northern hemiſphere, repreſenting the figure of a woman almoſt naked, her feet at a diſtance from each other, and her arms extended and chained; being one of the original 48 aſteriſms, or figures under which the ancients comprehended the ſtars, as derived to us from the Greeks, who probably had them from the Egyptians or Indians, and who, it is ſuſpected, altered their names, and accompanied them with fabulous ſtories of their own. According to them, Cepheus, the father of Andromeda, was obliged to give her up to be devoured by a monſter, to preſerve his kingdom from the plague; but that ſhe was delivered by Perſeus, who ſlew the monſter, and eſpouſed her. And the family were all tranſlated by Minerva to heaven, the mother being the conſtellation Caſſiopeia.

She is ſometimes called, in Latin, *Perſea, Mulier catenata, Virgo devota*, &c. The Arabians, whoſe religion did not permit them to draw the figure of the human body on any occaſion whatever, have changed this conſtellation into the figure of a ſea-calf. Schickard has changed the name for that of the ſcripture name *Abigail.* And Schiller has alſo changed the figure of the conſtellation, for that of a ſepulchre, and calls it the *Holy Sepulchre.*

This conſtellation contains about 27 ſtars that are viſible to the naked eye; of which the principal are, α *Andromeda's head;* β in the girdle, and called *mirach* or *mizar;* γ on the ſouth foot, and named *alamak*, and ſometimes *alhames.*

The number of ſtars placed in this conſtellation by the catalogue of Ptolemy is 23, by that of Tycho Brahe alſo 23, by that of Hevelius 47, and by that of Flamſteed 66.

ANEMOMETER, an inſtrument for meaſuring the force of the wind.

An inſtrument of this ſort, it ſeems, was firſt invented by Wolfius in the year 1708, and firſt publiſhed in his *Areometry* in 1709, alſo in the *Acta Eruditorum* of the ſame year; afterwards in his *Mathematical Dictionary*, and in his *Elem. Matheſeos.* He ſays he tried the goodneſs of it, and obſerves that the internal ſtructure may be preſerved, ſo as to meaſure the force of running water, or that of men or horſes when they draw or pull. The machine conſiſts of ſails, A, B, C, like thoſe of a wind-mill, againſt which the wind blows, and by turning them about, raiſes an arm K with a weight L upon it, to different angles of elevation, ſhewn by the index M, according to the force of the wind. (Plate III. fig. 3)

In the Philoſ. Tranſ. another anemometer is deſcribed, in which the wind being ſuppoſed to blow directly againſt a flat ſide, or board, which moves along the graduated arch of a quadrant, the number of degrees it advances ſhews the comparative forceof the wind.

In the ſame Tranſactions, for the year 1766, Mr. Alex. Brice deſcribes a method, ſucceſsfully practiſed by him, of meaſuring the velocity of the wind, by means of that of the ſhadow of clouds paſſing over a plane upon the earth.

Alſo in the ſame Tranſactions, for the year 1775, Dr. Lind gives a deſcription of a very ingenious portable *Wind-Gauge*, by which the force of the wind is eaſily meaſured; a brief deſcription of the principal parts of which here follows. This ſimple inſtrument conſiſts of two glaſs tubes, AB, CD, (Plate III. fig. 4.) which ſhould not be leſs than 8 or 9 inches long, the bore of each being about $\frac{4}{10}$ of an inch diameter, and connected together by a ſmall bent glaſs tube *ab*, only of about $\frac{1}{10}$ of an inch diameter, to check the undulations of the water cauſed by a ſudden guſt of wind. On the upper end of the leg AB is fitted a thin metal tube, which is bent perpendicularly outwards, and having its mouth open to receive the wind blowing horizontally into it. The two tubes, or rather the two branches of the tube, are connected to a ſteel ſpindle KL by ſlips of braſs near the top and bottom, by the ſockets of which at *e* and *f* the whole inſtrument turns eaſily about the ſpindle, which is fixed into a block by a ſcrew in its bottom, by the wind blowing in at the orifice at F. When the inſtrument is uſed, a quantity of water is poured in, till the tubes are about half full; then expoſing the inſtrument to the wind, by blowing in at the orifice F, it forces the water down lower in the tube AB, and raiſes it ſo much higher in the other tube; and the diſtance between the ſurfaces of the water in the two tubes, eſtimated by a ſcale of inches and parts HI, placed by the ſides of the tubes, will be the height of a column of water whoſe weight is equal to the force or momentum of the wind blowing or ſtriking againſt an equal baſe. And as a cubic foot of water weighs 1000 ounces, or $62\frac{1}{2}$ pounds, the 12th part of which is $5\frac{5}{24}$ or $5\frac{1}{5}$ pounds nearly, therefore for every inch the ſurface of the water is raiſed, the force of the wind will be equal to ſo many times $5\frac{1}{5}$ pounds on a ſquare foot. Thus, ſuppoſe the water ſtand 3 inches higher in the one tube, than in the other; then 3 times $5\frac{1}{5}$ or $15\frac{3}{5}$ pounds is equal to the preſſure or force of the wind on the ſurface of a foot ſquare.

This inſtrument of Dr. Lind's, meaſures only the force or momentum of the wind, but not its velocity. However the velocity of the wind may be deduced from its force ſo obtained, by help of ſome experiments performed by me at the Royal Military Academy, in the years 1786, 1787, and 1788; from which experiments it appears that a plane ſurface of a ſquare foot ſuffers a reſiſtance of 12 ounces from the wind, when blowing with a velocity of 20 feet per ſecond; and that the force is nearly as the ſquare of the velocity. Hence then, taking the force of $15\frac{3}{5}$ pounds, above found for the force of the wind when it ſuſtains 3 inches of water, and taking the ſquare roots of the forces, it will be, as $\sqrt{12} : \sqrt{15\frac{3}{5}} :: 20 : 22\frac{4}{5}$ the 4th proportional, that is a velocity of $22\frac{4}{5}$ feet per ſecond, or $15\frac{1}{2}$ miles per hour, is the rate or velocity at which the wind blows, when it raiſes the water 3 inches higher in the one tube than the other. And farther, as the ſaid height is as the force, and the force as the ſquare of the velocity, we ſhall have the force and velocity, correſponding to ſeveral heights of the water in the one tube, above that in the other, as in the following table.

Table of the correſponding height of water, force on a ſquare foot, and velocity of wind.

Height of water.	Force of wind.	Velocity of wind per hour.
Inches.	Pounds.	Miles.
$0\frac{1}{4}$		4·5
$0\frac{1}{2}$		6·4
1	5·2	9·0
2	10·4	12·7
3	15·6	15·5
4	20·8	18·0
5	26·0	20·1
6	31·25	22·0
7	36·5	23·8
8	41·7	25·4
9	46·9	27·0
10	52·1	28·4
11	57·3	29·8
12	62·5	31·0

In one inſtance Dr. Lind found that the force of the wind was ſuch as to be equal $34\frac{9}{10}$ pounds, on a ſquare foot; and this by proportion, in the following table, will be found to anſwer to a velocity of $23\frac{1}{4}$ miles per hour.

Mr. Leutmann improved upon Wolſius's anemometer, by placing the ſails horizontal, inſtead of vertical, which are eaſier to move, and turn what way ſoever the wind blows.

Mr. Benjamin Martin alſo (Plate III. fig. 5) improved upon the ſame. He made the axis like the fuſee of a watch, having a cord winding upon it, with two weights at the ends which make always a balance to the force of the wind on the ſails. See his Philoſ. Britan.

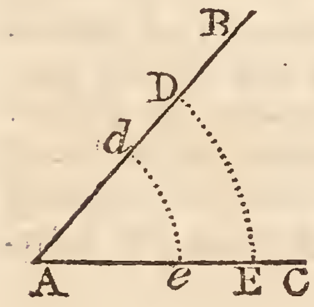
And M. D'Ons-en-Bray invented a new anemometer, which of itſelf expreſſes on paper, not only the ſeveral winds that have blown during the ſpace of 24 hours, and at what hour each began and ended, but alſo the different ſtrength or velocity of each. See Mem. Acad. Scienc. an. 1734. See alſo the article *Wind-Gauge*.

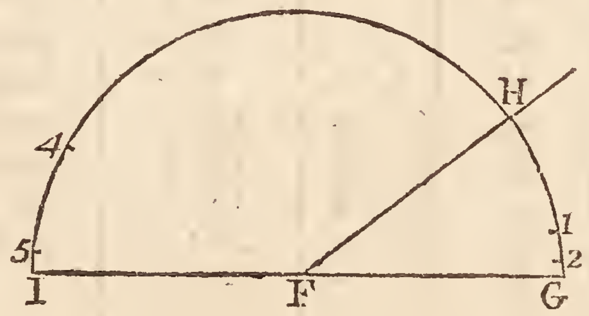
ANEMOSCOPE, is ſometimes uſed to denote a machine invented to foretell the changes of the wind, or weather; and ſometimes for an inſtrument ſhewing by an index what the preſent direction of the wind is. Of this latter ſort, it ſeems, was that uſed by the ancients, and deſcribed by Vitruvius; and we have many of them at preſent in large or public buildings, where an index withinſide a room or hall, points to the name of the quarter from whence the wind blows without; which is ſimply effected by connecting an index to the lower end of the ſpindle of a weather-cock.

It has been obſerved that hygroſcopes made of catgut, or ſuch like, prove very good anemoſcopes; ſeldom failing, by the turning of the index, to foretell the ſhifting of the wind. See accounts of two different anemoſcopes; one by Mr. Pickering, vol. 43 Philoſ. Tranſ. the other by Mr. B. Martin, vol. 2 of his Philoſ. Britan.

Otto Gueric alſo gave the title anemoſcope to a machine invented by him to foretell the change of the weather, as to rain and fair. It conſiſted of the ſmall wooden figure of a man, which roſe and fell in a glaſs tube, as the atmoſphere was more or leſs heavy. Which was only an application of the common barometer, as ſhewn by M. Couriers in the *Acta Eruditorum* for 1684.

ANGLE, *Angulus*, in *Geometry*, the opening or mutual inclination of two lines, or two planes, or three planes, meeting in a point called the vertex or angular point. Such as the angle formed by, or between, the two lines AB and AC, at the vertex or angular point A.—Alſo the two lines AB and AC, are called the *legs* or the *ſides* of the angle.





Angles are ſometimes denoted, or named, by the ſingle letter placed at the angular point, as the angle A; and ſometimes by three letters, placing always that of the vertex in the middle. The former method is uſed when only one angle has the ſame vertex; and the latter method it is neceſſary to uſe when ſeveral angles have the ſame vertex, to diſtinguiſh them from one another.

The meaſure of an angle, by which its quantity or magnitude is expreſſed, is an arch, as DE deſcribed from the centre A, with any radius at pleaſure, and contained between its legs AB and AC.—Hence angles are compared and diſtinguiſhed by the ratio of the arcs which ſubtend them, to the whole circumference of the circle; or by the number of degrees contained in the arc DE by which they are meaſured, to 360, the number of degrees in the whole circumference of the circle. And thus an angle is ſaid to be of ſo many degrees, viz, as are contained in the arc DE.

Hence it matters not, with what radius the arc is deſcribed, by which an angle is meaſured, when great or ſmall, as AD, or A*d*, or any other: for the arcs DE, *de*, being ſimilar, have the ſame ratio as their reſpective radii or circumferences, and therefore they contain the ſame number of degrees.—Hence it follows, that the quantity or magnitude of the angle remains ſtill the ſame, though the legs be ever ſo much increaſed or diminiſhed.—And thus, in ſimilar figures, the like or correſponding angles are equal.

The *taking* or *meaſuring* of *angles*, is an operation of great uſe and extent in ſurveying, navigation, geography, aſtronomy, &c. And the inſtruments chiefly uſed for this purpoſe, are quadrants, ſextants, octants, theodolites, circumferentors, &c. Mr. Hadley invented an excellent inſtrument for taking the *larger* ſort of angles, where much accuracy is required, or where the motion of the object, or any circumſtance cauſing an unſteadineſs in the common inſtruments, renders the obſervations difficult, or uncertain. And Mr. Dollond contrived an inſtrument for meaſuring *ſmall* angles. See *Hadley's Quadrant*, *Micrometer*, and the Philoſ. Tranſ. Numbers 420, 425, and vol. 48.

To meaſure the Quantity of an Angle.

1. *On paper.* Apply the centre of a protractor to the vertex A of the angle, ſo that the radius may coincide with one leg, as AB; then the degree on the arch that is cut by the other leg AC, will give the meaſure of the angle required.

Or thus, by a line of chords. Take off the chord of 60 with a pair of compaſſes; and with that radius, from the centre A, deſcribe an arc as DE. Then take this arc DE between the compaſſes, and apply the extent to the ſcale of chords, which will give the degrees in the angle as before.

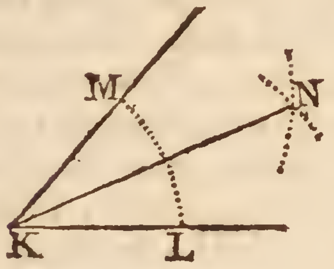
M. De Lagny gave, in ſeveral memoirs of the Royal Academy of Sciences, a new method of meaſuring angles, which he called *Goniometry*. The method conſiſts in meaſuring, with a pair of compaſſes, the arc which ſubtends the propoſed angle, not by applying its extent to a pre-conſtructed ſcale, like chords, but in the following manner: From the angular point as a centre, with a pretty large radius, deſcribe a circle, producing one leg of the angle backwards to cut off a ſemicircle; then ſearch out what part of the ſemicircle the arc is which meaſures the given angle, in this manner; viz, take the extent of this arc with a very fine pair of compaſſes, and apply it ſeveral times to the arc of the ſemicircle, to find how often it is contained, with a ſmall part remaining over; in the ſame manner take the extent of this ſmall part, and apply it to the firſt arc, to find how often it is contained in it; and what remains this 2d time, apply in like manner to the firſt remainder; then the 3d remainder apply to the 2d, and ſo on, always counting how often the laſt remainder is contained in the next foregoing, till nothing remain, or till the remainder is inſenſible, and too ſmall to be meaſured: Then, beginning at the laſt, and returning backwards, make a ſeries of fractions of which the numerators are always 1, and the denominators are the number of times each remainder is contained in its next remainder, with the fractional part more, as derived from the following remainder; then the laſt fraction, thus obtained, will ſhew what part the given angle is of 180° or the ſemicircle; and being turned into degrees &c, will be the meaſure of the angle, and nearer, it is aſſerted, than it can be obtained by any other means; whether it be meaſuring, or calculating by trigonometrical tables.—Thus, if it be required to meaſure the angle GFH: With a large radius deſcribe the ſemicircle GHI, meeting the leg FG produced in I; then take the extent of the arc GH in the compaſſes, and applying it from G upon the ſemicircle, ſuppoſe it contains 4 times to the point 4, and the part 4 I over; take 4 I and apply it from H to 1, ſo that HG contains 4 I once, and 1 G over; alſo apply this remainder to the former 4 I, and it contains 5 times, from 4 to 5, and 5 I over;

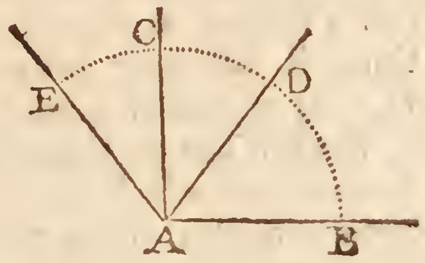
and laſtly the remainder 5 I is juſt two times contained in the former remainder 1 G or 1 2, without any remainder. Here then, the ſeries of quotients, or numbers of times contained, are 4, 1, 5, 2; therefore, beginning at the laſt, the firſt fraction is $\frac{1}{2}$, or the laſt remainder is half the preceding one; and the 2d fraction is $\frac{1}{5\frac{1}{2}}$ or $\frac{2}{11}$; the 3d is $\frac{1}{1\frac{2}{11}}$ or $\frac{11}{13}$; and the fourth is $\frac{1}{4\frac{11}{13}}$ or $\frac{13}{63}$; that is, the arc GH is $\frac{13}{63}$ of a ſemicircle, or the angle GFH is $\frac{13}{63}$ of two right angles, or of 180°, which is equivalent to $37\frac{1}{7}$ degrees, or $37° \, 8' \, 34'' \frac{2}{7}$.

2. *On the ground.* Place a ſurveying inſtrument with its centre over the angular point to be meaſured, turning the inſtrument about till o, the beginning of its arch, fall in the line or direction of one leg of the angle; then turn the index about to the direction of the other leg, and it will cut off from the arch the degrees anſwering to the given angle.

To plot or lay down any given angle, either on paper or on the ground, may be performed in the ſame manner; and the method is farther explained under the articles PLOTTING and PROTRACTING, and under the names of the ſeveral inſtruments.

To biſect a given angle, as ſuppoſe the angle LKM. From the centre K, with any radius, deſcribe the arc LM; then with the centres L and M, deſcribe two arcs interſecting in N; and draw the line KN, which will biſect the given angle LKM, dividing it into the two equal angles LKN, MKN.





To triſect an angle, ſee TRISECTION.

Pappus, in his Mathematical Collections, book 4, treats of angular ſections, but particularly and more largely, of triſections. He alſo treats of any ſection in general, in the 36th and following propoſitions.

ANGLES are of various kinds and denominations. With regard to the form of their legs, they are divided into *rectilinear*, *curvilinear*, and *mixed*.

Rectilinear, or *right-lined* ANGLE, is that whoſe legs are both right lines; as the foregoing angle CAB.

Curvilinear ANGLE, is that whoſe legs are both of them curves.

Mixt, or *mixtilinear* ANGLE, is that of which one leg is a right line, and the other a curve.

With regard to their *magnitude*, angles are again divided into *right* and *oblique*, *acute* and *obtuſe*.

Right ANGLE, is that which is formed by one line perpendicular to another; or that which is ſubtended by a quadrant of a circle. As the angle BAC.—Therefore the meaſure of a right angle is a quadrant of a circle, or 90°; and conſequently all right angles are equal to each other.

Oblique ANGLE, is a common name for any angle that is not a right one; and it is either *acute* or *obtuſe*.

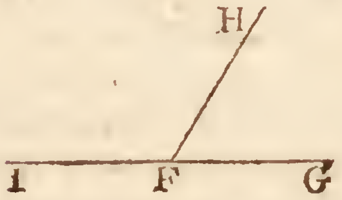
Acute ANGLE, is that which is leſs than a right angle, or leſs than 90 degrees; as the angle BAD. And

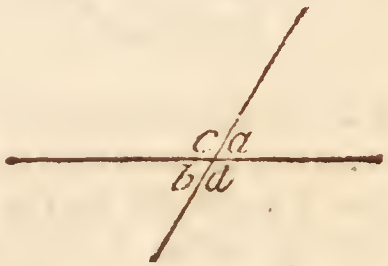
Obtuſe ANGLE, is greater than a right angle, or whoſe meaſure exceeds 90 degrees; as the angle BAE.

With regard to their ſituation in reſpect of each other, angles are diſtinguiſhed into *contiguous*, *adjacent*, *vertical*, *oppoſite*, and *alternate*.

Contiguous ANGLES, are ſuch as have the ſame vertex, and one leg common to both. As the angles BAD, CAD, which have AD common.

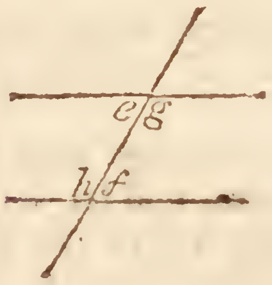
Adjacent ANGLES, are thoſe of which a leg of the one produced forms a leg of the other: as the angles GFH and IFH, which have the legs IF and FG in a ſtraight line.—Hence adjacent angles are ſupplements to each other, making together 180 degrees. And therefore if one of theſe be given, the other will be known by ſubtracting the given one from 180 degrees. Which property is uſeful in ſurveying, to find the quantity of an inacceſſible angle; viz, meaſure its adjacent acceſſible one, and ſubtract this from 180 degrees.

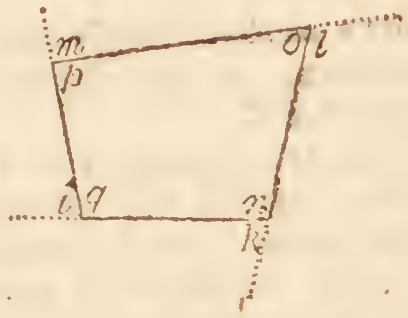




Vertical or *oppoſite* ANGLES, are ſuch as have their legs mutually continuations of each other; as the two angles *a* and *b*, or *c* and *d*.—The property of theſe is, that the vertical or oppoſite angles are always equal to each other, viz, $\angle a = \angle b$, and $\angle c = \angle d$. And hence the quantity of an inacceſſible angle of a field, &c, may be found, by meaſuring its acceſſible oppoſite angle.

Alternate ANGLES, are thoſe made on the oppoſite ſides of a line cutting two parallel lines; ſo, the angles *e* and *f*, or *g* and *h*, are alternates. And theſe are always equal to each other; viz, the $\angle e = \angle f$, or $\angle g = \angle h$.



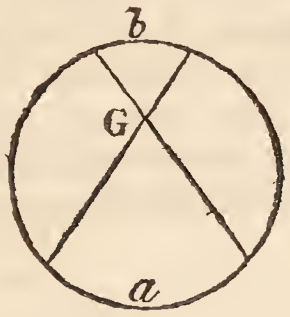


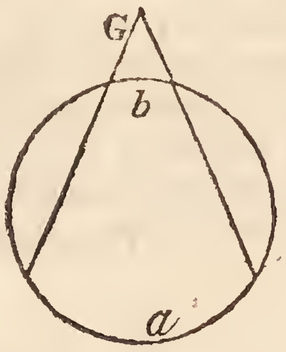
External ANGLES, are the angles of a figure made without it, by producing its ſides outwards; as the angles *i*, *k*, *l*, *m*. All the external angles of any right-lined figure, taken together, are equal to 4 right angles; and the external angle of a triangle is equal to both the internal oppoſite ones taken together; alſo any external angle of a trapezium inſcribed in a circle, is equal to the internal oppoſite angle.

Internal ANGLES, are the angles within any figure, made by the ſides of it; as the angles *n*, *o*, *p*, *q*.—In any right-lined figure, an internal angle as *n*, and its adjacent external angle *k*, together make two right angles, or 180 degrees; and all the internal angles *n*, *o*, *p*, *q*, taken together, make twice as many right angles, wanting 4 right angles; alſo any two oppoſite internal angles of a trapezium inſcribed in a circle, taken together, make two right angles, or 180 degrees.

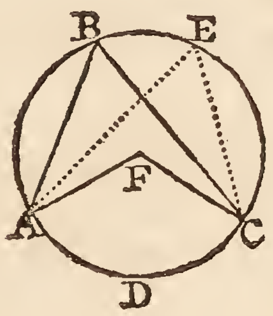
Homologous, or *like* Angles, are ſuch angles in two figures, as retain the ſame order from the firſt, in both figures.

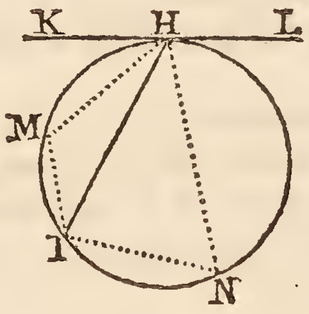
Angle *out of the centre*, as G, is one whoſe vertex is not in the centre of the circle.—And its meaſure is half the ſum $\frac{a+b}{2}$ of the arcs intercepted by its legs when it is within the circle, or half the difference $\frac{a-b}{2}$ when it is without.





Angle *at the centre*, is an angle whoſe vertex is in the centre; as the angle AFC, formed by two radii AF, FC, and meaſured by the arc ADC.—An angle at the centre, as AFC, is always double of the angle ABC at the circumference, ſtanding upon the ſame arc ADC; and all angles at the centre are equal that ſtand upon the ſame or equal arcs: alſo all angles at the centre, are proportional to the arcs they ſtand upon; and ſo alſo are all angles at the circumference.





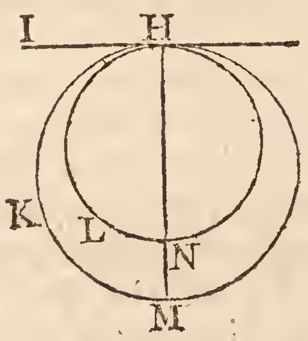
Angle *at the circumference*, is an angle whoſe vertex is ſomewhere in the circumference of a circle; as the angle ABC.

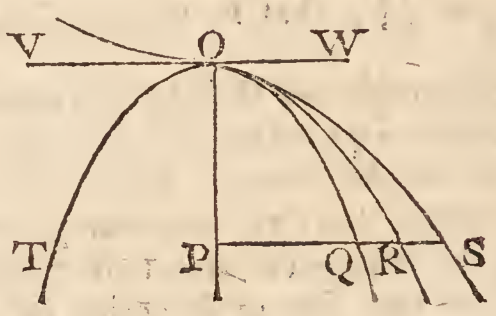
Angle *in a ſegment*, is an angle whoſe legs meet the extremities of the baſe of the ſegment, and its vertex is anywhere in its arch; as the angle B is in the ſegment ABC, or ſtanding upon the ſupplemental ſegment ADC; and is comprehended between two chords AB and BC.—An angle at the circumference is meaſured by half the arc ADC upon which it ſtands; and all the angles ABC, AEC, in the ſame ſegment, are equal to each other.

Angle *in a ſemicircle* is an angle at the circumference contained in a ſemicircle, or ſtanding upon a ſemicircle, or on a diameter.—An angle in a ſemicircle, is always a right angle; in a greater ſegment, the angle is leſs, and in a leſs ſegment the angle is greater than a right angle.

Angle *of a ſegment*, is that made by a chord with a tangent, at the point of contact. So IHK is the angle of the leſs ſegment IMH, and IHL, the angle of the greater ſegment INH.—And the meaſure of each of theſe angles, is half the alternate or ſupplemental ſegment, or equal to the angle in it; viz, the ∠ IHK = ∠ INH, and the ∠ IHL = ∠ IMH.

Angle *of a ſemicircle*, is the angle which the diameter of a circle makes with the circumference. And Euclid demonſtrates that this is leſs than a right angle, but greater than any acute angle.





Angle *of contact*, is that made by a curve line and a tangent to it, at the point of contact; as the angle IHK. It is proved by Euclid, that the angle of contact between a right line and a circle, is leſs than any right-lined angle whatever; though it does not therefore follow that it is of no magnitude or quantity. This has been the ſubject of great diſputes amongſt geometricians, in which Peletarius, Clavius, Taquet, Wallis, &c, bore a conſiderable ſhare; Peletarius and Wallis contending that it is no angle at all, againſt Clavius, who rightly aſſerts that it is not abſolutely nothing in itſelf, but only of no magnitude in compariſon with a right-lined angle, being a quantity of a different kind or nature; like as a line in reſpect to a ſurface, or a ſurface in reſpect to a ſolid, &c. And ſince his time, it has been proved by Sir I. Newton, and others, that angles of contact can be compared to each other, though not to right-lined angles, and what are the proportions which they bear to each other. Thus, the circular angles of contact IHK, IHL, are to each other in the reciprocal ſubduplicate ratio of the diameters HM, HN. And hence the circular angle of contact may be divided, by deſcribing intermediate circles, into any number of parts, and in any proportion. And if, inſtead of circles, the curves be parabolas, and the point of contact H the common vertex of their axes; the angles of contact would then be reciprocally in the ſubduplicate ratio of their parameters. But in ſuch elliptical and hyperbolical angles of contact, theſe will be reciprocally in the ſubduplicate of the ratio compounded of the ratios of the parameters, and the tranſverſe axes. Moreover, if TOQ be a common parabola, to the axis OP, and tangent VOW, and whoſe equation is $1 \times x = y^2$, or $x = y^2$, where x is the abſciſs OP, and y the ordinate PQ, the parameter being 1: and if OR, OS, &c, be other parabolas to the ſame axis, tangent, and parameter, their ordinate y being PR, or PS, &c, and their equations $x = y^3$, $x = y^4$, $x = y^5$, &c: then the ſeries of angles of contact will be in ſucceſſion infinitely greater than each other, viz, the angle of contact WOQ infinitely greater than WOR, and this infinitely greater than WOS, and ſo on infinitely.

And farther, between the angles of contact of any two of this kind, may other angles of contact be found *ad infinitum*, which ſhall infinitely exceed each other, and yet the greateſt of them be infinitely leſs than the ſmalleſt right-lined angle. So alſo $x^2 = y^3$, $x^3 = y^4$, $x^4 = y^5$, &c, denote a ſeries of curves, of which every ſucceeding one makes an angle with its tangent, infinitely greater than the preceding one; and the leaſt of theſe, viz, that whoſe equation is $x^2 = y^3$, or the ſemicubical parabola, is infinitely greater than any circular angle of contact.

ANGLES are again divided into *plane*, *ſpherical*, and *ſolid*.

Plane ANGLES, are all thoſe above treated of; which are defined by the inclination of two lines in a plane, meeting in a point.

Spherical ANGLE, is an angle formed on the ſurface of a ſphere by the interſection of two great circles; or, it is the inclination of the planes of the two great circles.

The meaſure of a ſpherical angle, is the arc of a great circle of the ſphere, intercepted between the two planes which form the angle, and which cuts the ſaid planes at right angles. For their properties, &c, ſee SPHERE, SPHERICAL, and SPHERICAL TRIGONOMETRY.

Solid ANGLE, is the mutual inclination of more than two planes, or plane angles, meeting in a point, and not contained in the ſame plane; like the angles or corners of ſolid bodies. For their meaſure, properties, &c, ſee SOLID *Angle*.

Angles of other leſs uſual kinds and denominations, are alſo to be found in ſome books of Geometry. As,

Horned ANGLE, *angulus cornutus*, that which is made by a right line, whether a tangent or ſecant, with the circumference of a circle.

Lunular ANGLE, *angulus lunularis*, is that which is formed by the interſection of two curve lines, the one concave, and the other convex.

Ciſſoid ANGLE, *angulus ciſſoides*, the inner angle made by two ſpherical convex lines interſecting each other.

Siſtroid ANGLE, *angulus ſiſtroides*, is that which is in form of a ſiſtrum.

Pelecoid ANGLE, *angulus pelecoides*, is that in form of a hatchet.

ANGLE, in *Trigonometry*. See TRIANGLE, TRIGONOMETRY, SINE, TANGENT, *&c*.

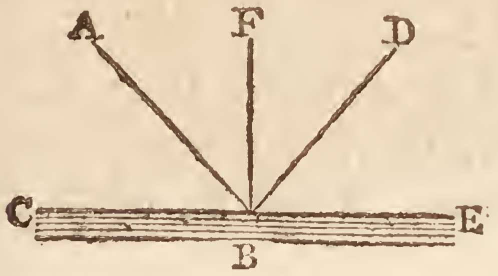
ANGLE, in *Mechanics*.—*Angle of Direction*, is that which is comprehended between the lines of direction of two conſpiring forces.

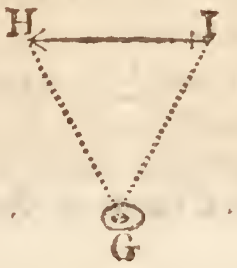
ANGLE *of Elevation*, is that which is comprehended between the line of direction and any plane upon which the projection is made, whether horizontal or oblique.

ANGLE *of Incidence*, is that made by the line of direction of an impinging body, at the point of impact. As the angle ABC.

ANGLE *of Reflection*, is that made by the line of direction of the reflected body, at the point of impact. As the angle DBE.

Inſtead of the angles of incidence and reflection being eſtimated from the plane on which the body impinges, ſometimes the complements of theſe are underſtood, viz, as eſtimated from a perpendicular to the reflecting plane; as the two angles ABF and DBF.





ANGLE in *Optics*.—*Viſual* or *Optic Angle*, is the angle included between the two rays drawn from the two extreme points of an object to the centre of the pupil of the eye: as the angle HGI. The apparent magnitude of objects is greater or leſs, according to the angle under which they appear.—Objects ſeen under the ſame or an equal angle, always appear equal.—The leaſt *viſible* angle, or leaſt angle under which a body can be ſeen, according to Dr. Hook, is one minute; but Dr. Jurin ſhews, that at the time of his debate with Hevelius on this ſubject, the latter could probably diſcover a ſingle ſtar under ſo ſmall an angle as 20″. But bodies are viſible under ſmaller angles as they are more bright or luminous. Dr. Jurin ſtates the grounds of this controverſy, and diſcuſſes the queſtion at large, in his Eſſay upon diſtinct and indiſtinct Viſion, publiſhed in Smith's Optics, pa. 148, *& ſeq*.

ANGLE *of the interval*, of two places, is the angle ſubtended by two lines directed from the eye to thoſe places.

ANGLE of *incidence*, or *reflection*, or *refraction*, &c. See the reſpective words INCIDENCE, REFLECTION, REFRACTION, &c.

ANGLE in *Aſtronomy*.—*Angle of Commutation*. See COMMUTATION.

ANGLE *of elongation*, or *Angle at the Earth*. See ELONGATION.

Parallactic ANGLE, or the *parallax*, is the angle made at the centre of a ſtar, the ſun, &c, by two lines drawn, the one to the centre of the earth, and the other to its ſurface. See PARALLACTIC, and PARALLAX.

ANGLE *of the poſition of the ſun*, *of the ſun's apparent ſemi-diameter*, &c. See the reſpective words.

ANGLE *at the ſun*, is the angle under which the diſtance of a planet from the ecliptic, is ſeen from the ſun.

ANGLE *of the Eaſt*. See NONAGESIMAL.

ANGLE *of obliquity*, of the ecliptic, or the angle of inclination of the axis of the earth, to the axis of the ecliptic, is now nearly 23° 28′. See OBLIQUITY, and ECLIPTIC.

ANGLE *of longitude*, is the angle which the circle of a ſtar's longitude makes with the meridian, at the pole of the ecliptic.

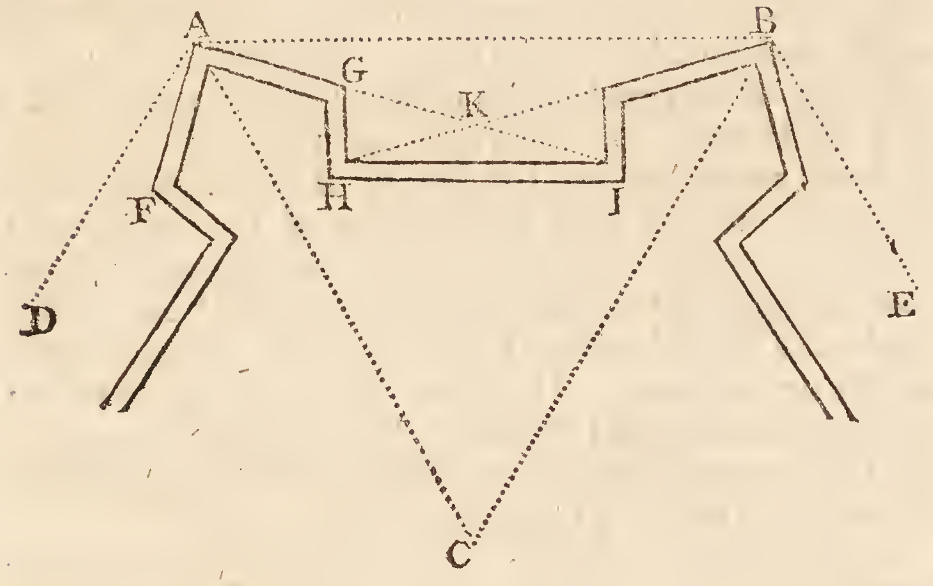
ANGLE *of right aſcenſion*, is the angle which the circle of a ſtar's right aſcenſion makes with the meridian at the pole of the equator.

ANGLE in *Navigation*. ANGLE *of the rhumb*, or *loxodromic angle*. See RHUMB and LOXODROMIC.

ANGLES, in *Fortification*, are underſtood of thoſe formed by the ſeveral lines uſed in fortifying, or making a place defenſible.

Theſe are of two ſorts; *real* and *imaginary*.—*Real angles* are thoſe which actually exiſt and appear in the works. Such as the *flanked angle*, the *angle of the epaule*, *angle of the flank*, and the *re-entering angle of the counterſcarp*. Imaginary, or *occult angles*, are thoſe which are only ſubſervient to the conſtruction, and which exiſt no more after the fortification is drawn. Such as the *angle of the centre*, *angle of the polygon*, *flanking angle*, *ſaliant angle of the counterſcarp*, &c.

ANGLE *of*, or *at*, *the centre*, is the angle formed at the centre of the polygon, by two radii drawn from the centre to two adjacent angles, and ſubtended by a ſide of of it, as the angle ACB. This is found by dividing 360 degrees by the number of ſides in the regular polygon.



Angle *of the Polygon*, is the angle intercepted between two ſides of the polygon; as DAB, or ABE. This is the ſupplement of the angle at the centre, and is therefore found by ſubtracting the angle C from 180 degrees.

Angle *of the Triangle*, is half the angle of the polygon; as CAB or CBA; and is therefore half the ſupplement of the angle C at the centre.

Angle *of the Baſtion*, is the angle FAG made by the two faces of the baſtion. And is otherwiſe called the flanked angle.

Diminiſhed Angle, is the angle BAG made by the meeting of the exterior ſide of the polygon with the face AG of the baſtion.

Angle *of the curtin*, or *of the flank*, is the angle GHI made between the curtin and the flank.

Angle *of the epaule*, or *ſhoulder*, is the angle AGH made by the flank and the face of the baſtion.

Angle *of the tenaille*, or *exterior flanking angle*, is the angle AKB made by the two raſant lines of defence, or the faces of two baſtions produced.

Angle *of the counterſcarp*, is the angle made by the two ſides of the counterſcarp, meeting before the middle of the curtin.

Angle, *flanking inward*, is the angle made by the flanking line with the curtin.

Angle *forming the flank*, is that conſiſting of one flank and one demigorge.

Angle *forming the face*, is that compoſed of one flank and one face.

Angle *of the moat*, is that made before the curtin, where it is interſected.

Re-entering, or *re-entrant* Angle, is that whoſe vertex is turned inwards, towards the place; as H or I.

Saliant, or *ſortant* Angle, is that turned outwards, advancing its point towards the field; as A or G.

Dead Angle, is a re-entering angle, which is not flanked or defended.

Angle *of a wall*, in *Architecture*, is the point or corner where the two ſides or faces of a wall meet.

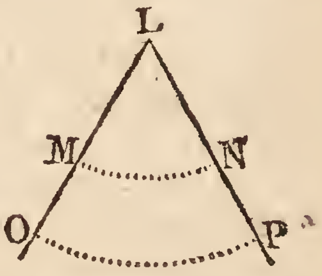
Angles, in *Aſtrology*, denote certain houſes of a figure, or ſcheme of the heavens. So the horoſcope of the firſt houſe, is termed the *angle of the eaſt*.

ANGUINEAL *Hyperbola*, a name given by Sir I. Newton to four of his curves of the ſecond order, viz, ſpecies 33, 34, 35, 36, expreſſed by the equation $xy^2 + ey = -ax^3 + bx^2 + cx + d$; being hyperbolas of a ſerpentine figure. See Curves.

ANGULAR, ſomething relating to, or that hath angles.

At a diſtance, angular bodies appear round; the angles and ſmall inequalities diſappearing at a much leſs diſtance than the bulk of the body.

ANGULAR *Motion*, is the motion of a body which moves circularly about a point; or the variation in the angle deſcribed by a line, or radius, connecting a body with the centre about which it moves.—Thus, a pendulum has an angular motion about its centre of motion; and the planets have an angular motion about the ſun.—Two moveable points M and O, of which the one deſcribes the arc MN, and the other the arc OP, in the ſame time, have an equal, or the ſame angular motion, although the real motion of the point O be much greater than that of the point M, viz, as the arc OP is greater than the arc MN. The angular motions of revolving bodies, as of the planets about the ſun, are reciprocally proportional to their periodic times. And they are alſo, as their real or abſolute motions directly, and as their radii of motion inverſely.



Angular *motion*, is alſo a kind of compound motion compoſed of a circular and a rectilinear motion; like the wheel of a coach, or other vehicle.

ANIMATED *needle*, a needle touched with a magnet or load-ſtone.

ANNUAL, in *Aſtronomy*, ſomething that returns every year, or which terminates with the year.

Annual *motion of the earth*. See Earth.

Annual *argument of longitude*. See Argument.

Annual *epacts*. See Epact.

Annual *equation* of the mean motion of the ſun and moon, and of the moon's apogee and nodes. See Equation.

ANNUITIES, a term for any periodical income, ariſing from money lent, or from houſes, lands, ſalaries, penſions, &c; payable from time to time; either annually, or at other intervals of time.

Annuities may be divided into ſuch as are *certain*, and ſuch as depend on ſome *contingency*, as the continuance of a life, &c.

Annuities are alſo divided into annuities in *poſſeſſion*, and annuities in *reverſion*; the former meaning ſuch as have commenced; and the latter ſuch as will not commence till ſome particular event has happened, or till ſome given period of time has elapſed.

Annuities may be farther conſidered as payable either *yearly*, or *half yearly*, or *quarterly*, &c.

The *preſent value* of an annuity, is that ſum, which, being improved at intereſt, will be ſufficient to pay the annuity.

The *preſent value* of an *annuity certain*, payable yearly, is calculated in the following manner.—Let the annuity be 1, and let r denote the amount of 1*l.* for a year, or 1*l.* increaſed by its intereſt for one year. Then, 1 being the preſent value of the ſum r, and having to find the preſent value of the ſum 1, it will be, by proportion thus, $r : 1 :: 1 : \frac{1}{r}$ the preſent value of 1*l.* due a year hence. In like manner $\frac{1}{r^2}$ will be the preſent value of 1*l.* due 2 years hence; for $r : 1 :: \frac{1}{r} : \frac{1}{r^2}$. In like manner $\frac{1}{r^3}, \frac{1}{r^4}, \frac{1}{r^5}$, &c, will be the preſent value of 1*l.* due at the end of 3, 4, 5, &c, years reſpectively; and in general, $\frac{1}{r^n}$ will be the value of 1*l.* to be received after the expiration of *n* years. Conſequently the ſum of all theſe, or $\frac{1}{r} + \frac{1}{r^2} + \frac{1}{r^3} + \frac{1}{r^4} +$ &c, continued to *n* terms, will be the preſent value of all the *n* years annuities. And the value of the perpetuity, is the ſum of the ſeries continued *ad infinitum*.

But this ſeries, it is evident, is a geometrical progreſſion, whoſe firſt term and common ratio are each $\frac{1}{r}$, and the number of its terms *n*; and therefore the ſum *s* of all the terms, or the preſent value of all the annual payments, will be $s = \frac{1}{r-1} - \frac{1}{r-1} \times \frac{1}{r^n}$.

When the annuity is a perpetuity, it is plain that the laſt term $\frac{1}{r^n}$ vaniſhes, and therefore $\frac{1}{r-1} \times \frac{1}{r^n}$ alſo vaniſhes; and conſequently the expreſſion becomes barely $s = \frac{1}{r-1}$; that is, any annuity divided by its intereſt for one year, is the value of the perpetuity. So, if the rate of intereſt be 5 per cent; then $\frac{100}{5} = 20$ is the value of the perpetuity at 5 per cent. Alſo $\frac{100}{4} = 25$ is the value of the perpetuity at 4 per cent. And $\frac{100}{3} = 33\frac{1}{3}$ is the value of the perpetuity at 3 per cent. intereſt. And ſo on.

If the annuity is not to be entered on immediately, but after a certain number of years, as *m* years; then the preſent value of the reverſion is equal to the difference between two preſent values, the one for the firſt term of *m* years, and the other for the end of the laſt term *n*: that is, equal to the difference between $\frac{1}{r-1} - \frac{1}{r-1} \times \frac{1}{r^n}$ and $\frac{1}{r-1} - \frac{1}{r-1} \times \frac{1}{r^m}$, or $= \frac{1}{r-1} \times \overline{\frac{1}{r^m} - \frac{1}{r^n}}$.

Annuities certain differ in value, as they are made payable *yearly*, *half-yearly*, or *quarterly*. And by proceeding as above, uſing the intereſt or amount of a half year, or a quarter, as thoſe for the whole year were uſed, the following ſet of theorems will ariſe; where *r* denotes, as before, the amount of 1*l.* and its intereſt for a year, and *n* the number of years, during which, any annuity is to be paid; alſo P denotes the perpetuity $\frac{1}{r-1}$, Y denotes $\frac{1}{r-1} - \frac{1}{r-1} \times \frac{1}{r^n}$ the value of the annuity ſuppoſed payable yearly, H the value of the ſame when it is payable half-yearly, and Q the value when payable quarterly; or univerſally, M the value when it is payable every *m* part of a year.

THEOR. 1. $Y = P - P \times \left(\frac{1}{r}\right)^n$.

THEOR. 2. $H = P - P \times \left(\frac{2}{r+1}\right)^{2n}$.

THEOR. 3. $Q = P - P \times \left(\frac{4}{r+3}\right)^{4n}$.

THEOR. 4. $M = P - P \times \left(\frac{m}{r+m-1}\right)^{mn}$.

Example 1.

Let the rate of intereſt be 4 per cent, and the term 5 years; and conſequently $r = 1{\cdot}04$, $n = 5$, $P = 25$; alſo let $m = 12$, or the intereſt payable monthly in theorem 4: then the preſent value of ſuch annuity of 1*l.* a year, for 5 years, according as it is ſuppoſed payable 1*l.* yearly, or $\frac{1}{2}$*l.* every half year, or $\frac{1}{4}$*l.* every quarter, or $\frac{1}{12}$*l.* every month or $\frac{1}{12}$th part of a year, will be as follows:

$Y = 25 - 25 \times {\cdot}821928 = 4{\cdot}4518$
$H = 25 - 25 \times {\cdot}820348 = 4{\cdot}4913$
$Q = 25 - 25 \times {\cdot}819543 = 4{\cdot}5114$
$M = 25 - 25 \times {\cdot}818996 = 4{\cdot}5251$

EXAMPLE 2. Suppoſing the annuity to continue 25 years, the rate of intereſt and every thing elſe being as before; then the values of the annuities for 25 years will be

$Y = 25 - 25 \times {\cdot}375118 = 15{\cdot}6221$
$H = 25 - 25 \times {\cdot}371527 = 15{\cdot}7118$
$Q = 25 - 25 \times {\cdot}369709 = 15{\cdot}7573$
$M = 25 - 25 \times {\cdot}368477 = 15{\cdot}7881$

EXAMPLE 3. And if the term be 50 years, the values will be

$Y = 25 - 25 \times {\cdot}140713 = 21{\cdot}4822$
$H = 25 - 25 \times {\cdot}138032 = 21{\cdot}5492$
$Q = 25 - 25 \times {\cdot}136685 = 21{\cdot}5829$
$M = 25 - 25 \times {\cdot}135775 = 21{\cdot}6056$

EXAMPLE 4. Alſo if the term be 100 years, the values will be

$Y = 25 - 25 \times {\cdot}019800 = 24{\cdot}5050$
$H = 25 - 25 \times {\cdot}019053 = 24{\cdot}5237$
$Q = 25 - 25 \times {\cdot}018683 = 24{\cdot}5329$
$M = 25 - 25 \times {\cdot}018435 = 24{\cdot}5391$

Hence the difference in the value by making periods of payments ſmaller, for any given term of years, is the more as the intervals are ſmaller, or the periods more frequent. The ſame difference is alſo variable, both as the rate of intereſt varies, and alſo as the whole term of years *n* varies; and, for any given rate of intereſt, it is evident that the difference, for any periods m of payments, firſt increaſes from nothing as the term n increaſes, when n is 0, to ſome certain finite term or value of n, when the difference D is the greateſt or a maximum; and that afterwards, as n increaſes more, that difference will continually decreaſe to nothing again, and vaniſh when n is infinite: alſo the term or value of n, for the maximum of the difference, will be different according to the periods of payment, or value of m. And the general value of n, when the difference is a maximum between the yearly payments and the payments of m times in a year, is expreſſed by this formula, viz,

$$n = \frac{l.\dfrac{ml.\frac{m+r-1}{m}}{l.\,r}}{ml.\frac{m+r-1}{m} - l.\,r},$$ where $l.$ denotes the logarithm of the quantity following it. Hence, taking the different values of m, viz, 2 for half years, 4 for quarters, 12 for monthly payments, &c, and ſubſtituting in the general formula, the term or value of n for each caſe, when the difference in the preſent worths of the annuities, will be as follows, reckoning intereſt at 4 per cent, viz,

$$n = \frac{l.\dfrac{2l.\frac{r+1}{2}}{l.\,r}}{2l.\frac{r+1}{2} - l.\,r} = 25·3777 \text{ Years for half-yearly payments,}$$

$$n = \frac{l.\dfrac{4l.\frac{r+3}{4}}{l.\,r}}{4l.\frac{r+3}{4} - l.\,r} = 25·3200 \text{ for quarterly payments,}$$

$$n = \frac{l.\dfrac{12l.\frac{r+11}{12}}{l.\,r}}{12l.\frac{r+11}{12} - l.\,r} = 25·2643 \text{ for monthly payments.}$$

Annuities may alſo be conſidered as in arrears, or as forborn, for any number of years; in which caſe each payment is to be conſidered as a ſum put out to intereſt for the remainder of the term after the time it becomes due. And as 1*l.* due at the end of 1 year, amounts to r at the end of another year, and to r^2 at the end of the 3d year, and to r^3 at the end of the 4th year, and ſo on; therefore by adding always the laſt year's annuity, or 1, to the amounts of all the former years, the ſum of all the annuities and their intereſts, will be the ſum of the following geometrical ſeries, $1 + r + r^2 + r^3 + r^4$ - - - - - - - to r^{n-1}, continued till the laſt term be r^{n-1}, or till the number of terms be n, the number of years the annuity is forborn. But the ſum of this geometrical progreſſion is $\frac{r^n - 1}{r - 1}$, which therefore is the amount of 1*l.* annuity forborn for n years. And this quantity being multiplied by any other annuity a, inſtead of 1, will produce the amount for that other annuity.

But the amounts of annuities, or their preſent values, are eaſieſt found by the two following tables of numbers for the annuity of 1*l.* ready computed from the foregoing principles.

TABLE I.

The Amount of an Annuity of 1*l. at Comp. Intereſt.*

Yrs.	at 3 per cent.	3½ per cent.	4 per cent.	4½ per cent.	5 per cent.	6 per cent.
1	1·00000	1·00000	1·00000	1·00000	1·00000	1·00000
2	2·03000	2·03500	2·04000	2·04500	2·05000	2·06000
3	3·09090	3·10623	3·12160	3·13703	3·15250	3·18360
4	4·18363	4·21494	4·24646	4·27819	4·31013	4·37462
5	5·30914	5·36247	5·41632	5·47071	5·52563	5·63709
6	6·46841	6·55015	6·63298	6·71689	6·80191	6·97532
7	7·66246	7·77941	7·89829	8·01915	8·14201	8·39384
8	8·89234	9·05169	9·21423	9·38001	9·54911	9·89747
9	10·15911	10·36850	10·58280	10·80211	11·02656	11·49132
10	11·46388	11·73139	12·00611	12·28821	12·57789	13·18079
11	12·80780	13·14199	13·48635	13·84118	14·20679	14·97164
12	14·19203	14·60196	15·02581	15·46403	15·91713	16·86994
13	15·61779	16·11303	16·62684	17·15991	17·71298	18·88214
14	17·08632	17·67699	18·29191	18·93211	19·59863	21·01507
15	18·59891	19·29568	20·32359	20·78405	21·57856	23·27597
16	20·15688	20·97103	21·82453	22·71934	23·65749	25·67253
17	21·76159	22·70502	23·69751	24·74171	25·84037	28·21288
18	23·41444	24·49969	25·64541	26·85508	28·13238	30·90565
19	25·11687	26·35718	27·67123	29·06356	30·53900	33·75999
20	26·87037	28·27968	29·77808	31·37142	33·06595	36·78559
21	28·67649	30·26947	31·96920	33·78314	35·71925	39·99273
22	30·53678	32·32890	34·24797	36·30338	38·50521	43·39229
23	32·45288	34·46041	36·61789	38·93703	41·43048	46·99583
24	34·42647	36·66653	39·08260	41·68920	44·50200	50·81558
25	36·45926	38·94986	41·64591	44·56521	47·72710	54·86451
26	38·55304	41·31310	44·31174	47·57064	51·11345	59·15638
27	40·70963	43·75906	47·08421	50·71132	54·66913	63·70577
28	42·93092	46·29063	49·96758	53·99333	58·40258	68·52811
29	45·21885	48·91080	52·96629	57·42303	62·32271	73·63980
30	47·57542	51·62268	56·08494	61·00707	66·43885	79·05819
31	50·00268	54·42947	59·32834	64·75239	70·76079	84·80168
32	52·50276	57·33450	62·70147	68·66625	75·29883	90·88978
33	55·07784	60·34121	66·20953	72·75623	80·06377	97·34316
34	57·73018	63·45315	69·85791	77·03026	85·06696	104·18375
35	60·46208	66·67401	73·65222	81·49662	90·32031	111·43478
36	63·27594	70·00760	77·59831	86·16397	95·83632	119·12087
37	66·17422	73·45787	81·70225	91·04134	101·62814	127·26812
38	69·15945	77·02889	85·97034	96·13820	107·70955	135·90421
39	72·23423	80·72491	90·40915	101·46442	114·09502	145·05846
40	75·40126	84·55028	95·02552	107·03032	120·79977	154·76197
41	78·66330	88·50954	99·82654	112·84669	127·83976	105·04768
42	82·02320	92·60737	104·81960	118·92479	135·23175	175·95054
43	85·48389	96·84863	110·01238	125·27640	142·99334	187·50758
44	89·04841	101·23833	115·41288	131·91384	151·14301	199·75803
45	92·71986	105·78167	121·02939	138·84997	159·70016	212·74351
46	96·50146	110·48403	126·87057	146·09821	168·68516	226·50812
47	100·39650	115·35097	132·94539	153·67263	178·11942	241·09861
48	104·40840	120·38826	139·26321	161·58790	188·02539	256·56453
49	108·54065	125·60185	145·83373	169·85936	198·42666	272·95840
50	112·79687	130·99791	152·66708	178·50303	209·34800	290·33590
51	117·18077	136·58284	159·77377	187·53566	220·81540	308·75606
52	121·69620	142·36324	167·16472	196·97477	232·85617	328·28142
53	126·34708	148·34595	174·85131	206·83863	245·49897	348·97831
54	131·13750	154·53806	182·84536	217·14637	258·77392	370·91701

Table II.

The preſent Value of an Annuity of 1l.

Yrs.	at 3 per cent.	3½ per cent.	4 per cent.	4½ per cent.	5 per cent.	6 per cent.
1	0·97087	0·96618	0·96154	0·95694	0·95238	0·94340
2	1·91347	1·89969	1·88610	1·87267	1·85941	1·83339
3	2·82861	2·80164	2·77509	2·74896	2·72325	2·67301
4	3·71710	3·67308	3·62990	3·58753	3·54595	3·46511
5	4·57971	4·51505	4·45182	4·38998	4·32948	4·21236
6	5·41719	5·32855	5·24214	5·15787	5·07569	4·91732
7	6·23028	6·11454	6·00205	5·89270	5·78637	5·58238
8	7·01969	6·87396	6·73274	6·59589	6·46321	6·20979
9	7·78611	7·60769	7·43533	7·26879	7·10782	6·80169
10	8·53020	8·31661	8·11090	7·91272	7·72173	7·36009
11	9·25262	9·00155	8·76048	8·52892	8·30541	7·88687
12	9·95400	9·66333	9·38507	9·11858	8·86325	8·38384
13	10·63496	10·30274	9·98565	9·68285	9·39357	8·85268
14	11·29607	10·92052	10·56312	10·22283	9·89864	9·29498
15	11·93794	11·51741	11·11839	10·73955	10·37966	9·71225
16	12·56110	12·09412	11·65230	11·23402	10·83777	10·10590
17	13·16612	12·65132	12·16567	11·70719	11·27407	10·47726
18	13·75351	13·18968	12·65930	12·15999	11·68959	10·82760
19	14·32380	13·70984	13·13394	12·59329	12·08532	11·15812
20	14·87747	14·21240	13·59033	13·00794	12·46221	11·46992
21	15·41502	14·69797	14·02916	13·40472	12·82115	11·76408
22	15·93692	15·16712	14·45112	13·78442	13·16300	12·04158
23	16·44361	15·62041	14·85684	14·14777	13·48857	12·30338
24	16·93554	16·05837	15·24696	14·49548	13·79864	12·55036
25	17·41315	16·48151	15·62208	14·82821	14·09394	12·78336
26	17·87684	16·89035	15·98277	15·14661	14·37519	13·00317
27	18·32703	17·28536	16·32959	15·45130	14·64303	13·21053
28	18·76411	17·66702	16·66306	15·74287	14·89813	13·40616
29	19·18845	18·03577	16·98371	16·02189	15·14107	13·59072
30	19·60044	18·39205	17·29203	16·28889	15·37245	13·76483
31	20·00043	18·73628	17·58849	16·54439	15·59281	13·92909
32	20·38877	19·06887	17·87355	16·78889	15·80268	14·08404
33	20·76579	19·39021	18·14765	17·02286	16·00255	14·23023
34	21·13184	19·70068	18·41120	17·24676	16·19290	14·36814
35	21·48722	20·00066	18·66461	17·46101	16·37419	14·49825
36	21·83225	20·29049	18·90828	17·66604	16·54685	14·62099
37	22·16724	20·57053	19·14258	17·86224	16·71129	14·73678
38	22·49246	20·84109	19·36786	18·04999	16·86789	14·84602
39	22·80822	21·10250	19·58448	18·22966	17·01704	14·94907
40	23·11477	21·35507	19·79277	18·40158	17·15909	15·04630
41	23·41240	21·59910	19·99305	18·56611	17·29437	15·13802
42	23·70136	21·83488	20·18563	18·72355	17·42321	15·22454
43	23·98190	22·06269	20·37079	18·87421	17·54591	15·30617
44	24·25427	22·28279	20·54884	19·01838	17·66277	15·38318
45	24·51871	22·49545	20·72004	19·15635	17·77407	15·45583
46	24·77545	22·70092	20·88465	19·28837	17·88007	15·52437
47	25·02471	22·89944	21·04294	19·41471	17·98102	15·58903
48	25·26671	23·09124	21·19513	19·53561	18·07716	15·65003
49	25·50166	23·27656	21·34147	19·65130	18·16872	15·70757
50	25·72976	23·45562	21·48218	19·76201	18·25593	15·76186
51	25·95123	23·62862	21·61749	19·86795	18·33898	15·81308
52	26·16624	23·79576	21·74758	19·96933	18·41807	15·86139
53	26·37499	23·95726	21·87267	20·06634	18·49340	15·90697
54	26·57766	24·11330	21·99296	20·15918	18·56515	15·94998

THE USE OF TABLE I.

To find the Amount of an annuity forborn any number of years. Take out the amount from the 1ſt table, for the propoſed years and rate of intereſt; then multiply it by the annuity in queſtion; and the product will be its amount for the ſame number of years, and rate of intereſt.

And the converſe to find the rate or time.

Exam. 1. To find how much an annuity of 50l. will amount to in 20 years at 3½ per cent. compound intereſt.—On the line of 20 years, and in the column of 3½ per cent, ſtands 28·27968, which is the amount of an annuity of 1l. for the 20 years; and therefore 28·27968 multiplied by 50, gives 1413·984l. or 1413l. 19s. 8d. for the anſwer.

Exam. 2. In what time will an annuity of 20l. amount to 1000l. at 4 per cent. compound intereſt?—Here the amount of 1000l. divided by 20l. the annuity, gives 50, the amount of 1l. annuity for the ſame time and rate. Then, the neareſt tabular number in the column of 4 per cent. is 49·96758, which ſtanding on the line of 28, ſhews that 28 years is the anſwer.

Exam. 3. If it be required to find at what rate of intereſt an annuity of 20l. will amount to 1000l. forborn for 28 years.—Here 1000 divided by 20 gives 50 as before. Then looking along the line of 28 years, for the neareſt to this number 50, I find 49·96758 in the column of 4 per cent. which is therefore the rate of intereſt required.

THE USE OF TABLE II.

Exam. 1. To find the preſent value of an annuity of 50l. which is to continue 20 years, at 3½ per cent.—By the table, the preſent value of 1l. for the ſame rate and time, is 14·21240; therefore 14·2124 × 50 = 710·62l. or 710l. 12s. 4d. is the preſent value ſought.

Exam. 2. To find the preſent value of an annuity of 20l. to commence 10 years hence, and then to continue for 40 years, or to terminate 50 years hence, at 4 per cent. intereſt.—In ſuch caſes as this, it is plain we have to find the difference between the preſent values of two equal annuities, for the two given times; which therefore will be effected by ſubtracting the tabular value of the one term from that of the other, and multiplying by the annuity. Thus,

tabular value for 50 years	21·48218
tabular value for 10 years	8·11090
the difference	13·37128
mult. by -	20
gives -	267·4256
or	267l. 8s. 6d. the anſwer.

The foregoing obſervations, rules, and tables, contain all that is important in the doctrine of *annuities certain*. And for farther information, reference may be had to arithmetical writings, particularly Malcolm's Arithmetic, page 595; Simpſon's Algebra, ſect. 16; Dodſon's Mathematical Repoſitory, page 298, &c; Jones's Synopſis, ch. 10; Philoſ. Tranſ. vol. lxvi, page 109.

For what relates to the doctrine of *annuities on lives*, ſee ASSURANCE, COMPLEMENT, EXPECTATION, LIFE ANNUITIES, REVERSIONS, &c.

ANNULETS, in *Architecture*, are ſmall ſquare members, in the Doric capital, placed under the quarter round.

Annulet is alſo uſed for a narrow flat moulding, common to other parts of a column, as well as to the capital; and ſo called, becauſe it encompaſſes the column around. In which ſenſe annulet is frequently uſed for *baguette*, or little *aſtragal.*

ANNULUS, a ſpecies of VOLUTA. See alſo RING.

ANOMALISTICAL *Year*, in *Aſtronomy*, called alſo *periodical year*, is the ſpace of time in which the earth, or a planet, paſſes through its orbit. The anomaliſtical, or common year, is ſomewhat longer than the tropical year; by reaſon of the preceſſion of the equinox.

And the apſes of all the planets have a like progreſſive motion; by which it happens that a longer time is neceſſary to arrive at the aphelion, which has advanced a little, than to arrive at the ſame fixed ſtar. For example, the tropical revolution of the ſun, with reſpect to the equinox, is - 365da 5^{h} 48^{m} 45^{s};
but the ſidereal, or return to the ſame
ſtar, - - 365 6 9 11;
and the anomaliſtic revolution is 365 6 15 20,
becauſe the ſun's apogee advances each year 65″½ with reſpect to the equinoxes, and the ſun cannot arrive at the apogee till he has paſſed over the 65″½ more than the revolution of the year anſwering to the equinoxes.

To find the anomaliſtic revolution, ſay, As the whole ſecular motion of a planet *minus* the motion of its aphelion, is to 100 years or 3155760000 ſeconds, ſo is 360°, to the duration of the anomaliſtic revolution.

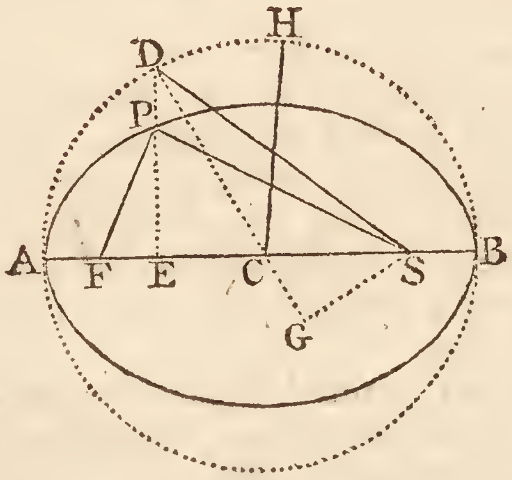
ANOMALOUS, is ſomething irregular, or that deviates from the ordinary rule and method of other things of the ſame kind.

ANOMALY, in *Aſtronomy*, is an irregularity in the motion of a planet, by which it deviates from the aphelion or apogee; or it is the angular diſtance of the planet from the aphelion or apogee; that is, the angle formed by the line of the apſes, and another line drawn through the planet.

Kepler diſtinguiſhes three kinds of anomaly; *mean*, *eccentric*, and *true*.

Mean or *Simple* ANOMALY, in the ancient aſtronomy, is the diſtance of a planet's mean place from the apogee. Which Ptolomy calls the angle of the mean motion.

But in the modern aſtronomy, in which a planet P is conſidered as deſcribing an ellipſe APB about the ſun S, placed in one focus, it is the time in which the planet moves from its aphelion A, to the mean place or point of its orbit P.



Hence, as the elliptic area ASP is proportional to the time in which the planet deſcribes the arc AP, that area may repreſent the mean anomaly.—Or, if PD be drawn perpendicular to the tranſverſe axis AB, and meet the circle in D deſcribed on the ſame axis; then the mean anomaly may alſo be repreſented by the circular trilineal ASD, which is always proportional to the elliptic one ASP, as is proved in my Menſuration, pr. 8, page 296, ſecond edition.—Or, drawing SG perpendicular to the radius DC produced; then the mean anomaly is alſo proportional to SG + the circular arc AD, as is demonſtrated by Keil in his *Lect. Aſtron.*—Hence, taking DH = SG, the arc AH, or angle ACH will be the mean anomaly in practice, as expreſſed in degrees of a circle, the number of thoſe degrees being to 360°, as the elliptic trilineal area ASP, is to the whole area of the ellipſe; the degrees of mean anomaly, being thoſe in the arc AH, or angle ACH.

Eccentric ANOMALY, or *of the centre*, in the modern aſtronomy, is the arc AD of the circle ADB intercepted between the apſis A and the point D determined by the perpendicular DPE to the line of the apſes, drawn through the place P of the planet. Or it is the angle ACD at the centre of the circle.—Hence the eccentric anomaly is to the mean anomaly, as AD to AD + SG, or as AD to AH, or as the angle ACD to the angle ACH.

True or *Equated* ANOMALY, is the angle ASP at the ſun, which the planet's diſtance AP from the aphelion, appears under; or the angle formed by the radius vector or line SP drawn from the ſun to the planet, with the line of the apſes.

The true anomaly being given, it is eaſy from thence to find the mean anomaly. For the angle ASP, which is the true anomaly, being given, the point P in the ellipſe is given, and thence the proportion of the area ASP to the whole ellipſe, or of the mean anomaly to 360 degrees. And for this purpoſe, the following eaſy rules for practice are deduced from the properties of the ellipſe, by M. de la Caille in his Elements of Aſtronomy, and M. de la Lande, art. 1240 &c of his aſtronomy: 1ſt, As the ſquare root of SB the perihelion diſtance, is to the ſquare root of SA the aphelion diſtance, ſo is the tangent of half the true anomaly ASP, to the tangent of half the eccentric anomaly ACD. 2nd, The difference DH or SG between the eccentric and mean anomaly, is equal to the product of the eccentricity CS, by the ſine of SCG the eccentric anomaly juſt found. And in this caſe, it is proper to expreſs the eccentricity in ſeconds of a degree, which will be found by this proportion, as the mean diſtance 1 : the eccentricity : : 206264·8 ſeconds, or 57° 17′ 44″·8, in the arch whoſe length is equal to the radius, to the ſeconds in the arc which is equal to the eccentricity CS; which being multiplied by the ſine of the eccentric anomaly, to radius 1, as above, gives the ſeconds in SG, or in the arc DH, being the difference between the mean and eccentric anomalies. 3d, To find the radius vector SP, or diſtance of the planet from the ſun, ſay either, as the ſine of the true anomaly is to the ſine of the eccentric anomaly, ſo is half the leſs axis of the orbit, to the radius vector SP; or as the ſine of half the true anomaly is to the ſine of half the eccentric anomaly, ſo is the ſquare root of the perihelion diſtance SB, to the ſquare root of the radius vector or planet's diſtance SP.

But the mean anomaly being given, it is not ſo eaſy to find the true anomaly, at leaſt by a direct proceſs; Kepler, who firſt propoſed this problem, could not find a direct a direct way of reſolving it, and therefore made uſe of an indirect one, by the rule of falſe poſition, as may be ſeen page 695 of Kepler's *Epitom. Aſtron. Copernic.* See alſo §628 Wolfius *Elem. Aſtron.* Now the eaſieſt method of performing this operation, would be to work firſt for the eccentric anomaly, viz, aſſume it nearly, and from it ſo aſſumed compute what would be its mean anomaly by the rule above given, and find the difference between this reſult and the mean anomaly given; then aſſume another eccentric anomaly, and proceed in the ſame way with it, finding another computed mean anomaly, and its difference from the given one; and treating theſe differences as in the rule of poſition for a nearer value of the eccentric anomaly: repeating the operation till the reſult comes out exact. Then, from the eccentric anomaly, thus found, compute the true anomaly by the 1ſt rule above laid down.

Of this problem, Dr. Wallis firſt gave the geometrical ſolution by means of the protracted cycloid; and Sir Iſaac Newton did the ſame at prop. 31 *lib.* 1 *Princip.* But theſe methods being unfit for the purpoſe of the practical aſtronomer, various ſeries for approximation have been given, viz, ſeveral by Sir Iſaac Newton in his *Fragmenta Epiſtolarum*, page 26, as alſo in the Schol. to the prop. above-mentioned, which is his beſt, being not only fit for the planets, but alſo for the comets, whoſe orbits are very eccentric. Dr. Gregory, in his *Aſtron.* lib. 3, has alſo given the ſolution by a ſeries, as well as M. Reyneau, in his *Analyſe Demontrée*, page 713, &c. And a better ſtill for converging is given by Keil in his *Prælect. Aſtron.* page 375; he ſays, if the arc AH be the mean anomaly, calling its ſine e, coſine f, the eccentricity g, alſo putting $z = ge$, and $a = 1 + fg$; then the eccentric anomaly AD will be $= \frac{rz}{a} \times (1 - \frac{z^2}{a^2}$ &c), ſuppoſing $r = 57{\cdot}29578$ degrees; of which the firſt term $\frac{rz}{a}$ is ſufficient for all the planets, even for Mars itſelf, where the error will not exceed the 200th part of a degree; and in the orbit of the earth, the error is leſs than the 10000th part of a degree.

Dr. Seth Ward, in his *Aſtronomia Geometrica*, takes the angle AFP at the other focus, where the ſun is not, for the mean anomaly, and thence gives an elegant ſolution. But this method is not ſufficiently accurate when the orbit is very eccentric, as in that of the planet Mars, as is ſhewn by Bullialdus, in his defence of the *Philolaic. Aſtron.* againſt Dr. Ward. However, when Newton's correction is made, as in the Schol. above-mentioned, and the problem reſolved according to Ward's hypotheſis, Sir Iſaac affirms that, even in the orbit of Mars, there will ſcarce ever be an error of more than one ſecond.

ANSÆ, Anses, in *Aſtronomy*, thoſe ſeemingly prominent parts of the ring of the planet Saturn, diſcovered in its opening, and appearing like handles to the body of the planet; from which appearance the name *anſæ* is taken.

ANSER, in *Aſtronomy*, a ſmall ſtar, of the 5th or 6th magnitude, in the milky-way, between the eagle and ſwan, firſt brought into order by Hevelius.

ANTARCTIC *pole*, denotes the ſouthern pole, or ſouthern end of the earth's axis.—The ſtars near the antarctic pole never appear above our horizon in theſe latitudes.

ANTARCTIC *circle*, is a ſmall circle parallel to the equator, at the diſtance of 23° 28′ from the antarctic or ſouth pole.—At one time of the year the ſun never riſes above the horizon of any part within this circle; and at other times he never ſets.

ANTARES, in Aſtronomy, the ſcorpion's heart; a fixed ſtar of the firſt magnitude, in the conſtellation *Scorpio*.

ANTECANIS is uſed by ſome aſtronomers, to denote the conſtellation otherwiſe called *canis minor*, or the ſtar *procyon*. It is ſo called, as preceding, or being the forerunner of the *canis major*, and riſing a little before it.

ANTECEDENT, *of a ratio*, denotes the firſt of the two terms of the ratio, or that term which is compared with the other. Thus, if the ratio be 2 to 3, or a to b; then 2 or a is the antecedent.

ANTECEDENTAL *Method*, is a branch of general geometrical proportion, or univerſal compariſon, and is derived from an examination of the Antecedents of ratios, having given conſequents, and a given ſtandard of compariſon, in the various degrees of augmentation and diminution, which they undergo by compoſition and decompoſition. This is a method invented by Mr. James Glenie, and publiſhed by him in 1793; a method which he ſays he always uſed inſtead of the fluxional and differential methods, and which is totally unconnected with the ideas of motion and time. See the author's treatiſe above-mentioned, and alſo his Doctrine of Univerſal Compariſon, or General Proportion, 1789, upon which it is founded.

ANTECEDENTIA, a term uſed by aſtronomers when a planet &c moves weſtward, or contrary to the order of the ſigns aries, taurus, &c.—Like as when it moves eaſtward, or according to the order of the ſigns aries, taurus, &c, it is then ſaid to move *in conſequentia*.

ANTECIANS, or Antoeci, in *Geography*, the inhabitants of the earth which occupy the ſame ſemicircle of the ſame meridian, but equally diſtant from the equator, the one north and the other ſouth; as Peloponneſus and the Cape of Good Hope.

Theſe have their noon, or midnight, or any other hour at the ſame time; but their ſeaſons are contrary, being ſpring to the one, when it is autumn with the other; and ſummer with the one, when it is winter with the other; alſo the length of the day to the one, is equal to the length of night to the other.

ANTES, in Architecture, are ſmall pilaſtres placed at the corners of buildings.

ANTICS, in Architecture, figures of men, beaſts, &c, placed as ornaments to buildings.

ANTICUM, in Architecture, a porch before a door; alſo that part of a temple, which is called the outer temple, and lies between the body of the temple and the portico.

ANTILOGARITHM, the complement of the logarithm of a ſine, tangent, ſecant, &c, to that of the radius. This is found by beginning at the left hand, ſubtracting each figure from 9, and the laſt figure from 10.

ANTINOUS, in *Aſtronomy*, a part of the conſtellation *aquila*, or the eagle.

ANTIOCHIAN *Sect*, or *Academy*, a name given to the fifth academy or branch of academics. It took its name from being founded by Antiochus, a philoſopher contemporary with Cicero; and it ſucceeded the Philonian academy. Though Antiochus was really a ſtoic, and only nominally an academic.

ANTIOCHIAN *epocha*, a method of computing time from the proclamation of liberty granted to the city of Antioch, about the time of the battle of Pharſalia.

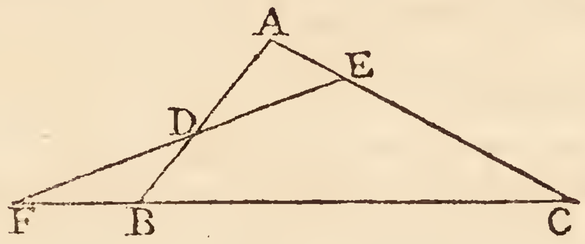
ANTIPARALLELS, in *Geometry*, are thoſe lines which make equal angles with two other lines, but contrary ways; that is, calling the former pair the firſt and 2d lines, and the latter pair the 3d and 4th lines, if the angle made by the 1ſt and 3d lines be equal to the angle made by the 2d and 4th, and contrariwiſe the angle made by the 1ſt and 4th equal to the angle made by the 2d and 3d; then each pair of lines are antiparallels with reſpect to each other, viz, the firſt and 2d, and the 3d and 4th. So, if AB and AC be any two lines, and FC and FE be two others, cutting them ſo,

that the angle B is equal to the angle E,
and the angle C is equal to the angle D;

then BC and DE are antiparallels with reſpect to AB and AC; alſo theſe latter are antiparallels with regard to the two former.—See alſo SUBCONTRARY.

It is a property of theſe lines, that each pair cuts the other into proportional ſegments, taking them alternately,

viz AB : AC : : AE : AD : : DB : EC,
and FE : FC : : FB : FD : : DE : BC.



ANTIPODES, in Geography, are the inhabitants of two places on the earth which lie diametrically oppoſite to each other, or that walk feet to feet; that is, if a line be continued down from our feet, quite through the centre of the earth, till it arrive at the ſurface on the other ſide, it will fall on the feet of our Antipodes, and *vice verſa*.——Antipodes are 180 degrees diſtant from each other every way on the ſurface of the globe; they have equal latitudes, the one north and the other ſouth, but they differ by 180 degrees of longitude: they have therefore the ſame climates or degrees of heat and cold, with the ſame ſeaſons and length of days and nights; but all of theſe at contrary times, it being day to the one, when it is night to the other, ſummer to the one when it is winter to the other, &c: they have alſo the ſame horizon, the one being as far diſtant on the one ſide, as the other on the other ſide, and therefore when the ſun, &c, riſes to the one, it ſets to the other. The Antipodes to London are a part a little ſouth of New Zealand.

It has been ſaid that Plato firſt ſtarted the notion of Antipodes, and gave them the name; which is likely enough, as he conceived that the earth was of a globular figure. But there have been many diſputes upon this point, and the fathers of the church have greatly oppoſed it, eſpecially Lactantius and Auguſtine, who laughed at it, and were greatly perplexed to think how men and trees ſhould hang pendulous in the air with their feet uppermoſt, as he thought they muſt do, in the other hemiſphere.

ANTISCIANS, or ANTISCII, in Geography, are people who dwell in the oppoſite hemiſpheres of the earth, as to north and ſouth, and whoſe ſhadows at noon fall in contrary directions. This term is more general than *antœci*, with which it is often confounded. The *Antiſcians* ſtand contradiſtinguiſhed from *Periſcians*.

ANTISCII is alſo uſed ſometimes, among *Aſtrologers*, for two points of the heavens equally diſtant from the tropics. Thus the ſigns Leo and Taurus are accounted *antiſcii* to each other.

ANTŒCI, ſee ANTECIANS.

APERTURE, in *Geometry*, is uſed for the ſpace left between two lines which mutually incline towards each other, to form an angle.

APERTURE, in Optics, is the hole next the object-glaſs of a teleſcope or microſcope, through which the light and the image of the object come into the tube, and are thence conveyed to the eye.

Aperture is alſo underſtood of that part of the object-glaſs itſelf which covers the former, and which is left pervious to the rays.

A great deal depends upon having a juſt aperture.—To find it experimentally: apply ſeveral circles of dark paper, of various ſizes, upon the face of the glaſs, from the breadth of a ſtraw, to ſuch as leave only a ſmall hole in the glaſs; and with each of theſe, ſeparately, view ſome diſtinct objects, as the moon, ſtars, &c; then that aperture is to be choſen through which they appear the moſt diſtinctly.

Huygens firſt found the uſe of apertures to conduce much to the perfection of teleſcopes; and he found by experience (*Dioptr.* prop. 56.) that the beſt aperture for an object-glaſs, for example of 30 feet, is to be determined by this proportion, as 30 to 3, ſo is the ſquare root of 30 times the diſtance of the focus of any lens, to its proper aperture: and that the focal diſtances of the eye-glaſſes are proportional to the apertures. And M. Auzout ſays he found, by experience, that the apertures of teleſcopes ought to be nearly in the ſubduplicate ratio of their lengths. It has alſo been found by experience that object-glaſſes will admit of greater apertures, if the tubes be blacked within ſide, and their paſſage furniſhed with wooden rings.

It is to be noted, that the greater or leſs aperture of an object-glaſs, does not increaſe or diminiſh the viſible area of the object; all that is effected by this, is the admittance of more or fewer rays, and conſequently the more or leſs bright the appearance of the object. But the largeneſs of the aperture or focal diſtance, cauſes the irregularity of its refractions. Hence, in viewing Venus through a teleſcope, a much leſs aperture is to be uſed than for the moon, or Jupiter, or Saturn, becauſe her light is ſo bright and glaring. And this circumſtance ſomewhat invalidates and diſturbs Azout's proportion, as is ſhewn by Dr. Hook, Philoſ. Tranſ. No. 4.

APHELION, or APHELIUM, in Aſtronomy, that point in the orbit of the earth, or a planet, in which it is at the greateſt diſtance from the ſun. Which is the point A (in the fig. to the art. ANOMALY) or extremity of the tranſverſe axis, of the elliptic orbit, fartheſt from the focus S, where the ſun is placed; and diametrically oppoſite to the perihelion B, or nearer extremity of the ſame axis. In the Ptolemaic ſyſtem, or in the ſuppoſition that the ſun moves about the earth, the aphelion becomes the *apogee*.

The times of the aphelia of the primary planets, may be known by their apparent diameter appearing the ſmalleſt, and alſo by their moving ſloweſt in a given time. Calculations and methods of finding them have been given by many aſtronomers, as Ricciolli, *Almag. Nov.* lib. 7, ſect. 2 and 3; Wolſius, *Elem. Aſtron.* § 659; Dr. Halley, *Philoſ. Tranſ.* No. 128; Sir I. Newton, *Princip.* lib. 3, prop. 14; Dr. Gregory, *Aſtron.* lib. 3, prop. 14; Keil, *Aſtron. Lect.*; De la Lande, *Memoires de l'Acad.* 1755, 1757, 1766, and in his *Aſtron.* liv. 22; alſo in the writings of MM. Euler, D'Alembert, Clairaut, &c, upon attraction.

The aphelia of the planets are not fixed: for their mutual actions upon one another keep thoſe points of their orbits in a continual motion, which is greater or leſs in the different planets. This motion is made in conſequentia, or according to the order of the ſigns; and Sir I. Newton ſhews that it is in the ſeſquiplicate ratio of the diſtance of the planet from the ſun, that is, as the ſquare root of the cube of the diſtance.

The quantities of this motion, as well as the place of the aphelion for a given time, are variouſly given by different authors. Kepler ſtates them, for the year 1700 as in the following table.

Planets.	Aphelion.				Annual Motion.	
Mercury	♑	8°	25′	30″	1′	45″
Venus	♒	3	24	27	1	18
Mars	♏	0	51	29	1	7
Jupiter	♎	8	10	40	0	47
Saturn	♐	28	3	48	1	10
The Earth	♋	8	25	30		

By De la Hire they are given as follows, for the ſame year 1700.

Planets.	Aphelion.				Annual Motion.	
Mercury	♑	13°	3′	40″	1′	39″
Venus	♒	6	56	10	1	26
Mars	♏	0	35	25	1	7
Jupiter	♎	10	17	14	1	34
Saturn	♐	29	14	41	1	22

And De la Lande ſtates them as follows, for the year 1750.

Planets.	Aphelion.			Secular Motion.		
Mercury	8^s	13°	33′	1°	57′	40″
Venus	10	8	13	4	10	0
Mars	5	1	28	1	51	40
Jupiter	6	10	22	1	43	20
Saturn	8	29	53	2	23	20
The Earth	9	8	38	1	49	10

Of the new planet, Herſchel, or Georgium Sidus, the aphelion for 1790 was $11^s\,23^\circ\,29'\,42''$, and its annual motion $50''\frac{3}{8}$. See Connoiſſance des Temps, 1786 and 1787.

APHRODISIUS, in Chronology, denotes the eleventh month in the Bythinian year, commencing on the 25th of July in ours.

APIAN or APPIAN (PETER), called in German *Bienewitz*, a celebrated aſtronomer and mathematician, was born at Leiſnig or Leipſick in Miſnia, 1495, and made profeſſor of mathematics at Ingolſtadt, in 1524, where he died in the year 1552, at 57 years of age.

Apian wrote treatiſes upon many of the mathematical ſciences, and greatly improved them; more eſpecially aſtronomy and aſtrology, which in that age were much the ſame thing; alſo geometry, geography, arithmetic, &c. He particularly enriched aſtronomy with many inſtruments, and obſervations of eclipſes, comets, &c. His principal work was the *Aſtronomicum Cæſareum*, publiſhed in folio at Ingolſtadt in 1540, and which contains a number of intereſting obſervations, with the deſcriptions and diviſions of inſtruments. In this work he predicts eclipſes, and conſtructs the figures of them in plano. In the 2d part of the work, or the *Meteoroſcopium Planum*, he gives the deſcription of the moſt accurate aſtronomical quadrant, and its uſes. To it are added obſervations of five different comets, viz, in the years 1531, 1532, 1533, 1538, and 1539: where he firſt ſhews that the tails of comets are always projected in a direction from the ſun.

Apian alſo wrote a treatiſe on *Coſmography*, or *Geographical Inſtruction*, with various mathematical inſtruments. This work Voſſius ſays he publiſhed in 1524, and that Gemma Friſius republiſhed it in 1540. But Weidler ſays he wrote it only in 1530, and that Gemma Friſius publiſhed it at Antwerp in 1550 and 1584, with obſervations of many eclipſes. The truth may be, that perhaps all theſe editions were publiſhed.

In 1533 he made, at Norimberg, a curious inſtrument, which from its figure he called *Folium Populi*; which, by the ſun's rays, ſhewed the hour in all parts of the earth, and even the unequal hours of the Jews.

In 1534 he publiſhed his *Inſcriptiones Orbis*.

In 1540, his *Inſtrumentum Sinuum, ſive Primi Mobilis*, with 100 problems.

Beſide theſe, Apian was the author of many other works: among which may be mentioned the *Ephemerides* from the year 1534 to 1570: Books upon *Shadows*: *Arithmetical Centilogues*: Books upon *Arithmetic*, with the *Rule of Coſs* (Algebra) demonſtrated: Upon *Gauging*: *Almanacs*, with Aſtrological directions: A book upon *Conjunctions*: Ptolemy, with very correct figures, drawn in a quadrangular form: Ptolemy's works in Greek: Books of Eclipſes: the works of Azoph, a very ancient aſtrologer: the works of Gebre: the Perſpective of Vitello: of Critical Days, and of the Rainbow: a new Aſtronomical and Geometrical Radius, with various uſes of Sines and Chords: Univerſal Aſtrolabe of Numbers: Maps of the World, and of particular countries: &c, &c.

Apian left a ſon, who many years afterwards taught mathematics at Ingolſtadt, and at Tubinga. Tycho has preſerved (Progymn. p. 643) his letter to the Landgrave of Heſſe, in which he gives an opinion on the new ſtar in Caſſiopeia, of the year 1572.

One of the comets obſerved by Apian, viz, that of 1532, had its elements nearly the ſame as of one obſerved 128¼ years after, viz, in 1661, by Hevelius and other Aſtronomers; from hence Dr. Halley judged that they were the ſame comet, and that therefore it might be expected to appear again in the beginning of the year 1789. But it was not found that it returned at this period, although the aſtronomers then looked anxiouſly for it; and it is doubtful whether the diſappointment might be owing to its paſſing unobſerved, or to any errors in the obſervations of Apian, or to its period being diſturbed and greatly altered by the actions of the ſuperior planets, &c.

APIS, *muſca*, the *Bee*, or *Fly*, in Aſtronomy, one of the ſouthern conſtellations, containing 4 ſtars.

APOCATASTASIS, in Aſtronomy, is the period of a planet, or the time employed in returning to the ſame point of the zodiac from whence it ſet out.

APOGEE, *Apogæum*, in Aſtronomy, that point in the orbit of the ſun, moon, &c, which is fartheſt diſtant from the earth. It is at the extremity of the line of the apſides; and the point oppoſite to it is called the *perigee*, where the diſtance from the earth is the leaſt.

The ancient aſtronomers, conſidering the earth as the centre of the ſyſtem, chiefly regarded the apogee and perigee: but the moderns, placing the ſun in the centre, change theſe terms for the *aphelion* and *perihelion*.—The apogee of the ſun, is the ſame thing as the aphelion of the earth; and the perigee of the ſun is the ſame as the perihelion of the earth.

The manner of finding the apogee of the ſun or moon, is ſhewn by Ricciolus, *Almag. Nov.* lib. 3, cap. 24; by Wolfius in *Elem. Aſtr.* § 618; by Caſſini, De la Hire, and many others: ſee alſo *Memoires de l'Academie*, the *Philoſ. Tranſ.* vol. 5, 47, &c.

The quantity of motion in the apogee may be found by comparing two obſervations of it made at a great diſtance of time; converting the difference into minutes, and dividing them by the number of years elapſed between the two obſervations; the quotient gives the annual motion of the apogee. Thus, from an obſervation made by Hipparchus in the year before Chriſt 140, by which the ſun's apogee was found 5° 30′ of ♊; and another made by Ricciolus, in the year of Chriſt 1646, by which it was found 7° 26′ of ♋; the annual motion of the apogee is found to be 1′ 2″. And the annual motion of the moon's apogee is about $1^{s}\ 10^{\circ}\ 39'\ 52''$.

But the moon's apogee moves unequably. When ſhe is in the ſyzygy with the ſun, it moves forwards; but in the quadratures, backwards; and theſe progreſſions and regreſſions are not equable, but it goes forward ſlower when the moon is in the quadratures, or perhaps goes retrograde; and when the moon is in the ſyzygy, it goes forward the faſteſt of all.—See alſo Newton's Theory of the Moon for more upon this ſubject.

APOLLODORUS, a celebrated architect, under Trajan and Adrian, was born at Damaſcus, and flouriſhed about the year of Chriſt 100. He had the direction of the ſtone bridge which Trajan ordered to be built over the Danube in the year 104, which was eſteemed the moſt magnificent of all the works of that emperor. Adrian, one day as Trajan was diſcourſing with this architect upon the buildings he had raiſed at Rome, would needs give his judgment, in which he ſhewed that he knew nothing of the matter. Apollodorus turned upon him bluntly, and ſaid to him, Go paint Citruls, for you are very ignorant of the ſubject we are talking upon. Adrian at this time boaſted of his painting Citruls well. This was the firſt ſtep towards the ruin of Apollodorus; a ſlip which he was ſo far from attempting to retrieve, that he even added a new offence, and that too after Adrian was advanced to the empire, upon the following occaſion: Adrian ſent to him the plan of a temple of Venus; and though he aſked his opinion, yet to ſhew that he had no need of him, and that he did not mean to be directed by it, the temple was already built. Apollodorus wrote his opinion very freely, and remarked ſuch eſſential faults in it, as the emperor could neither deny nor remedy. He ſhewed that it was neither high nor large enough; that the ſtatues in it were diſproportioned to its bulk: for, ſaid he, if the goddeſſes ſhould have a mind to riſe and go out, they could not do it. This put Adrian into a great paſſion, and prompted him to the deſtruction of Apollodorus. He baniſhed him at firſt; then under the pretext of certain ſuppoſed crimes, of which he had him accuſed, he at laſt put him to death.

APOLLONIUS, of Perga, a city in Pamphilia, was a celebrated geometrician who flouriſhed in the reign of Ptolemy Euergetes, about 240 years before Chriſt; being about 60 years after Euclid, and 30 years later than Archimedes. He ſtudied a long time in Alexandria under the diſciples of Euclid; and afterwards he compoſed ſeveral curious and ingenious geometrical works, of which only his books of Conic Sections are now extant, and even theſe not perfect. For it appears from the author's dedicatory epiſtle to Eudemus, a geometrician in Pergamus, that this work conſiſted of 8 books; only 7 of which however have come down to us.

From the Collections of Pappus, and the Commentaries of Eutocius, it appears that Apollonius was the author of various pieces in geometry, on account of which he acquired the title of the Great Geometrician. His *Conics* was the principal of them. Some have thought that Apollonius appropriated the writings and diſcoveries of Archimedes; Heraclius, who wrote the life of Archimedes, affirms it; though Eutocius endeavours to refute him. Although it ſhould be allowed a groundleſs ſuppoſition, that Archimedes was the firſt who wrote upon Conics, notwithſtanding his treatiſe on Conics was greatly eſteemed; yet it is highly probable that Apollonius would avail himſelf of the writings of that author, as well as others who had gone before him; and, upon the whole, he is allowed the honour of explaining a difficult ſubject better than had been done before; having made ſeveral improvements both in Archimedes's problems, and in Euclid. His work upon Conics was doubtleſs the moſt perfect of the kind among the ancients, and in ſome reſpects among the moderns alſo. Before Apollonius, it had been cuſtomary, as we are informed by Eutocius, for the writers on Conics to require three different ſorts of cones to cut the three different ſections from, viz, the parabola from a right angled cone, the ellipſe from an acute, and the hyperbola from an obtuſe cone; becauſe they always ſuppoſed the ſections made by a plane cutting the cones to be perpendicular to the ſide of them: but Apollonius cut his ſections all from any one cone, by only varying the inclination or poſition of the cutting plane; an improvement that has been followed by all other authors ſince his time. But that Archimedes was acquainted with the ſame manner of cutting any cone, is ſufficiently proved, againſt Eutocius, Pappus, and others, by Guido Ubaldus, in the beginning of his Commentary on the 2d book of Archimedes's Equiponderantes, publiſhed at Piſa in 1588.

The firſt four books of Apollonius's Conics only have come down to us in their original Greek language; but the next three, the 5th, 6th, and 7th, in an Arabic verſion; and the 8th not at all. Theſe have been commented upon, tranſlated, and publiſhed by various authors. Pappus, in his Mathematical Collections, has left ſome account of his various works, with notes and lemmas upon them, and particularly on the Conics. And Eutocius wrote a regular elaborate commentary on the propoſitions of ſeveral of the books of the Conics.

The firſt four books were badly tranſlated by Joan. Baptiſta Memmius. But a better tranſlation of theſe in Latin was made by Commandine, and publiſhed at Bononia in 1566.—Voſſius mentions an edition of the Conics in 1650; the 5th, 6th, and 7th books being recovered by Golius.—Claude Richard, Profeſſor of mathematics in the imperial college of his order at Madrid, in the year 1632, explained, in his public lectures, the firſt four books of Apollonius, which were printed at Antwerp in 1655, in folio.—And the Grand Duke Ferdinand the 2d, and his brother Prince Leopold de Medicis, employed a profeſſor of the Oriental languages at Rome to tranſlate the 5th, 6th, and 7th books into Latin. Theſe were publiſhed at Florence in 1661, by Borelli, with his own notes, who alſo maintains that theſe books are the genuine production of Apollonius, by many ſtrong authorities, againſt Mydorgius and others, who ſuſpected that theſe three books were not the real production of Apollonius.

As to the 8th book, ſome mention is made of it in a book of Golius's, where he had written that it had not been tranſlated into Arabic; becauſe it was wanting in the Greek copies, from whence the Arabians tranſlated the others. But the learned Merſenne, in the preface to Apollonius's Conics, printed in his Synopſis of the Mathematics, quotes the Arabic philoſopher Aben Nedin for a work of his about the year 400 of Mahomet, in which is part of that 8th book, and who aſſerts that all the books of Apollonius are extant in his language, and even more than are enumerated by Pappus; and Voſſius ſays he has read the ſame; *De Scientiis Mathematicis*, pa. 55.—A neat edition of the firſt four books in Latin was publiſhed by Dr. Barrow, in 4to, at London in 1675.—A magnificent edition of all the 8 books, was publiſhed in folio, by Dr. Halley, at Oxford in 1710; together with the Lemmas of Pappus, and the Commentaries of Eutocius. The firſt four in Greek and Latin, but the latter four in Latin only, the 8th book being reſtored by himſelf.

The other writings of Apollonius, mentioned by Pappus, are,

1. The Section of a Ratio, or Proportional Sections, two books.
2. The Section of a Space, in two books.
3. Determinate Section, in two books.
4. The Tangencies, in two books.
5. The Inclinations, in two books.
6. The Plane Loci, in two books.

The contents of all theſe are mentioned by Pappus, and many lemmas are delivered relative to them; but none, or very little of theſe books themſelves have deſcended down to the moderns. From the account however that has been given of their contents, many reſtorations have been made of theſe works, by the modern mathematicians, as follow: viz,

Vieta, Apollonius Gallus. The Tangencies. Paris, 1600, in 4to.

Snellius, Apollonius Batavus. Determinate Section. Lugd. 1601, 4to.

Snellius, Sectio Rationis & Spatii. 1607.

Ghetaldus, Apollonius Redivivus. The Inclinations. Venice, 1607, 4to.

Ghetaldus, Supplement to the Apollonius Redivivus. Tangencies. 1607.

Ghetaldus, Apollonius Redivivus, lib. 2. 1613.

Alex. Anderſon, Supplem. Apol. Redivivi. Inclin. Paris, 1612, 4to.

Alex. Anderſon, Pro Zetetico Apolloniani problematis a ſe jam pridem edito in Supplemento Apollonii Redivivi. Paris, 1615, 4to.

Schooten, Loca Plana reſtituta. Lug. Bat. 1656.

Fermat, Loca Plana, 2 lib. Toloſ. 1679, folio.

Halley, Apol. de Sectione Rationis libri duo ex Arabico MS. Latine verſi duo reſtituti. Oxon. 1706, 8vo.

Simſon, Loca Plana, libri duo. Glaſg. 1749, 4to.

Simſon, Sectio Determinat. Glaſg. 1776, 4to.

Horſley, Apol. Inclinat. libri duo. Oxon. 1770, 4to.

Lawſon, The Tangencies, in two books. Lond. 1771, 4to.

Lawſon, Determinate Section, two books. Lond. 1772, 4to.

Wales, Determinate Section, two books. Lond. 1772, 4to.

Burrow, The Inclinations. Lond. 1779, 4to.

APONO (PETER *de*), a learned aſtronomer and philoſopher, was born at Apono near Padua, about the year 1250. He deſcribed the *Aſtrolabium Planum*, by which were ſhewn the equations of the celeſtial houſes for any hour and minute, and for any part of the world: it was publiſhed at Venice in 1502. He acquired the name of the *Conciliator*, on account of a book of his, in which he reconciles the writings of the ancient philoſophers and phyſicians: the book was publiſhed at Venice in 1483. He reſided at Padua, where, from his practiſing medicine, and his ſkill in aſtronomy, he fell under the ſuſpicion of magic. He died in 1316, at 66 years of age.

APOPHYGE, in Architecture, is a concave part or ring of a column, lying above or below the flat member; and it owes its origin to the ring by which the ends of wooden columns were hooped, to prevent them from ſplitting.

APOTOME, the remainder or difference between two lines or quantities which are only commenſurable in power. Such is the difference between 1 and $\sqrt{2}$, or the difference between the ſide of the ſquare and its diagonal.

The term is uſed by Euclid; and a pretty full explanation of ſuch quantities is given in the tenth book of his Elements, where he diſtinguiſhes ſix kinds of apotomes, and ſhews how to find them all geometrically.

Apotome *Prima*, is when the greater term is rational, and the difference of the ſquares of the two is a ſquare number; as the difference $3-\sqrt{5}$.

Apotome *Secunda*, is when the leſs number is rational, and the ſquare root of the difference of the ſquares of the two terms, has to the greater term, a ratio expreſſible in numbers; ſuch is $\sqrt{18}-4$, becauſe the difference of the ſquares 18 and 16 is 2, and $\sqrt{2}$ is to $\sqrt{18}$ as $\sqrt{1}$ to $\sqrt{9}$ or as 1 to 3.

Apotome *Tertia*, is when both the terms are irrational, and, as in the ſecond, the ſquare root of the difference of their ſquares, has to the greater term, a rational ratio: as $\sqrt{24}-\sqrt{18}$; for the difference of their ſquares 24 and 18 is 6, and $\sqrt{6}$ is to $\sqrt{24}$ as $\sqrt{1}$ to $\sqrt{4}$ or as 1 to 2.

Apotome *Quarta*, is when the greater term is a rational number, and the ſquare root of the difference of the ſquares of the two terms, has not a rational ratio to it: as $4-\sqrt{3}$, where the difference of the ſquares 16 and 3 is 13, and $\sqrt{13}$ has not a ratio in numbers to 4.

Apotome *Quinta*, is when the leſs term is a rational number, and the ſquare root of the difference of the ſquares of the two, has not a rational ratio to the greater: as $\sqrt{6}-2$, where the difference of the ſquares 6 and 4 is 2, and $\sqrt{2}$ to $\sqrt{6}$ or $\sqrt{1}$ to $\sqrt{3}$ or 1 to $\sqrt{3}$ is not a rational ratio.

Apotome *Sexta*, is where both terms are irrational, and the ſquare root of the difference of their ſquares has not a rational ratio to the greater: as $\sqrt{6}-\sqrt{2}$; where the difference of the ſquares 6 and 2 is 4, and $\sqrt{4}$ to $\sqrt{6}$ or 2 to $\sqrt{6}$, is not a rational ratio.

The doctrine of apotomes, in lines, as delivered by Euclid in the tenth book, is a very curious ſubject, and has always been much admired and cultivated by all mathematicians who have rightly underſtood this part of the elements; and therefore Peter Ramus has greatly expoſed his judgment by cenſuring that book. And the firſt algebraical writers in Europe commonly employed a conſiderable portion of their works on an algebraical expoſition of that book, which led them to the doctrine of ſurd quantities; as Lucas de Burgo, Cardan, Tartalea, Stifelius, Peletarius, &c, &c. See alſo Pappus, lib. 4, prop. 3, and the introduc. to lib. 7. And Dr. Wallis's Algebra, pa. 109.

Apotome, in *Muſic*, is the difference between a greater and leſs ſemitone, being expreſſed by the ratio of 128 to 125.

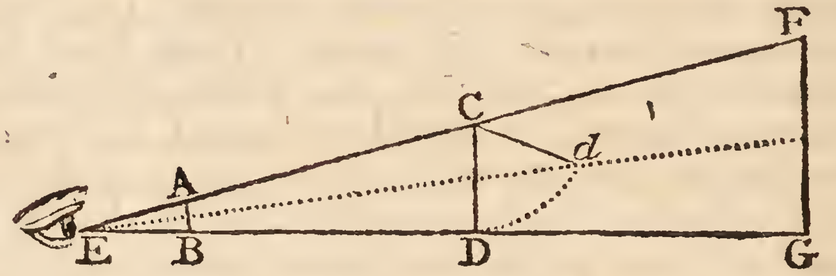
APPARENT, that which is viſible, or evident to the eye, or the underſtanding.

Apparent *conjunction* of the planets, is when a right line, ſuppoſed to be drawn through their centres, paſſes through the eye of the ſpectator, and not through the centre of the earth.—And, in general, the apparent conjunction of any objects, is when they appear or are placed in the ſame right line with the eye.

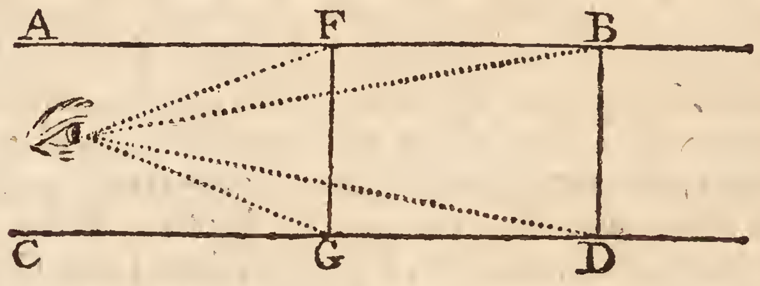
Apparent *Altitude*, *Diameter*, *Diſtance*, *Horizon*, *Magnitude*, *Motion*, *Place*, *Time*, &c. See the reſpective ſubſtantives, for the quantity and meaſure of it.

The apparent ſtate of things, is commonly very different from their real ſtate, either as to diſtance, figure, magnitude, poſition, &c, &c. Thus,

Apparent *Diameter*, or *Magnitude*, as for example of the heavenly bodies, is not the real length of the diameter, but the angle which they ſubtend at the eye, or under which they appear. And hence, the angle, or apparent extent, diminiſhing with the diſtance of the object, a very ſmall object, as AB, may have the ſame apparent diameter as a very large one FG; and indeed the objects have all the ſame apparent diameter, that are contained in the ſame angle FEG. And if theſe are parallel, the real magnitudes are directly proportional to their diſtances.



But the apparent magnitude varies not only by the diſtance, but alſo by the poſition of it. So, if the object CD be changed from the direct poſition to the oblique one C*d*, its apparent magnitude would then be only the angle CE*d*, inſtead of the angle CED.



If the eye E be placed between two parallels AB, CD, theſe parallels will appear to converge or come nearer and nearer to each other the farther they are continued out, and at laſt they will appear to coincide in that point where the ſight terminates, which will happen when the optic angle BED becomes equal to about one minute of a degree, the ſmalleſt angle under which an object is viſible.—Alſo the apparent magnitudes of the ſame object FG or BD, ſeen at different diſtances, that is the angles FEG, BED, are in a leſs ratio than the reciprocal ratio of the diſtances, or the diſtance increaſes in a greater ratio than the angle or apparent magnitude diminiſhes. But when the object is very remote, or the optic angle is very ſmall, as one degree or thereabouts, the angle then varies nearly as the diſtance reciprocally.

But although the optic angle be the uſual or ſenſible meaſure of the apparent magnitude of an object, yet habit, and the frequent experience of looking at diſtant objects, by which we know that they are larger than they appear, has ſo far prevailed upon the imagination and judgment, as to cauſe this too to have ſome ſhare in our eſtimation of apparent magnitudes; ſo that theſe will be judged to be more than in the ratio of the optic angles.

The apparent magnitude of the ſame object, at the ſame diſtance, is different to different perſons, and different animals, and even to the ſame perſon, when viewed in different lights, all which may be occaſioned by the different magnitudes of the eye, cauſing the optic angle to differ as that is greater or leſs: and ſince, in the ſame perſon, the more light there comes from an object, the leſs is the pupil of the eye, looking at that object; therefore the optic angle will alſo be leſs, and conſequently the apparent magnitude of the object. Every one muſt have experienced the truth of this, by looking at another perſon in a room, and afterwards abroad in the ſunſhine, when he always appears ſmaller than in a room where the light is leſs. So alſo, objects up in the air, having more light coming from them than when they are upon the ground, or near it, may appear leſs in the former caſe than in the latter; like as the ball of the croſs on the top of St. Paul's church, which is 6 feet in diameter, appears leſs than an object of the ſame diameter ſeen at the ſame diſtance below, near the ground. And this may be the chief reaſon why the ſun and moon appear ſo much larger when ſeen in the horizon, where their beams are weak, then when they are raiſed higher, and their light is more bright and glaring.

Again, if the eye be placed in a rare medium, and view an object through a denſer, as glaſs or water, having plane ſurfaces; the object will appear larger than it is: and contrariwiſe, ſmaller. And hence it is that fiſhes, and other objects, ſeen in the water, by an eye in the air, always appear larger than in the air.—In like manner, an object will appear larger when viewed through a globe of glaſs or water, or any convex ſpherical ſegments of theſe; and, on the contrary, it will appear ſmaller when viewed through a concave of glaſs or water.

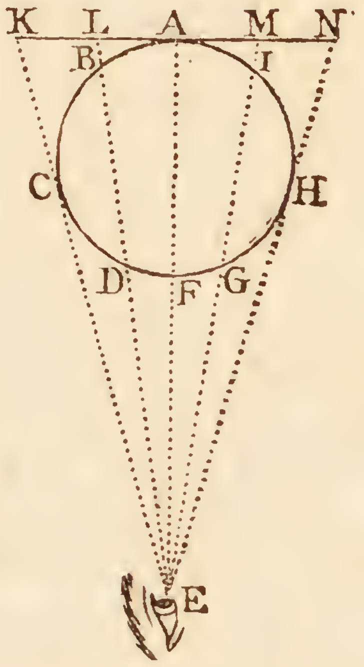
APPARENT *Diſtance*, is that diſtance which we judge an object to be from us, when ſeen afar off. This is commonly very different from the true diſtance; becauſe we are apt to think that all very remote objects, whoſe parts cannot well be diſtinguiſhed, and which have no other viſible objects near them, are at the ſame diſtance from us; though perhaps they may be thouſands or millions of miles off; as in the caſe of the ſun and moon. The apparent diſtances of objects are alſo greatly altered by the refraction of the medium through which they are ſeen.

APPARENT *Figure*, is the figure or ſhape which an object appears under when viewed at a diſtance; and is often very different from the true figure. For a ſtraight line, viewed at a diſtance, may appear but as a point; a ſurface, as a line; and a ſolid, as a ſurface. Alſo theſe may appear of different magnitudes, and the ſurface and ſolid of different figures, according to their ſituation with reſpect to the eye: thus, the arch of a circle may appear a ſtraight line; a ſquare, a trapezium, or even a triangle; a circle, an ellipſis; angular magnitudes, round; and a ſphere, a circle. Alſo all objects have a tendency to roundneſs and ſmoothneſs, or appear leſs angular, as their diſtance is greater: for, as the diſtance is increaſed, the ſmaller angles and aſperities firſt diſappear, by ſubtending a leſs angle than one minute; after theſe, the next larger diſappear, for the ſame reaſon; and ſo on continually, as the diſtance is more and more increaſed; the object ſeeming ſtill more and more round and ſmooth. So, a triangle, or ſquare, at a great diſtance, ſhews only as a round ſpeck; and the edge of the moon appears round to the eye, notwithſtanding the hills and valleys on her ſurface. And hence it is alſo, that near objects, as a range of lamps, or ſuch like, ſeen at a great diſtance, appear to be contiguous, and to form one uniform continued magnitude, by the intervals between them diſappearing, from the ſmallneſs of the angles ſubtended by them.

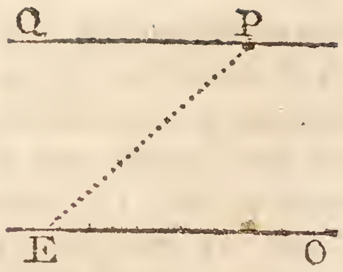
APPARENT *Motion*, is either that motion which we perceive in a diſtant body that moves, the eye at the ſame time being either in motion or at reſt; or that motion which an object at reſt ſeems to have, while the eye itſelf only is in motion.

The motions of bodies at a great diſtance, though really moving equally, or paſſing over equal ſpaces in equal times, may appear to be very unequal and irregular to the eye, which can only judge of them by the mutation of the angle at the eye. And motions, to be equally viſible, or appear equal, muſt be directly proportional to the diſtances of the objects moving. Again, very ſwift motions, as thoſe of the luminaries, may not appear to be any motions at all, but like that of the hour hand of a clock, on account of the great diſtance of the objects: and this will always happen, when the ſpace actually paſſed over in one ſecond of time, is leſs than about the 14000th part of its diſtance from the eye; for the hour hand of a clock, and the ſtars about the earth, move at the rate of 15 ſeconds of a degree in one ſecond of time, which is only the 13751 part of the radius or diſtance from the eye. On the other hand, it is poſſible for the motion of a body to be ſo ſwift, as not to appear any motion at all; as when through the whole ſpace it deſcribes there conſtantly appears a continued ſurface or ſolid as it were generated by the motion of the object, like as when any thing is whirled very ſwiftly round, deſcribing a ring, &c.

Alſo, the more oblique the eye is to the line which a diſtant body moves in, the more will the apparent motion differ from the true one. So, if a body revolve with an equable motion in the circumference of the circle ABCD &c, and the eye be at E in the plane of the circle; as the body moves from A to B and C, it ſeems to move ſlower and ſlower along the line ALK, till when the body arrives at C, it appears at reſt at K; then while it really moves from C by D to F, it appears to move quicker and quicker from K by L to A, where its motion is quickeſt of all; after this it appears to move ſlower and ſlower from A to N while the body moves from F to H: there becoming ſtationary again, it appears to return from N to A in the ſtraight line, while it really moves from H by I to A in the circle. And thus it appears to move in the line KN by a motion continually varying between the leaſt, or nothing, at the extremes K and N, and the greateſt of all at the middle point A. Or, if the motion be referred to the concave ſide of the circle, inſtead of the line KN, the appearances will be the ſame.



If an eye move directly forwards from E to O, &c; any remote object at reſt at P, will appear to move the



contrary way, or from P to Q, with the ſame velocity. But if the object P move the ſame way, and with the ſame velocity as the eye; it will ſeem to ſtand ſtill. If the object have a leſs velocity than the eye, it will appear to move back towards Q with the difference of the velocities; and if it move faſter than the eye, it will appear to move forwards from Q, with the ſame difference of the velocities. And ſo likewiſe when the object P moves contrary to the motion of the eye, it appears to move backwards with the ſum of the motions of the two. And the truth of all this is experienced by perſons ſitting in a boat moving on a river, or in a wheel-carriage when running faſt, and viewing houſes or trees, &c, on the ſhore or ſide of the road, or other boats or wheel-carriages alſo in motion.

Apparent Place *of an object*, in Optics, is that in which it appears, when ſeen in or through glaſs, water, or other refracting mediums; which is commonly different from the true place. So, if an object be ſeen in or through glaſs, or water, either plane or concave, it will appear nearer to the eye than its true place; but when ſeen through a convex glaſs, it appears more remote from the eye than the real place of it.

Apparent Place *of the Image of an object*, in Catoptrics, is that where the image of an object made by the reflexion of a ſpeculum appears to be; and the optical writers, from Euclid downwards, give it as a general rule that this is where the reflected rays meet the perpendicular to the ſpeculum drawn from the object: ſo that if the ſpeculum be a plane, the apparent place of the image will be at the ſame diſtance behind the ſpeculum as the eye is before it; if convex, it will appear behind the glaſs nearer to the ſame; but if concave, it will appear before the ſpeculum. And yet in ſome caſes there are ſome exceptions to this rule, as is ſhewn by Kepler in his *Paralipomena in Vitellionem*, *prop.* 18. See alſo Wolfius *Catoptr.* § 51, 188, 233, 234.

Apparent *Place of a Planet*, &c, in Aſtronomy, is that point in the ſurface of the ſphere of the world, where the centre of the luminary appears from the ſurface of the earth.

APPARITION, in Aſtronomy, denotes a ſtar's or other luminary's becoming viſible, which before was hid. So, the heliacal riſing, is rather an apparition than a proper riſing.

Circle of perpetual Apparition. See Circle *of perpetual apparition*.

APPEARANCE, in Perſpective, is the repreſentation or projection of a figure, body, or the like object, on the perſpective plane.—The appearance of an objective right line, is always a right line. See Perspective.—Having given the appearance of an opake body, and of a luminary, to find the appearance of the ſhadow; ſee Shadow.

Appearance *of a ſtar or planet*. See Apparition.

Appearances, in Aſtronomy, &c, are more uſually called *phænomena* and *phaſes*.—In *Optics*, the term *direct appearance* is uſed for the view or ſight of any object by direct rays; without either refraction or reflexion.

APPIAN. See Apian.

APPLICATE, Applicata, *Ordinate* Applicate, in Geometry, is a right line drawn to a curve, and biſected by its diameter. This is otherwiſe called an Ordinate, which ſee.

Applicate *Number*. See Concrete.

Application, the act of applying one thing to another, by approaching or bringing them nearer together. So a longer ſpace as meaſured by the continual *application* of a leſs, as a foot or yard by an inch, &c. And motion is determined by a ſucceſſive application of any thing to different parts of ſpace.

Application is ſometimes uſed, both in Arithmetic and Geometry, for the rule or operation of diviſion, or what is ſimilar to it in geometry. Thus 20 applied to, or divided by 4, gives 5. And a rectangle *ab*, applied to a line *c*, gives the 4th proportional $\frac{ab}{c}$, or another line which, with the given line *c*, will contain another rectangle which ſhall be equal to the given rectangle *ab*. And this is the ſenſe in which Euclid uſes the term, *lib.* 6, *pr.* 28.

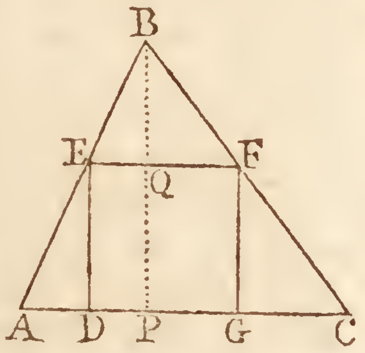
Application, in Geometry, is alſo uſed for the act or ſuppoſition of putting or placing one figure upon another, to find whether they be equal or unequal; which ſeems to be the primary way in which the mind firſt acquires both the idea and proof of equality. And in this way Euclid, and other geometricians, demonſtrate ſome of the firſt or leading properties in geometry. Thus, if two triangles have two ſides in the one triangle equal to two ſides in the other, and alſo the angle included by the ſame ſides equal to each other; then are the two triangles equal in all reſpects: for by conceiving the one triangle placed on the other, it is proved that they coincide or exactly agree in all their parts. And the ſame happens if, of two triangles, one ſide and the two adjacent angles of the one triangle, are equal, reſpectively, to one ſide and the two correſponding angles of the other. Thus alſo it may be proved that the diameter of a circle divides it into two equal parts, as alſo that the diagonal of a ſquare or parallelogram biſects or divides it into two equal parts.

Application of one ſcience to another, as of Algebra to Geometry, is ſaid of the uſe made of the principles and properties of the one for augmenting and perfecting the other. Indeed all arts and ſciences mutually receive aid from each other. But the application here meant, is of a more expreſs and immediate nature; as will appear by what follows.

Application *of Algebra* or *of Analyſis to Geometry*. The firſt and principal applications of algebra, were to arithmetical queſtions and computations, as being the firſt and moſt uſeful ſcience in all the concerns of human life. Afterwards algebra was applied to geometry and all the other ſciences in their turn. The application of algebra to geometry, is of two kinds; that which regards the plane or common geometry, and that which reſpects the higher geometry, or the nature of curve lines.

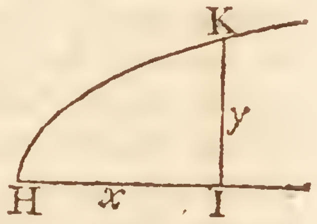
The firſt of theſe, or the application of algebra to common geometry, is concerned in the algebraical ſolution of geometrical problems, and finding out theorems in geometrical figures, by means of algebraical inveſtigations or demonſtrations. This kind of application has been made from the time of the moſt early writers on algebra, as Diophantus, Lucas de Burgo, Cardan, Tartalea, &c, &c, down to the preſent times. Some of the beſt precepts and exerciſes of this kind of application, are to be met with in Newton's *Univerſal Arithmetic*, and in Thomas Simpſon's *Algebra* and *Select Exerciſes.* Geometrical Problems are commonly reſolved more directly and eaſily by algebra, than by the geometrical analyſis, eſpecially by young beginners; but then the ſyntheſis, or conſtruction and demonſtration, is moſt elegant as deduced from the latter method. Now it commonly happens that the algebraical ſolution ſucceeds beſt in ſuch problems as reſpect the ſides and other lines in geometrical figures, and on the contrary, thoſe problems in which angles are concerned, are beſt effected by the geometrical analyſis. Newton gives theſe, among many other remarks on this branch. Having any problem propoſed; compare together the quantities concerned in it; and, making no difference between the known and unknown quantities, conſider how they depend upon, or are related to, one another; that we may perceive what quantities, if they are aſſumed, will, by proceeding ſynthetically, give the reſt, and that in the ſimpleſt manner. And in this compariſon, the geometrical figure is to be feigned and conſtructed at random, as if all the parts were actually known or given, and any other lines drawn that may appear to conduce to the eaſier and ſimpler ſolution of the problem. Having conſidered the method of computation, and drawn out the ſcheme, names are then to be given to the quantities entering into the computation, that is, to ſome few of them, both known and unknown, from which the reſt may moſt naturally and ſimply be derived or expreſſed, by means of the geometrical properties of figures, till an equation be obtained, by which the value of the unknown quantity may be derived by the ordinary methods of reduction of equations, when only one unknown quantity is in the notation; or till as many equations are obtained as there are unknown letters in the notation.

For example, ſuppoſe it were required to inſcribe a ſquare in a given triangle. Let ABC be the given triangle; and feign DEFG to be the required ſquare; alſo draw the perpendicular BP of the triangle, which will be given, as well as all the ſides of it. Then, conſidering that the triangles BAC, BEF are ſimilar, it will be proper to make the notation as follows, viz, making the baſe AC = b, the perpendicular BP = p, and the ſide of the ſquare DE or EF = x. Hence then BQ = BP − ED = $p - x$; conſequently, by the proportionality of the parts of thoſe two ſimilar triangles, viz, BP : AC : : BQ : EF, it is $p : b :: p - x : x$; then, multiply extremes and means &c, there ariſes $px = bp - bx$, or $bx + px = bp$, and $x = \dfrac{bp}{b+p}$ the ſide of the ſquare ſought; that is, a fourth proportional to the baſe and perpendicular, and the ſum of the two, taking this ſum for the firſt term, or AC + BP : BP : : AC : EF.



The other branch of the application of algebra to geometry, was introduced by Deſcartes, in his Geometry, which is the new or higher geometry, and reſpects the nature and properties of curve lines. In this branch, the nature of the curve is expreſſed or denoted by an algebraic equation, which is thus derived: A line is conceived to be drawn, as the diameter or ſome other principal line about the curve; and upon any indefinite points of this line other lines are erected perpendicularly, which are called ordinates, whilſt the parts of the firſt line cut off by them, are called abſciſſes. Then, calling any abſciſs x, and its correſponding ordinate y, by means of the known nature, or relations of the other lines in the curve, an equation is derived, involving x and y, with other given quantities in it. Hence, as x and y are common to every point in the primary line, that equation, ſo derived, will belong to every poſition or value of the abſciſs and ordinate, and ſo is properly conſidered as expreſſing the nature of the curve in all points of it; and is commonly called the equation of the curve.

In this way it is found that any curve line has a peculiar form of equation belonging to it, and which is different from that of every other curve, either as to the number of the terms, the powers of the unknown letters x and y, or the ſigns or coefficients of the terms of the equation. Thus, if the curve line HK be a circle, of which HI is part of the diameter, and IK a perpendicular ordinate: then put HI = x, IK = y, and p = the diameter of the circle, the equation of the circle will be $px - x^2 = y^2$. But if HK be an ellipſe, an hyperbola, or parabola, the equation of the curve will be different, and for all the four curves, will be reſpectively as follows, viz,



for the circle - - $px - x^2 = y^2$,

for the ellipſe - $px - \dfrac{p}{t}x^2 = y^2$,

for the hyperbola - $px + \dfrac{p}{t}x^2 = y^2$,

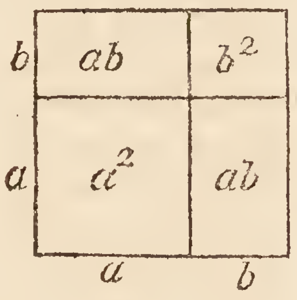
for the parabola - px - - $= y^2$;

where t is the tranſverſe axis, and p its parameter. And, in like manner for other curves.

This way of expreſſing the nature of curve lines, by algebraic equations, has given occaſion to the greateſt improvement and extenſion of the geometry of curve lines; for thus, all the properties of algebraic equations, and their roots, are transferred and added to the curve lines, whoſe abſciſſes and ordinates have ſimilar properties. Indeed the benefit of this ſort of application is mutual and reciprocal, the known properties of equations being transferred to the curves they repreſent; and, on the contrary, the known properties of curves transferred to their repreſentative equations. See Curves.

Application *of Geometry to Algebra.* Beſides the uſe and application of the higher geometry, namely, of curve lines, to detecting the nature and roots of equations, and to the finding the values of thoſe roots by the geometrical conſtruction of curve lines, even common geometry may be made ſubſervient to the purpoſes of algebra. Thus, to take a very plain and ſimple inſtance, if it were required to ſquare the binomial $a + b$;

by forming a ſquare, as in the annexed figure, whoſe ſide is equal to $a + b$, or the two lines or parts added together denoted by the letters a and b; and then drawing two lines parallel to the ſides, from the points where the two parts join, it will be immediately evident that the whole ſquare of the compound quantity $a + b$, is equal to the ſquares of both the parts, together with two rectangles under the two parts, or a^2 and b^2 and $2ab$, that is the ſquare of $a + b$ is equal to $a^2 + b^2 + 2ab$, as derived from a geometrical figure or conſtruction. And in this very manner it was, that the Arabians, and the early European writers on algebra, derived and demonſtrated the common rule for reſolving compound quadratic equations. And thus alſo, in a ſimilar way, it was, that Tartalea and Cardan derived and demonſtrated all the rules for the reſolution of cubic equations, uſing cubes and parallelopipedons inſtead of ſquares and rectangles. And many other inſtances might be given of the uſe and application of geometry in algebra.



Application *of Algebra and Geometry to Mechanics.* This is founded on the ſame principles as the application of algebra to geometry. It conſiſts principally in repreſenting by equations the curves deſcribed by bodies in motion, by determining the equation between the ſpaces which the bodies deſcribe, when actuated by any forces, and the times employed in deſcribing them, &c. A familiar inſtance alſo of the application of geometry to mechanics, may be ſeen at the article Acceleration, where the perpendiculars of triangles repreſent the times, the baſes the velocities, and the areas the ſpaces deſcribed by bodies in motion; a method firſt given by Galileo. In ſhort, as velocities, times, forces, ſpaces, &c, may be repreſented by lines and geometrical figures; and as theſe again may be treated algebraically; it is evident how the principles and properties, of both algebra and geometry, may be applied to mechanics, and indeed to all the other branches of mixt mathematics.

Application *of Mechanics to Geometry.* This conſiſts chiefly in the uſe that is ſometimes made of the centre of gravity of figures, for determining the contents of ſolids deſcribed by thoſe figures.

Application *of Geometry and Aſtronomy to Geography.* This conſiſts chiefly in three articles. 1ſt, In determining the figure of the globe we inhabit, by means of geometrical and aſtronomical operations. 2d, In determining the poſitions of places, by obſervations of latitudes and longitudes. 3d, In determining, by geometrical operations, the poſitions of ſuch places as are not far diſtant from one another.

Geometry and Aſtronomy are alſo of great uſe in Navigation.

Application *of Geometry and Algebra to Phyſics or Natural Philoſophy.* This application we owe to Newton, whoſe philoſophy may therefore be called the geometrical or mathematical philoſophy; and upon this application are founded all the phyſico-mathematical ſciences. Here a ſingle obſervation or experiment will often produce a whole ſcience: ſo when we know, as we do by experience, that the rays of light, in reflecting, make the angle of incidence equal to the angle of reflexion; we thence deduce the whole ſcience of catoptrics: for that experiment once admitted, catoptrics become a ſcience purely geometrical, ſince it is reduced to the compariſon of angles and lines given in poſition. And the ſame in many other ſciences.

APPLICATION *of one thing to another*, in general, is employed to denote the uſe that is made of the former, to underſtand or to perfect the latter. Thus, the application of the cycloid to pendulums, means the uſe made of the cycloidal curve for improving the doctrine and uſe of pendulums.

APPLY. This term is uſed two different ways, in geometry.

1ſt, It ſignifies to transfer or place a given line, either in a circle or ſome other figure, ſo that the extremities of the line ſhall be in the perimeter of the figure.

2d, It is alſo uſed to expreſs diviſion in geometry, or to find one dimenſion of a rectangle, when the area and the other dimenſion are given. As the area ab applied to the line c, is $\frac{ab}{c}$.

APPROACH, the *curve of equable approach.* It was firſt propoſed by Leibnitz, namely, to find a curve, down which a body deſcending by the force of gravity, ſhall make equal approaches to the horizon in equal portions of time. It has been found by Bernoulli and others, that the curve is the ſecond cubical parabola, placed with its vertex uppermoſt, and which the deſcending body muſt enter with a certain determinate velocity.—Varignon rendered the queſtion general for any law of gravity, by which a body may approach towards a given point by equal ſpaces in equal times. And Maupertuis alſo reſolved the problem in the caſe of a body deſcending in a medium which reſiſts as the ſquare of the velocity. *See Hiſt. de l' Acad. des Sciences* for 1699 and 1730.

Method of Approaches, a name given by Dr. Wallis, in his Algebra, to a method of reſolving certain problems relating to ſquare numbers, &c. This is done by firſt aſſigning certain limits to the quantities required, and then approaching nearer and nearer till a coincidence is obtained.—In this ſenſe, the method of Trial-and-error, or double rule of Falſe Poſition, may be conſidered as a method of approaches.

APPROACHES, in *Fortification*, the ſeveral works made by the beſiegers, for advancing or getting nearer to a fortreſs or place beſieged. Such as the trenches, mines, ſaps, lodgments, batteries, galleries, epaulments, &c.

Approaches, or *Lines of* Approach, are particularly uſed for trenches dug in the ground, and the earth thrown up on the ſide next the place beſieged; under the defence or ſhelter of which, the beſiegers may approach without loſs, as near as poſſible to the place, to raiſe batteries and plant guns &c, to batter it.—The lines of approach are commonly carried on, in a zig-zag way, parallel to the oppoſite faces of the beſieged work, or nearly ſo, that they may not be enfiladed by the guns from the enemy's works. And they are alſo connected by parallels or lines of communication.—The beſieged commonly make counter-approaches, to interrupt and defeat the approaches of the beſiegers.

The ancients made their approaches towards the place beſieged, much after the ſame manner as the moderns. Folard ſhews, that they had their trenches, their parallels, ſaps, &c.; which, though uſually thought of modern invention, it appears, have been practiſed long before, by the Greeks, Romans, Aſiatics, &c.

APPROXIMATION, a continual approach, ſtill nearer and nearer, to a root or any quantity ſought. — Methods of continual approximation for the ſquare roots and cube roots of numbers, have been employed by algebraiſts and arithmeticians, from Lucas de Burgo down to the preſent time. And the later writers have given various approximations, not only for the roots of higher powers, or all ſimple equations, but for the roots of all ſorts of compound equations whatever: eſpecially Newton, Wallis, Raphſon, Halley, De Lagny, &c, &c; all of them forming a kind of infinite ſeries, either expreſſed or underſtood, converging nearer and nearer to the quantity ſought, according to the nature of the proceſs.

It is evident that if a number propoſed be not a true ſquare, then no exact ſquare root of it can be found, explicable by rational numbers, whether integers or fractions: therefore, in ſuch caſes, we muſt be content with approximations, or coming continually nearer and nearer to the truth. In like manner, for the cube and other roots, when the propoſed quantities are not exact cubes, or other powers.

The moſt eaſy and general method of approximation, is perhaps by the rule of Double Poſition, or, what is ſometimes called, the Method of Trial-and-error; which method ſee under its own name. And among all the methods for the roots of pure powers, of which there are many, I believe the beſt is that which was diſcovered by myſelf, and given in the firſt volume of my Mathematical Tracts, in point of eaſe, both of execution and for remembering it. The method is this: if N denote any number, out of which is to be extracted the root whoſe index is denoted by r, and if n be the neareſt root firſt taken; then ſhall $\frac{r + 1 . N + r - 1 . n^r}{r - 1 . N + r + 1 . n^r} \times n$ be the required root of N very nearly; or as $r - 1$ times the given number added to $r + 1$ times the neareſt power, is to $r + 1$ times the given number added to $r - 1$ times the neareſt power, ſo is the aſſumed root n, to the required root, very nearly. Then this laſt value of the root, ſo found, if one ſtill nearer is wanted, is to be uſed for n in the ſame theorem, to repeat the operation with it. And ſo on, repeating the operation as often as neceſſary. Which theorem includes all the rational formulæ of Halley and De Lagny.

For example, ſuppoſe it were required to double the cube, or to find the cube root of the number 2. Here $r = 3$; conſequently $r + 1 = 4$, and $r - 1 = 2$; and therefore the general theorem becomes

$\frac{4N + 2n^3}{2N + 4n^3} \times n$ or $\frac{2N + n^3}{N + 2n^3} \times n$ for the cube root of N;

or as $N + 2n^3 : 2N + n^3 :: n$: the root ſought nearly. Now, in this caſe, $N = 2$, and therefore the neareſt root n is 1, and its cube $n^3 = 1$ alſo: hence $N + 2n^3 = 2 + 2 = 4$, and $2N + n^3 = 4 + 1 = 5$; therefore, as $4 : 5 :: 1 : \frac{5}{4}$ or $1\frac{1}{4} = 1{\cdot}25$ the firſt approximation.

Again, taking $r = \frac{5}{4}$, and conſequently $r^3 = \frac{125}{64}$;

$$\text{hence } N + 2n^3 = 2 + \frac{250}{64} = \frac{378}{64},$$

$$\text{and } 2N + n^3 = 4 + \frac{125}{64} = \frac{381}{64};$$

therefore as $378 : 381$, or as $126 : 127 :: \frac{5}{4} : \frac{635}{504} = 1{\cdot}259921$, which is the cube root of 2, true in all the figures. And by taking $\frac{635}{504}$ for a new value of n, and repeating the proceſs again, a great many more figures may be found.

Of the Roots of Equations by APPROXIMATION.— Stevinus and Vieta gave methods for finding values, always nearer and nearer, of the roots of equations. And Oughtred and others purſued and improved the ſame. Theſe however were very tedious and imperfect, and required a different proceſs for every degree of equations. But Newton introduced, not only general methods for expreſſing radical quantities by approximating infinite ſeries, but alſo for the roots of all ſorts of compound equations whatever, which are both eaſy and expeditious: which will be more particularly deſcribed under each reſpective word or article. His method for approximating of roots, is in ſubſtance this: Firſt take a value of the root as near as may be, by trials, either greater or leſs; then aſſuming another letter to denote the unknown difference between this and the true value, ſubſtitute into the equation the ſum or difference of the approximate root and this aſſumed letter, inſtead of the unknown letter or root of the equation, which will produce a new equation having only the aſſumed ſmall difference for its root or unknown letter; and, by any means, find, from this equation, a near value of this ſmall aſſumed quantity. Aſſume then another letter for the ſmall difference between this laſt value and the true one, and ſubſtitute the ſum or difference of them into the laſt equation, by which will ariſe a third equation, involving the ſecond aſſumed quantity; whoſe near value is found as before. Proceeding thus as far as we pleaſe, all the near values, connected together by their proper ſigns, will form a ſeries approaching ſtill nearer and nearer to the true value of the root of the firſt or propoſed equation. The approximate values of the ſeveral ſmall aſſumed differences, may be found in different ways: Newton's method is this: As the quantity ſought is ſmall, its higher powers decreaſe more and more, and therefore neglecting them will not lead to any great error, Newton therefore neglects all the terms having in them the 2d and higher powers, leaving only the 1ſt power and the abſolute known term; from which ſimple equation he always finds the value of the aſſumed unknown letter nearly, in a very ſimple and eaſy manner. Halley's method of doing the ſame thing, was to neglect all the terms above the ſquare or 2d power, and then to find the root of the remaining quadratic equation; which would indeed be a nearer value of the aſſumed letter than Newton's was, but then it is much more troubleſome to perform.—Raphſon has another way, which is a little varied from that of Newton's again, which is this: having found a near value of the firſt aſſumed ſmall quantity or difference, by this he corrects the firſt approximation to the root of the propoſed equation; and then, aſſuming another letter for the next, or ſmaller difference, he introduces it into the original equation in the ſame way as before. And thus he proceeds, from one correction to another, employing always the firſt propoſed equation to find them, inſtead of the ſucceſſive new equations uſed by Newton.

For example, let it be required to find the root of the equation $x^2-5x=31$, or $x^2-5x-31=0$:—Here the root x, it is evident, is nearly $=8$; for x therefore take $8+z$, and ſubſtitute $8+z$ for x in the given equation, and the terms will be thus;

$$\begin{aligned} x^2 &= 64+16z+z^2\\ -5x &= -40-5z\\ -31 &= -31 \end{aligned}$$

the ſum is $-7+11z+z^2=0$.

Then, rejecting z^2, it is $11z-7=0$, and $z=\frac{7}{11}={\cdot}6363$, &c, or $={\cdot}6$ nearly.

Aſſume now $z={\cdot}6+y$: then

$$\begin{aligned} z^2 &= {\cdot}36+1{\cdot}2y+y^2\\ 11z &= 6{\cdot}6+11y\\ -7 &= -7 \end{aligned}$$

the ſum $-{\cdot}04+12{\cdot}2y+y^2=0$,

where. $y=\frac{{\cdot}04}{12{\cdot}2}={\cdot}003278$ nearly.

Aſſume it $y={\cdot}003278-v$: then

$$\begin{aligned} y^2 &= {\cdot}0000107452 84-{\cdot}006556v+v^2\\ 12{\cdot}2y &= {\cdot}0399916 \qquad -12.2\text{-}\text{-}v\\ -{\cdot}04 &= -04 \end{aligned}$$

the ſum ${\cdot}000002345284-12{\cdot}206556v+v^2=0$,

where $v=\frac{{\cdot}000002345284}{12{\cdot}206556}={\cdot}000000192133$.

Hence, then, collecting all the aſſumed differences, with their ſigns, it is found that $x=8+z+y-v=8+{\cdot}6+{\cdot}003278-{\cdot}000000192133=8{\cdot}6003277807867$ the root of the equation required, by Newton's method.

The ſame by Raphſon's way.

Firſt $x=8+z$;

$$\begin{aligned} \text{then } x^2 &= 64+16z+z^2\\ -5x &= -40-5z\\ -31 &= -31 \end{aligned}$$

the ſum $-7+11z+z^2=0$;

hence $z=\frac{7}{11}={\cdot}6$ nearly,

and $x=8+z=8{\cdot}6$ nearly.

Aſſume it $x=8{\cdot}6+y$;

$$\begin{aligned} \text{then } x^2 &= 73{\cdot}96+17{\cdot}2y+y^2\\ -5x &= -43 \qquad -5y\\ -31 &= -31 \end{aligned}$$

the ſum $-{\cdot}04+12{\cdot}2y+y^2=0$;

hence $y=\frac{{\cdot}04}{12{\cdot}2}={\cdot}003278$ nearly,

and $x=8{\cdot}6+y=8{\cdot}603278$ nearly.

Aſſume it $x=8{\cdot}603278-v$;

$$\begin{aligned} \text{then } x^2 &= 74{\cdot}016392345284-17{\cdot}206556v+v^2\\ -5x &= -43{\cdot}016390 \qquad +5v\\ -31 &= -31 \end{aligned}$$

the ſum ${\cdot}00000234528412{\cdot}206556v+v^2=0$;

hence $v={\cdot}000000192133$,

and conſeq. $x=8{\cdot}6032777807867$, as before.

Example 2. Again, taking the cubic equation $y^3-2y-5=0$; Newton proceeds thus:

y is nearly $=2$; take it therefore $y=2+p$;

$$\begin{aligned} \text{then } y^3 &= 8+12p+6p^2+p^3\\ -2y &= -4-2p\\ -5 &= -5 \end{aligned}$$

the ſum $-1+10p+6p^2+p^3=0$;

hence $p=\frac{1}{10}={\cdot}1$ nearly.

Aſſume it $p={\cdot}1+q$;

$$\begin{aligned} \text{then } p^3 &= 0{\cdot}001+0{\cdot}03q+0{\cdot}3q^2+q^3\\ +6p^2 &= 0{\cdot}06 \quad +1{\cdot}2 \quad +6\\ +10p &= 1 \quad +10\\ -1 &= -1 \end{aligned}$$

the ſum $0{\cdot}061+11{\cdot}23q+6{\cdot}3q^2+q^3=0$;

hence $q=-0{\cdot}0054$ nearly.

Aſſume it $q=-0{\cdot}0054+r$;

$$\begin{aligned} \text{then } q^3 &= -0{\cdot}000000157464+0{\cdot}00008748r, \text{ \&c.}\\ +6{\cdot}3q^2 &= +0{\cdot}000183708 \qquad -0{\cdot}06804r, \text{ \&c.}\\ +11{\cdot}23q &= -0{\cdot}060642 \qquad +11{\cdot}23r\\ +0{\cdot}061 &= +0{\cdot}061 \end{aligned}$$

the ſum $+0{\cdot}000541550536+11{\cdot}16204748r$;

hence $r=-0{\cdot}000048517$, &c.

$$\begin{aligned} \text{Hence, } y &= 2+p+q+r\\ &= 2+0{\cdot}1-0{\cdot}0054-0{\cdot}000048517\\ &= 2{\cdot}094551483, \end{aligned}$$

the root of the equation $y^3-2y=5$. And in the ſame manner Newton performs the approximation for the roots of literal equations, that is, equations having literal coefficients; ſo the root of this equation

$y^3+axy+a^2y-x^3-2a^3=0$, is

$$y=a-\frac{x}{4}+\frac{x^2}{64a}+\frac{131x^3}{512a^2}+\frac{509x^4}{16384a^3}\ \text{\&c.}$$

See alſo a memoir on this method by the Marquis de Courtivron, in the *Memoires de l'Academie* for 1744.

Other Methods of Approximation. Beſides the foregoing general methods, other particular ways of approximating, for various purpoſes, have been given by many other perſons.—As for example, methods of approximating, by ſeries, to the roots of cubic equations belonging to the irreducible caſe, by Nicole in the ſame *Memoirs*, by M. Clairaut in his Algebra, and by myſelf in the *Philoſ. Tranſ.* for 1780. See alſo ſeveral parts of Simpſon's works, and my Tracts vol. I. Alſo the methods of infinite ſeries by Wallis, Newton, Gregory, Mercator, &c, may be conſidered as approximations, in quadratures, and other branches of the mathematics, many inſtances of which may be ſeen in Wallis's Algebra, and other books:—Likewiſe the method of exhauſtions of the ancients, by which Archimedes and others have approximated to the quadrature and rectification of the circle, &c, which was performed by continually biſecting the ſides of polygons, both inſcribed in a circle and circumſcribed about it; by which means the ſum of the ſides of the like polygons approach continually nearer and nearer together, and the circumference of the circle is nearly a mean between the two ſums. See alſo Equations.

APPULSE, in Aſtronomy, means the actual contact of two luminaries, according to ſome authors; but others deſcribe it as their near approach to each other, ſo as to be ſeen, for inſtance, within the ſame teleſcope.

The appulſes of the planets to the fixed ſtars have always been very uſeful to aſtronomers, as ſerving to fix and determine the places of the former. The ancients, wanting an eaſy method of comparing the planets with the ecliptic, which is not viſible, had ſcarce any other way of fixing their ſituations, but by obſerving their track among the fixed ſtars, and marking their appulſes to ſome of thoſe viſible points. See Hiſt. Acad. Scienc. for 1710, pa. 417. And Philoſ. Tranſ. No. 369, where Dr. Halley has given a method of determining the places of the planets, by obſerving their near appulſes to the fixed ſtars. See alſo Philoſ. Tranſ. No. 76, pa. 361, and Mem. Acad. Scienc. for 1708, where Flamſteed and De la Hire have given obſervations of the moon's appulſes to the Pleiades. See alſo Flamſteed's Hiſtoria Cœleſtis, where a multitude of obſervations of appulſes, or ſmall diſtances, of the moon and planets, from the fixed ſtars, are recorded. And Dr. Halley has publiſhed a map or planiſphere of the ſtarry zodiac, in which are accurately laid down all the ſtars to which the moon's appulſe has ever been obſerved in any part of the world. See Philoſ. Tranſ. No. 369; or Abridg. vol. vi. pa. 170.

APRIL, the 4th month of the year according to the common computation, and the 2d from the vernal equinox.—The word is derived from *Aprilis*, of *aperio*, *I open*; becauſe the earth, in this month, begins to open her boſom for the production of vegetables.—In this month the ſun travels through part of the ſigns Aries and Taurus.

APRON, in Gunnery, a piece of thin or ſheet lead, uſed to cover the vent or touch-hole of a cannon.

APSES, in Aſtronomy, are the two points in the orbits of planets, where they are at their greateſt and leaſt diſtance, from the ſun or the earth. The point at the greateſt diſtance being called the *higher apſis*, and that at the neareſt diſtance the *lower apſis*. And the two apſes are alſo called *auges*. Alſo the higher apſis is more particularly called the *aphelion*, or the *apogee*; and the lower apſis, the *perihelion*, or the *perigee*. The diameter which joins theſe two points, is called the *line of the apſes* or of the *apſides*; and it paſſes through the centre of the orbit of the planet, and the centre of the ſun or the earth; and in the modern aſtronomy this line makes the longer or tranſverſe axis of the elliptical orbit of the planet. In this line is counted the excentricity of the orbit; being the diſtance between the centre of the orbit and the focus, where is placed the ſun or the earth.

The foregoing definitions ſuppoſe the lines of the greateſt and leaſt diſtances to lie in the ſame ſtraight line; which is not always preciſely the caſe; as they are ſometimes out of a right line, making an angle greater or leſs than 180 degrees, and the difference from 180 degrees is called the motion of the line of the apſes: when the angle is leſs than 180 degrees, the motion of the apſes is ſaid to be contrary to the order of the ſigns; on the other hand, when the angle exceeds 180 degrees, the motion is according to the order of the ſigns.

Different means have been employed to determine the motion of the *apſes*. Dr. Keil explains, in his Aſtronomy, the method uſed by the ancients, who ſuppoſed the orbits of the planets to be perfectly circular, and the ſun out of the centre. But ſince it has been diſcovered that they deſcribe elliptical orbits, various other methods have been deviſed for determining it. Halley has given one, which ſuppoſes to be known only the time of the planet's revolution, or periodic time. Seth Ward has alſo given a determination from three different obſervations of a planet, in any three places of its orbit: but his method being founded on an hypotheſis not ſtrictly true, Euler has given one much more exact in vol. 7. of the *Peterſburgh Commentaries*. See various ways explained in the Aſtronomy of Keil and Mounier.

Newton has alſo given, in the *Principia*, an excellent method of determining the ſame motion, on the ſuppoſition that the orbits of the planets differ but little from circles, which is the caſe nearly. That great philoſopher ſhews, that if the ſun be immoveable, and all the planets gravitate towards him in the inverſe ratio of the ſquares of their diſtances, then the apſes will be fixed, or their motion nothing; that is, the lines of greateſt and leaſt diſtance will form one right line, and the apſes will be directly oppoſite, or at 180 degrees diſtance from each other. But, becauſe of the mutual tendency of the planets towards each other, their gravitation towards the ſun is not preciſely in that ratio; and hence it happens, that the apſes are not always exactly in a right line with the ſun. And Newton has given a very elegant method of determining the motion of the *apſes*, on the ſuppoſition that we know the force which is thereby added to the gravitation of the planet towards the ſun, and that this additional force is always in that direction.

APUS or Apous, *Avis Indica*, in Aſtronomy, a conſtellation of the ſouthern hemiſphere, ſituated near the ſouth pole, between the *triangulum auſtrale* and the chameleon, and ſuppoſed to repreſent the bird of paradiſe. Alſo ſuppoſed to be one of the birds named *Apodes*, as having no feet.

The number of ſtars contained in this conſtellation, are 11 in the Britiſh Catalogue, in Bayer's Maps 12, and a ſtill greater number in La Caille's Catalogue; the principal ſtar being but of the 4th or 5th order of magnitude. See *Cælum Auſtrale Stelliferum*, and the *Memoires de l'Acad.* for 1752, pa. 569.

AQUARIUS, in Aſtronomy, one of the celeſtial conſtellations, being the eleventh ſign in the zodiac, reckoning from Aries, and is marked by the character ♒, repreſenting part of a ſtream of water, iſſuing from the veſſel of Aquarius, or the water-pourer. This ſign alſo gives name to the eleventh part of the ecliptic, through which the ſun moves in part of the months of January and February.

The poets feign that Aquarius was Ganymede, whom Jupiter raviſhed under the ſhape of an eagle, and carried away into Heaven to ſerve as a cup-bearer, inſtead of Hebe and Vulcan; whence the name. Others hold, that the ſign was thus called, becauſe that when it appears in the horizon, the weather commonly proves rainy.

The ſtars in the conſtellation Aquarius, are, in Ptolemy's Catalogue, 45; in Tycho's 41; in Hevelius's 47; and in Flamſteed's 108. See the article Constellation; alſo Catalogue.

AQUEDUCT, or Aquæduct, as much as to ſay *ductus aquæ*, *a conduit of water*, is a conſtruction of ſtone or timber built on uneven ground, to preſerve the level of water, and convey it, by a canal, from one place to another.—Some aqueducts are under ground, being conducted through hills, &c; and others are raiſed above ground, and ſupported on arches, to conduct the water over vallies, &c.

The Romans were very magnificent in their aqueducts; having ſome that extended a hundred miles, or more. Frontinus, a man of conſular dignity, who had the direction of the aqueducts under the emperor Nerva, ſpeaks of nine that emptied themſelves through 13594 pipes, of an inch diameter. And it is obſerved by Vigenere, that in the ſpace of 24 hours, Rome received from theſe aqueducts not leſs than 500000 hogſheads of water. The chief aqueducts now in being, are theſe: 1ſt, that of the Aqua Virginia, repaired by pope Paul IV; 2d, the Aqua Felice, conſtructed by pope Sixtus V, and is called from the name he aſſumed before he was exalted to the papal throne; 3d, the Aqua Paulina, repaired by pope Paul V, in the year 1611; and 4thly, the aqueduct built by Lewis XIV, near Maintenon, to convey the river Bure to Verſailles, which is perhaps the largeſt in the world; being 7000 fathoms long, elevated 2560 fathoms in height, and containing 242 arcades. See Philoſ. Tranſ. for 1685, No. 171; or Abridg. vol. I. pa. 594.

AQUEOUS HUMOUR, or the *watry humour* of the eye, is the firſt or outermoſt, and the rareſt of the three humours of the eye. It is tranſparent and colourleſs, like water; and it fills up the ſpace that lies between the cornea tunica, and the cryſtalline humour.

AQUILA, *the Eagle*, or the *Vulture* as it is ſometimes called, is a conſtellation of the northern hemiſphere, uſually joined with Antinous. It is one of the 48 old conſtellations, according to the diviſion of which Hipparchus made his Catalogue of the Fixed Stars, and which are deſcribed by Ptolemy. The number of ſtars in Aquila, and thoſe near it, now in the later-formed conſtellation Antinous, amount to 15 in Ptolemy's Catalogue, to 19 in Tycho's, to 42 in that of Hevelius, and to 71 in Flamſteed's. But in Aquila alone, Tycho counts only 12 ſtars, and Hevelius 23; the principal ſtar being Lucida Aquila, and is between the 1ſt and 2d magnitude. The Greeks, as uſual, relate various fables of this conſtellation, to make the ſcience appear as of their own invention.

ARA, the *Altar*, one of the 48 old conſtellations, mentioned by the ancient aſtronomers, and is ſituated in the ſouthern hemiſphere; containing only 7 ſtars in Ptolemy's Catalogue, and 9 in that of Flamſteed; none of which exceed the 4th magnitude.

ARATUS, celebrated for his Greek poem intitled Φαινόμενα, the *Phenomena*, flouriſhed about the 127th Olympiad, or near 300 years before Chriſt, while Ptolomy Philadelphus reigned in Egypt. Being educated under Dionyſius Heracleotes, a Stoic philoſopher, he eſpouſed the principles of that ſect, and became phyſician to Antigonus Gonatus, the ſon of Demetrius Poliorcetes, King of Macedon. The *Phenomena* of Aratus gives him a title to the character of an aſtronomer, as well as a poet. In this work he deſcribes the nature and motion of the ſtars, and ſhews their various diſpoſitions and relations; he deſcribes the figures of the conſtellations, their ſituations in the ſphere, the origin of the names which they bear in Greece and in Egypt, the fables which have given riſe to them, the riſing and ſetting of the ſtars, and he indicates the manner of knowing the conſtellations by their reſpective ſituations.

The poem of Aratus was commented upon and tranſlated by many authors: of whom among the ancients were Cicero, Germanicus Cæſar, and Feſtus Avienus, who made Latin tranſlations of it; a part of the former of which is ſtill extant. Aratus muſt have been much eſteemed by the ancients, ſince we find ſo great a number of ſcholiaſts and commentators upon him; among whom are Ariſtarchus of Samos, the Aryſtylli the geometricians, Apollonius, the Evæneti, Crates, Numenius the grammarian, Pyrrhus of Magneſia, Thales, Zeno, and many others, as may be ſeen in Voſſius, p. 156. Suidas aſcribes ſeveral other works to Aratus. Virgil, in his Georgics, has tranſlated or imitated many paſſages from this author: Ovid ſpeaks of him with admiration, as well as many others of the poets: And St. Paul has quoted a paſſage from him; which is in his ſpeech to the Athenians (Acts xvii. 28.) where he tells them that ſome of their own poets have ſaid, *For we are alſo his offſpring*, theſe words being the beginning of the 5th line of the Phenomena of Aratus.

His modern editors are as follow: Henry Stephens publiſhed his poem at Paris in 1566, in his collection of the poets, in folio. Grotius publiſhed an edition of the Phenomena at Leyden in quarto, 1600, in Greek and Latin, with the fragments of Cicero's verſion, and the tranſlations of Germanicus and Avienus; all which the editor has illuſtrated with curious notes. Alſo a neat and correct edition of Aratus was publiſhed at Oxford, 1672, in 8vo. with the Scholia.

ARÆOMETER, ſee AREOMETER.

ARC, or ARCH; which ſee.

ARCADE, in Architecture, denotes an opening in the wall of a building formed by an arch.

ARC-BOUTANT, is a kind of arched buttreſs, formed of a flat arch, or part of an arch, abutting againſt the feet or ſides of another arch or vault, to ſupport them and prevent them from burſting or giving way.

ARCAS, a name by which ſome of the old writers call the ſtar Arcturus; a ſingle and very bright ſtar of the firſt magnitude, between the legs of the conſtellation Bootes. They ſay Arcas, the ſon of Caliſto by Jupiter, when he was about to have killed his mother in the ſhape of a bear, was, together with her, ſnatched up into Heaven; where ſhe was converted into the conſtellation of the Great Bear, near the north pole, and the youth into this ſingle ſtar.

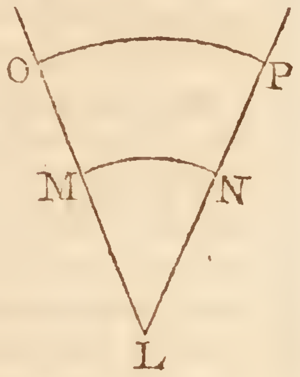
ARCH, ARC, *Arcus*, in Geometry, a part of any curve line; as, of a circle, or ellipſis, or the like.

It is by means of circular arcs, or arches, that all angles are meaſured; the arc being deſcribed from the angular point as a centre. For this purpoſe, every circle is ſuppoſed to be divided into 360 degrees, or equal parts; and an arch, or the angle it ſubtends and meaſures, is eſtimated according to the number of thoſe degrees it contains: thus, an arc, or angle, is ſaid to be of 30 or 80, or 100 degrees.—Circular arcs are alſo of great uſe in finding of fluents.

Concentric ARCS, are ſuch as have the ſame centre.

Equal ARCS, are ſuch arcs, of the ſame circle, or of equal circles, as contain the ſame number of degrees. Theſe have alſo equal chords, ſines, tangents, &c.

Similar ARCS, of unequal circles, &c, are ſuch as contain the ſame number of degrees, or that are the like part or parts of their reſpective whole circles. Hence, in concentric circles, any two radii cut off, or intercept, ſimilar arcs MN and OP.—Similar arcs are proportional to the radii LM, LO, or to the whole circumferences.—Similar arcs of other like curves, are alſo like parts of the wholes, or determined by like parts alike poſited.



Of the Length of Circular ARCS. The lengths of circular arcs, as found and expreſſed in various ways, may be ſeen in my large Treatiſe on Menſuration, pa. 118, & ſeq. 2d edition: ſome of which are as follow. The radius of a circle being 1; and of any arc a, if the tangent be t, the ſine s, the coſine c, and the verſed ſine v: then the arc a will be truly expreſſed by ſeveral ſeries, as follow, viz, the arc

$$a = t - \frac{1}{3}t^3 + \frac{1}{5}t^5 - \frac{1}{7}t^7 + \frac{1}{9}t^9 \text{ \&c.}$$

$$a = \frac{s}{c} - \frac{1}{3}\cdot\frac{s^3}{c^3} + \frac{1}{5}\cdot\frac{s^5}{c^5} - \frac{1}{7}\cdot\frac{s^7}{c^7} \text{ \&c.}$$

$$a = s + \frac{1}{2\cdot3}s^3 + \frac{1\cdot3}{2\cdot4\cdot5}s^5 + \frac{1\cdot3\cdot5}{2\cdot4\cdot6\cdot7}s^7 \text{ \&c.}$$

$$a = \sqrt{2v} \times \left(1 + \frac{1}{2.3}\cdot\frac{v}{2} + \frac{1.3}{2.4.5}\cdot\frac{v^2}{2^2} + \frac{1.3.5}{2.4.6.7}\cdot\frac{v^3}{2^3} \text{ \&c.}\right.$$

$$a = \frac{3\cdot14159 \text{ \&c.}}{180}d = \cdot01745329 \text{ \&c} \times d;$$

where d denotes the number of degrees in the given arc. Alſo $a = \frac{8c - C}{3}$ nearly; where C is the chord of the arc, and c the chord of half the arc; whatever the radius is.

To inveſtigate the length of the arc of any curve. Put x = the abſciſs, y = the ordinate, of the arc z, of any curve whatever. Put $\dot{z} = \sqrt{\dot{x}^2 + \dot{y}^2}$; then, by means of the equation of the curve, find the value of $\dot{x}$ in terms of $\dot{y}$, or of $\dot{y}$ in terms of $\dot{x}$, and ſubſtitute that value inſtead of it in the above expreſſion $\dot{z} = \sqrt{\dot{x}^2 + \dot{y}^2}$; hence, taking the fluents, they will give the length of the arc z, in terms of x or y.

ARCH, in Aſtronomy. Of this, there are various kinds. Thus, the latitude, elevation of the pole, and the declination, are meaſured by an arch of the meridian; and the longitude, by an arch of a parallel circle, &c.

Diurnal ARCH of the ſun, is part of a circle parallel to the equator, deſcribed by the ſun in his courſe from his riſing to the ſetting. And his *Nocturnal* ARCH is of the ſame kind; excepting that it is deſcribed from ſetting to riſing.

ARCH *of Progreſſion*, or *Direction*, is an arch of the ecliptic, which a planet ſeems to paſs over, when its motion is direct, or according to the order of the ſigns.

ARCH of *Retrogradation*, is an arch of the ecliptic, deſcribed while a planet is retrograde, or moves contrary to the order of the ſigns.

ARCH *between the Centres*, in eclipſes, is an arch paſſing from the centre of the earth's ſhadow, perpendicular to the moon's orbit, meeting her centre at the middle of an eclipſe.——If the aggregate of this arch and the apparent ſemi-diameter of the moon, be equal to the ſemi-diameter of the ſhadow, the eclipſe will be total for an inſtant, or without any duration; and if that ſum be leſs than the radius of the ſhadow, the eclipſe will be total, with ſome duration; but if greater, the eclipſe will be only partial.

ARCH *of Viſion*, is that which meaſures the ſun's depth below the horizon, when a ſtar, before hid by his rays, begins to appear again.—The quantity of this arch is not always the ſame, but varies with the latitude, declination, right aſcenſion, or deſcenſion, and diſtance, of any planet or ſtar. Ricciol. Almag. v. 1, pa. 42. However, the following numbers will ſerve nearly for the ſtars and planets.

TABLE exhibiting the *Arch of Viſion* of the PLANETS and FIXED STARS.

PLANETS.			FIXED STARS. Magnitude.	
Mercury - -	10°	0′	1 - - - -	12°
Venus - - -	5	0	2 - - - -	13
Mars - -	11	30	3 - - - -	14
Jupiter - -	10	0	4 - - - -	15
Saturn - -	11	0	5 - - - -	16
			6 - - - -	17

ARCH, in Architecture, is a concave ſtructure, raiſed or turned upon a mould, called the centering, in form of the arch of a curve, and ſerving as the inward ſupport of ſome ſuperſtructure. Sir Henry Wotton ſays, An arch is nothing but a narrow or contracted vault; and a vault is a dilated arch.

Arches are uſed in large intercolumnations of ſpacious buildings; in porticoes, both within and without temples; in public halls, as ceilings, the courts of palaces, cloiſters, theatres, and amphitheatres. They are alſo uſed to cover the cellars in the foundations of houſes, and powder magazines; alſo as buttreſſes and counter-forts, to ſupport large walls laid deep in the earth; for triumphal arches, gates, windows, &c; and, above all, for the foundations of bridges and aqueducts. And they are ſupported by piers, butments, impoſts, &c.

Arches are of ſeveral kinds, and are commonly denominated from the figure or curve of them; as circular, elliptical, cycloidal, catenarian, &c, according as their curve is in the form of a circle, ellipſe, cycloid, catenary, &c.

There are alſo other denominations of circular arches, according to the different parts of a circle, or manner of placing them. Thus,

Semicircular ARCHES, which are thoſe that make an exact ſemicircle, having their centre in the middle of the ſpan or chord of the arch; called alſo by the French builders, perfect arches, and *arches en plein centre*. The arches of Weſtminſter Bridge are ſemicircular.

Scheme ARCHES, or *ſkene*, are thoſe which are leſs than ſemicircles, and are conſequently flatter arches;

containing 120, or 90, or 60, degrees, &c. They are alſo called *imperfect* and *diminiſhed arches*.

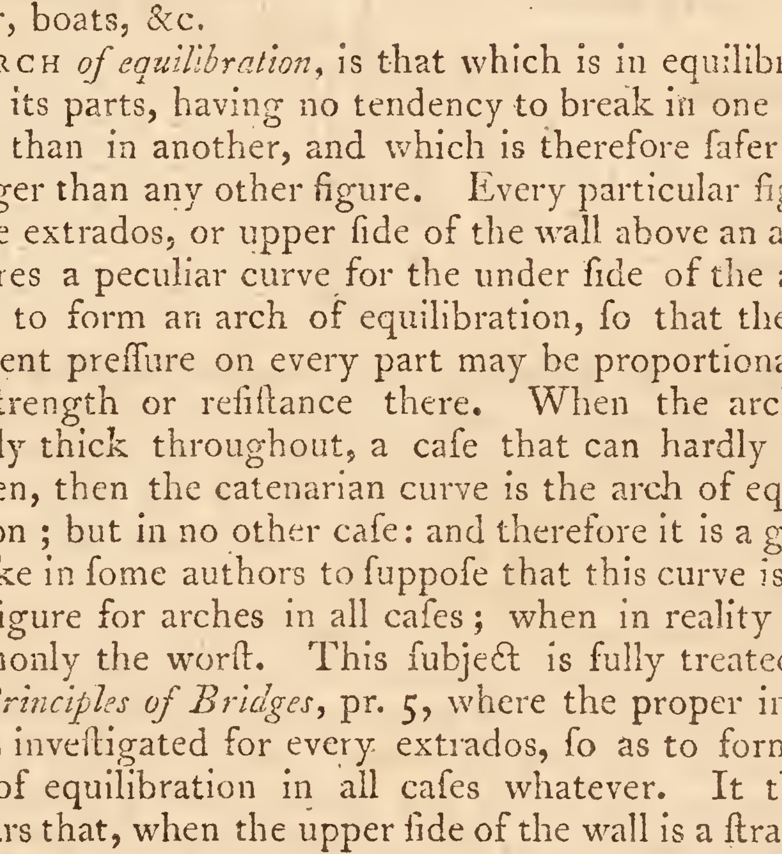
ARCHES *of the third and fourth point*, or *Gothic arches*; or, as the Italians call them, *di terzo* and *quarto acuto*, becauſe they always meet in an acute angle at top. Theſe conſiſt of two excentric circular arches, meeting in an angle above, and are drawn from the diviſion of the chord into three or four or more parts at pleaſure. Of this kind are many of the arches in churches and other old Gothic buildings.

Elliptical ARCHES, uſually conſiſt of ſemi-ellipſes; and were formerly much uſed inſtead of mantle-trees in chimnies; and are now much uſed, from their bold and beautiful appearance, for many purpoſes, and particularly for the arches of a bridge, like that at Black-Friars, both for their ſtrength, beauty, convenience, and cheapneſs.

Straight ARCHES, are thoſe which have their upper and under edges parallel ſtraight lines, inſtead of curves. Theſe are chiefly uſed over doors and windows; and have their ends and joints all pointing towards one common centre.

ARCH is particularly uſed for the ſpace between the two piers of a bridge, intended for the paſſage of the water, boats, &c.

ARCH *of equilibration*, is that which is in equilibrium in all its parts, having no tendency to break in one part more than in another, and which is therefore ſafer and ſtronger than any other figure. Every particular figure of the extrados, or upper ſide of the wall above an arch, requires a peculiar curve for the under ſide of the arch itſelf, to form an arch of equilibration, ſo that the incumbent preſſure on every part may be proportional to the ſtrength or reſiſtance there. When the arch is equally thick throughout, a caſe that can hardly ever happen, then the catenarian curve is the arch of equilibration; but in no other caſe: and therefore it is a great miſtake in ſome authors to ſuppoſe that this curve is the beſt figure for arches in all caſes; when in reality it is commonly the worſt. This ſubject is fully treated in my *Principles of Bridges*, pr. 5, where the proper intrados is inveſtigated for every extrados, ſo as to form an arch of equilibration in all caſes whatever. It there appears that, when the upper ſide of the wall is a ſtraight horizontal line, as in the annexed figure, the equation of the curve is thus expreſſed,



$$y = h \times \frac{\text{log. of } \dfrac{a + x + \sqrt{2ax + xx}}{a}}{\text{log. of } \dfrac{a + r + \sqrt{2ar + rr}}{a}};$$

where $x = DP$, $y = PC$, $r = DQ$, $h = AQ$, and $a = DK$. And hence, when a, h, r, are any given numbers, a table is formed for the correſponding values of x and y, by which the curve is conſtructed for any particular occaſion. Thus ſuppoſing a or $DK = 6$, h or $AQ = 50$, and r or $DQ = 40$; then the correſponding values of KI and IC, or horizontal and vertical lines, will be as in this table.

Table for conſtructing the Curve of Equilibration.

Value of KI.	Value of IC.	Value of KI.	Value of IC.	Value of KI.	Value of IC.
0	6·000	21	10.381	36	21·774
2	6·035	22	10·858	37	22·948
4	6·144	23	11·368	38	24·190
6	6·324	24	11·911	39	25·505
8	6·580	25	12·489	40	26·894
10	6·914	26	13·106	41	28·364
12	7·330	27	13·761	42	29·919
13	7·571	28	14·457	43	31·563
14	7·834	29	15·196	44	33·299
15	8·120	30	15·980	45	35·135
16	8·430	31	16·811	46	37·075
17	8·766	32	17·693	47	39·126
18	9·168	33	18·627	48	41·293
19	9·517	34	19·617	49	43·581
20	9·934	35	20·665	50	46·000

The doctrine and uſe of arches are neatly delivered by Sir Henry Wotton, though he is not always mathematically accurate in the principles. He ſays; *Firſt*, All matter, unleſs impeded, tends to the centre of the earth in a perpendicular line. *Secondly*; All ſolid materials, as bricks, ſtones, &c, in their ordinary rectangular form, if laid in numbers, one by the ſide of another, in a level row, and their extreme ones ſuſtained between two ſupporters; thoſe in the middle will neceſſarily ſink, even by their own gravity, much more if forced down by any ſuperincumbent weight. To make them ſtand, therefore, either their figure or their poſition muſt be altered.—*Thirdly*; Stones, or other materials, being figured *cuneatim*, or wedge-like, broader above than below, and laid in a level row, with their two extremes ſupported as in the laſt article, and pointing all to the ſame centre; none of them can ſink, till the ſupporters or butments give way, becauſe they want room in that ſituation to deſcend perpendicularly. But this is a weak ſtructure; becauſe the ſupporters are ſubject to too much impulſion, eſpecially where the line is long; for which reaſon the form of ſtraight arches is ſeldom uſed, excepting over doors and windows, where the line is ſhort and the ſide walls ſtrong. In order to fortify the work, therefore, we muſt change not only the figure of the materials, but alſo their poſition.—*Fourthly*; If the materials be ſhaped wedgewiſe, and be diſpoſed in form of an arch, and pointing to ſome centre; in this caſe, neither the pieces of the ſaid arch can ſink downwards, for want of room to deſcend perpendicularly; nor can the ſupporters or butments ſuffer much violence, as in the preceding flat form: for the convexity will always make the incumbent rather reſt upon the ſupporters, than thruſt or puſh them outwards. His reaſoning, however, afterwards, on the effect of circular and other arches, is not accurate, as he attends only to the ſide preſſure, without conſidering the effect of different vertical preſſures.

The chief properties of arches of different curves, may be ſeen in the 2d ſect. of my *Principles of Bridges*, above quoted. It there appears that none, except the mechanical curve of the arch of equilibration, can admit of a horizontal line at top: that this arch is of a form both graceful and convenient, as it may be made higher or lower at pleaſure, with the ſame ſpan or opening: that all other arches require extrados that are curved, more or leſs, either upwards or downwards: of theſe, the elliptical arch approaches the neareſt to that of equilibration for equality of ſtrength and convenience; and it is alſo the beſt form for moſt bridges, as it can be made of any height to the ſame ſpan, its hanches being at the ſame time ſufficiently elevated above the water, even when it is very flat at top: elliptical arches alſo look bolder and lighter, are more uniformly ſtrong, and much cheaper than moſt others, as they require leſs materials and labour. Of the other curves, the cycloidal arch is next in quality to the elliptical one, for thoſe properties, and, laſtly, the circle. As to the others, the parabola, hyperbola, and catenary, they are quite inadmiſſible in bridges that conſiſt of ſeveral arches; but may, in ſome caſes, be employed for a bridge of one ſingle arch which may be intended to riſe very high, as in ſuch caſes as they are not much loaded at the hanches.

Arch *Mural*. See Mural *arch*.

ARCHER, or *Sagittarius*, one of the conſtellations of the northern hemiſphere, and one of the twelve ſigns of the zodiac, placed between the Scorpion and Capricorn. See Sagittarius.

ARCHIMEDES, one of the moſt celebrated mathematicians among the ancients, who flouriſhed about 250 years before Chriſt, being about 50 years later than Euclid. He was born at Syracuſe in Sicily, and was related to Hiero, who was then king of that city. The mathematical genius of Archimedes ſet him with ſuch diſtinguiſhed excellence in the view of the world, as rendered him both the honour of his own age, and the admiration of poſterity. He was indeed the prince of the ancient mathematicians, being to them what Newton is to the moderns, to whom in his genius and character he bears a very near reſemblance. He was frequently loſt in a kind of reverie, ſo as to appear hardly ſenſible; he would ſtudy for days and nights together, neglecting his food; and Plutarch tells us that he uſed to be carried to the baths by force. Many particulars of his life, and works, mathematical and mechanical, are recorded by ſeveral of the ancients, as Polybius, Livy, Plutarch, Pappus, &c. He was equally ſkilled in all the ſciences, aſtronomy, geometry, mechanics, hydroſtatics, optics, &c, in all of which he excelled, and made many and great inventions.

Archimedes, it is ſaid, made a ſphere of glaſs, of a moſt ſurpriſing contrivance and workmanſhip, exhibiting the motions of the heavenly bodies in a very pleaſing manner. Claudian has an epigram upon this invention, which has been thus tranſlated:

> When in a glaſs's narrow ſpace confin'd,
> Jove ſaw the fabric of th' almighty mind;
> He ſmil'd, and ſaid, Can mortals' art alone,
> Our heavenly labours mimic with their own?
> The Syracuſian's brittle work contains
> Th' eternal law, that through all nature reigns.
> Fram'd by his art ſee ſtars unnumber'd burn,
> And in their courſes rolling orbs return:
> His ſun through various ſigns deſcribes the year;
> And every month his mimic moons appear.
> Our rival's laws his little planets bind,
> And rule their motions with a human mind.
> Salmoneus could our thunder imitate,
> But Archimedes can a world create.

Many wonderful ſtories are told of his diſcoveries, and of his very powerful and curious machines, &c. Hiero once admiring them, Archimedes replied, theſe effects are nothing, "But give me, ſaid he, ſome other place to fix a machine on, and I ſhall move the earth." He fell upon a curious device for diſcovering the deceit which had been practiced by a workman, employed by the ſaid king Hiero to make a golden crown. Hiero, having a mind to make an offering to the gods of a golden crown, agreed for one of great value, and weighed out the gold to the artificer. After ſome time he brought the crown home of the full weight; but it was afterwards diſcovered or ſuſpected that a part of the gold had been ſtolen, and the like weight of ſilver ſubſtituted in its ſtead. Hiero, being angry at this impoſition, deſired Archimedes to take it into conſideration, how ſuch a fraud might be certainly diſcovered. While engaged in the ſolution of this difficulty, he happened to go into the bath; where obſerving that a quantity of water overflowed, equal to the bulk of his body, it preſently occurred to him, that Hiero's queſtion might be anſwered by a like method: upon which he leaped out, and ran homeward, crying out εὗρηκα! εὗρηκα! I have found it! I have found it! He then made two maſſes, each of the ſame weight as the crown, one of gold and the other of ſilver: this done, he filled a veſſel to the brim with water, and put the ſilver maſs into it, upon which a quantity of water overflowed equal to the bulk of the maſs; then taking the maſs of ſilver out he filled up the veſſel again, meaſuring the water exactly, which he put in; this ſhewed him what meaſure of water anſwered to a certain quantity of ſilver. Then he tried the gold in like manner, and found that it cauſed a leſs quantity of water to overflow, the gold being leſs in bulk than the ſilver, though of the ſame weight. He then filled the veſſel a third time, and putting in the crown itſelf, he found that it cauſed more water to overflow than the golden maſs of the ſame weight, but leſs than the ſilver one; ſo that, finding its bulk between the two maſſes of gold and ſilver, and that in certain known proportions, he hence computed the real quantities of gold and ſilver in the crown, and ſo maniſeſtly diſcovered the fraud.

Archimedes alſo contrived many machines for uſeful and beneficial purpoſes: among theſe, engines for launching large ſhips; ſcrew pumps, for exhauſting the water out of ſhips, marſhes or overflowed lands, as Egypt, &c, which they would do from any depth.

But he became moſt famous by his curious contrivances, by which the city of Syracuſe was ſo long defended, when beſieged by the Roman conſul Marcellus; ſhowering upon the enemy ſometimes long darts, and ſtones of vaſt weight and in great quantities; at other times lifting their ſhips up into the air, that had come near the walls, and daſhing them to pieces by letting them fall down again; nor could they find their ſafety in removing out of the reach of his cranes and levers, for there he contrived to fire them with the rays of the ſun reflected from burning glaſſes.

However, notwithſtanding all his art, Syracuſe was at length taken by ſtorm, and Archimedes was ſo very intent upon ſome geometrical problem, that he neither heard the noiſe, nor minded any thing elſe, till a ſoldier that found him tracing of lines, aſked him his name, and upon his requeſt to begone, and not diſorder his figures, ſlew him. "What gave Marcellus the greateſt concern, ſays Plutarch, was the unhappy fate of Archimedes, who was at that time in his muſeum; and his mind, as well as his eyes, ſo fixed and intent upon ſome geometrical figures, that he neither heard the noiſe and hurry of the Romans, nor perceived the city to be taken. In this depth of ſtudy and contemplation, a ſoldier came ſuddenly upon him, and commanded him to follow him to Marcellus; which he refuſing to do, till he had finiſhed his problem, the ſoldier, in a rage, drew his ſword, and ran him through." Livy ſays he was ſlain by a ſoldier, not knowing who he was, while he was drawing ſchemes in the duſt: that Marcellus was grieved at his death, and took care of his funeral; and made his name a protection and honour to thoſe who could claim a relationſhip to him. His death it ſeems happened about the 142 or 143 Olympiad, or 210 years before the birth of Chriſt.

When Cicero was queſtor for Sicily, he diſcovered the tomb of Archimedes, all overgrown with buſhes and brambles; which he cauſed to be cleared, and the place ſet in order. There was a ſphere and cylinder cut upon it, with an inſcription, but the latter part of the verſes quite worn out.

Many of the works of this great man are ſtill extant, though the greateſt part of them are loſt. The pieces remaining are as follow: 1. Two books on the Sphere and Cylinder.—2. The Dimenſion of the Circle, or proportion between the diameter and the circumference.—3. Of Spiral lines.—4. Of Conoids and Spheroids.—5. Of Equiponderants, or Centres of Gravity.—6. The Quadrature of the Parabola.—7. Of Bodies floating on Fluids.—8. Lemmata.—9. Of the Number of the Sand.

Among the works of Archimedes which are loſt, may be reckoned the deſcriptions of the following inventions, which may be gathered from himſelf and other ancient authors. 1. His account of the method which he employed to diſcover the mixture of gold and ſilver in the crown, mentioned by Vitruvius.—2. His deſcription of the Cochleon, or engine to draw water out of places where it is ſtagnated, ſtill in uſe under the name of Archimedes's Screw. Athenæus, ſpeaking of the prodigious ſhip built by the order of Hiero, ſays, that Archimedes invented the cochleon, by means of which the hold, notwithſtanding its depth, could be drained by one man And Diodorus Siculus ſays, that he contrived this machine to drain Egypt, and that by a wonderful mechaniſm it would exhauſt the water from any depth.—3. The Helix, by means of which, Athenæus informs us, he launched Hiero's great ſhip.—4. The Triſpaſton, which, according to Tzetzes and Oribaſius, could draw the moſt ſtupendous weights.—5. The machines, which, according to Polybius, Livy, and Plutarch, he uſed in the defence of Syracuſe againſt Marcellus, conſiſting of Tormenta, Baliſtæ, Catapults, Sagittarii, Scorpions, Cranes, &c.—6. His Burning Glaſſes, with which he ſet fire to the Roman gallies.—7. His Pneumatic and Hydroſtatic engines, concerning which ſubjects he wrote ſome books, according to Tzetzes, Pappus, and Tertullian.—8. His Sphere, which exhibited the celeſtial motions. And probably many others.

A whole volume might be written upon the curious methods and inventions of Archimedes, that appear in his mathematical writings now extant only. He was the firſt who ſquared a curvilineal ſpace; unleſs Hypocrates muſt be excepted on account of his lunes. In his time the conic ſections were admitted into geometry, and he applied himſelf cloſely to the meaſuring of them, as well as other figures. Accordingly he determined the relations of ſpheres, ſpheroids, and conoids, to cylinders and cones; and the relations of parabolas to rectilineal planes whoſe quadratures had long before been determined by Euclid. He has left us alſo his attempts upon the circle: he proved that a circle is equal to a right-angled triangle, whoſe baſe is equal to the circumference, and its altitude equal to the radius; and conſequently, that its area is equal to the rectangle of half the diameter and half the circumference; thus reducing the quadrature of the circle to the determination of the ratio between the diameter and circumference; which determination however has never yet been done. Being diſappointed of the *exact* quadrature of the circle, for want of the rectification of its circumference, which all his methods would not effect, he proceeded to aſſign an uſeful approximation to it: this he effected by the numeral calculation of the perimeters of the inſcribed and circumſcribed polygons: from which calculation it appears that the perimeter of the circumſcribed regular polygon of 192 ſides, is to the diameter, in a leſs ratio than that of $3\frac{1}{7}$ or $3\frac{10}{70}$ to 1; and that the perimeter of the inſcribed polygon of 96 ſides, is to the diameter, in a greater ratio than that of $3\frac{10}{71}$ to 1; and conſequently that the ratio of the circumference to the diameter, lies between theſe two ratios. Now the firſt ratio, of $3\frac{1}{7}$ to 1, reduced to whole numbers, gives that of 22 to 7, for $3\frac{1}{7} : 1 :: 22 : 7$; which therefore is nearly the ratio of the circumference to the diameter. From this ratio between the circumference and the diameter, Archimedes computed the approximate area of the circle, and he found that it is to the ſquare of the diameter, as 11 is to 14. He determined alſo the relation between the circle and ellipſe, with that of their ſimilar parts. And it is probable that he likewiſe attempted the hyperbola; but it is not to be expected that he met with any ſucceſs, ſince approximations to its area are all that can be given by the various methods that have ſince been invented.

Beſide theſe figures, he determined the meaſures of the ſpiral, deſcribed by a point moving uniformly along a right line, the line at the ſame time revolving with a uniform angular motion; determining the proportion of its area to that of the circumſcribed circle, as alſo the proportion of their ſectors.

Throughout the whole works of this great man, we every where perceive the deepeſt deſign, and the fineſt invention. He ſeems to have been, with Euclid, exceedingly careful of admitting into his demonſtrations nothing but principles perfectly geometrical and unexceptionable: and although his moſt general method of demonſtrating the relations of curved figures to ſtraight ones, be by inſcribing polygons in them; yet to determine thoſe relations, he does not increaſe the number, and diminiſh the magnitude, of the ſides of the polygon *ad infinitum;* but from this plain fundamental principle, allowed in Euclid's Elements, (viz, that any quantity may be ſo often multiplied, or added to itſelf, as that the reſult ſhall exceed any propoſed finite quantity of the ſame kind,) he proves that to deny his figures to have the propoſed relations, would involve an abſurdity. And when he demonſtrated many geometrical properties, particularly in the parabola, by means of certain progreſſions of numbers, whoſe terms are ſimilar to the inſcribed figures; this was ſtill done without conſidering ſuch ſeries as continued *ad infinitum*, and then collecting or ſumming up the terms of ſuch infinite ſeries.

There have been various editions of the exiſting writings of Archimedes. The whole of theſe works, together with the commentary of Eutocius, were found in their original Greek language, on the taking of Conſtantinople, from whence they were brought into Italy; and here they were found by that excellent mathematician John Muller, otherwiſe called Regiomontanus, who brought them into Germany: where they were, with that Commentary, publiſhed long afterwards, viz, in 1544, at Baſil, being moſt beautifully printed in folio, both in Greek and Latin, by Hervagius, under the care of Thomas Gechauff Venatorius.—A Latin tranſlation was publiſhed at Paris 1557, by Paſcalius Hamellius.—Another edition of the whole, in Greek and Latin, was publiſhed at Paris 1615, in folio, by David Rivaltus, illuſtrated with new demonſtrations and commentaries: a life of the author is prefixed; and at the end of the volume is added ſome account, by way of reſtoration, of our author's other works, which have been loſt; viz, The Crown of Hiero; the Cochleon or Water Screw; the Helicon, a kind of endleſs ſcrew; the Triſpaſton, conſiſting of a combination of wheels and axles; the Machines employed in the defence of Syracuſe; the Burning Speculum; the Machines moved by Air and Water; and the Material Sphere.—In 1675, Dr. Iſaac Barrow publiſhed a neat edition of the works, in Latin, at London, in 4to; illuſtrated, and ſuccinctly demonſtrated in a new method.—But the moſt complete of any, is the magnificent edition, in folio, lately printed at the Clarendon preſs, Oxford, 1792. This edition was prepared ready for the preſs by the learned Joſeph Torelli, of Verona, and in that ſtate preſented to the Univerſity of Oxford. The Latin tranſlation is a new one. Torelli alſo wrote a preface, a commentary on ſome of the pieces, and notes on the whole. An account of the life and writings of Torelli is prefixed, by Clemens Sibiliati. And at the end a a large appendix is added, in two parts; the firſt being a Commentary on Archimedes's paper upon Bodies that float on Fluids, by the Rev. Abram Robertſon of Chriſt Church College; and the latter is a large collection of various readings in the Manuſcript works of Archimedes, found in the library of the late king of France, and of another at Florence, as collated with the Baſil edition above mentioned.

There are alſo extant other editions of certain parts of the works of Archimedes. Thus, Commandine publiſhed, in 4to, at Bologna 1565, the two books concerning Bodies that Float upon Fluids, with a Commentary. Commandine publiſhed alſo a tranſlation of the Arenarius. And Borelli publiſhed, in folio, at Florence 1661, Archimedes's *Liber Aſſumptorum*, tranſlated into Latin from an Arabic manuſcript copy. This is accompanied with the like tranſlation, from the Arabic, of the 5th, 6th, and 7th books of Apollonius's Conics. Mr. G. Anderſon publiſhed (in 8vo. Lond. 1784) an Engliſh tranſlation of the Arenarius of Archimedes, with learned and ingenious notes and illuſtrations. Dr. Wallis publiſhed a tranſlation of the Arenarius. And there may be other editions beſide the above, but theſe are all that I have got, or know of.

ARCHIMEDES's *Screw*. See SCREW *of Archimedes*.

ARCHIMEDES's *Burning-glaſs*. See BURNING-*glaſs*.

ARCHITECT, a perſon ſkilled in architecture, or the art of building; who forms plans and deſigns for edifices, conducts the work, and directs the various artificers employed in it.

The moſt celebrated architects are, Vitruvius, Palladio, Scamozzi, Serlio, Vignola, Barbaro, Cataneo, Alberti, Viola, Inigo Jones, De Lorme, Perrault, S. Le Clerc, Sir Chriſtopher Wren, and the Earl of Burlington.

ARCHITECTURE, *Architectura*, the art of planning and building or erecting any edifice, ſo as properly to anſwer the end propoſed, for ſolidity, conveniency, and beauty; whether houſes, temples, churches, bridges, halls, theatres, &c, &c.—Architecture is divided into civil, military, and naval or marine.

Civil ARCHITECTURE, is the art of deſigning and erecting edifices of every kind for the uſes of civil life in every capacity; as churches, palaces, private houſes, &c; and it has been divided into five orders or manners of building, under the names of the *Tuſcan*, *Doric*, *Ionic*, *Corinthian*, and *Compoſite*.

There were many authors on architecture among the Greeks and Romans, before Vitruvius; but he is the firſt whoſe work is entire and extant. He lived in the reigns of Julius Cæſar and Auguſtus, and compoſed a complete ſyſtem of architecture, in ten books, which he dedicated to this prince. The principal authors on architecture ſince Vitruvius, are Philander, Barbarus, Salmaſius, Baldus, Alberti, Gauricus, Demonioſius, Perrault, De l'Orme, Rivius, Wotton, Serlio, Palladio, Strada, Vignola, Scamozzi, Dieuſſart, Catanei, Freard, De Cambray, Blondel, Goldman, Sturmy, Wolfius, De Roſi, Deſgodetz, Baratteri, Mayer, Gulielmus, Ware, &c, &c. See alſo ARCHITECT.

Military ARCHITECTURE, otherwiſe more uſually called *Fortification*, is the art of ſtrengthening and fortifying places, to ſcreen them from the inſults or attacks of enemies, and the violence of arms; by erecting forts, caſtles, and other fortreſſes, with ramparts, baſtions, &c.—The authors who have chiefly excelled in this art, are Coehorn, Pagan, Vauban, Scheiter, Blondel, and Montalembert.

Naval ARCHITECTURE, or *ſhip-building*, is the art of conſtructing ſhips, galleys, and other veſſels proper to float on the water.

ARCHITRAVE, is that part of a column which bears immediately upon the capital. It is the loweſt member of the entablature, and is ſuppoſed to repreſent the principal beam in timber buildings, in which it is ſometimes called the *reaſon piece*, or *maſter-piece*. Alſo, in chimneys it is called the *mantle-piece;* and the *hyperthyron* over the jaumbs of doors, or lintels of windows.

Architrave *Corniche*. See Corniche.

Architrave *doors*, are thoſe which have an architrave on the jaumbs, and over the door; upon the cap piece if ſtraight; or on the arch, if the top be curved.

Architrave *windows*, of timber, are uſually an ogee raiſed out of the ſolid timber, with a liſt over it: though ſometimes the mouldings are ſtruck, and laid on; and ſometimes they are cut in brick.

ARCHIVOLT, the contour of an arch; or a band or frame adorned with mouldings, running over the faces of the vouſſoirs or arch-ſtones, and bearing upon the impoſts.

ARCHYTAS, of Tarentum, a celebrated mathematician, coſmographer, and Pythagorean philoſopher, whom Horace calls

> ———Maris ac Terræ, numeroque carentis Arenæ
> Menſorem.

He flouriſhed about 400 years before Chriſt; and was the maſter of Plato, Eudoxus, and Philolaus. He gave a method of finding two mean proportionals between two given lines, and thence the Duplication of the Cube, by means of the conic ſections. His ſkill in Mechanics was ſuch, that he was ſaid to be the inventor of the crane and the ſcrew; and he made a wooden pigeon that could fly about, when it was once ſet off, but it could not riſe again of itſelf, after it reſted. He wrote ſeveral works, though none of them are now extant, particularly a treatiſe περὶ τȣ̃ Παντὸς, *de Univerſo*, cited by Simplicius in Ariſtot. Categ. It is ſaid he invented the ten categories. He acquired great reputation both in his legiſlative and military capacity; having commanded an army ſeven times without ever being defeated. He was at laſt ſhipwrecked, and drowned in the Adriatic ſea.

ARCTIC *Circle*, is a leſſer circle of the ſphere, parallel to the equator, and paſſing through the north pole of the ecliptic, or diſtant from the north or arctic pole, by a quantity equal to the obliquity of the ecliptic, which was formerly eſtimated at 23° 30′, but its mean quantity is now 23° 28′ nearly. This, and its oppoſite, the antarctic circle, are alſo called the polar circles, where the longeſt day and longeſt night are 24 hours, and within all the ſpace of theſe circles, at one time of the year, the ſun never ſets, and at the oppoſite ſeaſon he never riſes for ſome days, more or leſs according as the place is nearer the pole.

Arctic *Pole*, the north pole of the world, and ſo called from αρκτος, *urſa*, the *bear*, from its proximity to the conſtellation of that name.

ARCTOPHYLAX, a conſtellation otherwiſe called Bootes. Which ſee.

ARCTURUS, a fixed ſtar of the firſt magnitude, between the thighs of the conſtellation Bootes. So called from αρκτος, *bear*, and ȣρα, *tail;* as being near the *bear's tail*.

This ſtar is twice mentioned in the book of Job, viz, ix. 9, and xxxviii. 32, by the name Aiſh if the tranſlation be right; and by many of the ancients under its Greek name *Arcturus*. The Greeks gave the fabulous hiſtory of this ſtar, or conſtellation, to this purport: Caliſto, who was afterwards, in form of the great bear, raiſed up into a conſtellation, they tell us, brought forth a ſon to Jupiter, whom they called Arcas. That Lyacon, when Jupiter afterwards came to viſit him, cut the boy in pieces, and ſerved him up at table. Jupiter, in revenge, as well as by way of puniſhment, called down lightning to conſume the palace, and turned the monarch into a wolf. The limbs of the boy were gathered up, to which the god gave life again, and he was taken and educated by ſome of the people. His mother, who was all this time a bear in the woods, fell in his way: he chaſed her, ignorant of the fact, and, to avoid him, ſhe threw herſelf into the temple of Jupiter: he followed her thither to deſtroy her; and this being death by the laws of the country, Jupiter took them both up into heaven, to prevent the puniſhment, making her the conſtellation of the great bear, and converting the youth into this ſingle ſtar behind her.

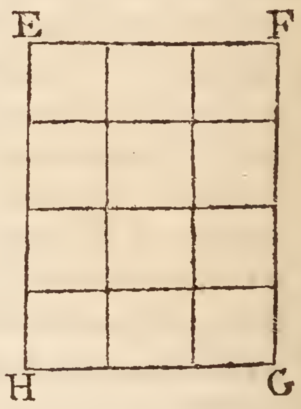
Dr. Hornſby, the Savilian Profeſſor of Aſtronomy, concludes that Arcturus is the neareſt ſtar to our ſyſtem viſible in the northern hemiſphere, becauſe the variation of its place, in conſequence of a proper motion of its own, is more remarkable than that of any other of the ſtars; and by comparing a variety of obſervations reſpecting both the quantity and direction of the motion of this ſtar, he infers, that the obliquity of the ecliptic decreaſes at the rate of 58″ in 100 years; a quantity which nearly correſponds to the mean of the computations framed by Euler and De la Lande, upon the principles of attraction. *Philoſ. Tranſ.* v. 63.

ARCTUS, a name given by the Greeks to two conſtellations of the northern hemiſphere; by the Latins called Ursa *major* and *minor*, and by us the *greater* and *leſſer* bear.

AREA, in general, denotes any plain ſurface to walk upon; and derived from *arere*, *to be dry*.

Area, in *Architecture*, denotes the ſpace or ſcite of ground on which an edifice ſtands. It is alſo uſed for inner courts, and ſuch like portions of ground.

Area, in *Geometry*, denotes the ſuperficial content of any figure. The areas of figures are eſtimated in ſquares and parts of ſquares. Thus, ſuppoſe a rectangle EFGH have its length EH equal to 4 inches, or feet, or yards, &c, and its breadth EF equal to 3; its area will then be 3 times 4, or 12 ſquares, each ſide of which is reſpectively one inch, or foot, or yard, &c. The areas of other particular figures may be ſeen under their reſpective names.



The areas of all ſimilar figures, are in the duplicate ratio, or as the ſquares of their like ſides, or of any like linear dimenſions.——Alſo the law by which the planets move round the ſun, is regulated by the areas deſcribed by a line connecting the ſun and planet; that is, the time in which the planet deſcribes, or paſſes over, any arc of its elliptic orbit, is proportional to the elliptic area deſcribed in that time by the ſaid line, or the ſector contained by the ſaid arc and two radii drawn from its extremities to the focus in which the ſun is placed.

Area, in *Optics*. See Field.

ARENARIUS, the name of a book of Archimedes, in which he demonſtrated, that not only the ſands of the earth, but even a greater quantity of particles than could be contained in the immenſe ſphere of the fixed ſtars, might be expreſſed by numbers, in a way by him invented and deſcribed. This notation proceeds by certain geometrical progreſſions; and in denoting and producing certain very diſtant terms of the progreſſion, he here firſt of any one makes uſe of a property ſimilar to that of logarithms, viz, adding the indices of the terms, to find the index of the product of them. See Archimedes.

AREOMETER, Aræometrum, an inſtrument to meaſure the denſity or gravity of fluids.

The areometer, or water-poiſe, is commonly made of glaſs; conſiſting of a round hollow ball, which terminates in a long ſlender neck, hermetically ſealed at top; having firſt as much running mercury put into it, as will ſerve to balance or keep it ſwimming in an erect poſition. The ſtem, or neck, is divided into degrees or parts which are numbered, to ſhew, by the depth of its deſcent into any liquor, the lightneſs or denſity of it: for that fluid is heavieſt in which it ſinks leaſt, and lighteſt in which it ſinks deepeſt.

Another inſtrument of this kind is deſcribed by Homberg of Paris, in the Memoirs of the Acad. of Sciences for the year 1699; alſo in the Philoſ. Tranſ. N° 262, where a table of numbers is given, expreſſing the denſity of various fluids, as determined by this inſtrument both in ſummer and winter. By this table it appears that the denſity, or ſpecific gravity of quickſilver and diſtilled water, in the two ſeaſons, were as follow, viz,

in ſummer as - 13·61 to 1,
in winter as - 13·53 to 1;
and the medium of theſe two is as 13·57 to 1.

See alſo the Philoſ. Tranſ. vol. 36, or Abridg. vol. 6, for the deſcription and uſe of another new areometer.

AREOMETRY, the ſcience of meaſuring the lightneſs and denſity of fluids.—See the Philoſ. Tranſ. vol. 68, for an eſſay on areometry, &c.

AREOSTYLE, in Architecture, a ſort of intercolumniation in which the columns were placed at a great diſtance from one another.

ARGENTICOMUS, among Ancient Aſtrologers, denotes a kind of ſilver-haired comet, of uncommon luſtre, ſuppoſed to be the cauſe of great changes in the planetary ſyſtem.

ARGETENAR, a ſtar of the fourth magnitude, in the flexure of the conſtellation Eridanus.

ARGO NAVIS, or *the ſhip*, is a conſtellation of fixed ſtars, in the ſouthern hemiſphere, being one of the 48 old conſtellations. The number of ſtars in this conſtellation, are, in Ptolemy's catalogue 45, in Tycho Brahe's 11, in Flamſteed's 64.

The Greeks tell us, that this was the famous ſhip in which the Argonauts performed that celebrated expedition, which has been ſo famous in all their hiſtory.

ARGUMENT, in Aſtronomy, is an arch given, by which another arch is found in ſome proportion to it. Hence,

Argument *of Inclination*, or Argument *of Latitude*, of any planet, is an arch of a planet's orbit, intercepted between the aſcending node, and the place of the planet from the ſun, numbered according to the ſucceſſion of the ſigns.

Menſtrual Argument *of Latitude*, is the diſtance of the moon's true place from the ſun's true place.—By this is found the quantity of the real obſcuration in eclipſes, or how many digits are darkened in any place.

Annual Argument *of the moon's apogee*, or ſimply, *Annual Argument*, is the diſtance of the ſun's place from the place of the moon's apogee; that is, the arc of the ecliptic compriſed between thoſe two places.

ARIES, or the *Ram*, in *Aſtronomy*, one of the conſtellations of the northern hemiſphere, and the firſt of the old twelve ſigns of the zodiac, and marked ♈ in imitation of a ram's head. It gives name to a twelfth part of the ecliptic, which the ſun enters commonly about the 20th of March.—The ſtars of this conſtellation in Ptolemy's catalogue are 18, in Tycho Brahe's 21, in Hevelius's 27, and in Flamſteed's 66: but they are moſtly very ſmall, only one being of the 2d magnitude, two of the 3d magnitude, and all the reſt ſmaller.

The fabulous account of this conſtellation, as given by the Greeks, is to this effect. That Nephele gave Phryxus, her ſon, a ram, which bore a golden fleece, as a guard againſt the greateſt dangers. Juno, the ſtep-mother both of him and Helle, laid deſigns againſt their lives; But Phryxus, remembering the admonition of his mother, took his ſiſter with him, and getting upon the back of the ram, they were carried to the ſea. The ram plunged in, and the youth was carried over; but Helle dropped off, and was drowned, and ſo gave name to the Helleſpont. When he arrived in Colchis, Æeta, the king, received him kindly; and, ſacrificing the ram to Jupiter, dedicated the fleece to the god; which was afterwards carried off by Jaſon. The animal itſelf, they ſay, Jupiter ſnatched up into the heavens, and made of it the conſtellation Aries. They have other fables alſo to account for its origin. But it is moſt probable that the inventors of this ſign, placed it there as the father of thoſe animals which are brought forth about the time the ſun approaches to that part of the heavens, and ſo marking the beginning of ſpring.

Aries alſo denotes a *battering ram*; being a military engine with an iron head, much uſed by the ancients, to batter and beat down the walls of places beſieged. See Ram, and Battering *ram*.

ARISTARCHUS, a celebrated Greek philoſopher and aſtronomer, and a native of the city of Samos; but of what date is not exactly known; it muſt have been however before the time of Archimedes, as ſome parts of his writings and opinions are cited by that author, viz, in his Arenarius; he probably flouriſhed about 420 years before Chriſt. He held the opinion of Pythagoras as to the ſyſtem of the world, but whether before or after him, is uncertain, teaching that the ſun and ſtars were fixed in the heavens, and that the earth moved in a circle about the ſun, at the ſame time that it revolved about its own centre or axis. He taught alſo, that the annual orbit of the earth, compared with the diſtance of the fixed ſtars, is but as a point. On this head Archimedes ſays, "Ariſtarchus, the Samian, confuting the notions of aſtrologers, laid down certain poſitions, from whence it follows, that the world is much larger than is generally imagined; for he lays it down, that the fixed ſtars and the ſun are immoveable; and that the earth is carried round the ſun in the circumference of a circle." On which account, although he might not ſuffer perſecution and impriſonment like Galileo, yet he did not eſcape cenſure for his ſuppoſed impiety; for it is ſaid Cleanthus was of opinion, that Greece ought to have tried Ariſtarchus for irreligion, for endeavouring to preſerve the regular appearance of the heavenly bodies, by ſuppoſing that the heavens themſelves ſtood ſtill; but that the earth revolved in an oblique circle, and at the ſame time turned round its own axis.

Ariſtarchus invented a peculiar kind of ſun-dials, mentioned by Vitruvius. There is extant of his works only a treatiſe upon the magnitude and diſtance of the ſun and moon: this was tranſlated into Latin, and commented upon by Commandine, who firſt publiſhed it with Pappus's Explanations, in 1572. Dr. Wallis afterwards publiſhed it in Greek, with Commandine's Latin verſion, in 1688, and which he inſerted again in the 3d volume of his Mathematical Works, printed in folio at Oxford, 1699. The piece was animadverted upon by Mr. Foſter, in his Miſcellanies. There is another piece which has gone under the name of Ariſtarchus, of the *Mundane Syſtem*, its parts, and motions, publiſhed in Latin by Robervale, and by Merſenne, in his *Mathematical Synopſis*. But this piece is cenſured, by Menagius (in Diog. Laert.), and Deſcartes, in his Epiſtles, as a fictitious piece of Robervale's, and not the genuine work of Ariſtarchus.

ARISTOTELIAN, ſomething that relates to the philoſopher *Ariſtotle*. Thus we ſay, an *Ariſtotelian* dogma, the *Ariſtotelian* ſchool, &c.

Aristotelian *Philoſophy*, the philoſophy taught by *Ariſtotle*, and maintained by his followers. It is otherwiſe called the *peripatetic* philoſophy, from their practice of teaching while they were walking.—The principles of Ariſtotle's philoſophy, the learned agree, are chiefly laid down in the four books *de Cœlo*. Inſtead of the more ancient ſyſtems, he introduced *matter*, *form*, and *privation*, as the principles of all things; but it does not ſeem that he derived much benefit from them in natural philoſophy. And his doctrines are, for the moſt part, ſo obſcurely expreſſed, that it has not yet been ſatisfactorily aſcertained, what were his ſentiments on ſome of the moſt important ſubjects. He attempted to refute the Pythagorean doctrine concerning the two-fold motion of the earth; and pretended to demonſtrate, that the matter of the heavens is ungenerated, incorruptible, and not ſubject to any alteration: and he ſuppoſed that the ſtars were carried round the earth in ſolid orbs.

ARISTOTELIANS, a ſect of philoſophers, ſo called, from their leader *Ariſtotle*, and are otherwiſe called *Peripatetics*.—The Ariſtotelians and their dogmata prevailed for a long while, in the ſchools and univerſities; even in ſpite of all the efforts of the Carteſians, Newtonians, and other corpuſcularians. But the ſyſtems of the latter have at length gained the aſcendency; and the Newtonian philoſophy in particular is now very generally received.

ARISTOTLE, a Grecian philoſopher, the ſon of Nicomachus, phyſician to Amyntas king of Macedonia, was born 384 years before Chriſt, at Stagira, a town of Macedonia, or, as others ſay, of Thrace; from which he is alſo called the Stagirite. Not ſucceeding in the profeſſion of arms, to which it ſeems he firſt applied himſelf, he turned his views to philoſophy, and at 17 years of age entered himſelf a diſciple of Plato, and attended in the academy till the death of that philoſopher. Ariſtotle then retired to Atarna, where the prince Hermias gave him his daughter to wife. Repairing afterwards to the court of king Philip, he became preceptor to his ſon, Alexander the Great, whoſe education he attended for the ſpace of 8 years; and by the magnificent encouragement of this prince he was afterwards enabled to procure all ſorts of animals, from the inſpection of which to write the hiſtory of them. On his quitting Macedon, he ſettled at Athens, where he eſtabliſhed his ſchool, having the Lyceum aſſigned him, by the magiſtrates, for the place of his inſtruction or diſputation; where he became the head and founder of the ſect called after his name, as alſo Peripatetics, from the circumſtance of his giving inſtructions while walking. But being here accuſed of impiety by Eurymedon, prieſt of Ceres, and fearing the fate of Socrates, he retired to Chalcis, where he died at 63 years of age, and 322 years before Chriſt. Some ſay that he poiſoned himſelf, others that he died of a cholic, and others again pretend that he threw himſelf into the ſea for grief that he could not diſcover the cauſe of the flux and reflux of the waters. Laertius, in his life of Ariſtotle, eſtimates his books at the number of 4000; of which however ſcarce 20 have come down to us: theſe may be compriſed under five heads; the firſt, relating to poetry and rhetoric; the ſecond, to logics; the third, to ethics and politics; the fourth, to phyſics; and the fifth, to metaphyſics. In the ſchools, Ariſtotle has been called the philoſopher, and the prince of philoſophers. And ſuch was the veneration paid to him, that his opinion was allowed to ſtand on a level with reaſon itſelf: nor was any appeal from it admitted, the parties, in every diſpute, being obliged to ſhew, that their concluſions were no leſs conformable to the doctrine of Ariſtotle than to truth.

ARITHMETIC, the art and ſcience of numbers; or, that part of mathematics which conſiders their powers and properties, and teaches how to compute or calculate truly, and with eaſe and expedition. It is by ſome authors alſo defined the ſcience of diſcrete quantity. Arithmetic conſiſts chiefly in the four principal rules or operations of Addition, Subtraction, Multiplication, and Diviſion; to which may perhaps be added involution and evolution, or raiſing of powers and extraction of roots. But beſides theſe, for the facilitating and expediting of computations, mercantile, aſtronomical, &c, many other uſeful rules have been contrived, which are applications of the former, ſuch as, the rules of proportion, progreſſion, alligation, falſe poſition, fellowſhip, intereſt, barter, rebate, equation of payments, reduction, tare and tret, &c. Beſides the doctrine of the curious and abſtract properties of numbers.

Very little is known of the origin and invention of arithmetic. In fact it muſt have commenced with mankind, or as ſoon as they began to hold any ſort of commerce together; and muſt have undergone continual improvements, as occaſion was given by the extenſion of commerce, and by the diſcovery and cultivation of other ſciences. It is therefore very probable that the art has been greatly indebted to the Phœnicians or Tyrians; and indeed Proclus, in his commentary on the firſt book of Euclid, ſays, that the Phœnicians, by reaſon of their traffic and commerce, were accounted the firſt inventors of Arithmetic. From Aſia the art paſſed into Egypt, whither it was carried by Abraham, according to the opinion of Joſephus. Here it was greatly cultivated and improved; inſomuch that a conſiderable part of the Egyptian philoſophy and theology ſeems to have turned altogether upon numbers. Hence thoſe wonders related by them about unity, trinity, with the numbers 4, 7, 9, &c. In effect, Kircher, in his Oedip. Ægypt. ſhews, that the Egyptians explained every thing by numbers; Pythagoras himſelf affirming, that the nature of numbers pervades the whole univerſe; and that the knowledge of numbers is the knowledge of the deity.

From Egypt arithmetic was tranſmitted to the Greeks, by means of Pythagoras and other travellers; amongſt whom it was greatly cultivated and improved, as appears by the writings of Euclid, Archimedes, and others: with theſe improvements it paſſed to the Romans, and from them it has deſcended to us.

The nature of the arithmetic however that is now in uſe, is very different from that above alluded to; this art having undergone a total alteration by the introduction of the Arabic notation, about 800 years ſince, into Europe: ſo that nothing now remains of uſe from the Greeks, but the theory and abſtract properties of numbers, which have no dependence on the peculiar nature of any particular ſcale or mode of notation. That uſed by the Hebrews, Greeks, and Romans, was chiefly by means of the letters of their alphabets. The Greeks, particularly, had two different methods; the firſt of theſe was much the ſame with the Roman notation, which is ſufficiently well known, being ſtill in common uſe with us, to denote dates, chapters and ſections of books, &c. Afterwards they had a better method, in which the firſt nine letters of their alphabet repreſented the firſt numbers, from one to nine, and the next nine letters repreſented any number of tens, from one to nine, that is, 10, 20, 30, &c, to 90. Any number of hundreds they expreſſed by other letters, ſupplying what they wanted with ſome other marks or characters: and in this order they went on, uſing the ſame letters again, with ſome different marks, to expreſs thouſands, tens of thouſands, hundreds of thouſands, &c: In which it is evident that they approached very near to the more perfect decuple ſcale of progreſſion uſed by the Arabians, and who acknowledge that they had received it from the Indians. Archimedes alſo invented another peculiar ſcale and notation of his own, which he employed in his Arenarius, to compute the number of the ſands. In the 2d century of chriſtianity lived Cl. Ptolemy, who, it is ſuppoſed, invented the ſexageſimal diviſion of numbers, with its peculiar notation and operations: a mode of computation ſtill uſed in aſtronomy &c, for the ſubdiviſions of the degrees of circles. Thoſe notations however were ill adapted to the practical operations of arithmetic; and hence it is that the art advanced but very little in this part; for, ſetting aſide Euclid, who has given many plain and uſeful properties of numbers in his Elements, and Archimedes, in his Arenarius, they moſtly conſiſt in dry and tedious diſtinctions and diviſions of numbers; as appears from the treatiſes of Nicomachus, ſuppoſed to be written in the 3d century of Rome, and publiſhed at Paris in 1538; as alſo that of Boethius, written at Rome in the 6th century of Chriſt. A compendium of the ancient arithmetic, written in Greek, by Pſellus, in the 9th century, was publiſhed in Latin by Xylander, in 1556. A ſimilar work was written ſoon after in Greek by Jodocus Willichius; and a more ample work of the ſame kind was written by Jordanus, in the year 1200, and publiſhed with a comment by Faber Stapulenſis in 1480.

Since the introduction of the Indian notation into Europe, about the 10th century, arithmetic has greatly changed its form, the whole algorithm, or practical operations with numbers, being quite altered, as the notation required; and the authors of arithmetic have gradually become more and more numerous. This method was brought into Spain by the Moors or Saracens; whither the learned men from all parts of Europe repaired, to learn the arts and ſciences of them. This, Dr. Wallis proves, began about the year 1000; particularly that a monk, called Gilbert, afterwards pope, by the name of Sylveſter II, who died in the year 1003, brought this art from Spain into France, long before the date of his death: and that it was known in Britain before the year 1150, where it was brought into common uſe before 1250, as appears by the treatiſe of arithmetic of Johannes de Sacro Boſco, or Halifax, who died about 1256. Since that time, the principal writers on this art have been, Barlaam, Lucas de Burgo, Tonſtall, Aventinus, Purbach, Cardan, Scheubelius, Tartalia, Faber, Stifelius, Recorde, Ramus, Maurolycus, Hemiſchius, Peletarius, Stevinus, Xylander, Kerſey, Snellius, Tacquet, Clavius, Metius, Gemma Friſius, Buteo, Urſinus, Romanus, Napier, Ceulen, Wingate, Kepler, Briggs, Ulacq, Oughtred, Cruger, Van Schooten, Wallis, Dee, Newton, Morland, Moore, Jeake, Ward, Hatton, Malcolm, &c, &c; the particular inventions or excellencies of whom, will be noticed under the articles of the ſeveral ſpecies or kinds of arithmetic here following, which may be included under theſe heads, viz, *theoretical*, *practical*, *inſtrumental*, *logarithmical*, *numerous*, *ſpecious*, *univerſal*, *common* or *decadal*, *fractional*, *radical or of ſurds*, *decimal*, *duodecimal*, *ſexageſimal*, *dynamical or binary*, *tetractycal*, *political*, &c.

Theoretical ARITHMETIC, is the ſcience of the properties, relations, &c, of numbers, abſtractedly conſidered; with the reaſons and demonſtrations of the ſeveral rules. Such is that contained in the 7th, 8th, and 9th books of Euclid's Elements; the *Logiſtics* of Barlaam the monk, publiſhed in Latin by J. Chambers, in 1600; the *Summa Arithmetica* of Lucas de Burgo, printed 1494, who gives the ſeveral diviſions of numbers from Nicomachus, and their properties from Euclid, with the algorithm, both in integers, fractions, extraction of roots, &c; Malcolm's *New Syſtem of Arithmetic, theoretical and practical*, in 1730, in which the ſubject is very completely treated, in all its branches, &c.

Practical ARITHMETIC, is the art or practice of numbering or computing; that is, from certain numbers given, to find others which ſhall have any propoſed relation to the former. As, having the two numbers 4 and 6 given; to find their ſum, which is 10; or their difference, which is 2; or their product, 24; or their quotient, 1½; or a third proportional to them, which is 9; &c.—Lucas de Burgo's works contain the whole practice of arithmetic, then uſed, as well as the theory. Tunſtall gave a neat practical treatiſe of Arithmetic in 1526; as did Stifelius, in 1544, both on the practical and other parts. Tartalea gave an entire body of practical arithmetic, which was printed at Venice in 1556, conſiſting of two parts; the former, the application of arithmetic to civil uſes; the latter, the grounds of Algebra. And moſt of the authors in the liſt before enumerated, joined the practice of arithmetic with the theory.

Binary or *Dyadic* ARITHMETIC, is that in which only two figures are uſed, viz 1 and 0. See BINARY.—Leibnitz and De Lagny both invented an arithmetic of this ſort, about the ſame time: and Dangicourt, in the Miſcel. Berol. gives a ſpecimen of the uſe of it in arithmetical progreſſions; where he ſhews, that the laws of progreſſion may be more eaſily diſcovered by it, than by any other method where more characters are uſed.

Common or *Vulgar* ARITHMETIC, is that which is concerning integers and vulgar fractions.

Decimal or *Decadal* ARITHMETIC, is that which is performed by a ſeries of ten characters or figures, the progreſſion being ten-fold, or from 1 to 10's, 100's, &c; which includes both integers and decimal fractions, in the common ſcale of numbers; and the characters uſed are the ten Arabic or Indian figures 0, 1, 2, 3, 4, 5, 6, 7, 8, 9. This method of arithmetic was not known to the Greeks and Romans; but was borrowed from the Moors while they poſſeſſed a great part of Spain, and who acknowledge that it came to them from the Indians. It is probable that this method took its origin from the ten fingers of the hands, which were uſed in computations before arithmetic was brought into an art. The Eaſtern miſſionaries aſſure us, that to this day the Indians are very expert at computing on their fingers, without any uſe of pen and ink. And it is aſſerted, that the Peruvians, who perform all computations by the different arrangements of grains of maize outdo any European, both for certainty and diſpatch, with all his rules.

Duodecimal ARITHMETIC, is that which proceeds from 12 to 12, or by a continual ſubdiviſion according to 12. This is greatly uſed by moſt artificers, in calculating the quantity of their work; as Bricklayers, Carpenters, Painters, Tilers, &c.

Fractional ARITHMETIC, or *of fractions*, is that which treats of fractions, both vulgar and decimal.

Harmonical ARITHMETIC, is ſo much of the doctrine of numbers, as relates to the making the compariſons, reductions, &c of muſical intervals.

ARITHMETIC *of Infinites*, is the method of ſumming up a ſeries of numbers, of which the numbers of terms is infinite. This method was firſt invented by Dr. Wallis, as appears by his treatiſe on that ſubject; where he ſhews its uſes in geometry, in finding the areas of ſuperficies, the contents of ſolids, &c. But the method of fluxions, which is a kind of univerſal arithmetic of infinites, performs all theſe more eaſily; as well as a great many other things, which the former will not reach.

Inſtrumental ARITHMETIC, is that in which the common rules are performed by inſtruments, or ſome ſort of tangible or palpable ſubſtance. Such are the methods of computing by the ten fingers and the grains of maize, by the Eaſt Indians and Peruvians, above-mentioned; by the Abacus or Shwanpan of the Chineſe; the ſeveral ſorts of ſcales and ſliding rules; Napier's bones or rods; the arithmetical machine of Paſcal, and others; Sir Samuel Morland's inſtrument, deſcribed in 1666; that of Leibnitz, deſcribed in the Miſcell. Berol.; that of Polenus, publiſhed in the Venetian Miſcellany, 1709; and that of Dr. Saunderſon, of Cambridge, deſcribed in the introduction to his algebra.

Integral ARITHMETIC, or *of integers*, is that which reſpects integers, or whole numbers.

Literal or *Algebra* ARITHMETIC, is that which is performed by letters, which repreſent any numbers indefinitely.

Logarithmical ARITHMETIC is performed by the tables of logarithms. Theſe were invented by baron Napier; and the beſt treatiſe on the ſubject, is Briggs's Arithmetica Logarithmica, 1624.

Logiſtical ARITHMETIC. See LOGISTICAL.

Numerous or *Numeral* ARITHMETIC, is that which teaches the calculus of numbers, or of abſtract quantities; and is performed by the common numeral or Arabic characters.

Political ARITHMETIC, is the application of arithmetic to political ſubjects; ſuch as, the ſtrength and revenues of nations, the number of people, births, burials, &c. See POLITICAL Arithmetic. To this head may alſo be referred the doctrine of Chances, Gaming, &c.

ARITHMETIC of *Radicals*, *Rationals*, and *Irrationals*: See RADICAL, &c.

Sexageſimal or *Sexagenary* ARITHMETIC, is that which proceeds by ſixties; or the doctrine of ſexageſimal fractions: a method which, it is ſuppoſed, was invented by Ptolemy, in the 2d century; at leaſt they were uſed by him. In this notation, the integral numbers from 1 to 59 were expreſſed in the common way, by the alphabetical letters: then ſixty was called a *ſexagena prima*, and marked with a daſh to the character, thus I′; twice ſixty, or 120, thus II′; and ſo on to 59 times 60, or 3540, which is LIX′. Again, 60 times 60, or 3600, was called *ſexagena ſecunda*, and marked with two daſhes, thus I″; twice 3600, thus II″; and ten times 3600, thus X″; &c. And in this way the notation was continued to any length. But when a number leſs than ſixty was to be joined with any of the ſexageſimal integers, their proper expreſſion was annexed without the daſh: thus 4 times 60 and 25, is IV′XXV; the ſum of twice 60 and 10 times 3600 and 15, is X″II′XV. So near did the inventor of this method approach to the Arabic notation: inſtead of the ſexageſimal progreſſion, he had only to ſubſtitute decimal; and to make the ſigns of numbers, from 1 to 9, ſimple characters, and to introduce another character, which ſhould ſignify nothing by itſelf, but ſerving only to fill up places.—The *ſexagenæ integrorum* were ſoon laid aſide, in ordinary calculations, after the introduction of the Arabic notation; but the ſexageſimal fractions continued till the invention of decimals, and indeed are ſtill uſed in the ſubdiviſions of the degrees of circular arcs and angles.

Sam. Reyher has invented a kind of ſexagenal rods, in imitation of Napier's bones, by means of which the ſexageſimal arithmetic is eaſily performed.

Specious Arithmetic, is that which gives the calculus of quantities as deſigned by the letters of the alphabet: a method which was more generally introduced into algebra, by Vieta; being the ſame as literal arithmetic, or algebra.—Dr. Wallis has joined the numeral with the literal calculus; by which means he has demonſtrated the rules for fractions, proportions, extraction of roots, &c; of which a compendium is given by himſelf, under the title of Elementa Arithmeticæ, in the year 1698.

Tabular Arithmetic, is that in which the operations of multiplication, diviſion, &c, are performed by means of tables calculated for that purpoſe: ſuch as thoſe of Herwart, in 1610; and my tables of powers and products, publiſhed by order of the Commiſſioners of Longitude, in 1781.

Tetractic Arithmetic, is that in which only the four characters 0, 1, 2, 3 are uſed. A treatiſe of this kind of arithmetic is extant, by Erhard or Echard Weigel. But both this, and binary arithmetic, are little better than curioſities, eſpecially with regard to practice; as all numbers are much more compendiouſly and conveniently expreſſed by the common decuple ſcale.

Vulgar, or *Common* Arithmetic, is that which is converſant about integers and vulgar fractions.

Univerſal Arithmetic, is the name given by Newton to the ſcience of algebra; of which he left at Cambridge an excellent treatiſe, being the text-book drawn up for the uſe of his lectures, while he was profeſſor of Mathematics in that Univerſity.

ARITHMETICAL, ſomething relating to or after the manner of arithmetic.

Arithmetical *Complement*, of a logarithm, is what the logarithm wants of 10·00000 &c; and the eaſieſt way to find it is, beginning at the left hand, to ſubtract every figure from 9, and the laſt from 10. So, the arithmetical complement of 8·2501396 is 1·7498604.—It is commonly uſed in trigonometrical calculations, when the firſt term of a proportion is not radius; in that caſe, adding all together, the logarithms of the 3d, 2d, and arithmetical complement of the 1ſt term.

Arithmetical *Inſtruments*, or *Machines*, are inſtruments for performing arithmetical computations; ſuch as Napier's bones, ſcales, ſliding rules, Paſcal's machine, &c.

Arithmetical *Mean*, or *Medium*, is the middle term of three quantities in arithmetical progreſſion; and is always equal to half the ſum of the extremes. So, an arithmetical mean between 3 and 7, is 5; and between a and b, is $\frac{a+b}{2}$, or $\frac{1}{2}a + \frac{1}{2}b$.

Arithmetical *Progreſſion*, is a ſeries of three or more quantities that have all the ſame common difference: as 3, 5, 7, &c, which have the common difference 2; and a, $a + d$, $a + 2d$, &c, which have all the ſame difference d.

In an arithmetical progreſſion, the chief properties are theſe: 1ſt, The ſum of any two terms, is equal to the ſum of every other two that are taken at equal diſtances from the two former, and equal to double the middle term when there is one equally diſtant between thoſe two: ſo, in the ſeries 0, 1, 2, 3, 4, 5, 6, &c, $0 + 6 = 1 + 5 = 2 + 4 =$ twice 3 or 6.—2d, The ſum of all the terms of any arithmetical progreſſion, is equal to the ſum of as many terms of which each is the arithmetical mean between the extremes; or equal to half the ſum of the extremes multiplied by the number of terms: ſo, the ſum of theſe ten terms 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, is $\frac{0+9}{2} \times 10$, or 9×5, which is 45: and the reaſon of this will appear by inverting the terms, ſetting them under the former terms, and adding each two together, which will make double the ſame ſeries;

thus 0, 1, 2, 3, 4, 5, 6, 7, 8, 9,
inverted 9, 8, 7, 6, 5, 4, 3, 2, 1, 0,
ſums 9, 9, 9, 9, 9, 9, 9, 9, 9 9;

where the double ſeries being the ſame number of 9's, or ſum of the extremes, the ſingle ſeries muſt be the half of that ſum.—3d, The laſt, or any term, of ſuch a ſeries, is equal to the firſt term, with the product added of the common difference multiplied by 1 leſs than the number of terms, when the ſeries aſcends or increaſes; or the ſame product ſubtracted when the ſeries deſcends or decreaſes: ſo, of the ſeries 1, 2, 3, 4, &c, whoſe common difference is 1, the 50th term is $1 + 1 \times 49$, or $1 + 49$, that is 50; and of the ſeries 50, 49, 48, &c, the 50th term is $50 - 1 \times 49$, or $50 - 49$, which is 1. Alſo, if a denote the leaſt term,
z the greateſt term,
d the common difference,
n the number of the terms,
and s the ſum of them all;

then the principal properties are expreſſed by theſe equations, viz,

$$z = a + d.\overline{n - 1},$$

$$a = z - d.\overline{n - 1},$$

$$s = \overline{a + z}\,.\tfrac{1}{2}n,$$

$$s = \overline{z - \tfrac{1}{2}d.\overline{n - 1}}\,.n,$$

$$s = \overline{a + \tfrac{1}{2}d.\overline{n - 1}}\,.n.$$

Moreover, when the firſt term a, is 0 or nothing; the theorems become $z = d.\overline{n - 1}$.
and $s = \frac{1}{2}zn$.

Arithmetical *Proportion*, is when the difference between two terms, is equal to the difference between other two terms. So, the four terms, 2, 4, 10, 12, are in arithmetical proportion, becauſe the difference between 2 and 4, which is 2, is equal to the difference between 10 and 12.—The principal property, beſides the above, and which indeed depends upon it, is this, that the ſum of the firſt and laſt, is equal to the ſum of the two means: ſo $2 + 12$, or the ſum of 2 and 12, is equal $4 + 10$, or the ſum of 4 and 10, which is 14.

ARITHMETICAL *Ratio*, is the ſame as the difference of any two terms: ſo, the arithmetical ratio of the ſeries 2, 4, 6, 8, is 2; and the arithmetical ratio of a and b, is $a - b$.

ARITHMETICAL *Scales*, a name given by M. de Buffon, in the *Memoirs of the Acad.* for 1741, to different progreſſions of numbers, according to which, arithmetical computations might be made. It has already been remarked above, that our common decuple ſcale of numbers was probably derived from the number of fingers on the two hands, by means of which the earlieſt and moſt natural mode of computation was performed; and that other ſcales of numbers, formed in a ſimilar way, but of a different number of characters, have been deviſed; ſuch as the binary and tetractic ſcales of arithmetic. In the memoir above cited, Buffon gives a ſhort and ſimple method to find, at once, the manner of writing down a number given in any ſcale of numbers whatever; with remarks on different ſcales. The general effect of any number of characters, different from ten, is, that by a ſmaller number of characters, any given number would require more places of figures to expreſs or denote it by, but then arithmetical calculations, by multiplication and diviſion, would be eaſier, as the ſmall numbers 2, 3, 4, &c, are eaſier to uſe than the larger 7, 8, 9; and by employing more than ten characters, although any given number would be expreſſed by fewer of them, yet the calculations in arithmetic would be more difficult, as by the larger numbers 11, 12, 13, &c. It is therefore concluded, upon the whole, that the ordinary decuple ſcale is a good convenient medium amongſt them all, the numbers expreſſed being tolerably ſhort and compendious, and no ſingle character repreſenting too large a number. The ſame might alſo be ſaid, and perhaps more, of a duodecimal ſcale, by twelve characters, which would expreſs all numbers in a more compendious way than the decuple one, and yet no ſingle character would repreſent a number too large to compute by; as is confirmed by the now common practice of extending the multiplication table, in ſchool books, to 12 numbers or dimenſions, each way, inſtead of 10; and every perſon is taught, with ſufficient eaſe, to multiply and divide by 11 and 12 as eaſily as by 8 or 9 or 10. Another convenience might be added, namely, that the number 12 admitting of more ſubmultiples than the number 10, there would be fewer expreſſions of interminate fractions in that way than in decimals. So that on all accounts, it is very probable that the duodecimal would be the beſt of any ſcale of numbers whatever.

ARITHMETICAL *Triangle*. See *Arithmetical* TRIANGLE.

ARMED. A magnet or loadſtone is ſaid to be armed, when it is capped, caſed, or ſet in iron or ſteel; to make it take up a greater weight; and alſo readily to diſtinguiſh its poles.

It is ſurpriſing, that a little iron faſtened to the poles of a magnet, ſhould ſo greatly improve its power, as to make it even 150 times ſtronger, or more, than it is naturally, or when unarmed. The effect however, it ſeems, is not uniform; but that ſome magnets, by arming, gain much more, and others much leſs, than one would expect; and that ſome magnets even loſe ſome of their efficacy by arming. In general, however, the thickneſs of the iron armour ought to be nearly proportioned to the natural ſtrength of the magnet; giving thick irons to a ſtrong magnet, and to the weaker ones thinner: ſo that a magnet may eaſily be over-loaded.

The uſual armour of a load-ſtone, in form of a right-angled parallelopipedon, conſiſts of two thin pieces of iron or ſteel, of a ſquare figure, and of a thickneſs proportioned to the goodneſs of the ſtone; the proper thickneſs being found by trials; always filing it thinner and thinner, till the effect be found to be the greateſt poſſible.—The armour of a ſpherical load-ſtone, conſiſts of two ſteel ſhells, faſtened together by a joint, and covering a good part of the convexity of the ſtone. This alſo is to be filed away, till the effect is found to be the greateſt.

Kircher, in his book *de Magnete*, ſays, that the beſt way to arm a load-ſtone, is to drill a hole through the ſtone, from pole to pole, in which is to be placed a ſteel rod of a moderate length: this rod, he aſſerts, will take up more weight at the end, than the ſtone itſelf when armed in the common way. And Gaſſendus and Cabæus preſcribe the ſame method of arming. But Muſchenbroek found, by repeated trials, that the uſual armour already mentioned, is preferable to Kircher's; and he gives the following directions for preparing it. When, by means of ſteel filings and a ſmall needle, the poles of a magnet have been diſcovered, he directs that the adjacent parts ſhould be rubbed or ground into parallel planes, without ſhortening the polar axis; and the magnet may be afterwards ſhaped into the figure of a cube or parallelopipedon, or any other figure that may be more convenient. Plates of the ſofteſt iron are then prepared, of the ſame length and breadth with the whole polar ſides of the magnet: the thickneſs of which plates, ſo as that they may admit and convey the greateſt quantity of the magnetic virtue, is to be previouſly determined by experiment, in a manner which he preſcribes for the purpoſe. A thicker piece of iron is to be annexed at right angles to theſe plates, which is called *pes armaturæ*, the foot or baſe of the armour: then the plates, nicely ſmoothed and poliſhed, are to be firmly attached to each of the polar ſides, whilſt the thicker part or baſe is brought into cloſe contact with the lower part of the magnet. In this way, he ſays, almoſt all the magnetic virtue iſſuing from the poles, enters into the armour, is directed to the baſe, and condenſed by means of its roundneſs, ſo as to ſuſtain the greateſt weight of iron. Phyſ. Exper. and Geom. Diſſert. 1729, pa. 131.

ARMILLARY *Sphere*, a name given to the artificial ſphere, compoſed of a number of circles of metal, wood, or paper, which repreſent the ſeveral circles of the ſyſtem of the world, put together in their natural order. It ſerves to aſſiſt the imagination to conceive the diſpoſition of the heavens, and the motion of the celeſtial bodies.

This ſphere is repreſented at Plate II, Fig. 6, where P and Q repreſent the poles of the world, AD the equator, EL the ecliptic and zodiac, PAGD the meridian, or the ſolſtitial colure, T the earth, FG the tropic of cancer, HT the tropic of capricorn, MN the arctic circle, OV the antarctic, N and O the poles of the ecliptic, and RS the horizon.

The Armillary ſphere conſtructed not long ſince by Dr. Long, in Pembroke-hall, Cambridge, is 18 feet in diameter; and will contain more than 30 perſons ſitting within it, to view, as from a centre, the repreſentation of the celeſtial ſpheres. The lower part of the ſphere, which is not viſible to England, is cut off; and the whole apparatus is ſo contrived, that it may be turned round with as little labour as is employed to wind up a common jack.

See alſo Mr. Ferguſon's ſphere in his Lectures, p. 194.

ARMILLARY *Trigonometer*, an inſtrument firſt contrived by Mr. Mungo Murray, and improved by Mr. Ferguſon, conſiſting of five ſemicircles; viz, meridian, vertical circle, horizon, hour circle, and equator; ſo adapted to each other by joints and hinges, and ſo divided and graduated, as to ſerve for expeditiouſly reſolving many problems in aſtronomy, dialling, and ſpherical trigonometry. The drawing, deſcription, and method of uſing it, may be ſeen in Ferguſon's Tracts, pa. 80, &c.

ARTIFICIAL *Numbers*, *Sines*, *Tangents*, &c, are the ſame as the Logarithms of the natural numbers, ſines, tangents, &c.

ARTILLERY, the heavy equipage of war; comprehending all ſorts of large fire-arms, with their appurtenances; as cannon, mortars, howitzers, balls, ſhells, petards, muſquets, carbines, &c; being what is otherwiſe called *Ordnance*. The term is alſo applied to the larger inſtruments of war uſed by the ancients, as the catapult, baliſta, battering ram, &c.

The term *Artillery*, or *Royal Artillery*, is alſo applied to the perſons employed in that ſervice; and likewiſe to the art or ſcience itſelf; and formerly it was uſed for what is otherwiſe called *pyrotechnia*, or the art of fireworks, with the apparatus and inſtruments belonging to the ſame.

There have been many authors on the ſubject of artillery; the principal of which are, Bucherius, Braunius, Tartalea, Collado, Sardi, Ufano, Hanzelet, Digges, Moretti, Simienowitz, Mieth, d'Avelour, Maneſſon, Mallet, St. Julien; and the later authors, of ſtill more conſequence, are Belidor, St. Remy, le Blond, Valiere, Morogue, Puget, Coudray, Robins, Muller, Euler, Antoni, Tignola, Scheele; to which may be added the extenſive and accurate experiments publiſhed in my 1ſt vol. of Tracts, and in the Philoſ. Tranſ. for 1778.

Park of ARTILLERY, is that place in a camp which is ſet apart for the Artillery, or large fire arms.

Traile or *Train* of ARTILLERY, a number of pieces of ordnance, mounted on carriages, with all their furniture fit for marching. To this commonly belong mortars, cannon, balls, ſhells, &c.—There are trains of Artillery in moſt of the royal magazines; as in the Tower, at Portſmouth, Plymouth, &c, but, above all, at Woolwich, from whence the ſhips commonly receive their ordnance, and where they are all completely proved before they are received into the public ſervice.

The officers and men of the artillery were formerly called alſo the Train of Artillery, but are now called the Royal Regiment of Artillery; conſiſting at preſent of four battalions, beſides a battalion of invalids, and four troops of Horſe or Cavalry Artillery.

ASCENDANT, in *Aſtrology*, denotes the horoſcope; or the degree of the ecliptic which riſes upon the horizon, at the time of the birth of any one.—This, it is ſuppoſed, has an influence on the perſon's life and fortune, by giving him a bent and propenſity to one thing more than other.—In the ſcience of Aſtrology, this is called the firſt houſe, the Oriental angle, or angle of the Eaſt, and the ſignificator of life: and the aſtrologer ſays, ſuch a planet ruled in his Aſcendant, or Jupiter was in his Aſcendant, &c.

ASCENDING, in Aſtronomy, a term uſed to denote any ſtar, or degree, or other point of the heavens, riſing above the horizon.

ASCENDING *Latitude*, is the latitude of a planet when going towards the north.

ASCENDING *Node*, is that point of a planet's orbit where it croſſes the ecliptic, in proceeding northward. It is otherwiſe called the Northern Node, and is denoted by this character ☊, repreſenting a node, or knot, with the larger part upwards; like as the ſame character reverſed is uſed to denote the oppoſite, or Deſcending Node.

ASCENDING *Signs*, are ſuch as are upon their aſcent, or riſe, from the nadir or loweſt point of the heavens, towards the zenith, or higheſt point.

ASCENSION-DAY, otherwiſe called *Holy Thurſday*, is a feſtival of the church, held 10 days before Whitſunday, in memory of our Saviour's Aſcenſion.

ASCENSION, in Aſtronomy, is either Right or Oblique.

Right ASCENSION of the ſun, or of a ſtar, is that degree of the equinoctial, accounted from the beginning of Aries, which riſes with them, in a right ſphere.—Or, *Right Aſcenſion*, is that point of the equinoctial, counted as before, which comes to the meridian with the ſun or ſtar, or other point of the heavens. And the reaſon of thus referring it to the meridian, is, becauſe this is always at right angles to the equinoctial; whereas the horizon is ſo only in a right or direct ſphere.—The right aſcenſion, ſtands oppoſed to the right deſcenſion; and is ſimilar to the longitude of places on the earth. All the fixed ſtars, &c, which have the ſame right-aſcenſion, that is, which are at the ſame diſtance from the firſt point of Aries, or, which comes to the ſame thing, which are in the ſame meridian, riſe at the ſame time in a right ſphere, namely to the people who live at the equator. And if they be not in the ſame meridian, the difference between their times of riſing, or of coming to the meridian of any place, is the preciſe difference of their right aſcenſion.—But, in an oblique ſphere, where the horizon cuts all the meridians obliquely, different points of the ſame meridian never riſe or ſet together: ſo that two or more ſtars on the ſame meridian, or having the ſame right aſcenſion, never riſe or ſet at the ſame time in an oblique ſphere: and the more oblique the ſphere is, the greater is the interval of time between them.

To find the right aſcenſion of the ſun, ſtars, &c, by Trigonometry, ſay, As radius is to the coſine of the ſun's greateſt declination, or obliquity of the ecliptic, ſo is the tangent of the ſun's or ſtar's longitude, to the tangent of the right aſcenſion.

RIGHT ASCENSION *of the Mid-heaven*, often uſed by aſtronomers, eſpecially in calculating eclipſes by means of the nonageſimal degree, is the right aſcenſion of that point of the equator which is in the meridian; and it is equal to the ſum of the ſun's right aſcenſion and the horary angle or true time reduced to degrees, or to the ſum of the mean longitude and mean time.

Oblique ASCENSION, is an arch of the equator intercepted between the firſt point of Aries, and that point of the equator which riſes together with the ſtar, &c, in an oblique ſphere.—The Oblique Aſcenſion is counted from weſt to eaſt; and is greater or leſs, according to the various obliquity of the ſphere.—To find the Oblique Aſcenſion of the ſun, ſee ASCENSIONAL and GLOBE.

The *Arch of Oblique Aſcenſion*, is an arch of the horizon intercepted between the beginning of Aries, and the point of the equator which riſes with a ſtar or planet, in an oblique ſphere; and it varies with the latitude of the place.

Refraction of ASCENSION *and Deſcenſion*. See REFRACTION.

ASCENSIONAL DIFFERENCE, is the difference between the right and oblique aſcenſion of the ſame point on the ſurface of the ſphere. Or it is the time the ſun riſes or ſets before or after 6 o'clock.

To find the Aſcenſional Difference, having given the ſun's declination and the latitude of the place, ſay, As radius is to the tangent of the latitude, ſo is the tangent of the ſun's declination to the ſine of the Aſcenſional Difference ſought. The ſun's Aſcenſional Difference converted into time, ſhews how much he riſes before, or ſets after, 6 o'clock. When the ſun has north declination, the right aſcenſion is greater than the oblique; but the contrary, when the ſun has ſouth declination; and the difference, in either caſe, is the Aſcenſional Difference.

ASCENT, the motion of a body from below tending upwards; or the continual receſs of a body from the earth, or from ſome other centre of force. And it is oppoſed to *deſcent*, or motion downwards.

The Peripatetics attributed the ſpontaneous aſcent of bodies to a principle of levity, inherent in them. But the moderns deny that there is any ſuch thing as ſpontaneous levity; and they ſhew, that whatever aſcends, does ſo by virtue of ſome external impulſe or extruſion. Thus it is that ſmoke, and other rare bodies, aſcend in the atmoſphere; and oil, light woods, &c, in water: not by any inherent principle of levity; but by the ſuperior gravity, or tendency downwards of the medium in which they aſcend and float.

The aſcent of light bodies in heavy mediums, is produced after the ſame manner as the aſcent of the lighter ſcale of a balance. It is not that ſuch ſcale has an internal principle, by which it immediately tends upwards; but it is impelled upwards by the preponderancy of the other ſcale; the exceſs of the weight in the one having the ſame effect, by augmenting its impetus downwards, as ſo much real levity in the other: becauſe the tendencies mutually oppoſe each other, and action and reaction are always equal.—See this farther illuſtrated under the articles FLUID, and SPECIFIC GRAVITY.

ASCENT *of Bodies on Inclined Planes*. See the doctrine and laws of them under INCLINED PLANE.

ASCENT *of Fluids*, is particularly underſtood of their riſing above their own level, between the ſurfaces of nearly contiguous bodies, or in ſlender capillary glaſs tubes, or in veſſels filled with ſand, aſhes, or the like porous ſubſtances. Which is an effect that takes place as well *in vacuo*, as in the open air, and in crooked, as well as ſtraight tubes. Indeed ſome fluids aſcend ſwifter than others, as ſpirit of wine, and oil of turpentine; and ſome riſe after a different manner from others. The phenomenon, with its cauſes, &c, in the inſtance of capillary tubes, will be treated more at large under CAPILLARY *Tube*.

As to planes: Two ſmooth poliſhed plates of glaſs, metal, ſtone, or other matter, being placed almoſt contiguous, have the effect of ſeveral capillary tubes, and the fluid riſes in them accordingly: the like may be ſaid of a veſſel filled with ſand, &c; the various ſmall interſtices of which form, as it were, a kind of capillary tubes. So that the ſame principle accounts for the appearance in them all. And to the ſame cauſe may probably be aſcribed the aſcent of the ſap in vegetables. And on this ſubject Sir I. Newton ſays, "If a large pipe of glaſs be filled with ſifted aſhes, well preſſed together, and one end dipped into ſtagnant water, the fluid will aſcend ſlowly in the aſhes, ſo as in the ſpace of a week or fortnight, to reach the height of 30 or 40 inches above the ſtagnant water. This aſcent is wholly owing to the action of thoſe particles of the aſhes which are upon the ſurface of the elevated water; thoſe within the water attracting as much downwards as upwards: it follows, that the action of ſuch particles is very ſtrong; though being leſs denſe and cloſe than thoſe of glaſs, their action is not equal to that of glaſs, which keeps quickſilver ſuſpended to the height of 60 or 70 inches, and therefore acts with a force which would keep water ſuſpended to the height of above 60 feet. By the ſame principle, a ſpunge ſucks in water, and the glands in the bodies of animals, according to their ſeveral natures and diſpoſitions, imbibe various juices from the blood." Optics, pa. 367.

Again, if a drop of water, oil, or other fluid, be dropped upon a glaſs plane, perpendicular to the horizon, ſo as to ſtand without breaking, or running off; and another plane touching it at one end, be gradually inclined towards the former, till it touch the drop; then will the drop break and move along towards the touching end of the planes; and it will move the faſter in proportion as it proceeds farther, becauſe the diſtance between the planes is conſtantly diminiſhing. And after the ſame manner, the drop may be brought to any part of the planes, either upward or downward, or ſideways, by altering the angle of inclination.

Laſtly, if the ſame perpendicular planes be ſo placed, as that two of their ſides meet, and form a ſmall angle, the other two being only kept apart by the interpoſition of ſome thin body; and thus immerged in a fluid, tinged with ſome colour to render it viſible; the fluid will aſcend between the planes, and that the higheſt where the planes are neareſt; ſo as to form a curve line which is found to be a true hyperbola, of which one of the aſymptotes is the line of the fluid, the other being a line drawn along the touching ſides.

And the phyſical cauſe of all theſe phenomena, is the ſame power of attraction.

ASCENT *of Vapour*. See CLOUD and VAPOUR.

ASCENT, in Aſtronomy, &c. See ASCENSION.

ASCII, are thoſe inhabitants of the globe, who, at certain times of the year, have no ſhadow. Such are the inhabitants of the torrid zone, who twice a year having the ſun at noon in their zenith, have then no ſhadow.

ASELLI, two fixed ſtars of the fourth magnitude, in the conſtellation Cancer.

ASH-WEDNESDAY, the firſt day of Lent, ſuppoſed to have been ſo called from a cuſtom in the church, of ſprinkling aſhes that day on the heads of penitents then admitted to penance.

ASPECT, is the ſituation of the ſtars and planets in reſpect of each other. Or, in Aſtrology, it denotes a certain configuration and mutual relation between the planets, ariſing from their ſituations in the zodiac, by which it is ſuppoſed that their powers are mutually either increaſed or diminiſhed, as they happen to agree or diſagree in their active or paſſive qualities. Though ſuch configurations may be varied and combined a thouſand ways, yet only a few of them are conſidered. Hence, Wolfius more accurately defines aſpect to be, the meeting of luminous rays emitted from two planets to the earth, either poſited in the ſame right line, or including an angle which is an aliquot part, or ſome number of aliquot parts, of four right angles, or of 360 degrees.

The doctrine of aſpects was introduced by the aſtrologers, as the foundation of their predictions. And hence Kepler defines aſpect to be, an angle formed by the rays of two planets meeting on the earth, capable of exciting ſome natural power or influence.

The ancients reckoned five aſpects, viz, *conjunction*, *ſextile*, *quartile*, *trine*, and *oppoſition*.

Conjunction is denoted by this character ☌, and is when the planets are in the ſame ſign and degree, or have the ſame longitude.

Sextile is denoted by ⚹, and is when the planets are diſtant by the 6th part of a circle, or 2 ſigns, or 60 degrees.

Quartile is denoted by □, and is when the planets are diſtant $\frac{1}{4}$ of the circle, or 90 degrees, or 3 ſigns.

Trine is denoted by △, and is when the planets are diſtant by $\frac{1}{3}$ of the circle, or 4 ſigns, or 120 degrees. And

Oppoſition is denoted by ☍, and is when the planets are in oppoſite points of the circle, or differ by $\frac{1}{2}$ the circle, or 6 ſigns, or 180 degrees of longitude.

Or their characters and diſtances are as in this following tablet.

NAME.	CHARACTER.	DISTANCE.
Conjunction -	☌	0°
Sextile - - -	⚹	60
Quartile - -	□	90
Trine - - -	△	120
Oppoſition -	☍	180

Theſe intervals are reckoned according to the longitudes of the planets; ſo that the aſpects are the ſame, whether the planet be in the ecliptic or out of it.

To the five ancient aſpects, modern writers have added ſeveral more: as *decile*, for the 10th part of a circle; *tridecile*, or $\frac{3}{10}$; and biquintile, or $\frac{2}{5}$ of a circle. And Kepler adds others, from meteorological obſervations, as he tells us: as the *ſemiſextile*, being $\frac{1}{12}$, or 30°; and *quincunx*, or $\frac{5}{12}$, or 150°. Laſtly, to the aſtrological phyſicians, &c, we owe *octile*, or $\frac{1}{8}$; *trioctile*, or $\frac{3}{8}$; and *quintile*, $\frac{1}{5}$ of the circle.

The aſpects are divided with regard to their ſuppoſed influences, into *benign*, *malign*, and *indifferent*. The trine and ſextile aſpects being eſteemed benign or friendly; the quartile and oppoſition, malign or unfriendly; and the conjunction an indifferent aſpect.

Aſpects are alſo diſtinguiſhed into *partile* and *platic*.

Partile ASPECT, is when the planets are juſt ſo many degrees diſtant, as are expreſſed above. And theſe only are the proper aſpects.

Platic ASPECT, is when the planets do not regard each other exactly from theſe very degrees of diſtance; but the one exceeding as much as the other falls ſhort. So that the one does not caſt its rays immediately on the body of the other, but only on its orb or ſphere of light.

ASPERITY, ſignifies the inequality or roughneſs of the ſurface of any body; by which ſome parts of it are more prominent than the reſt, ſo as to hinder the hand, &c, from paſſing over it with eaſe and freedom; and thus producing what is called friction.—Aſperity, or roughneſs, ſtands oppoſed to ſmoothneſs, evenneſs, politure, &c.

According to the relations of Vermauſen, the blind man ſo celebrated for diſtinguiſhing colours by the touch, it ſeems that every colour has its peculiar degree and kind of aſperity. He makes black the rougheſt, as it is the darkeſt of colours; but the others are not ſmoother in proportion as they are lighter; that is, the rougheſt do not always reflect the leaſt light: for, according to him, yellow is two degrees rougher than blue, and as much ſmoother than green. See COLOURS.

ASSIGNABLE *Magnitude*, is uſed for any finite magnitude that can be expreſſed or denoted. And,

ASSIGNABLE *Ratio*, for any expreſſible ratio.

ASSUMPTION, a feaſt celebrated in the Romiſh church, in honour of the miraculous aſcent of the Holy Virgin, as they deſcribe it, body and ſoul, into heaven. It is kept on the 15th of Auguſt.

ASSURANCE *on Lives*, a compact by which ſecurity is granted for the payment of a certain ſum of money on the expiration of the life on which the policy is granted, in conſideration of ſuch a previous payment made to the aſſurer as is accounted a ſufficient compenſation for the loſs and hazard to which he expoſes himſelf.

The ſum at which this compenſation ſhould be valued, depends principally on theſe two circumſtances, viz, 1ſt, On the rate of intereſt given for the uſe of money; and 2d, On the probability of the duration of the life aſſured, and the values of annuities. For, 1ſt, If the intereſt of money be high, the value of the aſſurance will be proportionally low, & *è contra*; becauſe the higher the rate of intereſt, the leſs will be the preſent value which amounts to a certain propoſed ſum in any given time. Alſo, if the probability of the duration of life be high, the value of the aſſurance will again be proportionably low, & *è contra*; becauſe the longer the time is, the leſs will be the principal which will amount to any aſſigned ſum. Thus, if it be required to know the premium or preſent value, to be given for 100 pounds to be received at the end of any time, as ſuppoſe 10 years; then, if the intereſt of money be at the rate of 5 per cent. the anſwer, or preſent premium, would be 61l. 7s. 10d; but at four per cent. it would be 67l. 11s. 1d; and at 3 per cent. it would amount to 74l. 8s. 2d. Again, ſuppoſe it were required to aſſure 100l. on a life, for any time, for inſtance 1 year; that is, let 100l. be ſuppoſed to be payable a year hence, provided a life of a given age fails in that time: here it is evident that, whatever be the rate of intereſt, the leſs the probability of the life failing within the year, the leſs the riſk is, and the leſs the premium ought to be. In effect, the rate of intereſt being 5 per cent, if it were ſure that the life would fail in that year, the value of the aſſurance would be the ſame as the preſent value of 100l. payable at the end of the year, which is 95l. 4s. 9d. But, if it be an equal chance whether the life does or does not fail in the year, in which caſe the probability of failing is $\frac{1}{2}$; then the value of the aſſurance will be but half the former value, or 47l. 12s. 4½d. Or if the odds againſt its failing be as 2 to 1, that is, if one perſon out of every 3 die at the age of the propoſed life, the probability of dying being only $\frac{1}{3}$, the value of the aſſurance will be $\frac{1}{3}$ of the firſt value, or 31l. 14s. 11d. And if the odds be 19 to 1, or one perſon die out of 20, of that age, the probability of dying will be $\frac{1}{20}$, and the value of the aſſurance will alſo be $\frac{1}{20}$ of the firſt value, or 4l. 15s. 3d. nearly. Laſtly, if only one perſon die out of 50 at the given age, the probability of dying will be $\frac{1}{50}$, and the value of the aſſurance will be accordingly only $\frac{1}{50}$ of the firſt ſum, or 1l. 18s. 1d: the intereſt of money being all along conſidered as after the rate of 5 per cent.—Now, according to Dr. Halley's table of obſervations, one perſon dies out of 3, at the age of 87; one in 20 at the age of 64; and one in 50 at the age of 39: It follows, therefore, that the value of the aſſurance of 100l. for one year, on a life aged 87, is 31l. 14s. 11d; on a life aged 64, it is 4l. 15s. 3d; and on a life aged 39, it is 1l. 18s. 1d: reckoning intereſt at 5 per cent. But if intereſt were rated at 3 per cent. theſe values would be 32l. 7s. 3d, and 4l. 17s. 1d, and 1l. 18s. 10d.

The aſſurances moſt commonly practiſed, are ſuch as theſe, on ſingle lives, and for ſingle years. But many private aſſurers, and even ſome large aſſuring offices, either from ignorance or impoſition, pay no regard to any difference of age, but demand 5l. from all ages indiſcriminately, for the aſſurance of 100l. for one year: a practice very abſurd and inequitable; for it appears that this is more than the value of the aſſurance of a life of 64 years of age, and even more than double the value of the aſſurance of a life of 39 years of age; allowing the aſſured to make 5 per cent. of the money he advances.

When a life is aſſured for any number of years; the premium or value may be paid, either in one ſingle preſent payment; in conſequence of which the ſum aſſured will become payable without any farther compenſation, whenever, within the given term, the life ſhall happen to drop: or the value may be paid in annual payments, to be continued till the failure of the life, ſhould that happen within the term; or, if not, till the determination of the term. And the determination of the value of aſſurance, in all caſes, is to be made out from the rules for computing annuities on lives; the principal writers on which are Halley, De Moivre, Simpſon, Smart, Kerſſeboom, De Parcieux, Price, Morgan, and Maſeres. See alſo LIFE ANNUITIES, REVERSION, &c.

Aſſurances may be made either on *ſingle lives*; as above explained; or they may be made on any number of *joint* lives, or on the *longeſt* of any lives; that is, an aſſurer may bind himſelf to pay any ſums at the extinction of any *joint* lives, or the *longeſt* of any lives, or at the extinction of any one or two of any number of lives. There are further aſſurances on ſurvivorſhips; by which is meant an obligation, for the value received, to pay a given ſum or annuity, provided a given life ſhall ſurvive any other given life or lives. For which ſee SURVIVORSHIP.

The principal offices for making theſe inſurances, in England, are the "London and the Royal Exchange Aſſurance Offices;" "the Amicable Society, incorporated for a perpetual Aſſurance Office;" "the Society for equitable Aſſurances on Lives and Survivorſhips;" and "the Weſtminſter Society for granting Annuities and inſuring Money on Lives."

The firſt two of theſe offices, having chiefly in view aſſurances on ſhips and houſes, deal but little in the way of aſſurances on lives; and all the buſineſs they tranſact in this way, is at 5l. for every 100l. aſſured on a ſingle life for a ſingle year, without paying any regard to the ages of the lives aſſured.

The next, or Amicable Society at Serjeant's Inn, requires an annual payment of 5l. from every member during life, payable quarterly. The whole annual income, hence ariſing, is equally divided among the nominees, or heirs, of ſuch members as die every year. But this ſociety engages that the dividends ſhall not be leſs than 150l. to each claimant, though they may be more. No members are admitted whoſe ages are greater than 45, or leſs than 12; nor is any difference of contribution allowed on account of difference of age. The ſociety has ſubſiſted ever ſince the year 1706, and its credit and uſefulneſs are well eſtabliſhed.

The Equitable Society for Aſſurances on Lives and Survivorſhips, which meets at Black-Friars' Bridge, was eſtabliſhed in the year 1762, in conſequence of propoſals which had been made, and lectures, recommending ſuch a deſign, which had been read by Mr. Dodſon, author of the Mathematical Repoſitory. It aſſures any ſums or reverſionary annuities, on any life or lives for any number of years, as well as for the whole continuance of the lives; and in any manner that may be beſt adapted to the views of the perſons aſſured: that is, either by making the aſſured ſums payable certainly at the failure of any given lives; or on condition of ſurvivorſhip; and alſo, either by taking the price of the aſſurance in one preſent payment, or in annual payments, during any ſingle or joint lives, or any terms, leſs than the whole poſſible duration of the lives. In ſhort, there are no kinds of aſſurances on lives and ſurvivorſhips, which this ſociety does not make.

In doing this, the Society follows the rules which have been given by the beſt mathematical writers on the doctrine of Life Annuities and Reverſions, particularly Mr. Thomas Simpſon, profeſſor of Mathematics in the Royal Military Academy. It is to be obſerved however that the Society takes the advantage of making its calculations on the ſuppoſition that the intereſt of money is at ſo low a rate as 3 per cent, inſtead of the uſual intereſt of 4 per cent; which conſequently raiſes the inſurance proportionally higher; and it alſo founds its calculations on the tables of the probabilities and values of lives in London; another circumſtance which ſecures a very advantageous profit to the Society, as experience has proved that the deaths are really in a much lower proportion than according to thoſe tables, and even lower than thoſe of Dr. Halley, which are founded on the bills of mortality of Breſlaw. By theſe means the Society finding itſelf, by experience, well ſecured againſt future hazards, and being unwilling to take from the public an extravagant profit, have determined to reduce all the future payments for aſſurances, one tenth, and alſo generouſly to return, to the perſons now aſſured, one tenth of all the payments they have made: and it ſeems there is reaſon to expect that this will be only a preparation to farther reduction.

From the foregoing account of this ſociety, it is manifeſt that its buſineſs is ſuch, that none but ſkilful mathematicians are qualified to conduct it. The intereſt of the ſociety therefore requires, that it ſhould make the places of thoſe who manage its buſineſs ſufficiently advantageous, to induce the ableſt mathematicians to accept them: and this will render it the more neceſſary for the ſociety to take care, in filling up any future vacancies, to pay no regard to any other conſiderations than the ability and integrity of the candidates. The conſequence of granting good pay, will be a multitude of ſolicitations on every vacancy, from perſons who, however unqualified, will hope for ſucceſs from their connexions, and the intereſt they are able to make. And ſhould the ſociety, in any future time, be led by ſuch cauſes to truſt its buſineſs in the hands of perſons not poſſeſſed of ſufficient ability, as mathematicians and calculators, ſuch miſtakes may be committed, as may prove, in the higheſt degree, detrimental and dangerous. There is reaſon to believe, that at preſent the ſociety is in no danger of this kind; and one of the great public advantages attending it, is, that it has eſtabliſhed an office, where not only the buſineſs above deſcribed, is tranſacted with faithfulneſs and ſkill; but where alſo all perſons, who want ſolutions of any queſtions relating to life annuities and reverſions, may apply, and be ſure of receiving juſt anſwers. The following is a

Table of the rates of aſſurance on ſingle lives in the Society for Equitable Aſſurances. The Sum aſſured 100*l.*

Age.	For one year.			For ſeven years at an annual payment of			For the whole life at an annual payment of		
	l.	s.	d.	l.	s.	d.	l.	s.	d.
10	1	9	6	1	10	7	2	2	10
15	1	11	0	1	12	7	2	6	6
20	1	13	11	1	16	0	2	12	10
25	1	17	7	2	0	2	3	0	6
30	2	2	6	2	6	0	3	8	11
35	2	8	7	2	14	2	3	17	9
40	2	19	2	3	5	1	4	7	11
45	3	11	0	3	18	6	5	0	0
50	4	4	8	4	11	2	5	12	11
55	5	0	9	5	11	7	6	9	3
60	5	19	1	6	16	10	7	17	7
65	7	0	11	8	13	0	10	3	9

Theſe rates are 10 per cent. lower than the true values, according to the decrements of life in London, reckoning intereſt at 3 per cent; but at the ſame time, it is to be obſerved that, for all ages under 50, they are near one third higher than all the true values, according to Dr. Halley's table of the decrements of life at Breſlaw, and Dr. Price's tables of the decrements of life at Northampton and Norwich. But as the ſociety has lately found that the decrements of life among its members have hitherto been lower than even thoſe given in theſe laſt tables, it may reaſonably be expected, that they will in time reduce their rates of aſſurance to the true values, as determined by theſe tables.

As to the *Weſtminſter Society for granting Annuities, and inſuring Money on Lives*, lately eſtabliſhed, viz. in the year 1789, from the number and reſpectability of its members, the equitable terms upon which it propoſes to deal, and the known ability and accuracy of the mathematicians and calculators employed in conducting it, there is every reaſon to expect an honourable and equitable treatment of the public, and a permanent continuance of its uſefulneſs.

ASTERISM, the ſame with conſtellation, or a collection of many ſtars, which are uſually repreſented on globes by ſome particular image or figure, to diſtinguiſh the ſtars which compoſe this conſtellation from thoſe of others.

ASTRÆA, a name given by ſome to the ſign Virgo, by others called Erigone, and ſometimes Iſis. The poets feign that Juſtice quitted heaven to reſide on earth, in the golden age; but, growing weary of the iniquities of mankind, ſhe left the earth, and returned to heaven, placing herſelf in that part of the zodiac called Virgo, where ſhe became a conſtellation of ſtars, and from her orb ſtill looks down on the ways of men. Ovid. Metam. lib. i. ver. 149.

ASTRAGAL, in Architecture, a ſmall round moulding, which encompaſſes the top of the ſuſt or ſhaft of a column, like a ring or bracelet. The ſhaft always terminates at top with an aſtragal, and at bottom with a fillet, which in this place is called *ozia*.

Astragal, in Gunnery, is a kind of ring or moulding on a piece of ordnance, at about half a foot diſtance from the muzzle or mouth; ſerving as an ornament to the gun, as the former does to a column.

ASTRAL, ſomething belonging to or depending on the ſtars.

Astral *Year*, or *Sidereal Year*. See Year.

ASTRODICTICUM, an aſtronomical inſtrument invented by M. Weighel, by means of which many perſons ſhall be able to view the ſame ſtar at the ſame time.

ASTROGNOSIA, the art of knowing the fixed ſtars, their names, ranks, ſituations in the conſtellations, and the like.

ASTROLABE, from αϛηρ, ſtar, and λαμϐανω, I take; alluding to its uſe in taking, or obſerving, the ſtars. The Arabians call it in their tongue *aſtharlab*; a word formed by corruption from the common Greek name.

This name was originally uſed for a ſyſtem or aſſemblage of the ſeveral circles of the ſphere, in their proper order and ſituation with reſpect to each other. And the ancient inſtruments were much the ſame as our armillary ſpheres.

The firſt, and moſt celebrated of this kind, was that of Hipparchus, which he made at Alexandria, the capital of Egypt, and lodged in a ſecure place, where it ſerved for divers aſtronomical operations. Ptolemy made the ſame uſe of it: but as the inſtrument had ſeveral inconveniences, he contrived to change its figure, though perfectly natural, and agreeable to the doctrine of the ſphere; and to reduce the whole Aſtrolabe to a plane ſurface, to which he gave the name of the *Planiſphere.* Hence,

ATSROLABE is uſed among the moderns for a *Planiſphere*, or a ſtereographic projection of the circles of the ſphere upon the plane of one of the great circles; which is uſually either the plane of the equinoctial, the eye being then placed in the pole of the world; or that of the meridian, the eye being ſuppoſed in the point of interſection of the equinoctial and horizon; or on that of the horizon.

The Aſtrolabe has been treated at large by Stoffler, Gemma Friſius, Clavius, &c. And for a farther account of the nature and kinds of it, ſee the article PLANISPHERE.

ASTROLABE, or SEA ASTROLABE, more particularly denotes an inſtrument chiefly uſed for taking altitudes at ſea; as the altitude of the pole, the ſun, or the ſtars.

The common Aſtrolabe, repreſented Plate II, *fig.* 7, conſiſts of a large braſs ring, about 15 inches in diameter, whoſe limb, or a convenient part of it, is divided into degrees and minutes. It is fitted with a moveable label or index, which turns upon the centre, and carries two ſights; and having a ſmall ring, at A, to hang it by in time of obſervation.

To make uſe of the Aſtrolabe in taking altitudes; ſuſpend it by the ring A, and turn it to the ſun, &c, ſo as that the rays may paſs freely through both the ſights F and G; then will the label cut or point out the altitude on the divided limb. There are many other uſes of the Aſtrolabe; of which Clavius, Henrion, and others have written very largely.

The Aſtrolabe, though now grown into diſuſe, is by many eſteemed equal to any other inſtrument for taking the altitude at ſea; eſpecially between the tropics, where the ſun comes near the zenith.

ASTROLOGER, a perſon profeſſing or practiſing the art of Aſtrology.

ASTROLOGICAL, ſomething relating to Aſtrology.

ASTROLOGY, the art of foretelling future events, from the poſitions, aſpects, and influences of the heavenly bodies.

The word is compounded of αϛηρ, *ſtar*, and λογος, *diſcourſe;* whence, in the literal ſenſe of the term, Aſtrology ſhould ſignify no more than the *doctrine* or *ſcience of the ſtars;* which indeed was its original acceptation, and conſtituted the ancient Aſtrology; which conſiſted formerly of both the branches now called Aſtronomy and Aſtrology, under the name of the latter only; and for the ſake of making judiciary predictions it was that aſtronomical obſervations, properly ſo called, were chiefly made by the ancients. And though the two branches be now perfectly ſeparated, and that of Aſtrology almoſt univerſally rejected by men of real learning, this has but lately been the caſe, as their union ſubſiſted, in ſome degree, from Ptolemy till Kepler, who had a ſtrong bias towards the ancient aſtrology.

Aſtrology may be divided into two branches, *natural* and *judiciary.*

To NATURAL ASTROLOGY belongs the predicting of natural effects; ſuch as the changes of weather, winds, ſtorms, hurricanes, thunder, floods, earthquakes, &c. But this art properly belongs to *Phyſiology*, or *Natural Philoſophy;* and is only to be deduced, *à poſteriori*, from phenomena and obſervations. And for this ſort of Aſtrology it is that Mr. Boyle makes an apology in his Hiſtory of the Air. Its foundation and merits may be gathered from what is ſaid under the articles AIR, ATMOSPHERE, and WEATHER.

Judicial or *Judiciary* ASTROLOGY, which is what is commonly and properly called *Aſtrology*, is that which profeſſes to foretel moral events, or ſuch as have a dependence on the free will and agency of man; as if they were produced or directed by the ſtars.

The profeſſors of this kind of Aſtrology maintain, "That the heavens are one great volume or book, wherein God has written the hiſtory of the world; and in which every man may read his own fortune, and the tranſactions of his time. The art, ſay they, had its riſe from the ſame hands as Aſtronomy itſelf: while the ancient Aſſyrians, whoſe ſerene unclouded ſky favoured their celeſtial obſervations, were intent on tracing the paths and periods of the heavenly bodies, they diſcovered a conſtant, ſettled relation of analogy, between them and things below; and hence were led to conclude theſe to be the *Parcæ*, the Deſtinies ſo much talked of, which preſide at our births, and diſpoſe of our future fate."

"The laws therefore of this relation being aſcertained by a ſeries of obſervations, and the ſhare each planet has therein; by knowing the preciſe time of any perſon's nativity, they were enabled, from their knowledge in aſtronomy, to erect a ſcheme or horoſcope of the ſituation of the planets, at that point of time; and hence, by conſidering their degrees of power and influence, and how each was either ſtrengthened or tempered by ſome other, to compute what muſt be the reſult thereof."

Judicial Aſtrology, it is commonly ſaid, was invented in Chaldæa, and from thence tranſmitted to the Egyptians, Greeks, and Romans; though ſome inſiſt that it was of Egyptian origin, and aſcribe the invention to Cham. But it is to the Arabs that we owe it. At Rome the people were ſo infatuated with it, that the aſtrologers, or, as they were then called, the mathematicians, maintained their ground in ſpite of all the edicts of the emperors to expel them out of the city. See GENETHLIACI.

Among the Indians, the Bramins, who introduced and practiſed this art in the Eaſt, have hereby made themſelves the arbiters of good and evil hours, which, gives them great authority: they are conſulted as oracles; and they have taken care always to ſell their anſwers at good rates.

The ſame ſuperſtition has prevailed in more modern ages and nations. The French hiſtorians remark, that in the time of queen Catharine de Medicis, Aſtrology was ſo much in vogue, that the moſt inconſiderable thing was not to be done without conſulting the ſtars. And in the reigns of king Henry III. and IV. of France, the predictions of Aſtrologers were the common theme of the court converſation. And this predominant humour in that court was well rallied by Barclay, in his Argenis, *lib.* 2, on occaſion of an Aſtrologer, who had undertaken to inſtruct king Henry in the event of a war which was then threatened by the faction of the Guiſes.

ASTROMETEOROLOGIA, the art of foretelling the weather, and its changes, from the aſpects and configurations of the moon and planets: a ſpecies of Aſtrology diſtinguiſhed by ſome under the denomination of *meteorological aſtrology*.

ASTRONOMICAL, ſomething relating to Aſtronomy.

Astronomical *Calendar*, *Characters*, *Column*, *Horizon*, *Hours*, *Month*, *Quadrant*, *Ring-Dial*, *Sector*, *Tables*, *Teleſcope*, *Time*, *Year*. See the ſeveral ſubſtantives.

Astronomical *Obſervations*. Of theſe there are records, or mention, in almoſt all ages. It is ſaid that the Chineſe have obſervations for a courſe of many thouſand years. But of theſe, as well as thoſe of the Indians, we have never yet had any benefit. But the obſervations of moſt of the other ancients, as Babylonians, Greeks, &c, amongſt which thoſe of Hipparchus make a principal figure, are carefully preſerved by Ptolemy, in his Almageſt.

About the year 880, Albategni, a Saracen, applied himſelf to the making of obſervations; in which he was followed by others of the ſame nation, as well as Perſians and Tartars; among whom were Naſſir-Eddin-Ettuſi, Arzachel, who alſo conſtructed a table of ſines, and Ulug Beigh. In 1457 Regiomontanus undertook the province at Norimberg; and his diſciples, J. Werner and Ber. Walther, continued the ſame from 1475 to 1504. Their obſervations were publiſhed together in 1544.—In 1509, Copernicus, and after him the landgrave of Heſſe, with his aſſiſtants Rothman and Byrge, obſerved; and after them Tycho Brahe, aſſiſted by the celebrated Kepler, from 1582 to 1601.—All the foregoing obſervations, together with Tycho's apparatus of inſtruments, are contained in the Hiſtoria Cœleſtis, publiſhed in 1672, by order of the emperor Ferdinand.—In 1651, was publiſhed at Bononia, by Ricciolus, Almageſtum Novum, being a complete body of ancient and modern obſervations, which he ſo named after the work of the ſame nature by Ptolemy.—Soon after, Hevelius, with a magnificent and well-contrived apparatus of inſtruments, deſcribed in his Machina Cœleſtis, began a courſe of obſervations. It has been objected to him, that he only uſed plain ſights, and could never be brought to take the advantage of teleſcopic ones; which occaſioned Dr. Hook to write animadverſions on Hevelius's inſtruments, printed in 1674, in which he too raſhly deſpiſes them, on account of their inaccuracy: but Dr. Halley, who at the inſtance of the Royal Society went over to Dantzick in the year 1679, to inſpect his inſtruments, approved of their juſtneſs, as well as of the obſervations made with them. See Sights.—Our two countrymen Jer. Horrox and Will. Crabtree, are celebrated for their obſervations from the year 1635 to 1645, who firſt obſerved the tranſit of Venus over the ſun in the year 1639.—They were followed by Flamſteed, Caſſini the father and ſon, Halley, de la Hire, Roemer, and Kirchius.—The obſervations of the celebrated Dr. Bradley have not yet been publiſhed, though long expected. We have alſo now publiſhed, from time to time, the accurate obſervations of the preſent Britiſh Aſtronomer Royal: as alſo thoſe of the French and other obſervatories, with the obſervations of many ingenious private aſtronomers, which are to be found in the Tranſactions and Memoirs of the various Philoſophical Societies.—There have been alſo obſervations of many other eminent aſtronomers; as, Galileo, Huygens, and our countryman Harriot, whoſe very intereſting obſervations have lately been brought to light by the earl of Egremont, and count Bruhl, by whoſe means they may come to be publiſhed. Other publications of celeſtial obſervations, are thoſe of Caſſini, La Caille, Monnier, &c.—See farther under Celestial *Obſervations*, Catalogue, Observatory, &c.

Astronomical *Place* of a ſtar or planet, is its longitude, or place in the ecliptic, reckoned from the beginning of Aries, *in conſequentia*, or according to the order of the ſigns.

ASTRONOMICALS, a name uſed by ſome writers for ſexageſimal fractions; on account of their uſe in aſtronomical calculations.

ASTRONOMICUS *Radius*. See Radius.

ASTRONOMY, the doctrine of the heavens, and their phenomena.

Aſtronomy is properly a mixed mathematical ſcience, by which we become acquainted with the celeſtial bodies, their motions, periods, eclipſes, magnitudes, diſtances, and other phenomena. Some, however, underſtand the term aſtronomy in a more extenſive ſenſe, as compriſing in it the theory of the univerſe, with the primary laws of nature: in which ſenſe it ſeems to be rather a branch of phyſics than of mathematics.

Hiſtory of Aſtronomy.

The invention of aſtronomy has been variouſly given, and aſcribed to ſeveral perſons, ſeveral nations, and ſeveral ages. Indeed it is probable that mankind never exiſted without ſome knowledge of aſtronomy amongſt them. For, beſides the motives of mere curioſity, which are ſufficient of themſelves to have excited men to a contemplation of the glorious and varying celeſtial canopy, it is obvious that ſome parts of the ſcience anſwer ſuch eſſential purpoſes to mankind, as to make the cultivation of it a matter of indiſpenſable neceſſity. Accordingly we find traces of it, in different degrees of improvement, among all nations.

Adam, in his ſtate of innocence, it is ſuppoſed by ſome of the Jewiſh rabbins, was endowed with a knowledge of the nature, influence, and uſes of the heavenly bodies; and Joſephus aſcribes to Seth and his poſterity a conſiderable knowledge of aſtronomy: he ſpeaks of two pillars, the one of ſtone and the other of brick, called the pillars of Seth, upon which they engraved the principles of the ſcience; and he ſays that the former was ſtill entire in his time. But be this as it may, it is evident that the great length of the antediluvian lives would afford ſuch excellent opportunities for obſerving the heavenly bodies, that we cannot but ſuppoſe that the ſcience of aſtronomy was conſiderably advanced before the flood. Indeed Joſephus ſays that longevity was beſtowed upon them for the very purpoſe of cultivating the ſciences of geometry and aſtronomy; obſerving that the latter could not be learned in leſs than 600 years; "for that period, he adds, is the *grand year*."

An expreſſion remarkable enough; and by which it may be ſuppoſed is meant the period in which the ſun and moon come again into the ſame ſituation in which they were at the beginning of it, with regard to the nodes, apogee of the moon, &c. "This period, ſays Caſſini, of which we find no intimation in any monument of any other nation, is the fineſt period that ever was invented: for it brings out the ſolar year more exactly than that of Hipparchus and Ptolemy; and the lunar month within about one ſecond of what is determined by modern aſtronomers." If the Antediluvians had ſuch a period of 600 years, they muſt have known the motions of the ſun and moon more exactly than their deſcendants knew them ſome ages after the flood.

On the building of the Tower of Babel, it is ſuppoſed that Noah retired with his children, born after the flood, to the north-eaſtern part of Aſia, where his deſcendants peopled the vaſt empire of China. And this, ſays Dr. Long, "may perhaps account for the Chineſe having ſo early cultivated the ſtudy of aſtronomy, &c." It is ſaid that the Jeſuit miſſionaries have found traditional accounts among the Chineſe, of their having been taught this ſcience by their firſt emperor Fo-hi, who is ſuppoſed to be the ſame with Noah; and Kempfer aſſerts that Fo-hi diſcovered the motions of the heavens, divided time into years and months, and invented the 12 ſigns into which they divide the zodiac, and which they diſtinguiſh by theſe names following; 1, the mouſe; 2, the ox or cow; 3, the tiger; 4, the hare; 5, the dragon; 6, the ſerpent; 7, the horſe; 8, the ſheep; 9, the monkey; 10, the cock or hen; 11, the dog; and, 12, the boar. They divide the heavens into 28 conſtellations, or claſſes of ſtars, allotting 4 to each of the 7 planets; ſo that the year always begins with the ſame planet; and their conſtellations anſwer to the 28 lunar manſions uſed by the Arabian aſtronomers. Theſe conſtellations however they do not mark with the figures of animals, like moſt other nations, but by connecting the ſtars by ſtraight lines, and denoting the ſtars themſelves by ſmall circles: ſo, for inſtance, the great bear would be marked thus,

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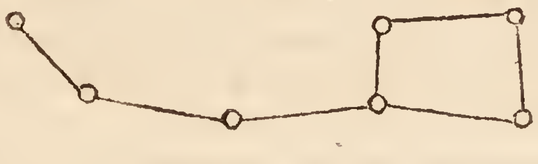


The Chineſe themſelves have many records and traditions of the high antiquity of their aſtronomy; though not without ſuſpicion of great miſtakes. But, on more certain authority, it is aſſerted by F. Gaubil, that at leaſt 120 years before Chriſt, the Chineſe had determined by obſervation the number and extent of their conſtellations as they now ſtand; the ſituation of the fixed ſtars with reſpect to the equinoctial and ſolſtitial points; and the obliquity of the ecliptic; with the theory of eclipſes: and that they were, long before that, acquainted with the true length of the ſolar year, the method of obſerving meridian altitudes of the ſun by the ſhadow of a gnomon, and of deducing from thence his declination, and the height of the pole. The ſame miſſionary alſo ſays, that the Chineſe have yet remaining ſome books of aſtronomy, which were written about 200 years before Chriſt; from which it appears, that the Chineſe had known the daily motion of the ſun and moon, and the times of the revolutions of the planets, many years before that period.

Du Halde informs us, that Tcheou-cong, the moſt ſkilful aſtronomer that ever China produced, lived more than a thouſand years before Chriſt; that he paſſed whole nights in obſerving the celeſtial bodies, and arranging them into conſtellations, &c. At preſent however, the ſtate of aſtronomy is but very low in that country, although it be cultivated at Peking, by public authority, in like manner as in moſt of the capital cities of Europe.

The inhabitants of Japan, of Siam, and of the Mogul's empire, have alſo been acquainted with aſtronomy from time immemorial; and the celebrated obſervatory at Benares, is a monument both of the ingenuity of the people, and of their ſkill in that ſcience.

According to Porphyry, aſtronomy muſt have been of very ancient ſtanding in the Eaſt. He informs us that, when Babylon was taken by Alexander, there were brought from thence celeſtial obſervations for the ſpace of 1903 years; which therefore muſt have commenced within 115 years after the flood, or within 15 years after the building of Babel.—Epigenes, according to Pliny, affirmed that the Babylonians had obſervations of 720 years engraven on bricks.—Again, Achilles Tatius aſcribes the invention of aſtronomy to the Egyptians; and adds, that their knowledge of that ſcience was engraven on pillars, and by that means tranſmitted to poſterity.

M. Bailly, in his elaborate Hiſtory of ancient and modern aſtronomy, endeavours to trace the origin of this ſcience among the Chaldeans, Egyptians, Perſians, Indians and Chineſe, to a very early period. And thence he maintains, that it was cultivated in Egypt and Chaldea 2800 years before Chriſt; in Perſia, 3209; in India, 3101; and in China, 2952 years before that æra. He alſo apprehends, that aſtronomy had been ſtudied even long before this diſtant period, and that we are only to date its revival from thence.

In inveſtigating the antiquity and progreſs of aſtronomy among the Indians, M. Bailly examines and compares four different ſets of aſtronomical tables of the Indian philoſophers, namely that of the Siameſe, explained by M. Caſſini in 1689; that brought from India by M. le Gentil of the Academy of Sciences; and two other manuſcript tables, found among the papers of the late M. de Liſle; all of which he found to accord together, and all referring to the meridian of Benares, above-mentioned. It appears that the fundamental epoch of the Indian aſtronomy, is a conjunction of the ſun and moon, which took place at the amazing diſtance of 3102 years before Chriſt: and M. Bailly informs us that, by our moſt accurate aſtronomical tables, ſuch a conjunction did really happen at that time. He further obſerves that, at preſent, the Indians calculate eclipſes by the mean motions of the ſun and moon obſerved 5000 years ſince; and that their accuracy, with regard to the ſolar motion, far exceeds that of the beſt Grecian aſtronomers. They had alſo ſettled the lunar motions by computing the ſpace through which that luminary had paſſed in 1,600,984 days, or a little more than 4383 years. M. Bailly alſo informs us, that they make uſe of the cycle of 19 years, the ſame as that aſcribed by the Greeks to Meton; that their theory of the planets is much better than Ptolemy's, as they do not ſuppoſe the earth in the centre of the celeſtial motions, and believe that Venus and Mercury move round the ſun; and that their aſtronomy agrees with the moſt modern diſcoveries as to the decreaſe of the obliquity of the ecliptic, the acceleration of the motion of the equinoctial points, &c.

In the 2d vol. of the Tranſactions of the Royal Society of Edinburgh is alſo a learned and ingenious diſſertation on the aſtronomy of the Brahmins of India, by Mr. Profeſſor Playfair; in which the great accuracy and high antiquity of the ſcience, among them, is reduced to the greateſt probability. It hence appears that their tables and rules of computation, have peculiar reference to an epoch, and to obſervations, 3 or 4 thouſand years before Chriſt; and many other inſtances are there adduced, of their critical knowledge in the other mathematical ſciences, employed in their precepts and calculations.

Aſtronomy, it ſeems, too, was not unknown to the Americans; though in their diviſion of time, they made uſe only of the ſolar, and not of the lunar motions. And that the Mexicans, in particular, had a ſtrange predilection for the number 13, by means of which they regulated almoſt every thing: their ſhorteſt periods conſiſted of 13 days; their cycle of 13 months, each containing 20 days; and their century of 4 periods, of 13 years each: and this exceſſive veneration for the number 13, aroſe, according to Siguenza, from its being the number of their greater gods. And it is very remarkable, that the Abbé Clavigero aſſerts it as a fact, that, having diſcovered the exceſs of a few hours in the ſolar above the lunar year, they made uſe of intercalary days, to bring them to an equality, as eſtabliſhed by Julius Cæſar in the Roman Calendar; but with this difference, that, inſtead of one day every 4 years, they interpoſed 13 days every 52 years, which produces the ſame effect.

Moſt authors however fix the origin of aſtronomy and aſtrology, either in Chaldea or in Egypt; and accordingly among the ancients we find the word Chaldean often uſed for aſtronomer, or, which was the ſame thing, aſtrologer. Indeed both of theſe nations pretended to a very high antiquity, and claimed the honour of producing the firſt cultivators of this ſcience. The Chaldeans boaſted of their temple or tower of Belus, and of Zoroaſter, whom they placed 5000 years before the deſtruction of Troy; while the Egyptians boaſted of their colleges of prieſts, where aſtronomy was taught, and of the monument of Oſymandyas, in which, it is ſaid, there was a golden circle of 365 cubits in circumference, and one cubit thick, divided into 365 equal parts according to the days of the year, &c.

It is, indeed, evident, that both Chaldea and Egypt were countries very proper for aſtronomical obſervations, on account of the extended flatneſs of the country, and the purity and ſerenity of the air. The tower of Belus, or of Babel itſelf, of a great height, was probably an aſtronomical obſervatory; and the lofty pyramids of Egypt, whatever they were originally deſigned for, might perhaps anſwer the ſame purpoſe; and at leaſt they ſhew the ſkill of this people in practical aſtronomy, as they are all placed with their four fronts exactly facing the cardinal points of the compaſs. The Chaldeans certainly began to make obſervations ſoon after the confuſion of languages, as appears from the obſervations found there on the taking of Babylon by Alexander; and it is probable they began much earlier. It hence appears that they had determined, with tolerable exactneſs, the length both of a periodical and ſynodical month. They had alſo diſcovered, that the motion of the moon was not uniform; and they even attempted to aſſign thoſe parts of the orbit in which the motion is quicker or ſlower. We are alſo aſſured by Ptolemy that they were not unacquainted with the motion of the moon's apogee and nodes, the latter of which they ſuppoſed made a complete revolution in 6585⅓ days, or a little more than 18 years, and contained 223 complete lunations, which period is called the Chaldean *Saros*. From Hipparchus, the ſame author alſo gives us ſeveral obſervations of lunar eclipſes made at Babylon above 720 years before Chriſt. And Ariſtotle informs us, that they had many occultations of the planets and fixed ſtars by the moon; a circumſtance which led them to conceive that eclipſes of the ſun were to be attributed to the ſame cauſe. They had alſo no inconſiderable ſhare in arranging the ſtars into conſtellations. Nor had even thoſe eccentric bodies the comets eſcaped their obſervation: for both Diodorus Siculus and Appollinus Myndicus, Seneca informs us, accounted theſe to be permanent bodies, having ſtated revolutions as well as the planets, but in much more extenſive orbits: although others of them were of opinion, that the comets were only meteors raiſed very high in the air, which, blazing for a while, diſappear when the matter of which they conſiſt is conſumed or diſperſed. The branch of dialling was alſo practiſed among them long before the Greeks were acquainted with that ſcience.

The Egyptians, it appears from various circumſtances, were much of the ſame ſtanding in Aſtronomy as the Chaldeans. Herodotus aſcribes their knowledge in the ſcience to Seſoſtris; probably not the ſame whom Newton makes contemporary with Solomon, as they were acquainted with aſtronomy at leaſt many hundred years before that æra. We learn, from the teſtimony of ſome ancient authors, many particulars relative to the ſtate of their knowledge in aſtronomy; ſuch as, that they believed the figure of the earth was ſpherical; that the moon was eclipſed by paſſing through the earth's ſhadow, though it does not certainly appear that they had any knowledge of the true ſyſtem of the univerſe; that they attempted to meaſure the magnitude of the earth and ſun, though their methods of aſcertaining the latter were very erroneous; and that they even pretended to foretel the appearance of comets, as well as earthquakes and inundations; and the ſame is alſo aſcribed to the Chaldeans; though theſe muſt probably have been rather a kind of aſtrological predictions, than obſervations drawn from aſtronomy, properly ſo called.

This ſcience however fell into great decay with the Egyptians, and in the time of the emperor Auguſtus, it was entirely extinct among them.

From Chaldea and Egypt the ſcience of aſtronomy paſſed into Phenicia, which this people applied to the purpoſes of navigation, ſteering their courſe by the north polar ſtar; and hence they became maſters of the ſea, and of almoſt all the commerce in the world.

The Greeks, it is probable, derived their aſtronomical knowledge chiefly from the Egyptians and Phenicians, by means of ſeveral of their countrymen who viſited theſe nations, for the purpoſe of learning the different ſciences. Newton ſuppoſes that moſt of the conſtellations were invented about the time of the Argonautic expedition; but it is more probable that they were, at leaſt moſt part of them, of a much older date, and derived from other nations, though cloathed in fables of their own invention or application. Several of the conſtellations are mentioned by Heſiod and Homer, the two moſt ancient writers among the Greeks, and who lived about 870 years before Chriſt. Their knowledge in this ſcience however was greatly improved by Thales the Mileſian, and other Greeks, who travelled into Egypt, and brought from thence the chief principles of the ſcience. Thales was born about 640 years before Chriſt; and he, firſt of all among the Greeks, obſerved the ſtars, the ſolſtices, the eclipſes of the ſun and moon, and predicted the ſame. And the ſame was farther cultivated and extended by his ſucceſſors Anaximander, Anaximanes, and Anaxagoras; but moſt eſpecially by Pythagoras, who was born 577 years before Chriſt, and having reſided for ſeveral years in Egypt, &c, brought from thence the learning of theſe people, taught the ſame in Greece and Italy, and founded the ſect of the Pythagoreans. He taught that the ſun was in the centre of the univerſe; that the earth was round, and people had antipodes; that the moon reflected the rays of the ſun, and was inhabited like the earth; that comets were a kind of wandering ſtars, diſappearing in the further parts of their orbits; that the white colour of the milky-way was owing to the united brightneſs of a great multitude of ſmall ſtars; and he ſuppoſed that the diſtances of the moon and planets from the earth, were in certain harmonic proportions to one another.

Philolaus, a Pythagorean, who flouriſhed about 450 years before Chriſt, aſſerted the annual motion of the earth about the ſun; and not long after, the diurnal motion of the earth on her own axis, was taught by Hicetas, a Syracuſan. About the ſame time flouriſhed at Athens, Meton and Euctemon, where they obſerved the ſummer ſolſtice 432 years before Chriſt, and obſerved the riſings and ſettings of the ſtars, and what ſeaſons they anſwered to. Meton alſo invented the cycle of 19 years, which ſtill bears his name.

Eudoxus the Cnidian lived about 370 years before Chriſt, and was accounted one of the moſt ſkilful aſtronomers and geometricians of antiquity, being accounted the inventor of many of the propoſitions in Euclid's Elements, and having introduced geometry into the ſcience of aſtronomy. He travelled into Aſia, Africa, Sicily, and Italy, for improvements in aſtronomy; and we are informed by Pliny, that he determined the annual year to contain 365 days 6 hours, that he determined alſo the periodical times of the planets, and made other important obſervations and diſcoveries.

Calippus flouriſhed ſoon after Eudoxus, and his celeſtial ſphere is mentioned by Ariſtotle; but he is better known by a period of 76 which he invented, containing 4 corrected Metonic periods, and which commenced at the ſummer ſolſtice in the year 330 before Chriſt. About his time the knowledge of the Pythagorean ſyſtem was carried into Italy, Gaul, and Egypt, by certain colonies of Greeks.

However, the introduction of Aſtronomy into Greece is repreſented by Vitruvius in a manner ſomewhat different. He maintains, that Beroſus, a Babylonian, brought it immediately from Babylon itſelf, and opened an aſtronomical ſchool in the iſle of Cos. And Pliny ſays, that in conſideration of his wonderful predictions, the Athenians erected him a ſtatue in the gymnaſium, with a gilded tongue. But if this Beroſus be the ſame with the author of the Chaldaic hiſtories, he muſt have lived before Alexander.

After the death of this conqueror, the ſciences flouriſhed chiefly in Egypt, under the auſpices of Ptolemy Philadelphus and his ſucceſſors. He founded a ſchool there, which continued to be the grand ſeminary of learning, till the invaſion of the Saracens in the year of Chriſt 650. From the founding of that ſchool, the ſcience of aſtronomy advanced conſiderably. Ariſtarchus, about 270 years before Chriſt, ſtrenuouſly aſſerted the Pythagorean ſyſtem, and gave a method of determining the ſun's diſtance by the dichotomy of the moon.—Eratoſthenes, who was born at Cyrene in the year 271 before Chriſt, meaſured the circumference of the earth by means of a gnomon; and being invited to Alexandria, from Athens, by Ptolemy Euergetes, and made keeper of the royal library there, he ſet up for that Prince thoſe armillary ſpheres, which Hipparchus and Ptolemy the aſtronomer afterwards employed ſo ſucceſsfully in obſerving the heavens. He alſo determined the diſtance between the tropics to be $\frac{11}{83}$ of the whole meridian circle, which makes the obliquity of the ecliptic in his time to be 23° 51′ $\frac{1}{3}$.—The celebrated Archimedes, too, cultivated aſtronomy, as well as geometry and mechanics: he determined the diſtances of the planets from one another, and conſtructed a kind of planetarium or orrery, to repreſent the phenomena and motions of the heavenly bodies.

To paſs by ſeveral others of the ancients, who practiſed or cultivated aſtronomy, more or leſs, we find that Hipparchus, who flouriſhed about 140 years before Chriſt, was the firſt who applied himſelf to the ſtudy of every part of aſtronomy, and, as we are informed by Ptolemy, made great improvements in it: he diſcovered that the orbits of the planets are eccentric, that the moon moved ſlower in the apogee than in her perigee, and, that there was a motion of anticipation of the moon's nodes: he conſtructed tables of the motions of the ſun and moon, collected accounts of ſuch eclipſes, &c, as had been made by the Egyptians and Chaldeans, and calculated all that were to happen for 600 years to come: he diſcovered that the fixed ſtars changed their places, having a ſlow motion of their own from weſt to eaſt: he corrected the Calippic period, and pointed out ſome errors in the method of Eratoſthenes for meaſuring the circumference of the earth: he computed the ſun's diſtance more accurately than any of his predeceſſors: but his chief work is a catalogue which he made of the fixed ſtars, to the number of 1022, with their longitudes, latitudes, and apparent magnitudes; which, with moſt of his other obſervations, are preſerved by Ptolemy in his Almageſt.

There was but little progreſs made in aſtronomy from the time of Hipparchus to that of Ptolemy, who was born at Peluſium in Egypt, in the firſt century of chriſtianity, and who made the greateſt part of his obſervations at the celebrated ſchool of Alexandria in that country. Profiting of thoſe of Hipparchus and other ancient aſtronomers, he formed a ſyſtem of his own, which, though erroneous, was followed for many ages by all nations. He compiled a great work, called the Almageſt, which contained the obſervations and collections of Hipparchus and others his predeceſſors in aſtronomy, on which account it will ever be valuable to the profeſſors of that ſcience. This work was preſerved from the grievous conflagration of the Alexandrine library by the Saracens, and tranſlated out of Greek into Arabic in the year 827, and from thence into Latin in 1230. The Greek original was not known in Europe till the beginning of the 15th century, when it was brought from Conſtantinople, then taken by the Turks, by George, a monk of Trapezond, by whom it was tranſlated into Latin; and various other editions have been ſince made.

During the long period from the year 800 till the beginning of the 14th century, the weſtern parts of Europe were immerſed in groſs ignorance and barbarity, while the Arabians, profiting by the books they had preſerved from the wreck of the Alexandrine library, cultivated and improved all the ſciences, and particularly that of aſtronomy, in which they had many able profeſſors and authors. The caliph Al Manſur firſt introduced a taſte for the ſciences into his empire. His grandſon Al Mamun, who aſcended the throne in 814, was a great encourager and improver of the ſciences, and eſpecially of aſtronomy. Having conſtructed proper inſtruments, he made many obſervations; determined the obliquity of the ecliptic to be 23° 35′; and under his auſpices a degree of the circle of the earth was meaſured a ſecond time in the plain of Singar, on the border of the Red Sea. About the ſame time Alferganus wrote elements of aſtronomy; and the ſcience was from hence greatly cultivated by the Arabians, but principally by Albategnius, who flouriſhed about the year 880, and who greatly reformed aſtronomy, by comparing his own obſervations with thoſe of Ptolemy: hence he computed the motion of the ſun's apogee from Ptolemy's time to his own; ſettled the preceſſion of the equinoxes at one degree in 70 years; and fixed the obliquity of the ecliptic at 23° 35′. The tables which he compoſed, for the meridian of Aracta, were long eſteemed by the Arabians. After his time, though the Saracens had many eminent aſtronomers, ſeveral centuries elapſed without producing any very valuable obſervations, excepting thoſe of ſome eclipſes obſerved by Ebn Younis, aſtronomer to the caliph of Egypt, by means of which the quantity of the moon's acceleration ſince that time may be determined.

Other eminent Arabic aſtronomers, were, Arzachel a Moor of Spain, who obſerved the obliquity of the ecliptic: he alſo improved Trigonometry by conſtructing tables of ſines, inſtead of chords of arches, dividing the diameter into 300 equal parts. And Alhazen, his contemporary, who wrote upon the twilight, the height of the clouds, the phenomenon of the horizontal moon, and who firſt ſhewed the importance of the theory of refractions in aſtronomy.

Ulug Beg, grandſon of the celebrated Tartar prince Tamerlane, was a great proficient in practical aſtronomy; he had very large inſtruments, particularly a quadrant of about 180 feet high, with which he made good obſervations. From theſe he determined the latitude of Samercand, his capital, to be 39° 37′ 23″; and compoſed aſtronomical tables for the meridian of the ſame ſo exact, that they differ very little from thoſe conſtructed afterwards by Tycho Brahe; but his principal work was his catalogue of the fixed ſtars, made alſo from his own obſervations in the year 1437.

During this period, almoſt all Europe was immerſed in groſs ignorance. But the ſettlement of the Moors in Spain introduced the ſciences into Europe; from which time they have continued to improve, and to be communicated from one people to another, to the preſent time, when aſtronomy and all the ſciences, have arrived at a very eminent degree of perfection. The emperor Frederick II, about 1230, firſt began to encourage learning; reſtoring ſome decayed univerſities, and founding a new one in Vienna: he alſo cauſed the works of Ariſtotle, and Ptolemy's Almageſt, to be tranſlated into Latin; and from the tranſlation of this work we may date the revival of aſtronomy in Europe. Two years after this, John de Sacro Boſco, that is, of Halifax, compiled, from Ptolemy, Albategnius, Alferganus, and other Arabic aſtronomers, his work *De Sphæra*, which was held in the greateſt eſtimation for 300 years after, and was honoured with commentaries by Clavius and other learned men. In 1240, Alphonſo, king of Caſtile, not only cultivated aſtronomy himſelf, but greatly encouraged others; and by the aſſiſtance of ſeveral learned men he corrected the tables of Ptolemy, and compoſed thoſe which were denominated from him the Alphonſine Tables. About the ſame time alſo, Roger Bacon, an Engliſh monk, wrote ſeveral tracts relative to aſtronomy, particularly of the lunar aſpects, the ſolar rays, and the places of the fixed ſtars. And, about the year 1270, Vitello, a Polander, compoſed a treatiſe on optics, in which he ſhewed the uſe of refractions in aſtronomy.

Little other improvement was made in aſtronomy till the time of Purbach, who was born in 1423. He compoſed new tables of ſines for every 10 minutes, making the radius 60, with four ciphers annexed. He conſtructed ſpheres and globes, and wrote ſeveral aſtronomical tracts; as, a commentary on Ptolemy's Almageſt; ſome treatiſes on Arithmetic and Dialling, with tables for various climates; new tables of the fixed ſtars reduced to the middle of that century; and he corrected the tables of the planets, making new equations to them where the Alphonſine tables were erroneous. In his ſolar tables, he placed the ſun's apogee in the beginning of Cancer; but retained the obliquity of the ecliptic 23° 33½′, as determined by the lateſt obſervations. He alſo obſerved ſome eclipſes, made new tables for computing them, and had juſt finiſhed a theory of the planets, when he died in 1462, being only 39 years of age.

Purbach was ſucceeded in his aſtronomical and mathematical labours by his pupil and friend, John Muller, commonly called Regiomontanus, from Monteregio, or Koningſberg, a town of Franconia, where he was born. He completed the epitome of Ptolemy's Almageſt, which Purbach had begun; and after the death of his friend, was invited to Rome, where he made many aſtronomical obſervations. Being returned to Nuremberg in 1471, by the encouragement of a wealthy citizen named Bernard Walther, he made ſeveral inſtruments for aſtronomical obſervations, among which was an armillary aſtrolabe, like that uſed at Alexandria by Hipparchus and Ptolemy, with which he made many obſervations, uſing alſo a good clock, which was then but a late invention. He made ephemerides for 30 years to come, ſhewing the lunations, eclipſes, &c; and, the art of printing having then been lately invented, he printed the works of many of the moſt celebrated ancient aſtronomers. He wrote the Theory of the Planets and Comets, and a treatiſe on triangles, ſtill in repute for ſeveral good theorems; computing the table of ſines for every ſingle minute, to the radius 1000000, and introducing the uſe of tangents alſo into trigonometry. After his death, which happened at Rome in 1476, being only 40 years of age, Walther collected his papers, and continued the aſtronomical obſervations till his own death alſo. The obſervations of both were collected by order of the ſenate of Nuremberg, and publiſhed there in 1544 by John Schoner: they were alſo afterwards publiſhed in 1618 by Snellius, at the end of the obſervations made by the Landgrave of Heſſe; and laſtly with thoſe of Tycho Brahe in 1666.

Walther was ſucceeded, as aſtronomer at Nuremberg, by John Werner, a clergyman. He obſerved the motion of the comet in 1500; and wrote ſeveral tracts on geometry, aſtronomy, and geography, in a maſterly manner; the moſt remarkable of which, are thoſe concerning the motion of the 8th ſphere, or of the fixed ſtars; in this tract, by comparing his own obſervations, made in 1514, with thoſe of Ptolemy, Alphonſus, and others, he ſhewed that the motion of the fixed ſtars, ſince called the preceſſion of the equinoxes, is 1° 10′ in 100 years. He made alſo the firſt ſtar of Aries 26° diſtant from the equinoctial point, and the obliquity of the ecliptic only 23° 28′. He conſtructed a planetarium, repreſenting the celeſtial motions according to the Ptolemaic hypotheſis; and he publiſhed a tranſlation of Ptolemy's Geography, with a commentary, in which he firſt propoſed the method of finding the longitude at ſea by obſerving the moon's diſtance from the fixed ſtars; now ſo ſucceſsfully practiſed for that purpoſe. Werner died in 1528, at 60 years of age.

Nicolaus Copernicus was the next who made any conſiderable figure in aſtronomy, by whom indeed the old Pythagorean ſyſtem of the world was reſtored, which had been till now ſet aſide from the time of Ptolemy. About the year 1507 Copernicus conceived doubts of this ſyſtem, and entertained notions about the true one, which he gradually improved by a ſeries of aſtronomical obſervations, and the contemplation of former authors. By theſe he formed new tables, and completed his work in the year 1530, containing theſe, and his renovation of the true ſyſtem of the univerſe, in which all the planets are conſidered as revolving about the ſun, placed in the centre. But the work was only printed in 1543, under the care of Schoner and Oſiander, by the title of *Revolutiones Orbium Cæleſtium*; and the author juſt received a copy of the work a few hours before his death, which happened on the 23d of May 1543, at 70 years of age.

After the death of Copernicus, the ſcience and practice of aſtronomy were greatly improved by many other perſons, as Schoner, Nonius, Appian, Gemma Friſius, Rothman, Byrgius, the Landgrave of Heſſe, &c.—Schoner reformed and explained the calendar, improved the methods of making celeſtial obſervations, and publiſhed a treatiſe on coſmography; but he died 4 years after Copernicus.—Nonius wrote ſeveral works on mathematics, aſtronomy and navigation, and invented ſome uſeful and more accurate inſtruments than formerly; one of theſe was the aſtronomical quadrant, on which he divided the degrees into minutes by a number of concentric circles; the firſt of which was divided in 90 equal parts or degrees, the ſecond into 89, the third into 88, and ſo on, to 46; ſo that, the index of the quadrant always falling upon or near one of the diviſions, the minutes would be known by an eaſy computation.—The chief work of Appian, *The Cæſarean Aſtronomy*, was publiſhed at Ingoldſtat in 1540; in which he ſhews, how to obſerve the places of the ſtars and planets by the aſtrolabe; to reſolve aſtronomical problems by certain inſtruments; to predict eclipſes, and to deſcribe the figures of them; and the method of dividing and uſing an aſtronomical quadrant: at the end are added obſervations of 5 comets, one of which has been ſuppoſed the ſame with that obſerved by Hevelius, and if ſo, it ought to have returned again in the year 1789;—but it was not obſerved then. Gemma Friſius wrote a commentary on Appian's *Coſmography*, accompanied with many obſervations of eclipſes: he alſo invented the aſtronomical ring, and ſeveral other inſtruments, uſeful in taking obſervations at ſea; and he was the firſt who recommended a time-keeper for determining the longitude at ſea.—Rheticus gave up his profeſſorſhip of mathematics at Wittemberg, that he might attend the aſtronomical lectures of Copernicus; and, for improving aſtronomical calculations, he began a very extenſive work, being a table of ſines, tangents and ſecants, to a very large radius, and to every 10 ſeconds, or $\frac{1}{6}$ of a minute; which was completed by his pupil Valentine Otho, and publiſhed in 1594.

About the year 1561, William IV, Landgrave of Heſſe Caſſel, applied himſelf to the ſtudy of aſtronomy, having furniſhed himſelf with the beſt inſtruments that could then be made: with theſe he made a great number of obſervations, which were publiſhed by Snellius in 1618, and which were preferred by Hevelius to thoſe of Tycho Brahe. From theſe obſervations he formed a catalogue of 400 ſtars, with their latitudes and longitudes, adapted to the beginning of the year 1593.

Tycho Brahe, a noble Dane, began his obſervations about the ſame time with the Landgrave of Heſſe, abovementioned, and he obſerved the great conjunction of Jupiter and Saturn: but finding the uſual inſtruments very inaccurate, he conſtructed many others much larger and exacter, with which he applied himſelf diligently to obſerve the celeſtial phenomena. In 1571 he diſcovered a new ſtar in the chair of Caſſiopeia; which induced him, like Hipparchus on a ſimilar occaſion, to make a new catalogue of the ſtars; which he compoſed to the number of 777, and adapted their places to the year 1600. In the year 1576, by favour of the king of Denmark, he built his new obſervatory, called Uraniburg, on the ſmall iſland Huenna, oppoſite to Copenhagen, and which he very amply furniſhed with many large inſtruments, ſome of them ſo divided as to ſhew ſingle minutes, and in others the arch might be read off to 10 ſeconds. One quadrant was divided according to the method invented by Nonius, that is, by 47 concentric circles; but moſt of them were divided by diagonals; a method of diviſion invented by a Mr. Richard Chanceler, an Engliſhman. Tycho employed his time at Uraniburg to the beſt advantage, till the death of the king, when, falling into diſcredit, he was obliged to remove to Holſtein; and he afterwards found means of introducing himſelf to the Emperor Rodolph, with whom he continued at Prague till the time of his death in 1601.—It is well known that Tycho was the inventor of a ſyſtem of aſtronomy, a kind of Semi-Ptolemaic, which he vainly endeavoured to eſtabliſh inſtead of the Copernican or true ſyſtem. His works, however, which are very numerous, ſhew that he was a man of great abilities; and his diſcoveries, together with thoſe of Purbach and Regiomontanus, were collected and publiſhed together in 1621, by Longomontanus, the favourite diſciple of Tycho.

While Tycho reſided at Prague with the emperor, he prevailed on Kepler to leave the univerſity of Glatz, and to come to him, which he did with his family and library in 1600: but Tycho dying in 1601, Kepler enjoyed all his life the title of mathematician to the Emperor, who ordered him to finiſh the tables of Tycho Brahe, which he did accordingly, and publiſhed them in 1627, under the title of Rodolphine. He died about the year 1630 at Ratiſbon, where he was ſoliciting the arrears of his penſion. From his own obſervations, and thoſe of Tycho, Kepler diſcovered ſeveral of the true laws of nature, by which the motions of the celeſtial bodies are regulated. He diſcovered that all the planets revolved about the ſun, not in circular, but in elliptical orbits, having the ſun in one of the foci of the ellipſe; that their motions were not equable, but varying, quicker or ſlower as they were near to the ſun or farther from him; but that this motion was ſo regulated, that the areas deſcribed by the variable line drawn from the planet to the ſun, are equal in equal times, and always proportional to the times of deſcribing them. He alſo diſcovered, by trials, that the cubes of the diſtances of the planets from the ſun, were in the ſame proportion as the ſquares of their periodical times of revolution. By obſervations alſo on comets, he concluded that they are freely carried about among the orbits of the planets, in paths that are nearly rectilinear, but which he could not then determine. Beſides many other diſcoveries, which are to be found in his writings.

In Kepler's time there were many other good proficients in aſtronomy; as Edward Wright, baron Napier, John Bayer, &c. Wright made ſeveral good meridional obſervations of the ſun, with a quadrant of 6 feet radius, in the years 1594, 1595, and 1596; from which he greatly improved the theory of the ſun's motion, and computed more accurately his declination, than any perſon had done before. In 1599 he publiſhed alſo an excellent work entitled "Certain Errors in Navigation diſcovered and detected," containing a method which has commonly, though erroneouſly, been aſcribed to Mercator.—To Napier we owe ſome excellent theorems and improvements in ſpherics, beſides the ever memorable invention of logarithms, one of the moſt uſeful ever made in the art of numbering, and of the greateſt uſe in all the other mathematical ſciences.—Bayer, a German, publiſhed his *Uranometria*, being a complete celeſtial atlas, or the figures of all the conſtellations viſible in Europe, with the ſtars marked on them, and the ſtars alſo accompanied by names, or the letters of the Greek alphabet; a contrivance by which the ſtars may eaſily be referred to with diſtinctneſs and preciſion. —About the ſame time too, aſtronomy was cultivated by many other perſons; namely, abroad by Mercator, Maurolycus, Maginus, Homelius, Schultet, Stevin, Galileo, &c; and in England by Thomas and Leonard Digges, John Dee, Robert Flood, Harriot, &c.

The beginning of the 17th century was particularly diſtinguiſhed by the invention of teleſcopes, and the application of them to aſtronomical obſervations; an invention to which we owe the moſt brilliant diſcoveries, and all the accuracy to which the ſcience is now brought. The more diſtinguiſhed early obſervations with the teleſcope, were made by Galileo, Harriot, Huygens, Hook, Caſſini, &c. It is ſaid that from report only, and before he had ſeen one, Galileo made for himſelf teleſcopes, by which he diſcovered inequalities in the moon's ſurface, Jupiter's ſatellites, and the ring of Saturn; alſo ſpots on the ſun, by which he found out the revolution of that luminary on his axis; and he diſcovered that the nebulæ and milky way were full of ſmall ſtars. Harriot alſo, who has hitherto been known only as an algebraiſt, made much the ſame diſcoveries as Galileo, and as early, if not more ſo, as appears by his papers not yet printed, in the poſſeſſion of the earl of Egremont.

Mr. Horrox, a young aſtronomer of great talents, made conſiderable diſcoveries and improvements. In 1633 he found out that the planet Venus would paſs over the ſun's diſc on the 24th of November, 1639; an event which he announced only to his friend Mr. Crabtree; and theſe two were the only perſons in the world that obſerved this tranſit, which was alſo the firſt time it had ever been ſeen by mortal eyes. Mr. Horrox made alſo many other uſeful obſervations, and had even formed a new theory of the moon, taken notice of by Newton; but his early death, in the beginning of the year 1640, put a ſtop to his uſeful and valuable labours.

About the ſame time flouriſhed Hevelius, Burgomaſter of Dantzic, who furniſhed an excellent obſervatory in his own houſe, where he obſerved the ſpots and phaſes of the moon, from which obſervations he compiled his *Selenographia*; and an account of his apparatus is contained in his work entitled *Machina Cœleſtis*, a book now very ſcarce, as moſt of the copies were accidentally burnt, with the whole houſe and apparatus, in 1679. Hevelius died in 1688, aged 76.

Dr. Hook, a contemporary of Hevelius, invented inſtruments with teleſcopic ſights, and cenſured thoſe of the latter; which occaſioned a ſharp diſpute between them, and to ſettle which the celebrated Dr. Halley was ſent over to Hevelius to examine his inſtruments. The two aſtronomers made ſeveral obſervations together, very much to their ſatisfaction, and amongſt them was one of an occultation of Jupiter by the moon, when they determined the diameter of the latter to be 30′ 33.″

Before the middle of the 17th century the conſtruction of teleſcopes had been greatly improved, particularly by Huygens and Fontana. The former conſtructed one of 123 feet, with which he long obſerved the moon and planets, and diſcovered that Saturn was encompaſſed with a ring. With teleſcopes too, of 200 and 300 feet focus, Caſſini ſaw five ſatellites of Saturn, with his zones or belts, and the ſhadows of Jupiter's ſatellites paſſing over his body. In 1666 Azout applied a micrometer to teleſcopes, to meaſure the diameters of the planets, and other ſmall diſtances in the heavens: but an inſtrument of this kind had been invented before, by Mr. Gaſcoigne, though it was but little known abroad.

To obviate the difficulties of the great lengths of refracting teleſcopes and the aberration of the rays, it is ſaid that Merſennus firſt ſtarted the idea of making teleſcopes of reflectors, inſtead of lenſes, in a letter to Deſcartes; and in 1663 James Gregory of Aberdeen ſhewed how ſuch a teleſcope might be conſtructed. After ſome time ſpent alſo by Newton, on the conſtruction of both ſorts of teleſcopes, he found out the great inconvenience which ariſes to refractors from the different refrangibility of the rays of light, for which he could not then find a remedy; and therefore, purſuing the other kind, in the year 1672 he preſented to the Royal Society two reflectors, which were conſtructed with ſpherical ſpeculums, as he could not procure other figures. The inconveniences however ariſing from the different refrangibility of the rays of light, have ſince been fully obviated by the ingenious Mr. Dollond. Towards the latter part of the 17th, and beginning of the 18th century, practical aſtronomy it ſeems rather languiſhed. But at the ſame time the ſpeculative part was carried to the higheſt perfection by the immortal Newton in his Principia, and by the Aſtronomy of David Gregory.

Soon after this however, great improvements of aſtronomical inſtruments began to take place, particularly in Britain. Mr. Graham, a celebrated mechanic and watchmaker, not only improved clocks and watch work, but alſo carried the accuracy of aſtronomical inſtruments to a ſurpriſing degree. He conſtructed the old 8 feet mural arch at the Royal Obſervatory Greenwich, and a ſmall equatorial ſector for making obſervations out of the meridian; but he is chiefly remarkable for contriving the zenith ſector of 24 feet radius, and afterwards one of 12½ feet, with which Dr. Bradley diſcovered the aberration of the fixed ſtars. The reflecting teleſcope of Gregory and Newton, was greatly improved by Mr. Hadley, who preſented a very powerful inſtrument of that kind to the Royal Society in 1719. The ſame gentleman has alſo immortalized his memory by the invention of the reflecting quadrant or ſector, now called by his name, which he preſented to the ſociety in 1731, and which is now ſo univerſally uſeful at ſea, eſpecially where nice obſervations are required. It appears however that an inſtrument ſimilar to this in its principles, had been invented by Newton; and a deſcription with a drawing of it given by him to Dr. Halley, when he was preparing for his voyage in 1701, to diſcover the variation of the needle: it has alſo been aſſerted that a Mr. Godfrey of Philadelphia in America, made the ſame diſcovery, and the firſt inſtrument of this kind. About the middle of this century, the conſtructing and dividing of large aſtronomical inſtruments were carried to great perfection by Mr. John Bird; and reflecting teleſcopes were not leſs improved by Mr. Short, who alſo firſt executed the divided object-glaſs micrometer, which had been propoſed and deſcribed by M. Louville and others. Mr. Dollond alſo brought refracting teleſcopes to the greateſt perfection, by means of his acromatic glaſſes; and lately the diſcoveries of Herſchel are owing to the amazing powers of reflectors of his own conſtruction.

Thus the aſtronomical improvements in the preſent century, have been chiefly owing to the foregoing inventions and improvements in the inſtruments, and to the eſtabliſhment of regular obſervatories in England, France, and other parts of Europe. Roemer, a celebrated Daniſh Aſtronomer, firſt made uſe of a meridional teleſcope; and, by obſerving the eclipſes of Jupiter's ſatellites, he firſt diſcovered the progreſſive motion of light, concerning which he read a diſſertation before the Academy of Sciences at Paris in 1675.—Mr. Flamſteed was appointed the firſt Aſtronomer Royal at Greenwich in 1675. He obſerved, for 44 years, all the celeſtial phenomena, the ſun, moon, planets, and fixed ſtars, of all which he gave an improved theory and tables, viz, a catalogue of 3000 ſtars with their places, to the year 1689; alſo new ſolar tables, and a theory of the moon according to Horrox; likewiſe, in Sir Jonas Moore's Syſtem of Mathematics, he gave a curious tract on the doctrine of the ſphere, ſhewing how, geometrically to conſtruct eclipſes of the ſun and moon, as well as occultations of the fixed ſtars by the moon. And it was upon his tables that were conſtructed, both Halley's tables, and Newton's theory of the moon.—Caſſini alſo, the firſt French Aſtronomer Royal, very much diſtinguiſhed himſelf, making many obſervations on the ſun, moon, planets and comets, and greatly improved the elements of their motions. He alſo erected the gnomon, and drew the celebrated meridian line in the church of Petronia at Bologna.

In 1719 Mr. Flamſteed was ſucceeded by Dr. Halley, as Aſtronomer Royal at Greenwich. The Doctor had been ſent at the early age of 21 to the iſland of St. Helena, to obſerve the ſouthern ſtars, and make a catalogue of them, which was publiſhed in 1679. In 1705 he publiſhed his *Synopſis Aſtronomiæ Cometicæ*, in which he ventured to predict the return of a comet in 1758 or 1759. He was the firſt who diſcovered the acceleration of the moon, and he gave a very ingenious method for finding her parallax by three obſerved phaſes of a ſolar eclipſe. He publiſhed, in the Philoſophical Tranſactions, many learned papers, and amongſt them ſome that were concerning the uſe that might be made of the next tranſit of Venus in determining the diſtance of the ſun from the earth. He compoſed tables of the ſun, moon, and all the planets, which are ſtill in great repute; with which he compared the obſervations he made of the moon at Greenwich, amounting to near 1500, and noted the differences. He recommended the method of determining the longitude by the moon's diſtances from the ſun and certain fixed ſtars; a method which had before been noticed, and which has ſince been carried into execution, more particularly at the inſtance of the preſent Aſtronomer Royal.

About this time a diſpute aroſe concerning the figure of the earth. Sir Iſaac Newton had determined, from a conſideration of the laws of gravity, and the diurnal motion of the earth, that the figure of it was an oblate ſpheroid, and flatted at the poles: but Caſſini had determined, from the meaſures of Picart, that the figure was an oblong ſpheroid, or lengthened at the poles. To ſettle this diſpute, it was reſolved, under Lewis XV, to meaſure two degrees of the meridian; one near the equator, and the other as near the pole as poſſible. For this purpoſe, the Royal Academy of Sciences ſent to Lapland, Meſs. Maupertuis, Clairault, Camus, and Le Monier; being alſo accompanied by the Abbé Outhier, and by M. Celſus, profeſſor of anatomy at Upſal. And on the ſouthern expedition were ſent Meſs. Godin, Condamine, and Bouguer, to whom the king of Spain joined Don George Juan and Don Antonio de Ulloa. Theſe ſet out in 1735, and returned at different times in 1744, 1745, and 1746; but the former party, who ſet out only in 1736, returned the year following; having both fulfilled their commiſſions. Picart's meaſure was alſo reviſed by Caſſini and De la Caille, which after his errors were corrected, was found to agree very well with the other two; and the reſult of the whole ſerved to confirm the determination of the figure before laid down by Newton.—On the ſouthern expedition, it was found that the attraction of the great mountains of Peru had a ſenſible effect on the plumb-line of one of their largeſt inſtruments, deflecting it 7 or 8 ſeconds from the true perpendicular.

On the death of Dr. Halley, in 1742, he was ſucceeded by Dr. Bradley, as Aſtronomer Royal at Greenwich. The accuracy of his obſervations enabled him to detect the ſmaller inequalities in the motions of the planets and fixed ſtars. The conſequence of this accuracy was, the diſcovery of the aberration of light, the nutation of the earth's axis, and a much greater degree of perfection in the lunar tables. He alſo obſerved the places, and computed the elements of the comets which appeared in the years 1723, 1736, 1743, and 1757. He made new and accurate tables of the motions of Jupiter's ſatellites; and from a multitude of obſervations of the luminaries, he conſtructed the moſt accurate table of refractions yet extant. Alſo, with a very large tranſit inſtrument, and a new mural quadrant of 8 feet radius, conſtructed by Mr. Bird in 1750, he made an immenſe number of obſervations for ſettling the places of all the ſtars in the Britiſh catalogue, together with near 1500 places of the moon, the greater part of which he compared with Mayer's tables. Dr. Bradley died in 1762.

In the mean time the mathematicians and aſtronomers elſewhere were aſſiduous in their endeavours to promote the ſcience of aſtronomy. The theory of the moon was particularly conſidered by Meſſrs Clairault, D'Alembert, Euler, Meyer, Simpſon, and Walmſly, and eſpecially Clairault, Euler, and Mayer, who computed complete ſets of lunar tables; thoſe of the laſt of theſe authors, for their ſuperior accuracy, were rewarded with a premium of 3000 pounds by the Board of Longitude, who brought them into uſe in the computation of the Nautical Ephemeris, publiſhed by that Board.—The moſt accurate tables of the ſatellites of Jupiter were compoſed, from obſervations by Mr. Wargentin, an excellent Swediſh aſtronomer.—Among the many French aſtronomers who contributed to the advancement of the ſcience, it was particularly indebted to M. De la Caille, for an excellent ſet of ſolar tables. And in 1750 he went to the Cape of Good Hope to make obſervations in concert with the moſt celebrated aſtronomers in Europe, for determining the parallax of Mars and the moon, and thence, that of the ſun, which it was concluded did not much exceed 10 ſeconds. Here he re-examined and adjuſted, with great accuracy, the ſtars about the ſouthern pole; and alſo meaſured a degree of the meridian. —In Italy the ſcience was aſſiduouſly cultivated by Bianchini, Boſcovich, Friſi, Manfredi, Zanotti, and many others; in Sweden by Wargentin already mentioned, Blingenſtern, Mallet, and Planman; and in Germany by the Eulers, Mayer, Lambert, Griſchow, and others.

In the year 1760 all the learned Societies in Europe made preparations for obſerving the tranſit of Venus over the ſun, which had been predicted by Dr. Halley more than 80 years before, and the uſe that might be made of it in determining the ſun's parallax, and the diſtances of the planets from the ſun. And the ſame exertions were repeated, to obſerve the tranſit in 1769, by ſending obſervers to different parts of the world, for the more convenience in obſerving. And from the whole, Mr. Short computed that the ſun's parallax was nearly 8⅗ ſeconds, and conſequently the diſtance of the ſun from the earth about 24114 of the earth's diameters, or 96 millions of miles.

Dr. Bradley was ſucceeded, in 1762, in his office of Aſtronomer Royal, by Mr. Bliſs, Savilian profeſſor of aſtronomy; who being in a declining ſtate of health, did not long enjoy it. But, dying in 1765, was ſucceeded by the learned Nevil Maſkelyne, D. D. the preſent Aſtronomer Royal, who has diſcharged the duties of that office with the greateſt honour to himſelf, and benefit to the ſcience. In January 1761 this gentleman was ſent by the Royal Society, at a very early age, to the Iſland of St. Helena, to obſerve the tranſit of Venus over the ſun, and the parallax of the ſtar Sirius. The firſt of theſe objects partly failed, by clouds preventing the ſight of the 2d internal contact; and the 2d alſo, owing to Mr. Short having ſuſpended the plumbline by a *loop* from the neck of the central pin. However, our aſtronomer indemnified himſelf by many other valuable obſervations: Thus, he obſerved at St. Helena, the tides; the horary parallaxes of the moon; and the going of a clock, to find, by compariſon with its previous going which had been obſerved in England, the difference of gravity at the two places: alſo, in going out and returning, he practiſed the method of finding the longitude by the lunar diſtances taken with a Hadley's Quadrant, making out rules for the uſe of ſeamen, and taught the method to the officers on board the ſhip; which he afterwards explained in a letter to the Secretary of the Royal Society, inſerted in the Philoſ. Tranſ. for the year 1762, and ſtill more fully afterwards, in the Britiſh Mariner's Guide, which he publiſhed in the year 1763. He returned from St. Helena in the ſpring of 1762, after a ſtay there of 10 months; and in September 1763 ſailed for the iſland of Barbadoes, to ſettle the longitude of the place, and to compare Mr. Harriſon's watch with the time there when this gentleman ſhould bring it out: another object was alſo to try Mr. Irwin's marine chair, which he did in his way out. While at Barbadoes, he alſo made many other obſervations, and amongſt them, many relating to the moon's horary parallaxes, not yet publiſhed. Returning to England in the latter part of the year 1764, he was appointed in 1765 to ſucceed Mr. Bliſs as Aſtronomer Royal, and immediately recommended to the Board of Longitude the lunar method of finding the longitude, and propoſed to them the project of a Nautical Almanac, to be calculated and publiſhed to facilitate that method; this they agreed to, and the firſt vol. was publiſhed for 1767, and it has continued ever ſince under his direction, to the great benefit of navigation and univerſal commerce.

A multitude of other uſeful writings by this gentleman are inſerted in the volumes of the Philoſ. Tranſ. and particularly in conſequence of a propoſal, made by him to the Royal Society, the noble project was formed of meaſuring very accurately the effect of ſome mountain on the plumb line, in deflecting it from the perpendicular; and the mountain of Schehallien, in Scotland, having been found the moſt convenient in this iſland for the purpoſe, at the requeſt of the Society, he went into Scotland to conduct the buſineſs, which he performed in the moſt accurate manner, ſhewing that the ſum of the deflections on the two oppoſite ſides was about $11\frac{3}{5}$ ſeconds of a degree; and proving, to the ſatisfaction of the whole world, the univerſal attraction of all matter. From the data reſulting from theſe meaſures I have computed the mean denſity of the whole matter in the earth, which I have found to be about $4\frac{1}{2}$ times that of common water. Beſides many learned and valuable papers in the Philoſophical Tranſactions, and the moſt aſſiduous exertions in the duties of the obſervatory, as abundantly appears by the curious and voluminous obſervations which he has publiſhed, the world is particularly obliged to his endeavours with the Board of Longitude, for the publication of the Nautical Ephemeris, and the method of obſerving the longitude, by the diſtances of the moon and ſtars, now adopted by all nations, and by which the practice of Navigation has been brought to the greateſt perfection.

Finally, the diſcoveries of Dr. Herſchel form a new æra in aſtronomy. He firſt, in 1781, began with obſervations on the periodical ſtar *in Collo Ceti*, and a new method of meaſuring the lunar mountains, none of which he made more than half a mile in height: and, having conſtructed teleſcopes vaſtly more powerful than any former ones, he proceeded to other obſervations, concerning which he has had ſeveral papers printed in the Philoſophical Tranſactions; as, on the rotation of the planets round their axes; On the parallax of the fixed ſtars; Catalogues of double, triple, &c ſtars; On the proper motion of the ſun and ſolar ſyſtem; On the remarkable appearances of the ſolar regions of the planet Mars, &c. And, above all, his diſcovery of a new primary planet, on the 13th of March 1781, which he calls the *Georgian Planet*, but it is named the *Planet Herſchel* by the French and other foreign aſtronomers; by which, and its two ſatellites, which he has alſo diſcovered ſince that time, he has greatly enlarged the bounds of the ſolar ſyſtem, this new planet being more than twice as far from the ſun as the planet Saturn.

Liſts and hiſtorical accounts of the principal writings and authors on aſtronomy are contained in Weidler's Hiſtory of Aſtronomy, which is brought down to the year 1737. There is alſo Bailly's Hiſtory of aſtronomy, ancient and modern. For this purpoſe, conſult alſo the following authors, viz, Adam, Voſſius, Bayle, Chauffepié, Niceron, Perraut, the chronological table of Riccioli, and that of Sherburn, at the end of his edition of Manilius; alſo the firſt volume of De la Lande's aſtronomy. The more modern, and popular books on aſtronomy, are very numerous, and well known: as thoſe of Ferguſon, Long, Emerſon, Vince, De la Lande, Leadbetter, Brent, Keil, Whiſton, Wing, Street, &c, &c.

ASTROSCOPE, a kind of aſtronomical inſtrument, compoſed of two cones, on the ſurface of which are delineated the conſtellations, with their ſtars, by means of which theſe may eaſily be known in the heavens. The aſtroſcope was invented by William Shukhard, profeſſor of mathematics at Tubingen, upon which he publiſhed a treatiſe in 1698.

ASTROSCOPIA, the art of obſerving and examining the ſtars, by means of teleſcopes, to diſcover their nature and properties.

ASTROTHEMATA, the places or poſitions of the ſtars, in an aſtrological ſcheme of the heavens.

ASTROTHESIA, is uſed by ſome for a conſtellation or collection of ſtars in the heavens.

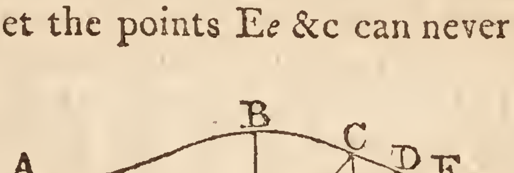
ASTRUM, or Astron, a conſtellation, or aſſemblage of ſtars: in which ſenſe it is diſtinguiſhed from *Aſter*, which denotes a ſingle ſtar. Some apply the term, in a more particular ſenſe, to the Great Dog, or rather to the large bright ſtar in his mouth.

ASYMMETRY, the want of proportion, otherwiſe called *incommenſurability*, or the relation of two quantities which have no common meaſure, as between 1 and $\sqrt{2}$, or the ſide and diagonal of a ſquare.

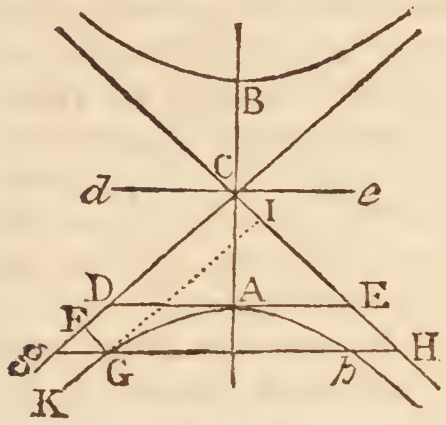
ASYMPTOTE, is properly a right line, which approaches continually nearer and nearer to ſome curve, whoſe aſympote it is ſaid to be, in ſuch ſort, that when they are both indefinitely produced, they are nearer together than by any aſſignable finite diſtance; or it may be conſidered as a tangent to the curve when conceived to be produced to an infinite diſtance. Two curves are alſo ſaid to be aſymptotical, when they thus continually approach indefinitely to a coincidence: thus, two parabolas, placed with their axes in the ſame right line, are aſymptotes to one another.

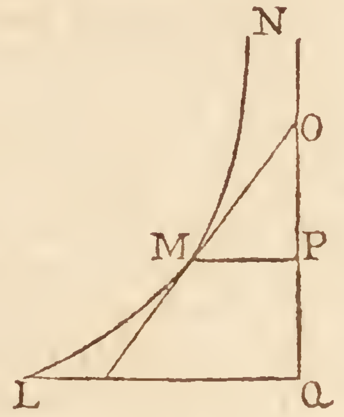
Of lines of the ſecond kind, or curves of the firſt kind, that is the conic ſections, only the hyperbola has aſymptotes, which are two in number. All curves of the ſecond kind have at leaſt one aſymptote; but they may have three. And all curves of the third kind may have four aſymptotes. The conchoid, ciſſoid, and logarithmic curve, though not reputed geometrical curves, have each one aſymptote. And the branch or leg of a curve that has an aſymptote, is ſaid to be of the hyperbolic kind.

The nature of aſymptotes will be eaſily conceived from the inſtance of the aſymptote to the conchoid. Thus, if, ABC &c be part of a conchoid, and the line MN be ſo drawn that the parts FB, GC, HD, IE, &c, of right lines, drawn from the pole P, be equal to each other; then will the line MN be the aſymptote of the curve: becauſe the perpendicular C*c* is ſhorter than FB, and D*d* than C*c*, &c; ſo that the two lines continually approach; yet the points E*e* &c can never coincide.



Asymptotes *of the* Hyperbola are thus deſcribed. Suppoſe a right line DE drawn to touch the curve in any point A, and equal to the conjugate *de* of the diamete ACB drawn to that point A, viz, AD or AE equal to the ſemiconjugate C*d* or C*e*; then the two lines CDF, CEH, drawn from the centre C, through the points D and E, are the two aſymptotes of the curve.





The parts of any right line, lying between the curve of the common hyperbola and its aſymptotes, are equal to one another on both ſides, that is, $gG = hH$. In like manner, in hyperbolas of the ſecond kind, if there be drawn any right line cutting both the curve and its three aſymptotes in three points, the ſum of the two parts of that right line extended in the ſame direction from any two of the aſymptotes to two points of the curve, is equal to the third part which extends in the contrary direction from the third aſymptote to the third point of the curve.

If AGK be an hyperbola of any kind, whoſe nature, with regard to the curve and aſymptote, is expreſſed by this general equation $x^m y^n = a^{m+n}$, where x is $=$ CF, and $y =$ FG drawn any where parallel to the other aſymptote CH; and the parallelogram CFGI be completed: Then $m - n$ is to n, as this parallelogram CFGI is to the hyperbolic ſpace FGK, contained under the determinate line FG, with the aſymptote FK and the curve GK, both indefinitely continued towards K. So that, if m be greater than n, the ſaid aſymptotic ſpace is finite and quadrable: but when $m = n$, as in the common or conic hyperbola, then $m - n = 0$, the ratio of that ſpace to the ſaid parallelogram, is as n to 0: that is, the hyperbolic ſpace is infinitely great, in reſpect of the finite parallelogram: and when m is leſs than n, then, $m - n$ being negative, the aſymptotic ſpace is to the determinate parallelogram, as a poſitive number is to a negative one, and is what Dr. Wallis calls more than infinite.

Asymptote *of the* Logarithmic Curve. If LMN be the logarithmic curve, QON an aſymptote, LQ and MP ordinates, MO a tangent, and PO the ſubtangent, which in this curve is a conſtant quantity. Then the indeterminate ſpace LMNQ is equal to LQ × PO, the rectangle under the ordinate LQ and the conſtant ſubtangent PO; and the ſolid generated by the rotation of that curve ſpace about the aſymptote NQ, is equal to half the cylinder, whoſe altitude is the ſaid conſtant ſubtangent PO, and the radius of its baſe is LQ.

Asymptotes, by ſome, are diſtinguiſhed into various orders. The aſymptote is ſaid to be of the firſt order, when it coincides with the baſe of the curvilinear figure: of the 2d order, when it is a right line parallel to the baſe: of the 3d order, when it is a right line oblique to the baſe: of the 4th order, when it is the common parabola, having its axis perpendicular to the baſe: and, in general, of the $n + 2$ order, when it is a parabola whoſe ordinate is always as the n power of the baſe. The aſymptote is oblique to the baſe, when the ratio of the firſt fluxion of the ordinate to the fluxion of the baſe, approaches to an aſſignable ratio, as its limit; but it is parallel to the baſe, or coincides with it, when this limit is not aſſignable.

The doctrine and determination of the aſymptotes of curves, is a curious part of the higher geometry. Fontenelle has given ſeveral theorems relating to this ſubject, in his *Geometrie de l'Infini.* See alſo Stirling's *Lineæ Tertii Ordinis*, prop. vi, where the ſubject of Aſymptotes is learnedly treated; and Cramer's *Introduction à l'analyſe des lignes courbes*, art. 147 *& ſeq.* for an excellent theory of aſymptotes of geometrical curves and their branches. Likewiſe Maclaurin's Algebra, and his Fluxions, book i, chap. 10, where he has carefully avoided the modern paradoxes concerning infinites and infiniteſimals. But the eaſieſt way of determining aſymptotes, it ſeems, is by conſidering them as tangents to the curves at an infinite diſtance from the beginning of the abſciſs, that is when the abſciſs x is infinite in the equation of the curve, and in the proportion of x to y, or in that of the ſubtangent to the ordinate.

The areas bounded by curves and their aſymptotes, though indefinitely extended, have ſometimes limits to which they may approach indefinitely near: and this happens in hyperbolas of all kinds, except the firſt or Apollonian, and in the logarithmic curve; as was obſerved above. But in the common hyperbola, and many other curves, the aſymptotical area has no ſuch limit, but is infinitely great.—Solids, too, generated by hyperbolic areas, revolving about their aſymptotes, have ſometimes their limits; and ſometimes they may be produced till they exceed any given ſolid.—Alſo the ſurface of ſuch ſolid, when ſuppoſed to be infinitely produced, is either finite or infinite, according as the area of the generating figure is finite or infinite.

ATLANTIDES, a name given to the Pleiades, or ſeven ſtars, ſometimes alſo called Vergiliæ. They were thus called, as being ſuppoſed by the poets to have been the daughters either of Atlas or his brother Heſperus, who were tranſlated to heaven.

ATMOSPHERE, a term uſed to ſignify the whole of the fluid maſs, conſiſting of air, aqueous and other vapours, electric fluids, &c, which ſurrounds the earth to a conſiderable height, and partaking of all its motions, both annual and diurnal.

The compoſition of that part of our atmoſphere properly called air, was till lately but very little known. Formerly it was ſuppoſed to be a ſimple, homogeneous, and elementary fluid. But the experiments of Dr. Prieſtley and others, have diſcovered, that even the pureſt kind of air, which they call dephlogiſticated, is in reality a compound, and might be artificially produced in various ways. This dephlogiſticated air, however, is but a ſmall part of the compoſition of our atmoſphere. By accurate experiments, the air we uſually breathe, is compoſed of only one-fourth part of this dephlogiſticated air, or perhaps leſs; the other three parts, or more, conſiſting of what Dr. Prieſtley calls *phlogiſticated*, and M. Lavoiſier *mephitic air.*

Beſide theſe ſorts of air, it is obvious that the whole maſs of the atmoſphere contains a great deal of water, together with a vaſt heterogeneous collection of particles raiſed from all bodies of matter on the ſurface of the earth, by effluvia, exhalations, &c; ſo that it may be conſidered as a chaos of the particles of all ſorts of matter confuſedly mingled together. And hence the atmoſphere has been conſidered as a large chemical veſſel, in which the matter of all kinds of ſublunary bodies is copiouſly floating; and thus expoſed to the continual action of that immenſe ſurface the ſun; from whence proceed innumerable operations, ſublimations, ſeparations, compoſitions, digeſtions, fermentations, putrefractions, &c.

There is, however, one ſubſtance, namely the electrical fluid, which is very diſtinguiſhable in the maſs of the atmoſphere. To meaſure the abſolute quantity of this fluid, either in the atmoſphere or any other ſubſtance, is perhaps impoſſible: and all that we know on this ſubject is, that the electric fluid pervades the atmoſphere; that it appears to be more abundant in the ſuperior than the inferior regions; that it ſeems to be the immediate bond of connection between the atmoſphere and the water which is ſuſpended in it; and that by its various operations, the phenomena of hail, rain, ſnow, lightning, and the other kinds of meteors are occaſioned. See thoſe reſpective articles; and ſee alſo Beccaria's Eſſay on Atmoſpheric Electricity, annexed to the Engliſh tranſlation of his Artificial Electricity.

Uſes of the ATMOSPHERE.—The uſes of the atmoſphere are ſo many and great, that it ſeems indeed abſolutely neceſſary, not only to the comfort and convenience of men, but even to the exiſtence of all animal and vegetable life, and to the very conſtitution of all kinds of matter whatever, and without which they would not be what they are: for by it we live, breathe, and have our being; and by inſinuating itſelf into all the vacuities of bodies, it becomes the great ſpring of moſt of the mutations here below; as generation, corruption, diſſolution, &c; and without which none of theſe operations could be carried on. Without the atmoſphere, no animal could exiſt, or indeed be produced; neither any plant, all vegetation ceaſing without its aid; there would be neither rain nor dews to moiſten the face of the ground; and though we might perceive the ſun and ſtars like bright ſpecks, we ſhould be in utter darkneſs, having none of what we call day light or even twilight: nor would either fire or heat exiſt without it. In ſhort, the nature and conſtitution of all matter would be changed and ceaſe; wanting this univerſal bond and conſtituting principle.

By the mechanical force of the atmoſphere too, as well as its chemical virtues, many neceſſary purpoſes are anſwered. We employ it as a moving power, in the motion of ſhips, to turn mills, and for other ſuch uſes. And it is one of the great diſcoveries of the modern philoſophers, that the ſeveral motions attributed by the ancients to a *fuga vacui*, are really owing to the preſſure of the atmoſphere. Galileo, having obſerved that there was a certain ſtandard altitude, beyond which no water could be elevated by pumping, took an occaſion from thence to call in queſtion the doctrine of the ſchools, which aſcribed the aſcent of water in pumps, to the *fuga vacui*, and inſtead of it he happily ſubſtituted the hypotheſis of the weight and preſſure of the air. It was with him, indeed, little better than an hypotheſis, ſince it had not then thoſe confirmations from experiment, afterwards found out by his pupil Torricelli, and other ſucceeding philoſophers, particularly Mr Boyle.

Nor have the attempts to fly or float through the air been altogether without ſucceſs. F. De Lana thought he had contrived an aeronautic machine for navigating the atmoſphere: and Sturmius, who examined it, aſſerted that it was not impracticable: though Dr Hook was of a different opinion, and detected the fallacy of the contrivance. Roger Bacon, long before, propoſed ſomething of the ſame kind. The great ſecret of this art is to contrive a machine ſo much lighter than the air, that it will riſe up and float in the atmoſphere, and together with itſelf, buoy up and carry men along with it. The principle on which it is to be effected is, by means of an air pump, to exhauſt the air from a very thin and light, yet firm, metalline veſſel. But the hopes of ſucceſs in ſuch an enterprize will appear very ſmall if it be conſidered, that if a globe were formed of braſs of the thickneſs only of $\frac{1}{12}$ of an inch, ſuch a globe would require to be about 277 feet in diameter to float in the air: and if, as De Lana ſuppoſes, the diameter of the globe were but 25 feet, the thickneſs of the metal could not exceed $\frac{1}{33}$ of an inch. See Herman's Phoronomia pa. 158. However, what is not to be expected from metalline globes or ſhells, has now been ſucceſsfully accompliſhed by the balloons of cloth, ſilk, or ſkin, of Montgolfier and others. See the article *Aeroſtation*, &c.

Salubrity of the ATMOSPHERE. On the tops of mountains the air is generally more ſalubrious than in pits or very deep places. Indeed denſe air is always more proper for reſpiration, as to the mere quality of denſity only, than that which is rarer. But then the air on mountains, though rarer, is freer from phlogiſtic vapours than that of pits; and hence it has been found that people can live very well on the tops of mountains, even when the air is but about half the denſity of that below. But it would ſeem that at ſome intermediate height between the two extremes, the air is the moſt ſalubrious and proper for animal life; and this height, according to M. de Sauſſure, is about 500 or 600 yards above the level of the ſea.

Beſides the difference ariſing from the mere difference of altitude, the ſalubrity of the atmoſphere is greatly affected by many other circumſtances. The air, when confined or ſtagnant, is commonly more impure than when agitated and ſhifted: thus, all cloſe places are unhealthy, and even the air in a bed chamber is leſs ſalubrious in a morning, after it has been ſlept in, than in the evening. Dr White, in vol. 68 Philoſ. Tranſ. gives an account of experiments on this quality of the air, and remarks one inſtance when the air was particularly impure, viz September 13, 1777; when the barometer ſtood at 30·30, the thermometer at 69°; the air being then dry and ſultry, and no rain having fallen for more than two weeks. A ſlight ſhock of an earthquake was perceived that day. In vol. 70 of the ſame Tranſactions Dr. Ingenhouſz gives an account of ſome experiments on this head, made in various places and ſituations;

ſituations: he finds," That the air at ſea, and cloſe to it, is in general purer, and fitter for animal life, than the air on the land:" but the Doctor did not find much difference between the air of the towns and of the country, nor between one town and another. The Abbé Fontana, made nearly the ſame concluſions, from accurate experiments, aſſerting, "that the difference between the air of one country and that of another, at different times, is much leſs than what is commonly believed; and yet that this difference in the purity of the air at different times, is much greater than the difference between the air of the different places obſerved by him." Finally M. Fontana concludes, that "Nature is not ſo partial as we commonly believe. She has not only given us an air almoſt equally good every where at every time, but has allowed us a certain latitude, or a power of living and being in health in qualities of air which differ to a certain degree. By this I do not mean to deny the exiſtence of certain kinds of noxious air in ſome particular places; but only ſay, that in general the air is good every where, and that the ſmall differences are not to be feared ſo much as ſome people would make us believe. Nor do I mean to ſpeak here of thoſe vapours and other bodies which are accidentally joined to the common air in particular places, but do not change its nature and intrinſical property. This ſtate of the air cannot be known by the teſt of nitrous air; and thoſe vapours are to be conſidered in the ſame manner as we ſhould conſider ſo many particles of arſenic ſwimming in the atmoſphere. In this caſe it is the arſenic, and not the degenerated air, that would kill the animals who ventured to breathe it."

Figure of the ATMOSPHERE.—As the atmoſphere envelops all parts of the ſurface of our globe, if they both continued at reſt, and were not endowed with a diurnal motion about their common axis, then the atmoſphere would be exactly globular, according to the laws of gravity; for all the parts of the ſurface of a fluid in a ſtate of reſt, muſt be equally removed from its centre. But as the earth and the ambient parts of the atmoſphere revolve uniformly together about their axis, the different parts of both have a centrifugal force, the tendency of which is more conſiderable, and that of the centripetal leſs, as the parts are more remote from the axis; and hence the figure of the atmoſphere muſt become an oblate ſpheroid; ſince the parts that correſpond to the equator are father removed from the axis, than the parts which correſpond to the poles. Beſides, the figure of the atmoſphere muſt, on another account, repreſent a flattened ſpheroid, namely becauſe the ſun ſtrikes more directly the air which encompaſſes the equator, and is comprehended between the two tropics, than that which pertains to the polar regions: for, from hence it follows, that the maſs of air, or part of the atmoſphere, adjoining to the poles, being leſs heated, cannot expand ſo much, nor reach ſo high. And yet, notwithſtanding, as the ſame force which contributes to elevate the air, diminiſhes its gravity and preſſure on the ſurface of the earth, higher columns of it about the equatorial parts, all other circumſtances being the ſame, may not be heavier than thoſe about the poles.

In the Tranſactions of the Royal Iriſh Academy for 1788 Mr Kirwin has an ingenious diſſertation on the figure, height, weight, &c, of the atmoſphere. He obſerves that, in the natural ſtate of the atmoſphere, that is, when the barometer would every where, at the level of the ſea, ſtand at 30 inches, the weight of the atmoſphere, at the ſurface of the ſea, muſt be equal all over the globe; and in order to produce this equality, as the weight proceeds from its denſity and height, it muſt be loweſt where the denſity is greateſt, and higheſt where the denſity is leaſt; that is, higheſt at the equator and loweſt at the poles, with ſeveral intermediate gradations.

Though the equatorial air however be leſs denſe to a certain height than the polar, yet at ſome greater heights it muſt be more denſe: for ſince an equatorial and polar column are equal in total weight or maſs, the lower part of the equatorial column, being more expanded by heat &c than that of the polar, muſt have leſs maſs, and therefore a proportionably greater part of its maſs muſt be found in its ſuperior ſection; ſo that the lower extremity of the ſuperior ſection of the equatorial column is more compreſſed, and conſequently denſer, than the correſponding part of the polar column. The ſame thing is to be underſtood alſo of the extra-tropical columns with reſpect to each other, where differences of heat prevail.

Hence, in the higheſt regions of the atmoſphere, the denſer equatorial air, not being ſupported by the collateral extra-tropical columns, gradually flows over, and rolls down to the north and ſouth.

Theſe ſuperior tides conſiſt chiefly of inflammable air, as it is much lighter than any other, and is generated in great plenty between the tropics; it furniſhes the matter of the auroræ borealis and auſtralis, by whoſe combuſtion it is deſtroyed, elſe its quantity would in time become too great, and the weight of the atmoſphere annually increaſed; but its combuſtion is the primary ſource of the greateſt perturbations of the atmoſphere.

Weight or Preſſure of the ATMOSPHERE.—It is evident that the maſs of the atmoſphere, in common with all other matter, muſt be endowed with weight and preſſure; and this principle was aſſerted by almoſt all philoſophers, both ancient and modern. But it was only by means of the experiments made with pumps and the barometrical tube, by Galileo and Torricelli, that we came to the proof, not only that the atmoſphere is endued with a preſſure, but alſo what the meaſure and quantity of that preſſure is. Thus, it is found that the preſſure of the atmoſphere ſuſtains a column of quickſilver, in the tube of the barometer, of about 30 inches in height; it therefore follows, that the whole preſſure of the atmoſphere is equal to the weight of a column of quickſilver, of an equal baſe, and 30 inches height: and becauſe a cubical inch of quickſilver is found to weigh nearly half a pound averdupois, therefore the whole 30 inches, or the weight of the atmoſphere on every ſquare inch of ſurface, is equal to 15 pounds. Again, it has been found that the preſſure of the atmoſphere balances, in the caſe of pumps &c, a column of water of about 34½ feet high; and, the cubical foot of water weighing juſt 1000 ounces, or 62½ pounds, 34½ times 62½, or 2158lb, will be the weight of the column of water, or of the atmoſphere on a baſe of a ſquare foot; and conſequently the 144th part of this, or 15lb, is the weight of the atmoſphere on a ſquare inch;

inch'; the ſame as before. Hence Mr Cotes computed that the preſſure of this ambient fluid on the whole ſurface of the earth, is equivalent to that of a globe of lead of 60 miles in diameter. And hence alſo it appears, that the preſſure upon the human body muſt be very conſiderable; for as every ſquare inch of ſurface ſuſtains a preſſure of 15 pounds, every ſquare foot will ſuſtain 144 times as much, or 2160 pounds; then, if the whole ſurface of a man's body he ſuppoſed to contain 15 ſquare feet, which is pretty near the truth, he muſt ſuſtain 15 times 2160, or 32400 pounds, that is nearly 14½ tons weight, for his ordinary load. By this enormous preſſure we ſhould undoubtedly be cruſhed in a moment, if all parts of our bodies were not filled either with air or ſome other elaſtic fluid, the ſpring of which is juſt ſufficient to counterbalance the weight of the atmoſphere. But whatever this fluid may be, it is certain that it is juſt able to counteract the weight of the atmoſphere, and no more: for, if any conſiderable preſſure be ſuperadded to that of the air, as by going into deep water, or the like, it is always ſeverely felt let it be ever ſo equable, at leaſt when the change is made ſuddenly; and if, on the other hand, the preſſure of the atmoſphere be taken off from any part of the human body, as the hand for inſtance, when put over an open receiver, from whence the air is afterwards extracted, the weight of the external atmoſphere then prevails, and we imagine the hand ſtrongly ſucked down into the glaſs.

The difference in the weight of the air which our bodies ſuſtain at one time more than another, is alſo very conſiderable, from the natural changes in the ſtate of the atmoſphere. This change takes place chiefly in countries at ſome diſtance from the equator; and as the barometer varies at times from 28 to 31 inches, or about one tenth of the whole quantity, it follows that this difference amounts to about a ton and a half on the whole body of a man, which he therefore ſuſtains at one time more than at another. On the increaſe of this natural weight, the weather is commonly fine, and we feel ourſelves what we call braced and more alert and active; but, on the contrary, when the weight of the air diminiſhes, the weather is bad, and people feel a liſtleſsneſs and inactivity about them. And hence it is no wonder that perſons ſuffer very much in their health, from ſuch changes in the atmoſphere, eſpecially when they take place very ſuddenly, for it is to this circumſtance chiefly that a ſenſation of uneaſineſs and indiſpoſition is to be attributed; thus, when the variations of the barometer and atmoſphere are ſudden and great, we feel the alteration and effect on our bodies and ſpirits very much; but when the change takes place by very ſlow degrees, and by a long continuance, we are ſcarcely ſenſible of it, owing, undoubtedly, to the power with which the body is naturally endowed, of accommodating itſelf to this change in the ſtate of the air, as well as to the change of many other circumſtances of life, the body requiring a certain interval of time to effect the alteration in its ſtate, proper to that of the air &c. Thus, in going up to the tops of mountains, where the preſſure of the atmoſphere is diminiſhed two or three times more than on the plain below, little or no inconvenience is felt from the rarity of the air, if it is not mixed with other noxious vapours &c; becauſe that, in the aſcent the body has had ſufficient time to accommodate itſelf gradually to the ſlow variation in the ſtate of the atmoſphere: but, when a perſon aſcends with a balloon, very rapidly to a great height in the atmoſphere, he feels a difficulty in breathing and an uneaſineſs of body; and the ſame is ſoon felt by an animal when incloſed in a receiver, and the air ſuddenly drawn or pumped out of it. So alſo, on the condenſation of the air, we feel little or no alteration in ourſelves, except when the change happens ſuddenly, as in very rapid changes in the weather, and in deſcending to great depths in a diving bell, &c. I have often heard the late unfortunate Mr. Spalding ſpeak of his experience on this point: he always found it abſolutely neceſſary to deſcend with the bell very ſlowly, and that only from one depth to another, reſting a while at each depth before he began to deſcend farther: he firſt deſcended ſlowly for about 5 or 6 fathom, and then ſtopped a while; he felt an uneaſineſs in his head and ears, which increaſed more and more as he deſcended, till he was obliged to ſtop at the depth above mentioned, where the denſity of the air was nearly doubled; having remained there a while, he felt his ears give a ſudden crack, and after that he was ſoon relieved from any uneaſineſs in that part, and it ſeemed as if the denſity of the air was not altered. He then deſcended other 5 fathoms or 30 feet more, with the ſame precaution and the ſame ſenſations as before, being again relieved, in the ſame manner, after remaining awhile ſtationary at the next ſtage of his deſcent, where the denſity of the air was tripled. And thus he continued proceeding to a great depth, always with the ſame circumſtances, repeated at every 5 or 6 fathoms, and adding the preſſure of one more atmoſphere at every period of the progreſs.

It is not eaſy to aſſign the true reaſon for the variations that happen in the gravity of the atmoſphere in the ſame place. One cauſe of it however, either immediate or otherwiſe, it ſeems, is the heat of the ſun; for where this is uniform, the changes are ſmall and regular; thus between the tropics it ſeems the change depends on the heat of the ſun, as the barometer conſtantly ſinks about half an inch every day, and riſes again to its former ſtation in the night time. But in the temperate zones the barometer ranges from 28 to near 31 inches, ſhewing, by its various altitudes, the changes that are about to take place in the weather. If we could know therefore, the cauſes by which the weather is influenced; we ſhould alſo know thoſe by which the gravity of the atmoſphere is affected. Theſe may perhaps be reduced to immediate ones, viz, an emiſſion of latent heat from the vapour contained in the atmoſphere, or of electric fluid from the ſame, or from the earth; as it is obſerved that they both produce the ſame effect with the ſolar heat in the tropical climates, viz, to rarefy the air, by mixing with it, or ſetting looſe a lighter fluid, which did not before act in ſuch large proportion in any particular place.

With regard to the alteration of heat and cold in the atmoſphere, many reaſons and hypotheſes have been given, and many experiments made; as may be ſeen by conſulting the authors upon this ſubject, viz, M. Bouguer's obſervations in Peru, Lambert, De Luc, Sauſſure's journeys on the Alps, Sex's and Darwin's experiments in vol. 78 Philoſ. Tranſ. This laſt gentleman hence infers, "There is good reaſon to conclude that in all circumſtances where air is mechanically expanded, it becomes capable of attracting the fluid matter of heat from other bodies in contact with it. Now, as the vaſt region of air which ſurrounds our globe is perpetually moving along its ſurface, climbing up the ſides of mountains, and deſcending into the valleys ; as it paſſes along it muſt be perpetually varying the degree of heat according to the elevation of the country it traverſes : for, in riſing to the ſummits of mountains, it becomes expanded, having ſo much of the preſſure of the ſuperincumbent atmoſphere taken away ; and when thus expanded, it attracts or abſorbs heat from the mountains in contiguity with it ; and, when it deſcends into the valleys and is compreſſed into leſs compaſs, it again gives out the heat it has acquired to the bodies it comes in contact with. The ſame thing muſt happen to the higher regions of the atmoſphere, which are regions of perpetual froſt, as has lately been diſcovered by the aerial navigators. When large diſtricts of air, from the lower parts of the atmoſphere, are raiſed two or three miles high, they become ſo much expanded by the great diminution of the preſſure over them, and thence become ſo cold, that hail or ſnow is produced by the precipitation of the vapour : and as there is, in theſe high regions of the atmoſphere, nothing elſe for the expanded air to acquire heat from after it has parted with its vapour, the ſame degree of cold continues till the air, on deſcending to the earth, acquires its former ſtate of condenſation and of warmth. The Andes, almoſt under the line, reſts its baſe on burning ſands : about its middle height is a moſt pleaſant and temperate climate covering an extenſive plain, on which is built the city of Quito ; while its forehead is encircled with eternal ſnow, perhaps coeval with the mountain. Yet, according to the accounts of Don Ulloa, theſe three diſcordant climates ſeldom encroach much on each other's territories. The hot winds below, if they aſcend, become cooled by their expanſion ; and hence they cannot affect the ſnow upon the ſummit ; and the cold winds that ſweep the ſummit, become condenſed as they deſcend, and of temperate warmth before they reach the fertile plains of Quito."

Height and Denſity of the ATMOSPHERE. Various attempts have been made to aſcertain the height to which the atmoſphere is extended all round the earth. Theſe commenced ſoon after it was diſcovered by means of the Torricellian tube, that air is endued with weight and preſſure. And had not the air an elaſtic power, but were it every where of the ſame denſity, from the ſurface of the earth to the extreme limit of the atmoſphere, like water, which is equally denſe at all depths it would be a very eaſy matter to determine its height from its denſity and the column of mercury which it would counterbalance in the barometer tube : for, it having been obſerved that the weight of the atmoſphere is equivalent to a column of 30 inches or 2½ feet of quickſilver, and the denſity of the former to that of the latter, as 1 to 11040 ; therefore the height of the uniform atmoſphere would be 11040 times 2½ feet, that is 27600 feet, or little more than 5 miles and a quarter. But the air, by its elaſtic quality, expands and contracts ; and it being found by repeated experiments in moſt nations of Europe, that the ſpaces it occupies, when compreſſed by different weights, are reciprocally proportional to thoſe weights themſelves ; or, that the more the air is preſſed, ſo much the leſs ſpace it takes up ; it follows that the air in the upper regions of the atmoſphere muſt grow continually more and more rare, as it aſcends higher ; and indeed that, according to that law, it muſt neceſſarily be extended to an indefinite height. Now, if we ſuppoſe the height of the whole divided into innumerable equal parts; the quantity of each part will be as its denſity ; and the weight of the whole incumbent atmoſphere being alſo as its denſity ; it follows, that the weight of the incumbent air, is every where as the quantity contained in the ſubjacent part; which cauſes a difference between the weights of each two contiguous parts of air. But, by a theorem in arithmetic, when a magnitude is continually diminiſhed by the like part of itſelf, and the remainders the ſame, theſe will be a ſeries of continued quantities decreaſing in geometrical progreſſion : therefore if, according to the ſuppoſition, the altitude of the air, by the addition of new parts into which it is divided, do continually increaſe in arithmetical progreſſion, its denſity will be diminiſhed, or, which is the ſame thing, its gravity decreaſed, in continued geometrical proportion. And hence, again, it appears that, according to the hypotheſis of the denſity being always proportional to the compreſſing force, the height of the atmoſphere muſt neceſſarily be extended indefinitely. And, farther, as an arithmetical ſeries adapted to a geometrical one, is analogous to the logarithms of the ſaid geometrical one ; it follows therefore that the altitudes are proportional to the logarithms of the denſities, or weights of air ; and that any height taken from the earth's ſurface, which is the difference of two altitudes to the top of the atmoſphere, is proportional to the difference of the logarithms of the two denſities there, or to the logarithm of the ratio of thoſe denſities, or their correſponding compreſſing forces, as meaſured by the two heights of the barometer there. This law was firſt obſerved and demonſtrated by Dr. Halley, from the nature of the hyperbola ; and afterwards by Dr. Gregory, by means of the logarithmic curve. See Philoſ. Tranſ. N°. 181, or Abridg. vol. 2, p. 13, and Greg. Aſtron. lib. v, prop. 3.

It is now eaſy, from the foregoing property, and two or three experiments, or barometrical obſervations, made at known altitudes, to deduce a general rule to determine the abſolute height anſwering to any denſity, or the denſity anſwering to any given altitude above the earth. And accordingly, calculations were made upon this plan by many philoſophers, particularly by the French ; but it having been found that the barometrical obſervations did not correſpond with the altitudes as meaſured in a geometrical manner, it was ſuſpected that the upper parts of the atmoſpherical regions were not ſubject to the ſame laws with the lower ones, in regard to the denſity and elaſticity. And indeed, when it is conſidered that the atmoſphere is a heterogeneous maſs of particles of all ſorts of matter, ſome elaſtic, and others not, it is not improbable but this may be the caſe, at leaſt in the regions very high in the atmoſphere, which it is likely may more copiouſly abound with the electrical fluid. Be this however as it may, it has lately been diſcovered that the law above given, holds very well for all ſuch altitudes as are within our reach, or as far as to the tops of the higheſt mountains on the earth, when a correction is made for the difference of the heat or temperature of the air only, as was fully evinced by M. De Luc, in a long ſeries of obſervations, in which he determined the altitudes of hills both by the barometer, and by geometrical meaſurement, from which he deduced a practical rule to allow for the difference of temperature. See his Treatiſe on the Modifications of the Atmoſphere. Similar rules have alſo been deduced from accurate experiments, by Sir George Shuckburgh and General Roy, both concurring to ſhew, that ſuch a rule for the altitudes and denſities, holds true for all heights that are acceſſible to us, when the elaſticity of the air is corrected on account of its denſity: and the reſult of their experiments ſhewed, that the difference of the logarithms of the heights of the mercury in the barometer, at two ſtations, when multiplied by 10000, is equal to the altitude in Engliſh fathoms, of the one place above the other; that is, when the temperature of the air is about 31 or 32 degrees of Fahrenheit's thermometer; and a certain quantity more or leſs, according as the actual temperature is different from that degree.

But it may here be ſhewn, that the ſame rule may be deduced independent of ſuch a train of experiments as thoſe above, merely by the denſity of the air at the ſurface of the earth alone. Thus, let D denote the denſity of the air at one place, and d the denſity at the other; both meaſured by the column of mercury in the barometrical tube: then the difference of altitude between the two places, will be proportional to the log. of D − the log. of d, or to the log. of $\frac{D}{d}$. But as this formula expreſſes only the relation between different altitudes, and not the abſolute quantity of them, aſſume ſome indeterminate, but conſtant quantity h, which multiplying the expreſſion log. $\frac{D}{d}$, may be equal to the real difference of altitude a, that is, $a = h \times$ log. of $\frac{D}{d}$. Then, to determine the value of the general quantity h, let us take a caſe in which we know the altitude a which correſponds to a known denſity d; as for inſtance, taking $a = 1$ foot, or 1 inch, or ſome ſuch ſmall altitude: then becauſe the denſity D may be meaſured by the preſſure of the whole atmoſphere, or the uniform column of 27600 feet, when the temperature is 55°; therefore 27600 feet will denote the denſity D at the lower place, and 27599 the leſs denſity d at 1 foot above it; conſequently $1 = h \times$ log. of $\frac{27600}{27599}$, which, by the nature of logarithms, is nearly $= h \times \frac{\cdot 43429448}{27600}$ or $\frac{1}{63551}$ nearly; and hence we find $h = 63551$ feet; which gives us this formula for any altitude a in general, viz, $a = 63551 \times$ log. of $\frac{D}{d}$, or $a = 63551 \times$ log. of $\frac{M}{m}$ feet, or $10592 \times$ log. of $\frac{M}{m}$ fathoms; where M denotes the column of mercury in the tube at the lower place, and m that at the upper This formula is adapted to the mean temperature of the air 55°: but it has been found, by the experiments of Sir Geo. Shuckburgh and General Roy, that for every degree of the thermometer, different from 55°, the altitude a will vary by its 435th part; hence, if we would change the factor h from 10592 to 10000, becauſe the difference 592 is the 18th part of the whole factor 10592, and becauſe 18 is the 24th part of 435; therefore the change of temperature, anſwering to the change of the factor h, is 24°, which reduces the 55° to 31°. So that, $a = 10000 \times$ log. of $\frac{M}{m}$ fathoms, is the eaſieſt expreſſion for the altitude, and anſwers to the temperature of 31°, or very nearly the freezing point: and for every degree above that, the reſult muſt be increaſed by ſo many times its 435th part, and diminiſhed when below it.

From this theorem it follows, that, at the height of $3\frac{1}{2}$ miles, the denſity of the atmoſphere is nearly 2 times rarer than it is at the ſurface of the earth; at the height of 7 miles, 4 times rarer; and ſo on, according to the following table:

Height in miles.	Number of times rarer.
$3\frac{1}{2}$	2
7	4
14	16
21	64
28	256
35	1024
42	4096
49	16384
56	65536
63	262144
70	1048576

And, by purſuing the calculations in this table, it might be eaſily ſhewn, that a cubic inch of the air we breathe would be ſo much rarefied at the height of 500 miles, that it would fill a ſphere equal in diameter to the orbit of Saturn.

Hence we may perceive how very ſoon the air becomes ſo extremely rare and light, as to be utterly imperceptible to all experience; and that hence, if all the planets have ſuch atmoſpheres as our earth, they will, at the diſtances of the planets from one another, be ſo extremely attenuated, as to give no ſenſible reſiſtance to the planets in their motion round the ſun for many, perhaps hundreds or thouſands of ages to come. Even at the height of about 50 miles, it is ſo rare as to have no ſenſible effect on the rays of light; for it was found by Kepler, and De la Hire after him, who computed the height of the ſenſible atmoſphere from the duration of twilight, and from the magnitude of the terreſtrial ſhadow in lunar eclipſes, that the effect of the atmoſphere to reflect and intercept the light of the ſun, is only ſenſible to the altitude of between 40 and 50 miles: and at that altitude we may collect, from what has been already ſaid, that the air is above 10000 times rarer than at the ſurface of the earth. It is well known that the twilight begins and ends when the centre of the ſun is about 18 degrees below the horizon, or only 17° 27′, by ſubtracting 33′ for refraction, which raiſes the ſun ſo much higher than he would be. And a ray coming from the ſun in that poſition, and entering the earth's atmoſphere, is refracted and bent into a curve line in paſſing through it to the eye. M. de la Hire took great pains to demonſtrate, that, ſuppoſing the denſity of the atmoſphere proportional to its weight, this curve is a cycloid: he alſo ſays, that if the ray be a tangent to the atmoſphere, the diameter of its generating circle will be the height of the atmoſphere; and that this diameter increaſes, till at laſt, when the rays are perpendicular, it becomes infinite, or the circle degenerates into a right line. This reaſoning ſuppoſes that the refracting ſurface of the atmoſphere is a plane; but ſince it is in reality a curve, he obſerves that theſe cycloids become in fact epicycloids. But Herman detected the error of M. de la Hire, and ſhewed that this curve is infinitely extended, and has an aſymptote. And it is obſerved by Dr. Brook Taylor, in his Methodus Increm. pa. 168, &c, that this curve is one of the moſt intricate and perplexed that can well be propoſed. The ſame ingenious author computes, that the refractive power of the air is to the force of gravity at the ſurface of the earth, as 320 millions to 1.

Conſidering the extreme rarity of the atmoſphere at only 40 or 50 miles in height, it ſeems to be ſurprizing that ſome meteors ſhould be enflamed at ſuch great heights as they have been obſerved at. A very remarkable one of this kind was obſerved by Dr. Halley in the month of March 1719, the altitude of which he computed at between 69 and 73½ Engliſh miles; its diameter 2800 yards, or more than a mile and a half; and its velocity about 350 miles in a minute. Others, apparently of the ſame kind, but whoſe altitude and velocity were ſtill greater, have been obſerved; particularly that very remarkable one, of Auguſt 18th 1783, whoſe diſtance from the earth could not be leſs than 90 miles, its diameter at leaſt as large as the former, while its velocity was certainly not leſs than 1000 miles in a minute. Now, from analogy of reaſoning, it ſeems very probable, that the meteors which appear at ſuch great heights in the air, are not eſſentially different from thoſe which are ſeen on or near the ſurface of the earth. The difficulty with regard to the former is, that at the great heights above-mentioned, the atmoſphere ought not to have any denſity ſufficient to ſupport flame, or to propagate ſound; and yet ſuch meteors are commonly ſucceeded by one exploſion or more, and it is ſaid are even ſometimes accompanied with a hiſſing noiſe as they paſs over our heads. The meteor of 1719 was not only very bright, ſeeming for a ſhort time to turn night into day, but was attended with an exploſion heard over all the iſland of Britain, cauſing a violent concuſſion in the atmoſphere, and ſeeming to ſhake the earth itſelf: And yet, in the regions in which this meteor moved, the air ought to have been 300 thouſand times rarer than the air we breathe, or 1000 times rarer than the vacuum commonly made by a good air-pump. Dr. Halley offers a conjecture, indeed, that the vaſt magnitude of ſuch bodies might compenſate for the thinneſs of the medium in which they moved. But appearances of this kind are, by ſome others, attributed to electricity; though the circumſtances of them cannot be reconciled to that cauſe; for the meteors move with all different degrees of velocity; and though the electrical fire eaſily pervades the vacuum of an air-pump, yet it does not in that caſe appear in bright well-defined ſparks, as in the open air, but rather in long ſtreams reſembling the aurora borealis; and from ſome late experiments it has been concluded that the electric fluid cannot even penetrate a perfect vacuum.

Of the Refractive and Reflective Power of the ATMOSPHERE. It has been obſerved above, that the atmoſphere has a refractive power, by which the rays of light are bent from the right lined direction, as in the caſe of the twilight; and many other experiments manifeſt the ſame virtue, which is the cauſe of many phenomena. Alhazen, the Arabian, who lived about the year 1100, it ſeems was more inquiſitive into the nature of refraction than former writers. But neither Alhazen, nor his follower Vitello, knew any thing of its juſt quantity, which was not known, to any tolerable degree of exactneſs, till Tycho Brahe, with great diligence, ſettled it. But neither did Tycho nor Kepler diſcover in what manner the rays of light were refracted by the atmoſphere. Tycho thought the refraction was chiefly cauſed by denſe vapours, very near the earth's ſurface: while Kepler placed the cauſe wholly at the top of the atmoſphere, which he thought was uniformly denſe; and thence he determined its altitude to be little more than that of the higheſt mountains. But the true conſtitution of the denſity of the atmoſphere, deduced afterwards from the Torricellian experiment, afforded a juſter idea of theſe refractions, eſpecially after it appeared, by a repetition of Mr. Lowthorp's experiment, that the refractive power of the air is proportional to its denſity. By this variation in the denſity and refractive power of the air, a ray of light, in paſſing through the atmoſphere, is continually refracted at every point, and thereby made to deſcribe a curve, and not a ſtraight line, as it would have done were there no atmoſphere, or were its denſity uniform.

The atmoſphere, or air, has alſo a reflective power; and this power is the means by which objects are enlightened ſo uniformly on all ſides. The want of this power would occaſion a ſtrange alteration in the appearance of things; the ſhadows of which would be ſo very dark, and their ſides enlightened by the ſun ſo very bright, that probably we could ſee no more of them than their bright halves; ſo that for a view of the other halves, we muſt turn them half round, or if immoveable, muſt wait till the ſun could come round upon them. Such a pellucid unreflective atmoſphere would indeed have been very commodious for aſtronomical obſervations on the courſe of the ſun and planets among the fixed ſtars, viſible by day as well as by night; but then ſuch a ſudden tranſition from darkneſs to light, and from light to darkneſs, immediately upon the riſing and ſetting of the ſun, without any twilight, and even upon turning to or from the ſun at noon day, would have been very inconvenient and offenſive to our eyes. However, though the atmoſphere be greatly aſſiſtant in the illumination of objects, yet it muſt alſo be obſerved that it ſtops a great deal of light. By M. Bouguer's experiments, it ſeems that the light of the moon is often 2000 times weaker in the horizon, than at the altitude of 66 degrees; and that the proportion of her light at the altitudes of 66 and 19 degrees, is about 3 to 2; and the lights of the ſun muſt bear the ſame proportion to each other at thoſe heights; which Bouguer made choice of, as being the meridian heights of the ſun, at the ſummer and winter ſolſtices, in the latitude of Croiſic in France. Smith's optics. Rem. 95.

For the Atmoſphere of the ſun, moon, and planets, ſee the reſpective articles.

Atmosphere *of Solid or Conſiſtent Bodies*, is a kind of ſphere formed by the effluvia, or minute corpuſcles, emitted from them. Mr. Boyle endeavours to ſhew, that all bodies, even the hardeſt and moſt coherent, as gems, &c, have their atmoſpheres.

Atmosphere, *in Electricity*, denotes that medium which is conceived to be diffuſed over the ſurface of electrified bodies, and to ſome diſtance around them, and conſiſting of effluvia iſſuing from them; by which, other bodies immerged in it become endued with an electricity contrary to that of the body to which the atmoſphere belongs. This was firſt noticed at a very early period in the hiſtory of this ſcience by Otto Guericke, and afterwards by the academicians *del Cimento*, who contrived to render the electric atmoſphere viſible, by means of ſmoke attracted by a piece of amber, and gently riſing from it, but vaniſhing as the amber cooled. Dr. Franklin exhibited this electric atmoſphere with greater advantage, by dropping roſin on hot iron plates held under electrified bodies, from which the ſmoke aroſe and encompaſſed the bodies, giving them a very beautiful appearance. But the theory of electric atmoſpheres was not well explained and underſtood for a conſiderable time; and the inveſtigation led to many curious experiments and obſervations. The experiments of Mr. Canton and Dr. Franklin prepared the way for the concluſion that was afterwards drawn from them by Meſſ. Wilcke and Epinus, though they retained the common opinion of electric atmoſpheres, and endeavoured to explain the phenomena by it. The concluſion was, that the electric fluid, when there is a redundancy of it in any body, repels the electric fluid in any other body, when they are brought within the ſphere of each other's influence, and drives it into the remote parts of the body, or quite out of it, if there be any outlet for that purpoſe.

By Atmoſphere, M. Epinus ſays, no more is to be underſtood than the ſphere of action belonging to any body, or the neighbouring air electrified by it. Sig. Beccaria agrees in the ſame opinion, that electrified bodies have no other atmoſphere than the electricity communicated to the neighbouring air, and which goes with the air, and not with the electrified bodies. Mr. Canton alſo, having relinquiſhed the opinion that electrical atmoſpheres were compoſed of effluvia from excited or electrified bodies, maintained that they only reſult from an alteration in the ſtate of the electric fluid contained in it, or belonging to the air ſurrounding theſe bodies to a certain diſtance; for inſtance, that excited glaſs repels the electric fluid from it, and conſequently beyond that diſtance makes it more denſe; whereas excited wax attracts the electric fluid exiſting in the air nearer to it, making it rarer than it was before. In the courſe of experiments that were performed on this occaſion, Meſſ. Wilcke and Epinus ſucceeded in charging a plate of air, by ſuſpending large boards of wood covered with tin, with the flat ſides parallel to one another, and at ſome inches aſunder: for they found, upon electrifying one of the boards poſitively, that the other was always negative; and a ſhock was produced by forming a communication between the upper and lower plates. Beccaria has largely conſidered the ſubject of electric atmoſpheres, in his Artificial Electricity, pa. 179 &c, Eng. edit. See alſo Dr Prieſtley's Hiſt. of Electricity, vol. ii. ſect. 5. and Cavallo's Electricity, pa. 241.

Atmosphere, *Magnetic, &c*, is underſtood of the ſphere within which the virtue of the magnet, &c, acts.

ATOM, a particle of matter indiviſible on account of its ſolidity, hardneſs, and impenetrability; which preclude all diviſion, and leave no vacancy for the admiſſion of any foreign force to ſeparate or diſunite its parts. As atoms are the firſt matter, it is neceſſary they ſhould be indiſſolvable, that they may be incorruptible. Newton adds, it is alſo required that they be immutable, that the world may continue in the ſame ſtate, and bodies be of the ſame nature now as formerly.

ATOMICAL Philosophy, or the doctrine of atoms, a ſyſtem which accounted for the origin and formation of things, from the hypotheſis that atoms are endued with weight and motion. This philoſophy was firſt taught by Moſchus, ſome time before the Trojan war: but it was moſt cultivated by Epicurus; whence it is called the Epicurean philoſophy.

ATTRACTION, or Attactive Power, a general term uſed to denote the cauſe, power, or principle, by which all bodies mutually tend towards each other, and cohere, till ſeparated by ſome other power. The laws, phenomena, &c, of attraction, form the chief ſubject of Newton's philoſophy, being the principal agent of nature, in almoſt all her wonderful operations.

The principle of attraction, in the Newtonian ſenſe of it, it ſeems was firſt ſurmiſed by Copernicus. “As for gravity, ſays he, I conſider it as nothing more than a certain natural appetence *(appetentia)* that the Creator has impreſſed upon all the parts of matter, in order to their uniting or coaleſcing into a globular form, for their better preſervation; and it is probable that the ſame power is alſo inherent in the ſun and moon, and planets, that thoſe bodies may conſtantly retain that round form in which we ſee them.” *De Revol. Orb. Cæleſt.* lib. i, cap. 9. Kepler calls gravity a corporeal and mutual affection between ſimilar bodies, in order to their union. *Aſt. Nov. in Introd.* And he pronounced more poſitively that no bodies whatever were abſolutely light, but only relatively ſo; and conſequently that all matter was ſubjected to the power and law of gravitation. *Ibid.*

The firſt in this country who adopted the notion of attraction, was Dr. Gilbert, in his book *De Magnete*; and the next was the celebrated Lord Bacon, in his *Nov. Organ.* lib. ii, aphor. 36, 45, 48. *Sylv.* cent. i, exp. 33; alſo in his treatiſe *De Motu*, particularly under the articles of the 9th and the 13th ſorts of Motion. In France it was received by Fermat and Roberval; and in Italy by Galileo and Borelli. But till Newton appeared, this principle was very imperfectly defined and applied.

It muſt be obſerved, that though this great author makes uſe of the word attraction, in common with the ſchool philoſophers, yet he very ſtudiouſly diſtinguiſhes between the ideas. The ancient attraction was conceived to be a kind of quality inherent in certain bodies themſelves, and ariſing from their particular or ſpecific forms. But the Newtonian attraction is a more indefinite principle; denoting not any particular kind or mode of action; nor the phyſical cauſe of ſuch action; but only a general tendency, a *conatus accedendi*, to whatever cauſe, phyſical or metaphyſical, ſuch effect be owing; whether to a power inherent in the bodies themſelves, or to the impulſe of an external agent. Accordingly, that author remarks, in his *Philoſ. Nat. Prin. Math.* “ that he uſes the words *attraction*, *impulſe*, and *propenſion* to the centre, indifferently; and cautions the reader not to imagine that by attraction he expreſſes the modus of the action, or its efficient cauſe, as if there were any proper powers in the centres, which in reality are only mathematical points; or as if centres could attract.” Lib. 1, pa. 5. So, he “ conſiders centripetal powers as attractions, though, phyſically ſpeaking, it were perhaps more juſt to call them impulſes. Ib. pa. 147. He adds, “ that what he calls attraction may poſſibly be effected by impulſe, though not a common or corporeal impulſe, or after ſome other manner unknown to us. ” *Optic.* p. 322.

Attraction, if conſidered as a quality ariſing from the ſpecific forms of bodies, ought, together with ſympathy, antipathy, and the whole tribe of occult qualities, to be exploded. But when theſe are ſet aſide, there will remain innumerable phenomena of nature, and particularly the gravity or weight of bodies, or their tendency to a centre, that argue a principle of action ſeemingly diſtinct from impulſe; where, at leaſt, there is no ſenſible impulſion concerned. Nay, what is more, this action, in ſome reſpects, differs from all impulſion we know of; impulſe being always found to act in proportion to the ſurfaces of bodies; whereas gravity acts according to their ſolid content, and conſequently it muſt ariſe from ſome cauſe that penetrates or pervades the whole ſubſtance of it. This unknown principle, unknown we mean in reſpect of its cauſe, for its phenomena and effects are moſt obvious, with all its ſpecies and modifications, is called attraction; being a general name, under which may be ranged all mutual tendencies, where no phyſical impulſe appears, and which conſequently cannot be accounted for upon any known laws of nature.

And hence ariſe divers particular kinds of attraction; as *Gravity*, *Magnetiſm*, *Electricity*, *&c*, which are ſo many different principles, acting by different laws; and only agreeing in this, that we do not perceive any phyſical cauſes of them: but that, as to our ſenſes, they may really ariſe from ſome power or efficacy in ſuch bodies, by which they are enabled to act even upon diſtant bodies; though our reaſon abſolutely diſallows of any ſuch action.

Attraction may be divided, with reſpect to the law it obſerves, into two kinds.

1. That which extends to a ſenſible diſtance. As the attraction of gravity, which is found in all bodies; and the attractions of magnetiſm and electricity, found only in particular bodies. The ſeveral laws and phenomena of each, ſee under their reſpective articles.

The attraction of gravity, called alſo among mathematicians the *centripetal force*, is one of the greateſt and moſt univerſal principles of all nature. We ſee and feel it operate on bodies near the earth, and find by obſervation that the ſame power (i. e. a power which acts in the ſame manner, and by the ſame rules, viz, aways proportionally to the quantities of matter, and inverſely as the ſquares of the diſtances) does alſo obtain in the moon, and the other planets, both primary and ſecondary, as well as in the comets; and even that this is the very power by which they are all retained in their orbits, &c. And hence, as gravity is found in all the bodies which come under our obſervation, it is eaſily inferred, by one of the eſtabliſhed rules of philoſophizing, that it obtains in all others. And ſince it is found to be proportional to the quantity of matter in any body, it muſt exiſt in every particle of it: and hence it is proved that every particle in nature attracts every other particle.

From this attraction ariſes all the motion, and conſequently all the mutation, in the great world. By this heavy bodies deſcend, and light ones are made to aſcend: by this projectiles are directed, vapours and exhalations riſe, and rains &c fall: by this rivers glide, the ocean ſwells, the air preſſes, &c. In ſhort, the motions and forces ariſing from this principle, conſtitute the ſubject of that extenſive branch of mathematics, called *mechanics* or *ſtatics*, with the parts or appendages of it, as hydroſtatics, pneumatics, hydraulics, &c.

2. That which does not extend to ſenſible diſtances. Such is found to obtain in the minute particles of which bodies are compoſed, attracting each other at or extremely near the point of contact, with forces often much ſuperior to that of gravity, but which at any diſtance decreaſe much faſter than the power of gravity. This power a late ingenious author calls the *attraction of coheſion*, as being that by which the atoms or inſenſible particles of bodies are united into ſenſible maſſes.

This kind of attraction owns Sir Iſaac Newton for its diſcoverer; as the former does for its improver. The laws of motion, percuſſion, &c, in ſenſible bodies, under various circumſtances, as falling, projected, &c, aſcertained by the later philoſophers, do not reach thoſe more recluſe, inteſtine motions in the component particles of the ſame bodies, on which depend the changes in the texture, colour, properties, &c, of bodies. So that our philoſophy, if it were only founded on the principle of gravitation, and even carried as far as this would lead us, would ſtill be very deficient.

But beſides the common laws of ſenſible maſſes, the minute parts they are compoſed of are found ſubject to ſome others, which have but lately been noticed, and are even yet imperfectly known. Newton himſelf, to whoſe happy penetration we owe the hint, limits himſelf with eſtabliſhing that there are ſuch motions in the *minima naturæ*, and that they flow from certain powers or forces, not reducible to any of thoſe in the great world. He ſhews that, by virtue of theſe powers, "the ſmall particles act on one another even at a diſtance; and that many of the phenomena of nature reſult from it. Senſible bodies, we have already obſerved, act on one another divers ways; and as we thus perceive the tenor and courſe of nature, it appears highly probable that there may be other powers of the like kind; nature being very uniform and conſiſtent with herſelf. Thoſe juſt mentioned, reach to ſenſible diſtances, and ſo have been obſerved by vulgar eyes; but there may be others which reach to ſuch ſmall diſtances as have hitherto eſcaped obſervation; and it is probable electricity may reach to ſuch diſtances, even without being excited by friction."

The great author juſt mentioned proceeds to confirm the reality of theſe ſuſpicions from a great number of phenomena and experiments, which plainly argue ſuch powers and actions between the particles, for example of ſalts and water, oil of vitriol and water, aquafortis and iron, ſpirit of vitriol and ſaltpetre. He alſo ſhews, that theſe powers, &c, are unequally ſtrong between different bodies; ſtronger, for inſtance, between the particles of ſalt of tartar and thoſe of aquafortis than thoſe of ſilver, between aquafortis and lapis calaminaris than iron, between iron than copper, and copper than ſilver or mercury. So ſpirit of vitriol acts on water, but more on iron or copper, &c. And the other experiments are innumerable which countenance the exiſtence of ſuch principle of attraction in the particles of matter.

Theſe actions, by virtue of which the particles of the bodies above-mentioned tend towards each other, the author calls by a general indefinite name, *attraction*; a name equally applicable to all actions by which bodies tend towards one another, whether by impulſe, or by any other more latent power: and from hence he accounts for an infinity of phenomena, otherwiſe inexplicable, to which the principle of gravity is inadequate.

Thus, adds our author, "will nature be found very conformable to herſelf, and very ſimple; performing all the great motions of the heavenly bodies by the attraction of gravity, which intercedes thoſe bodies, and almoſt all the ſmall ones of their parts, by ſome other attractive power diffuſed through their particles. Without ſuch principles, there never would have been any motion in the world; and without the continuance of it, motion would ſoon periſh, there being otherwiſe a great decreaſe or diminution of it, which is only ſupplied by theſe active principles."

It need not be ſaid how unjuſt it is in the generality of foreign philoſophers to declare againſt a principle which furniſhes ſo beautiful a view, for no other reaſon but becauſe they cannot conceive how one body ſhould act on another at a diſtance. It is indeed true, that philoſophy allows of no action but what is by immediate contact and impulſion; for how can a body exert any active power where it does not exiſt? yet we ſee effects, without perceiving any ſuch impulſe; and where effects are obſerved, there muſt exiſt cauſes whether we ſee them or not. But we may contemplate ſuch effects, without entering into the conſideration of the cauſes, as indeed it ſeems the buſineſs of a philoſopher to do: for to exclude a number of phenomena which we do ſee, would be to leave a great chaſm in the hiſtory of nature; and to argue about actions which we do not ſee, would be to raiſe caſtles in the air. It follows therefore, that the phenomena of attraction are matter of phyſical conſideration, and as ſuch intitled to a ſhare in the ſyſtem of phyſics; but that their cauſes will only become ſo when they become ſenſible, that is when they appear to be the effect of ſome other higher cauſes; for a cauſe is no otherwiſe ſeen than as itſelf is an effect, ſo that the firſt cauſe muſt needs be always inviſible: we are therefore at liberty to ſuppoſe the cauſes of attractions what we pleaſe, without any injury to the effects. The illuſtrious author himſelf ſeems to be a little indetermined as to the cauſes; inclining ſometimes to attribute gravity to the action of an immaterial cauſe (*Optics*, pa. 343 &c), and ſometimes to that of a material one, *Ib.* pa. 325.

In his philoſophy, the reſearch into cauſes is the laſt thing, and never comes under conſideration till the laws and phenomena of the effect be ſettled; it being to theſe phenomena that the cauſe is to be accommodated. The cauſe even of any, the groſſeſt and moſt ſenſible action, is not adequately known. How impulſe or percuſſion itſelf produces its effects, that is how motion is communicated from body to body, confounds the deepeſt philoſophers; yet is impulſe received not only into philoſophy, but into mathematics: and accordingly the laws and phenomena of its effects make the chief part of common mechanics.

The other ſpecies of attraction, therefore, in which no impulſe is obſervable, when their phenomena are ſufficiently aſcertained, have the ſame title to be promoted from phyſical to mathematical conſideration; and this without any previous inquiry into their cauſes, to which our conceptions may not be proportionate.

Our great philoſopher, then, far from adulterating ſcience with any thing foreign or metaphyſical, as many have reproached him with doing, has the glory of having thrown every thing of this kind out of his ſyſtem, and of having opened a new ſource of ſublimer mechanics, which, duly cultivated, might be of far greater extent than all the mechanics yet known. Hence it is alone that we muſt expect to learn the manner of the changes, productions, generations, corruptions, &c, of natural things; with all that ſcene of wonders opened to us by the operations of chemiſtry.

Some of our own countrymen have proſecuted the diſcovery with laudable zeal. Dr. Keil particularly has endeavoured to deduce ſome of the laws of this new action, and applied them in reſolving ſeveral of the more general phenomena of matter, as coheſion, fluidity, elaſticity, ſoftneſs, fermentation, coagulation, &c: and Dr. Freind, ſeconding his endeavours, has made a farther application of the ſame principles, at once to account for almoſt all the phenomena that chemiſtry preſents. So that ſome philoſophers are inclined to think that the new mechanics ſhould ſeem already raiſed to a complete ſcience, and that nothing now can occur but what we have an immediate ſolution of, from the principles of attractive forces.

But this ſeems a little too precipitate: a principle ſo fertile ſhould have been further explained; its particular laws, limits, &c, more induſtriouſly detected and laid down, before we had proceeded to the application. Attraction in the groſs is ſo complex a thing, that it may ſolve a thouſand different phenomena alike. The notion is but one degree more ſimple and preciſe than action itſelf; and, till its properties are more fully aſcertained, it were better to apply it leſs, and ſtudy it more. It may be added, that ſome of Newton's followers have been charged with falling into that error which he induſtriouſly avoided, viz, of conſidering attraction as a cauſe or active property in bodies, not merely as a phenomenon or effect.

For the laws, properties, &c, of the different ſorts of Attraction, ſee their particular articles COHESION, GRAVITY, MAGNETISM, &c.

ATTRACTION, *Centre of*, See CENTRE *of Attraction*.

ATTRACTION *of Mountains*, See MOUNTAINS.

ATTRITION, the ſtriking or rubbing of bodies againſt one another, ſo as to throw off ſome of their ſuperficial particles: ſuch as the grinding and poliſhing of bodies. Or ſimply the act of rubbing: as when amber and other electric bodies are rubbed, to make them attract, or emit their electric force.

AVANT-FOSS, or Ditch of the Counterſcarp, in Fortification, is a wet ditch ſurrounding the counterſcarp, on the outer ſide, next to the country, at the foot of the glacis. It would not be proper to have ſuch a ditch if it could be laid dry, as it would then ſerve as a lodgment for the enemy.

AVERROES, or ABEN-ROES, a very ſubtile Arabian philoſopher, who flouriſhed about the end of the 11th century, when the Moors had poſſeſſion of part of Spain. He was the ſon of the high prieſt and chief judge of Corduba or Cordova in Spain: but he was educated in the univerſity of Morocco, where he was profeſſor, and where he died in 1206, having there ſtudied natural philoſophy, medicine, mathematics, law, and divinity. After the death of his father, he enjoyed his poſts in Spain, to which was afterward added that of judge of Morocco and Mauritania, where having ſettled deputies, he returned to his duty in Spain. Notwithſtanding he was very rich, and had a very great income, his liberality to men of letters in neceſſity, whether they were his friends or his enemies, kept him always in debt. He was afterwards ſtripped of all his poſts, and thrown into priſon, for hereſy, by the inſtigations of bad men, his enemies; but the oppreſſions of the judge who ſucceeded him, cauſed him to be reſtored to his former employments.

He was exceſſively fat, though he eat but once a day, and ſpent moſt part of the night in the ſtudy of philoſophy, when he was fatigued, amuſing himſelf with reading poetry or hiſtory. He was never ſeen to play at any game, or to partake in any diverſion. He was extremely fond of Ariſtotle's works, and wrote commentaries upon them; whence he was ſtyled *the Commentator*, by way of eminence. He wrote many other pieces; among them a work on the Whole Art of Phyſic; an Epitome of Ptolomy's Almageſt, which Voſſius dates about the year 1149; alſo a Treatiſe of Aſtrology, which was tranſlated into Hebrew by R. Jacob Ben Samſon, and ſaid to be extant in the French king's library. He wrote alſo ſeveral poems, and many amorous verſes, but theſe laſt he threw into the fire when he grew old. His other poems are loſt, except a ſmall piece, in which he ſays, "that when he was young, he acted againſt his reaſon; but that when he was in years, he followed its dictates;" upon which he utters this wiſh, "Would to God I had been born old, and that in my youth I had been in a ſtate of perfection!" As to religion, his opinions were, that Chriſtianity is abſurd; Judaiſm, the religion of children; Mahometaniſm, the religion of ſwine.

AVICENA, AVICENNE, or AVICENES, has been accounted the prince of Arabian philoſophers and phyſicians. He was born at Aſſena, near Bokhara, in 978; and died at Hamadan in 1036, being 58 years of age.

The firſt years of Avicena were employed on the ſtudy of the Belles Lettres, and the Koran, and at 10 years of age he was perfect maſter of the hidden ſenſes of that book. Then applying to the ſtudy of logic, philoſophy and mathematics, he quickly made a rapid progreſs. After ſtudying under a maſter the firſt principles of logic, and the firſt 5 or 6 propoſitions of Euclid's elements, he became diſguſted with the ſlow manner of the ſchools, applied himſelf alone, and ſoon accompliſhed all the reſt by the help of the commentators only.

Poſſeſſed with an extreme avidity to be acquainted with all the ſciences, he ſtudied medicine alſo. Perſuaded that this art conſiſts as much in practice as in theory, he ſought all opportunities of ſeeing the ſick; and afterwards confeſſed that he had learned more from ſuch experience than from all the books he had read. Being now in his 16th year, and already celebrated for being the light of his age, he determined to reſume his ſtudies of philoſophy, which medicine, &c, had made him for ſome time neglect: and he ſpent a year and a half in this painful labour, without ever ſleeping all this time a whole night together. At the age of 21, he conceived the bold deſign of incorporating, in one work, all the objects of human knowledge; and he carried it into execution in an Encyclopedia of 20 volumes, to which he gave the title of the *Utility of Utilities*.

Many wonderful ſtories are related of his ſkill in medicine, and the cures which he performed. Several princes had been taken dangerouſly ill, and Avicenes was the only one that could know their ailments, and cure them. His reputation increaſed daily, and all the princes of the eaſt deſired to retain him in their families, and in fact he paſſed through ſeveral of them. But the irregularities of his conduct ſometimes loſt him their favour, and threw him into great diſtreſſes. His exceſſes in pleaſures, and his infirmities, made a poet ſay, who wrote his epitaph, that the profound ſtudy of philoſophy had not taught him good morals, nor that of medicine the art of preſerving his own health.

After his death however, he enjoyed ſo great a reputation, that till the 12th century he was preferred for the ſtudy of philoſophy and medicine to all his predeceſſors. Even in Europe his works were the only writings in vogue in the ſchools. They were very numerous, and various, the titles of which are as follow:

1. Of the Utility and Advantage of the Sciences, in 20 books.—2. Of Innocence and Criminality, 2 books.—3. Of Health and Remedies, 18 books.—4. On the Means of preſerving Health, 3 books.—5. Canons of Phyſic, 14 books.—6. On Aſtronomical Obſervations, 1 book.—7. On Mathematical Sciences.—8. Of Theorems, or Mathematical and Theological Demonſtrations, 1 book.—9. On the Arabic Language, and its Properties, 10 books.—10. On the Laſt Judgment.—11. On the Origin of the Soul, and the Reſurrection of Bodies.—12. On the end we ſhould propoſe to ourſelves in Harangues and Philoſophical Argumentations.—18. Demonſtration of the Collateral Lines in the Sphere.—14. Abridgment of Euclid.—15. On Finity and Infinity.—16. On Phyſics and Metaphyſics.—17. On Animals and Vegetables, &c.—18. Encyclopedie, 20 volumes.

AUGUST, the 8th month of the year, containing 31 days. In the antient Roman calendar this was called *ſextilis*, as being the 6th month from March, with which their year began; but changed to its preſent name by the emperor Auguſtus, calling it after his own name on account of his having obtained many victories and honours in that month.

AVOIRDUPOIS *Weight*, a weight uſed in England for weighing all the larger and coarſer ſorts of goods; as groceries, cheeſe, butter, fleſh, wool, ſalt, hops, &c, and all metals except gold and ſilver. Avoirdupois weight is thus divided, viz.

16 dr. or drams make	1 ounce, marked	*oz.*
16 oz. - - - -	1 pound, - -	*lb.*
112 lb. - - - -	1 hundred weight,	*cwt.*
20 cwt. - -	1 ton - -	*ton.*

The Avoirdupois ounce is leſs than the Troy ounce, in the proportion of 700 to 768, but the Avoirdupois pound greater than the Troy pound in the proportion of 700 to 576;

for 1lb Avoird. is = 7000 grains Troy,
but 1lb Troy is = 5760 grains Troy,
alſo 1 oz Avoird. is = $437\frac{1}{2}$ grains Troy,
and 1 oz Troy is = 480 grains Troy.

AURIGA, the *Waggoner*, a conſtellation in the northern hemiſphere, conſiſting of 14 ſtars in Ptolemy's catalogue; but in Tycho's, 27; in Hevelius's, 40; and in the Britannic catalogue, 66.

This is one of the 48 old aſteriſms, mentioned by all the moſt ancient aſtronomers. It is repreſented by the figure of an old man, in a poſture ſomewhat like ſitting, with a goat and her kids in his left hand, and a bridle in his right.

The Greeks probably received this and all their other conſtellations from the Egyptians; but, wanting to appear the inventors of them themſelves, and not underſtanding the meaning of the figures, they have cloathed them with ſome of their own fabulous dreſſes, to favour the deceit. They accordingly tell us that this figure of a waggoner was an honourable character, and Erichthonius, the inventor of coaches. Vulcan, ſay they, once fell in love with Minerva, and when he could not prevail with her to marry him, he would have obtained her upon leſs honourable terms. There was a ſtruggle between them, and ſome way or other Erichthonius was begotten, though it does not ſeem that Minerva had much ſhare in it: ſhe took care of the offspring however. Some have ſuppoſed it was only a ſerpent; but the graver authors ſay, Erichthonius was a man with legs only like the body of a ſerpent, and that to hide this monſtrous part of his figure he invented coaches to carry him about. They add, that Jupiter, doing him honour for an invention that was, in ſome degree, imitating the ſun's carriage on the earth, raiſed him up among the ſtars.

But others, ill ſatisfied with a ſtory which ſo badly agreed with the figure, have ſaid that it belonged to Myrtillus, a ſon of Mercury and Clytie, and charioteer to Aenomanus; they ſay, that at his death, his father Mercury, by permiſſion of his ſuperiors, raiſed him up into the ſkies. All this however does not at all account for the goat and her two kids in the hands of Auriga. To ſet this right, they afterward made Auriga to be Olenus, a ſon of Vulcan, and the father of Aega and Helice, two of the Cretan nymphs that nurſed the infant Jupiter. They talk of a goat that was uſed for giving milk to the young deity, and they ſuppoſe that this creature, and its two young ones, were placed in the hands of the father of the virgins, to commemorate the creature they took into their ſervice on that occaſion.

Beſides the Hœdi, this conſtellation contains alſo another of thoſe ſtars which the ancients honoured with peculiar names, the goat Capra, and Amalthæa Capra: this is the bright one near the ſhoulder, and ſuppoſed to be the mother of the Hœdi, and the nurſe of Jupiter.

Although the whole conſtellation of Auriga is not mentioned among thoſe from which the ancients formed preſages of the ſucceeding weather, the two ſtars in his arm were of the foremoſt in that rank. It is theſe they called by the name Hœdi, and dreaded ſo extremely on account of the ſtorms and tempeſts that ſucceeded their riſing, that it is ſaid they ſhut up the navigation of the ſea for their ſeaſon. And the day of their influence being over, we find, was celebrated as a feſtival with ſports and games, under the name of Natalis Navigationis. Germanicus calls them unfriendly ſtars to mariners; and Virgil couples them with Arcturus, mentioning their ſetting and its riſing as things of the moſt important preſage. Horace alſo puts them together as the moſt formidable of all the ſtars to thoſe who follow the traffic of the ſea. And to the ſame purpoſe ſpeak all the ancient writers, thus making a part of the conſtellation Auriga, if not the whole conſtellation, a thing to be obſerved with the utmoſt attention, and to be feared as much as the blazing Arcturus.

AURORA, the morning twilight; or that faint light which appears in the morning when the ſun is within 18 degrees of the horizon.

AURORA BOREALIS, NORTHERN LIGHT, or *Streamers*; a kind of meteor appearing in the northern part of the heavens, moſtly in the winter ſeaſon, and in froſty weather. It is uſually of a reddiſh colour, inclining to yellow, and ſends out frequent coruſcations of pale light, which ſeem to riſe from the horizon in a pyramidal undulating form, and ſhooting with great velocity up to the zenith. It appears often in form of an arch, which is partly bright, and partly dark, but generally tranſparent. And the matter of it is not found to have any effect on the rays of light, which paſs freely through it. Dr. Hamilton obſerves, that he could plainly diſcern the ſmalleſt ſpeck in the Pleiades through the denſity of thoſe clouds which formed part of the Aurora borealis in 1763, without the leaſt diminution of its ſplendour, or increaſe of twinkling. Philoſ. Eſſays, pa. 106.

Sometimes it produces an Iris. Hence M. Godin judges, that moſt of the extraordinary meteors and phenomena in the ſkies, related as prodigies by hiſtorians, as battles, and the like, may probably enough be reduced to the claſs of Auroræ boreales. Hiſt. Acad. R. Scienc. for 1762, pa. 405.

This kind of meteor never appears near the equator; but, it ſeems, is frequent enough towards the ſouth pole, like as towards the north, having been obſerved there by voyagers. See Philoſ. Tranſ. N° 461, and vol. 54; alſo Forſter's account of his voyage round the world with Captain Cook, where he deſcribes their appearance as obſerved for ſeveral nights together, in ſharp froſty weather, which was much the ſame as thoſe obſerved in the north, excepting that they were of a lighter colour.

It ſeems that meteors of this kind have appeared ſometimes more frequently than others. They were ſo rare in England, or elſe ſo little regarded, that none are recorded in our annals ſince that remarkable one of Nov. 14, 1574, till the ſurpriſing Aurora borealis of March 6, 1716, which appeared for three nights ſucceſſively, but by far more ſtrongly on the firſt: except that five ſmall ones were obſerved in the year 1707 and 1708. Hence it would ſeem, that the air, or earth, or both, are not at all times diſpoſed to produce this phenomenon.

The extent of theſe appearances is alſo amazingly great. That in March 1716 was viſible from the weſt of Ireland, to the confines of Ruſſia and the eaſt of Poland; extending at leaſt near 30 degrees of longitude, and from about the 50th degree in latitude, over almoſt all the north of Europe; and in all places, at the ſame time, it exhibited the like wondrous appearances. Father Boſcovich has determined the height of an aurora borealis, which was obſerved by the Marquis of Polini the 16th of December, 1737, and found it was 825 miles high; and Mr. Bergman, from a mean of 30 computations, makes the average height of the aurora borealis amount to 70 Swediſh, or 469 Engliſh miles. But Euler ſuppoſes the height to be ſeveral thouſands of miles; and Mairan alſo aſſigns to them a very elevated region.

Many attempts have been made to determine the cauſe of this phenomenon. Dr. Halley imagines that the watery vapours, or effluvia, exceedingly rarefied by ſubterraneous fire, and tinged with ſulphureous ſtreams, which many naturaliſts have ſuppoſed to be the cauſe of earthquakes, may alſo be the cauſe of this appearance: or that it is produced by a kind of ſubtile matter, freely pervading the pores of the earth, and which, entering into it nearer the ſouthern pole, paſſes out again with ſome force into the æther, at the ſame diſtance from the northern. This ſubtile matter, by becoming more denſe, or having its velocity increaſed, may perhaps be capable of producing a ſmall degree of light, after the manner of effluvia from electric bodies, which, by a ſtrong and quick friction, emit light in the dark; to which ſort of light this ſeems to have a great affinity. Philoſ. Tranſ. N° 347. See alſo Mr. Cotes's deſcription of this phenomenon, and his method of explaining it, by ſtreams emitted from the heterogeneous and fermenting vapours of the atmoſphere, in Smith's Optics, pa. 69; or Philoſ. Tranſ. abr. vol. 6, part 2.

The celebrated M. de Mairan, in an expreſs treatiſe on the Aurora Borealis, publiſhed in 1731, ſuppoſes its cauſe to be the zodiacal light, which, according to him, is no other than the ſun's atmoſphere: this light happening, on ſome occaſions, to meet the upper parts of our atmoſphere about the limits where univerſal gravity begins to act more forcibly towards the earth than towards the ſun, falls into our air to a greater or leſs depth, as its ſpecific gravity is greater or leſs, compared with the air through which it paſſes. See Tract. Phyſ. et Hiſt. de l'Aurore Boreale. Suite des Memoires de l'Acad. R. des Scien. 1731. Alſo Philoſ. Tranſ. N° 433, or Abridg. vol. 8, pa. 540.

However M. Euler thinks the cauſe of the aurora borealis not owing to the zodiacal light, as M. de Mairan ſuppoſes; but to particles of our atmoſphere, driven beyond its limits by the impulſe of the ſolar light. And on this ſuppoſition he endeavours to account for the phenomena obſerved concerning this light. He ſuppoſes the zodiacal light, and the tails of comets, to be owing to a ſimilar cauſe.

But ever ſince the identity of lightning and the electric matter has been determined, philoſophers have been naturally led to ſeek for the explication of aerial meteors in the principles of electricity; and there is now no doubt but moſt of them, and eſpecially the aurora borealis, are electrical phenomena. Beſides the more obvious and known appearances which conſtitute a reſemblance between this meteor and the electric matter by which lightning is produced, it has been obſerved that the aurora occaſions a very ſenſible fluctuation in the magnetic needle; and that when it has extended lower than uſual in the atmoſphere, the flaſhes have been attended with various ſounds of rumbling and hiſſing, eſpecially in Ruſſia and the other more northern parts of Europe; as noticed by Sig. Beccaria and M. Meſſier. Mr. Canton, ſoon after he had obtained electricity from the clouds, offered a conjecture, that the aurora is occaſioned by the daſhing of electric fire poſitive towards negative clouds at a great diſtance, through the upper part of the atmoſphere, where the reſiſtance is leaſt: and he ſuppoſes that the aurora which happens at the time when the magnetic needle is diſturbed by the heat of the earth, is the electricity of the heated air above it: and this appears chiefly in the northern regions, as the alteration in the heat of the air in thoſe parts is the greateſt. Nor is this hypotheſis improbable, when it is conſidered, that electricity is the cauſe of thunder and lightning; that it has been extracted from the air at the time of the aurora borealis; that the inhabitants of the northern countries obſerve it remarkably ſtrong when a ſudden thaw ſucceeds very cold ſevere weather; and that the tourmalin is known to emit and abſorb the electric fluid only by the increaſe or diminution of its heat. Poſitive and negative electricity in the air, with a proper quantity of moiſture to ſerve as a conductor, will account for this and other meteors, ſometimes ſeen in a ſerene ſky. Mr. Canton has ſince contrived to exhibit this meteor by means of the Torricellian vacuum, in a glaſs tube about 3 feet long, and ſealed hermetically. When one end of the tube is held in the hand, and the other applied to the conductor, the whole tube will be illuminated from end to end, and will continue luminous without interruption for a conſiderable time after it has been removed from the conductor. If, after this, it be drawn through the hand either way, the light will be remarkably intenſe through the whole length of the tube. And though a great part of the electricity be diſcharged by this operation, it will ſtill flaſh at intervals, when held only at one extremity, and kept quite ſtill; but if, at the ſame time, it be graſped by the other hand in a different place, ſtrong flaſhes of light will dart from one end to the other; and theſe will continue 24 hours or more, without a freſh excitation. Sig. Beccaria conjectures that there is a conſtant and regular circulation of the electric fluid from north to ſouth; and he thinks that the aurora borealis may be this electric matter performing its circulation in ſuch a ſtate of the atmoſphere as renders it viſible, or approaching nearer than uſual to the earth. Though probably this is not the mode of its operation, as the meteor is obſerved in the ſouthern hemiſphere, with the ſame appearances as in the northern. Dr. Franklin ſuppoſes, that the electric fire diſcharged into the polar regions, from many leagues of vaporiſed air raiſed from the ocean between the tropics, accounts for the aurora borealis; and that it appears firſt, where it is firſt in motion, namely in the moſt northern part; and the appearance proceeds ſouthward, though the fire really moves northward. Franklin's Exper. and Obſ. 1769, pa. 49. Philoſ. Tranſ. vol. 48, pa. 358, 784; *Ib.* vol. 51, pa. 403; Lettere dell' Ellettriciſmo, pa. 269; or Prieſtley's Hiſt. of Electricity. See alſo an ingenious ſolution of this phenomenon, on the ſame principles, by Dr. Hamilton, in his Philoſ. Eſſays. Mr. Kirwan (in the Tranſactions of the Royal Iriſh Academy, ann. 1788) has ſome ingenious remarks on the *auroræ borealis & auſtralis*. He gives his reaſons for ſuppoſing the rarefaction of the atmoſphere in the polar regions to proceed from them, and theſe from a combuſtion of inflammable air cauſed by electricity. He obſerves, that after an aurora borealis the barometer commonly falls, and high winds from the ſouth generally follow.

AURUM FULMINANS, a preparation from gold, which being thrown into the fire, it explodes with a violent noiſe, like thunder. The matter is produced by diſſolving gold in aqua regia, and precipitating the ſolution by oil of tartar *per deliquium*, or volatile ſpirit of ſal ammoniac. The powder being waſhed in warm water, and dried to the conſiſtence of a paſte, is afterwards formed into ſmall grains of the ſize of hempſeed.

It is inflammable, not only by fire, but alſo by a gentle warmth; and gives a report much louder than that of gunpowder. A ſingle grain laid on the point of a knife, and lighted at a candle, explodes with a greater report than a muſquet: and a ſcruple of this powder, it is ſaid, acts more loudly than half a pound of gunpowder; and yet it is ſaid that, by mixture, it does not increaſe the elaſtic force of fired gunpowder. Dr. Black attributes the increaſe of weight, and alſo the exploſive property, of this powder, to adhering fixable air.—This is a very dangerous preparation, and ſhould be uſed with great caution.

AUSTRAL, the ſame with *ſouthern*. Thus, Auſtral ſigns, are the laſt 6 ſigns of the zodiac; and are ſo called becauſe they are on the ſouth ſide of the equinoctial.

AUSTRALIS *Corona*; ſee CORONA *Auſtralis*.

AUSTRALIS PISCIS, the *Southern Fiſh*, is a conſtellation of the ſouthern hemiſphere. See PISCIS *Auſtralis*.

AUTOMATON, a ſeemingly ſelf-moving machine; or one ſo conſtructed, by means of weights, levers, pullies, ſprings, &c, as to move for a conſiderable time, as if it were endued with animal life. And according to this deſcription, clocks, watches, and all machines of that kind, are automata.

It is ſaid, that Archytas of Tarentum, 400 years before Chriſt, made a wooden pigeon that could fly; that Archimedes alſo made ſuch-like automatons; that Regiomontanus made a wooden eagle that flew forth from the city, met the emperor, ſaluted him, and returned; alſo that he made an iron fly, which flew out of his hand at a feaſt, and returned again after flying about the room; that Dr. Hook made the model of a flying chariot, capable of ſupporting itſelf in the air. Many other ſurprizing automatons we have been eye-witneſſes of, in the preſent age: thus, we have ſeen figures that could write, and perform many other actions in imitation of animals: M. Vaucanſon made a figure that played on the flute; the ſame gentleman alſo made a duck, which was capable of eating, drinking, and imitating exactly the voice of a natural one; and, what is ſtill more ſurprizing, the food it ſwallowed was evacuated in a digeſted ſtate, or conſiderably altered on the principles of ſolution; alſo the wings, viſcera, and bones were formed ſo as ſtrongly to reſemble thoſe of a living duck; and the actions of eating and drinking ſhewed the ſtrongeſt reſemblance, even to the muddling the water with its bill. M. Le Droz of la Chaux de Fonds, in the province of Neufchatel, has alſo executed ſome very curious pieces of mechaniſm: one was a clock, preſented to the king of Spain; which had, among other curioſities, a ſheep that imitated the bleating of a natural one, and a dog watching a baſket of fruit, that barked and ſnarled when any one offered to take it away; beſides a variety of human figures, exhibiting motions truly ſurpriſing. But all theſe ſeem to be inferior to M. Kempell's cheſs-player, which may truly be conſidered as the greateſt maſter-piece in mechanics that ever appeared in the world. See alſo Baptiſta Porta's *Magia Nat.* c. 19, and Scaliger's *Subtil.* 326.

AUTUMN, the third ſeaſon, when the harveſt and fruits are gathered in. This begins at the deſcending equinox, which, in the northern hemiſphere, is when the ſun enters the ſign Libra, or about the 22d day of Auguſt; and it ends, when winter commences, about the ſame day in December.

Autumnal, ſomething belonging to autumn. Thus,

Autumnal *Equinox*, the time when the ſun enters the deſcending point of the ecliptic, where it croſſes the equinoctial; and is ſo called, becauſe the nights and days are then equal.

Autumnal *Point*, the point of the ecliptic anſwering to the autumnal equinox.

Autumnal *Signs*, are the ſigns Libra, Scorpio, Sagittary, through which the ſun paſſes during the autumn.

AXIOM, a ſelf-evident truth, or a propoſition aſſented to by every perſon at firſt ſight. Such as, that the whole is greater than its part; that a thing cannot both be and not be at the ſame time; and that from nothing, nothing can ariſe.

Some Axioms are in effect, ſtrictly ſpeaking, no other than identical propoſitions. Thus, to ſay that all right angles are equal to each other, is as much as to ſay, all right angles are right angles; ſuch equality being implied in the very definition, or the very name or term itſelf.

Axiom is alſo an eſtabliſhed principle in ſome art or ſcience. Thus, it is an axiom in phyſics, that nature does nothing in vain; that effects are proportional to their cauſes; &c. It is an axiom in geometry, that two things equal to the ſame third thing, are alſo equal to each other; that if to equal things equals be added, the ſums will be equal. And it is an axiom in optics, that the angle of incidence is equal to the angle of reflection. In this ſenſe alſo the general laws of motion are called axioms; as, that all motion is rectilinear, that action and reaction are equal, &c.

AXE or AXIS, in *Geometry*, the ſtraight line in a plane figure, about which it revolves, to produce or generate a ſolid. Thus, if a ſemicircle be moved round its diameter at reſt, it will generate a ſphere, whoſe axis is that diameter. And if a right-angled triangle be turned about its perpendicular at reſt, it will deſcribe a cone, whoſe axis is that perpendicular.

Axis is yet more generally uſed for a right line conceived to be drawn from the vertex of a figure to the middle of the baſe. So the

Axis *of a circle or ſphere*, is any line drawn through the centre, and terminated at the circumference, on both ſides.

Axis *of a cone*, is the line from the vertex to the centre of the baſe.

Axis *of a cylinder*, is the line from the centre of the one end to that of the other.

Axis *of a conic ſection*, is the line from the principal vertex, or vertices, perpendicular to the tangent at that point. The ellipſe and hyperbola have each two axes, which are finite and perpendicular to each other; but the parabola has only one, and that infinite in length.

Tranſverſe Axis, in the Ellipſe and Hyperbola, is the diameter paſſing through the two foci, and the two principal vertices of the figure. In the hyperbola it is the ſhorteſt diameter, but in the ellipſe it is the longeſt.

Conjugate Axis, or *Second Axis*, in the Ellipſe and Hyperbola, is the diameter paſſing through the centre, and perpendicular to the tranſverſe axis; and is the ſhorteſt of all the conjugate diameters.

Axis, of a curve line, is ſtill more generally uſed for that diameter which has its ordinates at right angles to it, when that is poſſible. For, like as in the conic ſections, any diameter biſects all its parallel ordinates, making the two parts of them on both ſides of it equal; and that diameter which has ſuch ordinates perpendicular to it, is an Axis: So, in curves of the ſecond order, if any two parallel lines each meeting the curve in three points; the right line which cuts theſe two parallels ſo, that the ſum of the two parts on one ſide of the cutting line, between it and the curve, is equal to the third part terminated by the curve on the other ſide, then the ſaid line will in like manner cut all other parallels to the former two lines, viz, ſo that, of every one of them, the ſum of the two parts, or ordinates, on one ſide, will be equal to the third part or ordinate on the other ſide. Such cutting line then is a diameter; and that diameter whoſe parallel ordinates are at right angles to it, when poſſible, is an Axis. And the ſame for other curves of ſtill higher orders. Newton, Enumeratio Linearum Tertii Ordinis, ſect. 2, art. 1.

Axis, *in Aſtronomy*. As, the Axis *of the world*, is an imaginary right line conceived to paſs through the centre of the earth, and terminating at each end in the ſurface of the mundane ſphere. About this line, as an axis, the ſphere, in the Ptolomaic ſyſtem, is ſuppoſed daily to revolve.

Axis *of the Earth*, is the line connecting its two poles, and about which the earth performs its diurnal rotation, from weſt to eaſt. This is a part of the axis of the world, and always remains parallel to itſelf during the motion of the earth in its orbit about the ſun, and perpendicular to the plane of the equator.

Axis *of a Planet*, is the line paſſing through its centre, and about which the planet revolves.—The Sun, Earth, Moon, Jupiter, Mars, and Venus, it is known from obſervation, move about their ſeveral axes; and the like motion is eaſily inferred of the other three, Mercury, Saturn, and Georgian planet.

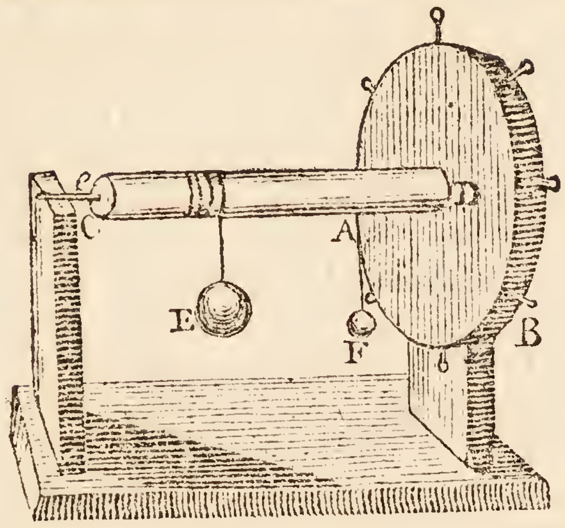
Axis of the *Horizon*, *Equator*, *Ecliptic*, *Zodiac*, &c, are right lines paſſing through the centres of thoſe circles, perpendicular to their planes.

Axis *of a Magnet*, or *magnetical* Axis, is a line paſſing through the middle of a magnet, lengthwiſe; in ſuch manner, that however the magnet be divided, provided the diviſion be made according to a plane paſſing through that line, the magnet will then be cut into two loadſtones. And the extremities of ſuch lines are called the poles of the ſtone.

Axis *in Mechanics*.—The axis of a balance, is the line upon which it moves or turns.

Axis *of Oſcillation*, is a line parallel to the horizon, paſſing through the centre about which a pendulum vibrates, and perpendicular to the plane in which it oſcillates.

Axis *in Peritrochio*, or *wheel and axle*, is one of the five mechanical powers, or ſimple machines; contrived chiefly for the raiſing of weights to a conſider-



able height, as water from a well, &c. This machine conſiſts of a circle AB, concentric with the baſe of a cylinder, and moveable together with it about its axis CD. This cylinder is called the *axis*; and the circle, the *peritrochium;* and the radii, or ſpokes, which are ſometimes fitted immediately into the cylinder, without any circle, the *ſcytalæ*. About the axis winds a rope, or chain, by means of which great weights are raiſed by turning the wheel.—The axis in peritrochio takes place in the motion of every machine, in which a circle may be conceived as deſcribed about a fixed axis, concentric with the plane of a cylinder about which it is placed; as in Crane wheels, Mill wheels, Capſtans, &c.

The chief properties of the Axe-in-peritrochio, are as follow:

1. If the power F applied in the direction AF a tangent to the circumference, or perpendicular to the ſpoke, be to a weight E, as the radius of the axis C*e* is to the radius of the wheel AD, or the length of the ſpoke; the power will juſt ſuſtain the weight; that is, the power and the weight will be in equilibrio, when they are in the reciprocal proportion of their diſtances from the centre.

2. When the wheel moves, with the power and weight; the velocities of their motion, and the ſpaces paſſed over by them, will be both in the ſame proportion as above, namely, directly proportional to their diſtances from the centre, and reciprocally proportional to their own weights when they are in equilibrio.

3. A power, and a weight, being given to conſtruct an axis-in-peritrochio, by which it ſhall be ſuſtained and raiſed. Let the axis be taken large enough to ſupport the weight and power without breaking: then, as the weight is to the power, ſo make the radius of the wheel to the radius of the axis. Hence, if the power be very ſmall in reſpect of the weight, the radius of the wheel will be vaſtly great. For example, ſuppoſe the weight 4050, and the power only 50; then the radius of the wheel will be 81 times that of the axis; which would be a very inconvenient ſize. But this inconvenience is provided againſt by increaſing the number of the wheels and axes; making one to turn another, by means of teeth or pinions. And to find the effect of a number of wheels and axes, thus turning one another, multiply together, all the radii of the axes, and all the radii of the wheels, and then it will be, as the product of the former is to the product of the latter, ſo is the power to the weight. So, if there be 4 wheels and axes, the radius of each axis being 1 foot, and the radius of each wheel 3 feet; then the continual product of all the wheels is $3 \times 3 \times 3 \times 3$ or 81 feet, and that of the axis only 1; therefore the effect is as 81 to 1, or the weight is 81 times the power. And, on the contrary, if it be required to find the diameter of each of four equal wheels, by which a weight of 4050lb ſhall be balanced by a power of 50lb, the diameter of each axis being 1 foot: dividing 4050 by 50, the quotient is 81; extract the 4th root of 81, or twice the ſquare root, and it will give 3, for the diameter of the four wheels ſought.

Axis *of a veſſel*, is that quieſcent right line paſſing through the middle of it, perpendicular to its baſe, and equally diſtant from its ſides.

Axis *in Optics.*—*Optic Axis*, or *viſual axis*, is a ray paſſing through the centre of the eye, or falling perpendicularly on the eye.

Axis *of a lens*, or *glaſs*, is the axis of the ſolid of which the lens is a ſegment. Or the axis of a glaſs, is the line joining the two vertices or middle points of the two oppoſite ſurfaces of the glaſs.

Axis *of Incidence*, in Dioptrics, is the line paſſing through the point of incidence, perpendicularly to the refracting ſurface.

Axis *of Refraction*, is the line continued from the point of incidence or refraction, perpendicularly to the refracting ſurface, along the further medium.

AZIMUTH, of the ſun, or ſtar, &c, is an arch of the horizon, intercepted between the meridian of the place, and the azimuth or vertical circle paſſing through the ſun or ſtar; and is equal to the angle at the zenith formed by the ſaid meridian and vertical circle. Or it is the complement to the eaſtern or weſtern amplitude.—The azimuth is thus found by trigonometry;

As radius is to the tangent of the latitude,
So is the tangent of the altitude of the ſun or ſtar,
To the coſine of the azimuth from the ſouth, at
the time of the equinox.

Azimuth, *magnetical*, an arch of the horizon contained between the magnetical meridian, and the azimuth or vertical circle of the object; or its apparent diſtance from the north or ſouth point of the compaſs. This is found by obſerving the ſun, or ſtar, &c, with an azimuth compaſs, when it is 10 or 15 degrees high, either before or after noon.

Azimuth Compass, an inſtrument for finding either the magnetical azimuth or amplitude of a celeſtial object. The deſcription and uſe of this inſtrument, ſee under the article Compass.

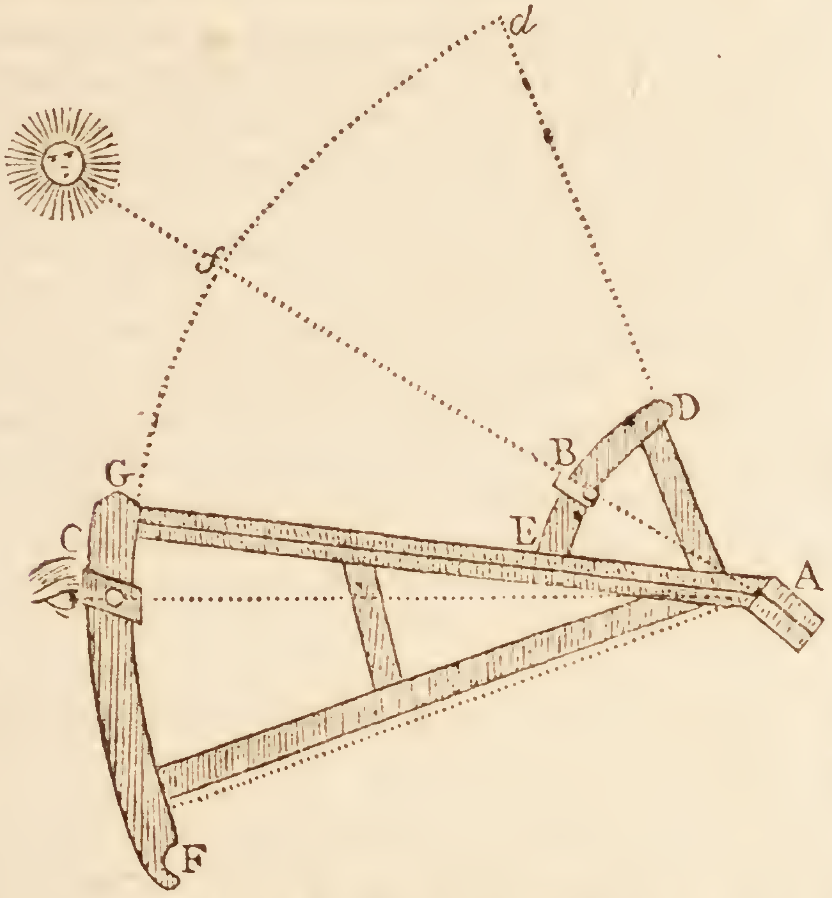
Azimuth Dial, a dial whoſe ſtile or gnomon is perpendicular to the plane of the horizon.

Azimuths, or *Vertical Circles*, are great circles of the ſphere interſecting each other in the zenith and nadir, and cutting the horizon at right angles.—Theſe azimuths are repreſented by the rhumbs on common ſea charts; and on the globe by the quadrant of altitude, when ſcrewed in the zenith. On theſe azimuths is counted the height of the ſun or ſtars, &c, when out of the meridian.

B.

BACK-STAFF, an inſtrument formerly uſed for taking the ſun's altitude at ſea; being ſo called becauſe the back of the obſerver is turned towards the ſun when he makes the obſervation. It was ſometimes called Davis's quadrant, from its inventor captain John Davis, a Welchman, and a celebrated navigator, who produced it about the year 1590.

This inſtrument conſiſts of two concentric arches of box-wood, and three vanes: the arch of the longer radius is of 30 degrees, and the other 60 degrees, making between them 90 degrees, or a quadrant: alſo the vane A at the centre is called the *horizon-vane*, that on the arch of 60° at B the *ſhade-vane*, and that on the other arch at C the *ſight-vane*.



To uſe the Back-Staff. The ſhade-vane is to be ſet upon the 60 arch, at an even degree of ſome latitude, leſs by 10 or 15 degrees than you judge the complement of the ſun's altitude will be; alſo the horizon-vane being put on at A, and the ſight-vane on the 30 arch F G, the obſerver turns his back to the ſun, lifts up the inſtrument, and looks through the ſight-vane, raiſing or falling the quadrant, till the ſhadow of the upper edge of the ſhade vane fall on the upper edge of the ſlit in the horizon-vane; and then if he can ſee the horizon though the ſaid ſlit, the obſervation is exact, and the vanes are right ſet: But if the ſea appear inſtead of the horizon, the ſight-vane muſt be moved downward towards F; or if the ſky appear, it muſt be moved upward towards G; thus trying till it comes right: the obſerver then examines how many degrees and minutes are cut by that edge of the ſight-vane that anſwers to the ſight hole, and to them he adds the degrees cut by the upper edge of the ſhade-vane; then the ſum is the ſun's diſtance from the zenith, or the complement of the altitude; that is, of his upper limb when the upper end of the ſhade-vane is uſed in the obſervation, or of his lower limb when the lower part of that vane is uſed; therefore in the former caſe add 16 minutes, the ſun's ſemidiameter, and ſubtract 16 minutes in the latter caſe, to give the zenith diſtance or co-altitude of the ſun's centre.

Mr. Flamſteed contrived a glaſs lens, or double convex, to be placed in the middle of the ſhade-vane, which throws a ſmall bright ſpot on the ſlit of the horizon-vane, inſtead of the ſhade; which is a great improvement, if the glaſs be truly made; for by this means, the inſtrument may be uſed in hazy weather, and a much more accurate obſervation made at all times.

BACON (ROGER), an Engliſh monk of the Franciſcan order, an amazing inſtance of genius and learning, was born near Ilcheſter in Somerſetſhire, in the year 1214. He commenced his ſtudies at Oxford; from whence he removed to the univerſity of Paris, which at that time was eſteemed the centre of literature; and where it ſeems he made ſuch progreſs in the ſciences, that he was eſteemed the glory of that univerſity, and was there greatly careſſed by ſeveral of his countrymen, particularly by Robert Groothead or Grouthead, afterwards biſhop of Lincoln, his great friend and patron. Having taken the degree of doctor, he returned to England in 1240, and took the habit of the Franciſcan order, being but about 26 years of age; but according to ſome he became a monk before he left France. He now purſued his favourite ſtudy of experimental philoſophy with unremitting ardour and aſſiduity. In this purſuit, in experiments, inſtruments, and in ſcarce books, he informs us he ſpent, in the courſe of 20 years, no leſs than 2000*l*, an amazing ſum in thoſe days, and which ſum it ſeems was generouſly furniſhed to him by ſome of the heads of the univerſity, to enable him the better to purſue his noble reſearches. By ſuch extraordinary talents, and amazing progreſs in the ſciences, which in that ignorant age were ſo little known to the reſt of mankind, while they raiſed the admiration of the more intelligent few, could not fail to excite the envy of his illiterate fraternity, whoſe malice he farther drew upon him by the freedom with which he treated the clergy in his writings, in which he ſpared neither their ignorance nor their want of morals: theſe therefore found no difficulty in poſſeſſing the vulgar with the notion of Bacon's dealing with the devil. Under this pretence he was reſtrained from reading lectures; his writings were confined to his convent; and at length, in 1278, he himſelf was impriſoned in his cell, at 64 years of age. However, being allowed the uſe of his books, he ſtill proceeded in the rational purſuit of knowledge, correcting his former labours, and writing ſeveral curious pieces.

When Bacon had been 10 years in confinement, Jerom de Aſcoli, general of his order, who had condemned his doctrine, was choſen pope by the name of Nicholas IV; and being reputed a perſon of great abilities, and one who had turned his thoughts to philoſophical ſtudies, Bacon reſolved to apply to him for his diſcharge; and to ſhew both the innocence and the uſefulneſs of his ſtudies, addreſſed to him a treatiſe *On the means of avoiding the infirmities of old age.* What effect this had on the pope does not appear; it did not at leaſt produce an immediate diſcharge: however, towards the latter end of his reign, by the interpoſition of ſome noblemen, Bacon obtained his liberty; after which he ſpent the remainder of his life in the college of his order, where he died in the year 1294, at 80 years of age, and was buried in the Franciſcan church. Such are the few particulars which the moſt diligent reſearches have been able to diſcover concerning the life of this very extraordinary man.

Bacon's printed works are, 1. *Epiſtola Fratris Rogeri Baconis de Secretis Operibus Artis et Naturæ, et de Nullitate Magiæ*: Paris, 1542, in 4to. Baſil, 1593, in 8vo. 2. *Opus Majus*: London, 1733, in fol. publiſhed by Dr. Jebb. 3. *Theſaurus Chemicus*: Francf. 1603 and 1620. Theſe printed works of Bacon contain a conſiderable number of eſſays, which have been conſidered as diſtinct books in the catalogue of his writings by Bale, Pitts, &c; but there remain alſo in different libraries ſeveral manuſcripts not yet publiſhed.

By an attentive peruſal of his works, the reader is aſtoniſhed to find that this great luminary of the 13th century was deeply ſkilled in all the arts and ſciences, and in many of them made the moſt important inventions and diſcoveries. He was, ſays Dr. Peter Shaw, beyond all compariſon the greateſt man of his time, and he might perhaps ſtand in competition with the greateſt that have appeared ſince. It is wonderful, conſidering the ignorant age in which he lived, how he came by ſuch a depth of knowledge on all ſubjects. His writings are compoſed with that elegance, conciſeneſs and ſtrength, and adorned with ſuch juſt and exquiſite obſervations on nature, that, among all the chemiſts, we do not know his equal. In his chemical writings, he attempts to ſhew how imperfect metals may be ripened into perfect ones; making, with Geber, mercury the common baſis of all metals, and ſulphur the cement.

His other phyſical writings ſhew no leſs genius and force of mind. In his treatiſe *Of the Secret Works of Art and Nature*, he ſhews that a perſon perfectly acquainted with the manner obſerved by nature in her operations, would be able to rival, and even to ſurpaſs her. In another piece, *Of the Nullity of Magic*, he ſhews with great ſagacity and penetration, whence the notion of it ſprung, and how weak all pretences to it are. From a peruſal of his works, adds the ſame author, we find Bacon was no ſtranger to many of the capital diſcoveries of the preſent and paſt ages. Gunpowder he certainly knew: thunder and lightning, he tells us, may be produced by art; for that ſulphur, nitre and charcoal, which when ſeparate have no ſenſible effect, yet when mixed together in due proportion, and cloſely confined, and fired, they yield a loud report. A more preciſe deſcription of gunpowder cannot be given in words. He alſo mentions a ſort of unextinguiſhable fire prepared by art: which ſhews he was not unacquainted with phoſphorus: and that he had a notion of the rarefaction of the air, and the ſtructure of an air-pump, is paſt contradiction. He was the miracle, ſays Dr. Freind, of the age he lived in, and the greateſt genius, perhaps, for mechanical knowledge, that ever appeared in the world ſince Archimedes. He appears likewiſe to have been maſter of the whole ſcience of optics: he has accurately deſcribed the uſes of reading-glaſſes, and ſhewn the way of making them. Dr. Freind adds, that he alſo deſcribes the camera obſcura, and all ſorts of glaſſes, which magnify or diminiſh any object, or bring it nearer to the eye, or remove it farther off. Bacon ſays himſelf, that he had great numbers of burning-glaſſes: and that there were none ever in uſe among the Latins, till his friend Peter de Mahara Curia applied himſelf to the making of them. That the teleſcope was not unknown to him, appears from a paſſage where he ſays, that he was able to form glaſſes in ſuch a manner, with reſpect to our ſight and the objects, that the rays ſhall be refracted and reflected wherever we pleaſe, ſo that we may ſee a thing under what angle we think proper, either near or at a diſtance, and be able to read the ſmalleſt letters at an incredible diſtance, and to count the duſt and ſand, on account of the greatneſs of the angle under which we ſee the objects; and alſo that we ſhall ſcarce ſee the greateſt bodies near us, on account of the ſmallneſs of the angle under which we view them. His ſkill in aſtronomy was amazing: he diſcovered that error which occaſioned the reformation of the calendar; one of the greateſt efforts, according to Dr. Jebb, of human induſtry: and his plan for correcting it was followed by pope Gregory the 13th, with this variation, that Bacon would have had the correction to begin from the birth of our Saviour, whereas Gregory's amendment reaches no higher than the Nicene council.

BACON (Francis), baron of Verulam, viſcount of St. Albans, and lord high chancellor of England under king James I. He was born in 1560, being ſon of Sir Nicholas Bacon lord keeper of the great ſeal in the reign of queen Elizabeth, by Anne daughter of Sir Anthony Cook, eminent for her ſkill in the Latin and Greek languages. He gave even in his infancy tokens of what he would one day become; and queen Elizabeth had many times occaſion to admire his wit and talents, and uſed to call him her young lord keeper. He ſtudied the philoſophy of Ariſtotle at Cambridge; where he made ſuch progreſs in his ſtudies, that at 16 years of age he had run through the whole circle of the liberal arts as they were then taught, and even began to perceive thoſe imperfections in the reigning philoſophy, which he afterwards ſo effectually expoſed, and thence not only overturned that tyranny which prevented the progreſs of true knowledge, but laid the foundation of that free and uſeful philoſophy which has ſince opened a way to ſo many glorious diſcoveries. On his leaving the univerſity, his father ſent him to France; where, before he was 19 years of age, he wrote a general view of the ſtate of Europe: but his father dying, he was obliged ſuddenly to return to England; where he applied himſelf to the ſtudy of the common law, at Gray's-inn. His merit at length raiſed him to the higheſt dignities in his profeſſion, attorney-general, and lord high chancellor. But being of an eaſy and liberal diſpoſition, his ſervants took advantage of that temper, and their ſituation under him, by accepting preſents in the line of his profeſſion. Being abandoned by the king, he was tried by the houſe of lords, for bribery and corruption, and by them ſentenced to pay a fine of 40,000*l.* and to remain priſoner in the Tower during the king's pleaſure. The king however ſoon after remitted the fine and puniſhment: but his misfortunes had given him a diſtaſte for public affairs, and he afterwards moſtly lived a retired life, cloſely purſuing his philoſophical ſtudies and amuſements, in which time he compoſed the greateſt part of his Engliſh and Latin works. Though even in the midſt of his honours and employments he forgot not his philoſophy, but in 1620 publiſhed his great work *Novum Organum*. After ſome years ſpent in his philoſophical retirement, he died in 1626, being 66 years of age.

The chancellor Bacon is one of thoſe extraordinary geniuſes who have contributed the moſt to the advancement of the ſciences. He clearly perceived the imperfection of the ſchool philoſophy, and he pointed out the only means of reforming it, by proceeding in the oppoſite way, from experiments to the diſcovery of the laws of nature. Addiſon has ſaid of him, That he had the ſound, diſtinct, comprehenſive knowledge of Ariſtotle, with all the beautiful light graces and embelliſhments of Cicero. Mr. Walpole calls him the *Prophet of Arts*, which Newton was afterwards to reveal; and adds, that his genius and his works will be univerſally admired as long as ſcience exiſts. He did not yet, ſaid another great man, underſtand nature, but he knew and pointed out all the ways that lead to her. He very early deſpiſed all that the univerſities called philoſophy; and he did every thing in his power that they ſhould not diſgrace her by their quiddities, their horrors of a vacuum, their ſubſtantial forms, and ſuch like impertinencies.

He compoſed two works for perfecting the ſciences. The former *On the Dignity and Augmentation of the Sciences*. He here ſhews the ſtate in which they then were, and points out what remains to be diſcovered for perfecting them; condemning the unnatural way of Ariſtotle, in reverſing the natural order of things. He here alſo propoſes his celebrated diviſion of the ſciences.

To remedy the faults of the common logic, Bacon compoſed his ſecond work, the *New Organ of Sciences*, above-mentioned. He here teaches a new logic, the chief end of which is to ſhew how to make a good inference, as that of Ariſtotle's is to make a ſyllogiſm. Bacon was 18 years in compoſing this work, and he always eſtemed it as the chief of his compoſitions.

The pains which Bacon beſtowed upon all the ſciences in general, prevented him from making any conſiderable applications to any one in particular: and as he knew that natural philoſophy is the foundation of all the other ſciences, he chiefly endeavoured to give perfection to it. He therefore propoſed to eſtabliſh a new ſyſtem of phyſics, rejecting the doubtful principles of the ancients. For this purpoſe he took the reſolution of compoſing every month a treatiſe on ſome branch of phyſics; he accordingly began with that of the winds; then he gave that of heat; next that of motion; and laſtly that of life and death. But as it was impoſſible that one man alone could ſo compoſe the whole circle of ſciences with the ſame preciſion, after having given theſe patterns, to ſerve as a model to thoſe who might chooſe to labour upon his principles, he contented himſelf with tracing in a few words the deſign of four other tracts, and with furniſhing the materials, in his Silva Silvarum, where he has amaſſed a vaſt number of experiments, to ſerve as a foundation for his new phyſics. In fact, no one before Bacon underſtood any thing of the experimental philoſophy; and of all the phyſical experiments which have been made ſince his time, there is ſcarcely one that is not pointed out in his works.

This great precurſor of philoſophy was alſo an elegant writer, an hiſtorian, and a wit. His moral eſſays are valuable, but are formed more to inſtruct than to pleaſe. There are excellent things too in his work *On the Wiſdom of the Ancients*, in which he has moralized the fables which formed the theology of the Greeks and Romans. He wrote alſo *The Hiſtory of Henry the VIIth king of England*, by which it appears that he was not leſs a great politician than a great philoſopher.

Bacon had alſo ſome other writings, publiſhed at different times; the whole of which were collected together, and publiſhed at Frankfort, in the year 1665, in a large folio volume, with an introduction concerning his life and writings. Another edition of his works was publiſhed at London in 1740; the enumeration of which is as below:

1. De Dignitate et Augmentis Scientiarum.
2. Novum Organum Scientiarum, ſive Judicia vera de Interpretatione Naturæ; cum Paraſceve ad Hiſtoriam Naturalem & Experimentalem.
3. Phænomena Univerſi, ſive Hiſtoria Naturalis & Experimentalis de Ventis; Hiſtoria Denſi & Rari; Hiſtoria Gravis & Levis; Hiſtoria Sympathiæ & Antipathiæ Rerum; Hiſtoria Sulphuris, Mercurii, & Salis; Hiſtoria Vitæ & Mortis; Hiſtoria Naturalis & Experimentalis de Forma Calidi; De Motus, ſive Virtutis activæ variis ſpeciebus; Ratio inveniendi cauſas Fluxus & Refluxus Maris; &c, &c.
4. Silva Silvarum, ſive Hiſtoria Naturalis.
5. Novus Atlas.
6. Hiſtoria Regni Henrici vii Angliæ Regis.
7. Sermones Fideles, Ethici, Politici, Oeconomici.
8. De Sapientia Veterum.

BACULE, in *Fortification*, a kind of portcullis, or gate, made like a pit-fall, with a counterpoiſe, and ſupported with two great ſtakes. It is uſually made before the *corps de garde*, not far from the gate of a place.

BACULOMETRY, the art of meaſuring either acceſſible or inacceſſible lines, by the help of *baculi*, ſtaves, or rods. Schwenter has explained this art in his *Geometria Practica*; and the rules of it are delivered by Wolfius, in his Elements: Ozanam alſo gives an illuſtration of the principles of Baculometry.

BAILLY (JEAN SYLVAIN), a celebrated French aſtronomer, hiſtoriographer, and politician, was born at Paris the 15th of September 1736, and has figured as one of the greateſt men of the age, being a member of ſeveral academies, and an excellent ſcholar and writer. He enjoyed for ſeveral years the office of keeper of the king's pictures at Paris. He publiſhed, in 1766, a volume in 4to, *An Eſſay on the Theory of Jupiter's Satellites*, preceded by a Hiſtory of the Aſtronomy of theſe Satellites. In the Journal Encyclopédique for May and July 1773, he addreſſed a letter to M. Bernoulli, Aſtromer Royal at Berlin, upon ſome diſcoveries relative to theſe ſatellites, which he had diſputed. In 1768 he publiſhed the Eulogy of Leibnitz, which obtained the prize at the Academy of Berlin, where it was printed. In 1770 he printed at Paris, in 8vo, the Eulogies of Charles the Vth, of de la Caille, of Leibnitz, and of Corneille. This laſt had the ſecond prize at the Academy of Rouen, and that of Moliere had the ſame honour at the French Academy.

M. Bailly was admitted into the Academy as Adjunct the 29th of January 1763, and as Aſſociate the 14th of July 1770.—In 1775 came out at Paris, in 4to, his *Hiſtory of the Ancient Aſtronomy*, in 1 volume: In 1779 the *Hiſtory of Modern Aſtronomy* in 2 volumes: and in 1787 the *Hiſtory of the Indian and Oriental Aſtronomy*, being the 2d vol. of the Ancient Aſtronomy.

M. Bailly's memoirs publiſhed in the volumes of the Academy, are as follow:

Memoir upon the Theory of the Comet of 1759.

Memoir upon the Epoques of the Moon's motions at the end of the laſt century.

Firſt, ſecond, and third Memoirs on the Theory of Jupiter's Satellites, 1763.

Memoir on the Comet of 1762: vol. for 1763.

Aſtronomical Obſervations, made at Noſlon: 1764.

On the Sun's Eclipſe of the 1ſt of April, 1764.

On the Longitude of Polling; 1764.

Obſervations made at the Louvre from 1760 to 1764: 1765.

On the cauſe of the Variation of the Inclination of the Orbit of Jupiter's ſecond Satellite; 1765.

On the Motion of the Nodes, and on the Variation of the Inclination of Jupiter's Satellites; 1766.

On the Theory of Jupiter's Satellites, publiſhed by M. Bailly, and according to the Tables of their Motions and of thoſe of Jupiter, publiſhed by M. Jeaurat; 1766.

Obſervations on the Oppoſition of the Sun and Jupiter; 1768.

On the Equation of Jupiter's Centre, and on ſome other Elements of the Theory of that Planet; 1768.

On the Tranſit of Venus over the Sun, the 3d of June 1769; and on the Solar Eclipſe the 4th of June the ſame year; 1769.

In the beginning of the revolution in France, in 1789, M. Bailly took an active part in that buſineſs, and was ſo popular and generally eſteemed, that he was choſen the firſt preſident of the States General, and of the National Aſſembly, and was afterwards for two years together the Mayor of Paris; in both which offices he conducted himſelf with great ſpirit, and gave general ſatisfaction.

He ſoon afterward however experienced a ſad reverſe of fortune; being accuſed by the ruling party of favouring the king, he was condemned for inciviſm and wanting to overturn the Republic, and died by the Guillotine at Paris on the eleventh day of November, 1793, at 57 years of age.

BAINBRIDGE (John), an eminent phyſician, aſtronomer, and mathematician. He was born in 1582, at Aſhby de la Zouch, Leiceſterſhire. He ſtudied at Cambridge, where having taken his degrees of Bachelor and Maſter of Arts, he returned to Leiceſterſhire, where for ſome years he kept a grammar-ſchool, and at the ſame time practiſed phyſic; employing his leiſure hours in ſtudying mathematics, eſpecially aſtronomy, which had been his favourite ſcience from his earlieſt years. By the advice of his friends, he removed to London, to better his condition, and improve himſelf with the converſation of learned men there; and here he was admitted a fellow of the college of phyſicians. His deſcription of the comet, which appeared in 1618, greatly raiſed his character, and procured him the acquaintance of Sir Henry Savile, who, in 1619, appointed him his firſt profeſſor of aſtronomy at Oxford. On his removal to this univerſity, he entered a maſter commoner of Merton college; the maſter and fellows of which appointed him junior reader of Linacer's lecture in 1631, and ſuperior reader in 1635. As he reſolved to publiſh correct editions of the ancient aſtronomers, agreeably to the ſtatutes of the founder of his profeſſorſhip, that he might acquaint himſelf with the diſcoveries of the Arabian aſtronomers, he began the ſtudy of the Arabic language when he was above 40 years of age. Before completing that work however he died, in the year 1643, at 61 years of age.

Dr. Bainbridge wrote many works, but moſt of them have never been publiſhed; thoſe that were publiſhed, were the three following, viz:

1. An Aſtronomical Deſcription of the late Comet, from the 18th of November 1618, to the 16th of December following; 4to, London, 1619.—This piece was only a ſpecimen of a larger work, which the author intended to publiſh in Latin, under the title of Cometographia.

2. Procli Sphæra, Ptolomæi de Hypotheſibus Planetarum liber ſingularis. To which he added Ptolomy's Canon Regnorum. He collated theſe pieces with ancient manuſcripts, and gave a Latin verſion of them, illuſtrated with figures: printed in 4to, 1620.

3. Canicularia. A treatiſe concerning the Dog-ſtar, and the Canicular Days: publiſhed at Oxford in 1648, by Mr. Greaves, together with a demonſtration of the heliacal riſing of Sirius, the dog-ſtar, for the parallel of Lower Egypt. Dr. Bainbridge undertook this work at the requeſt of archbiſhop Uſher, but he left it imperfect; being prevented by the breaking out of the civil war, or by death.

There were alſo ſeveral diſſertations of his prepared for and committed to the preſs the year after his death, but the edition of them was never completed. The titles of them are as follow:

1ſt, Antiprognoſticon, in quo Μαντικῆς Aſtrologicæ, Cœleſtium Domorum, et Triplicitatum Commentis, magniſque Saturni et Jovis (cujuſmodi anno 1623, et 1643, contigerunt, et viceſimo fere quoque deinceps anno, ratis naturæ legibus, recurrent) Conjunctionibus innixæ, Vanitas breviter detegitur.

2nd, De Meridionorum ſive Longitudinum Differentiis inveniendis Diſſertatio.

3d, De Stella Veneris Diatriba.

Beſide the foregoing, there were ſeveral other tracts, never printed, but left by his will to archbiſhop Uſher; among whoſe manuſcripts they are preſerved in the library of the college of Dublin. Among which are the following: 1. A Theory of the Sun. 2. A Theory of the Moon. 3. A Diſcourſe concerning the Quantity of the Year. 4. Two volumes of Aſtronomical Obſervations. 5. Nine or ten volumes of Miſcellaneous Papers relating to Mathematical ſubjects.

BAKER (Thomas), a mathematician of ſome eminence, was born at Ilton in Somerſetſhire, in 1625. He entered upon his ſtudies at Oxford in 1640, where he remained ſeven years. He was afterwards appointed vicar of Biſhop's-Nymmet in Devonſhire, where he lived a ſtudious and retired life for many years, chiefly purſuing the mathematical ſciences; of which he gave a proof of his critical knowledge, in the book he publiſhed, concerning the general conſtruction of biquadratic equations, by a parabola and a circle; the title of which book at full length is, "The Geometrical Key; or the Gate of Equations unlocked: or a new diſcovery of the Conſtruction of all Equations, howſoever affected, not exceeding the 4th degree, viz, of Linears, Quadratics, Cubics, Biquadratics, and the finding of all their Roots, as well Falſe as True, without the uſe of Meſolabe, Triſection of Angles, without Reduction, Depreſſion, or any other previous preparations of equations by a circle, and any (and that one only) Parabole, &c": 1684, 4to, in Engliſh and Latin.

There is ſome account of this work in the Philoſ. Tranſ. an. 1684. And a little before his death, the Royal Society ſent him ſome mathematical queries; to which he returned ſuch ſatisfactory anſwers, as procured the preſent of a medal, with an inſcription full of honour and reſpect.—Mr. Baker died at Biſhop's-Nymmet, 1690, in the 65th year of his age.

BAKER (Henry), an ingenious and diligent naturaliſt, was born in London about the beginning of the 18th century. He was brought up under an eminent bookſeller, but being of a philoſophical turn of mind, he quitted that line of buſineſs ſoon after the expiration of his apprenticeſhip, and took to the employment of teaching deaf and dumb perſons to ſpeak and write &c, in which occupation, in the courſe of his life he acquired a handſome fortune. For his amuſement he cultivated various natural and philoſophical ſciences, particularly botany, natural hiſtory, and microſcopical ſubjects, in which he eſpecially excelled, having, in the year 1744, obtained the Royal Society's gold medal, for his microſcopical experiments on the cryſtallizations and configurations of ſaline particles. He had various papers publiſhed in the Philoſ. Tranſ. of the Royal Society, of which he was a worthy member, as well as of the Societies of Antiquaries, and of Arts. He was author of many pieces, on various ſubjects, the principal of which were, his Treatiſe on the Water Polype, and two Treatiſes on the Microſcope, viz, *The Microſcope made eaſy*, and *Employment for the Microſcope*, which have gone through ſeveral editions.

Mr. Baker married Sophia, youngeſt daughter of the celebrated Daniel Defoe, by whom he had two ſons, who both died before him. He terminated an honourable and uſeful life, at his apartments in the Strand on the 25th of November 1774, being then upwards of 70 years of age.

BAKER's Central Rule, *for the Conſtruction of Equations*, is a method of conſtructing all equations, not exceeding the 4th degree, by means of a given parabola and a circle, without any previous reduction of them, or firſt taking away their ſecond term. See Central *Rule*.

BALANCE, one of the ſix ſimple powers in mechanics, chiefly uſed in determining the equality or difference of weight in heavy bodies, and conſequently their maſſes or quantities of matter.

The balance is of two kinds, the *ancient* and *modern*. The ancient or Roman, called alſo *Statera Romana*, or *Steelyard*, conſiſts of a lever or beam, moveable on a centre, and ſuſpended near one of its extremities. The bodies to be weighed are ſuſpended from the ſhorter end, and their weight is ſhewn by the diviſion marked on the beam, where the power or conſtant weight, which is moveable along the lever, keeps the ſteelyard in equilibrio. This balance is ſtill in common uſe for weighing heavy bodies.

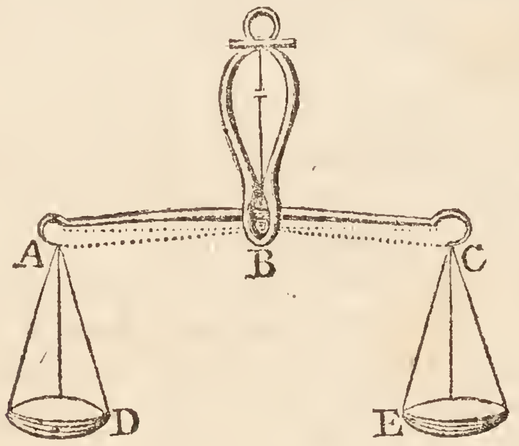
The modern balance, now commonly uſed, conſiſts of a lever or beam ſuſpended exactly in the middle, and having ſcales ſuſpended from the two extremities, to receive the weights to be weighed.

In either caſe the lever is called the *jugum* or the *beam*, and its two halves on each ſide the axis, the *brachia* or *arms*; alſo the line on which the beam turns, or which divides it in two, is called the *axis*; and when conſidered with regard to the length of the brachia, is eſteemed only a point, and called the *centre of the balance*, or *centre of motion*: the extremities where the weights are applied, are the *points of application* or *ſuſpenſion*; the handle by which the balance is held, or by which the whole apparatus is ſuſpended, is called *trutina*; and the ſlender part perpendicular to the beam, by which is determined either the equilibrium or preponderancy of bodies, is called the *tongue of the balance*.

From theſe deſcriptions we eaſily gather the characteriſtic diſtinction between the Roman balance and the common one, viz, that in the Roman balance, there is one conſtant weight uſed as a counterpoiſe, the point where it is ſuſpended being varied; but, on the contrary, in the common balance or ſcales, the points of ſuſpenſion remain the ſame, and the counterpoiſe is varied. The principle of both of them may be eaſily underſtood from the general properties of the lever, and the following obſervations.

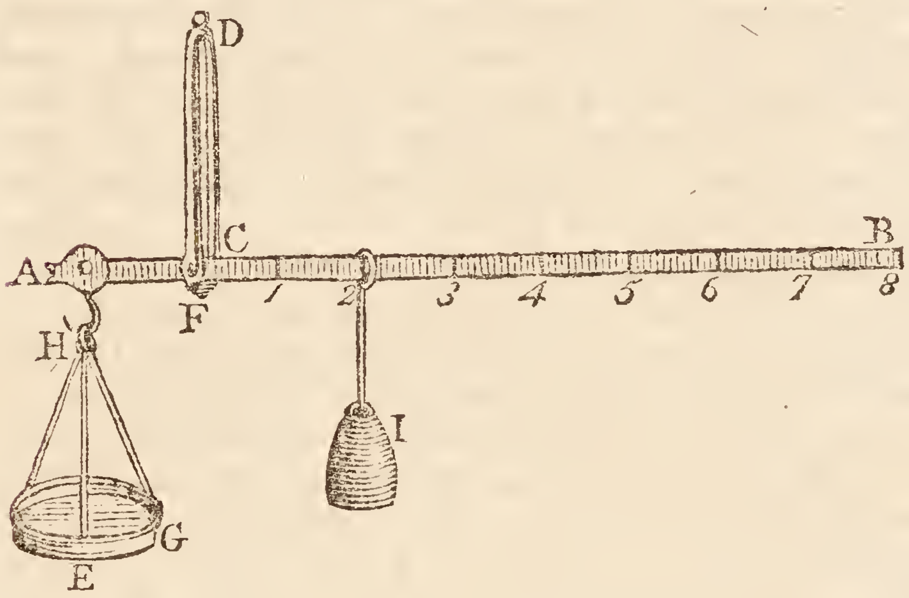
The beam ABC, the principal part of the balance, is a lever of the firſt kind; but inſtead of reſting on a fulcrum, it is ſuſpended by a handle, &c, faſtened to its centre of motion B: and hence the mechaniſm of the balance depends on the ſame theorems as that of the lever. Conſequently as the diſtance between the centre of motion and the place of the unknown weight, is to the diſtance between the ſame centre and the place of the known weight, ſo is the latter weight, to the former. So that the unknown weight is diſcovered by means of the known one, and their diſtances from the common centre of motion; viz, if the diſtances from the centre be equal, then the two weights will be equal alſo, as in the common balance; but if the diſtances be unequal, then the weights will alſo be unequal, and in the very ſame proportion, alternately, the leſs weight having ſo much the greater diſtance, as in the ſteelyard.

The Common Balance or Scales.



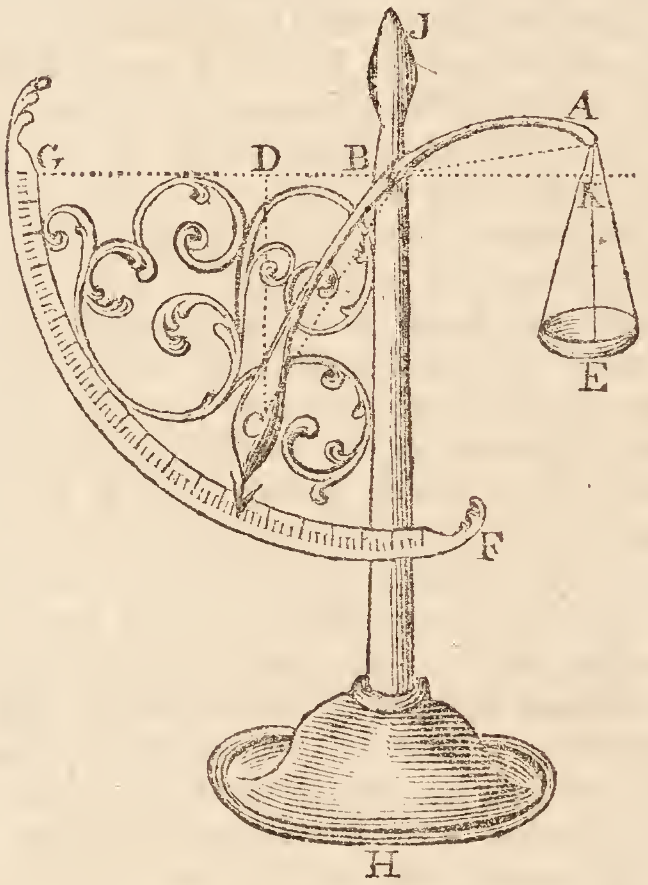
The two brachia AB, BC, ſhould be exactly equal in length, and in weight alſo when their ſcales D and E are fixed on their ends; the beam ſhould hang exactly level or horizontal in the caſe of an equipoiſe; and for this purpoſe the centre of gravity of the whole ſhould fall a little below the centre of motion, and but a little, that the balance be ſufficiently ſenſible to the leaſt variation of weight: the friction on the centre ſhould alſo be as ſmall as poſſible.

The Steel Yard.



Having made a proper bar of ſteel AB, tapering at the longer end, and very ſtrong at the other, ſuſpend it by a centre C near the ſhorter or thicker end, ſo that it may exactly balance itſelf in equilibrio, and prepare a conſtant weight I to weigh with: then hang on any weight, as one pound for inſtance, at the ſhorter arm, and ſlide the conſtant weight backwards and forwards upon the longer arm, till it be juſt in equilibrio with the former; and there make a notch and number 1, for the place of 1 pound: take off the 1lb, and hang a two pound weight in its ſtead at the ſhorter arm; then ſlide the conſtant weight back on the longer arm, till the whole come again into equilibrio, making a notch at the place of the conſtant weight and the number 2, for the place of 2lb. Proceed in the ſame manner for all other weights 3, 4, 5, &c; as alſo for the intermediate halves and quarters, &c, if it be neceſſary; always ſuſpending the variable weights at the end of the ſhorter arm, ſhifting the conſtant weight ſo as to balance them, and marking and numbering the places on the longer arm where the conſtant weight always makes a counterpoiſe. The uſe of the Steelyard is hence very evident: the thing whoſe weight is required being ſuſpended by a hook at the ſhort end, move the conſtant weight backwards and forwards on the longer arm, till the beam is balanced horizontally: then look what notch the conſtant weight is placed at, and its number will ſhew the weight of the body that was required. DC is the handle and tongue; F the centre of motion; EG a ſcale ſometimes hung on at the end by the hook H.

The Bent-Lever Balance.



This inſtrument operates by a fixed weight, C, increaſing in power as it aſcends along the arc FG of a circle, and pointing by an index to the number or diviſion of the arc which denotes the weight of any body put into the ſcale at E. And thus one conſtant weight ſerves to weigh all others, by only varying the poſition of the arms of the balance, inſtead of varying the places or points of ſuſpenſion in the arms themſelves.

The Deceitful Balance. This operates in the ſame manner as the ſteelyard, and cheats or deceives by having one arm a little longer than the other; though the deception is not perceived, becauſe the ſhorter arm is made ſomewhat heavier, ſo as to compenſate for its ſhortneſs, by which means the beam of the balance, when no weights are in the ſcales, hangs horizontal in equilibrio. The conſequence of this conſtruction is, that any commodity put in the ſcale of the longer arm, requires a greater weight in the other ſcale to balance it; and ſo the body is fallaciouſly accounted heavier than it really is. But the trick will eaſily be detected by making the body and the weight change places, removing them to the oppoſite ſcales, when the weight will immediately be ſeen to preponderate.

Aſſay-Balance. This is a very nice balance, uſed in determining the exact weights of very ſmall bodies. Its ſtructure is but little different from the common ſort;

except that it is made of the beſt and hardeſt ſteel, and made to turn with the ſmalleſt weight.

Hydroſtatical Balance. This is an inſtrument for determining the ſpecific gravity of bodies. See Hydrostatical, and Specific *Gravity*.

Balance, in Aſtronomy, the ſame as *Libra*.

Balance *of a Clock or Watch*, is that part which, by its motion, regulates and determines the beats. The circular part of it is called the *rim*, and its ſpindle the *verge;* there belong to it alſo two pallets or nuts, that play in the fangs of the crown-wheel: in pocket watches, that ſtrong ſtud in which the lower pivot of the verge plays, and in the middle of which one pivot of the crown-wheel runs, is called the *potence:* the wrought piece which covers the balance, and in which the upper pivot of the balance plays, is the *cock;* and the ſmall ſpring in the new pocket watches, is called the *regulator*.

BALCONY, a projecture in the front of a houſe, or other building, commonly ſupported by pillars or conſoles, and encompaſſed by a balluſtrade.

BALL, any ſpherical, globular, or round body.

Ball, in the military art, ſignifies all ſorts of bullets for fire arms, from the piſtol up to the largeſt cannon. Cannon balls are made of caſt iron; but the muſket and piſtol balls of lead, as theſe are both heavier under the ſame bulk, and do not furrow the barrels of the pieces.

Ball *of a Pendulum*, is the weight at the bottom of it; and is ſometimes, eſpecially in ſhorter pendulums, called the *bob*.

Balls *of Fire*, in Meteorology. See Fire *balls*.

Balls, in Electricity, invented by Mr. Canton, are two pieces of cork, or pith of elder tree, nicely turned in a lathe to the ſize of a ſmall pea, and ſuſpended by fine linen threads. They are uſed as electrometers, and are of excellent uſe to diſcover ſmall degrees of electricity; and to obſerve its changes from poſitive to negative, or the reverſe; as alſo to eſtimate the force of a ſhock before the diſcharge, ſo that the operator ſhall always be able to tell very nearly before hand what the exploſion will be, by knowing how high he has charged his jars.

BALLISTA, a military engine much uſed by the ancients for throwing ſtones, darts, and javelins; and ſomewhat reſembling our croſs-bows, but much larger and ſtronger. The word is Latin, ſignifying a croſs-bow; but is derived from the Greek βαλλω, to *ſhoot*, or *throw*.

Vegetius informs us, that the balliſta diſcharged darts with ſuch violence and rapidity, that nothing could reſiſt their force: and Athenæus adds, that Agiſtratus made one of little more than 2 feet in length, that ſhot darts 500 paces. Authors have often confounded the balliſta with the catapulta, attributing to the one what belongs to the other. According to Vitruvius, the balliſta was made after divers manners, though all were uſed to the ſame purpoſe: one ſort was framed with levers and bars; another with pullies; ſome with a crane; and others again with a toothed wheel. Marcellinus deſcribes the balliſta thus; A round iron cylinder is faſtened between two planks, from which reaches a hollow ſquare beam placed croſs-wiſe, faſtened with cords, to which are added ſcrews. At one end of this ſtands the engineer, who puts a wooden ſhaft, or arrow, with a large head, into the cavity of the beam; this done, two men bend the engine, by drawing ſome wheels; when the top of the head is drawn to the utmoſt end of the cords, the ſhaft is driven out of the balliſta, &c.

The balliſta is ranked by the ancients in the ſling kind, and its ſtructure and effect reduced to the principles of that inſtrument; whence it is called by Hero and others, *funda*, and *fundibulus*. Gunther calls it *Balearica machina*, as a ſling peculiar to the Balearic iſlands.—Perrault, in his notes on Vitruvius, gives a new contrivance of a like engine for throwing bombs without powder.

Fig. 1, Plate v, repreſents the balliſta uſed in ſieges, according to Folard: where 2, 2, denote the baſe of the balliſta; 3, 4, upright beams; 5, 6, tranſverſe beams; 7, 7, the two capitals in the upper tranſverſe beam, (the lower tranſverſe beam has alſo two ſimilar capitals, which cannot be ſeen in this tranſverſe figure); 9, 9, two poſts or ſupports for ſtrengthening the tranſverſe beams; 10, 10, two ſkains of cords faſtened to the capitals; 11, 11, two arms inſerted between the two ſtrands, or parts of the ſkains; 12, a cord faſtened to the two arms; 13, darts which are ſhot by the balliſta; 14, 14, curves in the upright beams, and in the concavity of which cuſhions are faſtened, in order to break the force of the arms, which ſtrike againſt them with great force when the dart is diſcharged; 16, the arbor of the machine, in which a ſtraight groove or canal is formed to receive the darts, in order to their being ſhot by the balliſta; 17, the nut of the trigger; 18, the roll or windlaſs, about which the cord is wound; 19, a hook, by which the cord is drawn towards the centre, and the balliſta cocked; 20, a ſtage or table on which the arbor is in part ſuſtained.

Ballista, in Practical Geometry, the ſame as the geometrical croſs, called alſo *Jacob's Staff*. See Cross-*Staff*.

BALLISTIC Pendulum, an ingenious machine invented by Benjamin Robins, for aſcertaining the velocity of military projectiles, and conſequently the force of fired gun-powder. It conſiſts of a large block of wood, annexed to the end of a ſtrong iron ſtem, having a croſs ſteel axis at the other end, placed horizontally, about which the whole vibrates together like the pendulum of a clock. The machine being at reſt, a piece of ordnance is pointed ſtraight towards the wooden block, or ball of this pendulum, and then diſcharged: the conſequence is this; the ball diſcharged from the gun ſtrikes and enters the block, and cauſes the pendulum to vibrate more or leſs according to the velocity of the projectile, or the force of the blow; and by obſerving the extent of the vibration, the force of that blow becomes known, or the greateſt velocity with which the block is moved out of its place, and conſequently the velocity of the projectile itſelf which ſtruck the blow and urged the pendulum.

The more minute and particular deſcription may be ſeen in my Tracts, vol. 1, where are given all the rules for uſing it, and for computing the velocities, with a multitude of accurate experiments performed with cannon balls, by means of which the moſt uſeful and important concluſions have been deduced in military projectiles and the nature of phyſics. I have alſo ſince that publication, made many other experiments of the ſame kind, by diſcharging cannon balls at various diſtances from the block; from which have reſulted the diſcovery of a complete ſeries of the reſiſtances of the air to balls paſſing through it with all degrees of velocity, from 0 up to 2000 feet in a ſecond of time.

Other writers on this ſubject are Euler, Antoni, Le Roy, Darcy, &c. See alſo Robins's Mathematical Tracts.

BALLISTICS, is uſed by ſome for projectiles, or the art of throwing heavy bodies. Merſennus has publiſhed a treatiſe on the projection of bodies, under this title.

BALLOON, or BALLON, in a general ſenſe, ſignifies any ſpherical hollow body. Thus, with chemiſts, it denotes a round ſhort-necked veſſel, uſed to receive what is diſtilled by means of fire: in architecture, a ball or globe on the top of a pillar, &c: and among engineers, a kind of bomb made of paſteboard, and played off in fireworks, in imitation of a real iron bomb-ſhell.

Air-BALLOON. See AEROSTATION and AIR-*Balloon*.

BALLUSTER, a ſmall kind of column or pillar, uſed for balluſtrades.

BALLUSTRADE, a ſeries or row of balluſters, joined by a rail; ſerving for a reſt to the arms, or as a fence or incloſure to balconies, altars, ſtaircaſes, &c.

BAND, in Architecture, denotes any flat low member, or moulding, that is broad, but not very deep. The word *lace* ſometimes means the ſame thing.

BANQUET, or BANQUETTE, in Fortification, a little foot-bank, or elevation of earth, forming a path along the inſide of a parapet, for the ſoldiers to ſtand upon to diſcover the counterſcarp, or to fire on the enemy, in the moat, or in the covert way. It is commonly about 3 feet wide, and a foot and a half high.

BARLOWE (WILLIAM), an eminent mathematician and divine, in the 16th century. He was born in Pembrokeſhire, his father (William Barlowe) being then biſhop of St. David's. In 1560 he was entered commoner of Baliol college in Oxford; and in 1564, having taken a degree in arts, he left the univerſity, and went to ſea; but in what capacity is uncertain: however he thence acquired conſiderable knowledge in the art of navigation, as his writings afterwards ſhewed. About the year 1573, he entered into orders, and became prebendary of Wincheſter, and rector of Eaſton, near that city. In 1588 he was made prebendary of Litchfield, which he exchanged for the office of treaſurer of that church. He afterwards was appointed chaplain to prince Henry, eldeſt ſon of king James the firſt; and, in 1614, archdeacon of Saliſbury. Barlowe was remarkable, eſpecially for having been the firſt writer on the nature and properties of the loadſtone, 20 years before Gilbert publiſhed his book on that ſubject. He was the firſt who made the inclinatory inſtrument tranſparent, and to be uſed with a glaſs on both ſides. It was he alſo who ſuſpended it in a compaſs-box, where, with 2 ounces weight, it was made fit for uſe at ſea. He alſo found out the difference between iron and ſteel, and their tempers for magnetical uſes. He likewiſe diſcovered the proper way of touching magnetical needles; and of piecing and cementing of loadſtones; and alſo why a loadſtone, being double-capped, muſt take up ſo great a weight.

Barlowe died in the year 1625.—His works are as follow:

1. *The Navigator's Supply, containing many things of principal importance belonging to Navigation, and uſe of diverſe Inſtruments framed chiefly for that purpoſe.* Lond. 1597, 4to; dedicated to Robert earl of Eſſex.

2. *Magnetical Advertiſement, or Diverſe Pertinent Obſervations and improved Experiments concerning the nature and properties of the Loadſtone.* Lond. 1616, 4to.

3. *A Brief Diſcovery of the idle Animadverſions of Mark Ridley, M. D. upon a treatiſe entitled Magnetical Advertiſements.* Lond. 1618, 4to.

In the firſt of theſe pieces, Barlowe gave a demonſtration of Wright's or Mercator's diviſion of the meridian line, as communicated by a friend; obſerving that "This manner of carde has been publiquely extant in print theſe thirtie yeares at leaſt [he ſhould have ſaid 28 only], but a cloude (as it were) and thicke miſte of ignorance doth keepe it hitherto concealed: And ſo much the more, becauſe ſome who were reckoned for men of good knowledge, have by glauncing ſpeeches (but never by any one reaſon of moment) gone about what they could to diſgrace it."

This work of Barlowe's contains deſcriptions of ſeveral inſtruments for the uſe of Navigation, the principal of which is an Azimuth Compaſs, with two upright ſights; and as the author was very curious in making experiments on the loadſtone, he treats well and fully upon the Sea-Compaſs. And he treated ſtill farther on the ſame inſtrument in his ſecond work, the Magnetical Advertiſement.

BAROMETER, an inſtrument for meaſuring the weight or preſſure of the atmoſphere; and by that means the variations in the ſtate of the air, foretelling the changes in the weather, and meaſuring heights or depths, &c.

This inſtrument is founded on what is called the Torricellian experiment, related below, and commonly conſiſts of a glaſs tube, open at one end; which being firſt filled with quickſilver, and then inverted with the open end downwards into a baſon of the ſame, the mercury deſcends in the tube till it remains at about the height of 29 or 30 inches, according to the weight or preſſure of the atmoſphere at the time, which is juſt equal to the weight of that column of the quickſilver. Hence it follows that, if by any means the preſſure of the air be altered, it will be indicated by the riſing or falling of the mercury in the tube; or if the barometer be carried to a higher ſtation, the quickſilver will deſcend lower in the tube, but when carried to a lower place, it will riſe higher in the tube, according to the difference in elevation between the two places.

Hiſtory of the Barometer.—About the beginning of the laſt century, when the doctrine of a plenum was in vogue, it was a common opinion among philoſophers, that the aſcent of water in pumps was owing to what they called nature's abhorrence of a vacuum; and that thus fluids might be raiſed by ſuction to any height whatever. But an accident having juſt diſcovered, that water could not be raiſed in a pump unleſs the ſucker reached to within 33 feet of the water in the well, it was conjectured by Galileo, who flouriſhed about that time, that there might be ſome other cauſe of the aſcent of water in pumps, or at leaſt that this abhorrence was limited to the finite height of 33 feet. Being unable to ſatisfy himſelf on this head, he recommended the conſideration of the difficulty to Torricelli, who had been his diſciple. After ſome time Torricelli fell upon the ſuſpicion that the preſſure of the atmoſphere was the cauſe of the aſcent of water in pumps; that a column of water 33 feet high was a juſt counterpoiſe to a column of air, of the ſame baſe, and which extended up to the top of the atmoſphere; and that this was the true reaſon why the water did not follow the ſucker any farther. And this ſuſpicion was ſoon after confirmed by various experiments. Torricelli conſidered, that if a column of water 33 feet high were a counterpoiſe to a whole column of the atmoſphere, then a column of mercury of about 2 feet and a half high would alſo be a counterpoiſe to it, ſince quickſilver is near 14 times heavier than water, and ſo the 14th part of the height, or near 2 feet and a half, would be as heavy as the column of water. This reaſoning was ſoon verified; for having filled a glaſs tube with quickſilver, and inverted it into a baſon of the ſame, the mercury preſently deſcended till its height, above that in the baſon, was about two feet and a half, juſt as he expected. And this is what has, from him, been called the Torricellian experiment.

The new opinion, with this confirmation of it, was readily acquieſced in by moſt of the philoſophers, who repeated the experiment in various ways. Others however ſtill adhered to the old doctrine, and raiſed ſeveral pretended objections againſt the new one; ſuch as that there was a film or imperceptible *rope of mercury*, extended through the upper part of the tube, which ſuſpended the column of mercury, and kept it from falling into that in the baſon. This and other objections were however ſoon overcome by additional confirmations of the true doctrine, particularly by varying the elevation of the place. It was hinted by Deſcartes and Paſcal, that if the mercury be ſuſtained in the tube by the preſſure of the atmoſphere, by carrying it to a higher ſituation, it would deſcend lower in the tube, having a ſhorter column of the atmoſphere to ſuſtain it, and vice verſa. And Paſcal engaged his brother-in-law, M. Perier, to try that experiment for him, being more conveniently ſituated for that purpoſe than he was at Paris. This he accordingly executed, by obſerving the height of the quickſilver in the tube, firſt at the bottom of a mountain in Auvergne, and then at ſeveral ſtations, or different altitudes, in aſcending, by which it was found that the mercury fell lower and lower all the way to the top of the mountain; and ſo confirming the truth of the doctrine relating to the univerſal preſſure of the atmoſphere, and the conſequent ſuſpenſion of the mercury in the tube of the barometer. Thus, by the united endeavours of Torricelli, Deſcartes, Paſcal, Merſenne, Huygens, and others, the cauſe of the ſuſpenſion of the quickſilver in the tube of the barometer, became pretty generally eſtabliſhed.

It was ſome time however after this general conſent, before it was known that the preſſure of the air was various at different times, in the ſame place. This could not however remain long unknown, as the frequent meaſuring of the column of mercury, muſt ſoon ſhew its variations in altitude; and experience and obſervation would preſently ſhew that thoſe variations in the mercurial column, were always ſucceeded by certain changes in the weather, as to rain, wind, froſts, &c. Hence this inſtrument ſoon came into uſe as the means of foretelling the changes of the weather; and on this account it obtained the name of the *weather-glaſs*, as it did that of *barometer* from its being the meaſure of the weight or preſſure of the air. We may now proceed to take a view of its various forms and uſes.

The Common Barometer. This is repreſented at fig. 1, plate iv, ſuch as it was invented by Torricelli. AB is a glaſs tube, of $\frac{1}{4}$, or $\frac{1}{3}$, or $\frac{1}{2}$ inch wide, the more the better, and about 34 inches long, being cloſe at the top A, and the open end B immerſed in a baſon of quickſilver CD, which is the better the wider it is. To fill this, or any other barometer; take a clean new glaſs tube, of the dimenſions as above, and pour into it well purified quickſilver, with a ſmall funnel either of glaſs or paper, in a fine continued ſtream, till it wants about half an inch or an inch of being full; then ſtopping it cloſe with the finger, invert it ſlowly, and the air in the empty part will aſcend gradually to the other end, collecting into itſelf ſuch other ſmall air bubbles as unavoidably get into the tube among the mercury, in filling it with the funnel: and thus continue to invert it ſeveral times, turning the two ends alternately upwards, till all the air bubbles are collected, and brought up to the open end of the tube, and when the part filled ſhall appear, without ſpeck, like a fine poliſhed ſteel rod. This done, pour in a little more quickſilver, to fill the empty part quite full, and ſo exclude all air from the tube: then, ſtopping the orifice again with the finger, invert the tube, and immerſe the finger and end, thus ſtopped, into a baſon of like purified quickſilver; in this poſition withdraw the finger, ſo ſhall the mercury deſcend in the tube to ſome place as E, between 28 and 31 inches above that in the baſon at F, as theſe are the limits between which it always ſtands in this country on the common ſurface of the earth. Then meaſure, from the ſurface of the quickſilver in the baſon at F, 28 inches to G, and 31 inches to H, dividing the ſpace between them into inches and tenths, which are marked on a ſcale placed againſt the ſide of the tube; and the tenths are ſubdivided into hundredth parts of an inch by a ſliding index carrying a vernier or nonius. Theſe 3 inches, between 28 and 31, ſo divided, will anſwer for all the ordinary purpoſes of a ſtationary or chamber barometer; but for experiments on altitudes and depths, it is proper to have the diviſions carried on a little higher up, and a great deal lower down. In the proper filling and otherwiſe fitting up of the barometer, ſeveral circumſtances are to be carefully noted; as, that the bore of the tube be pretty wide, to allow the freer motion of the quickſilver, without being impeded by an adheſion to the ſides; that the baſon below it be alſo pretty large, in order that the ſurface of the mercury at F may not ſenſibly riſe or fall with that in the tube; that the bottom of the tube be cut off rather obliquely, that when it reſts on the bottom of the baſon there may be a free paſſage for the quickſilver; and that, to have the quickſilver very pure, it is beſt to boil it in the tube, which will expel all the air from it. This barometer is commonly fitted up in a neat mahogany caſe, together with a thermometer and hygrometer, as repreſented in plate 4, fig. 13.

As the ſcale of variation is but ſmall, being included within 3 inches in the common barometer, ſeveral contrivances have been deviſed to enlarge the ſcale, or to render the motion of the quickſilver more ſenſible.

Deſcartes firſt ſuggeſted a method of increaſing the ſenſibility, which was executed by Huygens. This was effected by making the barometrical tube end in a pretty large cylindrical veſſel at top, into which was inſerted alſo the lower or open end of a much finer tube than the former, which was partly filled with water, to give little obſtruction by its weight to the motion of the mercury, while it moved through a pretty long ſpace of the very fine tube by a ſmall variation of the mercury below it, and ſo rendered the ſmall changes in the ſtate of the air very ſenſible. But the inconvenience was this, that the air contained in the water gradually diſengaged itſelf, and eſcaped through into the vacuum in the top of the ſmall tube, till it was collected in a body there, and by its elaſticity preventing the free riſe of the fluids in the tubes, ſpoiled the inſtrument as a barometer. And this, it may be obſerved by-the-bye, is the reaſon why a water barometer cannot ſucceed. This barometer is here repreſented in fig. 2, where CD is the veſſel, in which are united the upper or ſmall water tube AC, with the lower or mercurial one CB.

To remedy this inconvenience, Huygens thought of placing the mercury at top, and the water at bottom, which he thus contrived. ADG (fig. 3) is a bent tube hermetically ſealed at A, but open at G, of about one line in diameter, and paſſing through the two equal cylindrical veſſels BC, EF, which are about 20 inches apart, and of 15 lines diameter, their length being 10. The mercury being put into the tube, will ſtand between the middle of the veſſels EF and BC, the remaining ſpace to A being void both of air and mercury. Laſtly, common water, tinged with a 6th part of aqua regis, to prevent its freezing, is poured into the tube FG, till it riſes a foot above the mercury in DF. To prevent the water from evaporating, a drop of oil of ſweet almonds floats on the top of it. But the column of water will be ſenſibly affected by heat and cold, which ſpoils the accuracy of the inſtrument. For which reaſon other contrivances have been made, as below.

The Horizontal or Rectangular Barometer, fig. 4, was invented by J. Bernoulli and Caſſini; where AB is a pretty wide cylindrical part at the top of the tube, which tube is bent at right angles at C, the lower part of it CD being turned into the horizontal direction, and cloſe above at A, but open at the lower end D, where however the mercury cannot run out, being there oppoſed by the preſſure of the atmoſphere. This and the foregoing contrivance of Huygens are founded on the theorem in hydroſtatics, that fluids of the ſame baſe preſs according to their perpendicular altitude, not according to the quantity of their matter; ſo that the ſame preſſure of the atmoſphere ſuſtains the quickſilver that fills the tube ACD, and the ciſtern B, as would ſupport the mercury in the tube alone. Hence, having fixed upon the ſize of the ſcale, as ſuppoſe the extent of 12 inches, inſtead of the 3, in the common barometer from 28 to 31, that is 4 times as long; then the area of a ſection of the cylinder AB muſt be 4 times that of the tube, and conſequently its diameter double, ſince the areas of circles are as the ſquares of their diameters: then for every natural variation of an inch in the cylinder AB, there will be a variation of four inches in the tube CD.—But on account of the attrition of the mercury againſt the ſides of the glaſs, and the great momentum from the quick motion in CD, the quickſilver is apt to break, and the riſe and fall is no longer equable; and beſides, the mercury is apt to be thrown out of the orifice at D by ſudden motions of the machine.

The Diagonal Barometer of Sir Samuel Moreland, fig. 5, is another method of enlarging the natural ſcale of three inches perpendicular, or CD, by extending it to any length BC in an oblique direction. This is liable in ſome degree to the ſame inconvenience, from friction and breaking, as the horizontal one; and hence it is found that the diagonal part BC cannot properly be bent from the perpendicular more than in an angle of 45°, which only increaſes the ſcale nearly in the proportion of 7 to 5.

Doctor Hook's Wheel Barometer, fig. 6. This was invented about 1668, and is meant to render the alterations in the air more ſenſible. Here the barometer tube has a large ball AB at top, and is bent up at the lower or open end, where an iron ball G floats on the top of the mercury in the tube, to which is connected another ball H by a cord, hanging freely over a pulley, turning an index KL about its centre. When the mercury riſes in the part FG, it raiſes the ball, and the other ball deſcends and turns the pulley with the index round a graduated circle from N towards M and P; and the contrary way when the quickſilver and the ball ſink in the bent part of the tube. Hence the ſcale is eaſily enlarged 10 or 12 fold, being increaſed in proportion of the axis of the pulley to the length of the index KL. But then the friction of the pulley and axis is ſome obſtruction to the free motion of the quickſilver. Contrivances to leſſen the friction &c, may alſo be ſeen in the *Philoſ. Tranſ.* vol. 52, art. 29, and vol. 60, art. 10.

The Steelyard Barometer, for ſo that may be called which is repreſented by fig. 7, which enlarges the ſcale in the proportion of the ſhorter to the longer arm of a ſteelyard. AB is the barometer tube, cloſe at A and open at B, immerſed in a cylindrical glaſs ciſtern CD, which is but very little wider than the tube AB is. The barometer tube is ſuſpended to the ſhorter arm of an index like a ſteelyard, moving on the fulcrum E, and the extremity of its longer arm pointing to the diviſions of a graduated arch, with which index the tube is nearly in equilibrio. When the preſſure of the atmoſphere is leſſened, the mercury deſcends out of the tube into the ciſtern which raiſes the tube and the ſhorter arm of the index, and conſequently the extremity of the longer moves downwards, and paſſes over a part of the graduated arch. And on the contrary this moves upwards when the preſſure of the atmoſphere increaſes.

The Pendant Barometer, fig. 8, was invented by M. Amontons, in 1695. It conſiſts of a ſingle conical tube AB, hung up by a thread, the larger or open end downwards, and having no veſſel or ciſtern, becauſe the conical figure ſupplies that, and the column of mercury ſuſtained is always equal to that in the common barometer tube; which is effected thus; when the preſſure of the air is leſs, the mercury ſinks down to a lower and wider part of the tube, and conſequently the altitude of its column will be leſs; and on the contrary, by a greater preſſure of the atmoſphere the mercury is forced up to a higher and narrower part, till the length of the column CD be equal to that in the tube of the common barometer.—The inconvenience of this barometer is, that as the bore muſt be made very ſmall, to prevent the mercury from falling out by an accidental ſhake, the friction and adheſion to the ſides of the tube prevent the free motion of the mercury.

Mr. Rowning's Compound Barometers. This gentleman has ſeveral contrivances for enlarging the ſcale, and that in any proportion whatever. One of theſe is deſcribed in the *Philoſ. Tranſ.* N° 427, and alſo in his *Nat. Philoſ.* part 2; and another in the ſame part, which is here repreſented at fig. 9. ABC is a compound tube, hermetically ſealed at A, and open at C; empty from A to D, filled with mercury from thence to B, and from hence to E with water. Hence by varying the proportions of the two tubes AF and FC, the ſcale of variation may be changed in any degree.

The Marine Barometer. This was firſt invented by Dr. Hook, to be uſed on board of ſhip, being contrived ſo as not to be affected or injured by the motion of the ſhip. His contrivance conſiſted of a double thermometer, or a couple of tubes half filled with ſpirit of wine; the one ſealed at both ends, with a quantity of air included; the other ſealed at one end only. The former of theſe is affected only by the warmth of the air; but the other is affected both by the external warmth and by the variable preſſure of the atmoſphere. Hence, conſidering the ſpirit thermometer as a ſtandard, the exceſs of the riſe or fall of the other above it will ſhew the increaſe or decreaſe of the preſſure of the atmoſphere. This inſtrument is deſcribed by Dr. Halley, in the *Philoſ. Tranſ.* N° 269, where he ſays of it, "I had one of theſe barometers with me in my late ſouthern voyage, and it never failed to prognoſticate and give early notice of all the bad weather we had, ſo that I depended thereon, and made proviſion accordingly; and from my own experience I conclude, that a more uſeful contrivance hath not for this long time been offered for the benefit of navigation."

Mr. Nairne, an ingenious artiſt in London, has lately invented a new kind of *Marine Barometer;* which differs from the common barometer by having the lower part of the tube, for about 2 feet long, made very ſmall, to check the vibrations of the mercury, which would otherwiſe ariſe from the motions of the ſhip. This is alſo aſſiſted by being hung in gimbals, by a part which ſubjects it to be the leaſt affected by ſuch motions.

Another ſort of Marine Barometer has alſo been invented by M. Paſſemente, an ingenious artiſt at Paris. This contrivance conſiſts only in twiſting the middle of the tube into a ſpiral of two revolutions; by which contrivance the impulſes which the mercury receives from the motions of the ſhip are deſtroyed, by being tranſmitted in contrary directions.

The Statical Baroſcope, or *Barometer*, of Mr. Boyle, &c. This conſiſts of a large glaſs bubble, blown very thin, and then balanced by a ſmall braſs weight. Hence theſe two bodies being of unequal bulk, the larger will be very much affected by a change of the denſity of the medium, but the leſs not at all as to ſenſe: So that, when the atmoſphere becomes denſer, the ball loſes more of its weight, and the braſs weight preponderates; and contrariwiſe when the air grows lighter.

Mr. Caſwell's Baroſcope, or *Barometer.* This is deſcribed in the *Philoſ. Tranſ.* vol. 24, and ſeems to be the moſt ſenſible and exact of any. It is thus deſcribed: Suppoſe ABCD, (fig. 10) is a bucket of water, in which is the baroſcope *xrezyoſm*, which conſiſts of a body *xrſm*, and a tube *ezyo*, which are both concave cylinders, made of tin, or rather glaſs, and communicating with each other. The bottom of the tube *zy* has a leaden weight to ſink it, ſo that the top of the body may juſt ſwim even with the ſurface of the water by the addition of ſome grain weights on the top. When the inſtrument is forced with its mouth downwards, the water aſcends into the tube to the height *yu*. To the top is added a ſmall concave cylinder, or pipe, to keep the inſtrument from ſinking down to the bottom: *md* is a wire: and *mS*, *dc* are two threads oblique to the ſurface of the water, which perform the office of diagonals: for while the inſtrument ſinks more or leſs by an alteration in the gravity of the air, where the ſurface of the water cuts the thread is formed a ſmall bubble, which aſcends up the thread while the mercury of the common baroſcope aſcends, and vice verſa.

It appears from a calculation which the author makes, that this inſtrument ſhews the alterations in the air 1200 times more accurately than the common barometer. He obſerves, that the bubble is ſeldom known to ſtand ſtill even for a minute; that a ſmall blaſt of wind, which cannot be heard in a chamber, will ſenſibly make it ſink; and that a cloud paſſing over it always makes it deſcend, &c.

While ſome have been increaſing the ſenſibility of the barometer by enlarging the variations, others have endeavoured to make it more convenient by reducing the length of the tube. *M. Amontons*, in 1688, firſt propoſed this alteration in the ſtructure of barometers, by joining ſeveral tubes to one another, alternately filled with mercury and with air, or ſome other fluid; and the number of theſe tubes may be increaſed at pleaſure: but the contrivance is perhaps more ingenious than uſeful.

M. Mairan's reduced Barometer, which is only 3 inches long, ſerves the purpoſe of a manometer, in ſhewing the dilatations of the air in the receiver of an air-pump; and inſtruments of this kind are now commonly applied to this uſe.

The Portable Barometer, is ſo contrived that it may be carried from one place to another without being diſordered. The end of the tube is tied up in a leathern bag not quite full of mercury; which being preſſed by the air, forces the mercury into the tube, and keeps it ſuſpended at its proper height: This bag is uſually incloſed in a box, through the bottom of which paſſes a ſcrew, by means of which the mercury may be forced up to the top of the tube, and prevented from breaking it by daſhing againſt the top when the inſtrument is removed from one ſtation to another. It ſeems Mr. Patrick firſt made a contrivance of this kind: but the portable barometer has received various improvements ſince; and the moſt complete of this kind has been deſcribed by M. De Luc, in his Recherches, vol. 2, pa. 5 &c, together with the apparatus belonging to it, the method of conſtruction and uſe, and the advantages attending it. Improvements have alſo been ſuggeſted by Sir George Shuckburgh, and Col. Roy, which have been carried into execution, with farther improvements alſo, by Mr. Ramſden, and other ingenious artiſts in London.

Fig. 11 repreſents this inſtrument, as incloſed in its mahogany caſe by means of three metallic rings *a a a*. This caſe is a hollow cone, ſo ſhaped within as to contain ſteadily the body of the barometer, and is divided into three branches from *b* to *c*, forming three legs or ſupports for the inſtrument when obſervations are making, and ſuſtaining it at the part *d* of the caſe, as it appears in Fig. 12, by an improved kind of gimbals, in which its own weight renders it ſufficiently ſteady at any time. In the part of the frame *f g* where the barometer tube appears, is made a long ſlit or opening, that the column of mercury may be ſeen againſt the light, and the vernier piece *f* brought down to coincide very nicely with the edge of the mercury. When the inſtrument is fixed in its ſtand, the ſcrew *l* is to be turned to let the mercury down to its proper ſtation, and a peg at *i* muſt be looſened, to admit the external air to act upon the mercury contained in the box *k*. The proper adjuſtment, or mode of obſerving what is called the *zero* or 0 diviſion of the column of mercury, is by obſerving it in the tranſparent part of the box *k*, which has a glaſs tube or reſervoir for the quickſilver, and an edged piece of metal attached to the external part of it; with the edge of which the mercury is to be brought into contact by turning the ſcrew *l* to the right or left as occaſion requires. The vernier piece at *f*, which determines the altitude of the mercurial column, is firſt brought down by the hand to a near contact, and then accurately adjuſted by turning the ſcrew *e* at the top. The diviſions annexed to the tube of this inſtrument may be of any ſort, or of any degree of ſmallneſs, according to the purpoſes it is intended to ſerve. To accommodate it to the uſe of foreigners as well as natives, there are commonly added ſcales of both French and Engliſh inches, with their ſubdiviſions to any extent required. It is uſual to place the French ſcale of inches on the right ſide at *f g*, from 19 to 31 inches, meaſured from the zero or ſurface of the mercury in the box *k* below; each inch being divided into lines or 12th parts, and each line ſubdivided by the vernier into 10th parts, or 120th parts of inches; by means of which therefore the length of the mercurial column may be determined to the 120th part of a French inch. The other ſcale, which is placed on the left ſide of the inſtrument, is divided into Engliſh inches, and each inch into 20th parts, which by a vernier are ſubdivided into 25th parts, or 500th parts of inches; by this means ſhewing the height of the mercury to the 500th part of an Engliſh inch. But this vernier is figured double or each diviſion is accounted 2, which reduces the meaſures to 1000ths of an inch for the conveniency of calculation, in meaſuring altitudes of hills &c.

A thermometer is always attached to the inſtrument, as a neceſſary appendage to it, being faſtened to the body at *h*, and ſunk into the ſurface of the frame, to preſerve it from injury: the degrees of this thermometer are marked on two ſcales, one on each ſide of it, viz, the ſcale of Fahrenheit, and that of Reaumur; the freezing point of the former being at 32, and of the latter at 0. Alſo on the right hand ſide of theſe two ſcales there is a third, called a ſcale of *correction*, placed oppoſitely to that of Fahrenheit, with the word *add* and *ſubtract* marked; which ſhews the neceſſary correction of the obſerved altitude of the mercury at any given temperature of the air, indicated by the thermometer.

There are ſeveral other pieces of mechaniſm about the inſtrument, which will be evident by inſpection; and the manner of making the obſervations, with the neceſſary calculations, are fully explained in M. de Luc's *Recherches ſur les Modifications de l'Atmoſphere*, and the *Philoſ. Tranſ.* vol. 67 and 68, before cited.

The Common Chamber Weatherglaſs, is alſo uſually fitted up in a neat mahogany frame, and other embelliſhments, to make it an ornamental piece of furniture. It conſiſts of the common tube barometer, with a thermometer by the ſide of it, and an hygrometer at the top, as exhibited in fig. 13.

To the foregoing may be added a new ſort of *Barometer*, or *Weather Inſtrument by the Sound of a Wire*. This is mentioned by M. Lazowſki in his Tour through Switzerland: it is as yet but in an imperfect ſtate, and was lately diſcovered there by accident. It ſeems that a clergyman, though near-ſighted, often amuſed himſelf with firing at a mark, and contrived to ſtretch a wire ſo as to draw the mark to him to ſee how he had aimed. He obſerved that the wire ſometimes ſounded as if it vibrated like a muſical cord; and that after ſuch ſoundings, a change always enſued in the ſtate of the atmoſphere; from whence he came to predict rain or fine weather. On making farther experiments, it was found that the ſounds were moſt diſtinct when extended in the plane of the meridian. And according to the weather which was to follow, it was found that the ſounds were more or leſs ſoft, or more or leſs continued; alſo fine weather, it is ſaid, was announced by the tones of counter-tenor, and rain by thoſe of baſs. It has been ſaid that M. Volta mounted 15 chords in this way at Pavia, to bring this method to ſome precision, but no accounts have yet appeared of the ſucceſs of his obſervations.

The Phænomena and Obſervations of the Barometer *. The phænomena of the barometer are various; but authors are not yet agreed upon the cauſes of them; nor is the uſe of it, as a weather-glaſs, yet perfectly aſcertained, though daily obſervations and experience lead us ſtill nearer to preciſion. Mr. Boyle obſerves that the phænomena of the barometer are ſo precarious, that it is exceedingly difficult to form any certain general rules concerning the riſe and fall of the mercury. Even that rule fails which ſeems to hold the moſt generally, viz, that the mercury is low in high winds. The beſt rules however that have been deduced by ſeveral authors are as follow.

Dr Halley's Rules for judging of the Weather.

1. In calm weather, when the air is inclined to rain, the mercury is commonly low.

2. In ſerene, good, and ſettled weather, the mercury is generally high.

3. Upon very great winds, though they be not accompanied with rain, the mercury ſinks loweſt of all, according to the point of the compaſs the wind blows from.

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* An ingenious author obſerves that, by means of barometers we may regain the knowledge that ſtill reſides in brutes, and which we forfeited by not continuing in the open air, as they moſtly do; and, by our intemperance, corrupting the *craſis* of our organs of ſenſe.

4. The greateſt heights of the mercury are found upon eaſterly or north-eaſterly winds, other circumſtances alike.

5. In calm froſty weather, the mercury commonly ſtands high.

6. After very great ſtorms of wind, when the mercury has been very low, it generally riſes again very faſt.

7. The more northerly places have greater alterations of the barometer than the more ſoutherly, near the equator.

8. Within the tropics, and near them, there is little or no variation of the barometer, in all weathers. For inſtance, at St. Helena it is little or nothing, at Jamaica 3-10ths of an inch, and at Naples the variation hardly ever exceeds an inch; whereas in England it amounts to 2 inches and a half, and at Peterſburgh to $3\frac{1}{3}$ nearly.

Dr Beal, who followed the opinion of M. Paſcal, obſerves that, *cæteris paribus*, the mercury is higher in cold weather than in warm: and in the morning and evening uſually higher than at mid-day.—That in ſettled and fair weather, the mercury is higher than either a little before or after, or in the rain; and that it commonly deſcends lower after rain than it was before it. And he aſcribes theſe effects to the vapours with which the air is charged in the former caſe, and which are diſperſed by the falling rain in the latter. If it chance to riſe higher after rain, it is uſually followed by a ſettled ſerenity. And that there are often great changes in the air, without any perceptible alteration in the barometer.

Mr Patrick's Rules for judging of the Weather. Theſe are eſteemed the beſt of any general rules hitherto made:

1. The riſing of the mercury preſages, in general, fair weather; and its falling, foul weather, as rain, ſnow, high winds, and ſtorms.

2. In very hot weather, the falling of the mercury indicates thunder.

3. In winter, the riſing preſages froſt: and in froſty weather, if the mercury falls 3 or 4 diviſions, there will certainly follow a thaw. But in a continued froſt, if the mercury riſes, it will certainly ſnow.

4. When foul weather happens ſoon after the falling of the mercury, expect but little of it; and on the contrary, expect but little fair weather when it proves fair ſhortly after the mercury has riſen.

5. In foul weather, when the mercury riſes much and high, and ſo continues for 2 or 3 days before the foul weather is quite over, then expect a continuance of fair weather to follow.

6. In fair weather, when the mercury falls much and low, and thus continues for 2 or 3 days before the rain comes; then expect a great deal of wet, and probably high winds.

7. The unſettled motion of the mercury, denotes uncertain and changeable weather.

8. You are not ſo ſtrictly to obſerve the words engraved on the plates, as the mercury's riſing and falling; though in general it will agree with them. For if it ſtands at *much rain*, and then riſes up to *changeable*, it preſages fair weather; though not to continue ſo long as if the mercury had riſen higher. And ſo, on the contrary, if the mercury ſtood at *fair*, and falls to *changeable*, it preſages foul weather; though not ſo much of it as if it had ſunk lower.

Upon theſe rules of Mr Patrick, the following *Remarks* are made by *Mr Rowning*. That it is not ſo much the abſolute height of the mercury in the tube that indicates the weather, as its motion up and down: wherefore, to paſs a right judgment of what weather is to be expected, we ought to know whether the mercury is actually riſing or falling; to which end the following rules are of uſe.

1. If the ſurface of the mercury is convex, ſtanding higher in the middle of the tube than at the ſides, it is a ſign that the mercury is then riſing.

2. But if the ſurface be concave, or hollow in the middle, it is then ſinking. And,

3. If it be plain, or rather a very little convex, the mercury is ſtationary: for mercury being put into a glaſs tube, eſpecially a ſmall one, naturally has its ſurface a little convex, becauſe the particles of mercury attract one another more forcibly than they are attracted by glaſs. Farther,

4. If the glaſs be ſmall, ſhake the tube; then if the air be grown heavier, the mercury will riſe about half a 10th of an inch higher than it ſtood before; but if it be grown lighter, it will ſink as much. And, it may added, in the wheel or circular barometer, tap the inſtrument gently with the finger, and the index will viſibly ſtart forwards or backwards according to the tendency to riſe or fall at that time. This proceeds from the mercury's ſticking to the ſides of the tube, which prevents the free motion of it till it be diſengaged by the ſhock: and therefore when an obſervation is to be made with ſuch a tube, it ought to be firſt ſhaken; for ſometimes the mercury will not vary of its own accord, till the weather is preſent which it ought to have indicated.

And to the foregoing may be added the following additional rules, more accurately drawn from later and more cloſe obſervation of the motions of the barometer, and the conſequent changes in the air in this country.

1. In winter, ſpring, and autumn, the ſudden falling of the mercury, and that for a large ſpace, denotes high winds and ſtorms; but in ſummer it denotes heavy ſhowers, and often thunder: and it always ſinks loweſt of all for great winds, though not accompanied with rain; though it falls more for wind and rain together than for either of them alone. Alſo, if, after rain, the wind change into any part of the north, with a clear and dry ſky, and the mercury riſe, it is a certain ſign of fair weather.

2. After very great ſtorms of wind, when the mercury has been low, it commonly riſes again very faſt. In ſettled fair and dry weather, except the barometer ſink much, expect but little rain; for its ſmall ſinking then, is only for a little wind, or a few drops of rain; and the mercury ſoon riſes again to its former ſtation. In a wet ſeaſon, ſuppoſe in hay-time and harveſt, the ſmalleſt ſinking of the mercury muſt be minded; for when the conſtitution of the air is much inclined to ſhowers, a little ſinking in the barometer then denotes more rain, as it never then ſtands very high. And, if

in ſuch a ſeaſon, it riſe ſuddenly, very faſt, and high, expect not fair weather more than a day or two, but rather that the mercury will fall again very ſoon, and rain immediately to follow: the ſlow gradual riſing, and keeping on for 2 or 3 days, being moſt to be depended on for a week's fair weather. And the unſettled ſtate of the quickſilver always denoting uncertain and changeable weather, eſpecially when the mercury ſtands any where about the word *changeable* on the ſcale.

3. The greateſt heights of the mercury, in this country, are found upon eaſterly and north-eaſterly winds; and it may often rain or ſnow, the wind being in theſe points, and the barometer ſink little or none, or it may even be in a riſing ſtate, the effect of thoſe winds counteracting. But the mercury ſinks for wind, as well as rain, in all the other points of the compaſs; but riſes as the wind ſhifts about to the north or eaſt, or between thoſe points: but if the barometer ſhould ſink with the wind in that quarter, expect it ſoon to change from thence; or elſe, ſhould the fall of the mercury be much, a heavy rain is then likely to enſue, as it ſometimes happens.

Cauſe of the Phænomena of the Barometer.

To account for the foregoing phænomena of the barometer, many hypotheſes have been framed, which may be reduced to two general heads, viz, *mechanical* and *chemical*. The chief writers upon theſe cauſes, are Paſcal, Beal, Wallis, Garcin, Garden, Liſter, Halley, Garſten, De la Hire, Mariotte, Le Cat, Woodward, Leibnitz, De Mairan, Hamberger, D. Bernoulli, Muſchenbroek, Chambers, De Luc, Black, &c; and an account of moſt of their hypotheſes may be ſeen at large in M. De Luc's *Recherches ſur les Modifications de l'Atmoſphere*, vol. 1. chap. 3; ſee alſo the *Philoſ. Tranſ.* and various other works on this ſubject. It may ſuffice to notice here ſlightly a few of the principal of them.

Dr. Liſter accounts for the changes of the barometer from the alterations by heat and cold in the mercury itſelf; contracting by cold, and expanding by heat. But this, it is now well known, is quite inſufficient to account for the whole of the effect.

The changes in the weight or preſſure of the atmoſphere muſt therefore be regarded as the principal cauſe of thoſe in the barometer. But then, the difficulty will be to aſſign the cauſe of that cauſe, or whence ariſe thoſe alterations that take place in the atmoſphere, which are ſometimes ſo great as to alter its preſſure by the 10th part of the whole quantity. It is probable that the winds, as driven about in different directions, have a great ſhare in them; vapours and exhalations, riſing from the earth, may alſo have ſome ſhare; and ſome perhaps the flux and reflux occaſioned in the air by the moon; as well as ſome chemical cauſes operating between the different particles of matter.

Dr. Halley thinks the winds and exhalations ſufficient; and on this principle gives a theory, the ſubſtance of which may be compriſed in what follows:

1ſt, That the winds muſt alter the weight of the air in any particular country; and this, either by bringing together a greater quantity of air, and ſo loading the atmoſphere of any place; which will be the caſe as often as two winds blow from oppoſite parts, at the ſame time, towards the ſame point: or by ſweeping away ſome part of the air, and giving room for the atmoſphere to expand itſelf; which will happen when two winds blow oppoſite ways from the ſame point at the ſame time: or laſtly by cutting off the perpendicular preſſure of the air; which is the caſe when a ſingle wind blows briſkly any way; it being found by experience, that a ſtrong blaſt of wind, even made by art, will render the atmoſphere lighter; and hence the mercury in a tube below it, as well as in others more diſtant, will conſiderably ſubſide. See *Philoſ. Tranſ.* N° 292.

2dly, That the cold nitrous particles, and even the air itſelf condenſed in the northern regions, and driven elſewhere, muſt load the atmoſphere, and increaſe its preſſure.

3dly, That heavy dry exhalations from the earth muſt increaſe the weight of the atmoſphere, as well as its elaſtic force; as we find the ſpecific gravity of menſtruums increaſed by diſſolved ſalts and metals.

4thly, That the air being rendered heavier by theſe and the like cauſes, is thence better able to ſupport the vapours; which being likewiſe intimately mixed with it, make the weather ſerene and fair. Again, the air being made lighter from the contrary cauſes, it becomes unable to ſupport the vapours with which it is replete; theſe therefore precipitating, are collected into clouds, the particles of which in their progreſs unite into drops of rain.

Hence he infers, it is evident that the ſame cauſes which increaſe the weight of the air, and render it more able to ſupport the mercury in the barometer, do likewiſe produce a ſerene ſky, and a dry ſeaſon; and that the ſame cauſes which render the air lighter, and leſs able to ſupport the mercury, do likewiſe generate clouds and rain.

But theſe principles, though well adapted to many of the particular caſes of the barometer, ſeem however to fall ſhort of ſome of the principal and moſt obvious ones, beſides being liable to ſeveral objections.

Leibnitz accounted for the fall of the mercury before rain by another principle, viz, That as a body ſpecifically lighter than a fluid, while it is ſuſtained by it, adds more weight to that fluid than when, by being reduced in bulk, it becomes ſpecifically heavier, and deſcends; ſo the vapour, after it is reduced into the form of clouds, and deſcends, adds leſs weight to the air than it did before; and hence the mercury ſinks in the tube.—But here, granting that the drops of rain formed from the vapours always increaſing in ſize as they fall lower, were continually accelerated alſo in their motion, and ſo the air ſuffer a continued loſs of their weight as they deſcend; it may however be objected, that by the deſcent of the mercury the rain is foretold a much longer time before it comes, than the vapour can be ſuppoſed to take up in falling: that many times, and in different places, there falls a great deal of rain, without any ſinking of the mercury at all; as alſo that there often happens a fall of the mercury without any rain enſuing: and that ſometimes the mercury will ſuddenly ſink, in a ſhort ſpace of time, half an inch or more, which anſwers to 7 inches of rain, or about one third of the whole quantity falling in the whole year.

Mr. *De Luc* ſuppoſes that the changes obſerved in the preſſure of the atmoſphere, are chiefly produced by the greater or leſs quantity of vapours floating in it: as others have attributed them to the ſame cauſe, but have given a different explanation of it. His opinion is, that vapours diminiſh the ſpecific gravity, and conſequently the abſolute weight, of thoſe columns of the atmoſphere into which they are received, and which, notwithſtanding this admixture, ſtill remain of the ſame height with adjoining columns that conſiſt of pure or dry air. He afterwards vindicates and more fully explains this theory, and applies it to the ſolution of the principal phenomena of the barometer, as depending on the varying denſity and weight of the atmoſphere.

Dr. *James Hutton*, in his *Theory of Rain*, printed in the Tranſactions of the Royal Society of Edinburgh, vol. 1, gives ingenious and plauſible reaſons for thinking that the leſſening the weight of the atmoſphere by the fall of rain, is not the cauſe of the fall of the barometer; but that the principal, if not the only cauſe, ariſes from the commotions in the atmoſphere, which are chiefly produced by ſudden changes of heat and cold in the air. "The barometer, ſays he, is an inſtrument neceſſarily connected with motions in the atmoſphere; but it is not equally affected with every motion in that fluid body. The barometer is chiefly affected by thoſe motions by which there are produced accumulations and abſtractions of this fluid, in places or regions of ſufficient extent to affect the preſſure of the atmoſphere upon the ſurface of the earth. But as every commotion in the atmoſphere may, under proper conditions, be a cauſe for rain, and as the want of commotion in the atmoſphere is naturally a cauſe of fair weather, this inſtrument may be made of great importance for the purpoſe of meteorological obſervations, although not in the certain and more ſimple manner in which it has been, with the increaſe of ſcience, ſo ſucceſsfully applied to the meaſuring of heights." See Rain.

In the *Encyclopædia Britannica* there is another theory of the changes in the barometer, as depending on the *heat* in the atmoſphere, not as producing commotions there, but as altering the ſpecific gravity of the air by the changes of heat and cold. The principles of this theory are, 1ſt, That vapour is formed by an intimate union between the elements of fire and water, by which the fire or heat is ſo totally enveloped, and its action ſo perfectly ſuſpended by the aqueous particles, that it not only loſes its properties of burning and of giving light, but becomes incapable of affecting the moſt ſenſible thermometer, in which caſe it is ſaid to be in a latent ſtate: and 2d, That if the atmoſphere be affected by any unuſual degree of heat, it thence becomes incapable of ſupporting ſo long a column of mercury as before; for which reaſon it is that the barometer ſinks.

From theſe axioms it would follow, that as vapour is formed by an union of fire with water, whether by attraction or a ſolution of the water in the fire, the vapour cannot be condenſed till this union, attraction, or ſolution, is at an end. Hence the beginning of the condenſation of the vapour, or the firſt ſigns of approaching rain, muſt be the ſeparation of the fire which is latent in the vapour. In the beginning, this may be either ſlow and partial, or it may be ſudden and violent: in the firſt caſe, the rain will come on ſlowly, and after a conſiderable time; but in the other, it will come very quickly, and in a great quantity. But Dr. Black has proved, that when fire quits its latent ſtate, however long it may have lain dormant and inſenſible, it always reaſſumes its proper qualities, and affects the thermometer juſt the ſame as if it had never been abſorbed. The conſequence of this is, that in proportion as the latent heat is diſcharged from the vapour, thoſe parts of the atmoſphere into which it is diſcharged muſt be ſenſibly affected by it; and in proportion to the heat communicated to thoſe parts, they will become ſpecifically lighter, and the mercury will ſink of courſe.

In the *Memoirs of the Literary Society of Mancheſter*, vol. 4, is alſo a curious paper on this ſubject, viz, *Meteorological Obſervations made on different Parts of the Weſtern Coaſt of Great Britain: arranged by T. Garnett, M. D.* This paper is compoſed of materials furniſhed by ſeveral obſervers; thoſe of Mr. Copland, ſurgeon at Dumfries, are of ſpecial importance. This gentleman is of opinion that the changes of the barometer indicate approaching hot and cold weather, with much more certainty than dry and wet. "Every remarkable elevation of the barometer, ſays he, where it is of any duration, is followed by very warm or dry weather, and moderate as to wind, or by all of them; but heat ſeems to have moſt influence and connexion; and when it is deficient, the continuance of the other two will be longer and more remarkable; therefore the calculation muſt be in a compound ratio of the exceſs and deficiency of the heat, and of the dryneſs of the weather in compariſon of the medium of the ſeaſon; and with regard to the want of ſtrong wind, it appears to be intimately connected with the laſt, as they ſhew that no precipitation is going on in any of the neighbouring regions."

In his 14th and 15th remarks, he had ſaid,

'14th, That the barometer being lower, and continuing ſo longer than what can be accounted for by immediate falls, or ſtormy weather, indicates the approach of very cold weather for the ſeaſon; and alſo, cold weather, though dry, is always accompanied by a low barometer, till near its termination.'

'15th, That warm weather is always preceded and moſtly accompanied by a high barometer; and the riſing of the barometer in the time of broken or cold weather, is a ſign of the approach of warmer weather: and alſo if the wind is in any of the cold points, a ſudden riſe of the barometer indicates the approach of a ſoutherly wind, which in winter generally brings rain with it.'

In the two following remarks, Mr. Copland had explained certain phenomena from a principle ſimilar to that on which Dr. Darwin has ſo much inſiſted: (Botanic Garden, I. notes p. 79, &c.)

'That the falling of the barometer may proceed from a decompoſition of the atmoſphere occurring around or near that part of the globe where we are placed, which will occaſion the electricity of the atmoſphere to be repelled upwards in fine lambent portions; or driven downwards or upwards in more compacted balls of fire; or laſtly, to be carried along with the rain, &c, in an imperceptible manner to the ſurface of the earth: the precipitation of the watery parts generally very ſoon takes place, which diminiſhes the real gravity of the atmoſphere, and alſo by the decompoſition of ſome of the more active parts, the air loſes part of that elaſtic and repulſive power which it ſo eminently poſſeſſed, and will therefore preſs with leſs force on the mercury of the barometer than before, by which means a fall enſues.

'That the cauſe of the currents of air, or winds, may alſo be this way accounted for: and in very ſevere ſtorms, where great decompoſitions of the atmoſphere take place, this is particularly evident, ſuch as generally occur in one or more of the Weſt India iſlands at one time, a great loſs of real gravity, together with a conſiderable diminution of the ſpring of the air immediately enſues; hence a current commences, firſt in that direction whence the air has moſt gravity, or is moſt diſpoſed to undergo ſuch a change; but it being ſoon relieved of its ſuperior weight or ſpring on that ſide, by the decompoſition going on as faſt as the wind arrives on the iſland, it immediately veers to another point, which then ruſhes in moſtly with an increaſe of force; thus it goes on till it has blown more than half way round the points of the compaſs during the continuation of the hurricane. For in this manner the Weſt India phenomena, as well as the alteration of the wind during heavy rains in this country, can only be properly accounted for.' See remark No. 4.

Mr. C.'s 4th aphoriſm is, 'That the heavieſt rains, when of long continuance, generally beign with the wind blowing eaſterly, when it gradually veers round to the ſouth; and that the rain does not then begin to ceaſe till the wind has got to the weſt, or rather a little to the northward of it, when, it may be added, it commonly blows with ſome violence.'

Many other obſervations on the barometer, the weather, &c. may be ſeen in various parts of the Philoſ. Tranſ. And for other curious papers on the ſame, and other ſubjects connected with the barometer, ſee the Gentleman's Magazine for 1789, p. 317; alſo Greu's Journal of Nat. Philoſ. printed at Leipzig 1792, for the influence of the ſun and moon upon the barometer.

The Barometer applied to the meaſuring of Altitudes.

The ſecondary character of the barometer, namely as an inſtrument for meaſuring acceſſible heights or depths, was firſt propoſed by Paſcal, and Deſcartes, as has been before obſerved; and ſucceeding philoſophers have been at great pains to aſcertain the proportion between the fall of the barometer and the height to which it is carried; as Halley, Mariotte, Maraldi, Scheuchzer, J. Caſſini, D. Bernoulli, Horrebow, Bouguer, Shuckburgh, Roy, and more eſpecially by De Luc, who has given a critical and hiſtorical detail of moſt of the attempts that have at different times been made for applying the motion of the mercury in the barometer to the meaſurement of acceſſible heights. And for this purpoſe ſerves the portable barometer, before deſcribed, (fig. 11 and 12, plate 4,) which ſhould be made with all the accuracy poſſible. Various rules have been given by the writers on this ſubject, for computing the height aſcended from the given fall of the mercury in the tube of the barometer, the moſt accurate of which was that of Dr. Halley, till it was rendered much more accurate by the indefatigable reſearches of De Luc, by introducing into it the corrections of the columns of mercury and air, on account of heat. And other corrections and modifications of the ſame may be ſeen inſerted under the article ATMOSPHERE, where the moſt correct rule is deduced from one ſingle experiment only. This rule is as follows:

The Rule for Computing Altitudes, is this,

Viz, 10000 × log. of $\frac{M}{m}$ is the altitude in fathoms, in the mean temperature of 31°; and for every degree of the thermometer above that, the reſult muſt be increaſed by ſo many times its 435th part, and diminiſhed when below it: in which theorem M denotes the length of the column of mercury in the barometer tube at the bottom, and *m* that at the top of the hill, or other eminence; which lengths may be expreſſed in any one and the ſame ſort of meaſures, whether feet, or inches, or tenths, &c, and either Engliſh, or French, or of any other nation; but the reſult is always in fathoms, of 6 Engliſh feet each.

And the *Precepts*, in words, for the practice of meaſurements by the barometer, are theſe following:

1ſt, Obſerve the height of the barometer at the bottom of any height or depth, propoſed to be meaſured; together with the temperature of the mercury by means of the thermometer attached to the barometer, and alſo the temperature of the air in the ſhade by another thermometer which is detached from the barometer.

2dly, Let the ſame thing be done alſo at the top of the ſaid height or depth, and as near to the ſame time with the former as may be. And let thoſe altitudes of mercury be reduced to the ſame temperature, if it be thought neceſſary, by correcting either the one or the other, viz, augmenting the height of the mercury in the colder temperature, or diminiſhing that in the warmer, by its 9600th part for every degree of difference between the two; and the altitudes of mercury ſo corrected, are what are denoted by M and *m*, in the algebraic formula above.

3dly, Take out the common logarithms of the two heights of mercury, ſo corrected, and ſubtract the leſs from the greater, cutting off from the right hand ſide of the remainder three places for decimals; ſo ſhall thoſe on the left be fathoms in whole numbers, the tables of logarithms being underſtood to be ſuch as have 7 places of decimals.

4thly, Correct the number laſt found, for the difference of the temperature of the air, as follows: viz, Take half the ſum of the two temperatures of the air, ſhewn by the detached thermometers, for the mean one; and for every degree which this differs from the ſtandard temperature of 31°, take ſo many times the 435th part of the fathoms above found, and add them if the mean temperature be more than 31°, but ſubtract them if it be below 31°; ſo ſhall the ſum or difference be the true altitude in fathoms, or being multiplied by 6, it will give the true altitude in Engliſh feet.

Example 1. Let the ſtate of the barometers and thermometers be as follows, to find the altitude: viz,

Thermometers.		Barometers.
detached.	attached.	
57	57	29·68 lower
42	43	25·28 upper
mean $49\frac{1}{2}$	dif. 14	

As 9600 : 14 : : 29·68 : ·04

	cor. ·04	logs.
mean $49\frac{1}{2}$	M = 29·64	- 4718782
ſtand. 31	*m* = 25·28	- 4027771
dif. $18\frac{1}{2}$	As 435 : $18\frac{1}{2}$: : 691·011 : 29·388	
		29·388
	the altitude ſought is { 720·399 fath. or 4322·394 feet.	

Example 2. To find the altitude of a hill, when the ſtate of the barometer and thermometer, as obſerved at the bottom and top of it, is as follows; viz,

Thermometers.		Barometers.
detached.	attached.	
35	41	29·45
31	38	26·82
mean 33	dif. 3	

As 9600 : 3 : : 29·45 : ·01

	·01	logs.
mean 33	M = 29·44	- 4689378
ſtand. 31	*m* = 26·82	- 4284588
dif. 2	As 435 : 2 : : 404·790 : 1·86	
		1·86
	the altitude ſought is { 406·65 fathoms. or 2439·90 feet.	

See this rule inveſtigated under the article Pneumatics, at the end.

N. B. The mean height of the barometer in London, upon an average of two obſervations in every day of the year, kept at the houſe of the Royal Society, for many years paſt, is 29·88; the medium temperature, or height of the thermometer, according to the ſame, being 58°. But the medium height at the ſurface of the ſea, according to Sir Geo. Shuckburgh (Philoſ. Tranſ. 1777, p. 586) is 30·04 inches, the heat of the barometer being 55°, and of the air 62°.

BAROSCOPE, a machine for ſhewing the alterations in the weight or preſſure of the atmoſphere. See Barometer.

BARREL, an Engliſh veſſel or caſk, containing 36 gallons of beer meaſure, or 32 gallons of ale meaſure. The barrel of beer, vinegar, or of liquor preparing for vinegar, ought to contain 34 gallons, according to the ſtandard of the ale quart.

Barrel, in Clock-work, is the cylinder about which the ſpring is wrapped.

BARRICADE, or Barricado, a military term for a fence, or retrenchment, haſtily made with veſſels, or baſkets of earth, carts, trees, ſtakes, or the like, to preſerve an army from the ſhot or aſſault of an enemy.

BARRIER, a kind of fence made at a paſſage, retrenchment, gate, or ſuch like, to ſtop it up againſt an enemy.

BARROW (Isaac), a very eminent mathematician and divine of the 17th century, was born at London in October, 1630, being the ſon of Thomas Barrow, then a linen-draper of that city, but deſcended from an ancient family in Suffolk. He was at firſt placed at the Charter-houſe ſchool for two or three years; where his behaviour afforded but little hopes of ſucceſs in the profeſſion of a ſcholar, being fond of fighting, and promoting it among his ſchool-fellows: but being removed to Felſted in Eſſex, his diſpoſition took a different turn; and having ſoon made a great progreſs in learning, he was firſt admitted a penſioner of Peter Houſe in Cambridge; but when he came to join the univerſity, in Feb. 1645, he was entered at Trinity college. He now applied himſelf with great diligence to the ſtudy of all parts of literature, eſpecially natural philoſophy. He afterward turned his attention to the profeſſion of phyſic, and made a conſiderable progreſs in anatomy, botany, and chemiſtry: he next ſtudied divinity; then chronology, aſtronomy, geometry, and the other branches of the mathematics; with what ſucceſs, his writings afterwards moſt eminently ſhewed.

When Dr. Duport reſigned the chair of Greek profeſſor, he recommended his pupil Mr. Burrow for his ſucceſſor, who, in his probation exerciſe, ſhewed himſelf equal to the character that had been given him by this gentleman; but being ſuſpected of favouring Arminianiſm, he was not preferred. This diſappointment it ſeems determined him to quit the college, and viſit foreign countries; but his finances were ſo low, that he was obliged to diſpoſe of his books, to enable him to execute that deſign.

He left England in June 1655, and viſited France, Italy, Turkey, &c. At ſeveral places, in the courſe of this tour, he met with kindneſs and liberal aſſiſtance from the Engliſh ambaſſadors, &c, which enabled him to benefit the more from it, by protracting his ſtay, and prolonging his journey. He ſpent more than a year in Turkey, and returned to England by way of Venice, Germany, and Holland, in 1659. At Conſtantinople he read over the works of St. Chryſoſtom, once biſhop of that ſee, whom he preferred to all the other fathers.

On his return home Barrow was epiſcopally ordained by biſhop Brownrig; and in 1660, he was choſen to the Greek profeſſorſhip at Cambridge. In July 1662, he was elected profeſſor of geometry in Greſham college: in which ſtation, he not only diſcharged his own duty, but ſupplied likewiſe the abſence of Dr. Pope the aſtronomy profeſſor. Among his lectures, ſome were upon the projection of the ſphere and perſpective, which are loſt; but his Latin oration, previous to his lectures, is ſtill extant. About this time Mr. Barrow was offered a good living; but the condition annexed, of teaching the patron's ſon, made him refuſe it, as thinking it too like a ſimonial contract. Upon the 20th of May 1663 he was elected a fellow of the Royal Society, in the firſt choice made by the council after their charter. The ſame year the executors of Mr. Lucas having, according to his appointment, founded a mathematical lecture at Cambridge, they ſelected Mr. Barrow for the firſt profeſſor; and though his two profeſſorſhips were not incompatible with each other, he choſe to reſign that of Greſham-college, which he did May the 20th, 1664. In 1669 he reſigned the mathematical chair to his learned friend Mr. Iſaac Newton, being now determined to quit the ſtudy of mathematics for that of divinity. On quitting his profeſſorſhip, he had only his fellowſhip of Trinity-college, till his uncle gave him a ſmall ſinecure in Wales, and Dr. Seth Ward biſhop of Saliſbury conferred upon him a prebend in his church. In the year 1670 he was created doctor in divinity by mandate; and, upon the promotion of Dr. Pearſon maſter of Trinity college to the ſee of Cheſter, he was appointed to ſucceed him by the king's patent bearing date the 13th of February 1672: upon which occaſion the king was pleaſed to ſay, "he had given it to the beſt ſcholar in England." In this, his majeſty did not ſpeak from report, but from his own knowledge; the doctor being then his chaplain, he uſed often to converſe with him, and, in his humourous way, to call him an "unfair preacher," becauſe he exhauſted every ſubject, and left no room for others to come after him. In 1675 he was choſen vice-chancellor of the univerſity; and he omitted no endeavours for the good of that ſociety, nor in the line of his profeſſion as a divine, for the promotion of piety and virtue; but his uſeful labours were abruptly terminated by a fever on the 4th of May 1677, in the 47th year of his age. He was interred in Weſtminſter abbey, where a monument, adorned with his buſt, was ſoon after erected, by the contribution of his friends.

Dr. Barrow's works are very numerous, and indeed various, mathematical, theological, poetical, &c, and ſuch as do honour to the Engliſh nation. They are principally as follow:

1. Euclidis Elementa. Cantab. 1655, in 8vo.
2. Euclidis Data. Cantab. 1657, in 8vo.
3. Lectiones Opticæ xviii, Lond. 1669, 4to.
4. Lectiones Geometricæ xiii, Lond. 1670, 4to.
5. Archimedis Opera, Apollonii Conicorum libri iv, Theodoſii Sphericorum lib. iii; nova methodo illuſtrata, et ſuccincte demonſtrata. Lond. 1675, in 4to.

The following were publiſhed after his deceaſe, viz:

6. Lectio, in qua theoremata Archimedis de ſphæra et cylindro per methodum indiviſibilium inveſtigata, ac breviter inveſtigata, exhibentur. Lond. 1678, 12mo.
7. Mathematicæ Lectiones habitæ in ſcholis publicis academiæ Cantabrigienſis, an. 1664, 5, 6, &c. Lond. 1683.
8. All his Engliſh works in 3 volumes, Lond. 1683, folio.—Theſe are all theological, and were publiſhed by Dr. John Tillotson.
9. Iſaaci Barrow Opuſcula, viz, Determinationes, Conciones ad Clerum, Orationes, Poemata, &c. volumen quartum. Lond. 1687, folio.

Dr. Barrow left alſo ſeveral curious papers on mathematical ſubjects, written in his own hand, which were communicated by Mr. Jones to the author of "The Lives of the Greſham Profeſſors," a particular account of which may be ſeen in that book, in the Life of Barrow.

Several of his works have been tranſlated into Engliſh, and publiſhed; as the Elements and Data of Euclid; the Geometrical Lectures, the Mathematical Lectures. And accounts of ſome of them were alſo given in ſeveral volumes of the Philoſ. Tranſ.

Dr. Barrow muſt ever be eſteemed, in all the ſubjects which exerciſed his pen, a perſon of the cleareſt perception, the fineſt fancy, the ſoundeſt judgment, the profoundeſt thought, and the cloſeſt and moſt nervous reaſoning. "The name of Dr. Barrow (ſays the learned Mr. Granger) will ever be illuſtrious for a ſtrength of mind and a compaſs of knowledge that did honour to his country. He was unrivalled in mathematical learning, and eſpecially in the ſublime geometry; in which he has been excelled only by his ſucceſſor Newton. The ſame genius that ſeemed to be born only to bring hidden truths to light, and to riſe to the heights or deſcend to the depths of ſcience, would ſometimes amuſe itſelf in the flowery paths of poetry, and he compoſed verſes both in Greek and Latin. He at length gave himſelf up entirely to divinity; and particularly to the moſt uſeful part of it, that which has a tendency to make men wiſer and better."

Several good anecdotes are told of Barrow, as well of his great integrity, as of his wit, and bold intrepid ſpirit and ſtrength of body. His early attachment to fighting when a boy is ſome indication of the latter; to which may be added the two following anecdotes: In his voyage between Leghorn and Smyrna the ſhip was attacked by an Algerine pirate, which after a ſtout reſiſtance they compelled to ſheer off, Barrow keeping his poſt at the gun aſſigned him to the laſt. And when Dr. Pope in their converſation aſked him, "Why he did not go down into the hold, and leave the defence of the ſhip to thoſe, to whom it did belong? He replied, It concerned no man more than myſelf: I would rather have loſt my life, than to have fallen into the hands of thoſe mercileſs infidels."

There is another anecdote told of him, which ſhewed not only his intrepidity, but an uncommon goodneſs of diſpoſition, in circumſtances where an ordinary ſhare of it would have been probably extinguiſhed. Being once on a viſit at a gentleman's houſe in the country, where the neceſſary was at the end of a long garden, and conſequently at a great diſtance from the room where he lodged; as he was going to it before day, for he was a very early riſer, a fierce maſtiff, that uſed to be chained up all day, and let looſe at night for the ſecurity of the houſe, perceiving a ſtrange perſon in the garden at that unuſual time, ſet upon him with great fury. The doctor caught him by the throat, grappled with him, and, throwing him down, lay upon him: once he had a mind to kill him; but he altered his reſolution, on recollecting that this would be unjuſt, ſince the dog did only his duty, and he himſelf was in fault for rambling out of his room before it was light. At length he called out ſo loud, that he was heard by ſome of the family, who came preſently out, and freed the doctor and the dog from the danger they both had been in.

Among other inſtances of his wit and vivacity, they relate the following rencontre between him and that wicked wit lord Rocheſter. Theſe two meeting one day at the court, while the doctor was king's chaplain in ordinary, Rocheſter, thinking to banter him, with a flippant air, and a low formal bow, accoſted him with, "Doctor, I am yours to my ſhoe-tie:" Barrow perceiving his drift, and determined upon defending himſelf, returned the ſalute, with, "My lord, I am yours to the ground." Rocheſter, on this, improving his blow, quickly returned it, with, "Doctor, I am yours to the centre;" which was as ſmartly followed up by Barrow, with, "My lord, I am yours to the antipodes." Upon which, Rocheſter, diſdaining to be foiled by a muſty old piece of divinity, as he uſed to call him, exclaimed, "Doctor, I am yours to the loweſt pit of hell;" upon which Barrow, turning upon his heel, with a ſarcaſtic ſmile, archly replied, "There, my lord, I leave you."

BARS, in Muſic, are the ſpaces quite through any compoſition, ſeparated by upright lines drawn acroſs the five horizontal lines, each of which either contains the ſame number of notes of the ſame kind, or ſo many other notes as will make up a like interval of time; for all the bars, in any piece, muſt be of the ſame length, and played in the ſame time.

BARTER, or Truck, is the exchanging of one commodity for another; and forms a rule in the commercial part of arithmetic, by which the commodities are properly calculated and equalled, by computing firſt the value of the commodity which is given, and then the quantity of the other which will amount to the ſame ſum.

BASE, Basis, in Architecture, denotes the lower part of a column or pedeſtal.

Base, in Geometry, the loweſt ſide of any figure. Any ſide of a figure may be conſidered as its baſe, according to the poſition in which it may be conceived as ſtanding; but commonly it is underſtood of the loweſt ſide: as the baſe of a triangle, of a cone, cylinder, &c.

Base Line, in Perſpective, denotes the common ſection of the picture and the geometrical plane.

Base Ring, of a Cannon, is the great ring next behind the vent or touch-hole.

Base, *alternate*. See Alternate.

BASEMENT, in Architecture, a continued baſe, extended a conſiderable length, as about a houſe, a room, or other piece of building.

BASILIC, in the ancient Architecture, was a large hall, or court of judicature, where the magiſtrates ſat to adminiſter juſtice.

BASILICA, or Basilicus, the ſame as *Regulus*, or *Cor Leonis*, being a fixed ſtar of the firſt magnitude in the conſtellation *Leo*.

BASILISK, in the older Artillery, was a large piece of ordnance ſo called from its reſemblance to the ſuppoſed ſerpent of that name. It threw an iron ball of 200 pounds weight; and was in great repute in the time of Solyman emperor of the Turks, in the wars of Hungary; but it is now grown out of uſe in moſt parts of Europe. Paulus Jovius relates the terrible ſlaughter made in a Spaniſh ſhip by a ſingle ball from one of theſe baſiliſks; after paſſing through the beams and planks in the ſhip's head, it killed upwards of 30 men. And Maffeus ſpeaks of baſiliſks made of braſs, each of which required 100 yoke of oxen to draw them.—More modern writers alſo give the name baſiliſk to a much ſmaller and ſizeable piece of ordnance, made of 15 feet long by the Dutch, but of only 10 by the French, and carrying a ball of 48 pounds. The largeſt ſize of cannon now uſed by the Engliſh, are the 32 pounders.

BASIS, in Geometry, the ſame as Base.

BASS, the loweſt in the four parts of muſic; by ſome eſteemed the baſis and principal part of all, and by others as ſcarcely neceſſary in ſome tunes.

BASSANTIN (James), a Scotch aſtronomer of the 16th century, born in the reign of James the 4th of Scotland. He was a ſon of the Laird of Baſſantin in the Merſe. After finiſhing his education at the univerſity of Glaſgow, he travelled through Germany and Italy, and then ſettled in the univerſity of Paris, where he taught mathematics with great applauſe. Having acquired ſome property in this employment, he returned to Scotland in 1562, where he died 6 years after.

From his writings it appears he was no inconſiderable aſtronomer, for the age he lived in; but, according to the faſhion of the times, he was not a little addicted to judicial aſtrology. It was doubtleſs to our author that Sir James Melvil alludes in his Memoirs, when he ſays that his brother Sir Robert, when he was uſing his endeavours to reconcile the two queens Elizabeth and Mary, met with one Baſſantin a man learned in the high ſciences, who told him, "that all his travel would be in vain; for, ſaid he, they will never meet together; and next, there will never be any thing but diſſembling and ſecret hatred for a while, and at length captivity and utter wreck to our queen from England." He added, "that the kingdom of England at length ſhall fall, of right, to the crown of Scotland: but it ſhall coſt many bloody battles; and the Spaniards ſhall be helpers, and take a part to themſelves for their labour." A prediction in which Baſſantin partly gueſſed right, which it is likely he was enabled to do from a judicious conſideration of probable circumſtances and appearances.

Baſſantin's works are,

1. *Aſtronomia Jacobi Baſſantini Scoti, opus abſolutiſſimum, &c; ter. edit. Latine et Gallice.* Genev. 1599, fol. This is the title given it by Tornœſius, who tranſlated it into Latin from the French, in which language it was firſt publiſhed.

2. *Paraphraſe de l'Aſtrolabe, avec une amplification de l'uſage de l'Aſtrolabe.* Lyons 1555. Paris 1617, 8vo.

3. *Mathematica Genethliaca.*

4. *Arithmetica.*

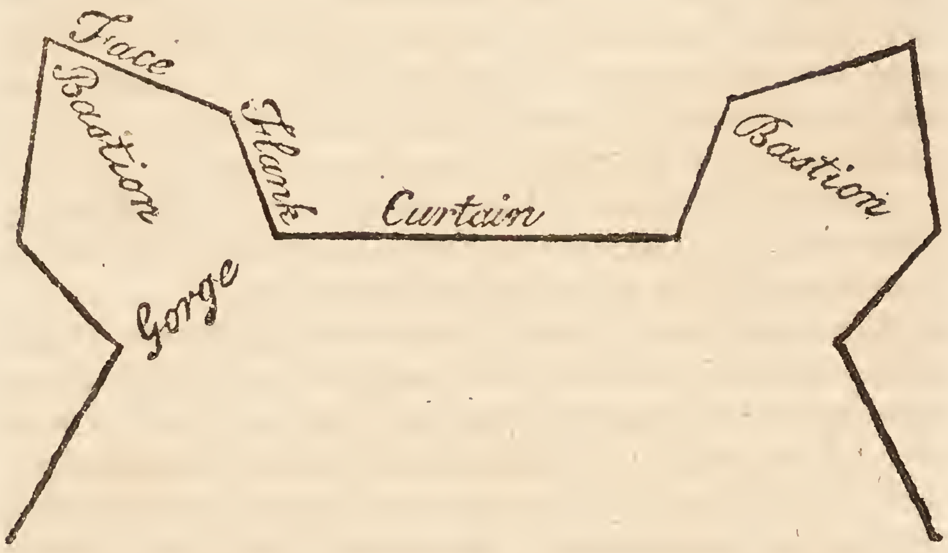
5. *Muſica ſecundum Platonem.*

6. *De Matheſi in Genere.*

BASSOON, a muſical inſtrument of the wind kind, ſerving for a baſs to the haut-boy. It is blown with a reed, and furniſhed with eleven holes.

BASS-VIOL, a baſs to the viol.

BASTION, in the modern fortification, a large maſs of earth at the angles of a work, connecting the curtains to each other; and anſwers to the bulwark of the ancients. It is formed by two faces, two flanks, and two demigorges. The two faces form the ſaliant angle, or angle of the baſtion; the two flanks form with the faces, the *epaules* or ſhoulders; and the union of the other two ends of the flanks with the curtains forms the two angles of the flanks.



Solid Bastion, are thoſe that are entirely filled up with earth to the height of the rampart, without any void ſpace towards the centre.

Void or *Hollow* Bastion, has the rampart and parapet ranging only round the flanks and ſpaces, ſo that a void ſpace is left within towards the centre, where the ground is ſo low that if the rampart be taken, no retrenchment can be made in the centre, but what will lie under the fire of the beſieged.

Regular Bastion, is that which has its due proportion of faces, flanks, and gorges.

Deformed or *Irregular* Bastion, is when the irregularity of the lines and angles throws the baſtion out of ſhape: as when it wants one of the demigorges, one ſide of the interior polygon being too ſhort, &c.

Demi Bastion, or *Half baſtion*, alſo otherwiſe called an *Epaulment*, has but one face and flank.

Double Bastion, is when one baſtion is raiſed within, and upon the plane of another baſtion.

Flat Bastion, is one built in the middle of the curtain, when it is too long to be defended by the uſual baſtions at the extremities.

Compoſed Bastion, is when the two ſides of the interior polygon are very unequal, which makes the gorges alſo unequal.

Cut Bastion, is that which has a re-entering angle at the point, and is ſometimes called a Bastion *with a Tenaille*, whoſe point is cut off, making an angle inwards, and two points outwards. This is uſed when the ſaliant angle would be too ſharp, or when water or ſome other impediment prevents it from being carried out to its full extent.

BASTON, or Batoon, in Architecture, a moulding in the baſe of a column, called alſo a *Tore* or *Torus*.

BATTEN, a name given by workmen to a ſcantling or piece of wooden ſtuff, about an inch thick, and from 2 to 4 inches broad; of a conſiderable but indeterminate length.

BATTERING, the attacking a place, work, or the like, with heavy artillery.

Battering-Ram, a military engine uſed for beating down walls, before the invention of gunpowder and the modern artillery. It was no other than a long heavy beam of timber, armed with an iron head, ſomething like the head of a ram. This being puſhed violently with conſtant ſucceſſive blows againſt a wall, gradually ſhakes it with a vibratory motion, till the ſtones are disjointed and the wall falls down. There were ſeveral kinds of the battering-ram, the firſt rude and plain, which the ſoldiers carried in their arms by main force, and ſo ſtruck the head of it againſt the wall. The ſecond was ſlung by a rope about the middle to another beam lying acroſs upon a couple of poſts; which was the kind deſcribed by Joſephus as uſed at the ſiege of Jeruſalem. A third ſort was covered over with a ſhell or ſcreen of boards, to defend the men from the ſtones and darts of the beſieged upon the walls, and thence called *teſtudo arietaria*. And Felibien deſcribes a fourth ſort of battering-ram, which ran upon wheels; and was the moſt perfect and effectual of any.

Vitruvius affirms, that the battering-ram was firſt invented by the Carthaginians, while they laid ſiege to Cadiz: yet Pliny aſſures us, that the ram was invented or uſed at the ſiege of Troy; and that it was this that gave occaſion to the fable of the wooden horſe. In fact there can be no doubt but that the uſe of ſome ſort of a battering-ram is as old as the art of war itſelf. And it has even been ſuſpected that the walls of Jericho, mentioned in the book of Joſhua, were beaten down by this inſtrument, the rams horns there mentioned, by means of which they were overthrown, being no other than the horns of the battering-rams. Pephaſmenos, a Tyrian, afterwards contrived to ſuſpend it with ropes; and laſtly, Polydus, the Theſſalian, mounted it on wheels, at the ſiege of Byzantium, under Philip of Macedon.

Plutarch relates, that Marc Anthony, in the Parthian war, made uſe of a ram 80 feet long: and Vitruvius affirms that they were ſometimes 106, and even 120 feet long; which muſt have given an amazing force to this engine. The ram required 100 ſoldiers to work and manage it at one time; who being exhauſted, another century relieved them; by which means in was kept playing continually without intermiſſion. See fig. 2, plate V, which repreſents the battering-ram ſuſpended in its open frame; in which 3 denotes the form of the head, faſtened to the enormous beam 2, by three or four bands (4) of iron, of about four feet in breadth. At the extremity of each of theſe bands was an iron chain (5), one end of which was faſtened to a hook (6), and to the laſt link at the other extremity was firmly bound a cable, which ran the whole length of the beam to the end of the ram 7, where theſe cables were bound all together as faſt as poſſible with ſmall ropes. To the end of theſe cables was faſtened another, that conſiſted of ſeveral ſtrong cords platted together to a certain length, and then running ſingle (8), at each of which were placed ſeveral men, to balance and work the machine. 10 Is the chain or cable by which the ram was hung to the croſs beam (11), fixed on the top of the frame; and 12 is the baſe of the machine.

The unſuſpended ram differed from this only in the manner of working it; as it moved on ſmall wheels upon another large beam, inſtead of being ſlung by a chain or cable.

BATTERY, in the Military Art, a place raiſed to plant cannon upon, to play with more advantage upon the enemy. It conſiſts of an epaulment or a breaſtwork, of about 8 feet high, and 18 or 20 feet thick.

In all batteries, the open ſpaces through which the muzzles of the cannon are pointed, are called *Embraſures*, and the diſtances between the embraſures, *merlons*. The guns are placed upon a platform of planks &c, aſcending a little from the parapet, to check the recoil, and that the gun may be the eaſier brought back again to the parapet: they are placed from 12 to 16 feet diſtant from one another, that the parapet may be ſtrong, and the gunners have room to work.

Mortar BATTERIES differ from the others, in that the ſlope of the parapet is inwards, and it is without embraſures, the ſhells being fired quite over the parapet, commonly at an angle of 45 degrees elevation.

Open BATTERY, is nothing more than a number of cannon, commonly field-pieces, ranged in a row abreaſt of one another, perhaps on ſome ſmall natural elevation of the ground, or an artificial bank a little raiſed for the purpoſe.

Covered or *Maſked* BATTERY, is when the cannon and gunners are covered by a bank or breaſt-work, commonly made of bruſh-wood, faggots, and earth, called a faſcine battery.

Sunk or *Buried* BATTERY, is when its platform is ſunk, or let down into the ground, ſo that trenches muſt be cut in the earth oppoſite the muzzles of the guns, to ſerve as embraſures to fire through. This is moſtly uſed on the firſt making of approaches in beſieging and battering a place.

Croſs BATTERIES, are two batteries playing athwart each other upon the ſame object, forming an angle there, and battering to more effect, becauſe what one battery ſhakes, the other beats down.

BATTERY *d'Enfilade*, is one that ſcours or ſweeps the whole length of a ſtraight line.

BATTERY *en Echarpe*, is one that plays obliquely.

BATTERY *de Reverſe*, or *Murdering Battery*, is one that plays upon the enemy's back.

Camerade or *Joint* BATTERY, is when ſeveral guns play upon one place at the ſame time.

BATTERY, *in Electricity*, is a combination of coated ſurfaces of glaſs, commonly jars, ſo connected together that they may be charged at once, and diſcharged by a common conductor. Mr. Gralath, a German electrician, firſt contrived to increaſe the ſhock by charging ſeveral phials at the ſame time.—Dr Franklin, having analyſed the Leyden phial, and found that it loſt at one ſurface the electric fire received at the other, conſtructed a battery of eleven large panes of ſaſh window glaſs, coated on both ſides, and ſo connected that the whole might be charged together, and with the ſame labour as one ſingle pane; then by bringing all the *giving* ſides into contact with one wire, and all the *receiving* ſides with another, he contrived to unite the force of all the plates, and to diſcharge them at once.—Dr. Prieſtley deſcribes a ſtill more complete battery. This conſiſts of 64 jars, each 10 inches long, and 2½ inches in diameter, all coated within an inch and a half of the top, forming in the whole about 32 ſquare feet of coated ſurface. A piece of very fine wire is twiſted about the lower end of the wire of each jar, to touch the inſide coating in ſeveral places; and it is put through a pretty large piece of cork, within the jar, to prevent any part of it from touching the ſide, by which a ſpontaneous diſcharge might be made. Each wire is turned round ſo as to make a loop at the upper end; and through theſe loops paſſes a pretty thick braſs rod with knobs, each rod ſerving for one row of the jars; and theſe rods are made to communicate together by a thick chain laid over them, or as many of them as may be wanted. The jars ſtand in a box, the bottom of which is covered with a tin plate; and a bent wire touching the plate paſſes through the box, and appears on the outſide. To this wire is faſtened any conductor deſigned to communicate with the outſide of the battery; and the diſcharge is made by bringing the braſs-knob to any of the knobs of the battery. When a very great force is required, the ſize or number of the jars may be increaſed, or two or more batteries may be uſed.—But the largeſt and moſt powerful battery of all, is that employed by Dr. Van Marum, to the amazing large electrical machine, lately conſtructed for Teyler's muſeum at Haarlem. This grand battery conſiſts of a great number of jars coated as above, to the amount of about 130 ſquare feet; and the effects of it, which are truly aſtoniſhing, are related by Dr. Van Marum in his deſcription of this machine, and of the experiments made with it, at Haarlem 1785, &c. See alſo Franklin's Exper. and Obſerv. and Prieſtley's Hiſtory of Electricity.

BATTLEMENTS, in Architecture, are notches or indentures in the top of a wall or other building, like embraſures, to look through.

BAY, in Geography, denotes a ſmall gulph, or an arm of the ſea ſtretching up into the land; being larger in the middle within, than at its entrance, which is called the mouth of the bay.

BAYER (JOHN), a German lawyer and aſtronomer of the latter part of the 16th and beginning of the 17th century, but in what particular year or place he was born, is not certainly known: however, his name will be ever memorable in the annals of aſtronomy, on account of that great and excellent work which he firſt publiſhed in the year 1603, under the title of *Uranometria*, being a complete celeſtial atlas, or large folio charts of all the conſtellations, with a nomenclature collected from all the tables of aſtronomy, ancient and modern, with the uſeful invention of denoting the ſtars in every conſtellation by the letters of the Greek alphabet, in their order, and according to the order of magnitude of the ſtars in each conſtellation. By means of theſe marks, the ſtars of the heavens may, with as great facility, be diſtinguiſhed and referred to, as the ſeveral places of the earth are by means of geographical tables; and as a proof of the uſefulneſs of this method, our celeſtial globes and atlaſſes have ever ſince retained it; and hence it is become of general uſe through all the literary world; aſtronomers, in ſpeaking of any ſtar in the conſtellation, denoting it by ſaying it is marked by Bayer, α, or β, or γ, &c.

Bayer lived many years after the firſt publication of this work, which he greatly improved and augmented by his conſtant attention to the ſtudy of the ſtars. At length, in the year 1627, it was republiſhed under a new title, viz, *Coelum Stellatum Chriſtianum*, that is, the *Chriſtian Stellated Heaven*, or the *Starry Heavens Chriſtianized*: for in this work, the heathen names and characters, or figures of the conſtellations, were rejected, and others, taken from the ſcriptures, were inſerted in their ſtead, to circumſcribe the reſpective conſtellations. This was the project of one Julius Schiller, a civilian of the ſame place. But this attempt was too great an innovation, to find ſucceſs, or a general reception, which might occaſion great confuſion. And, we even find in the later editions of this work, that the ancient figures and names were reſtored again; at leaſt ſo I find them in two editions, of the years 1654, and 1661, which are now before me.

BEAD, in Architecture, is a round moulding, carved in ſhort emboſſments, like beads in necklaces: and ſometimes an aſtragal is thus carved. There is alſo a ſort of plain bead often ſet on the edge of each facia of an architrave; as alſo ſometimes on the lining board of a door caſe, the upper edge of ſkirting boards, &c.

Bead, in aſſaying, the ſmall ball or maſs of pure metal ſeparated from the *ſcoriæ*, and ſeen diſtinct and pure in the middle of the coppel while in the fire.

BEAM, in Architecture, a large timber laid acroſs a building, into which the principal rafters are framed. Several ingenious authors have conſidered the force or ſtrength of beams, as ſupporting their own weight and any other additional weight; particularly Varignon, and Parent in the Memoir. Acad. R. Scien. an. 1708, and Mr. Emerſon, on the Strength and Streſs of Timber, in his Mechanics. Mr. Parent makes the proportion of the depth to the breadth of a beam to be as 7 to 5 when it is ſtrongeſt.

Beams *of a ſhip*, are the large, main, croſs timbers, ſtretched from ſide to ſide, to ſupport the decks, and keep the ſides of the ſhip from falling together.

Beam *of a balance*, is the horizontal piece of wood or iron ſupported on a pivot in the middle, and at the extremities of which the two ſcales are ſuſpended, for weighing any thing.

Beam-*Compaſs*, an inſtrument conſiſting of a wooden or braſs ſquare beam, having ſliding ſockets carrying ſteel or pencil points; and are uſed for deſcribing large circles, the radii of which are beyond the extent of the common compaſſes.

BEAR, in Aſtronomy, a name given to two conſtellations, called the *greater* and the *leſſer bear*, or Ursa *major* and *minor*. The pole ſtar is in the tail of the little bear, and is within leſs than 2 degrees of the north pole. See Ursa, *major* and *minor*.

BEARD, *of a Comet*, the rays which it emits in the direction in which it moves; as diſtinguiſhed from the tail, or the rays emitted or left behind it as it moves along, being always in a direction from the ſun.

BEARER, in Architecture, a poſt or brick wall, trimmed up between the two ends of a piece of timber, to ſhorten its bearing, or to prevent its bearing with the whole weight at the ends only.

BEARING, in Geography and Navigation, the ſituation of one place from another, with regard to the points of the compaſs; or an arch of the horizon between the meridian of a place and a line drawn through this and another place, or the angle formed by a line drawn through the two places and their meridians.—The bearings of places on the ground are uſually determined by the magnetic needle.

BEATS, in a Clock or Watch, are the ſtrokes made by the fangs or pallets of the ſpindle of the balance; or of the pads in a royal pendulum. For the number and uſe of the beats, ſee Derham's Artificial Clock Maker, pa. 14 and ſeq.

BED, *of a Great Gun*, a plank laid between the cheeks of the carriage, on the middle tranſum, for the breech of the gun to reſt upon.

Bed, or *Stool*, of a mortar, a thick and ſtrong planking on which a mortar is placed, hollowed a little to receive the breech and trunions.

BED-MOULDING, in Architecture, a term uſed by workmen for thoſe members in a cornice which are placed below the coronet, or crown. It uſually conſiſts of theſe four members, an ogee, a liſt, a large boultine, and another liſt under the coronet.

BELIDOR (Bernard Forest de), an engineer in the ſervice of France, but born in Catalonia in 1698. He was profeſſor in the new ſchool of artillery at la Fere, where he publiſhed his courſe of mathematics for the uſe of the artillery and engineers. He was the firſt who ſeriouſly conſidered the quantity of gunpowder proper for charges, and reduced it to 2-3ds the quantity. He was named Aſſociate in the Academy of Sciences in 1751; and died Sept. 8, 1761, at 63 years of age.

His works that have been publiſhed, are:

1. Sommaire d'un Cours d'Architecture militaire, civile & hydraulique, in 12mo, 1720.
2. Nouveau Cours de Mathematiques, &c. in 4to, 1725.
3. La Science des Ingénieurs, in 4to, 1729.
4. Le Bombardier Francois, in 4to, 1734.
5. Architecture Hydraulique, 4 vols. in 4to, 1737.
6. Dictionnaire portatif de l'Ingénieur, in 8vo.
7. Traité des Fortifications, 4 vols. in 4to.

Beſides ſeveral pieces inſerted in the volumes of the Memoirs of the Academy of Sciences, for the years 1737, 1750, 1753, and 1756.

BELLATRIX, in Aſtronomy, a ruddy, glittering ſtar of the 2d magnitude, in the left ſhoulder of Orion. Its name is from the Latin *bellum*, as being anciently ſuppoſed to have great influence in kindling wars, and forming warriors.

BELTS, *Faſciæ*, in Aſtronomy, two zones or girdles ſurrounding the planet Jupiter, brighter than the reſt of his body, and terminated by parallel lines. They are obſerved however to be ſometimes broader and ſometimes narrower, and not always occupying exactly the ſame part of the diſc. Jupiter's belts were firſt obſerved and deſcribed by Huygens, in his Syſt. Saturn. Dark ſpots have often been obſerved on the belts of Jupiter; and M. Caſſini obſerved a permanent one on the northern ſide of the moſt ſouthern belt, by which he determined the length of Jupiter's days, or the time in which the planet revolves upon its axis, which is 9h. 56m. Some aſtronomers ſuppoſe that theſe belts are ſeas, which alternately cover and leave bare large tracts of the planet's ſurface: and that the ſpots are gulphs in thoſe ſeas, which are ſometimes dry, and ſometimes full. But Azout conceived that the ſpots are protuberances of the belts; and others again are of opinion that the tranſparent and moveable ſpots are the ſhadows of Jupiter's ſatellites.

Caſſini alſo ſpeaks of the belts of Saturn; being three dark, ſtraight, parallel bands, or *faſciæ*, on the diſc of that planet. But it does not appear that Saturn's belts adhere to his body, as thoſe of Jupiter do; but rather that they are large dark rings ſurrounding the planet at a diſtance. Some imagine that they are clouds in the atmoſphere of Saturn, though it would ſeem that the middlemoſt is the ſhadow of his ring.

BENDING, the reducing a body to a curved or crooked form. The bending of boards, planks, &c, is effected by means of heat, whether by boiling or otherwiſe, by which their fibres are ſo relaxed that they may be bent into any figure. Bernoulli has a diſcourſe on the bending of ſprings, or elaſtic bodies. And Amontons gives ſeveral experiments concerning the bending of ropes. The friction of a rope bent or wound about an immoveable cylinder, is ſufficient, with a very ſmall power, to ſuſtain very great weights.

BERENICE's Hair; ſee Coma *Berenices*.

BERKELEY (George), the virtuous and learned biſhop of Cloyne in Ireland, was born in that kingdom, at Kilcrin, the 12th of March 1684. After receiving the firſt part of his education at Kilkenny ſchool, he was admitted a penſioner of Trinity College, Dublin, at 15 years old; and choſen fellow of that college in 1707.

The firſt public proof he gave of his literary abilities was, *Arithmetica abſque Algebra aut Euclide demonſtrata;* which from the preface it appears he wrote before he was 20 years old, though he did not publiſh it till 1707. It is followed by a Mathematical Miſcellany, containing obſervations and theorems inſcribed to his pupil Samuel Molineux.

In 1709 came out the *Theory of Viſion;* which of all his works it ſeems does the greateſt honour to his ſagacity; being, it has been obſerved, the firſt attempt that ever was made to diſtinguiſh the immediate and natural objects of ſight, from the concluſions we have been accuſtomed from infancy to draw from them. The boundary is here traced out between the ideas of ſight and touch; and it is ſhewn, that though habit hath ſo connected theſe two claſſes of ideas in the mind, that they are not without a ſtrong effort to be ſeparated from each other, yet originally they have no ſuch connection; inſomuch, that a perſon born blind, and ſuddenly made to ſee, would at firſt be utterly unable to tell how any object that affected his ſight would affect his touch; and particularly would not from ſight receive any idea of diſtance, or external ſpace, but would imagine all objects to be in his eye, or rather in his mind.

In 1710 appeared *The Principles of Human Knowledge;* and in 1713 *Dialogues between Hylas and Philonous:* the object of both which pieces is, to prove that the commonly received notion of the exiſtence of matter, is falſe; that ſenſible material objects, as they are called, are not external to the mind, but exiſt in it, and are nothing more than impreſſions made upon it by the immediate act of God, according to certain rules termed laws of nature.

Acuteneſs of parts and beauty of imagination were ſo conſpicuous in Berkeley's writings, that his reputation was now eſtabliſhed, and his company courted; men of oppoſite parties concurred in recommending him. For Steele he wrote ſeveral papers in the Guardian, and at his houſe became acquainted with Pope, with whom he always lived in friendſhip. Swift recommended him to the celebrated earl of Peterborough, who being appointed ambaſſador to the king of Sicily and the Italian States, took Berkeley with him as chaplain and ſecretary in 1713, with whom he returned to England the year following.

His hopes of preferment expiring with the fall of queen Anne's miniſtry, he ſome time after embraced an offer made him by Aſhe, biſhop of Clogher, of accompanying his ſon in a tour through Europe. In this he employed four years; and beſides thoſe places which fall within the grand tour, he viſited ſome that are leſs frequented, and with great induſtry collected materials for a natural hiſtory of thoſe parts, but which were unfortunately loſt in the paſſage to Naples. He arrived at London in 1721; and being much affected with the miſeries of the nation, occaſioned by the South-ſea ſcheme in 1720, he publiſhed the ſame year *An Eſſay towards preventing the ruin of Great Britain:* reprinted in his *Miſcellaneous Tracts*.

His way was now open into the very firſt company. Pope introduced him to lord Burlington, by whom he was recommended to the duke of Grafton, then appointed lord-lieutenant of Ireland, who took Berkeley over as one of his chaplains in 1721. The latter part of this year he accumulated the degrees of bachelor and doctor in divinity: and the year following he had a very unexpected increaſe of fortune from the death of Mrs. Vanhomrigh, the celebrated Vaneſſa, to whom he had been introduced by Swift: this lady had intended Swift for her heir; but perceiving herſelf to be ſlighted by him, ſhe left her fortune, of 8000l. between her two executors, of whom Berkeley was one. In 1724 he was promoted to the deanery of Derry, worth 1100l. a year.

In 1725 he publiſhed, and it has ſince been reprinted in his Miſcellaneous Tracts, *A Propoſal for converting the ſavage Americans to Chriſtianity, by a college to be erected in the Summer Iſles, otherwiſe called the Iſles of Bermuda.* The propoſal was well received, at leaſt by the king; and he obtained a charter for founding the college, with a parliamentary grant of 20,000l. toward carrying it into execution: but he could never get the money, it being otherwiſe employed by the miniſter; ſo that after two years ſtay in America on this buſineſs, he was obliged to return, and the ſcheme dropped.

In 1732 he publiſhed *The Minute Philoſopher*, in 2 volumes 8vo, againſt Freethinkers. In 1733 he was made biſhop of Cloyne; and might have been removed in 1745, by lord Cheſterfield, to Clogher; but declined it. He reſided conſtantly at Cloyne, where he faithfully diſcharged all the offices of a good biſhop, yet continued his ſtudies with unabated attention.

About this time he engaged in a controverſy with the mathematicians, which made a good deal of noiſe in the literary world; and the occaſion of it was this:

Addiſon had given the biſhop an account of the behaviour of their common friend Dr. Garth in his laſt illneſs, which was equally unpleaſing to both theſe advocates of revealed religion. For when Addiſon went to ſee the doctor, and began to diſcourſe with him ſeriouſly about another world, “Surely, Addiſon, replied he, I have good reaſon not to believe thoſe trifles, ſince my friend Dr. Halley, who has dealt ſo much in demonſtration, has aſſured me, that the doctrines of chriſtianity are incomprehenſible, and the religion itſelf an impoſture.” The biſhop therefore took up arms againſt Halley, and addreſſed to him, as to an Infidel Mathematician, a diſcourſe called *The Analyſt*; with a view of ſhewing that myſteries in faith were unjuſtly objected to by mathematicians, who he thought admitted much greater myſteries, and even falſhoods in ſcience, of which he endeavoured to prove that the doctrine of Fluxions furniſhed a clear example. This attack gave occaſion to *Robins's Diſcourſe concerning the Method of Fluxions*, to *Maclaurin's Fluxions*, and to other ſmaller works upon the ſame ſubject; but the direct anſwers to *The Analyſt* were made by a perſon under the name of Philalethes Cantabrigienſis, but commonly ſuppoſed to be Dr. Jurin, whoſe firſt piece was, *Geometry no Friend to Infidelity*, 1734. To this the biſhop replied in *A Defence of Freethinking in Mathematics; with an Appendix concerning Mr. Walton's Vindication*, 1735; which drew a ſecond anſwer the ſame year from Philalethes, under the title of *The Minute Mathematician, or the Freethinker no juſt Thinker*, 1735. Other writings in this controverſy, beſide thoſe before mentioned, were

1. A Vindication of Newton's Principles of Fluxions againſt the objections contained in the Analyſt, by J. Walton, Dublin, 1735.
2. The Catechiſm of the Author of the Minute Philoſopher fully anſwered, by J. Walton, Dublin, 1735.
3. Reaſons for not replying to Mr. Walton's Full Anſwer, in a letter to P. T. P. by the author of the Minute Philoſopher, Dublin, 1735.
4. An Introduction to the Doctrine of Fluxions, and Defence of the Mathematicians againſt the objections of the author of the Analyſt, &c. Lond. 1736.
5. A new Treatiſe of Fluxions; with anſwers to the principal objections in the Analyſt, by James Smith, A. M. Lond. 1737.
6. Mr. Robins's Diſcourſe of Newton's Methods of Fluxions, and of Prime and Ultimate Ratios, 1735.
7. Mr. Robins's Account of the preceding Diſcourſe, in the Repub. of Letters, for October 1735.
8. Philalethes's Conſiderations upon ſome paſſages contained in two letters to the author of the Analyſt &c, in Repub. of Letters, Novemb. 1735.
9. Mr. Robins's Review of ſome of the principal objections that have been made to the doctrine of Fluxions &c. Repub. of Letters for Decem. 1735.
10. Philalethes's Reply to ditto, in the Repub. of Letters, Jan. 1736.
11. Mr. Robins's Diſſertation, ſhewing that the account of the doctrines of Fluxions &c, is agreeable to the real ſenſe and meaning of their great Inventor, &c, Repub. of Letters, April 1736.
12. Philalethes's Conſiderations upon ditto, in Repub. of Letters, July 1736.
13. Mr. Robins's Remarks on ditto, in Repub. of Letters, Aug. 1736.
14. Mr. Robins's Remainder of ditto, in an Appendix to the Repub. of Letters, Sept. 1736.
15. Philalethes's Obſervations upon ditto, in an Appendix to the Repub. of Letters, Nov. 1736.
16. Mr. Robins's Advertiſement in Repub. of Letters, Decemb. 1736.
17. Philalethes's Reply to ditto, in an Appendix to the Repub. of Letters for Decem. 1736.
18. Some Obſervations on the Appendix to the Repub. of Letters for Decem. 1736, by Dr. Pemberton, in the Works of the Learned for Feb. 1737. With ſome ſmaller pieces in the ſame.

In 1736 biſhop Berkeley publiſhed *The Queriſt*, “a diſcourſe addreſſed to magiſtrates, occaſioned by the enormous licence and irreligion of the times;” and many other things afterward of a ſmaller kind. In 1744 came out his celebrated and curious book, “*Siris*; a Chain of Philoſophical Reflections and Inquiries concerning the virtues of Tar-water.” It had a ſecond impreſſion, with additions and emendations, in 1747; and was followed by *Farther Thoughts on Tar-water*, in 1752. In July the ſame year he removed, with his lady and family, to Oxford, partly to ſuperintend the education of a ſon, but chiefly to indulge the paſſion for learned retirement, which had always ſtrongly poſſeſſed him. He would have reſigned his biſhoprick for a canonry or headſhip at Oxford; but it was not permitted him. Here he lived highly reſpected, and collected and printed the ſame year all his ſmaller pieces in 8vo. But this happineſs did not long continue, being ſuddenly cut off by a palſy of the heart Jan. 14, 1753, in the 69th year of his age, while liſtening to a ſermon that his lady was reading to him.

The excellence of Berkeley's moral character is conſpicuous in his writings: he was certainly a very amiable as well as a very great man; and it is thought that Pope ſcarcely ſaid too much, when he aſcribed

> “To Berkeley every virtue under heaven.”

BERME, in Fortification, a ſmall ſpace of ground, 4 or 5 feet wide, left without the rampart, between it and the ſide of the moat, to receive the earth that rolls down from the rampart, and prevent its falling into the ditch and filling it up.—Sometimes, for greater ſecurity, the berme is palliſadoed.

BERNARD (Dr. EDWARD), a learned aſtronomer, critic and linguiſt, was born at Perry St. Paul, near Towceſter, the 2d of May 1638, and educated at Merchant-Taylor's ſchool, and at St. John's college Oxford. Having laid in a good fund of claſſical learning at ſchool, in the Greek and Latin languages, he applied himſelf very diligently at the univerſity to the ſtudy of hiſtory, the eaſtern languages, and mathematics under the celebrated Dr. Wallis. In 1668 he went to Leyden to conſult ſome Oriental manuſcripts left to that univerſity by Joſeph Scaliger and Levin Warner, and eſpecially the 5th, 6th, and 7th books of Apollonius's Conics, the Greek text of which is loſt, and this Arabic verſion having been brought from the eaſt by the celebrated

Golius, a tranſcript of which was thence taken by Bernard, and brought with him to Oxford, with intent to publiſh it there with a Latin tranſlation; but he was obliged to drop that deſign for want of encouragement. This however was afterwards carried into effect by Dr. Halley in 1710, with the addition of the 8th book, which he ſupplied by his own ingenuity and induſtry.

At his return to Oxford, Bernard examined and collated the moſt valuable manuſcripts in the Bodleian library. In 1669, the celebrated Chriſtopher Wren, Savilian profeſſor of aſtronomy at Oxford, having been appointed ſurveyor-general of his majeſty's works, and being much detained at London by this employment, obtained leave to name a deputy at Oxford, and pitched upon Mr. Bernard, which engaged the latter in a more particular application to the ſtudy of aſtronomy. But in 1673 he was appointed to the profeſſorſhip himſelf, which Wren was obliged to reſign, as, by the ſtatutes of the founder, Sir Henry Saville, the profeſſors are not allowed to hold any other office either eccleſiaſtical or civil.

About this time a ſcheme was ſet on foot at Oxford, of collecting and publiſhing the ancient mathematicians. Mr. Bernard, who had firſt formed the project, collected all the old books publiſhed on that ſubject ſince the invention of printing, and all the manuſcripts he could diſcover in the Bodleian and Savilian libraries, which he arranged in order of time, and according to the matter they contained; of this he drew up a ſynopſis or view; and as a ſpecimen he publiſhed a few ſheets of Euclid, containing the Greek text, and a Latin verſion, with Proclus's commentary in Greek and Latin, and learned ſcholia and corollaries. The ſynopſis itſelf was publiſhed by Dr. Smith, at the end of his life of our author, under the title of *Veterum Mathematicorum Græcorum, Latinorum, et Arabum, Synopſis.* And at the end of it there is a catalogue of ſome Greek writers, whoſe works are ſuppoſed to be loſt in their own language, but are preſerved in the Syriac or Arabic tranſlations of them.

Mr. Bernard undertook alſo an edition of the *Parva Syntaxis Alexandrina;* in which, beſides Euclid, are contained the ſmall treatiſes of Theodoſius, Menelaus, Ariſtarchus, and Hipſicles; but it never was publiſhed.

In 1676 he was ſent to France, as tutor to the dukes of Grafton and Northumberland, ſons to king Charles the 2d by the dutcheſs of Cleveland, who then lived with their mother at Paris: but the ſimplicity of his manners not ſuiting the gaiety of the dutcheſs's family, he returned about a year after to Oxford, and purſued his ſtudies; in which he made great proficiency, as appears by his many learned and critical works. In 1691, being preſented to the rectory of Brightwell in Berkſhire, he quitted his profeſſorſhip at Oxford, in which he was ſucceeded by David Gregory, profeſſor of mathematics at Edinburgh.

Toward the latter end of his life he was much afflicted with the ſtone; yet notwithſtanding this, and other infirmities, he undertook a voyage to Holland, to attend the ſale of Golius's manuſcripts, as he had once before done at the ſale of Heinſius's library. On his return to England, he fell into a languiſhing conſumption, which put an end to his life the 12th of January 1696, in the 58th year of his age.

Beſide the works of his before mentioned, he was author of many other compoſitions. He compoſed tables of the longitudes, latitudes, right-aſcenſions, &c. of the fixed ſtars: he wrote Obſervations on the Obliquity of the Ecliptic; and other pieces inſerted in the Philoſophical Tranſactions. He wrote alſo,

1. *A Treatiſe of the Ancient Weights and Meaſures.*
2. *Chronologiæ Samaritanæ Synopſis,* in two tables.
3. *Teſtimonies of the Ancients concerning the Greek Verſion of the Old Teſtament by the Seventy.*

And ſeveral other learned works. Beſides a great number of valuable manuſcripts left at his death.

BERNARD (Dr. James), profeſſor of philoſophy and mathematics, and miniſter of the Walloon church at Leyden, was born September the 1ſt 1658, at Nions in Dauphine. Having ſtudied at Geneva, he returned to France in 1679, and was choſen miniſter of Venterol, a village in Dauphine; but ſome time after he was removed to the church of Vinſobres in the ſame province. To avoid the perſecutions againſt the proteſtants in France, he went into Holland, where he was appointed one of the penſionary miniſters of Ganda. He here publiſhed ſeveral political and hiſtorical works. And in 1699 he began the *Nouvelles de la Republique des Lettres*, which continued till December 1710. In 1705 he was choſen miniſter of the Walloon church at Leyden; and about the ſame time, Mr. de Volder, profeſſor of philoſophy and mathematics at Leyden, having reſigned, Mr. Bernard was appointed his ſucceſſor; upon which occaſion the univerſity alſo preſented him with the degrees of doctor of philoſophy and maſter of arts. In 1716 he publiſhed a ſupplement to Moreri's dictionary in 2 volumes folio. The ſame year he reſumed his *Nouvelles de la Republique des Lettres;* which he continued till his death, which happened the 27th of April 1718, in the 60th year of his age.

BERNOULLI (James), a celebrated mathematician, born at Baſil the 27th of December 1654. Having taken his degrees in that univerſity, he applied himſelf to divinity at the entreaties of his father, but againſt his own inclination, which led him to aſtronomy and mathematics. He gave very early proofs of his genius for theſe ſciences, and ſoon became a geometrician, without a preceptor, and almoſt without books; for if one by chance fell into his hands, he was obliged to conceal it, to avoid the diſpleaſure of his father, who deſigned him for other ſtudies. This ſituation induced him to chooſe for his device, Phaeton driving the chariot of the ſun, with theſe words, *Invito patre ſidera verſo,* "I traverſe the ſtars againſt my father's will:" alluding particularly to aſtronomy, to which he then chiefly applied himſelf.

In 1676 he began his travels. When he was at Geneva, he fell upon a method to teach a young girl to write who had been blind from two months old. At Bourdeaux he compoſed univerſal gnomonic tables; but they were never publiſhed. He returned from France to his own country in 1680. About this time there appeared a comet, the return of which he foretold; and wrote a ſmall treatiſe upon it. Soon after this he went into Holland, where he applied himſelf to the ſtudy of the new philoſophy. Having viſited Flanders and Brabant, he paſſed over to England; where he formed an acquaintance with the moſt eminent men in the ſciences, and was frequent at their philoſophical meetings. He returned to his native country in 1682; and exhibited at Baſil a courſe of experiments in natural philoſophy and mechanics, which conſiſted of a variety of new diſcoveries. The ſame year he publiſhed his Eſſay on a new Syſtem of Comets; and the year following, his Diſſertation on the weight of the air. About this time Leibnitz having publiſhed, in the Acta Eruditorum at Leipſic, ſome eſſays on his new *Calculus Differentialis*, but concealing the art and method of it, Mr. Bernoulli and his brother John diſcovered, by the little which they ſaw, the beauty and extent of it: this induced them to endeavour to unravel the ſecret; which they did with ſuch ſucceſs, that Leibnitz declared that the invention belonged to them as much as to himſelf.

In 1687 James Bernoulli ſucceeded to the profeſſorſhip of mathematics at Baſil; a truſt which he diſcharged with great applauſe; and his reputation drew a great number of foreigners from all parts to attend his lectures. In 1699 he was admitted a foreign member of the Academy of Sciences of Paris; and in 1701 the ſame honour was conferred upon him by the Academy of Berlin: in both of which he publiſhed ſeveral ingenious compoſitions, about the years 1702, 3, and 4. He wrote alſo ſeveral pieces in the *Acta Eruditorum* of Leipſic, and in the *Journal des Sçavans*. His intenſe application to ſtudy brought upon him the gout, and by degrees a ſlow fever, which put a period to his life the 16th of Auguſt 1705, in the 51ſt year of his age. — Archimedes having found out the proportion of a ſphere and its circumſcribing cylinder, ordered them to be engraven on his monument: In imitation of him, Bernoulli appointed that a logarithmic ſpiral curve ſhould be inſcribed on his tomb, with theſe words, *Eadem mutata reſurgo;* in alluſion to the hopes of the reſurrection, which are in ſome meaſure repreſented by the properties of that curve, which he had the honour of diſcovering.

James Bernoulli had an excellent genius for invention and elegant ſimplicity, as well as a cloſe application. He was eminently ſkilled in all the branches of the mathematics, and contributed much to the promoting the new analyſis, infinite ſeries, &c. He carried to a great height the theory of the quadrature of the parabola; the geometry of curve lines, of ſpirals, of cycloids and epicycloids.

His works, that had been publiſhed, were collected, and printed in 2 volumes 4to, at Geneva in 1744. At the time of his death he was occupied on a great work entitled *De Arte Conjectandi*, which was publiſhed in 4to, in 1713. It contains one of the beſt and moſt elegant introductions to Infinite Series, &c. This poſthumous work is omitted in the collection of his works above mentioned.

BERNOULLI (John), the brother of James, laſt mentioned, and a celebrated mathematician, was born at Baſil the 7th of Auguſt 1667. His father intended him for trade; but his own inclination was at firſt for the Belles-Lettres, which however, like his brother, he left for mathematics. He laboured with his brother to diſcover the method uſed by Leibnitz, in his eſſays on the Differential Calculus, and gave the firſt principles of the Integral Calculus. Our author, with Meſſieurs Huygens and Leibnitz, was the firſt who gave the ſolution of the problem propoſed by James Bernoulli, concerning the catenary, or curve formed by a chain ſuſpended by its two extremities.

John Bernoulli had the degree of doctor of phyſic at Baſil, and two years afterward was named profeſſor of mathematics in the univerſity of Groningen. It was here that he diſcovered the mercurial phoſphorus or luminous barometer; and where he reſolved the problem propoſed by his brother concerning Iſoperimetricals.

On the death of his brother James, the profeſſor at Baſil, our author returned to his native country, againſt the preſſing invitations of the magiſtrates of Utrecht to come to that city, and of the univerſity of Groningen, who wiſhed to retain him. The Academic Senate of Baſil ſoon appointed him to ſucceed his brother, without aſſembling competitors, and contrary to the eſtabliſhed practice: an appointment which he held during his whole life.

In 1714 was publiſhed his treatiſe on the management of ſhips; and in 1730, his memoir on the elliptical figure of the planets gained the prize of the Academy of Sciences. The ſame academy alſo divided the prize, for their queſtion concerning the inclination of the planetary orbits, between our author and his ſon Daniel.

John Bernoulli was a member of moſt of the academies of Europe, and received as a foreign aſſociate of that of Paris in 1699. After a long life ſpent in conſtant ſtudy and improvement of all the branches of the mathematics, he died full of honours the 1ſt of January 1748, in the 81ſt year of his age. Of five ſons which he had, three purſued the ſame ſciences with himſelf. One of theſe died before him; the two others, Nicolas and Daniel, he lived to ſee become eminent and much reſpected in the ſame ſciences.

The writings of this great man were diſperſed through the periodical memoirs of ſeveral academies, as well as in many ſeparate treatiſes. And the whole of them were carefully collected and publiſhed at Lauſanne and Geneva, 1742, in 4 volumes, 4to.

BERNOULLI (Daniel), a celebrated phyſician and philoſopher, and ſon of John Bernoulli laſt mentioned, was born at Groningen Feb. the 9th, 1700, where his father was then profeſſor of mathematics. He was intended by his father for trade, but his genius led him to other purſuits. He paſſed ſome time in Italy; and at 24 years of age he declined the honour offered him of becoming preſident of an academy intended to have been eſtabliſhed at Genoa. He ſpent ſeveral years with great credit at Peterſburgh; and in 1733 returned to Baſil, where his father was then profeſſor of mathematics; and here our author ſucceſſively filled the chair of phyſic, of natural and of ſpeculative philoſophy.

In his work *Exercitationes Mathematicæ*, 1724, he took the only title he then had, viz, "Son of John Bernoulli," and never would ſuffer any other to be added to it. This work was publiſhed in Italy, while he was there on his travels; and it claſſed him in the rank of inventors. In his work, *Hydrodynamica*, publiſhed in 4to at Argentoratum or Straſbourg, in 1738, to the ſame title was alſo added that of *Med. Prof. Baſil.*

Daniel Bernoulli wrote a multitude of other pieces, which have been publiſhed in the Mem. Acad. of Sciences at Paris, and in thoſe of other Academies. He gained and divided ten prizes from the Academy of

Sciences, which were contended for by the moſt illuſtrious mathematicians in Europe. The only perſon who has had ſimilar ſucceſs of the ſame kind, is Euler, his countryman, diſciple, rival, and friend. His firſt prize he gained at 24 years of age. In 1734 he divided one with his father; which hurt the family union; for the father conſidered the conteſt itſelf as a want of reſpect; and the ſon did not ſufficiently conceal that he thought (what was really the caſe) his own piece better than his father's. And beſides, he declared for Newton, againſt whom his father had contended all his life. In 1740 our author divided the prize, "On the Tides of the Sea," with Euler and Maclaurin. The Academy at the ſame time crowned a fourth piece, whoſe chief merit was that of being Carteſian; but this was the laſt public act of adoration paid by the Academy to the authority of the author of the Vortices, which it had obeyed but too long. In 1748 Daniel Bernoulli ſucceeded his father John in the Academy of Sciences, who had ſucceeded his brother James; this place, ſince its firſt erection in 1699, having never been without a Bernoulli to fill it.

Our author was extremely reſpected at Baſil; and to bow to Daniel Bernoulli, when they met him in the ſtreets, was one of the firſt leſſons which every father gave every child. He was a man of great ſimplicity and modeſty of manners. He uſed to tell two little adventures, which he ſaid had given him more pleaſure than all the other honours he had received. Travelling with a learned ſtranger, who, being pleaſed with his converſation, aſked his name; "I am Daniel Bernoulli," anſwered he with great modeſty; "And I," ſaid the ſtranger (who thought he meant to laugh at him), "am Iſaac Newton." Another time having to dinner with him the celebrated Koenig the mathematician, who boaſted, with ſome degree of ſelf-complacency, of a difficult problem he had reſolved with much trouble, Bernoulli went on doing the honours of his table, and when they went to drink coffee he preſented Koenig with a ſolution of the problem more elegant than his own.

After a long, uſeful, and honourable life, Daniel Bernoulli died the 17th of March 1782, in the 83d year of his age.

BETELGEUSE, a fixed ſtar of the firſt magnitude in the right ſhoulder of Orion.

BEZOUT (Stephen), a celebrated French mathematician, Member of the Academies of Sciences and the Marine, and Examiner of the Guards of the Marine and of the Eleves of Artillery, was born at Nemours the 31ſt of March 1730. In the courſe of his ſtudies he met with ſome books of geometry, which gave him a taſte for that ſcience; and the Eloges of Fontenelle, which ſhewed him the honours attendant on talents and the love of the ſciences. His father in vain oppoſed the ſtrong attachment of young Bezout to the mathematical ſciences. April 8, 1758, he was named adjoint-mechanician in the French academy of ſciences; having before that ſent them two ingenious memoirs on the integral calculus, and given other proofs of his proficiency in the ſciences. In 1763, he was named to the new office of Examiner to the Marine, and appointed to compoſe a Courſe of Mathematics for their uſe; and in 1768, on the death of M. Camus, he ſucceeded as Examiner of the Artillery Eleves.

Bezout fixed his attention more particularly to the reſolution of algebraic equations; and he firſt found out the ſolution of a particular claſs of equations of all degrees. This method, different from all former ones, was general for the cubic and biquadratic equations, and juſt became particular only at thoſe of the 5th degree. Upon this work of finding the roots of equations, our author laboured from 1762 till 1779, when he publiſhed it. He compoſed two courſes of mathematics; the one for the Marine, the other for the Artillery: The foundation of theſe two works was the ſame; the applications only being different, according to the two different objects: theſe courſes have every where been held in great eſtimation. In his office of examiner he diſcharged the duties with great attention, care, and tenderneſs; a trait of his juſtice and zeal is remarkable in the following inſtance: During an examination which he held at Toulon, he was told that two of the pupils could not be preſent, being confined by the ſmall-pox: he himſelf had never had that diſeaſe, and he was greatly afraid of it; but as he knew that if he did not ſee theſe two young men, it would much impede their improvement; he ventured therefore to their bed-ſides, to examine them, and was happy to find them ſo deſerving of the hazard he put himſelf into for their benefit.

Mr. Bezout lived thus ſeveral years beloved of his family and friends, and reſpected by all, enjoying the fruits and the credit of his labours. But the trouble and fatigues of his offices, with ſome perſonal chagrines, had reduced his ſtrength and conſtitution; he was attacked by a malignant fever, of which he died Sept. 27, 1783, in the 54th year of his age, regretted by his family, his friends, the young ſtudents, and by all his acquaintance in general.

The books publiſhed by him, were:

1. Courſe of Mathematics for the uſe of the Marine, with a Treatiſe on Navigation; 6 vols. in 8vo, Paris, 1764.
2. Courſe of Mathematics for the Corps of Artillery; 4 vols. in 8vo, 1770.
3. General Theory of Algebraic Equations; 1779.

His papers printed in the volumes of the Memoirs of the Academy of Sciences, are:

1. On curves whoſe rectification depends on a given quantity; in the vol. for 1758.
2. On ſeveral claſſes of equations that admit of an algebraic ſolution; 1762.
3. Firſt vol. of a courſe of mathematics, 1764.
4. On certain equations, &c; 1764.
5. General reſolution of all equations; 1765.
6. Second vol. of a courſe of mathematics; 1765.
7. Third vol. of the ſame; 1766.
8. Fourth vol. of the ſame; 1767.
9. Integration of differentials, &c. vol. 3, Sav. Etr.
10. Experiments on cold; 1777.

BIANCHINI (Francis), a very learned Italian philoſopher and mathematician of the 17th century, was born at Verona the 13th of December 1662. He was much eſteemed by the learned, and was a member of ſeveral academies; and was even the founder of that at Verona, called the Academy of Aletofili, or

Lovers of Truth. He went to Rome in 1684; and was made librarian to cardinal Ottoboni, who was afterwards Pope by the name of Alexander the 8th. He entered into the church, and became canon of St. Mary de la Rotondo, and afterward of St. Lawrence in Damaſo.

Bianchini was author of ſeveral learned and ingenious diſſertations. In 1697 he publiſhed *La Iſtoria univerſale provata con monumenti, & figurata con ſimboli de gli antichi.* In 1701 pope Clement the 11th named him ſecretary of the conferences for the reformation of the calendar; and he publiſhed in 1703, *De Calendario & Cyclo Cæſaris, ac de Canone Paſchali ſancti Hyppoliti, Martyris, Diſſertationes duæ.* Bianchini was employed on the conſtruction of the large gnomon in the church of the Chartreux at Rome, upon which he publiſhed an ample diſſertation intitled, *De Nummo & Gnomone Clementino.* The reſearch concerning the parallax and the ſpots of Venus occupied him a long time; but his moſt remarkable diſcovery is that of the parallelism of the axis of Venus in her orbit. He propoſed to trace a meridian line through the whole extent of Italy. He was admitted a foreign Aſſociate in the Paris Academy of Sciences, in 1706; and he had many aſtronomical diſſertations inſerted in their Memoirs, particularly in thoſe of the years 1702, 1703, 1704, 1706, 1707, 1708, 1713, and 1718.—Bianchini died the 2d of March 1729, in the 67th year of his age.

BIMEDIAL *Line*, is the ſum of two Medials. Euclid reckons two of theſe bimedials, in pr. 38 and 39 lib. x; the firſt is when the rectangle is rational, which is contained by the two medial lines whoſe ſum makes the bimedial; and the ſecond when that rectangle is a medial, or contained under two lines that are commenſurable only in power.

BINARY Number, that which is compoſed of two units.

BINARY *Arithmetic*, that in which two figures or characters, viz, 1 and 0, only are uſed; the cipher multiplying every thing by 2, as in the common arithmetic by ten: thus, 1 is one, 10 is 2, 11 is 3, 100 is 4, 101 is 5, 110 is 6, 111 is 7, 1000 is 8, 1001 is 9, 1010 is ten; being founded on the ſame principles as common arithmetic.—This ſort of arithmetic was invented by Leibnitz, who pretended that it is better adapted than the common arithmetic, for diſcovering certain properties of numbers, and for conſtructing tables; but he does not venture to recommend it, for ordinary uſe, on account of the great number of places of figures requiſite to expreſs all numbers, even very ſmall ones. Joſ. Pelican of Prague has more largely explained the principles and practice of the binary arithmetic, in a book entitled *Arithmeticus Perfectus, qui tria numerare neſcit;* 1712. And De Lagni propoſed a new ſyſtem of logarithms, on the plan of the binary arithmetic; which he finds ſhorter, and more eaſy and natural than the common ones.

BINOCLE, or BINOCULAR TELESCOPE, is one by which an object is viewed with both eyes at the ſame time. It conſiſts of two tubes, each furniſhed with glaſſes of the ſame power, by which means it has been ſaid to ſhew objects larger and more clearly than a monocular or ſingle teleſcope; though this is probably only an illuſion, occaſioned by the ſtronger impreſſion which two equal images, alike illuminated, make upon the eyes; and they have been found more embarraſſing than uſeful in practice. This teleſcope has been chiefly treated of by the fathers Reita and Cherubin of Orleans.—There are alſo microſcopes of the ſame kind, though but little uſed; being ſubject to the ſame inconveniences as the teleſcopes.

BINOMIAL, a quantity conſiſting of two terms or members connected by the ſign + or −, viz, plus or minus; as $a + b$, or $3a - 2c$, or $a^2 + b$, or $x^2 - 2\sqrt{c}$, &c; denoting the ſum or the difference of the two terms. But the difference is alſo ſometimes named a *reſidual*, and by Euclid an *apotome*. The term binomial was firſt introduced by Robert Recorde; ſee his algebra, pa. 462.

BINOMIAL *Line*, or *Surd*, is that in which at leaſt one of the parts is a ſurd. Euclid enumerates ſix kinds of binomial lines or ſurds, in the 10th book of his Elements, which are exactly ſimilar to the ſix reſiduals or apotomes there treated of alſo, and of which an account is given under the art. APOTOME, which ſee. Thoſe apotomes become binomials by only changing the ſign of the latter term from minus to plus, which therefore are as below.

Euclid's 6 Binomial Lines.

Firſt binomial	$3 + \sqrt{5}$,
2d binomial	$\sqrt{18} + 4$,
3d binomial	$\sqrt{24} + \sqrt{18}$,
4th binomial	$4 + \sqrt{3}$,
5th binomial	$\sqrt{6} + 2$,
6th binomial	$\sqrt{6} + \sqrt{2}$.

To extract the Square Root of a Binomial, as of $a + \sqrt{b}$, or $\sqrt{c} + \sqrt{b}$. Various rules have been given for this purpoſe. The firſt is that of Lucas De Burgo, in his *Summa de Arith.* &c, which is this: When one part, as a, is rational, divide it into two parts ſuch, that their product may be equal to $\frac{1}{4}$th of the number under the radical b; then ſhall the ſum of the roots of thoſe parts be the root of the binomial ſought: or their difference is the root when the quantity is reſidual. That is, if $c + e = a$, and $ce = \frac{1}{4} b$; then is $\sqrt{c} + \sqrt{e} = \sqrt{a + \sqrt{b}}$ the root ſought. As if the binomial be $23 + \sqrt{448}$; then the parts of 23 are 16 and 7, and their product is 112, which is $\frac{1}{4}$th of 448; therefore the ſum of their roots $4 + \sqrt{7}$ is the root ſought of $23 + \sqrt{448}$.

De Burgo gives alſo another rule for the ſame extractions, which is this: The given binomial being, for example, $\sqrt{c} + \sqrt{b}$, its root will be $\sqrt{\frac{1}{2}\sqrt{c} + \frac{1}{2}\sqrt{c - b}} + \sqrt{\frac{1}{2}\sqrt{c} - \frac{1}{2}\sqrt{c - b}}$.—So in the foregoing example, $23 + \sqrt{448}$, here $\sqrt{c} = 23$, and $\sqrt{b} = \sqrt{448}$; hence $\frac{1}{2}\sqrt{c} = 11\frac{1}{2}$, and $\frac{1}{2}\sqrt{c - b} = \frac{1}{2}\sqrt{23^2 - 448} = \frac{1}{2}\sqrt{81} = 4\frac{1}{2}$;

theref. $\sqrt{\frac{1}{2}\sqrt{c} + \frac{1}{2}\sqrt{c - b}} = \sqrt{11\frac{1}{2} + 4\frac{1}{2}} - \sqrt{16} = 4$,

and $\sqrt{\frac{1}{2}\sqrt{c} - \frac{1}{2}\sqrt{c - b}} = \sqrt{11\frac{1}{2} - 4\frac{1}{2}} = \sqrt{7}$;

conſeq. $4 + \sqrt{7}$ is the root ſought, as before. Again, if the binomial be $\sqrt{18} + \sqrt{10}$; here $c = 18$, and $b = 10$; theref. $\frac{1}{2}\sqrt{c} = \frac{1}{2}\sqrt{18} = \frac{3}{2}\sqrt{2}$, and $\frac{1}{2}\sqrt{c-b} = \frac{1}{2}\sqrt{8} = \sqrt{2}$; hence,

$$\sqrt{\tfrac{1}{2}\sqrt{c}+\tfrac{1}{2}\sqrt{c-b}} = \sqrt{\tfrac{3}{2}\sqrt{2}+\sqrt{2}} = \sqrt{\tfrac{5}{2}\sqrt{2}} = \sqrt[4]{\frac{25}{2}},$$ and

$$\sqrt{\tfrac{1}{2}\sqrt{c}-\tfrac{1}{2}\sqrt{c-b}} = \sqrt{\tfrac{3}{2}\sqrt{2}-\sqrt{2}} = \sqrt{\tfrac{1}{2}\sqrt{2}} = \sqrt[4]{\tfrac{1}{2}};$$

confeq. $\sqrt[4]{\frac{25}{2}} + \sqrt[4]{\frac{1}{2}}$ or $\frac{\sqrt{5}+1}{\sqrt[4]{2}}$ is the root of $\sqrt{18} + \sqrt{10}$ ſought. And this latter rule has been uſed by all other authors, down to the preſent time.

To extract the Cubic and other higher Roots of a Binomial. This is uſeful in reſolving cubic and higher equations, and was introduced with the reſolution of thoſe equations by Tartalea and Cardan. The rules for ſuch extractions are in great meaſure tentative; and ſome of the principal ones are the following.

Tartalea's Rule for the Cube Root of a Binomial $p+q$. This rule is given in his 9th book of Miſcellaneous Queſtions, queſt. 40; and it is made out from either of the terms, p or q, of the binomial, taken ſingly, in this manner: Separate either term, as p, into two ſuch parts that the one of them may be a cubic number, and the other part diviſible by 3 without a remainder; then the cube root of the ſaid cubic part will be one term of the root, and the other term will be the ſquare root of the quotient ariſing from dividing the aforeſaid third part by the firſt term juſt found. So if p be divided into $r^3 + 3s$, then the root is $r + \sqrt{\frac{s}{r}}$. For example, to extract the cube root of $\sqrt{108} + 10$. Suppoſe the part 10 be taken: this ſeparates into the parts 1 and 9, the former of which is a cube, and the latter diviſible by 3; that is $r^3 = 1$, and $3s = 9$; hence $r = 1$, and $s = 3$; conſequently $r + \sqrt{\frac{s}{r}} = 1 + \sqrt{3}$ is the cubic root of $\sqrt{108} + 10$ ſought. Again, to uſe the other term $\sqrt{108}$: this divides into $\sqrt{27} + \sqrt{27}$, of which the former is a cube, and the latter diviſible by 3; that is, $r^3 = \sqrt{27}$, and $3s = \sqrt{27}$; therefore $r = \sqrt{3}$, and $s = \frac{1}{3}\sqrt{27} = \sqrt{3}$ alſo: hence $r + \sqrt{\frac{s}{r}} = \sqrt{3} \div \sqrt{\frac{\sqrt{3}}{\sqrt{3}}} = \sqrt{3} + 1$ the cube root, the ſame as before.

Bombelli's Rule for the Cubic Root of the Binomial $a + \sqrt{-b}$. Firſt find $\sqrt[3]{a^2 + b}$; then, by trials, ſearch out a number c, and a ſquare root $\sqrt{d}$, ſuch that the ſum of their ſquares $c^2 + d$ be $= \sqrt[3]{a^2 + b}$, and alſo $c^3 - 3cd$ be $= a$; then ſhall $c + \sqrt{-d}$ be the cube root of $a + \sqrt{-b}$ ſought. For example, to find the cube root of $2 + \sqrt{-121}$: here $\sqrt[3]{a^2 + b} = \sqrt[3]{125} = 5$; then taking $c = 2$, and $d = 1$, it is $c^2 + d = 5 = \sqrt[3]{a^2 + b}$, and $c^3 - 3cd = 8 - 6 = 2 = a$, as it ought; therefore $2 + \sqrt{-1}$ is the cube root of $2 + \sqrt{-121}$ ſought.—Bombelli gave alſo a rule for the cube root of the binomial $a + \sqrt{b}$, but it is good for nothing.

Albert Girard's Rule for the Cube Root of a Binomial. This is given in his *Invention Nouvelle en l'Algebre*, and is explained by him thus: Let $72 + \sqrt{5120}$ be the given binomial whoſe cubic root is ſought.

The ſquare of 72 the greater term is 5184
and of the leſs term is 5120
their difference 64
its cube root 4,

which 4 muſt be the difference between the ſquares of the two terms of the root ſought; and as the rational part 72 of the given binomial is the greater term, therefore the rational part of the required root will be the greater part alſo; conſequently the cubic root ſought muſt be one of the binominals here ſet in the margin, where the difference of the ſquares of the terms is always 4, as required; and to find out which of them it muſt be, proceed thus: The firſt, $2 + \sqrt{0}$ muſt be rejected, becauſe one term of it is 0 or nothing; alſo becauſe 5 exceeds the cube root of 72, or $\sqrt{20}$ exceeds the cube root of $\sqrt{5120}$, therefore $5 + \sqrt{20}$, and all after it muſt be rejected too; ſo that the root muſt be either $3 + \sqrt{5}$ or $4 + \sqrt{12}$, if the given quantity has a binomial root: to know which of theſe is to be taken, it muſt be conſidered that the rational term of the root muſt meaſure the rational term given; and alſo the irrational term of the root muſt meaſure the irrational term given; then, on examination it is found that both 3 and 4 meaſure or divide the 72 without a remainder, but that only the $\sqrt{5}$, and not $\sqrt{12}$, meaſures $\sqrt{5120}$; conſequently none but $3 + \sqrt{5}$ can be the cube root of the given quantity $72 + \sqrt{5120}$; which is found to anſwer, by cubing the ſaid root $3 + \sqrt{5}$.

$2 + \sqrt{0}$
$3 + \sqrt{5}$
$4 + \sqrt{12}$
$5 + \sqrt{20}$

Dr. Wallis's Rule for the Cube Root of a Binomial $a \pm m\sqrt{b}$ *or* $a \pm m\sqrt{-b}$. In theſe forms the greateſt rational part m is extracted out of the radical part, leaving only b the leaſt radical part poſſible under the radical ſign. He then obſerves that if the given quantity have a binomial root, it muſt be of this form $c \pm n\sqrt{b}$, with the ſame radical b. Then to find the value of c and n, he raiſes this root to the 3d power, which gives $c^3 + 3cn^2b \pm \overline{3c^2n + n^3b} \cdot \sqrt{b}$, which muſt be $= a \pm m\sqrt{b}$ the given quantity; hence putting the rational part of the one quantity equal to that of the other, and alſo the radical part of the one equal to that of the other, gives $c^3 + 3cn^2b = a$, and $3c^2n + n^3b = m$. Then aſſuming ſeveral values of n, from the laſt equation he finds the value of c; hence if theſe values of c and n, ſubſtituted in the firſt equation, make it juſt, they are right; but if not, another value of n is aſſumed, and ſo on, till the firſt equation hold true. And it is to be noted that n is always an integer or elſe the half of an integer. For example, if the cube root of $135 \pm \sqrt{1825}$ be required, or $135 \pm 78\sqrt{3}$; here $a = 135$, $m = 78$, and $b = 3$; hence $3c^2n + n^3b = m$ is $3c^2n + 3n^3 = 78$, or $c^2n + n^3 = 26$; then aſſuming $n = 1$, this laſt equation becomes $c^2 + 1 = 26$, from whence c is found $= 5$; which values of c and n being ſubſtituted in the firſt equation $c^3 + 3cn^2b = a$, makes $5^3 + 3 \cdot 5 \cdot 3 = 170$, but ought to be 135, ſhewing that c is too great, and conſequently n taken too little. Let n therefore be aſſumed $= 2$, ſo ſhall $2c^2 + 8 = 26$, and $c = 3$; and the firſt equation becomes $3^3 + 3 \cdot 3 \cdot 2^2 \cdot 3 = 27 \cdot 5 = 135 = a$ as it ought, which ſhews that the true value of n is 2, and that of c is 3; hence then the cube root of $135 \pm 78\sqrt{3}$ or $c \pm n\sqrt{b}$ is

$3 \pm 2\sqrt{3}$ or $3 \pm \sqrt{12}$. And in like manner is the proceſs inſtituted when the number in the radical is negative, as the cube root of $81 \pm 30\sqrt{-3}$, which is $\frac{9}{2} \pm \frac{1}{2}\sqrt{-3}$.

Another rule for extracting the cube root of an imaginary binomial was alſo given by Demoivre, at the end of Saunderſon's Algebra, by means of the triſection of an arc or angle.

Sir I. Newton's Rule for any Root of a Binomial $a \pm b$. In his Univerſal Arith. is given a rule for the ſquare root of a binomial, which is the ſame as the 2d of Lucas de Burgo, before given; and alſo a general rule for any root of a binomial, which I have not met with elſewhere; and it is this: Of the given quantity $a \pm b$, let a be the greater term, and c the index of the root to be extracted. Seek the leaſt number n whoſe power n^c can be divided by $aa-bb$ without a remainder, and let the quotient be q; Compute $\sqrt[c]{\overline{a+b}\,.\,\sqrt{q}}$ in the neareſt integer number, which call r; divide $a\sqrt{q}$ by its greateſt rational diviſor, calling the quotient s; and let the neareſt integer number above $\frac{r+\frac{n}{r}}{2s}$ be t: ſo ſhall $\frac{ts \pm \sqrt{t^2s^2-n}}{\sqrt[2c]{q}}$ be the root ſought, if the root can be extracted. And this rule is demonſtrated by *s'Graveſande* in his commentary on Newton's Arithmetic. And many numeral examples, illuſtrating this rule, are given in s'Graveſande's Algebra, abovementioned, pa. 160, as alſo in Newton's Univerſ. Arith. pa. 53 2d edit. and in Maclaurin's Algebra pa. 118. Other rules may be found in Schooten's Commentary on the Geometry of Deſcartes, and elſewhere.

Impoſſible or *Imaginary* BINOMIAL, is a binomial which has one of its terms an impoſſible or an imaginary quantity; as $a + \sqrt{-b}$.

BINOMIAL *Curve*, is a curve whoſe ordinate is expreſſed by a binomial quantity; as the curve whoſe ordinate is $x^a \times \overline{b+dx^c}|^{e}$. Stirling, Method. Diff. pa. 58.

BINOMIAL *Theorem*, is uſed to denote the celebrated theorem given by Sir I. Newton for raiſing a binomial to any power, or for extracting any root of it by an approximating infinite ſeries. It was known by Stifelius, and others, about the beginning of the 16th century, how to raiſe the integral powers, not barely by a continued multiplication of the binomial given, but Stifelius formed alſo a table of numbers by a continued addition, which ſhewed by inſpection the coefficients of the terms of any power of the binomial, contained within the limits of the table; but ſtill they could not independent of a table, and of any of the lower powers, raiſe any power of a binomial at once, by determining its terms one from another only, viz, the 2d term from the 1ſt, the 3d from the 2d, and ſo on as far as we pleaſe, by a general rule; and much leſs could they extract general algebraic roots in infinite ſeries by any rule whatever.

For although the nature and conſtruction of that table, which is a table of figurate numbers, was ſo early known, and employed in the raiſing of powers, and the extracting the roots of pure numbers; yet it was only by raiſing the numbers one from another by continual additions, and then taking them from the table when wanted, till Mr. Briggs firſt pointed out the way of raiſing any line in the table by itſelf, without any of the preceding lines; and thus teaching to raiſe the terms of any power of a binomial, independent of any of the other powers; and ſo gave the ſubſtance of the binomial theorem in words, wanting only the algebraic notation in ſymbols; as is ſhewn at large at pa. 75 of the hiſtorical introduction to my Mathematical Tables. Whatever was known however of this matter, related only to pure or integral powers, no one before Newton having thought of extracting roots by infinite ſeries. He happily diſcovered that, by conſidering powers and roots in a continued ſeries, roots being as powers having fractional exponents, the ſame binomial ſeries would equally ſerve for them all, whether the index ſhould be fractional or integral, or whether the ſeries be finite or infinite. The truth of this method however was long known only by trial in particular caſes, and by induction from analogy; nor does it appear that even Newton himſelf ever attempted any direct proof of it: however, various demonſtrations of the theorem have ſince been given by the more modern mathematicians, ſome of which are by means of the doctrine of fluxions, and others, more legally, from the pure principles of algebra only: for a full account of which, ſee pa. 71 &c, of my Mathematical Tracts, vol. I.

This theorem was firſt diſcovered by Sir I. Newton in 1669, and ſent in a letter of June 13, 1676, to Mr. Oldenburgh, Secretary to the Royal Society, to be by him communicated to Mr. Leibnitz; and it was in this form: $\overline{p+pq}|^{\frac{m}{n}} = p^{\frac{m}{n}} + \frac{m}{n}aq + \frac{m-n}{2n}bq + \frac{m-2n}{3n}cq + \frac{m-3n}{4n}dq +$ &c: where $p + pq$ ſignifies the quantity whoſe root, or power, or root of any power, is to be found; p being the firſt term of that quantity; q the quotient of all the reſt of the terms divided by that firſt term; and $\frac{m}{n}$ the numeral index of the power or root of the quantity $p + pq$, whether it be integral or fractional, poſitive or negative; and laſtly a, b, c, d, &c, are aſſumed to denote the ſeveral terms in their order as they are found, viz, $a =$ the firſt term $p^{\frac{m}{n}}$, $b =$ the 2d term $\frac{m}{n}aq$, $c =$ the 3d term $\frac{m-n}{2n}bq$, and ſo on. As Newton's general notation of indices was not commonly known, he takes this occaſion to explain it; and then he gives many examples of the application of this theorem, one of which is the following.

Ex. 1. To find the value of $\sqrt{c^2+x^2}$ or $\overline{c^2+x^2}|^{\frac{1}{2}}$, that is, to extract the ſquare root of c^2+x^2 in an infinite ſeries. Here $p = c^2$, $q = \frac{x^2}{c^2}$, $m = 1$, and $n = 2$; therefore $a = p^{\frac{m}{n}} = \overline{cc}|^{\frac{1}{2}} = c$, $b = \frac{m}{n}aq = \frac{x^2}{2c}$, $c = \frac{m-n}{2n}bq = \frac{-x^4}{8c^3}$, &c; and therefore the root ſought is $c + \frac{x^2}{2c} - \frac{x^4}{8c^3} + \frac{x^6}{16c^5} - \frac{5x^8}{128c^7}$ + &c.

A variety of other examples are alſo given in the ſame place, by which it is ſhewn that the theorem is of univerſal application to all ſorts of quantities whatever.

This theorem is ſometimes repreſented in other forms, as

$p^{\frac{m}{n}} \times : 1 + \frac{m}{n} q + \frac{m}{n} \cdot \frac{m-n}{2n} q^2 + \frac{m}{n} \cdot \frac{m-n}{2n} \cdot \frac{m-2n}{3n} q^3 +$ &c; which comes to the ſame thing. Or alſo thus $p^{\frac{m}{n}} + \frac{m}{n} p^{\frac{m-n}{n}} r + \frac{m}{n} \cdot \frac{m-n}{2n} p^{\frac{m-2n}{n}} r^2$ &c; where the binomial is $\overline{p + r}|^{\frac{m}{n}}$.

In another letter to Mr. Oldenburgh, of Oct. 24, 1676, Newton explains the train of reaſoning by which he obtained the ſaid theorem, as follows: "In the beginning of my mathematical ſtudies, when I was peruſing the works of the celebrated Dr. Wallis (ſee his Arith. of Infinites, prop. 118, and 121, alſo his Algebra chap. 82), and conſidering the ſeries by the interpolation of which he exhibits the area of the circle and hyperbola; for inſtance, in this ſeries of curves, whoſe common baſe or axis is x, and the ordinates reſpectively $\overline{1 - xx}|^{\frac{0}{2}}$, $\overline{1 - xx}|^{\frac{1}{2}}$, $\overline{1 - xx}|^{\frac{2}{2}}$, $\overline{1 - xx}|^{\frac{3}{2}}$, $\overline{1 - xx}|^{\frac{4}{2}}$, &c; I perceived that if the areas of the alternate curves, which are x,

$x - \frac{1}{3}x^3$,
$x - \frac{2}{3}x^3 + \frac{1}{5}x^5$,
$x - \frac{3}{3}x^3 + \frac{3}{5}x^5 - \frac{1}{7}x^7$,

&c; could be interpolated, we ſhould obtain the areas of the intermediate ones; the firſt of which, or $\overline{1 - xx}|^{\frac{1}{2}}$, is the area of the circle: now in order to this, it appeared that in all the ſeries the firſt term was x; that the 2d terms $\frac{0}{3}x^3$, $\frac{1}{3}x^3$, $\frac{2}{3}x^3$, $\frac{3}{3}x^3$, &c, were in arithmetical progreſſion; and conſequently that the firſt two terms of all the ſeries to be interpolated would be

$$x - \frac{\frac{1}{2}x^3}{3}, \quad x - \frac{\frac{3}{2}x^3}{3}, \quad x - \frac{\frac{5}{2}x^3}{3}, \text{ \&c.}$$

"Now for the interpolation of the reſt, I conſidered that the denominators 1, 3, 5, 7, &c, were in arithmetical progreſſion; and that therefore only the numeral coefficients of the numerators were to be inveſtigated. But theſe in the alternate areas, which are given, were the ſame with the figures of which the ſeveral powers of 11 conſiſt, viz, of 11^0, 11^1, 11^2, 11^3 &c; that is, the firſt 1,
the ſecond 1, 1
the third 1, 2, 1
the fourth 1, 3, 3, 1
the fifth 1, 4, 6, 4, 1
&c.

"I enquired therefore how, in theſe ſeries, the reſt of the terms may be derived from the firſt two being given; and I found that by putting m for the 2d figure or term, the reſt would be produced by the continued multiplication of the terms of this ſeries,

$$\frac{m-0}{1} \times \frac{m-1}{2} \times \frac{m-2}{3} \times \frac{m-3}{4} \times \frac{m-4}{5} \text{ \&c.}$$

"For inſtance, if the 2d term $m = 4$; then ſhall $4 \times \frac{m-1}{2}$, or 6, be the 3d term; and $6 \times \frac{m-2}{3}$, or 4, the 4th term; and $4 \times \frac{m-3}{4}$, or 1, the 5th term; and $1 \times \frac{m-4}{5}$, or 0, the 6th; which ſhews that in this caſe the ſeries terminates.

"This rule therefore I applied to the ſeries to be interpolated. And ſince, in the ſeries for the circle, the 2d term was $\frac{\frac{1}{2}x^3}{3}$ I put $m = \frac{1}{2}$, which produced the terms $\frac{1}{2} \times \frac{\frac{1}{2}-1}{2}$ or $-\frac{1}{8}$; $-\frac{1}{8} \times \frac{\frac{1}{2}-2}{3}$ or $+\frac{1}{16}$; $+\frac{1}{16} \times \frac{\frac{1}{2}-3}{4}$ or $-\frac{5}{128}$; and ſo on *ad infinitum*. And hence I found that the required area of the circular ſegment is

$$x - \frac{\frac{1}{2}x^3}{3} - \frac{\frac{1}{8}x^5}{5} - \frac{\frac{1}{16}x^7}{7} - \frac{\frac{5}{128}x^9}{9} - \text{ \&c.}$$

"And in the ſame manner might be produced the interpolated areas of the other curves: as alſo the area of the hyperbola and the other alternates in this ſeries $\overline{1 + xx}|^{\frac{0}{2}}$, $\overline{1 + xx}|^{\frac{1}{2}}$, $\overline{1 + xx}|^{\frac{2}{2}}$, $\overline{1 + xx}|^{\frac{3}{2}}$, &c. And in the ſame way alſo may other ſeries be interpolated, and that too if they ſhould be taken at the diſtance of two or more terms.

"This was the way then in which I firſt entered upon theſe ſpeculations; which I ſhould not have remembered, but that in turning over my papers a few weeks ſince, I chanced to caſt my eyes on thoſe relating to this matter.

"Having proceeded ſo far, I conſidered that the terms $\overline{1 - xx}|^{\frac{0}{2}}$, $\overline{1 - xx}|^{\frac{2}{2}}$, $\overline{1 - xx}|^{\frac{4}{2}}$, $\overline{1 - xx}|^{\frac{6}{2}}$, &c, that is, 1

$1 - x^2$
$1 - 2x^2 + x^4$
$1 - 3x^2 + 3x^4 - x^6$, &c, might be interpolated in the ſame manner as the areas generated by them: and for this, nothing more was required but to omit the denominators 1, 3, 5, 7, &c, in the terms expreſſing the areas; that is, the coefficients of the terms of the quantity to be interpolated, $\overline{1 - xx}|^{\frac{1}{2}}$, or $\overline{1 - xx}|^{\frac{3}{2}}$, or generally $\overline{1 - xx}|^{m}$, will be produced by the continued multiplication of the terms of this ſeries

$$m \times \frac{m-1}{2} \times \frac{m-2}{3} \times \frac{m-3}{4} \text{ \&c.}$$

"Thus, for example, there would be found

$$\overline{1 - xx}|^{\frac{1}{2}} = 1 - \tfrac{1}{2}x^2 - \tfrac{1}{8}x^4 - \tfrac{1}{16}x^6 \text{ \&c.}$$

$$\overline{1 - xx}|^{\frac{3}{2}} = 1 - \tfrac{3}{2}x^2 + \tfrac{3}{8}x^4 + \tfrac{1}{16}x^6 \text{ \&c.]}$$

$$\overline{1 - xx}|^{\frac{1}{3}} = 1 - \tfrac{1}{3}x^2 - \tfrac{1}{9}x^4 - \tfrac{5}{81}x^6 \text{ \&c.}$$

"Thus then I diſcovered a general method of reducing radical quantities into infinite ſeries, by the theorem which I ſent in the beginning of the former letter, before I knew the ſame by the extraction of roots.

"But having diſcovered that way, this other could not long remain unknown: for, to prove the truth of thoſe operations, I multiplied $1 - \frac{1}{2}x^2 - \frac{1}{8}x^4 - \frac{1}{16}x^6$ &c, by itſelf, and the product is $1 - x^2$, all the reſt of the terms vaniſhing after theſe, *in infinitum*. In like manner, $1 - \frac{1}{3}x^2 - \frac{1}{9}x^4 - \frac{5}{81}x^6$ &c, twice multiplied by itſelf, produced $1 - x^2$. But as this was a certain proof of thoſe concluſions, ſo I was naturally led to try converſely whether theſe ſeries, which were thus known to be the roots of the quantity $1 - x^2$, could not be extracted out of it after the manner of arithmetic; and upon trial I found it to ſucceed. The proceſs for the ſquare root is here ſet down

$$\begin{array}{l}
1 - x^2 \;\big(\, 1 - \frac{1}{2}x^2 - \frac{1}{8}x^4 - \frac{1}{16}x^6 \text{ \&c} \\
1 \\
\hline
0 - x^2 \\
\quad - x^2 + \frac{1}{4}x^4 \\
\hline
\qquad\quad - \frac{1}{4}x^4 \\
\qquad\quad - \frac{1}{4}x^4 + \frac{1}{8}x^6 + \frac{1}{64}x^8 \\
\hline
\qquad\qquad\qquad - \frac{1}{8}x^6 - \frac{1}{64}x^8
\end{array}$$

"Theſe methods being found, I laid aſide the other way by interpolation of ſeries, and uſed theſe operations only as a more genuine foundation. Neither was I ignorant of the reduction by diviſion, which is ſo much eaſier." See Collins's *Commercium Epiſtolicum*.

And this is all the account that Newton gives of the invention of this theorem, which is engraved on his monument in Weſtminſter Abbey, as one of his greateſt diſcoveries.

BIPARTIENT, is a number that divides another into two equal parts without a remainder. So 2 is a bipartient to 4, and 5 a bipartient to 10.

BIPARTITION, is a diviſion into two equal parts.

BIQUADRATE, or BIQUADRATIC *Power*, is the ſquared ſquare, or 4th power of any number or quantity. Thus 16 is the biquadrate or 4th power of 2, or it is the ſquare of 4 which is the 2d power of 2.

BIQUADRATIC *Root*, of any quantity, is the ſquare root of the ſquare root, or the 4th root of that quantity. So the biquadratic root of 16 is 2, and the biquadratic root of 81 is 3.

BIQUADRATIC *Equation*, is that which riſes to 4 dimenſions, or in which the unknown quantity riſes to the 4th power; as $x^4 + ax^3 + bx^2 + cx + d = 0$.

Any biquadratic equation may be conceived to be generated or produced from the continual multiplication of four ſimple equations,

as $\overline{x-p} \times \overline{x-q} \times \overline{x-r} \times \overline{x-s} = 0$;

or from that of two quadratic equations,

as $\overline{x^2 + px + q} \times \overline{x^2 + rx + s} = 0$;

or, laſtly, from that of a cubic and a ſimple equation,

as $\overline{x-p} \times \overline{x^3 + qx^2 + rx + s} = 0$: which was the invention of Harriot. And, on the contrary, a biquadratic equation may be reſolved into four ſimple equations, or into two quadratics, or into a cubic and a ſimple equation, having all the ſame roots with it.

1. *Ferrari's Method for Biquadratic Equations:*

The firſt reſolution of a biquadratic equation was given in Cardan's Algebra, chap. 39, being the invention of his pupil and friend Lewis Ferrari, about the year 1540. This is effected by means of a cubic equation, and is indeed a method of depreſſing the biquadratic equation to a cubic, which Cardan demonſtrates, and applies in a great variety of examples. The principle is very general, and conſiſts in completing one ſide of the equation up to a ſquare, by the help of ſome multiples or parts of its own terms and an aſſumed unknown quantity; which it is always eaſy to do; and then the other ſide is made to be a ſquare alſo, by aſſuming the product of its 1ſt and 3d terms equal to the ſquare of half the 2d term; for it conſiſts only of three terms, or three different denominations of the original letter; then this equality will determine the value of the aſſumed quantity by a cubic equation: other circumſtances depend on the artiſt's judgment. But the method will be farther explained by the following examples, extracted from Cardan's book.

Ex. 1. Given $x^4 + 6x^2 + 36 = 60x$, to be reſolved. Add $6x^2$ to both ſides of the equation, ſo ſhall

$x^4 + 12x^2 + 36$ or $\overline{x^2 + 6}|^2 = 6x^2 + 60x$.

Aſſume y, and add $\overline{x^2 + 6}\,.\,2y + y^2$ to both ſides, then is

$\overline{x^2 + 6}|^2 + \overline{x^2 + 6}\,.\,2y + y^2 = 6x^2 + 60x + \overline{x^2 + 6}\,.\,2y + y^2$, or $\overline{x^2 + 6 + y}|^2 = \overline{6 + 2y}\,.\,x^2 + 60x + 12y + y^2$.

Make now the 1ſt × 3d term = $\frac{1}{4}$ ſq. 2d,

this gives $\overline{6 + 2y}\,.\,\overline{12y + y^2}\,.\,x^2 = 900x^2$,

or $y^3 + 15y^2 + 36y = 450$; and hence

$y = \sqrt[3]{287\frac{1}{2} + \sqrt{80449\frac{1}{4}}} + \sqrt[3]{287\frac{1}{2} - \sqrt{80449\frac{1}{4}}} - 5$.

From which x may be found by a quadratic equation.

Ex. 2. Given $x^4 = x + 2$.

Before applying Ferrari's method to this example, Cardan reſolves it by another way as follows: ſubtract 1, then is $x^4 - 1 = x + 1$; divide by $x + 1$, then is $x^3 - x^2 + x - 1 = 1$, or $x^3 + x = x^2 + 2$; and hence

$$x = \sqrt[3]{\sqrt{\frac{2241}{2916}} + \frac{47}{54}} - \sqrt[3]{\sqrt{\frac{2241}{2916}} - \frac{47}{54}} + \frac{1}{3}.$$

But to reſolve it by Ferrari's rule:

Becauſe $x^4 = x + 2$. therefore

$x^4 + 2yx^2 + y^2$ or $\overline{x^2 + y}|^2 = 2yx^2 + x + \overline{2 + y^2}$;

hence $2y^3 + 4y = \frac{1}{4}$, or $y^3 + 2y = \frac{1}{8}$; and the root is

$$y = \sqrt[3]{\sqrt{\frac{2075}{6912}} + \frac{1}{16}} - \sqrt[3]{\sqrt{\frac{2075}{6912}} - \frac{1}{16}}:$$

by means of which x is found by a quadratic equation.

Ex. 3. Given $x^4 + 32x^2 + 16 = 48x$.—Add 240, then $x^4 + 32x^2 + 256$ or $\overline{x^2 + 16}|^2 = 48x + 240$; complete ſquare again, then

$\overline{x^2 + 16}|^2 + 2y\,.\,\overline{x^2 + 16} + y^2 = 2yx^2 + 48x + \overline{y^2 + 32y + 240}$;

make the laſt ſide a ſq. by the rule, which gives

$y^3 + 32y^2 + 240y = 24 \times 12 = 288$.

Put now $z = y + 10\frac{2}{3}$, and the laſt transforms to $z^3 = 101\frac{1}{3}z + 420\frac{16}{27}$; then the value of z found from this, gives the value of y, and hence the value of x, as before.

2. *Deſ-*

2. *Deſcartes's Rule for Biquadratic Equations.*

Another ſolution was given of biquadratic equations by Deſcartes, in the 3d book of his Geometry. In this ſolution he reſolved the given biquadratic equation into two quadratics, by means of a cubic equation, in this manner: Firſt, let the 2d term or 3d power be taken away out of the equation, after which it will ſtand thus, $x^4 + px^2 + qx + r = 0$. Find y in this cubic equation $y^6 + 2py^4 \left.\begin{matrix} +p^2 \\ -4r \end{matrix}\right\} y^2 - q^2 = 0$; and then the values of x in theſe two quadratics.

$$\left\{\begin{matrix} x^2 - yx + \frac{1}{2}y^2 + \frac{1}{2}p + \frac{q}{2y} = 0 \\ x^2 + yx + \frac{1}{2}y^2 + \frac{1}{2}p - \frac{q}{2y} = 0, \end{matrix}\right.$$

and theſe values of x will be the roots of the given biquadratic equation.

Ex. Let the equ. be $x^4 - 17x^2 - 20x - 6 = 0$.
Hence $p = -17$, $q = -20$, & $r = -6$;
and the cubic equ. is $x^6 - 34y^4 + 313y^2 - 400 = 0$,
the root of which is $y^2 = 16$, or $y = 4$;
hence the two quadratics are - $\left\{\begin{matrix} x^2 - 4x - 3 = 0 \\ x^2 + 4x + 2 = 0, \end{matrix}\right.$
the four roots of which are $2 \pm \sqrt{7}$ and $-2 \pm \sqrt{2}$.

3. *Euler's Method for Biquadratic Equations.*

The celebrated Leonard Euler gave, in the 6th volume of the Peterſburgh Ancient Commentaries, for the year 1738, an ingenious and general method of reſolving equations of all degrees, by means of the equation of the next lower degree, and among them of the biquadratic equation by means of the cubic; and this laſt was alſo given more at large in his treatiſe of Algebra, tranſlated from the German into French in 1774, in 2 volumes 8vo. The method is this: Let $x^4 - ax^2 - bx - c = 0$, be the given biquadratic equation, wanting the 2d term. Take $f = \frac{1}{2}a$, $g = \frac{1}{16}aa + \frac{1}{4}c$, and $h = \frac{1}{64}bb$; with which values of f, g, h, form the cubic equation $z^3 - fz^2 + gz - h = 0$. Find the three roots of this cubic equation, and let them be called p, q, r. Then ſhall the four roots of the propoſed biquadratic be theſe following, viz,

	When $\frac{1}{8}b$ is poſitive :	When $\frac{1}{8}b$ is negative :
1ſt.	$\sqrt{p} + \sqrt{q} + \sqrt{r}$	$\sqrt{p} + \sqrt{q} - \sqrt{r}$,
2d.	$\sqrt{p} + \sqrt{q} - \sqrt{r}$	$\sqrt{p} - \sqrt{q} + \sqrt{r}$,
3d.	$\sqrt{p} - \sqrt{q} + \sqrt{r}$	$-\sqrt{p} + \sqrt{q} + \sqrt{r}$,
4th.	$\sqrt{p} - \sqrt{q} - \sqrt{r}$	$-\sqrt{p} - \sqrt{q} - \sqrt{r}$.

Ex. Let the eq. be $x^4 - 25x^2 + 60x - 36 = 0$.
Here $a = 25$, $b = -60$, and $c = 36$;
theref. $f = \frac{25}{2}$, $g = \frac{625}{16} + 9 = \frac{769}{16}$, and $h = \frac{225}{4}$.
Conſeq. the cubic equation will be

$$z^3 - \frac{25}{2}z^2 + \frac{769}{16}z - \frac{225}{4} = 0.$$

The three roots of which are

$z = \frac{9}{4} = p$, and $z = 4 = q$, and $z = \frac{25}{4} = r$;
the roots of which are $\sqrt{p} = \frac{3}{2}$, $\sqrt{q} = 2$ or $\frac{4}{2}$, $\sqrt{r} = \frac{5}{2}$.
Hence, as the value of $\frac{1}{8}b$ is negative, the four roots are

1ſt. $x = \frac{3}{2} + \frac{4}{2} - \frac{5}{2} = 1$,
2d. $x = \frac{3}{2} - \frac{4}{2} + \frac{5}{2} = 2$,
3d. $x = -\frac{3}{2} + \frac{4}{2} + \frac{5}{2} = 3$,
4th. $x = -\frac{3}{2} - \frac{4}{2} - \frac{5}{2} = -6$.

4. *Simpſon's Rule for Biquadratic Equations.*

Mr. Simpſon gave alſo a general rule for the ſolution of biquadratic equations, in the 2d edit. of his Algebra, pa. 150, in which the given equation is alſo reſolved by means of a cubic equation, as well as the two former ways; and it is inveſtigated on the principle, that the given equation is equal to the difference between two ſquares; being indeed a kind of generalization of Ferrari's method.

Thus, he ſuppoſes the given equation, viz, $x^4 + px^3 + qx^2 + rx + s = \overline{x^2 + \frac{1}{2}px + A}|^2 - \overline{Bx + C}|^2$; then from a compariſon of the like terms, the values of the aſſumed letters are found, and the final equation becomes
$A^3 - \frac{1}{2}qA^2 + kA - \frac{1}{2}l = 0$,
where $k = \frac{1}{4}pr - s$, and $l = \frac{1}{4}r^2 + s.\overline{\frac{1}{4}p^2 - q}$.
The value of A being found in this cubic equation, from it will be had the values of B and C, which have theſe general values, viz, $B = \sqrt{2A + \frac{1}{4}p^2 - q}$, and $C = \frac{pA - r}{2B}$. Hence, finally, the root x will be obtained from the aſſumed equation $\overline{x^2 + \frac{1}{2}px + A}|^2 - \overline{Bx + C}|^2 = 0$, or $x^2 + \frac{1}{2}px + A = \pm Bx \pm C$, in four ſeveral values.

Ex. Given the equ. $x^4 - 6x^3 - 58x^2 - 114x - 11 = 0$.
Here $p = -6$, $q = -58$, $r = -114$, and $s = -11$, whence
k or $\frac{1}{4}pr - s = 182$, l or $\frac{1}{4}r^2 + s.\overline{\frac{1}{4}p^2 - q} = 2512$;
and therefore the cubic equation becomes
$A^3 + 29A^2 + 182A - 1256 = 0$,
the root of which is $A = 4$.
Hence then B or $\sqrt{2A + \frac{1}{4}p^2 - q} = \sqrt{75}$, and C or $\frac{pA - r}{2B} = \sqrt{27}$:
and the quadratic equation becomes
$x^2 - 3x + 4 = \pm x\sqrt{75} \pm \sqrt{27}$, the four roots of which are $x = 1\frac{1}{2} \pm 2\frac{1}{2}\sqrt{3} + \sqrt{17 \pm 11\frac{1}{2}\sqrt{3}}$,
and $x = 1\frac{1}{2} \pm 2\frac{1}{2}\sqrt{3} - \sqrt{17 \pm 11\frac{1}{2}\sqrt{3}}$.

Mr. Simpſon here ſubjoins an obſervation which it has ſince been found is erroneous, viz, that "The value of A, in this equation, will be *commenſurate* and *rational* (and therefore the eaſier to be diſcovered), not only when all the roots of the given equation are *commenſurate*, but when they are *irrational* and even *impoſſible*; as will appear from the examples ſubjoined." This is a ſtrange reaſon for Simpſon to give for the proof of a propoſition; and it is wonderful that he fell upon no examples that diſprove it, as the inſtances in which it holds true, are very few indeed, in compariſon with the number of thoſe in which it fails.

Note. In any biquadratic equation having all its terms, if $\frac{3}{8}$ of the ſquare of the coefficient of the 2d term be greater than the product of the coefficients of the 1ſt and 3d terms, or $\frac{3}{8}$ of the ſquare of the coefficient of the 4th term be greater than the product of the coefficients of the 3d and 5th terms, or $\frac{4}{9}$ of the ſquare of the coefficient of the 3d term greater than the product of the coefficients of the 2d and 4th terms; then all the roots of that equation will be real and unequal; but if either of the ſaid parts of thoſe ſquares be leſs than either of thoſe products, the equation will have imaginary roots.

For the conſtruction of biquadratic equations, ſee *Conſtruction.* See alſo *Deſcartes's Geometry*, with the *Commentaries of Schooten* and others; *Baker's Geometrical Key; Sluſius's Meſolabium; l'Hoſpital's Conic Sections; Wolfius's Elementa Matheſeos; &c.*

Biquadratic *Parabola*, a curve of the 3d order, having two infinite legs, and expreſſed by one of theſe three equations, viz,

$a^3x = y^4$, as in fig. 1,

$a^3x = y^4 - a^2 y^2$, as in fig. 2,

$a^3x = y^4 - \overline{a + b} \,. y^3 - aby^2$, as in fig. 3;

where $x = AP$ the abſciſs, $y = PQ$ the ordinate, $b = AB$, $c = AC$, and $a =$ a certain given quantity.

Fig 1

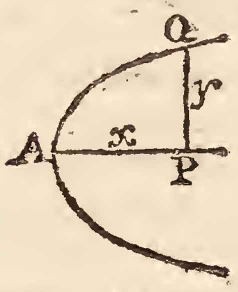


Fig 2

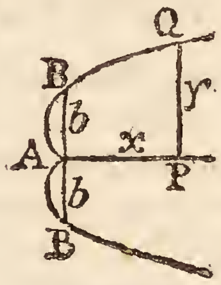
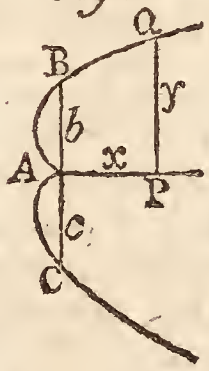
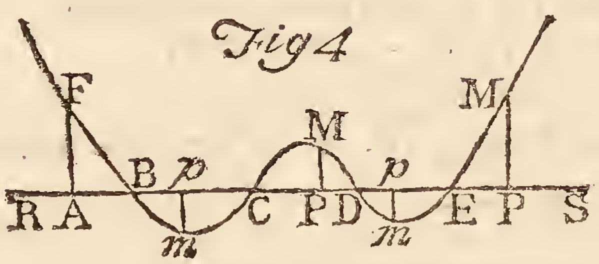


Fig 3



But the moſt general equation of this curve is the following, which belongs to fig. 4, viz,

Fig 4



$ay = x^4 + bx^3 + cx^2 + dx + e$;

where $x = Ap$ or AP the abſciſs, and $-y$ or $+y$ is the ordinate pm or PM, alſo a, b, c, d, e, are conſtant quantities; the beginning of the abſciſs being at any point A in the indefinite line AP.

But if the beginning of the abſciſs A be where this line interſects the curve, as in fig. 5, then the nature of the curve will be defined by this equation $a \times pm = Ap \times Bp \times Cp \times Dp$, wherever the point p is taken in the infinite line RS.

Fig 5

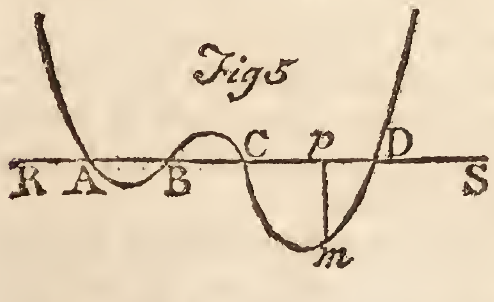
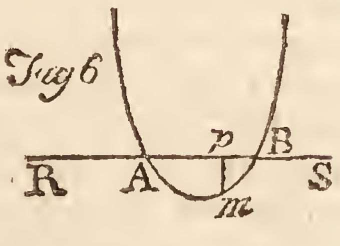


Fig 6



When the curve has no ſerpentine part, as fig. 6, the equation is more ſimple, being in this caſe barely $a \times pm = Ap^2 \times pB^2$. See Curve *Lines*, and Geometrical *Lines*.

BIQUINTILE, an aſpect of the planets when the diſtance between them is 144 degrees, or twice the 5th part of 360 degrees.

BISECTION, or Bissection, the diviſion of a quantity into two equal parts, otherwiſe called *bipartition.*

BISSEXTILE, or *Leap-year*, a year conſiſting of 366 days, and happening every 4th year, by the addition of a day in the month of February, which that year conſiſts of 29 days. And this is done to recover the 6 hours which the ſun takes up nearly in his courſe, more than the 365 days commonly allowed for it in other years.

The day thus added was by Julius Cæſar appointed to be the day before the 24th of February, which among the Romans was the 6th of the calends, and which on this occaſion was reckoned twice; whence it was called the *biſſextile.*

By the ſtatute *De anno biſſextile*, 21 Hen. III, to prevent miſunderſtandings, the intercalary day and that next before it are to be accounted as one day.

To find what year of the period any given year is; divide the given year by 4; then if 0 remains, it is leap year; but if any thing remain, the given year is ſo many after leap year.

But the aſtronomers concerned in reforming the calendar in 1582, by order of pope Gregory XIII, obſerving that in 4 years the biſſextile added 44 minutes more than the ſun ſpent in returning to the ſame point of the ecliptic; and computing that in 133 years theſe ſupernumerary minutes would form a day; to prevent any changes being thus inſenſibly introduced into the ſeaſons, directed, that in the courſe of 400 years there ſhould be three ſextiles retrenched; ſo that every centeſimal year, which is a leap year according to the Julian account, is a common year in the Gregorian account, unleſs the number of centuries can be divided by 4 without a remainder. So 1600 and 2000 are biſſextile; but 1700, 1800, and 1900 are common years.

The Gregorian computation has been received in moſt foreign countries ever ſince the reformation of the calendar in 1582; excepting ſome northern countries, as Ruſſia, &c. And by act of parliament, paſſed in 1751, it commenced in all the dominions under the crown of Great Britain in the year following; it being ordered by that act that the natural day next following the 2d of September, ſhould be accounted the 14th; omitting the intermediate 11 days of the common calendar. The ſupernumerary day, in leap years, being added at the end of the month February, and called the 29th of that month.

BLACK, a colour ſo called, or rather a privation of all colour. This, it ſeems, ariſes from ſuch a peculiar texture and ſituation of the ſuperficial parts of a black body, that they abſorb all or moſt of the rays of light, reflecting little or none to the eye: and hence it happens that black bodies, thus imbibing the rays, are always found to be hotter than thoſe of a lighter colour. Dr. Franklin obſerves that black cloaths heat more, and dry ſooner in the ſun than white cloaths; that therefore black is a bad colour for cloaths in hot climates; but a fit colour for the linings of ladies' ſummer hats;

and that a chimney painted black, when expoſed to the ſun, will draw more ſtrongly. Franklin's Experim. &c.—Dr. Watſon, the preſent biſhop of Landaff, covered the bulb of a thermometer with a black coating of Indian ink, and the conſequence was that the mercury roſe 10 degrees higher. Philoſ. Tranſ. vol. 63, pa. 40.—And a virtuoſo of unſuſpected credit aſſured Mr. Boyle, that in a hot climate by blacking the ſhells of eggs, and expoſing them to the ſun, he had ſeen them thus well roaſted in a ſhort time.

BLACKNESS, the quality of a black body, as to colour; ariſing from its ſtifling or abſorbing the rays of light, and reflecting little or none. In which ſenſe it ſtands directly oppoſed to whiteneſs; which conſiſts in ſuch a texture of parts, as indifferently reflects all the rays thrown upon it, of whatever colour they may be.

Deſcartes, it ſeems, firſt rightly diſtinguiſhed theſe cauſes of black and white, though he might be miſtaken with reſpect to the general nature of light and colours. —Sir Iſaac Newton ſhews, in his Optics, that to produce black colours, the corpuſcles muſt be ſmaller than for exhibiting the other colours; becauſe, where the ſizes of the component particles of a body are greater, the light reflected is too much for conſtituting this colour: but when they are a little ſmaller than is requiſite to reflect the white, and very faint blue of the firſt order, they will reflect ſo little light, as to appear intenſely black; and yet they may perhaps reflect it variouſly to and fro within them ſo long, till it be ſtifled and loſt.

And hence, it appears, why fire, or putrefaction, by dividing the particles of ſubſtances, turn them black: why ſmall quantities of black ſubſtances impart their colours very freely, and intenſely, to other ſubſtances, to which they are applied; the minute particles of theſe, on account of their very great number, eaſily overſpreading the groſs particles of others. Hence it alſo appears, why glaſs ground very elaborately on a copper-plate with ſand, till it be well poliſhed, makes the ſand, with what is rubbed off from the copper and glaſs, become very black: alſo why blacks commonly incline a little towards a blueiſh colour; as may be ſeen by illuminating white paper with light reflected from black ſubſtances, when the paper uſually appears of a blueiſh white; the reaſon of which is, that black borders on the obſcure blue of the firſt order of colours, and therefore reflects more rays of that colour than of any other: and laſtly why black ſubſtances do ſooner than others become hot in the ſun's light, and burn; an effect which may proceed partly from the multitude of refractions in a little ſpace, and partly from the eaſy commotion among ſuch minute particles.

BLAGRAVE (John), an eminent mathematician, who flouriſhed in the 16th and 17th centuries. He was the ſecond ſon of John Blagrave, of Bulmarſh-court near Sunning in Berkſhire, deſcended from an ancient family in that country. From a grammar ſchool at Reading he was ſent to St. John's college in Oxford, where he applied himſelf chiefly to the ſtudy of mathematics. From hence he retired to his patrimonial ſeat of Southcote-lodge near Reading, where he ſpent the reſt of his life, in a retired manner, without marrying, that he might have more leiſure to purſue his favourite ſtudies; which he did with great application and ſucceſs. After a life thus ſpent in ſtudy, and in acts of benevolence to all around him, he died in the year 1611; and was buried at Reading in the church of St. Lawrence, where a ſumptuous monument was erected to his memory.

He left the bulk of his fortune to the poſterity of his three brothers, which were very numerous. There have been mentioned various acts of his beneficence in private life, for the encouragement of learning, the reward of merit, and the relief of diſtreſs. Some of theſe were the reſult of a quaint, humorous diſpoſition, diſcovered chiefly in his legacies: One of theſe was 10 pounds left to be annually diſpoſed of in the following manner: On Good-friday, the church-wardens of each of the three pariſhes of Reading ſend to the town-hall "one virtuous maid who has lived five years with her maſter:" there, in the preſence of the magiſtrates theſe three virtuous maids throw dice for the ten pounds. The year following the two loſers are returned with a freſh one, and again the third year, till each has had three chances. He alſo left an annuity to 80 poor widows, who ſhould attend annually on Good-friday alſo, and hear a ſermon, for the preaching of which he left ten ſhillings to the miniſter. He took care alſo for the maintenance of his ſervants, rewarding their diligence and fidelity, and providing amply for their ſupport. Thus it appears he was not more remarkable for his ſcientific knowledge, than for his generoſity and philanthropy. His works are,

1. A Mathematical Jewel. Lond. 1585, folio.
2. Of the Making and Uſe of the Familiar Staff. Lond. 1590, 4to.
3. Aſtrolabium Uranicum Generale. Lond. 1596, 4to.
4. The Art of Dyalling. Lond. 1609, 4to.

BLAIR (John), an eminent chronologiſt, was educated at Edinburgh. Afterward, coming to London, he was for ſome time uſher of a ſchool in Hedge-lane. In 1754 he firſt publiſhed "The Chronology and Hiſtory of the World, from the Creation to the year of Chriſt 1753; illuſtrated in 56 tables. In 1755 he was elected a fellow of the Royal Society, and in 1761 of the Society of Antiquaries. In 1756 he publiſhed a 2d edition of his Chronological Tables; and in 1768 an improved edition of the ſame with the addition of 14 maps of Ancient and Modern Geography, for illuſtrating the Tables of Chronology and Hiſtory; to which is prefixed a Diſſertation on the Progreſs of Geography. In 1757 he was appointed chaplain to the Princeſs Dowager of Wales, and Mathematical Tutor to the Duke of York; whom he attended in 1763 in a tour to the continent, from which they returned the year after. Dr. Blair had ſucceſſively ſeveral good church livings: as, a prebendal ſtall at Weſtminſter, the vicarage of Hinckley, and the rectory of Burton Coggles in Lincolnſhire, all in 1761; the vicarage of St. Bride's in London, in 1771, in exchange for that of Hinckley; the rectory of St. John the Evangeliſt in Weſtminſter, in 1776, in exchange for the vicarage of St. Bride's; in the ſame year the rectory of Horton near Colebrooke, Bucks. Dr. Blair died the 24th of June, 1782.

BLIND, an epithet applied to an animal deprived of the uſe of eyes; or one from whom light, colours, and all the glorious objects of the viſible creation, are intercepted by ſome natural or accidental cauſe.

BLINDNESS, a privation of the ſenſe of ſight. The ordinary cauſes of blindneſs are, ſome external violence, vicious conformation, growth of a cataract, gutta ſerena, ſmall-pox, &c; or a decay of the optic nerve; an inſtance of which we have in the Academy of Sciences, where, upon opening the eye of a perſon long blind, it was found that the optic nerve was extremely ſhrunk and decayed, and without any medulla in it. The more extraordinary cauſes of blindneſs are malignant ſtenches, poiſonous juices dropped into the eye, baneful vermin, long confinement in the dark, or the like.

We find various recompenſes for blindneſs, or ſubſtitutes for the uſe of eyes, in the wonderful ſagacity of many blind perſons, related by Zahnius in his *Oculus Artificialis*, and others. In ſome, the defect has been ſupplied by a moſt excellent gift of remembering what they had ſeen before; others by a delicate noſe or the ſenſe of ſmelling; others by a very nice ear; and others again by an exquiſite touch, or ſenſe of feeling, which they have had in ſuch perfection, that as it has been ſaid of ſome, they learned to hear with their eyes, it may be ſaid of theſe that they taught themſelves to ſee with their hands.

Some have been able to perform all ſorts of curious works in the niceſt and moſt dexterous manner. Aldrovandus ſpeaks of a ſculptor, who had become blind at 20 years of age, and yet 10 years afterwards he made a perfect marble ſtatue of Coſmo II de Medicis, and another of clay like pope Urban VIII. Bartholin ſpeaks of a blind ſculptor in Denmark, who, by mere touch, diſtinguiſhed perfectly well all ſorts of wood, and even colours; and father Grimaldi relates an inſtance of the ſame kind; beſides the blind organiſt, lately living in Paris, who it was ſaid did the ſame thing. What ſeems more extraordinary ſtill, we are told, by authors of good report, of a blind guide, who uſed to conduct the merchants through the ſands and deſarts of Arabia: and a not leſs marvellous inſtance is now exiſting in this country, in one John Metcalf near Mancheſter, who became quite blind at a very early age; and yet paſſed many years of his life as a waggoner, and occaſionally, as a guide in different roads during the night, or when the paths were covered with ſnow; and, what is ſtranger ſtill, his preſent occupation is that of ſurveyor and projector of highways in difficult and mountainous parts, particularly about Buxton, and the Peak in Derbyſhire.

There are alſo many inſtances of blind men who have been highly diſtinguiſhed for their mental and literary talents, not to ſpeak of the poets Homer, Milton, Oſſian, &c; of which we have a remarkable inſtance in the late Dr. Sanderſon, profeſſor of mathematics in the univerſity of Cambridge, and in the preſent Dr. Henry Moyes, public lecturer in philoſophy, who both of them loſt their ſight by the ſmall pox at an age before they had any recollection; theſe men were well ſkilled in all branches of the mathematics, philoſophy, and optics, &c, which they taught with the greateſt reputation; beſides the monument of fame which the former has left behind him in his mathematical and philoſophical works.

The effects of a ſudden recovery of ſight in ſuch as have been born blind, are alſo very remarkable: devoid of the experience of diſtance and figure ariſing from ſight, they are liable to the greateſt miſtakes in this reſpect, in ſo much it has been ſaid that they could not diſtinguiſh by the mere ſight which was a cube and which a globe, without firſt touching them. Mr. Boyle mentions a gentleman of this ſort, who having been reſtored to ſight at eighteen years of age, was near going diſtracted with the joy: ſee Boyle's works abridg. vol. 1. pa. 4. See alſo a remarkable caſe of this kind in the Tatler, N° 55, vol. 1. And the gentleman couched by Mr. Cheſelden had no ideas of colour, ſhape, or diſtance: though he knew the colours aſunder in a good light during his blind ſtate; yet when he ſaw them after he had been couched, the faint ideas he had of them before, were not ſufficient for him to know them by afterwards: as to diſtance, his ideas were ſo deficient, that he thought all the objects he ſaw touched his eyes, as what he felt did his ſkin; and it was a conſiderable time before he could remember which was the dog and which the cat, though often informed, without feeling them.

BLINDS, or BLINDES, in Fortification, a kind of defence uſually made of oziers or branches interwoven, and laid acroſs between two rows of ſtakes, about a man's height, and 4 or 5 feet aſunder. They are uſed particularly at the heads of trenches, when theſe are extended in front towards the glacis; ſerving to defend the workmen, and prevent the enemy from overlooking them.

BLOCKADE, is the blocking up a place, by poſting troops all about it at the avenues, to prevent ſupplies of men and proviſions from getting into it; and thus ſtarving it out, without forming any regular ſiege or attacks.

BLONDEL (FRANCIS), a celebrated French mathematician and military engineer. He was born at Ribemond in Picardy in 1617. While he was yet but young, he was choſen Regius Profeſſor of Mathematics and Architecture at Paris. Not long after he was appointed governor to Lewis-Henry de Lomenix, Count de Brienne, whom he accompanied in his travels from 1652 to 1655, of which he publiſhed an account. He enjoyed many honourable employments, both in the navy and army; and was entruſted with the management of ſeveral negociations with foreign princes. He arrived at the dignity of marſhal de camp, and counſellor of ſtate, and had the honour to be appointed mathematical preceptor to the Dauphin. He was a member of the Royal Academy of Sciences, director of the Academy of Architecture, and lecturer to the Royal College: in all which he ſupported his character with dignity and applauſe. Blondel was no leſs verſed in the knowledge of the Belles Lettres than in the mathematical ſciences, as appears by the compariſon he publiſhed between Pindar and Horace. He died at Paris the 22d of February 1686, in the 69th year of his age. His chief mathematical works were,

1. Cours d'Architecture. Paris, 1675, in folio.
2. Reſolution des quatre principaux problemes d'Architecture. Paris, 1676, in folio.
3. Hiſtoire du Calendrier Romain. Paris, 1682, in 4to.

4. Cours de Mathematiques. Paris, 1683, in 4to.

5. L'Art de jetter des Bombes. La Haye, 1685, in 4to.

Beſides a New Method of fortifying Places, and other works.

Blondel had alſo many ingenious pieces inſerted in the Memoirs of the French Academy of Sciences, particularly in the year 1666.

BLOW, in a general ſenſe, denotes a ſtroke given either with the hand, a weapon, or an inſtrument. The effect of a blow is eſtimated like the force of percuſſion, and ſo is expreſſed by the velocity of the body multiplied by its weight.

BLOWING *of a Fire-arm*, is when the vent or touch-hole is run or gullied, and becomes wide, ſo that the powder will flame out.

BLUE, one of the ſeven primitive colours of the rays of light, into which they are divided when refracted through a glaſs priſm. See Newton's Optics &c. See alſo CHROMATICS.

BLUENESS, that quality of a body, as to colour, from whence it is called blue; depending on ſuch a ſize and texture of the parts that compoſe the ſurface of a body, as diſpoſes them to reflect only the blue or azure rays of light to the eye.

The blueneſs of the ſky is thus accounted for by De la Hire, after Da Vinci; viz, that a black body viewed through a thin white one, gives the ſenſation of blue, like the immenſe expanſe viewed through the air illuminated and whitened by the ſun. For the ſame reaſon he ſays it is, that ſoot mixed with white, makes a blue; for that white bodies, being always a little tranſparent, when mixing with a black behind, give the perception of blue. From the ſame principle too he accounts for the blueneſs of the veins on the ſurface of the ſkin, though the blood they are filled with be a deep red.

In the ſame manner was the blueneſs of the ſky accounted for by many other of the earlier writers, as Fromondus, Funceius, Otto Guericke, and many others, together with ſeveral of the more modern writers, as Wolfius, Muſchenbroek, &c. But in the explication of this phenomenon, Newton obſerves that all the vapours, when they begin to condenſe and coaleſce into natural particles, become firſt of ſuch a magnitude as to reflect the azure rays, before they can conſtitute clouds of any other colour. This being therefore the firſt colour they begin to reflect, muſt be that of the fineſt and moſt tranſparent ſkies, in which the vapours are not yet arrived at a groſſneſs ſufficient to reflect other colours.

Bouguer however aſcribes this blueneſs of the ſky to the conſtitution of the air itſelf, being of ſuch a nature that theſe fainter-coloured rays are incapable of making their way through any conſiderable tract of it. And as to the blue ſhadows which were firſt obſerved by Buffon in the year 1742, he accounts for them by the aerial colour of the atmoſphere, which enlightens theſe ſhadows, and in which the blue rays prevail; whilſt the red rays are not reflected ſo ſoon, but paſs on to the remoter regions of the atmoſphere. And the Abbé Mazeas accounts for the phenomenon of blue ſhadows by the diminution of light; obſerving that, of two ſhadows which were caſt upon a white wall from an opaque body, illuminated by the moon and by a candle at the ſame time, that from the candle was reddiſh, while the other from the moon was blue. See Newton's Optics pa. 228, Bouguer Traité d'Optique pa. 368, Edinb. Eſſ. vol. 2 pa. 75, or Prieſtley's Hiſt. of Viſion &c, pa. 436.

BOB *of a Pendulum*, the ſame as the ball, which ſee.

BODY, in Geometry, is a figure conceived to be extended in all directions, or what is uſually ſaid to conſiſt of length, breadth, and thickneſs; being otherwiſe called a *Solid*. A body is conceived to be formed or generated by the motion of a ſurface; like as a ſurface by the motion of a line, and a line by the motion of a point.—Similar bodies, or ſolids, are in proportion to each other, as the cubes of their like ſides, or linear dimenſions.

BODY, in Phyſics, or Natural Philoſophy, a ſolid, extended, palpable ſubſtance; of itſelf merely paſſive, being indifferent either to motion or reſt, and capable of any ſort of motion or figure.

Body is compoſed, according to the Peripatetics, of matter, form, and privation; according to the Epicureans and Corpuſcularians, of an aſſemblage of hooked, heavy atoms; according to the Carteſians, of a certain extenſion; and according to the Newtonians, of a ſyſtem or aſſociation of ſolid, maſſy, hard, impenetrable, moveable particles, ranged or diſpoſed in this or that manner; from which ariſe bodies of this or that form, and diſtinguiſhed by this or that name. Theſe elementary or component particles of bodies, they aſſert, muſt be perfectly hard, ſo as never to wear or break in pieces; which, Newton obſerves, is neceſſary, in order to the world's perſiſting in the ſame ſtate, and bodies continuing of the ſame nature and texture in ſeveral ages.

BODY *of a piece of Ordnance*, the part contained between the centre of the trunnions and the caſcabel. This ſhould always be more fortified or ſtronger than the reſt. See CANNON.

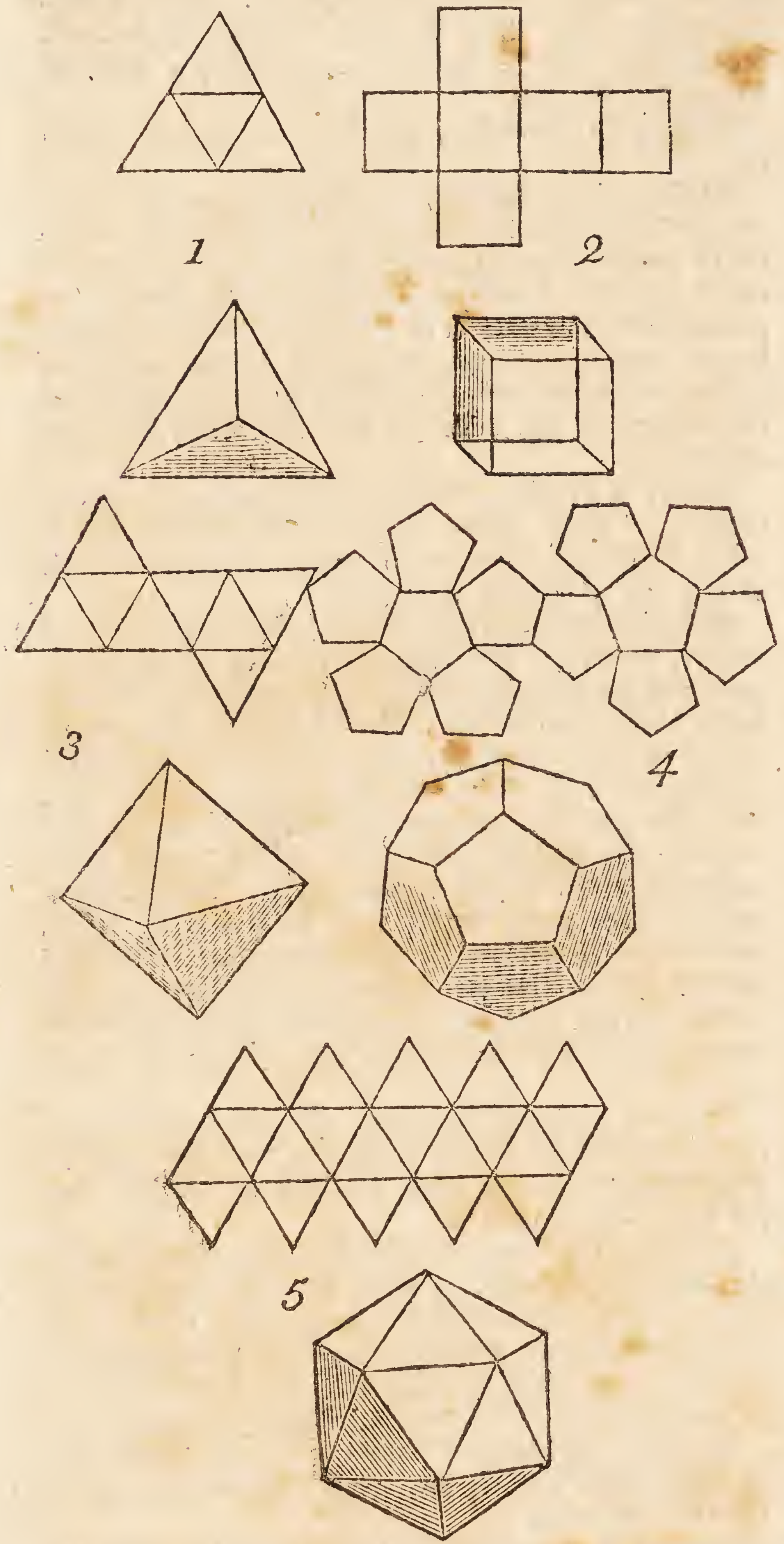
BODY *of the Place*, in Fortification, denotes either the buildings incloſed, or more generally the incloſure itſelf. Thus, to conſtruct the body of the place, is to fortify or incloſe the place with baſtions and curtains.

BODY *of a Pump*, the thickeſt part of the barrel or pipe of a pump, within which the piſton moves.

BODIES, *Regular* or *Platonic*, are thoſe which have all their ſides, angles, and planes, ſimilar and equal. Of theſe there are only 5; viz,
the *tetraedron*, contained by 4 equilateral triangles;
the *hexaedron* or *cube*, by 6 ſquares;
the *octaedron*, by 8 triangles;
the *dodecaedron*, by 12 pentagons; and
the *icoſaedron*, by 20 triangles.

To form the five Regular Bodies.

Let the annexed figures be exactly drawn on paſteboard, or ſtiff paper, and cut out from it by the extreme or bounding lines: then cut the others, or internal lines, only half through, ſo that the parts may be turned up by them, and then glued or otherwiſe faſtened together with paſte, ſealing-wax, &c; ſo ſhall they form the reſpective body marked with the correſponding number; viz, Nº 1 the tetraedron, Nº 2 the hexaedron or cube, Nº 3 the octaedron, Nº 4 the dodecaedron, and Nº 5 the icoſaedron.



To find the Superficies or Solidity of the Regular Bodies.

1. Multiply the proper tabular area (taken from the following table) by the ſquare of the linear edge of the ſolid, for the ſuperficies.

2. Multiply the tabular ſolidity by the cube of the linear edge, for the ſolid content.

Table of the Surfaces and Solidities of the five Regular Bodies, the linear edge being 1.

No. of Faces	Names	Surfaces	Solidities
4	Tetraedron	1·73205	0·11785
6	Hexaedron	6·00000	1·00000
8	Octaedron	3·46410	0·47140
12	Dodecaedron	20·64573	7·66312
20	Icoſaedron	8·66025	2·18169

For more particular properties, ſee each reſpective word. See alſo my large *Menſuration*, pa. 248, edit. 2.

Theſe bodies were called *platonic*, becauſe they were ſaid to have been invented, or firſt treated of, by *Plato*, who conceived certain myſteries annexed to them.

BOFFRAND (Germain), a celebrated French architect and engineer, was born at Nantes in Bretagne in 1667. He was brought up under Harduin Manſarad, who truſted him with conducting his greateſt works. Boffrand was admitted into the French Academy of Architecture in 1709. Many German princes choſe him for their architect, and raiſed conſiderable edifices on his plans. His manner of building approached that of Palladio; and there was much of grandeur in all his deſigns. As engineer and inſpector-general of bridges and highways, he directed and conſtructed a number of canals, ſluices, bridges, and other mechanical works. He publiſhed a curious and uſeful book, containing the general principles of his art; with an account of the plans, profiles, and elevations of the principal works, which he executed in France and other countries. Boffrand died at Paris in 1755, dean of the Academy of Architecture, firſt engineer and inſpector-general of the bridges and highways, architect and adminiſtrator of the general hoſpital.

BOILING, or Ebullition, the bubbling up of any fluid, by the application of heat. This is, in general, occaſioned by the diſcharge of an elaſtic vapour through the fluid that boils; whether that be common air, fixed air, or ſteam, &c. It is proved by Dr. Hamilton of Dublin, in his Eſſay on the aſcent of vapour, that the boiling of water is occaſioned by the lowermoſt particles of it being heated and rarefied into vapour, or ſteam; in conſequence of this diminution of their ſpecific gravity, they aſcend through the ſurrounding heavier fluid with great velocity, lacerating and throwing up the body of water in the aſcent, and ſo giving it the tumultuous motion called *boiling*.

That this is occaſioned by elaſtic ſteam, and not by particles of fire or air, as ſome have imagined, is eaſily proved by the following ſimple experiment: Take a common drinking glaſs filled with hot water, and invert it into a veſſel of the ſame: then, as ſoon as the water in the veſſel begins to boil, large bubbles will be ſeen to aſcend in the glaſs, by which the water in it will be diſplaced, and there will ſoon be a continued bubbling from under its edge: but if the glaſs be then drawn up, ſo that its mouth may juſt touch the water, and a cloth wetted in cold water be applied to the outſide, the elaſtic ſteam within it will be inſtantly condenſed, upon which the water will aſcend ſo as nearly to fill it again. Some ſmall parts of air &c, that may happen to be lodged in the fluid, may alſo perhaps be expelled, as well as the rarefied ſteam. And this is particularly recommended as a method of purifying quickſilver, for making more accurately barometers and thermometers.

We commonly annex the idea of a certain very great degree of heat to the boiling of liquids, though often without reaſon; for different liquids boil with different degrees of heat; and any one given liquid alſo, under different preſſures of the atmoſphere. Thus, a veſſel of tar being ſet over the fire till it boils, it is ſaid a perſon may then put his hand into it without injury: and by putting water under the receiver of an air-pump, and applying the flame of a candle or lamp under it, by gradually exhauſting, the water is made to boil with always leſs and leſs degrees of heat; and without applying any heat at all, the water, or even the moiſture about the bottom or edges of the receiver, will riſe in an elaſtic vapour up into it, when the exhauſtion is near completed.

Spirit of wine boils ſtill ſooner in vacuo than water. And Dr. Freind gives a table of the different times required to make ſeveral fluids boil by the ſame heat. See alſo Philoſ. Tranſ. N° 122.

BOMB, in Artillery, a ſhell, or hollow ball of caſt-iron, having a large vent, by which it is filled with gun-powder, and which is fitted with a fuze or hollow plug to give fire by, when thrown out of a mortar, &c: about the time when the ſhell arrives at the intended place, the compoſition in the pipe of the fuze ſets fire to the powder in the ſhell, which blows it all in pieces, to the great annoyance of the enemy, by killing the people, or firing the houſes, &c. They are now commonly called *ſhells* ſimply, in the Engliſh artillery.

Theſe ſhells, or bombs, are of various ſizes, from that of 17 or 18 inches diameter downwards. The very large ones are not uſed by the Engliſh, that of 13 inches diameter being the higheſt ſize now employed by them; the weight, dimenſions, and other circumſtances of them, and the others downwards, are as in the following table.

Diameter of the Shell.	Weight of the Shell.	Powder to fill them.		Powder to burſt them into moſt pieces.	
	lbs.	lb.	oz.	lb.	oz.
13 inch	195	9	4½	7	8
10	89	4	4½	3	4
8	46	2	3½	2	0
5⅘ Royal	14½	1	1½	0	14
4⅖ Cohorn	7½	0	8	0	7

Mr. Muller gives the following proportion for all ſhells. Dividing the diameter of the mortar into 30 equal parts, then the other dimenſions, in 30ths of that diameter, will be thus:

Diameter of the bore, or mortar - -	30
Diameter of the ſhell, - - - -	29½
Diameter of the hollow ſphere - -	21
Thickneſs of metal at the fuze hole -	3½
Thickneſs at the oppoſite part - -	5
Diameter of the fuze hole - -	4
Weight of ſhell empty - -	$\frac{10}{114}d$
Weight of powder to fill it - -	$\frac{2}{473}d$

where *d* denotes the cube of the diameter of the bore in inches.—But ſhells have alſo lately been made with the metal all of the ſame thickneſs quite around.

In general, the windage, or difference between the diameter of the ſhell and mortar, is $\frac{1}{20}$ of the latter; alſo the diameter of the hollow part of the ſhell is $\frac{7}{10}$ of the ſame.

Bombs are thrown out of mortars, or howitzers; but they may alſo be thrown out of cannon; and a very ſmall ſort are thrown by the hand, which are called granados: and the Venetians at the ſiege of Candia, when the Turks had poſſeſſed themſelves of the ditch uſed large bombs without any piece of ordnance, but barely rolled them down upon the enemy along a plank ſet aſlope, with ledges on the ſides to keep the bomb right forwards.

Mr. Blondel, in his *Art de jetter des Bombes*, ſays the firſt bombs were thoſe thrown into the city of Watchtendonch in Guelderland, in 1588; and they are deſcribed by our countryman Lucar, in his book on Artillery publiſhed this ſame year 1588; though it is pretended by others that they were in uſe near a century before, namely at the ſiege of Naples in 1495. They only came into common uſe, however, in 1634, and then only in the Dutch and Spaniſh armies. It is ſaid that one Malthus, an Engliſh engineer, was ſent for from Holland by Lewis the 14th, who uſed them for him with much ſucceſs, particularly at the ſiege of Cohoure in 1642.

The art of throwing bombs, or ſhells, forms a principal branch of Gunnery, founded on the theory of projectiles, and the quantities and laws of force of gunpowder. And the principal writers on this art are Meſſ. Blondel, Guiſnée, De Reſſons, De La Hire, &c.

Bomb-Chest, is a kind of cheſt uſually filled with bombs, and ſometimes only with gunpowder, placed under ground, to blow it up into the air with thoſe who ſtand upon it; being ſet on fire by means of a sauciſſee faſtened at one end. But they are now much out of uſe.

BOMBARD, an ancient piece of ordnance, now out of uſe. It was very ſhort and thick, with a large mouth; ſome of which it is ſaid threw balls of 300 pounds weight, requiring the uſe of cranes to load them. The Bombard is by ſome called *baſiliſk*, and by the Dutch *donderbus*.

To bombard, is to attack by throwing of bombs, or ſhells.

BOMBARDIER, a perſon employed about throwing bombs or ſhells. He adjuſts the fuze, and loads and fires the mortar.

BONES, *Napier's*. See Napier.

BONING, in Surveying and Levelling, &c, is the placing three or more rods or poles, all of the ſame length, in or upon the ground, in ſuch a manner that the tops of them be all in one continued ſtraight line, whether it be horizontal or inclined, ſo that the eye can look along the tops of them all, from one end of the line to the other.

BONNET, in Fortification, a ſmall work of two faces, having only a parapet, with two rows of paliſadoes at about 10 or 12 feet diſtance. It is commonly placed before the ſaliant angle of the counterſcarp, and having a communication with the covered way, by means of a trench cut through the glacis, and paliſadoes on each ſide.

Bonnet *à Prêtre*, or *Prieſt's Cap*, is an outwork, having three ſaliant angles at the head, beſides two inwards. It differs from the double tenaille only in this, that its ſides, inſtead of being parallel, grow narrower, or cloſer, at the gorge, and opening at the front; from whence it is called *queue d'aronde*, or ſwallow's tail.

BOOTES, a conſtellation of the northern hemiſphere, and one of the 48 old ones; having 23 ſtars in

Ptolemy's catalogue, 28 in Tycho's, 34 in Bayer's, 52 in Hevilus's, and 54 in Flamſteed's; of which one, in the ſkirt of his coat, is of the firſt magnitude, and called *Arcturus*.

Bootes is repreſented as a man in the poſture of walking; his right hand graſping a club, and his left extended upwards, and holding the cord of the two dogs which ſeem barking at the Great Bear.

The Greeks, contrary to their uſual cuſtom, do not give any certain account of the origin of this conſtellation. Thoſe who in very early days made the ſtars which were afterwards formed into the great bear repreſent a waggon drawn by oxen, made this Bootes the driver of them, from whence he was called the waggoner: others continued the office when the waggon was deſtroyed, and made a celeſtial bearward of Bootes, making it his office to drive the two bears round about the pole: and ſome, when the greater waggon was turned into the greater bear, were ſtill for preſerving the form of that machine in thoſe ſtars which conſtitute Bootes.

This conſtellation is called by various other names; as *Arcas*, *Arctophylax*, *Arcturus-Minor*, *Bubulcus*, *Bubulus*, *Canis-Latrans*, *Clamator*, *Icarus*, *Lycaon*, *Philometus*, *Plauſtri-Cuſtos*, *Plorans*, *Thegnis*, and *Vociferator*; by Heſychius it is called *Orion*, and by the Arabs *Aramech*, or *Archamech*. Schiller, inſtead of Bootes, makes the figure of St. Sylveſter; Schickhard, that of Nimrod; and Weigelius, the three Swediſh crowns. See Wolf. Lex Math. p. 266.

BORE, of a gun, or other piece of ordnance, is the chaſe, cylinder, or hollow part of the piece.

BOREAL Signs, are the firſt ſix ſigns of the Zodiac, or thoſe on the northern ſide of the equinoctial; viz, the ſigns ♈ aries, ♉ taurus, ♊ gemini, ♋ cancer, ♌ leo, ♍ virgo.

BOREALIS, Aurora. See Aurora *Borealis*.

BORELLI (John Alphonso), a celebrated philoſopher and mathematician, born at Naples the 28th of January 1608. He was profeſſor of philoſophy and mathematics in ſome of the moſt celebrated univerſities of Italy, particularly at Florence and Piſa, where he became highly in favour with the princes of the houſe of Medicis. But having been concerned in the revolt of Meſſina, he was obliged to retire to Rome, where he ſpent the remainder of his life under the protection of Chriſtina queen of Sweden, who honoured him with her friendſhip, and by her liberality towards him ſoftened the rigour of his hard fortune. He continued two years in the convent of the regular clergy of St. Pantaleon, called the *Pious Schools*, where he inſtructed the youth in mathematical ſtudies. And this ſtudy he proſecuted with great diligence for many years afterward, as appears by his correſpondence with ſeveral ingenious mathematicians of his time, and the frequent mention that has been made of him by others, who have endeavoured to do juſtice to his memory. He wrote a letter to Mr. John Collins, in which he diſcovers his great deſire and endeavours to promote the improvement of thoſe ſciences: he alſo ſpeaks of his correſpondence with, and great affection for, Mr. Henry Oldenburgh, Secretary of the Royal Society; of Dr. Wallis; of the then late learned Mr. Boyle, and lamented the loſs ſuſtained by his death to the commonwealth of learning. Mr. Baxter, in his *Enquiry into the Nature of the Human Soul*, makes frequent uſe of our author's book *De Motu Animalium*, and tells us, that he was the firſt who diſcovered that the force exerted within the body prodigiouſly exceeds the weight to be moved without, or that nature employs an immenſe power to move a ſmall weight. But he acknowledges that Dr. James Keil had ſhewn that Borelli was miſtaken in his calculation of the force of the muſcle of the heart; but that he nevertheleſs ranks him with the moſt authentic writers, and ſays he is ſeldom miſtaken: and, having remarked that it is ſo far from being true, that great things are brought about by ſmall powers, that, on the contrary, a ſtupendous power is manifeſt in the moſt ordinary operations of nature, he obſerves that the ingenious Borelli firſt obſerved this in animal motion; and that Dr. Stephen Hales, by a courſe of experiments in his *Vegetable Statics*, had ſhewn the ſame in the force of the aſcending ſap in vegetables.

After a courſe of unceaſing labours, Borelli died at Pantaleon of a pleuriſy, the 31ſt of December 1679, at 72 years of age.

Beſide ſeveral books on phyſical ſubjects, Borelli publiſhed the following mathematical ones: viz.

1. Apollonii Pergæi Conicorum Lib. 5, 6, & 7. Floren. 1661, fol.
2. Theoriæ Medicorum Planetarum ex cauſis phyſicis deductæ. Flor. 1666, 4to.
3. De Vi Percuſſionis. Bologna, 1667, 4to.—This piece was reprinted, with his celebrated treatiſe De Motu Animalium, and that other De Motionibus Naturalibus, in 1686.
4. Euclides Reſtitutus, &c. Piſa, 1668, 4to.
5. Oſſervatione intorno alla viſtu ineguali degli Occi.—This piece was inſerted in the Journal of Rome, for the year 1669.
6. De Motionibus Naturalibus de Gravitate pendentibus. Regio Julio, 1670, 4to.
7. Meteorologia Aetnea, &c. Regio Julio, 1670, 4to.
8. Oſſervatione dell' Eccliſſi Lunare, 11 Gennaro 1675.—Inſerted in the Journal of Rome 1675, p. 34.
9. Elementa Conica Appollonii Pergæi, & Archimedis Opera, nova & breviori methodo demonſtrata.—Printed at Rome in 1679, in 12mo, at the end of the 3d edition of his Euclides Reſtitutus.
10. De Motu Animalium. Pars prima in 1680, and Pars altera in 1681, 4to.—Theſe were reprinted at Leyden 1685; reviſed and purged from many errors; with the addition of John Bernoulli's Mathematical Meditations concerning the Motion of the Muſcles.
11. At Leyden, 1686, in 4to, a more correct and accurate edition, reviſed by J. Broen, M. D. of Leyden, of his two pieces, De Vi Percuſſionis, & De Motionibus de Gravitate pendentibus.

BOUGUER (Peter), a celebrated French mathematician, was born at Croiſic, in Lower Bretagne, the 10th of February 1698. He was the ſon of John Bouguer, Profeſſor Royal of Hydrography, a tolerably good mathematician, and author of A complete Treatiſe on Navigation. Young Bouguer was accuſtomed to learn mathematics from his father, from the time he was able to ſpeak, and thus became a proficient in thoſe ſciences while he was yet a child. He was ſent very early to the Jeſuits' college at Vannes, where he had the honour to inſtruct his regent in the mathematics, at eleven years of age.

Two years after this he had a public conteſt with a profeſſor of mathematics, upon a propoſition which the latter had advanced erroneouſly; and he triumphed over him; upon which the profeſſor, unable to bear the diſgrace, left the country.

Two years after this, when young Bouguer had not yet finiſhed his ſtudies, he loſt his father; whom he was appointed to ſucceed in his office of hydrographer, after a public examination of his qualifications; being then only 15 years of age; an occupation which he diſcharged with great reſpect and dignity at that early age.

In 1727, at the age of 29, he obtained the prize propoſed by the Academy of Sciences, for the beſt way of maſting of ſhips. This firſt ſucceſs of Bouguer was ſoon after followed by two others of the ſame kind; he ſucceſſively gained the prizes of 1729 and 1731; the former, for the beſt manner of obſerving at ſea the height of the ſtars, and the latter, for the moſt advantageous way of obſerving the declination of the magnetic needle, or the variation of the compaſs.

In 1729, he gave an *Optical Eſſay upon the Gradation of Light;* a ſubject quite new, in which he examined the intenſity of light, and determined its degrees of diminution in paſſing through different pellucid mediums, and particularly that of the ſun in traverſing the earth's atmoſphere. Mairan gave an extract of this firſt eſſay in the Journal des Savans, in 1730.

In this ſame year, 1730, he was removed from the port of Croiſic to that of Havre, which brought him into a nearer connection with the Academy of Sciences, in which he obtained, in 1731, the place of aſſociate geometrician, vacant by the promotion of Maupertuis to that of penſioner; and in 1735 he was promoted to the office of penſioner-aſtronomer. The ſame year he was ſent on the commiſſion to South America, along with Meſſieurs Godin, Condamine, and Jeuſſieu, to determine the meaſure of the degrees of the meridian, and the figure of the earth. In this painful and troubleſome buſineſs, of 10 years duration, chiefly among the lofty Cordelier mountains, our author determined many other new circumſtances, beſide the main object of the voyage; ſuch as the expanſion and contraction of metals and other ſubſtances, by the ſudden and alternate changes of heat and cold among thoſe mountains; obſervations on the refraction of the atmoſphere from the tops of the ſame, with the ſingular phenomenon of the ſudden increaſe of the refraction, when the ſtar can be obſerved below the line of the level; the laws of the denſity of the air at different heights, from obſervations made at different points of theſe enormous mountains; a determination that the mountains have an effect upon a plummet, though he did not aſſign the exact quantity of it; a method of eſtimating the errors committed by navigators in determining their route; a new conſtruction of the log for meaſuring a ſhip's way; with ſeveral other uſeful improvements.

Other inventions of Bouguer, made upon different occaſions, were as follow: The heliometer, being a teleſcope with two object glaſſes, affording a good method of meaſuring the diameters of the larger planets with eaſe and exactneſs: his reſearches on the figure in which two lines or two long ranges of parallel trees appear: his experiments on the famous reciprocation of the pendulum: and thoſe upon the manner of meaſuring the force of the light: &c, &c.

The cloſe application which Bouguer gave to ſtudy, undermined his health, and terminated his life the 15th of Auguſt 1758, at 60 years of age.—His chief works, that have been publiſhed, are,

1. The Figure of the Earth, determined by the obſervations made in South America; 1749, in 4to.
2. Treatiſe on Navigation and Pilotage; Paris, 1752, in 4to. This work has been abridged by M. La Caille, in 1 volume, 8vo, 1768.
3. Treatiſe on Ships, their Conſtruction and Motions; in 4to, 1756.
4. Optical Treatiſe on the Gradation of Light; firſt in 1729; then a new edition in 1760, in 4to.

His papers that were inſerted in the Memoirs of the Academy, are very numerous and important: as, in the Memoirs for 1726, Compariſon of the force of the ſolar and lunar light with that of candles.—1731, Obſervations on the curvilinear motion of bodies in mediums.—1732, Upon the new curves called the *lines of purſuit.*—1733, To determine the ſpecies of conoid, to be conſtructed upon a given baſe which is expoſed to the ſhock of a fluid, ſo that the impulſe may be the leaſt poſſible.—Determination of the orbit of comets.—1734, Compariſon of the two laws which the earth and the other planets muſt obſerve in the figure which gravity cauſes them to take.—On the curve lines proper to form the arches in domes.—1735, Obſervations on the equinoxes.—On the length of the pendulum.—1736, On the length of the pendulum in the torrid zone.—On the manner of determining the figure of the earth by the meaſure of the degrees of latitude and longitude.—1739, On the aſtronomical refractions in the torrid zone.—Obſervations on the lunar eclipſe, of the 8th of September 1737, made at Quito.—1744, Short account of the voyage to Peru, by the members of the Royal Academy of Sciences, to meaſure the degrees of the meridian near the equator, and from thence to determine the figure of the earth.—1745, Experiments made at Quito and divers other places in the torrid zone, on the expanſion and contraction of metals by heat and cold.—On the problem of the maſting of ſhips.—1746, Treatiſe on ſhips, their ſtructure and motions.—On the impulſe of fluids upon the fore parts of pyramidoids having their baſe a trapezium.—Continuation of the ſhort account given in 1744, of the voyage to Peru for meaſuring the earth.—1747, On a new conſtruction of the log, and other inſtruments for meaſuring the run of a ſhip.—1748, Of the diameters of the larger planets. The new inſtrument called a *heliometer*, proper for determining them; with obſervations of the ſun.—Obſervation of the eclipſe of the moon the 8th of Auguſt 1748.—1749, Second memoir on aſtronomical refractions, obſerved in the torrid zone, with remarks on the manner of conſtructing the tables of them.—

Figure of the earth determined by MM. Bouguer and Condamine, with an abridgment of the expedition to Peru.—1750, Obſervation of the lunar eclipſe of the 13th of December 1750.—1751, On the form of bodies moſt proper to turn about themſelves, when they are puſhed by one of their extremities, or any other point.—On the moon's parallax, with the eſtimation of the changes cauſed in the parallaxes by the figure of the earth.—Obſervation of the lunar eclipſe, the 2d of December 1751.—1752, On the operations made by ſeamen, called *Corrections*.—1753, Obſervation of the paſſage of Mercury over the ſun, the 6th of May 1753.—On the dilatations of the air in the atmoſphere.—New treatiſe of navigation, containing the theory and practice of pilotage, or working of ſhips.—1754, Operations, &c, for diſtinguiſhing, among the different determinations of the degree of the meridian near Paris, that which ought to be preferred.—On the direction which the ſtring of a plummet takes.—Solution of the chief problems in the working of ſhips.—1755, On the apparent magnitude of objects.—Second memoir on the chief problems in the working of ſhips.—1757, Account of the treatiſe on the working of ſhips.—On the means of meaſuring the light.—1758, His Eulogy.

In the volumes of the prizes given by the academy, are the following pieces by Bouguer:

In vol. 1, on the maſting of ſhips.—Vol. 2, On the method of exactly obſerving at ſea the height of the ſtars; and the variation of the compaſs. Alſo on the cauſe of the inclination of the planets' orbits.

BOULTINE, in Architecture, a convex moulding, of a quarter of a circle, and placed next below the plinth in the Tuſcan and Dorick capital.

BOW, an offenſive weapon made of wood, horn, ſteel, or other elaſtic matter, by which arrows are thrown with great force. This inſtrument was of very ancient and general uſe, and is ſtill found among all ſavage nations who have not the uſe of fire arms, by which it has been ſuperſeded among us. There are two ſpecies of the Bow, the *Long*, and the *Croſs* Bow.

The *Long Bow* is ſimply a bow, or a rod, with a ſtring faſtened to each end of it, to the middle of which the end of an arrow being applied, and then drawn by the hand, on ſuddenly quitting the hold, the bow returns by means of its elaſticity, and impels the arrow from the ſtring with great violence. The old Engliſh archers were famous for the long bow, by means of which they gained many victories in France and elſewhere.

The *Croſs Bow*, called alſo *arbaleſt* or *arbalet*, is a bow ſtrung and ſet in a ſhaft of wood, and furniſhed with a trigger; ſerving to throw bullets, darts, and large arrows, &c. The ancients had large machines for throwing many arrows at once, called *arbalets*, or *baliſtæ*.

The force of a bow may be calculated on this principle, that its ſpring, *i. e.* the power by which it reſtores itſelf to its natural poſition, is always proportional to the ſpace or diſtance it is bent or removed from it.

Bow, a mathematical inſtrument formerly uſed at ſea for taking the ſun's altitude. It conſiſted of a large arch divided into 90 degrees, fixed on a ſtaff, and furniſhed with three vanes, viz, a ſide vane, a ſight vane, and a horizon vane.

Bow-*Compaſs*, an inſtrument for drawing arches of very large circles, for which the common compaſſes are too ſmall. It conſiſts of a beam of wood or braſs, with three long ſcrews that govern or bend a lath of wood or ſteel to any arch.

BOX and Needle, the ſmall compaſs of a theodolite, circumferentor, or plain-table.

BOYAU, in Fortification, a ditch covered by a parapet, and ſerving as a communication between two trenches. It runs parallel to the works of the body of the place; and ſerves as a line of contravallation, both to hinder the ſallies of the beſieged, and to ſecure the miners. When it is a particular cut running from the trenches, to cover ſome ſpot of ground, it is drawn ſo as not to be enfiladed or ſcoured by the enemy's ſhot.

BOYLE (Robert), one of the greateſt philoſophers, as well as beſt men, that any country has ever produced, was the 7th ſon and the 14th child of Richard earl of Cork, and was born at Liſmore in the province of Munſter in Ireland, the 25th of January, 1626-7; the very year of the death of the learned Lord Bacon, whoſe plans of experimental philoſophy our author afterwards ſo ably ſeconded. While very young, he was inſtructed in his father's houſe to read and write, and to ſpeak French and Latin. In 1635, when only 8 years old, he was ſent over to England, to be educated at Eton ſchool. Here he ſoon diſcovered an extraordinary force of underſtanding, with a diſpoſition to cultivate and improve it to the utmoſt.

After remaining at Eton between 3 and 4 years, his father ſent our author and his brother Francis, in 1638, on their travels upon the continent. They paſſed through France to Geneva, where they ſettled for ſome time to purſue their ſtudies: here our author reſumed his acquaintance with the elements of the mathematics, which he had commenced at Eton when 10 years old, on occaſion of an illneſs which prevented his other uſual ſtudies.

In the autumn of 1641, he quitted Geneva, and travelled through Switzerland and Italy to Venice, from whence he returned again to Florence, where he ſpent the winter, ſtudying the Italian language and hiſtory, and the works of the celebrated aſtronomer Galileo, who died in a village near this city during Mr. Boyle's reſidence here.

About the end of March 1642, he ſet out from Florence, viſited Rome and other places in Italy; then returned to the ſouth of France. At Marſeilles, in May 1642, Mr. Boyle received letters from his father, which informed him that the rebellion had broken out in Ireland, and with how much difficulty he had procured 250l. then remitted to help him and his brother home. This remittance however never reached them, and they were obliged to return to Geneva with their governor Mr. Marcombes, who contrived on his own credit, and by ſelling ſome jewels, to raiſe money enough to ſend them to England, where they arrived in 1644. On their arrival they found that their father was dead, and had left our author the manor of Stalbridge in England, with ſome other conſiderable eſtates in Ireland.

From this time Mr. Boyle's chief reſidence, for ſome years at leaſt, was at his manor of Stalbridge, from whence he made occaſional excurſions to Oxford, London, &c; applying himſelf with great induſtry to various kinds of ſtudies, but eſpecially to philoſophy and chemiſtry; and ſeizing every opportunity of cultivating the acquaintance of the moſt learned men of his time. He was one of the members of that ſmall but learned body of men who, when all academical ſtudies were interrupted by the civil wars, ſecreted themſelves about the year 1645; and held private meetings, firſt in London, afterwards at Oxford, to cultivate ſubjects of natural knowledge upon that plan of experiment which Lord Bacon had delineated. They ſtyled themſelves then *The Philoſophic College*; but after the reſtoration, when they were incorporated, and diſtinguiſhed openly, they took the name of the *Royal Society*.

In the ſummer of 1654 he retired to ſettle at Oxford, the Philoſophical Society being removed from London to that place, that he might enjoy the converſation of the other learned members, his friends, who had retired thither, ſuch as Wilkins, Wallis, Ward, Willis, Wren, &c. It was during his reſidence here that he improved that admirable engine the air-pump; and by numerous experiments was enabled to diſcover ſeveral qualities of the air, ſo as to lay a foundation for a complete theory. He declared againſt the philoſophy of Ariſtotle, as having in it more of words than things; promiſing much, and performing little; and giving the inventions of men for indubitable proofs, inſtead of building upon obſervation and experiment. He was ſo zealous for this true method of learning by experiment, and ſo careful about it, that though the Carteſian philoſophy then made a great noiſe in the world, yet he could never be perſuaded to read the works of Deſcartes, for fear he ſhould be amuſed and led away by plauſible accounts of things founded on conjecture, and merely hypothetical. But philoſophy, and enquiries into nature, though they engaged his attention deeply, did not occupy him entirely; as he ſtill continued to purſue critical and theological ſtudies. He had offers of preferment to enter into holy orders, by the government, after the reſtoration. But he declined the offer, chooſing rather to purſue his ſtudies as a layman, in ſuch a manner as might be moſt effectual for the ſupport of religion; and began to communicate to the world the fruits of theſe ſtudies. Theſe fruits were very numerous and important, as well as various: the principal of which, as well as of ſome other memorable occurrences of his life, were nearly in the following order.

In 1660 came out, 1. New experiments, phyſico-mechanical, touching the ſpring of the air and its effects.—2. Seraphic love; or ſome motives and incentives to the love of God, pathetically diſcourſed of in a letter to a friend. A work which it has been ſaid was owing to his courtſhip of a lady, the daughter of Cary earl of Monmouth; though our author was never married.—3. Certain phyſiological eſſays and other tracts, in 1661.—4. Sceptical chemiſt, 1662; reprinted about the year 1679, with the addition of divers experiments and notes on the producibleneſs of chemical principles.

In the year 1663, the Royal Society being incorporated by king Charles the 2d, Mr. Boyle was named one of the council; and as he might juſtly be reckoned among the founders of that learned body, ſo he continued one of the moſt uſeful and induſtrious of its members during the whole courſe of his life. His next publications were, 5. Conſiderations touching the uſefulneſs of experimental natural philoſophy, 1663.—6. Experiments and conſiderations upon colours; to which was added a letter, containing Obſervations on a diamond that ſhines in the dark, 1663. This treatiſe is full of curious and uſeful remarks on the hitherto unexplained doctrine of light and colours; in which he ſhews great judgment, accuracy, and penetration; and which may be ſaid to have led the way to Newton, who made ſuch great diſcoveries in that branch of phyſics.—7. Conſiderations on the ſtyle of the holy ſcriptures, 1663. This was an extract from a larger work, intitled An eſſay on ſcripture; which was afterwards publiſhed by Sir Peter Pett, a friend of Mr. Boyle's.

In 1664 he was elected into the company of the royal mines; and was all this year occupied in proſecuting various good deſigns, which was probably the reaſon that he did not publiſh any works in this year. Soon after came out, 8. Occaſional reflections upon ſeveral ſubjects, 1665. This piece expoſed our author to the cenſure of the celebrated Dean Swift, who, to ridicule theſe diſcourſes, wrote *A pious meditation upon a broomſtick, in the ſtyle of the honourable Mr. Boyle.*—9. New experiments and obſervations upon cold, 1665.—10. Hydroſtatical paradoxes made out by new experiments, for the moſt part phyſical and eaſy, 1666.—11. The origin of forms and qualities, according to the corpuſcular philoſophy, 1666.—Both in this and the former year, our author communicated to his friend Mr. Oldenburgh, then ſecretary to the Royal Society, ſeveral curious and excellent ſhort pieces of his own, upon a great variety of ſubjects, and others tranſmitted to him by his learned friends, which are printed in the Philoſ. Tranſ.

In the year 1668 Mr. Boyle reſolved to ſettle in London for life; and for that purpoſe he removed to the houſe of his ſiſter, the lady Ranelagh, in Pall-Mall. This removal was to the great benefit of the learned in general, and particularly of the Royal Society, to whom he gave great and continual aſſiſtance, as abundantly appears by the ſeveral pieces communicated to them from time to time, and printed in their Tranſactions. To avoid improper waſte of time, he had ſet hours in the day appointed for receiving ſuch perſons as wanted to conſult him, either for their own aſſiſtance, or to communicate new diſcoveries to him: And he beſides kept up an extenſive correſpondence with the moſt learned men in Europe; ſo that it is wonderful how he could bring out ſo many new works as he did. His next publications were, 12. A continuation of new experiments touching the weight and ſpring of the air; to which is added, A diſcourſe of the atmoſphere of conſiſtent bodies, 1669.—13. Tracts about the coſmical qualities of things; coſmical ſuſpicions; the temperature of the ſubterraneous regions; the bottom of the ſea; to which is prefixed an introduction to the hiſtory of particular qualities, 1669.—14. Conſiderations on the uſefulneſs of experimental and natural philoſophy,

loſophy, the 2d part, 1671.—15. A collection of tracts upon ſeveral uſeful and important points of practical philoſophy, 1671.—16. An eſſay upon the origin and virtues of gems, 1672.—17. A collection of tracts upon the relation between flame and air; and ſeveral other uſeful and curious ſubjects, 1672. Beſides furniſhing, in this and the former year, a number of ſhort diſſertations upon a great variety of topics, addreſſed to the Royal Society, and inſerted in their Tranſactions.—18. Eſſays on the ſtrange ſubtilty, great efficacy, and determinate nature, of effluvia; with a variety of experiments on other ſubjects, 1673.—19. The excellency of theology compared with philoſophy, 1673. This diſcourſe was written in the year 1665, while our author, to avoid the great plague which then raged in London, was forced to go from place to place in the country, having little or no opportunity of conſulting his books.—20. A collection of tracts upon the ſaltneſs of the ſea, the moiſture of the air, the natural and preternatural ſtate of bodies; to which is prefixed a dialogue concerning cold, 1674.—21. A collection of tracts containing ſuſpicions about hidden qualities of the air; with an appendix touching celeſtial magnets; animadverſions upon Mr. Hobbes's problem about a vacuum; a diſcourſe of the cauſe of attraction and ſuction, 1674.—22. Some conſiderations about the reaſonableneſs of reaſon and religion; by T. E. (the final letters of his names). To which is annexed a diſcourſe about the poſſibility of the reſurrection; by Mr. Boyle, 1675. The ſame year ſeveral papers communicated to the Royal Society, among which were two upon quickſilver growing hot with gold.—23. Experiments and notes about the mechanical origin or production of particular qualities, in ſeveral diſcourſes on a great variety of ſubjects, and among the reſt on electricity, 1676.—He then communicated to Mr. Hook a ſhort memorial of ſome obſervations made upon an artificial ſubſtance that ſhines without any preceding illuſtration; publiſhed by Hook in his *Lectiones Cutlerianæ*.—24. Hiſtorical account of a degradation of gold made by an anti-elixir.—25. Aerial noctiluca; or ſome new phænomena, and a proceſs of a factitious ſelf-ſhining ſubſtance, 1680. This year the Royal Society, as a proof of the juſt ſenſe of his great worth, and of the conſtant and particular ſervices which through a courſe of many years he had done them, made choice of him for their preſident; but he being extremely, and, as he ſays, peculiarly tender in point of oaths, declined that honour.—26. Diſcourſe of things above reaſon; inquiring, whether a philoſopher ſhould admit any ſuch, 1681.—27. New experiments and obſervations upon the icy noctiluca; to which is added a chemical paradox, grounded upon new experiments, making it probable that chemical principles are tranſmutable, ſo that out of one of them others may be produced, 1682.—28. A continuation of new experiments, phyſico-mechanical, touching the ſpring and weight of the air, and their effects, 1682.—29. A ſhort letter to Dr. Beale, in relation to the making of freſh water out of ſalt, 1683.—30. Memoirs for the natural hiſtory of human blood, eſpecially the ſpirit of that liquor, 1684.—31. Experiments and conſiderations about the poroſity of bodies, 1684.—32. Short memoirs for the natural experimental hiſtory of mineral waters, &c, 1685.—33. An eſſay on the great effects of even languid and unheeded motion, &c, 1685.—34. Of the reconcileableneſs of ſpecific medicines to the corpuſcular philoſophy, &c, 1685.—35. Of the high veneration man's intellect owes to God, peculiarly for his wiſdom and power, 1685.—36. Free inquiry into the vulgarly received notion of nature, 1686.—37. The martyrdom of Theodora and Didymia, 1687. A work he had drawn up in his youth.—38. A diſquiſition about the final cauſes of natural things, and about vitiated light, 1688.

Mr. Boyle's health declining very much, he abridged greatly his time given to converſations and communications with other perſons, to have more time to prepare for the preſs ſome others of his papers, before his death, which were as follow:—39. *Medicina Hydroſtatica*, &c, 1690.—40. The Chriſtian virtuoſo, &c, 1690. 41. *Experimenta et Obſervationes Phyſicæ*, &c, 1691; which is the laſt work that he publiſhed.

Mr. Boyle died on the laſt day of December of the ſame year 1691, in the 65th year of his age, and was buried in St. Martin's church in the Fields, Weſtminſter; his funeral ſermon being preached by Dr. Gilbert Burnet biſhop of Saliſbury; in which he diſplayed the excellent qualities of our author, with many circumſtances of his life, &c. But as the limits of this work will not allow us to follow the biſhop in the copious and eloquent account he has given of this great man's abilities, we muſt content ourſelves with adding the following ſhort eulogium by the celebrated phyſician, philoſopher, and chemiſt, Dr. Boerhaave; who, after having declared lord Bacon to be the father of experimental philoſophy, aſſerts, that "Mr. Boyle, the ornament of his age and country, ſucceeded to the genius and inquiries of the great chancellor Verulam. Which, ſays he, of Mr. Boyle's writings ſhall I recommend? All of them. To him we owe the ſecrets of fire, air, water, animals, vegetables, foſſils: ſo that from his works may be deduced the whole ſyſtem of natural knowledge."

Mr. Boyle left alſo ſeveral papers behind him, which have been publiſhed ſince his death. Beautiful editions of all his works have been printed at London, in 5 volumes folio, and 6 volumes 4to. Dr. Shaw alſo publiſhed in 3 volumes 4to, the ſame works "abridged, methodized, and diſpoſed under the general heads of Phyſics, Statics, Pneumatics, Natural Hiſtory, Chymiſtry, and Medicine;" to which he has prefixed a ſhort catalogue of the philoſophical writings, according to the order of time when they were firſt publiſhed, &c, as follows:

Phyſico-mechanical experiments on the ſpring and weight of the air	1661
The Sceptical Chymiſt	1661
Phyſiological Eſſays	1662
Hiſtory of Colours	1663
Uſefulneſs of Experimental Philoſophy	1663
Hiſtory of Cold	1665
Hiſtorical Paradoxes	1666
Origin of Forms and Qualities	1666
Coſmical Qualities	1670
The admirable Rarefaction of the air	1670
The Origin and Virtues of Gems	1672
The Relation betwixt Flame and Air	1672

Effluviums	1673
Saltneſs of the Sea	1674
Hidden Qualities in the Air	1674
The Excellence &c of the Mechanical Hypotheſis	1674
Conſiderations on the Reſurrection	1675
Particular Qualities	1676
Aerial Noctiluca	1680
Icy Noctiluca	1680
Things above Reaſon	1681
Natural Hiſtory of Human Blood	1684
Poroſity of Bodies	1684
Natural Hiſtory of Mineral Waters	1684
Specific Medicines	1685
The High Veneration due to God	1685
Languid Motion	1685
The Notion of Nature	1685
Final Cauſes	1688
Medicina Hydroſtatica	1690
The Chriſtian Virtuoſo	1690
Experimenta & Obſervationes Phyſicæ	1691
Natural Hiſtory of the Air	1692
Medicinal Experiments	1718

BRADLEY (Dr. James), a celebrated Engliſh aſtronomer, the third ſon of William Bradley, was born at Sherborne in Glouceſterſhire in the year 1692. He was fitted for the univerſity at Northleach in the ſame county, at the boarding ſchool of Mr. Egles and Mr. Brice. From thence he was ſent to Oxford, and admitted a commoner of Baliol college March 15, 1710; where he took the degree of bachelor the 14th of October 1714, and of maſter of arts the 21ſt of January 1716. His friends intending him for the church, his ſtudies were regulated with that view; and as ſoon as he was of a proper age to receive holy orders, the biſhop of Hereford, who had conceived a great eſteem for him, gave him the living of Bridſtow, and ſoon after he was inducted to that of Landewy Welfry in Pembrokeſhire.

He was nephew to Mr. Pound, a gentleman well known in the learned world, by many excellent aſtronomical and other obſervations, and who would have enriched it much more, if the journals of his voyages had not been burnt at Pulo Condor, when the place was ſet on fire, and the Engliſh who were ſettled there cruelly maſſacred, Mr. Pound himſelf very narrowly eſcaping with his life. With this gentleman, at Wanſtead, Mr. Bradley paſſed all the time that he could ſpare from the duties of his function; being then ſufficiently acquainted with the mathematics to improve by Mr. Pound's converſation. It may eaſily be imagined that the example and converſation of this gentleman did not render Bradley more fond of his profeſſion, to which he had before no great attachment: he continued however as yet to fulfil the duties of it, though at this time he had made ſuch obſervations as laid the foundation of thoſe diſcoveries which afterward diſtinguiſhed him as one of the greateſt aſtronomers of his age. Theſe obſervations gained him the notice and friendſhip of the lord chancellor Macclesfield, Mr. Newton afterward Sir Iſaac, Mr. Halley, and many other members of the Royal Society, into which he was ſoon after elected a member.

Soon after, the chair of Savilian profeſſor of aſtronomy at Oxford became vacant, by the death of the celebrated Dr. John Keil; and Mr. Bradley was elected to ſucceed him on the 31ſt of October 1721, at 29 years of age: his colleague being Mr. Halley, who was profeſſor of geometry on the ſame foundation. Upon this appointment, Mr. Bradley reſigned his church livings, and applied himſelf wholly to the ſtudy of his favourite ſcience. In the courſe of his obſervations, which were innumerable, he diſcovered and ſettled the laws of the alterations of the fixed ſtars, from the progreſſive motion of light, combined with the earth's annual motion about the ſun, and the nutation of the earth's axis, ariſing from the unequal attraction of the ſun and moon on the different parts of the earth. The former of theſe effects is called the *aberration* of the fixed ſtars, the theory of which he publiſhed in 1727; and the latter the *nutation* of the earth's axis, the theory of which appeared in 1737: ſo that in the ſpace of about 10 years, he communicated to the world two of the fineſt diſcoveries in modern aſtronomy; which will for ever make a memorable epoch in the hiſtory of that ſcience. See ABERRATION and NUTATION.

In 1730 our author ſucceeded Mr. Whiteſide, as lecturer in aſtronomy and experimental philoſophy in the Muſeum at Oxford: which was a conſiderable emolument to him, and which he held till within a year or two of his death; when the ill ſtate of his health induced him to reſign it.

Our author always preſerved the eſteem and friendſhip of Dr. Halley; who, being worn out by age and infirmities, thought he could not do better for the ſervice of aſtronomy, than procure for Mr. Bradley the place of regius profeſſor of aſtronomy at Greenwich, which he himſelf had many years poſſeſſed with the greateſt reputation. With this view he wrote many letters, deſiring Mr. Bradley's permiſſion to apply for a grant of the reverſion of it to him, and even offered to reſign it in his favour, if it ſhould be thought neceſſary: but Dr. Halley died before he could accompliſh this kind object. Our author however obtained the place, by the intereſt of lord Macclesfield, who was afterward preſident of the Royal Society; and upon this appointment the univerſity of Oxford ſent him a diploma of doctor of divinity.

The appointment of aſtronomer royal at Greenwich, which was dated the 3d of February 1741-2, placed our author in his proper element; and he purſued his obſervations with unwearied diligence. However numerous the collection of aſtronomical inſtruments at that obſervatory, it was impoſſible that ſuch an obſerver as Dr. Bradley ſhould not deſire to increaſe them, as well to anſwer thoſe particular views, as in general to make obſervations with greater exactneſs. In the year 1748 therefore, he took the opportunity of the viſit of the Royal Society to the obſervatory, annually made to examine the inſtruments and receive the profeſſor's obſervations for the year, to repreſent ſo ſtrongly the neceſſity of repairing the old inſtruments, and providing new ones, that the ſociety thought proper to make application to the king, who was pleaſed to order 1000 pounds for that purpoſe. This ſum was laid out under the direction of our author, who, with the aſſiſtance of the late celebrated Mr. Graham and Mr. Bird, furniſhed the obſervatory with as complete a collection of aſtronomical inſtruments, as the moſt ſkilful and diligent obſerver could deſire. Dr. Bradley, thus furniſhed with ſuch aſſiſtance, purſued his obſervations with great aſſiduity during the reſt of his life; an immenſe number of which was found after his death, in 13 folio volumes, and were preſented to the univerſity of Oxford in the year 1776, on condition of their printing and publiſhing them; but which however, unfortunately for the improvement of aſtronomy, now after a lapſe of almoſt 20 years, has never yet been done.

During Dr. Bradley's reſidence at the Royal Obſervatory, the living of the church at Greenwich became vacant, and was offered to him: upon his refuſing to accept it, from a conſcientious ſcruple, “that the duty of a paſtor was incompatible with his other ſtudies and neceſſary engagements,” the king was pleaſed to grant him a penſion of 250l. over and above the aſtronomer's original ſalary from the Board of Ordnance, “in conſideration (as the ſign manual, dated the 15th Feb. 1752, expreſſes it) of his great ſkill and knowledge in the ſeveral branches of aſtronomy and other parts of the mathematics, which have proved ſo uſeful to the trade and navigation of this kingdom.” A penſion which has been regularly continued to the aſtronomers royal ever ſince.

About 1748, our author became entitled to biſhop Crew's benefaction of 30l. a year, to the lecture reader in experimental philoſophy at Oxford. He was elected a member of the Academy of Sciences at Berlin, in 1747; of that at Paris, in 1748; of that at Peterſburgh, in 1754; and of that at Bologna, in 1757. He was married in the year 1744, but never had more than one child, a daughter.

By too cloſe application to ſtudy and obſervations, Dr. Bradley became afflicted, for near two years before his death, with a grievous oppreſſion on his ſpirits; which interrupted his uſeful labours. This diſtreſs aroſe chiefly from an apprehenſion that he ſhould outlive his rational faculties: but this ſo much dreaded evil never came upon him. In June 1762 he was ſeized with a ſuppreſſion of urine, occaſioned by an inflammation in the reins, which terminated his exiſtence the 13th of July following. His death happened at Chalfont in Glouceſterſhire, in the 70th year of his age; and he was interred at Minchinhampton in the ſame county.

As to his character, Dr. Bradley was remarkable for a placid and gentle modeſty, very uncommon in perſons of an active temper and robuſt conſtitution. Although he was a good ſpeaker, and poſſeſſed the rare but happy art of expreſſing his ideas with the utmoſt preciſion and clearneſs, yet no man was a greater lover of ſilence, for he never ſpoke but when he thought it abſolutely neceſſary. Nor was he more inclined to write than to ſpeak, as he has publiſhed very little: he had a natural diffidence, which made him always afraid that his works might injure his character; ſo that he ſuppreſſed many which might have been worthy of publication.

His papers, which have been inſerted in the Philoſ. Tranſ. are,

1. Obſervations on the comet of 1703. Vol. 33, p. 41.
2. The longitude of Liſbon and of the fort of New York from Wanſted and London determined by the eclipſe of the firſt ſatellite of Jupiter. Vol. 34, p. 85.
3. An account of a new diſcovered Motion of the Fixed Stars. Vol. 35, p. 637.
4. On the Going of Clocks with Iſochronal Pendulums. Vol. 38, p. 302.
5. Obſervations on the Comet of 1736-7. Vol. 40, p. 111.
6. On the apparent Motion of the fixed Stars. Vol. 45, p. 1.
7. On the Occultation of Venus by the Moon, the 15th of April 1751. Vol. 46, p. 201.
8. On the Comet of 1757. Vol. 50, p. 408.
9. Directions for uſing the Common Micrometer. Vol. 62, p. 46.

BRADWARDIN (Thomas), archbiſhop of Canterbury, was born at Hartfield in Suſſex, about the cloſe of the 13th century. He was educated at Merton College, Oxford, where he took the degree of doctor of divinity; and acquired the reputation of a profound ſcholar, a ſkilful mathematician, and conſummate divine. It has been ſaid he was profeſſor of divinity at Oxford; that he was chancellor of the dioceſe of London, and confeſſor to Edward the 3d, whom he conſtantly attended during his war with France. After his return from the war, he was made prebendary of Lincoln, and afterward archbiſhop of Canterbury. He died at Lambeth in the year 1349, forty days after his conſecration. His works are, 1. *De Cauſa Dei*, printed London 1618, publiſhed by J. H. Savil.—2. *De Geometria ſpeculativa, &c.* Paris, 1495, 1512, 1530.—3. *De Arithmetica practica*, Paris, 1502, 1512.—4. *De Proportionibus*, Paris, 1495. Venice, 1505, folio.—5. *De Quadratura Circuli*, Paris, 1495, folio.

BRAHE (Tycho), a celebrated aſtronomer, deſcended from a noble family originally of Sweden but ſettled in Denmark, was born the 14th of December 1546, at Knudſtorp in the county of Schonen, near Helſimbourg. He was taught Latin when 7 years old, and ſtudied 5 years under private tutors. His father dying while our author was very young, his uncle, George Brahe, having no children, adopted him, and ſent him, in 1559, to ſtudy philoſophy and rhetoric at Copenhagen. The great eclipſe of the ſun, on the 21ſt of Auguſt 1560, happening at the preciſe time the aſtronomers had foretold, he began to conſider aſtronomy as ſomething divine; and purchaſing the tables of Stadius, he gained ſome notion of the theory of the planets. In 1562 he was ſent by his uncle to Leipſic to ſtudy the law, where his acquirements gave manifeſt indications of extraordinary abilities. His natural inclination however was to the ſtudy of the heavens, to which he applied himſelf ſo aſſiduouſly, that, notwithſtanding the care of his tutor to keep him cloſe to the ſtudy of the law, he made uſe of every means in his power for improving his knowledge of aſtronomy; he purchaſed with his pocket money whatever books he could meet with on the ſubject, and read them with great attention, procuring aſſiſtance in difficult caſes from Bartholomew Scultens his private tutor; and having procured a ſmall celeſtial globe, he took opportunities, when his tutor was in bed, and when the weather was clear, to examine the conſtellations in the heavens, to learn their names from the globe, and their motions from obſervation.

After a courſe of 3 years ſtudy at Leipſic, his uncle dying, he returned home in 1565. In this year a difference ariſing between Brahe and a Daniſh nobleman, they fought, and our author had part of his noſe cut off by a blow; a defect which he ſo artfully ſupplied with one made of gold and ſilver, that it was not perceivable. About this time he began to apply himſelf to chemiſtry, propoſing nothing leſs than to obtain the philoſopher's ſtone. But becoming greatly diſguſted to ſee the liberal arts deſpiſed, and finding his own relations and friends uneaſy that he applied himſelf to aſtronomy, as thinking it a ſtudy unſuitable to a perſon of his quality, he went to Wirtemberg in 1566, from whence the breaking out of the plague ſoon occaſioned his removal to Roſtock, and in 1569 to Augſburg, where he was viſited by Peter Ramus, then profeſſor of aſtronomy at Paris, and who greatly admired his uncommon ſkill in this ſcience.

In 1571 he returned to Denmark; and was favoured by his maternal uncle, Steno Billes, a lover of learning, with a convenient place at his caſtle of Herritzvad near Knudſtorp, for making his obſervations, and building a laboratory. And here it was he diſcovered, in 1573, a new ſtar in the conſtellation Caſſiopeia. But ſoon after, his marrying a country girl, beneath his rank, occaſioned ſo violent a quarrel between him and his relations, that the king was obliged to interpoſe to reconcile them.

In 1574, by the king's command, he read lectures at Copenhagen on the theory of the planets. The year following he began his travels through Germany, and proceeded as far as Venice. He then reſolved to remove his family, and ſettle at Baſil; but Frederic the 2d, king of Denmark, being informed of his deſign, and unwilling to loſe a man who was capable of doing ſo much honour to his country, he promiſed to enable him to purſue his ſtudies, and beſtowed upon him for life the iſland of Huen in the Sound, and promiſed that an obſervatory and laboratory ſhould be built for him, with a ſupply of money for carrying on his deſigns: and accordingly the firſt ſtone of the obſervatory was laid the 8th of Auguſt 1576, under the name of Uranibourg: The king alſo gave him a penſion of 2000 crowns out of his treaſury, a fee in Norway, and a canonry of Roſhild, which brought him in 1000 more. This ſituation he enjoyed for the ſpace of about 20 years, purſuing his obſervations and ſtudies with great induſtry: here he kept always in his houſe ten or twelve young men, who aſſiſted him in his obſervations, and whom he inſtructed in aſtronomy and mathematics. Here it was that he received a viſit from James the 6th, king of Scotland, afterward James the 1ſt of England, having come to Denmark to eſpouſe Anne, daughter of Frederick the 2d: James made our author ſome noble preſents, and wrote a copy of Latin verſes in his praiſe.

Brahe's tranquillity however in this happy ſituation was at length fatally interrupted. Soon after the death of king Frederick, by the aſperſions of envious and malevolent miniſters, he was deprived of his penſion, fee, and canonry, in 1596. Being thus rendered incapable of ſupporting the expences of his eſtabliſhment, he quitted his favourite Uranibourg, and withdrew to Copenhagen, with ſome of his inſtruments, and continued his aſtronomical obſervations and chemical experiments in that city, till the ſame malevolence procured from the new king, Charles the 4th, an order for him to diſcontinue them. This induced him to fall upon means of being introduced to the emperor Rodolphus, who was fond of mechaniſm and chemical experiments: and to ſmooth the way to an interview, Tycho now publiſhed his book, *Aſtronomia inſtaurata Mechanica*, adorned with figures, and dedicated it to the emperor. That prince received him at Prague with great civility and reſpect; gave him a magnificent houſe, till he could procure one for him more fit for aſtronomical obſervations; he alſo aſſigned him a penſion of 3000 crowns; and promiſed him a fee for himſelf and his deſcendants. Here then he ſettled in the latter part of 1598, with his ſons and ſcholars, and among them the celebrated Kepler, who had joined him. But he did not long enjoy this happy ſituation; for, about 3 years after, he died, on the 24th of October 1601, of a retention of urine, in the 55th year of his age, and was interred in a very magnificent manner in the principal church at Prague, where a noble monument was erected to him; leaving, beſide his wife, two ſons and four daughters. On the approach of death, he enjoined his ſons to take care that none of his works ſhould be loſt; exhorted the ſtudents to attend cloſely to their exerciſes; and recommended to Kepler the finiſhing of the Rudolphine tables he had conſtructed for regulating the motion of the planets.

Brahe's ſkill in aſtronomy is univerſally known; and he is famed for being the inventor of a new ſyſtem of the planets, which he endeavoured, though without ſucceſs, to eſtabliſh on the ruins of that of Copernicus. He was very credulous with regard to judicial aſtrology and preſages: If he met an old woman when he went out of doors, or a hare upon the road on a journey, he would turn back immediately, being perſuaded that it was a bad omen: Alſo, when he lived at Uranibourg, he kept at his houſe a madman, whom he placed at his feet at table, and fed himſelf; for as he imagined that every thing ſpoken by mad perſons preſaged ſomething, he carefully obſerved all that this man ſaid; and becauſe it ſometimes proved true, he fancied it might always be depended on. He was of a very irritable diſpoſition: a mere trifle put him in a paſſion; and againſt perſons of the firſt rank, whom he thought his enemies, he openly diſcovered his reſentment. He was very apt to rally others, but highly provoked when the ſame liberty was taken with himſelf.—The principal part of his writings, according to Gaſſendus, are,

1. An account of the New Star, which appeared Nov. 11th 1572, in Caſſiopeia; Copenh. 1573, in 4to.—2. An Oration concerning the Mathematical Sciences, pronounced in the univerſity of Copenhagen, in the year 1574: publiſhed by Conrad Aſlac, of Bergen in Norway.—3. A treatiſe on the Comet of the year 1577, immediately after it diſappeared. Nine years afterward, he reviſed it, and added a 10th chapter. Printed at Uranibourg, 1589.—4. Another treatiſe on the New Phenomena of the heavens. In the firſt part of which he treats of the Reſtitution, as he calls it, of the ſun, and of the fixed ſtars. And in the 2d part, of

a New Star, which then had made its appearance.—5. A collection of Aſtronomical Epiſtles: printed in 4to, at Uranibourg in 1596; Nuremberg in 1602; and at Franckfort in 1610. It was dedicated to Maurice landgrave of Heſſe; becauſe there are in it a conſiderable number of letters of the landgrave William his father, and of Chriſtopher Rothmann, the mathematician of that prince, to Tycho, and of Tycho to them.—6. The Mechanical Principles of Aſtronomy reſtored: Wandeſburg, 1598, in folio.—7. An Anſwer to the Letter of a certain Scotchman, concerning the comet, in the year 1577.—8. On the compoſition of an Elixir for the Plague; addreſſed to the emperor Rodolphus.—9. An elegy upon his Exile: Roſtock, 1614, 4to.—10. The Rudolphine Tables; which he had not finiſhed when he died; but were reviſed, and publiſhed by Kepler, as Tycho had deſired.—11. An accurate Enumeration of the Fixed Stars: addreſſed to the emperor Rodolphus.—12. A complete Catalogue of 1000 of the Fixed Stars; which Kepler has inſerted in the Rudolphine Tables.—13. *Hiſtoria Cœleſtis*; or a Hiſtory of the Heavens; in two parts: The 1ſt contains the Obſervations he had made at Uranibourg, in 16 books: The latter contains the Obſervations made at Wandeſburg, Wittenberg, Prague, &c; in 4 books.—14. Is an Epiſtle to Caſter Pucer; printed at Copenhagen 1668.

BRANCKER, or BRANKER (THOMAS), an eminent mathematician of the 17th century, ſon of Thomas Brancker ſome time bachelor of arts in Exeter College, Oxford, was born in Devonſhire in 1636, and was admitted butler of the ſaid college Nov. 8, 1652, in the 17th year of his age. In 1655, June the 15th, he took the degree of bachelor of arts, and was elected probationary fellow the 30th of the ſame month. In 1658, April the 22d, he took the degree of maſter of arts, and became a preacher; but after the reſtoration, refuſing to conform to the ceremonies of the church of England, he quitted his fellowſhip in 1662, and retired to Cheſter: But not long after, he became reconciled to the ſervice of the church, took orders from a biſhop, and was made a miniſter of Whitegate. He had however, for ſome time, enjoyed great opportunity and leiſure for purſuing the bent of his genius in the mathematical ſciences; and his ſkill both in the mathematics and chemiſtry procured him the favour of lord Brereton, who gave him the rectory of Tilſton. He was afterward choſen maſter of the well-endowed ſchool at Macclesfield, in that county, where he ſpent the remaining years of his life, which was terminated by a ſhort illneſs in 1676, at 40 years of age; and he was interred in the church at Macclesfield.

Brancker wrote a piece on the Doctrine of the Sphere, in Latin, which was publiſhed at Oxford in 1662; and in 1668, he publiſhed at London, in 4to, a tranſlation of Rhonius's Algebra, with the title of *An Introduction to Algebra*; which treatiſe having communicated to Dr. John Pell, he received from him ſome aſſiſtance towards improving it; which he generouſly acknowledges in a letter to Mr. John Collins; with whom, and ſome other gentlemen, proficients in this ſcience, he continued a correſpondence during his life.

BREACH, in Fortification, a gap or opening made in any part of the works of a town, by the cannon or mines of the beſiegers, with intent to ſtorm or attack the place.

BREAKING *Ground*, in Military Affairs, is beginning the works for carrying on the ſiege of a place; more eſpecially the beginning to dig trenches, or approaches.

BREECH *of a Gun*, the hinder part, from the caſcabel to the lower part of the bore.

BREREWOOD (EDWARD), a learned mathematician and antiquary, was the ſon of Robert Brerewood, a reputable tradeſman, who was three times mayor of Cheſter. Our author was born in that city in 1565, where he was educated in grammar learning at the free ſchool; and was afterward admitted, in 1581, of Brazen-noſe College, Oxford; where he ſoon acquired the character of a hard ſtudent; as he has ſhewn by the commentaries he wrote upon Ariſtotle's Ethics, which were written by him about the age of 21.

In the year 1596 he was choſen the firſt Profeſſor of Aſtronomy in Greſham College, being one of the two who, at the deſire of the electors, were recommended to them by the univerſity of Oxford. He loved retirement, and wholly devoted himſelf to the purſuit of knowledge. And though he never publiſhed any thing himſelf, yet he was very communicative, and ready to impart what he knew, to others, either in converſation or in writing. His retired ſituation at Greſham College being agreeable, it did not appear that he had any other views, but continued there the remainder of his life, which was terminated by a fever the 4th of November 1613, at 48 years of age, in the midſt of his purſuits, and before he had taken proper care to collect and digeſt his learned labours; which however were not loſt; being reduced to order, and publiſhed after his death. Theſe were little or nothing mathematical, being of a miſcellaneous nature, upon the ſeveral ſubjects of Weights, Money, Languages, Religion, Logic, the Sabbath, Meteors, the Eye, Ethics, &c.

BRIDGE, a work of carpentry or maſonry, built over a river, canal, or the like, for the convenience of paſſing from one ſide to the other; and may be conſidered as a road over water, ſupported by one or more arches, and theſe again ſupported by proper piers or buttments. Beſides theſe eſſential parts, may be added the paving at top, the banquet, or raiſed footway, on each ſide, leaving a ſufficient breadth in the middle for horſes and carriages, alſo the parapet wall either with or without a baluſtrade, or other ornamental and uſeful parts. The breadth of a bridge for a great city ſhould be ſuch, as to allow an eaſy paſſage for three carriages and two horſemen abreaſt in the middle way, and for 3 foot paſſengers in the ſame manner on each banquet: but for other ſmaller bridges, a leſs breadth.

Bridges are commonly very difficult to execute, on account of the inconvenience of laying foundations and walling under water. The earlieſt rules and inſtructions for building of bridges are given by Alberti, in his *Archit.* 1, 8. Other rules were afterwards laid down by Palladio, Serlio, and Scamozzi, which are collected by Blondel, in his *Cours d'Archit.* pa. 629 &c. The beſt of theſe rules were alſo given by Goldman, Baukhurſt, and in Hawkeſmoor's Hiſtory of

London Bridge. M. Gautier has a conſiderable volume expreſsly on bridges, antient and modern. See alſo Riou's Short Principles for the Architecture of Stone Bridges; as alſo Emerſon, Muller, Labelye, and my own Principles of Bridges.

The conditions required in a bridge are, That it be well deſigned, commodious, durable, and ſuitably decorated. It ſhould be of ſuch a height as to be quite convenient for the paſſage over it, and yet eaſily admitting through its arches the veſſels that navigate upon it, and all the water, even at high tides and floods: the neglect of this precept has been the ruin of many bridges. Bridges are commonly continued in a ſtraight direction perpendicular to the ſtream; though ſome think they ſhould be made convex towards the ſtream, the better to reſiſt floods, &c. And bridges of this ſort have been executed in ſome places, as Pont St. Eſprit near Lyons. Again, a bridge ſhould not be made in too narrow a part of a navigable river, or one ſubject to tides or floods: becauſe the breadth being ſtill more contracted by the piers, this will increaſe the depth, velocity, and fall of the water under the arches, and endanger the whole bridge and navigation. There ought to be an uneven number of arches, or an even number of piers; both that the middle of the ſtream or chief current may flow freely without the interruption of a pier; and that the two halves of the bridge, by gradually riſing from the ends to the middle, may there meet in the higheſt and largeſt arch; and alſo, that by being open in the middle, the eye in viewing it may look directly through there. When the middle and ends are of different heights, their difference however ought not to be great in proportion to the length, that the aſcent and deſcent may be eaſy; and in that caſe alſo it is more beautiful to make the top in one continued curve, than two ſtraight lines forming an angle in the middle. Bridges ſhould rather be of few and large arches, than of many ſmaller ones, if the height and ſituation will poſſibly allow of it; for this will leave more free paſſage for the water and navigation, and be a great ſaving in materials and labour, as there will be fewer piers and centres, and the arches &c will require leſs materials; a remarkable inſtance of which appears in the difference between the bridges of Weſtminſter and Blackfriars, the expence of the former being more than double the latter.

For the proper execution of a bridge, and making an eſtimate of the expence, &c, it is neceſſary to have three plans, three ſections, and an elevation. The three plans are ſo many horizontal ſections, viz, firſt a plan of the foundation under the piers, with the particular circumſtances attending it, whether of gratings, planks, piles, &c; the 2d is the plan of the piers and arches; and the 3d is the plan of the ſuperſtructure, with the paved road and banquet. The three ſections are vertical ones; the firſt of them, a longitudinal ſection from end to end of the bridge, and through the middle of the breadth; the 2d a tranſverſe one, or acroſs it, and through the ſummit of an arch; and the 3d alſo acroſs, but taken upon a pier. The elevation is an orthographic projection of one ſide or face of the bridge, or its appearance as viewed at a diſtance, ſhewing the exterior aſpect of the materials, with the manner in which they are diſpoſed &c.

For the figure of the arches, ſome prefer the ſemicircle, though perhaps without knowing any good reaſon why; others the elliptical form, as having many advantages over the ſemi-circular; and ſome talk of the catenarian arch, though its pretended advantages are only chimerical; but the arch of equilibration is the only perfect one, ſo as to be equally ſtrong in every part: ſee my Principles of Bridges. The piers are of diverſe thickneſs, according to the figure, ſpan, and height of the arches; as may be ſeen in the work above mentioned.

With the Romans, the repairing and building of bridges were committed to the prieſts, thence named *pontifices*; next to the cenſors, or curators of the roads; but at laſt the emperors took the care of the bridges into their own hands. Thus, the Pons Janiculenſis was built of marble by Antoninus Pius; the Pons Ceſtius was reſtored by Gordian; and Arian built a new one which was called after his own name. In the middle age, bridge-building was counted among the acts of religion; and, toward the end of the 12th century, St. Benezet founded a regular order of hoſpitallers, under the name of *pontifices*, or bridge-builders, whoſe office was to aſſiſt travellers, by making bridges, ſettling ferries, and receiving ſtrangers into hoſpitals, or houſes, built on the banks of rivers. We read of an hoſpital of this kind at Avignon, where the hoſpitallers reſided under the direction of their firſt ſuperior St. Benezet: and the Jeſuit Raynaldus has a treatiſe on St. John the bridge-builder.

Among the bridges of antiquity, that built by Trajan over the Danube, it is allowed, is the moſt magnificent. It was demoliſhed by his next ſucceſſor Adrian, and the ruins are ſtill to be ſeen in the middle of the Danube, near the city Warhel in Hungary. It had 20 piers, of ſquare ſtone, each of which was 150 feet high above the foundation, 60 feet in breadth, and 170 feet diſtant from one another, which is the ſpan or width of the arches; ſo that the whole length of the bridge was more than 1530 yards, or one mile nearly.

In France, the Pont de Garde is a very bold ſtructure; the piers being only 13 feet thick, yet ſerving to ſupport an immenſe weight of a triplicate arcade, and joining two mountains. It conſiſts of three bridges, one over another; the uppermoſt of which is an aqueduct.

The bridge of Avignon, which was finiſhed in the year 1188, conſiſts of 18 arches, and meaſures 1340 paces, or about 1000 yards in length.

The famous bridge at Venice, called the Rialto, paſſes for a maſter piece of art, conſiſting of only one very flat and bold arch, near 100 feet ſpan, and only 23 feet high above the water: it was built in 1591.—Poulet alſo mentions a bridge of a ſingle arch, in the city of Munſter in Bothnia, much bolder than that of the Rialto at Venice. Yet theſe are nothing to a bridge in China, built from one mountain to another, conſiſting only of a ſingle arch, 400 cubits long, and 500 cubits high, whence it is called the flying bridge; and a figure of it is given in the Philoſ. Tranſ. Kircher alſo ſpeaks of a bridge in the ſame country 360 perches long without any arch, but ſupported by 300 pillars.

There are many bridges of conſiderable note in our own country. The triangular bridge at Crowland in Lincolnſhire, it is ſaid, is the moſt ancient Gothic ſtructure remaining intire in the kingdom; and was erected about the year 860.

London bridge is on the old Gothick ſtructure, with 20 ſmall locks or arches, each of only 20 feet wide; but there are now only 18 open, two having lately been thrown into one in the centre, and another next one ſide is concealed or covered up. It is 900 feet long, 60 high, and 74 wide; the piers are from 25 to 34 feet broad, with ſtarlings projecting at the ends; ſo that the greateſt water-way, when the tide is above the ſtarlings, was 450 feet, ſcarce half the breadth of the river; and below the ſtarlings, the water-way was reduced to 194 feet, before the late opening of the centre. London bridge was firſt built with timber between the years 993 and 1016; and it was repaired, or rather new built with timber, 1163. The ſtone bridge was begun in 1176, and finiſhed in 1209. It is probable there were no houſes on this bridge for upwards of 200 years; ſince we read of a tilt and tournament held on it in 1395. Houſes it ſeems were erected on it afterwards; but being found of great inconvenience and nuiſance, they were removed in 1758, and the avenues to it enlarged and the whole made more commodious; the two middle arches were then thrown into one, by removing the pier from between them; the whole repairs amounting to above 80,000l.

There are ſtill ſome more bridges in England built in the old manner of London bridge; as the bridge at Rocheſter, and ſome others; alſo the late bridge at Newcaſtle upon Tyne, which was broken down by a great flood in the year 1771, for want of a ſufficient quantity of water-way through the arches.

The longeſt bridge in England is that over the Trent at Burton, built in the 12th century, of ſquared free-ſtone, and is ſtrong and lofty; it contains 34 arches, and the whole length is 1545 feet. But this falls far ſhort of the wooden bridge over the Drave, which according to Dr. Brown is at leaſt 5 miles long.

But one of the moſt ſingular bridges in Europe is that built over the Taaf in Glamorganſhire, by William Edward, a poor country maſon, in the year 1756. This remarkable bridge conſiſts of only one ſtupendous arch, which, though only 8 feet broad, and 35 feet high, is no leſs than 140 ſpan, being part of a circle of 175 feet diameter.

There is alſo a remarkable bridge of one arch, built at Colebrook Dale in 1779, of caſt iron: and another ſtill larger of the ſame metal, is now raiſing over the river Wear, at Sunderland, the arch being of 240 feet ſpan.

Of modern bridges, perhaps the two fineſt in Europe, are the Weſtminſter and Blackfriars bridges over the river Thames at London. The former is 1220 feet long, and 44 feet wide, having a commodious broad foot path on each ſide for paſſengers. It conſiſts of 13 large and two ſmall arches, all ſemicircular, with 14 intermediate piers. The arches all ſpring from about 2 feet above low-water mark; the middle arch is 76 feet wide, and the others on each ſide decreaſe always by 4 feet at a time. The two middle piers are each 17 feet thick at the ſpringing of the arches; and the others decreaſe equally on each ſide by one foot at a time; every pier terminating with a ſaliant right angle againſt either ſtream. This bridge is built of the beſt materials, and in a neat and elegant taſte, but the arches are too ſmall for the quantity of maſonry contained in it. This bridge was begun in 1738, and opened in 1750; and the whole ſum of money granted and paid for the erection of this bridge, with the purchaſe of houſes to take down, and widening the avenues, &c, amounted to 389,500 l.

Blackfriars bridge, nearly oppoſite the centre of the city of London, was begun in 1760, and was completed in 10 years and three quarters; and is an exceeding light and elegant ſtructure, but the materials unfortunately do not ſeem to be of the beſt, as many of the arch ſtones are decaying. It conſiſts of 9 large, elegant, elliptical arches; the centre arch being 100 feet wide, and thoſe on each ſide decreaſing in a regular gradation, to the ſmalleſt, at each extremity, which is 70 feet wide. The breadth of the bridge is 42 feet, and the length from wharf to wharf 995. The upper ſurface is a portion of a very large circle, which forms an elegant figure, and is of convenient paſſage over it. The whole expence was 150,840 l.

There are various ſorts of bridges, of ſtone, wood, or metal, of boats or floats, pendant or hanging bridges, draw bridges, flying bridges, &c, &c, and even natural bridges, or ſuch as are found formed by nature, of which kind a moſt wonderful one is deſcribed by Mr. Jefferſon, in his *State of Virginia*; and another, but ſmaller, is deſcribed by Don Ulloa, in the province of Angaraez in South America.

BRIGGS (Henry), one of the greateſt mathematicians in the 16th and 17th centuries, was born at Warleywood, near Halifax, in Yorkſhire, in 1556. From a grammar ſchool in that country he was ſent to St. John's College, Cambridge, 1579; where after taking both the degrees in arts, he was choſen fellow of his college in 1588. He applied himſelf chiefly to the ſtudy of the mathematics, in which he greatly excelled; in conſequence in 1592 he was made examiner and lecturer in that faculty; and ſoon after, reader of the phyſic lecture, founded by Dr. Linacer.

Upon the ſettlement of Greſham College, in London, he was choſen the firſt profeſſor of geometry there, in 1596. Soon after this, he conſtructed a table, for finding the latitude, from the variation of the magnetic needle being given. In the year 1609 he contracted an acquaintance with the learned Mr. James Uſher, afterwards archbiſhop of Armagh, which continued many years after by letters, two of Mr. Briggs being ſtill extant in the collection of Uſher's letters that were publiſhed: in the former of theſe, dated Auguſt 1610, he writes among other things, that he was engaged in the ſubject of eclipſes; and in the latter, dated the 10th of March 1615, that he was wholly taken up and employed about the noble invention of logarithms, which had come out the year before, and in the improvement of which he had afterwards ſo great a concern. For Briggs immediately ſet himſelf to the ſtudy and improvement of them; expounding them alſo to his auditors in his lectures at Greſham college. In theſe lectures he propoſed the alteration of the ſcale of logarithms, from the hyperbolic form which Napier had given them, to that in which 1 ſhould be the logarithm of the ratio of 10 to 1; and ſoon after he wrote to Napier to make the ſame propoſal to himſelf. In the year 1616 Briggs made a viſit to Napier at Edinburgh, to confer with him upon this change; and the next year he did the ſame alſo. In theſe conferences, the alteration was agreed upon accordingly, and upon Briggs's return from his ſecond viſit, in 1617, he publiſhed the firſt chiliad, or 1000 of his logarithms. See the Introduction to my Logarithms.

In 1619 he was made the firſt Savilian profeſſor of geometry; and reſigned the profeſſorſhip of Greſham college the 25th of July 1620. At Oxford he ſettled himſelf at Merton college, where he continued a moſt laborious and ſtudious life, employed partly in the duties of his office as geometry lecturer, and partly in the computation of the logarithms, and in other uſeful works. In the year 1622 he publiſhed a ſmall tract on the "North-weſt paſſage to the South Seas, through the continent of Virginia and Hudſon's Bay;" the reaſon of which was probably, that he was then a member of the company trading to Virginia. His next performance was his great and elaborate work, the *Arithmetica Logarithmica* in folio, printed at London in 1624; a ſtupendous work for ſo ſhort a time! containing the logarithms of 30 thouſand natural numbers, to 14 places of figures beſide the index. Briggs lived alſo to complete a table of logarithmic ſines and tangents for the 100th part of every degree, to 14 places of figures beſide the index; with a table of natural ſines for the ſame 100th parts to 15 places, and the tangents and ſecants for the ſame to ten places; with the conſtruction of the whole. Theſe tables were printed at Gouda in 1631, under the care of Adrian Vlacq, and publiſhed in 1633, with the title of *Trigonometria Britannica*. In the conſtruction of theſe two works, on the logarithms of numbers, and of ſines and tangents, our author, beſide extreme labour and application, manifeſts the higheſt powers of genius and invention; as we here for the firſt time meet with ſeveral of the moſt important diſcoveries in the mathematics, and what have hitherto been conſidered as of much later invention; ſuch as the Binomial Theorem; the Differential Method and Conſtruction of Tables by Differences; the Interpolation by Differences; with Angular Sections, and ſeveral other ingenious compoſitions: a particular account of which may be ſeen in the Introduction to my Mathematical Tables.

This truly great man terminated his uſeful life the 26 of January 1630, and was buried in the choir of the chapel of Merton College. As to his character, he was not leſs eſteemed for his great probity and other eminent virtues, than for his excellent ſkill in mathematics. Doctor Smith gives him the character of a man of great probity; eaſy of acceſs to all; free from arrogance, moroſeneſs, envy, ambition and avarice; a contemner of riches, and contented in his own ſituation; preferring a ſtudious retirement to all the ſplendid circumſtances of life. The learned Mr. Thomas Gataker, who attended his lectures when he was reader of mathematics at Cambridge, repreſents him as highly eſteemed by all perſons ſkilled in mathematics, both at home and abroad; and ſays, that deſiring him once to give his judgment concerning judicial aſtrology, his anſwer was, "that he conceived it to be a mere ſyſtem of groundleſs conceits." Oughtred calls him the mirror of the age, for his excellent ſkill in geometry. And one of his ſucceſſors at Greſham college, the learned Dr. Iſaac Barrow, in his oration there upon his admiſſion, has drawn his character more fully; celebrating his great abilities, ſkill, and induſtry, particularly in perfecting the invention of logarithms, which, without his care and pains, might have continued an imperfect and uſeleſs deſign.

His writings were more important than numerous: ſome of them were publiſhed by other perſons: the liſt of the principal part of them as follows.

1. *A Table to find the Height of the Pole; the Magnetical Declination being given.* This was publiſhed in Mr. Thomas Blundevile's Theoriques of the Seven Planets: London 1602, 4to.

2. *Tables for the improvement of Navigation.* Theſe conſiſt of, A table of declination of every minute of the ecliptic, in degrees, minutes and ſeconds: A table of the ſun's proſthaphaereſes: A table of equations of the ſun's ephemerides: A table of the ſun's declination: Tables to find the height of the pole in any latitude, from the height of the pole ſtar. Theſe tables are printed in the 2d edition of Edward Wright's treatiſe, intitled, Certain Errors in Navigation detected and corrected; London 1610, 4to.

3. *A deſcription of an Inſtrumental Table to find the Part Proportional, deviſed by Mr. Edward Wright.* This is ſubjoined to Napier's table of logarithms, tranſlated into Engliſh by Mr. Wright, and after his death publiſhed by Briggs, with a preface of his own: Lond. 1616 and 1618, 12mo.

4. *Logarithmorum chilias prima.* Lond. 1617, 8vo.

5. *Lucubrationes & Annotationes in opera poſthuma J. Neperi:* Edinb. 1619, 4to.

6. *Euclidis Elementorum vi libri priores: &c.* Lond. 1620, folio. This was printed without his name to it.

7. *A treatiſe of the North-weſt paſſage to the South Sea &c.* By H. B. Lond. 1622, 4to. This was reprinted in Purchas's Pilgrims, vol. 3, p. 852.

8. *Arithmetica Logarithmica, &c.* Lond. 1624, folio.

9. *Trigonometria Britannica, &c.* Goudæ 1633, folio.

10. *Two letters to archbiſhop Uſher.*

11. *Mathematica ab antiquis minus cognita.*—This is a ſummary account of the moſt obſervable inventions of modern mathematicians, communicated by Mr. Briggs to Dr. George Hakewill, and publiſhed by him in his *Apologie;* Lond. folio.

Beſide theſe publications, Briggs wrote ſome other pieces, that have not been printed: as,

(1). *Commentaries on the Geometry of Peter Ramus.*

(2). *Duæ Epiſtolæ ad celeberrimum virum, Chr. Sever. Longomontanum.* One of theſe letters contained ſome remarks on a treatiſe of Longomontanus, about ſquaring the circle; and the other a defence of arithmetical geometry.

(3). *Animadverſiones Geometricæ:* 4to.

(4). *De eodem Argumento:* 4to.—Theſe two were in the poſſeſſion of the late Mr. Jones. They both contain a great variety of geometrical propoſitions, concerning the properties of many figures, with ſeveral arithmetical computations, relating to the circle, angular ſections, &c.—The two following were alſo in poſſeſſion of Mr. Jones.

(5). *A treatiſe of Common Arithmetic;* folio.

(6). *A letter to Mr. Clarke of Graveſend,* dated 25 Feb. 1606; with which he ſends him the deſcription of a ruler, called Bedwell's ruler, with directions how to uſe it.

BRIGGS (William), an eminent phyſician in the latter part of the 17th century, was born at Norwich, for which town his father was four times member of parliament. He ſtudied at the univerſity of Cambridge. He afterwards travelled into France, where he attended the lectures of the famous anatomiſt Vieuſſens, at Montpelier. Upon his return he publiſhed his *Ophthalmographia,* in 1676. The year following he was made doctor of medicine at Cambridge, and ſoon after fellow of the college of phyſicians at London. In 1682 he reſigned his fellowſhip to his brother; and the ſame year his *Theory of Viſion* was publiſhed by Hook. The enſuing year he ſent to the Royal Society a continuation of that diſcourſe, which was publiſhed in their Tranſactions; and the ſame year he was appointed phyſician to St. Thomas's hoſpital. In 1684 he communicated to the Royal Society two remarkable caſes relating to viſion, which were likewiſe printed in their Tranſactions; and in 1685 he publiſhed a Latin verſion of his *Theory of Viſion,* at the deſire of Mr. Newton, afterwards Sir Iſaac, then profeſſor of mathematics at Cambridge, with a recommendatory epiſtle from him prefixed to it. He was afterwards made phyſician in ordinary to king William, and continued in great eſteem for his ſkill in his profeſſion till he died the 4th of September 1704.

Briggs's *Logarithms,* that ſpecies of them in which 1 is the logarithm of the ratio of 10 to 1, or the logarithm of 10. See Logarithms.

BROKEN *Number,* the ſame as *Fraction;* which ſee.

Broken *Ray,* or *Ray of Refraction,* in Dioptrics, is the line into which an incident ray is refracted or broken, in croſſing the ſecond medium.

BROUNCKER, or BROUNKER, (William), lord viſcount of Caſtle Lyons in Ireland, ſon of Sir William Brounker, afterwards made viſcount in 1645, was born about the year 1620. He very early diſcovered a genius for mathematics, in which he afterwards became very eminent. He was made doctor of phyſic at Oxford June 23, 1646. In 1657 and 1658, he was engaged in a correſpondence by letters on mathematical ſubjects with Dr. John Wallis, who publiſhed them in his *Commercium Epiſtolicum,* printed 1658, at Oxford. He was one of the nobility and gentry who ſigned the remarkable declaration concerning king Charles the 2d, publiſhed in April 1660.

After the reſtoration, lord Brounker was made chancellor and keeper of the great ſeal to the queen conſort, one of the commiſſioners of the navy, and maſter of St. Katherine's hoſpital near the tower of London. He was one of thoſe great men who firſt formed the Royal Society, of which he was by the charter appointed the firſt preſident in 1662: which office he held, with great advantage to the Society, and honour to himſelf, till the anniverſary election, Nov. 30, 1677. He died at his houſe in St. James's ſtreet, Weſtminſter, the 5th of April 1684; and was ſucceeded in his title by his younger brother Harry, who died in Jan. 1687.

Lord Brounker had ſeveral papers inſerted in the Philoſophical Tranſactions, the chief of which were, 1. Experiments concerning the Recoiling of Guns.—2. A Series for the Quadrature of the Hyperbola; which was the firſt ſeries of the kind upon that ſubject.—3. Several of his letters to archbiſhop Uſher were alſo printed in Uſher's letters; as well as ſome to Dr. Wallis, in his Commercium Epiſtolicum, above mentioned.

BROWN (Sir William), a noted phyſician and miſcellaneous writer, of the 18th century. He was ſettled originally at Lynn in Norfolk, where he publiſhed a tranſlation of Dr. Gregory's Elements of Catoptrics and Dioptrics; to which he added, 1. A Method for finding the Foci of all Specula and Lenſes univerſally; as alſo Magnifying or Leſſening a given object by a given Speculum or lens, in any aſſigned proportion.—2. A Solution of thoſe Problems which Dr. Gregory has left undemonſtrated.—3. A particular account of Microſcopes and Teleſcopes, from Mr. Huygens; with the diſcoveries made by Catoptrics and Dioptrics.

Having acquired a competence by his profeſſion, he removed to Queen's Square, Ormond Street, London, where he reſided till his death, in 1774, at 82 years of age; leaving by his will two prize-medals to be annually contended for by the Cambridge poets.

Sir William Brown was a very facetious man; and a great number of his lively eſſays, both in proſe and verſe, were printed and circulated among his friends. The active part taken by him in the conteſt with the licentiates, in 1768, occaſioned his being introduced by Mr. Foote in his *Devil upon Two Sticks.*—Upon Foote's exact repreſentation of him with his identical wig and coat, tall figure, and glaſs ſtiffly applied to his eye, he ſent him a card complimenting him on having ſo happily repreſented him; but as he had forgot his muff, he had ſent him his own.—This good-natured way of reſenting diſarmed Foote.—He uſed to frequent the annual ball at the ladies boarding-ſchool, Queen Square, merely as a neighbour, a good-natured man, and fond of the company of ſprightly young folks. A dignitary of the church being there one day to ſee his daughter dance, and finding this upright figure ſtationed there, told him he believed he was Hermippus redivivus who lived *anhelitu puellarum.*—When he lived at Lynn, a pamphlet was written againſt him; which he nailed up againſt his houſe-door.—At the age of 80, on St. Luke's day 1771, he came to Batſon's coffee-houſe in his laced coat and band, and fringed white gloves, to ſhew himſelf to Mr. Croſby, then lord mayor. A gentleman preſent obſerving that he looked very well, he replied, he had neither wife nor debts.

BULLIALD (Iſmael), an eminent aſtronomer and mathematician, was born at Laon in the Iſle of France in 1605. He travelled in his youth, for the ſake of improvement, and gave very early proofs of his aſtronomical genius; and his riper years rendered him beloved and admired. Riccioli ſtyled him, *Aſtronomus profundæ indaginis.* He firſt publiſhed his diſſertation intitled, *Philolaus, ſive de vero Syſtemate Mundi;* or his true ſyſtem of the world, according to Philolaus, an ancient philoſopher and aſtronomer. Afterward, in the year 1645, he ſet forth his *Aſtronomia Philolaica,* grounded upon the hypotheſis of the earth's motion, and the elliptical orbit deſcribed by the planet's motion about a cone. To which he added tables intitled, *Tabulæ Philolaicæ:* a work which Riccioli ſays ought to be attentively read by all ſtudents of aſtronomy.—He conſidered the hypotheſis, or approximation of biſhop Ward, and found it not to agree with the planet Mars; and ſhewed in his defence of the Philolaic aſtronomy againſt the biſhop, that from four obſervations made by Tycho on the planet Mars, that planet in the firſt and third quarters of the mean anomaly, was more forward than it ought to be according to Ward's hypotheſis; but in the 2d and 4th quadrant of the ſame, the planet was not ſo far advanced as that hypotheſis required. He therefore ſet about a correction of the biſhop's hypotheſis, and made it to anſwer more exactly to the orbits of the planets, which were moſt eccentric, and introduced what is called, by Street in his *Caroline Tables,* the Variation: for theſe tables were calculated from this correction of Bulliald's, and exceeded all in exactneſs that went before. This correction is, in the judgment of Dr. Gregory, a very happy one, if it be not ſet above its due place; and be accounted no more than a correction of an approximation to the true ſyſtem: For by this means we are enabled to gather the coequate anomaly *a priori* and directly from the mean, and the obſervations are well enough anſwered at the ſame time; which, in Mercator's opinion, no one had effected before.—It is remarkable that the ellipſis which he has choſen for a planet's motion, is ſuch a one as, if cut out of a cone, will have the axis of the cone paſſing through one of its foci, viz, that next the aphelion.

In 1657 was publiſhed his treatiſe *De Lineis Spiralibus, Exerc. Geom. & Aſtron.* Paris, 4to.—In 1682 came out at Paris, in folio, his large work intitled, *Opus novum ad Arithmeticam Infinitorum:* A work which is a diffuſe amplification of Dr. Wallis's Arithmetic of Infinites, and which Wallis treats of particularly in the 80th chapter of his hiſtorical treatiſe of Algebra.—He wrote alſo two Admonitions to Aſtronomers. The firſt, concerning a new ſtar in the neck of the Whale, appearing at ſome times, and diſappearing at others. The 2d, concerning a nebulous ſtar in the northern part of Andromeda's girdle, not diſcovered by any of the ancients. This ſtar alſo appeared and diſappeared by turns. And as theſe phenomena appeared new and ſurprizing, he ſtrongly recommended the obſerving them to all that might be curious in aſtronomy.

BURNING, the action of fire on ſome pabulum or fuel, by which its minute parts are put into a violent motion, and ſome of them, aſſuming the nature of fire, fly off *in orbem,* while the reſt are diſſipated in vapour or reduced to aſhes..

BURNING-*Glaſs,* or *Burning-Mirror,* a machine by which the ſun's rays are collected into a point; and by that means their force and effect are extremely heightened, ſo as to burn objects placed in it.

Burning glaſſes are of two kinds, *convex* and *concave.* The convex ones are lenſes, which acting according to the laws of refraction, incline the rays of light towards the axis, and unite them in a point or focus. The concave ones are mirrors or reflectors, whether made of poliſhed metal or ſilvered glaſs, and which acting by the laws of reflection, throw the rays back into a point or focus before the glaſs.

The uſe of burning glaſſes it appears is very ancient, many of the old authors relating ſome effects of them. Diodorus Siculus, Lucian, Dion, Zonaras, Galen, Anthemius, Euſtatius, Tzetzes, and others, relate that by means of them Archimedes ſet fire to the Roman fleet at the ſiege of Syracuſe. Tzetzes is ſo particular in his account of this matter, that his deſcription ſuggeſted to Kircher the method by which it was probably accompliſhed. That author ſays that "Archimedes ſet fire to Marcellus's navy by means of a burning glaſs compoſed of ſmall ſquare mirrors, moving every way upon hinges; which when placed in the ſun's rays, directed them upon the Roman fleet, ſo as to reduce it to aſhes at the diſtance of a bowſhot." And the burning power of reflectors is mentioned in Euclid's Optics, theor. 31. Again, Ariſtophanes, in his comedy of The Clouds, introduces Socrates as examining Strepſiades about a method he had diſcovered of getting clear of his debts. He replies, that "he thought of making uſe of a burning-glaſs which he had hitherto uſed in kindling his fire; for ſhould they bring a writ againſt me, I'll immediately place my glaſs in the ſun at ſome little diſtance from it, and ſet it on fire." Pliny and Lactantius have alſo ſpoken of glaſſes that burn by refraction. The former calls them *balls* or *globes* of *cryſtal* or *glaſs,* which being expoſed to the ſun, tranſmit a heat ſufficient to ſet fire to cloth, or corrode the dead fleſh of thoſe patients who ſtand in need of cauſtics; and the latter, after Clemens Alexandrinus, obſerves that fire may be kinkled by interpoſing glaſſes filled with water between the ſun and the object, ſo as to tranſmit the rays to it.

Among the ancients the moſt celebrated burning mirrors were thoſe of Archimedes and Proclus; by the former was burnt the fleet of Marcellus, as above mentioned; and by the latter, the navy of Vitellius, beſieging Byzantium, according to Zonaras was burnt to aſhes.

Among the moderns, the moſt remarkable burning-glaſſes, are thoſe of Magine of 20 inches diameter: of Sepatala of Milan, near 42 inches diameter, and which burnt at the diſtance of 15 feet; of Settala of Villette, of Tſchirnhauſen, of Buffon, of Trudaine, and of Parker.

Villette, a French artiſt at Lyons, made a large mirror, which was bought by Tavernier, and preſented to the king of Pruſſia; a ſecond, bought by the king of Denmark; a third, preſented to the Royal Academy by the king of France; and a 4th came to England, and was publicly ſhewn. This mirror is 47 inches wide, being a ſegment of a ſphere of 76 inches radius; ſo that its focus is about 38 inches from the vertex; and its ſubſtance is a compoſition of tin, copper, and tin-glaſs. Some of its effects were as follow:

	ſec.
A ſilver ſixpence melted in - -	7½
A George the 1ſt's halfpenny in -	16
and runs with a hole in - -	34
Tin melts in - - -	3
Caſt iron in - - - -	16
Slate in - - - -	3
A foſſil ſhell calcines in - -	7
Piece of Pompey's pillar vitrifies, the black part in	50
the white part in	54
Copper ore in - - -	8
Bone calcines in 4, and vitrifies in -	33

An emerald melts into a ſubſtance like a torquois ſtone; a diamond weighing 4 grains loſes ⅞ of its weight: the aſbeſtos vitrifies; as all other bodies will do if kept long enough in the focus; but when once vitrified, the mirror can go no farther with them. Philoſ. Tranſ. vol. iv. pa. 198.

Tſchirnhauſen's reflecting mirrors produced equally ſurprizing effects; as they may be ſeen deſcribed in the Acta Erudit. for 1687, pa. 52. And other perſons have made very good ones of wood, ſtraw, paper, ice, and other ſubſtances capable of taking a proper form and poliſh.

Every lens, whether convex, plano-convex, or convexo-convex, collects the ſun's rays, diſperſed over its convexity, into a point by refraction; and it is therefore a burning-glaſs. The moſt conſiderable of this kind is that made by Tſchirnhauſen, and deſcribed in the ſame Acta Erudit. The diameters of his lenſes are from 3 to 4 feet, having the focus at the diſtance of 12 feet, and its diameter an inch and a half. To make the focus more vivid, the rays are collected a ſecond time, by a ſecond lens parallel to the firſt, and placed at ſuch a diſtance that the diameter of the cone of rays formed by the firſt lens is equal to the diameter of the ſecond; ſo that it receives them all; and the focus is reduced from an inch and a half to half the quantity, and conſequently its force is quadrupled. This glaſs vitrifies tiles, ſlates, pumice-ſtones &c. in a moment. It melts ſulphur, pitch, and all roſins, under water; the aſhes of vegetables, woods and other matters, are tranſmuted into glaſs; and every thing applied to its focus is either melted, changed into a calx, or into fumes. The author obſerves that it ſucceeds beſt when the matter applied is laid on a hard charcoal well burnt.—But though the force of the ſolar rays be thus found ſo ſurprizing, yet the rays of the full moon, collected by the ſame burning-glaſs, do not ſhew the leaſt increaſe of heat.

Sir Iſaac Newton preſented a burning-glaſs to the Royal Society, conſiſting of 7 concave glaſſes, ſo placed that all their foci join in one phyſical point. Each glaſs is about 11½ inches diameter: ſix of them are placed contiguous to, and round the ſeventh, forming a kind of ſpherical ſegment, whoſe ſubtenſe is about 34½ inches: the common focus is about 22½ inches diſtant, and about an inch in diameter. This glaſs vitrifies brick or tile in 1 ſecond, and melts gold in 30 ſeconds.

M. Buffon alſo made a variety of very powerful burning-glaſſes, both as mirrors and as lenſes; but at length concluded with one which is probably of the ſame nature with that of Archimedes, and conſiſted of 400 mirrors reflecting their rays all to one point, and with which he could melt lead and tin at the diſtance of 140 feet; and with others he conſumed ſubſtances at the diſtance of 210 feet. See Philoſ. Tranſ. vol. 44; or Buffon's Hiſtoire Naturelle, Suppl. vol. 1; or Montucla's Hiſtoire des Math. vol. i. pa. 246.

It would ſeem there is no ſubſtance capable of reſiſting the efficacy of modern burning-glaſſes; though water &c. are not affected by them at all. Thus, Meſſrs Macquer and Baumé have ſucceeded in melting ſmall portions of platina by means of a concave glaſs, 22 inches diameter, and 28 inches focus; though this metal is not fuſible by the ſtrongeſt fires that can be excited in furnaces, or ſuſtained by any chemical apparatus. Yet it was long ſince obſerved, by the Academicians del Cimento, that ſpirit of wine could not be fired by any burning-glaſs which they uſed; and notwithſtanding the great improvements theſe inſtruments have ſince received, M. Nollet has not been able, by the moſt powerful burning mirrors, to ſet fire to any inflammable liquors whatever.

However, a large burning lens, for fuſing and vitrifying ſuch ſubſtances as reſiſt the fires of furnaces, and eſpecially for the application of heat in vacuo, and in certain other circumſtances in which heat cannot be applied by other means, has long been a deſideratum with perſons concerned in philoſophical experiments: and this it appears is now in a great meaſure accompliſhed by Mr. Parker, an ingenious glaſs manufacturer in Fleet-ſtreet, London. His lens is made of flint glaſs, and is 3 feet in diameter, but when fixed in its frame expoſes a ſurface of 32 inches in the clear; the length of the focus is 6 feet 8 inches, and its diameter one inch. The rays from this large lens are received and tranſmitted through a ſmaller, of 13 inches diameter in the clear within the frame, its focal length 29 inches, and diameter of its focus 3-8ths of an inch: ſo that this ſecond lens increaſes the power of the former more than 7 times, or as the ſquare of 8 to the ſquare of 3.

From a great number of experiments made with this lens, the following are ſelected to ſerve as ſpecimens of its powers:

Subſtances fuſed; with their weight, and time of fuſion.	Time in ſec.	Weight in grs.
Scoria of wrought iron - -	2	12
Common ſlate - - -	2	10
Silver, pure - - -	3	20
Platina, pure - - -	3	10
Nickell - - -	3	16
Caſt Iron, a cube - - -	3	10
Kearſh - - -	3	10
Gold, pure - - -	4	20
Cryſtal pebble - - -	6	7
Cauk, or terra ponderoſa - -	7	10
Lava - - -	7	10
Aſbeſtos - - -	10	10
Bar Iron, a cube - - -	12	10
Steel, a cube - - -	12	10
Garnet - - -	17	10
Copper, pure - - -	20	33
Onyx - - -	20	10

SUBSTANCES FUSED, &c.	Time in ſec.	Wgt. in grs.
Zeolites	23	10
Pumice Stone	24	10
Oriental Emerald	25	2
Jaſper	25	10
White Agate	30	10
Flint, oriental	30	10
Topaz, or chryſolite	45	3
Common Limeſtone	55	10
White rhomboidal Spar	60	10
Volcanic Clay	60	10
Corniſh Moorſtone	60	10
Rough Cornelian	75	10
Rotten Stone	80	10

Burning *Zone*, or *Torrid Zone*, the ſpace within $23\frac{1}{2}$ degrees of the equator, both north and ſouth.

BUSHEL, a meaſure of capacity for dry goods; as grain, pulſe, fruits, &c; containing 4 pecks, or 8 gallons, or $\frac{1}{8}$ of a quarter. By act of Parliament, made in 1697, it was ordained that "Every round buſhel with a plain and even bottom, being made $18\frac{1}{2}$ inches wide throughout, and 8 inches deep, ſhall be eſteemed a Legal Wincheſter Buſhel, according to the ſtandard in his majeſty's exchequer." Now a buſhel being thus made will contain 2150·42 cubic inches, and conſequently the corn gallon contains only $268\frac{4}{5}$ cubic inches.

BUTMENTS, are thoſe ſupporters, or props, by which the feet of arches, or the extremities of bridges are ſupported; and ſhould be made very ſtrong and firm.

BUTTRESS, is an arch, or a maſs of maſonry, ſerving to ſupport the ſides of a building, wall, or the like, on the outſide. See Arch, and Arch-boutant.

C.

CAILLE (Nicholas Lewis de la), an eminent French mathematician and aſtronomer, was born at Rumigny in the dioceſe of Rheims in 1713. His father having quitted the army, in which he had ſerved, amuſed himſelf in his retirement with ſtudying mathematics and mechanics, in which he proved the happy author of ſeveral inventions of conſiderable uſe to the public. From this example of his father, our author almoſt in his infancy took a fancy to mechanics, which proved of ſignal ſervice to him in his maturer years. At ſchool he diſcovered early tokens of genius. He next came to Paris in 1729; where he ſtudied the claſſics, philoſophy and mathematics. He afterwards ſtudied divinity in the college de Navarre, with the view of embracing the eccleſiaſtical life: however he never entered into prieſt's orders, apprehending that his aſtronomical ſtudies, to which he had become much devoted, might too much interfere with his religious duties. His turn for aſtronomy ſoon connected him with the celebrated Caſſini, who procured him an apartment in the obſervatory; where, aſſiſted by the counſels of this maſter, he ſoon acquired a name among the aſtronomers. In 1739 he was joined with M. Caſſini de Thury, ſon to M. Caſſini, in verifying the meridian through the whole extent of France: and in the ſame year he was named profeſſor of mathematics in the college of Mazarine. In 1741 our author was admitted into the Academy of Sciences as an adjoint member for aſtronomy; and had many excellent papers inſerted in their memoirs; beſide which he publiſhed ſeveral uſeful treatiſes, viz, Elements of Geometry, Aſtronomy, Mechanics, and Optics. He alſo carefully computed all the eclipſes of the ſun and moon that had happened ſince the chriſtian era, which were printed in the work entitled *l'Art de verifier les dates*, &c, Paris, 1750, in 4to. He alſo compiled a volume of aſtronomical ephemerides for the years 1745 to 1755; another for the years 1755 to 1765; and a third for the years 1765 to 1775: as alſo the moſt correct ſolar tables of any; and an excellent work entitled *Aſtronomiæ fundamenta noviſſimis ſolis & ſtellarum obſervationibus ſtabilita*.

Having gone through a ſeven years ſeries of aſtronomical obſervations in his own obſervatory in the Mazarine college, he formed the project of going to obſerve the ſouthern ſtars at the Cape of Good Hope: being countenanced by the court, he ſet out upon this expedition in 1750, and in the ſpace of two years he obſerved there the places of about 10 thouſand ſtars in the ſouthern hemiſphere that are not viſible in our latitudes, as well as many other important elements, viz, the parallaxes of the ſun, moon, and ſome of the planets, the obliquity of the ecliptic, the refractions, &c. Having thus executed the purpoſe of his voyage, and no preſent opportunity offering for his return, he thought of employing the vacant time in another arduous attempt; no leſs than that of taking the meaſure of the earth, as he had already done that of the heavens. This indeed had been done before by different ſets of learned men both in Europe and America; ſome determining the quantity of a degree at the equator, and others at the arctic circle: but it had not as yet been decided whether in the ſouthern parallels of latitude the ſame dimenſions obtained as in the northern. His labours were rewarded with the ſatisfaction he wiſhed for; having determined a diſtance of 410814 feet from a place called *Klip-Fontyn* to the

Cape, by means of a baſe of 38802 feet, three times actually meaſured: whence he diſcovered a new ſecret of nature, namely, that the radii of the parallels in ſouth latitude are not the ſame length as thoſe of the correſponding parallels in north latitude. About the 23d degree of ſouth latitude he found a degree on the meridian to contain 342222 Paris feet. The court of Verſailles alſo ſent him an order to go and fix the ſituation of the Iſles of France and of Bourbon. While at the Cape too he obſerved a wonderful effect of the atmoſphere in ſome ſtates of it: Although the ſky at the Cape be generally pure and ſerene, yet when the ſoutheaſt wind blows, which is pretty often, it is attended with ſome ſtrange and even terrible effects: the ſtars look larger, and ſeem to dance; the moon has an undulating tremor; and the planets have a ſort of beard like comets.

M. de la Caille returned to France in the autumn of 1754, after an abſence of about 4 years; loaded, not with the ſpoils of the eaſt, but with thoſe of the ſouthern heavens, before then almoſt unknown to aſtronomers. Upon his return, he firſt drew up a reply to ſome ſtrictures which the celebrated Euler had publiſhed relative to the meridian: after which he ſettled the reſults of the compariſon of his obſervations for the parallaxes, with thoſe of other aſtronomers: that of the ſun he fixed at 9½″; of the moon at 56′56″; of Mars in his oppoſition, 36″; of Venus 38″. He alſo ſettled the laws by which aſtronomical refractions are varied by the different denſity or rarity of the air, by heat or cold, and by dryneſs or moiſture. And laſtly he ſhewed an eaſy and practicable method of finding the longitude at ſea by means of the moon. His fame being now celebrated every where, M. de la Caille was ſoon elected a member of moſt of the academies and ſocieties of Europe, as London, Bologna, Peterſburgh, Berlin, Stockholm, and Gottingen.

In 1760 our author was attacked with a ſevere fit of the gout; which however did not interrupt the courſe of his ſtudies; for he then planned out a new and large work, no leſs than a hiſtory of aſtronomy through all ages, with a compariſon of the ancient and modern obſervations, and the conſtruction and uſe of the inſtruments employed in making them. Towards the latter part of 1761, his conſtitution became greatly reduced; though his mind remained unaffected, and he reſolutely perſiſted in his ſtudies to the laſt; death only putting an end to his labours the 21ſt of March 1762, at 49 years of age; after having committed his manuſcripts to the care and diſcretion of his eſteemed friend M. Maraldi.

Beſide the publications before mentioned, and perhaps ſome others alſo, he had a vaſt number inſerted in the volumes of the Memoirs of the French Academy of Sciences, much too numerous indeed, though very important, to be here all mentioned particularly; ſuffice it therefore juſt to diſtinguiſh the years of thoſe volumes in which his pieces are to be found, by the following liſt of them, viz, 1741, 1742, 1743, 1744, 1745, 1746, 1747, 1748, 1749, 1750, 1751, 1752, 1753, 1754, 1755, 1756, 1757, 1758, 1759, 1760, 1761, 1763; in all or moſt of which years there are two or three or more of his papers.

CAISSON, in Architecture, a kind of cheſt or flat-bottomed boat, in which the pier of a bridge is built, then ſunk to the bottom of the water, and the ſides looſened and taken off from the bottom, by a contrivance for that purpoſe; the bottom of the caiſſon being left under the pier as a foundation to it. The caiſſon is kept afloat till the pier is built to above the height of low-water mark; and for that purpoſe, its ſides are either made of more than that height at firſt, or elſe gradually raiſed to it as it ſinks by the weight of the work, ſo as always to keep its top above water. Mr. Labelye tells us, that the caiſſons in which he built ſome of the piers of Weſtminſter bridge, contained above 150 load of fir timber, of 40 cubic feet each, and that it was of more tonnage or capacity than a 40 gun ſhip of war.

CAISSON, in Military Affairs, is ſometimes uſed for a cheſt; and in particular for a bomb or ſhell cheſt, and is uſed as a ſuperficial mine, or fourneau. This is done by filling a cheſt either with gun powder and loaded ſhells, or elſe with ſhells alone, and burying it in a ſpot where an enemy, beſieging a place, is expected to come, and then firing it by a train to blow the men up.

CALCULATION, the act of computing ſeveral ſums, by adding, ſubtracting, multiplying, dividing, &c. From *calculus*, in alluſion to the practice of the ancients, who uſed *calculi*, or little ſtones, in making computations, in taking ſuffrages, and in keeping accounts, &c; as we now uſe counters, figures, &c. Calculation is more particularly uſed to ſignify the computations in aſtronomy, trigonometry, &c, for making tables of aſtronomy, of logarithms, ephemerides, finding the times of eclipſes, and ſuch like.

CALCULATOR, a perſon who makes or performs calculations.—It is alſo the name given by Mr. Ferguſon to a machine in the ſhape of an orrery, which he conſtructed for exhibiting the motions of the earth and moon, and reſolving a variety of aſtronomical problems. See his Aſtron. 4to pa. 265, or 8vo pa. 393.

CALCULATORES, were anciently accountants who reckoned their ſums by *calculi*, or little ſtones, or counters.—In ancient canons too we find a ſort of diviners or enchanters, cenſured under the denomination of *calculatores*.

CALCULUS denotes primarily a ſmall ſtone, pebble, or counter, uſed by the ancients in making calculations or computations, taking of ſuffrages, playing at tables, and the like.

CALCULUS denotes now a certain way of performing mathematical inveſtigations and reſolutions. Thus, we ſay the Arithmetical or Numeral Calculus, the Algebraical Calculus, the Differential Calculus, the Exponential Calculus, the Fluxional Calculus, the Integral Calculus, the Literal or Symbolical Calculus, &c; for which, ſee each reſpective word.

Arithmetical or *Numeral* **CALCULUS**, is the method of performing arithmetical computations by numbers. See ARITHMETIC, and NUMBER.

Algebraical, *Literal*, or *Symbolical* **CALCULUS**, is the method of performing algebraical calculations by letters or other ſymbols. See ALGEBRA.

Differential **CALCULUS**, is the arithmetic of the indefinitely ſmall differences of variable quantities; a mode of computation much uſed by foreign mathematicians, and introduced by Leibnitz, as ſimilar to the method of Fluxions of Newton. See DIFFERENTIAL &c.

Exponential CALCULUS, is the applying the fluxional or differential methods to exponential quantities; ſuch as a^x, or x^x, or ay^x, &c. See EXPONENTIAL.

Fluxional CALCULUS, is the method of fluxions, invented by Newton. See FLUXIONS.

Integral CALCULUS, or *Summatorius*, is a method of integrating, or ſumming up differential quantities; and is ſimilar to the finding of fluents. See INTEGRAL and FLUENT.

CALCULUS *Literalis*, or *Literal Calculus*, is the ſame with algebra, or ſpecious arithmetic, ſo called from its uſing the letters of the alphabet; in contradiſtinction to numeral arithmetic, in which figures are uſed.

CALENDAR, or KALENDAR, a diſtribution of time as accommodated to the uſes of life; or an Almanac, or table, containing the order of days, weeks, months, feaſts, &c, occurring in the courſe of the year: being ſo called from the word *Calendæ*, which among the Romans denoted the firſt days of every month, and anciently was written in large characters at the head of each month. See ALMANAC, CALENDS, MONTH, TIME, YEAR, &c.

In Calendars the days were originally divided into octoades, or eights; but afterwards, in imitation of the Jews, they were divided into hebdomades, or ſevens, for what we now call a week: which cuſtom, Scaliger obſerves, was not in uſe among the Romans till after the time of Theodoſius.

Divers calendars are eſtabliſhed in different countries, according to the different forms of the year, and diſtributions of time: As the Perſian, the Roman, the Jewiſh, the Julian, the Gregorian, &c, calendars.—The ancient Roman Calendar is given by Ricciolus, Struvius, Danet, and others; in which we perceive the order and number of the Roman holy-days and work-days.—The Jewiſh calendar was fixed by Rabbi Hillel, about the year 360; from which time the days of their year may be reduced to thoſe of the Julian calendar.—The three Chriſtian calendars are given by Wolfius in his Elements of Chronology; as alſo the Jewiſh and Mohamedan calendars. Other writers on the calendars are Vieta, Clavius, Scaliger, Blondel, &c.

The Roman CALENDAR was firſt formed by Romulus, who diſtributed time into ſeveral periods for the uſe of his followers and people. He divided the year into 10 months, of 304 days; beginning on the firſt of March, and ending with December.

Numa reformed the calendar of Romulus. He added the months of January and February, making it to commence on the firſt of January, and to conſiſt of 355 days. But as this was evidently deficient of the true year, he ordered an intercalation of 45 days to be made every 4 years, in this manner, viz, Every 2 years an additional month of 22 days, between February and March; and at the end of each two years more, another month of 23 days; the month thus interpoſed, being called Marcedonius, or the intercalary February.

Julius Cæſar, with the aid of Soſigenes, a celebrated aſtronomer of thoſe times, farther reformed the Roman calendar, from whence aroſe the Julian calendar, and the Julian or old ſtyle. Finding that the ſun performed his annual courſe in 365 days and a quarter nearly, he divided the year into 365 days, but every 4th year 366 days, adding a day that year before the 24th of February, which being the 6th of the calends, and being thus reckoned twice, gave occaſion to the name *biſſextile*, or what we alſo call leap-year.

This calendar was farther reformed by order of the pope Gregory XIII, from whence aroſe the term Gregorian calendar and ſtyle, or what we alſo call the new ſtyle, which is now obſerved by almoſt all European nations. The year of Julius was too long by nearly 11 minutes, which amounts to about 3 days in 400 years; the pope therefore, by the advice of Clavius and Ciaconius, ordained that there ſhould be omitted a day in every 3 centuries out of 4; ſo that every century, which would otherwiſe be a biſſextile year, is made to be only a common year, excepting only ſuch centuries as are exactly diviſible by 4, which happens once in 4 centuries. See BISSEXTILE. This reformation of the calendar, or the new ſtyle, as we call it, commenced in the countries under the popiſh influence, on the 4th of October 1582, when 10 days were omitted at once, which had been over-run ſince the time of the council of Nice, in the year 325, by the ſurplus of 11 minutes each year. But in England it only commenced in 1752, when 11 days were omitted at once, the 3d of September being accounted the 14th that year; as the ſurplus minutes had then amounted to 11 days.

Julian Chriſtian CALENDAR, is that in which the days of the week are determined by the letters A, B, C, D, E, F, G, by means of the ſolar cycle; and the new and full moons, particularly the paſchal full moon, with the feaſt of Eaſter, and the other moveable feaſts depending upon it, by means of golden numbers, or lunar cycles, rightly diſpoſed through the Julian year. See CYCLE, and GOLDEN NUMBER.

In this calendar, it is ſuppoſed that the vernal equinox is fixed to the 21ſt day of March; and that the golden numbers, or cycles of 19 years, conſtantly indicate the places of the new and full moons; though both are erroneous; and from hence aroſe a great irregularity in the time of Eaſter.

Gregorian CALENDAR, is that which, by means of Epacts, rightly diſpoſed through the ſeveral months, determines the new and full moons, with the time of Eaſter, and the moveable feaſts depending upon it, in the Gregorian year. This differs therefore from the Julian calendar, both in the form of the year, and in as much as epacts are ſubſtituted inſtead of golden numbers. See EPACT.

Though the Gregorian calendar be more accurate than the Julian, yet it is not without imperfections, as Scaliger and Calviſius have fully ſhewn; nor is it perhaps poſſible to deviſe any one that ſhall be quite perfect. Yet the Reformed Calendar, and that which is ordered to be obſerved in England, by act of Parliament made the 24th of George II, come very near to the point of accuracy: For, by that act it is ordered that "Eaſter-day, on which the reſt depend, is always the firſt Sunday after the full moon, which happens upon, or next after the 21ſt day of March; and if the full-moon happens upon a Sunday, Eaſter-day is the Sunday after."

Reformed, or *Corrected*, CALENDAR is that which, rejecting all the apparatus of golden numbers, epacts, and dominical letters, determines the equinox, and the paſchal full-moon, with the moveable feaſts depending upon it, by computation from aſtronomical tables. This calendar was introduced among the proteſtant ſtates of Germany in the year 1700, when 11 days were omitted in the month of February, to make the corrected ſtyle agree with the Gregorian. This alteration in the form of the year, they admitted for a time; in expectation that, the true quantity of the tropical year being at length more accurately determined by obſervation, the Romaniſts would agree with them on ſome more convenient intercalation.

French New CALENDAR, is a quite new form of calendar that commenced in France on the 22d of September 1792. At the time of printing this (viz, in July 1794), it does not certainly appear whether this new calendar will be made permanent or not; but merely as a curioſity in the ſcience of chronology, a very brief notice of it may here be added, as follows.

The year, in this calendar, commences at midnight the beginning of that day in which falls the true autumnal equinox for the obſervatory of Paris. The year is divided into 12 equal months, of 30 days each; after which 5 ſupplementary days are added, to complete the 365 days of the ordinary year: theſe 5 days do not belong to any month. Each month is divided into three decades of 10 days each; diſtinguiſhed by 1ſt, 2d, and 3d decade. All theſe are named according to the order of the natural numbers, viz, the 1ſt, 2d, 3d, &c, month, or day of the decade, or of the ſupplementary days. The years which receive an intercalary day, when the poſition of the equinox requires it, which we call emboliſmic or biſſextile, they call olimpic; and the period of four years, ending with an olimpic year, is called an olimpiade; the intercalary day being placed after the ordinary five ſupplementary days, and making the laſt day of the olimpic year. Each day, from midnight to midnight, is divided into 10 parts, each part into 10 others, and ſo on to the laſt meaſurable portion of time.

In this calendar too the months and days of them have new names. The firſt three months of the year, of which the autumn is compoſed, take their etymology, the firſt from the vintage which takes place from September to October, and is called *vendemaire;* the ſecond, *brumaire,* from the miſts and low fogs, which ſhew as it were the tranſudation of nature from October to November: the third, *frimaire,* from the cold, ſometimes dry and ſometimes moiſt, which is felt from November to December. The three winter months take their etymology, the firſt, *nivoſe,* from the ſnow which whitens the earth from December to January; the ſecond, *pluvioſe,* from the rains which uſually fall in greater abundance from January to February; the third, *ventoſe,* from the wind which dries the earth from February to March. The three ſpring months take their etymology, the firſt, *germinal,* from the fermentation and development of the ſap from March to April; the ſecond, *floreal,* from the blowing of the flowers from April to May: the third, *prairial,* from the ſmiling fecundity of the meadow crops from May to June. Laſtly, the three ſummer months take their etymology, the firſt, *meſſidor,* from the appearance of the waving ears of corn and the golden harveſts which cover the fields from June to July; the ſecond, *thermidor,* from the heat, at once ſolar and terreſtrial, which inflames the air from July to Auguſt; the third, *fructidor,* from the fruits gilt and ripened by the ſun from Auguſt to September. Thus, the whole 12 months are,

AUTUMN.	SPRING.
Vendemaire	Germinal
Brumaire	Floreal
Frimaire.	Prairial.
WINTER.	SUMMER.
Nivoſe	Meſſidor
Pluvioſe	Thermidor
Ventoſe.	Fructidor.

From theſe denominations it follows, that by the mere pronunciation of the name of the month, every one readily perceives three things and all their relations, viz, the kind of ſeaſon, the temperature, and the ſtate of vegetation: for inſtance, in the word *germinal,* his imagination will eaſily conceive, by the termination of the word, that the ſpring commences; by the conſtruction of the word, that the elementary agents are buſied; and by the ſignification of the word, that the buds unfold themſelves.

As to the names of the days of the week, or decade of 10 days each, which they have adopted inſtead of ſeven, as theſe bear the ſtamp of judicial aſtrology and heathen mythology, they are ſimply called from the firſt ten numbers; thus,

Primdi
Duodi
Tridi
Quartidi
Quintidi
Sextidi
Septidi
Octidi
Nonidi
Decadi.

In the almanac, or annual calendar, inſtead of the multitude of ſaints, one for each day of the year, as in the popiſh calendars, they annex to every day the name of ſome animal, or utenſil, or work, or fruit, or flower, or vegetable, &c, appropriate and moſt proper to the times.

Aſtronomical CALENDAR, an inſtrument engraven upon copper-plates, printed on paper, and paſted on board, with a braſs ſlider which carries a hair, and ſhews by inſpection, the ſun's meridian, altitude, right aſcenſion, declination, riſing, ſetting, amplitude, &c, to a greater exactneſs than can be ſhewn by the common globes.

CALENDS, *Calendæ,* in the Roman Chronology, denoted the firſt days of each month; being ſo named from καλεω, *calo, I call,* or *proclaim*; becauſe that, before the publication of the Roman *Faſti,* and counting their months by the motion of the moon, a prieſt was appointed to obſerve the firſt appearance of the new moon; who, having ſeen her, gave notice to the preſident of the ſacrifices to offer one; and calling the people together, he proclaimed unto them how they ſhould reckon the days until the nones; pronouncing the word *Caleo* 5 times if the nones ſhould happen on the 5th day, or ſeven times if they happened on the 7th day of the month.

The calends were reckoned backwards, or in a retrograde order: thus, for example, the firſt of May being the calends of May; the laſt or 30th day of April, was the *pridie calendarum*, or 2d of the calends of May; the 29th of April, the 3d of the calends, or before the calends: and ſo back to the 13th, where the ides commence; which are likewiſe numbered backwards to the 5th, where the nones begin; which are alſo reckoned after the ſame manner to the firſt da yof the month, which is the calends of April.

Hence comes this rule to find the day of the calends anſwering to any day of the month, viz, Conſider how many days of the month are yet remaining after the day propoſed, and to that number add 2, for the number of or from the calends. For example, ſuppoſe it were the 23d day of April, it would then be the 9th of the caledns of May: for April containing 30 days, from which 23 being taken, there remains 7; to which 2 being added, makes the ſum 9. And the reaſon for this addition of the conſtant number 2, is becauſe the laſt day of the month is called the 2d of the calends of the month following.

CALIBER, or Caliper, is the thickneſs or diameter of a round body, particularly the bore or width of a piece of ordnance, or that of its ball.

Caliber-*Compaſſes*, or Caliper-*Compaſſes*, or ſimple Calipers, a ſort of compaſſes made with bowed or arched legs, the better to take the diameter of any round body; as the diameters of balls, or the bores of guns; or the diameter, and even the length of caſks, and ſuch like. The beſt ſort of calipers uſually contain the following articles, viz, 1ſt, the meaſure of convex diameters in inches &c; 2d, of concave diameters: 3d, the weight of iron ſhot of given diameters; 4th, the weight of iron ſhot for given gun bores; 5th, the degrees of a ſemicircle; 6th, the proportion of troy and averdupois weight; 7th, the proportion of Engliſh and French feet and pounds weight; 8th, factors uſed in circular and ſpherical figures; 9th, tables of the ſpecific gravities and weights of bodies; 10th, tables of the quantity of powder neceſſary for the proof and ſervice of braſs and iron guns; 11th, rules for computing the number of ſhot or ſhells in a complete pile; 12th, rules for the fall or deſcent of heavy bodies; 13th, rules for the raiſing of water; 14th, rules for firing artillery and mortars; 15th, a line of inches; 16th, logarithmic ſcales of numbers, ſines, verſed ſines, and tangents; 17th, a ſectoral line of equal parts, or the line of lines; 18th, a ſectoral line of planes and ſuperficies; and 19th, a ſectoral line of ſolids.

Calippic *Period*, in Chronology, a period of 76 years, continually recurring; at every repetition of which, it was ſuppoſed, by its inventor Calippus, an Athenian aſtronomer, that the mean new and full moons would always return to the ſame day and hour.

About a century before, the golden number, or cycle of 19 years, had been invented by Meton, which Calippus finding to contain 19 of Nabonaſſar's year, 4 days and $\frac{3}{4}\frac{3}{5}\frac{1}{9}$, to avoid fractions he quadrupled it, and ſo produced his period of 76 years, or 4 times 19; after which he ſuppoſed all the lunations &c would regularly return to the ſame hour. But neither is this exact, as it brings them too late by a whole day in 225 years.

CAMBER-Beam, a piece of timber cut arch-wiſe, or with an obtuſe angle in the middle. They are commonly uſed in platforms, as for church-roofs, and other occaſions where long timbers are wanted to lie at a ſmall ſlope. A camber-beam is much ſtronger than another of the ſame dimenſions; for being laid with the hollow ſide downwards, and having good butments at the ends, they ſerve for a kind of arch.

CAMELEON, one of the conſtellations of the ſouthern hemiſphere, near the ſouth pole, and inviſible in our latitude. There are 10 ſtars marked in this conſtellation in Sharp's catalogue.

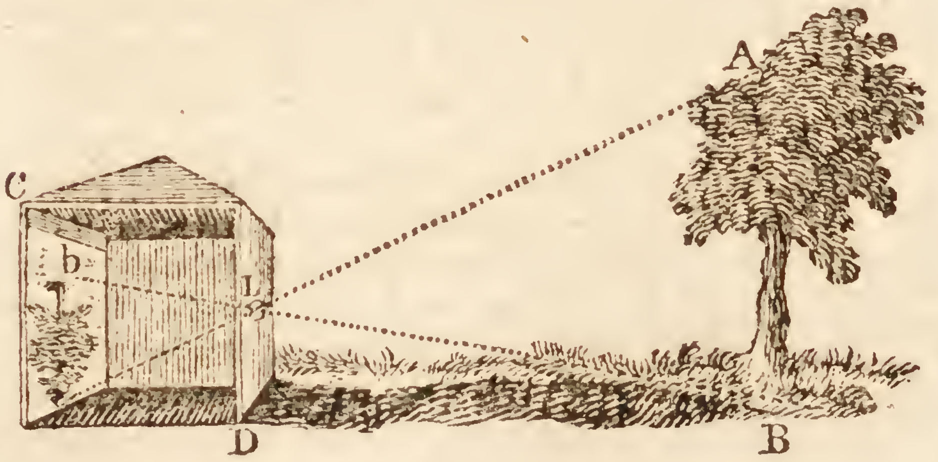
CAMELOPARDALUS, a new conſtellation of the northern hemiſphere, formed by Hevelius, conſiſting of 32 ſtars firſt obſerved by him. It is ſituated between Cepheus, Caſſiopeia, Perſeus, the Two Bears, and Draco; and it contains 58 ſtars in the Britiſh catalogue.

CAMERA *Æolia*, a name given by Kircher to a contrivance for blowing the fire, for the fuſion of ores, without bellows. This is effected by means of water falling through a funnel into a cloſe veſſel, which ſends from it ſo much air or vapour, as continually blows the fire. See Hook's Philoſ. Coll. n° 3, pa. 80.

Camera *Lucida*, a contrivance of Dr. Hook to make the image of any thing appear on a wall in a light room, either by day or night. See Philoſ. Tranſ. n° 38, pa. 741.

Camera *Obſcura*, or *Dark Chamber*, an optical machine or apparatus, repreſenting an artificial eye, by which the images of external objects, received through a double convex glaſs, are ſhewn diſtinctly, and in their native colours, on a white ground placed within the machine, in the focus of the glaſs. The firſt invention of the camera obſcura is aſcribed to John Baptiſta Porta. See his *Magia Naturalis*, *lib.* 17, *cap.* 6, where he largely deſcribes the effects of it. See alſo the end of s'Graveſande's Perſpective, and other optical writers, for the conſtruction and uſes of various ſorts of camera obſcuras.

This machine ſerves for many uſeful and entertaining purpoſes. For example, it is very uſeful in explaining the nature of viſion, repreſenting a kind of artificial eye: it exhibits very diverting ſights or ſpectacles; ſhewing images perfectly like their objects, clothed in their natural colours, but more intenſe and vivid, and at the ſame time accompanied with all their motions; an advantage which no art can imitate: and by this inſtrument, a perſon unacquainted with painting, or drawing, may delineate objects with the greateſt accuracy of drawing and colouring.



Theory of the Camera Obſcura. The theory and principle of this inſtrument may be thus explained. If any object AB radiate through a ſmall aperture L, upon a white ground oppoſite to it, within a darkened room, or box, &c; the image of the object will be painted on that ground in an inverted ſituation. For, by the ſmallneſs of the aperture, the rays from the object will croſs each other there, the image of the point A being at *a*, and that of B at *b*; ſo that the whole object AB will appear inverted, as at *ab*. And as the correſponding rays make equal angles on both ſides of the aperture, if the ground be parallel to the object, their heights will be to each other directly as their diſtances from the aperture.

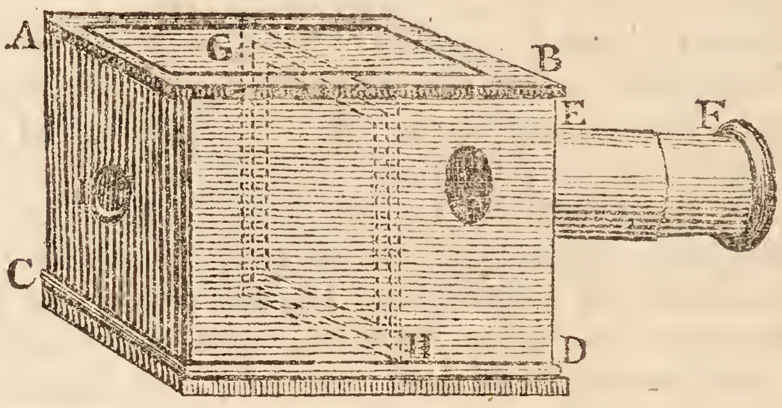
Conſtruction of a Camera Obſcura, by which the images of external objects ſhall be repreſented diſtinctly, and in their genuine colours. 1ſt, Darken a chamber that has one of its windows looking towards a place containing various objects to be viewed; leaving only a ſmall aperture open in one ſhutter. 2d, In this aperture fit a proper lens, either plano-convex, or convex on both ſides; the convexity forming a ſmall portion of a large ſphere. But note, that if the aperture be made very ſmall, as of the ſize of a pea, the objects will be repreſented even without any lens at all. 3d, At a proper diſtance to be determined by trials, ſtretch a paper or white cloth, unleſs there be a white wall at that diſtance, to receive the images of the objects: or the beſt way is to have ſome plaiſter of Paris caſt on a convex mould, ſo as to form a concave ſmooth ſurface, and of a curvature and ſize adapted to the lens, to be placed occaſionally at the proper diſtance. 4th, If it be rather deſired to have the objects appear erect, inſtead of inverted, this may be done either by placing a concave lens between the centre and the focus of the firſt lens; or by reflecting the image from a plane ſpeculum inclined to the horizon in an angle of 45 degrees; or by having two lenſes included in a draw-tube, inſtead of one.

That the images be clear and diſtinct, it is neceſſary that the objects be illuminated by the ſun's light ſhining upon them from the oppoſite quarter: ſo that, in a weſtern proſpect the images will be beſt ſeen in a forenoon, an eaſtern proſpect the afternoon, and a northern proſpect about noon; a ſouthern aſpect is the leaſt eligible of any. But the beſt way of any is, if the lens be fixed in a proper frame, on the top of a building, and made to move eaſily round in all directions, by a handle extended to the perſon who manages the inſtrument; the images being then thrown down into a dark room immediately below it, upon a horizontal round plaiſter of Paris ground: for thus a view of all the objects quite around may eaſily be taken in the ſpace of a few minutes; as is the caſe of the excellent camera obſcura placed on the top of the Royal Obſervatory at Greenwich.

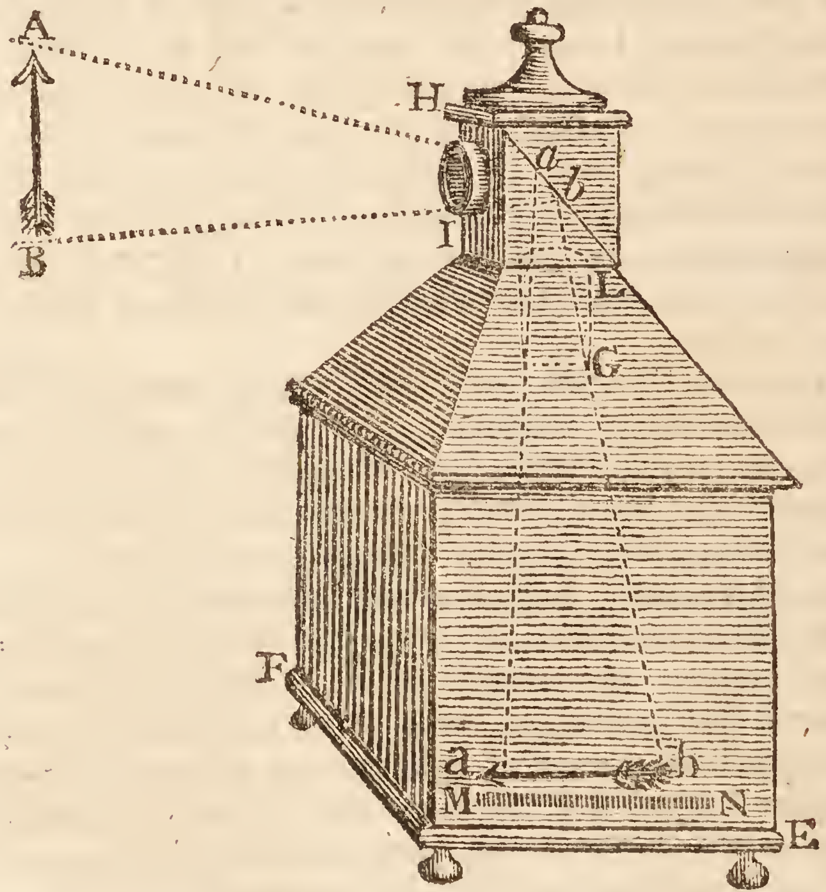
The objects will be ſeen brighter, if the ſpectator firſt wait a few minutes in the dark. Care ſhould alſo be taken, that no light eſcape through any chinks; and that the ground be not too much illuminated. It may further be obſerved, that the greater diſtance there is between the aperture and the ground, the larger the images will be; but then at the ſame time the brightneſs is weakened more and more with the increaſe of diſtance.

To conſtruct a Portable Camera Obſcura. 1ſt, Provide a ſmall box or cheſt of dry wood, and of about 10 inches broad, and 2 feet long or more, according to the ſize of the lenſes. 2d, In one ſide of it, as BD, fit a ſliding tube EF with two lenſes; or, to have the image at a leſs diſtance from the tube, with three lenſes, convex on both ſides; the diameter of the two outer ones to be about 7 inches, but that of the inner to be leſs, as 4¾ or 5 inches. 3d, At a proper diſtance, within the box, ſet up perpendicularly an oiled paper GH, ſo that images thrown upon it may be ſeen through. 4th, In the oppoſite ſide, at I, make a round hole, for a perſon to look conveniently through with both eyes. Then if the tube be turned towards the objects, and the lenſes be placed by trials at the proper diſtance, by ſliding the tube in and out, the objects will be ſeen delineated on the paper, erect as before.

The machine may be better accommodated for drawing, by placing a mirror to paſs from G to C; for this will reflect the image upon a rough glaſs plane, or an oiled paper, placed horizontally at AB; and a copy of it may there be ſketched out with a black-lead pencil.



Another Portable Camera Obſcura is thus made. 1ſt, On the top of a box or cheſt raiſe a little turret HI, open towards the object AB. 2d, Behind the aperture, incline a ſmall mirror *ab* at an angle of 45 degrees, to reflect the rays A*a* and B*b* upon a lens G convex on both ſides, and included in a tube GL. Or the lens may be fixed in the aperture. 3d, At the diſtance of the focus place a table, or board EF, covered with



with a white paper, to receive the image *ab*. Laſtly, In MN make an oblong aperture to look through ; and an opening may alſo be made in the ſide of the box, for the convenience of drawing.

This ſort of camera is eaſily changed into a ſhow-box, for viewing prints, &c : placing the print at the bottom of the box, with its upper part inwards, where it is enlightened through the front, left open for this purpoſe, either by day or candle-light ; and the print may be viewed through the aperture in HI.—A variety of contrivances for this purpoſe may be ſeen deſcribed in Harris's Optics, b. ii. ſect. 4. — Mr. Storer has alſo procured a patent for an inſtrument of this ſort, which he calls a delineator ; being formed of two double convex lenſes and a plane mirror, fitted into a proper box. One lens is placed cloſe to the mirror, making with it an angle of 45 degrees ; the other being placed at right angles to the former, and fixed in a moveable tube. If the moveable lens be directed towards the object, which is to be viewed or copied, and moved nearer to or farther from the mirror, till the image is diſtinctly formed on a greyed glaſs, laid upon that ſurface of the upper lens which is next the eye, it will be found more ſharp and vivid than thoſe formed in the common inſtruments ; becauſe the image is taken up ſo near the upper lens. And by increaſing the diameter and curvature of the lenſes, the effect will be much heightened.

CAMUS, *(Charles-Stephen-Lewis)*, a celebrated French mathematician, Examiner of the royal Schools of Artillery and Engineers, Secretary and Profeſſor of the Royal Academy of Architecture, Honorary member of that of the Marine, and fellow of the Royal Society of London, was born at Creſſy en Brie, the 25th of Auguſt 1699. His early ingenuity in mechanics and his own intreaties induced his parents to ſend him to ſtudy at a college in Paris, at 10 years of age ; where in the ſpace of two years his progreſs was ſo great, that he was able to give leſſons in mathematics, and thus to defray his own expences at the college, without any farther charge to his parents. By the aſſiſtance of the celebrated Varignon, young Camus ſoon ran through the courſe of the higher mathematics, and acquired a name among the learned. He made himſelf more particularly known to the Academy of Sciences in 1727, by his memoir upon the ſubject of the prize which they had propoſed for that year, viz, 'To determine the moſt advantageous way of maſting ſhips ;' in conſequence of which he was named that year Adjoint-Mechanician to the Academy ; and in 1730 he was appointed profeſſor of Architecture. In leſs than three years after, he was honoured with the ſecretaryſhip of the ſame ; and the 18th of April 1733, he obtained the degree of Aſſociate in the Academy, where he diſtinguiſhed himſelf greatly by his memoirs upon living forces, or bodies in motion acted upon by forces, on the figure of the teeth of wheels and pinions, on pump work, and ſeveral other ingenious memoirs.

In 1736 he was ſent, in company with Meſſieurs Clairaut, Maupertuis, and Monnier, upon the celebrated expedition to meaſure a degree at the north polar circle ; in which he rendered himſelf highly uſeful, not only as a mathematician, but alſo as a mechanician and an artiſt, branches for which he had a remarkable talent.

In 1741 Camus had the honour to be appointed Penſioner-Geometrician in the Academy ; and the ſame year he invented a gauging-rod and ſliding-rule proper at once to gauge all ſorts of caſks, and to calculate their contents. About the year 1747 he was named Examiner of the Schools of Artillery and Engineers : and, in 1756, one of the eight mathematicians appointed to examine by a new meaſurement, the baſe which had formerly been meaſured by Picard, between Villejuifve and Juviſi ; an operation in which his ingenuity and exactneſs were of great utility. In 1765 M. Camus was elected a fellow of the Royal Society of London ; and died the 4th of May 1768, in the 69th year of his age ; being ſucceeded by the celebrated d'Alembert in his office of Geometrician in the French Academy ; and leaving behind him a great number of manuſcript treatiſes on various branches of the mathematics.

The works publiſhed by M. Camus, are :

1. Courſe of Mathematics for the uſe of the Engineers, 4 vols. in 8vo.
2. Elements of Mechanics.
3. Elements of Arithmetic.

And his memoirs printed in the volumes of the Academy, are :

1. Of Accelerated Motions by living forces: vol. for 1728.
2. Solution of a Geometrical Problem of M. Cramer : 1732.
3. On the figure of the teeth and pinions in clocks : 1733.
4. On the action of a Muſket ball, piercing a pretty thick piece of wood, without communicating any conſiderable velocity to it : 1738.
5. On the beſt manner of employing buckets for raiſing water : 1739.
6. A Problem in Statics : 1740.
7. On an Inſtrument for gauging of veſſels : 1741.
8. On the Standard of the Ell Meaſure : 1746.
9. On the Tangents of Points common to ſeveral branches of the ſame curve : 1747.
10. On the Operations in meaſuring the diſtance between the centres of the pyramids of Villejuive and Juviſy, to diſcover the beſt meaſure of the degree about Paris : 1754.
11. On the Maſting of Ships : Prize Tom. 2.
12. The Manner of working Oars : Mach. tom. 2.
13. A Machine for moving many Colters at once : Mach. tom. 2.

CANCER, the *Crab*, one of the twelve ſigns of the zodiac, uſually drawn on the globe in the form of a crab, and in books of Aſtronomy denoted by a character reſembling the number ſixty-nine, turned ſideways, thus ♋.

This is one of the 48 old conſtellations; and, from the hieroglyphic mode of writing among the Egyptians &c, it is probable that they gave the name and figure to this conſtellation from the following circumſtance, viz, that as the crab is an animal that goes ſideling backwards, ſo the ſun, in his annual courſe through the zodiac, when he arrives at this part of the ecliptic, having reached his utmoſt limit northwards, begins there to return back again towards the ſouth. But the Greeks, who adapted ſome fable of their own to every thing of this kind, pretend that when Hercules was fighting with the Lernæan hydra, there was a crab upon the marſh which ſeized his foot. The hero cruſhed the reptile to pieces under his heel; but Juno, in gratitude for the offered ſervice, little as it was, raiſed the creature into the heavens.

The number of ſtars in the ſign cancer, Ptolemy makes 13, Tycho 15, Bayer and Hevelius 29, and Flamſteed 83.

Tropic of Cancer, a little circle of the ſphere parallel to the equinoctial, and paſſing through the beginning of the ſign cancer.

CANDLEMAS, or the *Purification*, a feaſt of the church, held on the 2d of February, in memory of the purification of the Virgin; taking its name of Candlemas, either from the number of lighted candles uſed by the Romiſh church, in the proceſſions of this day, or becauſe that the church then conſecrated candles for the whole year.

CANES *Venatici*, the *Hounds*, or the *Greyhounds*, one of the new conſtellations of the northern hemiſphere, which Hevelius has formed out of the unformed ſtars of the old catalogues. Theſe two dogs are farther diſtinguiſhed by the names of *aſterion* and *chara*. They contain 23 ſtars according to Hevelius, but 25 in the Britiſh catalogue.

CANICULA, a name given by many of the earlier aſtronomers to the conſtellation which we call the Leſſer Dog, and Canis Minor, but ſome Procyon and Antecanis. See Canis Minor.

It is alſo uſed for one of the ſtars of the conſtellation Canis Major; called alſo ſimply the Dog-ſtar; and by the Greeks Σειριος, Sirius. It is ſituated in the mouth of the conſtellation, and is the largeſt and brighteſt of all the ſtars in the heavens. From the heliacal riſing of this ſtar, that is, its emerſion from the ſun's rays, which now happens with us about the 11th of Auguſt, the ancients reckoned their *dies caniculares*, or dog-days.

The Egyptians and Ethiopians began their year at the heliacal riſing of Canicula; reckoning to its riſe again the next year, which is called the *Annus Canarius*.

CANICULAR *Days*, or *Dog-days*, denote a certain number of days, before and after the heliacal riſing of canicula, or the dog-ſtar, in the morning. The ancients imagined that this ſtar, ſo riſing, occaſioned the ſultry weather uſually felt in the latter part of the ſummer, or dog-days; with all the diſtempers of that ſickly ſeaſon: Homer's Il. lib. 5, v. 10, and Virgil's Æn. lib. 10, v. 270. Some authors ſay, from Hippocrates and Pliny, that the day this ſtar firſt riſes in the morning, the ſea boils, wine turns ſour, dogs begin to grow mad, the bile increaſes and irritates, and all animals grow languid; alſo that the diſeaſes it uſually occaſions in men, are burning fevers, dyſenteries, and phrenſies. The Romans too ſacrificed a brown dog every year to Canicula at his firſt riſing, to appeaſe its rage. All this however aroſe from a groundleſs idea that the dog-ſtar, ſo riſing, was the occaſion of the extreme heat and the diſeaſes of that ſeaſon; for the ſtar not only varies in its riſing, in any one year, as the latitude varies, but it is always later and later every year in all latitudes; ſo that in time the ſtar may, by the ſame rule, come to be charged with bringing froſt and ſnow, when he comes to riſe in winter.

The dog-days were commonly counted for about 40 days, viz, 20 days before and 20 days after the heliacal riſing; and almanac-makers have uſually ſet down the dog-days in their almanacs to the changing time of the ſtar's riſing, by which means they had at length fallen conſiderably after the hotteſt time of the year, till of late we have obſerved an alteration of them in the almanacs, and very properly, from July 3 to Auguſt 11. For, by the dog-days, the ancients meant to expreſs the hotteſt time of the year, which is commonly during the month of July, about which month the dog-ſtar roſe heliacally in the time of the moſt ancient aſtronomers that we know of: but the preceſſion of the equinoxes has carried this heliacal riſing into a much later and cooler part of the year; and becauſe Heſiod tells us that the hot time of the year ends on the 50th day after the ſummer ſolſtice, which brings us to about Auguſt 10 or 11, therefore the above alteration ſeems to be very proper.

Canicular *Year*, denotes the Egyptian natural year, which was computed from one heliacal riſing of canicula, to the next. This year was alſo called *annus canarius*, and *annus cynicus*; and by the Egyptians themſelves the *Sethic year*, from *Seth*, by which name they called Sirius. Some call it alſo the *heliacal year*. This year conſiſted ordinarily of 365 days, and every 4th year of 366; by which means it was accommodated to the civil year, like the Julian account. And the reaſon why they choſe this ſtar, in preference to others, to compute their time by, was not only the ſuperior brightneſs of that ſtar, but becauſe that in Egypt its heliacal riſing was a time of very ſingular note, as coinciding with the greateſt augmentation of the Nile, the reputed father of Egypt. Epheſtion adds, that from the aſpect of canicula, its colour &c, the Egyptians drew prognoſtics concerning the riſe of the Nile; and, according to Florus, predicted the future ſtate of the year. So that it is no wonder the firſt riſing of this ſtar was obſerved with great attention. Bainbrigge, *Canicul.* cap. 4. p. 26.

CANIS *Major*, the *Great Dog*, a conſtellation of the ſouthern hemiſphere, below the feet of Orion, and one of the old 48 conſtellations. The Greeks, as uſual, have many fables of their own about the exaltation of the dog into the ſkies; but the origin of this conſtellation, as well as its other name Sirius, lies more probably among the Egyptians, who carefully watched the riſing of this ſtar, and by it judged of the ſwelling of the Nile, calling the ſtar the ſentinel and watch of the year; and hence, according to their manner of hieroglyphic writing, repreſented it under the figure of a dog. They alſo called the Nile *Siris*; and hence their *Oſiris*.

The ſtars in this conſtellation, Ptolemy makes 29; Tycho however obſerved only 13, and Hevelius 21; but in Flamſteed's catalogue they are 31.

Canis *Minor*, a conſtellation of the northern hemiſphere, juſt below Gemini, and is one of the 48 old conſtellations. The Greeks fabled that this is one of Orion's hounds; but the Egyptians were moſt probably the inventors of this conſtellation, and they may have given it this figure to expreſs a little dog, or watchful creature, going before as leading in the larger, or riſing before it: and hence the Latins have called it Antecanis, the ſtar before the dog.

The ſtars in this conſtellation are, in Ptolomy's catalogue 2, the principal of which is the ſtar Procyon; in Tycho's 5, in Hevelius's 13; and in Flamſteed's 14.

CANNON, in Military Affairs, a long round hollow engine, made of iron or braſs, &c, for throwing balls, &c, by means of gunpowder. The length is diſtinguiſhed into three parts; the firſt reinforce, the ſecond reinforce, and the chaſe: the inſide hollow where the charge is lodged, being alſo called the chaſe, or bore. But for the ſeveral parts and members of a cannon, ſee ASTRAGAL, BASE-RING, BORE, BREECH, CASCABEL, CHASE, MUZZLE, OGEE, REINFORCE-RING, TRUNNIONS, &c. See alſo GUN, and GUNNERY.

Cannon were firſt made of ſeveral bars of iron adapted to each other lengthways, and hooped together with ſtrong iron rings. They were employed in throwing ſtones and metal of ſeveral hundred weight. Others were made of thin ſheets of iron rolled up, and hooped: and on emergencies they have been even made of leather, with plates of iron or copper. They are now made of caſt iron or braſs; being caſt ſolid, and the tube bored out of the middle of the ſolid metal.

Larrey makes braſs cannon the invention of J. Owen; and aſſerts that the firſt known in England, were in 1535; and farther that iron cannon were firſt caſt here in 1547. He acknowledged that cannon were known before; and remarks that at the battle of Creſſi, in 1346, there were 5 pieces of cannon in the Engliſh army, which were the firſt ever ſeen in France. Mezeray alſo obſerves that king Edward ſtruck terror into the French army, by 5 or 6 pieces of cannon; it being the firſt time they had met ſuch thundering machines.

In the liſt of aids raiſed for the redemption of king John of France, in 1368, mention is made of an officer in the French army called *maſter of the king's cannon*, and of his providing 4 large cannon for the garriſon of Harfleur. But father Daniel, in his life of Philip of Valois, produces a proof from the records of the chamber of accounts at Paris, that cannon and gunpowder were uſed in the year 1338. And Du-Cange even finds mention of the ſame engines in Froiſſart, and other French hiſtorians, ſome time earlier.

The Germans carry the invention of cannon farther back, and aſcribe it to Albertus Magnus, a Dominican monk, about the year 1250. But Iſaac Voſſius finds cannon in China upwards of 1700 years ago; being uſed by the emperor Kitey, in the year of Chriſt 85. The ancients too, of Europe and Aſia, had their fiery tubes, or *cannæ*, which being loaden with pitch, ſtones, and iron balls, were exploded with a vehement noiſe, ſmoke, and great effect.

Cannon were formerly made of a very great length, which rendered them exceedingly heavy, and their uſe very troubleſome and confined. But it has lately been found by experiment that there is very little added to the force of the ball by a great length of the cannon, and therefore they have very properly been much reduced both in their length and weight, and rendered eaſily manageable upon all occaſions. They were formerly diſtinguiſhed by many hard and terrible names, but are now only named from the weight of their ball; as a 6 pounder, a 12 pounder, a 24 pounder, or a 42 pounder, which is the largeſt ſize now uſed by the Engliſh for battering.

CANON, in Algebra, Arithmetic, Geometry, &c, is a general rule for reſolving all caſes of a like nature with the preſent enquiry. Thus the laſt ſtep of every equation is ſuch a canon, and if turned into words, becomes a rule to reſolve all caſes or queſtions of the ſame kind with that propoſed.

Tables of ſines, tangents, &c, whether natural or artificial, are alſo called canons.

CANON, in Ancient Muſic, is a rule or way of determining the intervals of muſical notes. Ptolomy, rejecting the Ariſtoxenian way of meaſuring the intervals in muſic by the magnitude of a tone, formed by the difference between a diapente and a diateſſaron, thought that they ſhould be diſtinguiſhed by the ratios which the ſounds terminating thoſe intervals bear to one another, when conſidered according to their degree of acuteneſs or gravity; which, before Ariſtoxenus, was the old Pythagorean way. He therefore made the diapaſon conſiſt in a double ratio; the diapente conſiſt in a ſeſquialterate; the diateſſaron, in a ſeſquitertian; and the tone itſelf, in a ſeſquioctave; and all the other intervals, according to the proportion of the ſounds that terminate them: wherefore, taking the canon, as it is called, for a determinate line of any length, he ſhews how this is to be cut, that it may repreſent the reſpective intervals: and this method anſwers exactly to experiments in the different lengths of muſical chords. From this canon, Ptolomy and his followers have been called *Canonici;* as thoſe of Ariſtoxenus were called *Muſici*.

Paſcal CANON, a table of the moveable feaſts, ſhewing the day of Eaſter, and the other feaſts depending upon it, for a cycle or period of 19 years. It is ſaid that the Paſcal Canon was the calculation of Euſebius of Cæſarea, and that it was made by order of the council of Nice.

CANOPUS, a name given by ſome of the old aſtronomers to a ſtar under the 2d bend of Eridanus. Theſe writers ſay that the river in the heavens is not the Eridanus, but the Nile, and that this ſtar commemorates an iſland made by that river, which was called by the ſame name.

CANOPUS is alſo the name of a bright ſtar of the firſt magnitude in the rudder of the ſhip Argo, one of the ſouthern conſtellations. Its ſituation, as given by ſeveral authors, at different times, is as follows:

Authors	Dates	Longit.	Lat.	Rt. Aſcen.	Declin. So.
F. Thomas	Jan. 1682	8° ♋ 52′	75° 15′	93° 32′ 20″	52° 31′ 33″
F. Noel	1697			93 54	52 29
Dr. Halley	170.	10 52	72 49		
F. Feuille	Mar. 1709				52 30 4

CANTALIVERS, in Architecture, are the ſame with modillions, except that the former are plain, and the latter carved. They are both a kind of cartouſes, ſet at equal diſtances under the corona of the cornice of a building.

CANTON (JOHN), an ingenious natural philoſopher, was born at Stroud, in Glouceſterſhire, in 1718; and was placed, when young, under the care of Mr Davis, an able mathematician of that place, with whom he had learned both vulgar and decimal arithmetic before he was quite 9 years of age. He next proceeded to higher parts of the mathematics, and particularly to algebra and aſtronomy, in which he had made a conſiderable progreſs when his father took him from ſchool, and ſet him to learn his own buſineſs, which was that of a broad cloth weaver. This circumſtance was not able to damp his zeal for acquiring knowledge. All his leiſure time was devoted to the aſſiduous cultivation of aſtronomical ſcience; by which he was ſoon able to calculate lunar eclipſes and other phenomena, and to conſtruct various kinds of ſun-dials, even at times when he ought to have ſlept, being done without the knowledge and conſent of his father, who feared that ſuch ſtudies might injure his health. It was during this prohibition, and at theſe hours, that he computed, and cut upon ſtone, with no better an inſtrument than a common knife, the lines of a large upright ſun-dial, on which, beſide the hour of the day, were ſhewn the ſun's riſing, his place in the ecliptic, and ſome other particulars. When this was finiſhed and made known to his father, he permitted it to be placed againſt the front of his houſe, where it excited the admiration of ſeveral neighbouring gentlemen, and introduced young Canton to their acquaintance, which was followed by the offer of the uſe of their libraries. In the library of one of theſe gentlemen he found Martin's Philoſophical Grammar, which was the firſt book that gave him a taſte for natural philoſophy. In the poſſeſſion of another gentleman he firſt ſaw a pair of globes; a circumſtance that afforded him great pleaſure, from the great eaſe with which he could reſolve thoſe problems he had hitherto been accuſtomed to compute.

Among other perſons with whom he became acquainted in early life, was Dr. Henry Miles of Tooting; who perceiving that young Canton poſſeſſed abilities too promiſing to be confined within the narrow limits of a country town, prevailed on his father to permit him to come up to London. Accordingly he arrived at the metropolis the 4th of March 1737, and reſided with Dr. Miles at Tooting till the 6th of May following; when he articled himſelf, for the term of 5 years, as a clerk to Mr. Samuel Watkins, maſter of the academy in Spital Square. In this ſituation his ingenuity, diligence, and prudence, were ſo diſtinguiſhed, that on the expiration of his clerkſhip in May 1742, he was taken into partnerſhip with Mr. Watkins for 3 years; which gentleman he afterward ſucceeded in the ſchool, and there continued during his whole life.

Towards the end of 1745, electricity received a great improvement by the diſcovery of the famous Leyden phial. This event turned the thoughts of moſt of the philoſophers of Europe to that branch of natural philoſophy; and our author, who was one of the firſt to repeat and to purſue the experiment, found his endeavours rewarded by many notable diſcoveries.—Towards the end of 1749, he was engaged with his friend, the late ingenious Benjamin Robins, in making experiments to determine the height to which rockets may be made to aſcend, and at what diſtance their light may be ſeen.—In 1750 was read at the Royal Society, Mr. Canton's “Method of making Artificial Magnets, without the uſe of, and yet far ſuperior to, any natural ones.” This paper procured him the honour of being elected a member of the Society, and the preſent of their gold medal. The ſame year he was complimented with the degree of M. A. by the univerſity of Aberdeen. And in 1751 he was choſen one of the council of the Royal Society; an honour which was twice repeated afterwards.

In 1752, our philoſopher was ſo fortunate as to be the firſt perſon in England who, by attracting the electric fire from the clouds during a thunder-ſtorm, verified Dr. Franklin's hypotheſis of the ſimilarity of lightning and electricity. Next year his paper intitled “Electrical Experiments, with an attempt to account for their ſeveral phenomena,” was read at the Royal Society. In the ſame paper Mr. Canton mentioned his having diſcovered, by many experiments, that ſome clouds were in a poſitive, and ſome in a negative ſtate of electricity: a diſcovery which was alſo made by Dr. Franklin in America much about the ſame time. This circumſtance, together with our author's conſtant defence of the doctor's hypotheſis, induced that excellent philoſopher, on his arrival in England, to pay Mr. Canton a viſit, and gave riſe to a friendſhip which ever after continued between them.—In the Ladies' Diary for 1756, our author anſwered the prize query that had been propoſed in the preceding year, concerning the meteor called ſhooting ſtars. The ſolution, though only ſigned A. M. was ſo ſatisfactory to his friend, the excellent mathematician Mr. Thomas Simpſon, who then conducted that ingenious and uſeful little work, that he ſent Mr. Canton the prize, accompanied with a note, in which he ſaid he was ſure that he was not miſtaken in the author of it, as no one beſides, that he knew of, could have given that anſwer.—Our philoſopher's next communication to the public, was a letter in the Gentleman's Magazine for September 1759, on the electrical properties of the tourmalin, in which the laws of that wonderful ſtone are laid down in a very conciſe and elegant manner. On the 13th of December in the ſame year was read at the Royal Society, “An attempt to account for the Regular Diurnal Variation of the Horizontal Magnetic Needle; and alſo for its Irregular Variation at the time of an Aurora Borealis.” A complete year's obſervations of the diurnal variations of the needle are annexed to the paper.—Nov. 5, 1761, our author communicated to the Royal Society an account of the Tranſit of Venus of the 6th of June that year, obſerved in Spital Square. His next communication to the Society, was a Letter, read the 4th of Feb. 1762, containing ſome remarks on Mr. Delaval's electrical experiments. On the 16th of Dec. the ſame year, another curious addition was made by him to philoſophical knowledge, in a paper, intitled, “Experiments to prove that Water is not Incompreſſible.” And on Nov. 8, the year following, were read before the Society, his farther “Experiments and Obſervations on the Compreſſibility of Water, and ſome other fluids.” Theſe experiments are a complete refutation of the famous Florentine experiment, which ſo many philoſophers have mentioned as a proof of the incompreſſibility of water. For this communication he had a ſecond time the Society's prize gold medal.

Another communication was made by our author to the Society, on Dec. 22, 1768, being “An eaſy method of making a phoſphorus that will imbibe and emit light like the Bolognian Stone; with experiments and obſervations.” When he firſt ſhewed to Dr. Franklin the inſtantaneous light acquired by ſome of this phoſphorus from the near diſcharge of an electrified bottle, the doctor immediately exclaimed, “And God ſaid let there be light, and there was light.”

The Dean and Chapter of St. Paul's having, in a letter, dated March 6, 1769, requeſted the opinion of the Royal Society relative to the beſt method of fixing electrical conductors to preſerve that cathedral from damage by lightning, Mr. Canton was one of the committee appointed to take the letter into conſideration, and to report their opinion upon it. The gentlemen joined with him in this buſineſs were, Mr. Delaval, Dr. Franklin, Dr. Watſon, and Mr. Wilſon. Their report was made on the 8th of June following: and the mode recommended by them has been carried into execution.—Our author's laſt communication to the Royal Society, was a paper read Dec. 21, 1769, containing "Experiments to prove that the Luminouſneſs of the Sea ariſes from the Putrefaction of its animal Subſtances."

Beſides the papers above mentioned, Mr. Canton wrote a number of others, both in the earlier and the later parts of his life, which appeared in ſeveral publications, and particularly in the Gentleman's Magazine.—He died of a dropſy, the 22d of March 1772, in the 54th year of his age.

CAPACITY, is the ſolid content of any body. Alſo our hollow meaſures for corn, beer, wine, &c, are called meaſures of capacity.

CAPE, or *Promontory*, is any high land, running out with a point into the ſea; as Cape Verde, Cape Horn, the Cape of Good Hope, &c.

CAPELLA, a bright ſtar of the firſt magnitude, in the left ſhoulder of Auriga.

CAPILLARY *Tubes*, in Phyſics, are very ſmall pipes, whoſe canals are exceedingly narrow; being ſo called from their reſemblance to a hair in ſmallneſs. Their uſual diameter may be from $\frac{1}{20}$ to $\frac{1}{50}$ of an inch: though Dr. Hook aſſures us that he drew tubes in the flame of a lamp much ſmaller, and reſembling a ſpider's thread.

The *Aſcent of Water &c*, in capillary tubes, is a noted phenomenon in philoſophy. Take ſeveral ſmall glaſs tubes, of different diameters, and open at both ends; immerſe them a little way into water, and the fluid will be ſeen to ſtand higher in the tubes than the ſurface of the water without, and higher as the tube is ſmaller, almoſt in the reciprocal ratio of the diameter of the tube; and that both in open air, and in vacuo. The greateſt height to which Dr. Hook ever obſerved the water to ſtand, in the ſmalleſt tubes, was 21 inches above the ſurface in the veſſel.

This does not however happen uniformly the ſame in all fluids; ſome ſtanding higher than others; and in quickſilver the contrary takes place, as that fluid ſtands lower within the tube than its ſurface in the veſſel, and the lower as the tube is ſmaller. See Philoſ. Tranſ. N° 355, or Abr. vol. 4, pa. 423, &c, or Cotes's Hydr. and Pneum. Lect. pa. 265.

Another phenomenon of theſe tubes is, that ſuch of them as would only naturally diſcharge water by drops, when electrified, yield a continued and accelerated ſtream; and the acceleration is proportional to the ſmallneſs of the tube: indeed the effect of electricity is ſo conſiderable, that it produces a continued ſtream from a very ſmall tube, out of which the water had not before been able to drop. Prieſtley's Hiſt. Electr. 8vo. vol. 1, pa. 171, ed. 3d.

This aſcent and ſuſpenſion of the water in the tube, is by Dr. Jurin, Mr. Haukſbee, and other philoſophers, aſcribed to the attraction of the periphery of the concave ſurface of the tube, to which the upper ſurface of the water is contiguous and adheres.

CAPITAL, in Architecture, the uppermoſt part of a column or pilaſter, ſerving as a head or crowning to it; being placed immediately over the ſhaft, and under the entablature. It is made differently in the different orders, and is that indeed which chiefly diſtinguiſhes the orders themſelves.

Capital *of a Baſtion*, is an imaginary line dividing any work into two equal and ſimilar parts; or a line drawn from the angle of the polygon to the point of the baſtion, or from the point of the baſtion to the middle of the gorge.

CAPONIERE, or Caponniere, in Fortification, is a paſſage made from one work to another, of 10 or 12 feet wide, and about 5 feet deep, covered on each ſide by a parapet, terminating in a glacis or ſlope. Sometimes it is covered with planks and earth.

CAPRA, or the *She-goat*, a name given to the ſtar Capella, on the left ſhoulder of Auriga; and ſometimes to the conſtellation Capricorn. Some again repreſent Capra as a conſtellation in the northern hemiſphere, conſiſting of 3 ſtars, compriſed between the 45th and 55th degree of latitude.

The poets fable her to be Amalthea's goat, which ſuckled Jupiter in his infancy.

CAPRICORN, the *Goat*, a ſouthern conſtellation, and the 10th ſign of the zodiac, as alſo one of the 48 original conſtellations received by the Greeks from the Egyptians. The figure of this ſign is drawn as having the fore part of a goat, but the hinder part of a fiſh; and ſometimes ſimply under the form of a goat. In writing, it is denoted by a character repreſenting the crooked horns of a goat's head, thus ♑.

As to the figure of this conſtellation, the Greeks pretend that Pan, to avoid the terrible giant Typhon, threw himſelf into the Nile, and was changed into the figure here drawn; in commemoration of which exploit, Jupiter took it up to heaven. But it is probable, as Macrobius obſerves, that the Egyptians marked the point of the ecliptic appropriated to this ſign, where the ſun begins again to aſcend up towards the north with the figure of a goat, an animal which is alwa climbing the ſides of mountains.

The ſtars in this conſtellation, in Ptolomy's an Tycho's catalogue, are 28; in that of Hevelius 29: though it is to be remarked that one of thoſe in th tail, of the 6th magnitude, marked the 27th in Tycho's book, was loſt in Hevelius's time. Flamſteed gives 51 ſtars to this ſign.

Tropic of Capricorn, a little circle of the ſphere, parallel to the equator, paſſing through the beginning of Capricorn, or the winter ſolſtice, or the point of the ſun's greateſt ſouth declination.

CAPSTAN, a large maſſy column ſhaped like a truncated cone; being ſet upright on the deck of a ſhip, and turned by levers or bars, paſſing through holes in its upper extremity. The capſtan is a kind of perpetual lever, or an axis-in-peritrochio, which, by means of a ſtrong rope or cable paſſed round, ſerves to raiſe very great weights; ſuch as to hoiſt ſails, to weigh the anchors, to draw the veſſels on ſhore, and hoiſt them up to be refitted, &c.

CAPUT Draconis, or dragon's head, a name given by ſome to a fixed ſtar of the firſt magnitude, in the head of the conſtellation Draco.

CARACT, or Carat, a name given to the weight which expreſſes the degree of goodneſs or fineneſs of gold. The whole quantity of metal is conſidered as conſiſting of 24 parts, which are the carats, ſo that the carat is the 24th part of the whole; this carat is divided into 4 equal parts, called grains of a carat, and the grain into halves and quarters.

When gold is purified to the utmoſt degree poſſible, ſo that it loſes no more by farther trials, it is conſidered as quite pure, and ſaid to be 24 carats fine; if it loſe 1 carat, or 1—24th in purifying, it was of 23 carats fine; and if it loſe 2 carats, it was 22 carats fine; and ſo on.

CARCASS, is a hollow caſe formed of ribs of iron, and covered over with pitched cloth &c, about the ſize of bomb-ſhells; or ſometimes made all of iron except two or three holes for the fire to blaze through. Theſe are filled with various matters and combuſtibles, to fire houſes, when thrown out of mortar pieces into beſieged places.

CARCAVI (Peter de), was born at Lyons, but in what year is not known. He was Counſellor to the Parliament of Toulouſe, afterward Counſellor to the Grand Council, and Keeper of the King's Library. He was appointed Geometrician to the French Academy of Sciences in 1666; and died at Paris in 1684. There are extant ſome letters of his, printed among thoſe of Deſcartes.

CARDAN (Hieronymus, or Jerom), one of the moſt extraordinary geniuſes of his age, was born at Pavia, in Italy, Sept. 24, 1501. At 4 years old he was carried to Milan, his father being an advocate and phyſician in that city: at the age of 20 he went to ſtudy in the univerſity of the ſame city; and two years afterward he explained Euclid. In 1524, he went to Padua: the ſame year he was admitted to the degree of maſter of arts; and the year following, that of doctor of phyſic. He married about the year 1531; and became profeſſor of mathematics, and practiſed medicine at Milan about 1533. In 1539 he was admitted a member of the college of phyſicians at Milan: in 1543 he read public lectures in medicine there; and the ſame at Pavia the year following; but he diſcontinued them becauſe he could not get payment of his ſalary, and returned to Milan.

In 1552 he went into Scotland, having been ſent for by the archbiſhop of St. Andrews, to cure him of a grievous diſorder, after trying the phyſicians of the king of France and of the emperor of Germany, without benefit. He began to recover from the day that Cardan preſcribed for him: our author took his leave of him at the end of ſix weeks and three days, leaving him preſcriptions which in two years wrought a complete cure. Upon this viſit Cardan paſſed through London, and calculated king Edward's nativity; for he was famous for his knowledge in aſtrology, as well as thoſe of mathematics and medicine. Returning to Milan, after four months abſence, he remained there till the beginning of Oct. 1552; and then went to Pavia, from whence he was invited to Bologna in 1562. He taught in this laſt city till the year 1570; at which time he was thrown into priſon; but ſome months after he was ſent home to his own houſe. He quitted Bologna in 1571; and went to Rome, where he lived for ſome time without any public employment. He was however admitted a member of the college of phyſicians, and received a penſion from the Pope, till the time of his death, which happened at Rome on the 21ſt of September 1575.

Cardan, at the ſame time that he was one of the greateſt geniuſes and moſt learned men of his age, in all the ſciences, was one of the moſt eccentric and fickle in conduct of all men that ever lived: deſpiſing all good principles and opinions, and without one friend in the world. The ſame capriciouſneſs that was remarkable in his outward conduct, is alſo obſervable in the compoſition of his numerous and elaborate works. In many of his treatiſes the reader is ſtopped almoſt every moment by the obſcurity of his text, or by digreſſions from the point in hand. In his arithmetical writings there are ſeveral diſcourſes on the motions of the planets, on the creation, on the Tower of Babel, and ſuch like. And the apology which he made for theſe frequent digreſſions is, that he might by that means enlarge and fill up his book, his bargain with the bookſeller being at ſo much per ſheet; and that he worked as much for his daily ſupport as for fame. The Lyons edition of his works, printed in 1663, conſiſts of no leſs than 10 volumes in folio.

In fact, when we conſider the tranſcendent qualities of Cardan's mind, it cannot be denied that he cultivated it with every ſpecies of knowledge, and made a greater progreſs in philoſophy, in the medical art, in aſtronomy, in mathematics, and the other ſciences, than the moſt part of his contemporaries who had applied themſelves but to one only of thoſe ſciences. In particular, he was perhaps the very beſt algebraiſt of his time, a ſcience in which he made great improvements; and his labours in cubic equations eſpecially have rendered his name immortal, the rules for reſolving them having ever ſince borne his name, and are likely to do ſo as long as the ſcience ſhall exiſt, although he received the firſt knowledge of them from another perſon; the account of which, and his diſputes with Tartalea, have been given at large under the article Algebra.

Scaliger affirms, that Cardan, having by aſtrology predicted and fixed the time of his death, abſtained from all food, that his prediction might be fulfilled, and that his continuance to live might not diſcredit his art. It is farther remarkable, that Cardan's father alſo died in this manner, in the year 1524, having abſtained from ſuſtenance for nine days. Our author too informs us that his father had white eyes, and could ſee in the night-time.

CARDINAL *Points*, in Geography, are the eaſt, weſt, north, and ſouth points of the horizon.

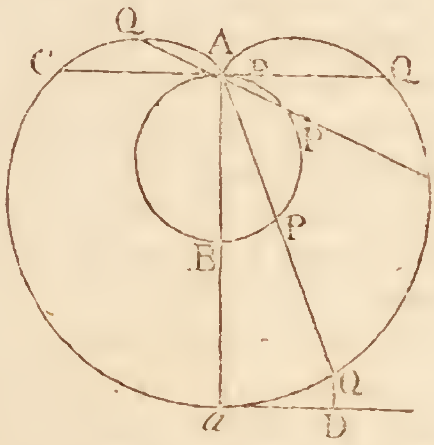
Cardinal *Points of the Heavens*, or *of a Nativity*, are the riſing and ſetting of the ſun, the zenith and nadir.

Cardinal *Signs*, are thoſe at the four quarters, or the equinoxes and ſolſtices, viz, the ſigns Aries, Libra, Cancer, and Capricorn.

Cardinal *Winds*, are thoſe that blow from the four cardinal points, viz, the eaſt, weſt, north, and ſouth winds.

CARDIOIDE, the name of a curve ſo called by Caſtilliani.—But it was firſt treated of by Koerſma, and by Carré. See Philoſ. Tranſ. 1741, and Memoires de l'Acad. 1705.

The Cardioide is thus generated. APB is a circle, and AB its diameter. Through one extremity A of the diameter draw a number of lines APQ, cutting the circle in P; upon theſe ſet off always PQ equal to the diameter AB; ſo ſhall the points Q be always in the curve of the cardioide.



From this generation of the curve, its chief properties are evident, viz, that,

everywhere PQ = AB,
CQ, or QQ is = Aa or 2 AB,
AQ = AB ± AP,
P always biſects QQ.

The cardioide is an algebraical curve, and the equation expreſſing its nature is thus:

Put a = AB the diameter,
x = aD perp. AB,
y = DQ perp. AD; then is

$$\left.\begin{array}{l} y^4 - 6ay^3 + 2x^2y^2 - 6ax^2y + x^4 \\ \quad + 12a^2y^2 - 8a^3y + 3a^2x^2 \end{array}\right\} = 0,$$

which is the equation of the curve.

Many properties of the cardioide may be ſeen in the places above cited.

CARRÉ (Lewis), was born in the year 1663, in the province of Brie in France. His father, a ſubſtantial farmer, intended him for the church. But young Carré, after going through the uſual courſe of education for that purpoſe, having an utter averſion to it, he refuſed to enter upon that function; by which he incurred his father's diſpleaſure. His reſources being thus cut off, he was obliged to quit the univerſity, and look out into the world for ſome employment. In this exigency he had the good fortune to be engaged as an amanuenſis by the celebrated father Malebranche; by which he found himſelf tranſported all at once from the mazes of ſcholaſtic darkneſs, to the ſource of the moſt brilliant and enlightened philoſophy. Under this great maſter he ſtudied mathematics and the moſt ſublime metaphyſics. After ſeven years ſpent in this excellent ſchool, M. Carré found it neceſſary, in order to procure himſelf ſome leſs precarious eſtabliſhment, to teach mathematics and philoſophy in Paris; but eſpecially that philoſophy which, on account of its tendency to improve our morals, he valued more than all the mathematics in the world. And accordingly his greateſt care was to make geometry ſerve as an introduction to his well-beloved metaphyſics.

Moſt of M. Carré's pupils were of the fair ſex. The firſt of theſe, who ſoon perceived that his language was rather the reverſe of elegant and correct, told him pleaſantly that, as an acknowledgement for the pains he took to teach her philoſophy, ſhe would teach him French; and he ever after owned that her leſſons were of great ſervice to him. In general he ſeemed to ſet more value upon the genius of women than that of men.

M. Carré, although he gave the preference to metaphyſics, did not neglect mathematics; and while he taught both, he took care to make himſelf acquainted with all the new diſcoveries in the latter. This was all that his conſtant attendance on his pupils would allow him to do, till the year 1697, when M. Varignon, ſo remarkable for his extreme ſcrupulouſneſs in the choice of his eleves, took M. Carré to him in that ſtation. Soon after, viz. in the year 1700, our author thinking himſelf bound to do ſomething that might render him worthy of that title, publiſhed the firſt complete work on the Integral Calculus, under the title of "A method of meaſuring Surfaces and Solids, and finding their Centres of Gravity, Percuſſion, and Oſcillation." He afterwards diſcovered ſome errors in the work, and was candid enough to own and correct them in a ſubſequent edition.

In a little time M. Carré became Aſſociate, and at length one of the Penſioners of the Academy. And as this was a ſufficient eſtabliſhment for one, who knew ſo well how to keep his deſires within juſt bounds, he gave himſelf up entirely to ſtudy; and as he enjoyed the appointment of Mechanician, he applied himſelf more particularly to mechanics. He took alſo a ſurvey of every branch relating to muſic; ſuch as the doctrine of ſounds, the deſcription of muſical inſtruments; though he deſpiſed the practice of muſic, as a mere ſenſual pleaſure. Some ſketches of his ingenuity and induſtry in this way may be ſeen in the Memoirs of the French Academy of Sciences. M. Carré alſo compoſed ſome treatiſes on other branches of natural philoſophy, and ſome on mathematical ſubjects; all which he bequeathed to that illuſtrious body; though it does not appear that any of them have yet been publiſhed. It is not unlikely that he was hindered from putting the laſt hand to them by a train of diſorders proceeding from a bad digeſtion, which, after haraſſing him during the ſpace of five or ſix years, at length brought him to the grave in 1711, at 48 years of age.

His memoirs printed in the volumes of the Academy, with the years of the volumes, are as below.

1. The Rectification of Curve Lines by Tangents: 1701.
2. Solution of a problem propoſed to Geometricians, &c. 1701.
3. Reflections on the Table of Equations: 1701.
4. On the Cauſe of the Refraction of Light: 1702.
5. Why the Tides are always augmenting from Breſt to St. Malo, and diminiſhing along the coaſts of Normandy: 1702.
6. The Number and the Names of Muſical Inſtruments: 1702.

7. On the Vinegar which cauſes ſmall ſtones to roll upon an inclined plane : 1703.

8. On the Rectification &c. of the Cauſtics by reflection : 1703.

9. Method for the Rectification of Curves : 1704.

10. Obſervations on the Production of Sound : 1704.

11. On a Curve formed from a Circle : 1705.

12. On the Refraction of Muſket-balls in water, and on the Reſiſtance of that fluid : 1705.

13. Experiments on Capillary Tubes : 1705.

14. On the Proportion of Pipes to have a determinate quantity of water : 1705.

15. On the Laws of Motion : 1706.

16. On the Properties of Pendulums ; with ſome new properties of the Parabola : 1707.

17. On the Proportion of Cylinders that their ſounds may form the muſical chords : 1709.

18. On the Elaſticity of the Air : 1710.

19. On Catoptrics : 1710.

20. On the Monochord : in the Machines, tom. 1. with ſome other pieces, not mathematical.

CARRIAGE, *of a Cannon*, is the machine upon which it is mounted ; ſerving to point or direct it for ſhooting, and to convey it from place to place.

Wheel CARRIAGE, one that is mounted and moved about upon wheels. Horſes draw in general, to moſt advantage, when the direction of their draft is parallel to the ground, or rather a little upwards. A carriage alſo goes eaſieſt when the centre of gravity is placed very high ; ſince then, when once put in motion, it continues it with very little labour to the horſes.

CARTES (RENE DES), one of the moſt eminent philoſophers and mathematicians of the 17th century, or indeed of any age whatever. He was deſcended of an ancient noble family in Touraine in France, being a younger ſon of a counſellor in the parliament of Rennes, and was born March 31, 1596. His father gave him a liberal education, and the more ſo as he obſerved in him the appearance of a promiſing genius, uſing to call him the philoſopher, on account of his inſatiable curioſity in aſking the reaſons of every thing that he did not underſtand.

Des Cartes was ſent to the Jeſuits college at La Fleche in 1604, and put under the tuition of Father Charlet. Here he made a great progreſs in the learned languages and polite literature ; but having paſſed through his courſe of philoſophy without any great ſatisfaction to himſelf, he left the college in 1612, and began to learn military arts, to ride and fence, and other ſuch like exerciſes. But notwithſtanding his inclination to military achievements, the weakneſs of his conſtitution not permitting him early to expoſe himſelf to the fatigues of war, he was ſent to Paris in 1613. Here he formed an acquaintance with ſeveral learned perſons, who helped to reclaim him from his intention of declining his ſtudies, particularly Father Merſenne, whoſe converſation revived in him a love for truth, and induced him to retire from the world to purſue his ſtudies without interruption ; which he did for two years : but in May 1616, at the repeated ſolicitations of his relations, he ſet out for Holland, and entered as a volunteer under the Prince of Orange.

Whilſt he lay in garriſon at Breda, during the truce between the Spaniſh and Dutch, an unknown perſon cauſed a problem in mathematics, in the Dutch language, to be fixed up in the ſtreets : when Des Cartes, ſeeing a concourſe of people ſtop to read it, deſired one who ſtood near him to explain it to him in Latin or French. The perſon promiſed to ſatisfy him, upon condition that he would engage to reſolve the problem ; and Des Cartes agreed to the condition with ſuch an air, that though he little expected ſuch a thing from a young military cadet, he gave him his addreſs, deſiring he would bring him the ſolution. Des Cartes next day viſited Beekman, principal of the college of Dort, who was the perſon that had tranſlated the problem to him. Beekman was ſurpriſed at his having reſolved it in ſuch a ſhort time ; but his wonder was much increaſed, to find, in the courſe of converſation, that the young man's knowledge was much ſuperior to his own in thoſe ſciences, in which he had employed his whole time for ſeveral years. During his ſtay at Breda, Des Cartes wrote in Latin, a treatiſe on Muſic, and laid the foundation of ſeveral of his other works.

In 1619, he entered himſelf a volunteer in the army of the duke of Bavaria. In 1621, he made the campaign in Hungary, under the count de Bucquoy ; but the loſs of his general, who was killed at a ſiege that year, determined him to quit the army. He ſoon after began his travels into the north, and viſited Sileſia, Poland, Pomerania, the coaſts of the Baltic, Brandenburgh, Holſtein, Eaſt Frieſland, Weſt Frieſland ; in his paſſage to which laſt place he was in danger of being murdered. The ſailors fancied he was a merchant, who had a large ſum of money about him ; and perceiving that he was a foreigner who had little acquaintance in the country, and a man of a mild diſpoſition, they reſolved to kill him, and throw his body into the ſea. They even diſcourſed of their deſign before his face, thinking he underſtood no language but French, in which he always ſpoke to his ſervant. Des Cartes ſuddenly ſtarted up ; and drawing his ſword, ſpoke to them in their own language, in ſuch a tone as ſtruck terror into them : upon which they behaved very civilly. The year following he went to Paris, where he cleared himſelf from the imputation of having been received among the Roſicruſians, whom he conſidered as a company of viſionaries and impoſtors.

Dropping the ſtudy of mathematics, he now applied himſelf again to ethics and natural philoſophy. The ſame year he took a journey through Switzerland to Italy. Upon his return he ſettled at Paris ; but his ſtudies being interrupted by frequent viſits, he went in 1628 to the ſiege of Rochelle. He returned to Paris in November ; but in the following ſpring he repaired to Amſterdam ; and from thence to a place near Franeker in Frieſland, where he began his Metaphyſical Meditations, and ſpent ſome time in Dioptrics ; about this time too he wrote his thoughts upon Meteors. After about ſix months he returned to Amſterdam.

Des Cartes imagined that nothing could more promote the temporal felicity of mankind, than the union of natural philoſophy with mathematics. But before he ſhould ſet himſelf to relieve men's labours, or multiply the conveniencies of life by mechanics, he thought it neceſſary to diſcover ſome means of ſecuring the human body from diſeaſe and debility: this led him to the ſtudy of anatomy and chemiſtry, in which he employed the winter at Amſterdam.

He now, viz, about 1630 or 1631, took a ſhort tour to England, and made ſome obſervations near London on the variation of the compaſs. In the ſpring of 1633 he removed to Deventer, where he completed ſeveral works that were left unfiniſhed the year before, and reſumed his ſtudies in aſtronomy. In the ſummer he put the laſt hand to his "Treatiſe of the World." The next year he returned to Amſterdam; but ſoon after took a journey into Denmark, and the lower parts of Germany. In autumn 1635 he went to Lewarden in Frieſland, where he remained till 1637, and wrote his "Treatiſe of Mechanics." The ſame year he publiſhed his four treatiſes concerning Method, Dioptrics, Meteors, and Geometry. About this time he received an invitation to ſettle in England from Sir Charles Cavendiſh, brother to the earl of Newcaſtle, with which he did not appear backward to comply, eſpecially upon being aſſured that the king was a catholic in his heart: but the breaking out of the civil wars in this country prevented his journey.

At the end of 1641, Lewis the 13th, of France, invited him to his court, upon very honourable terms; but he could not be perſuaded to quit his retirement. This year he publiſhed his Meditations concerning the Exiſtence of God, and the Immortality of the Soul. In 1645 he again applied to anatomy; but was a little diverted from this ſtudy, by the queſtion concerning the Quadrature of the Circle, which was at that time agitated. During the winter of the ſame year he compoſed a ſmall tract againſt Gaſſendus's Inſtitutes; and another on the Nature of the Paſſions. About this time he carried on an epiſtolary correſpondence with the princeſs Elizabeth, daughter to Frederick the 5th, elector palatine, and king of Bohemia, who had been his pupil in Holland.

A diſpute ariſing between Chriſtina, queen of Sweden, and M. Chanut, the reſident of France, concerning the following queſtion; When a man carries love or hatred to exceſs, which of theſe two irregularities is the worſt? The reſident ſent the queſtion to Des Cartes, who upon that occaſion drew up the diſſertation upon Love, that is publiſhed in the firſt volume of his letters, which proved highly ſatisfactory to the queen. In June 1647 he took a journey to France, where the king ſettled on him a penſion of 3000 livres; but he returned to Holland about the end of September. In November he received a letter from M. Chanut, in queen Chriſtina's name, deſiring his opinion of the ſovereign good; which he accordingly ſent her, with ſome letters upon the ſame ſubject formerly written to the princeſs Elizabeth, and his treatiſe on the Paſſions. The queen was ſo highly pleaſed with them, that ſhe wrote him a letter of thanks with her own hand, and invited him to come to Sweden. He arrived at Stockholm in Oct. 1648. The queen engaged him to attend her every morning at five o'clock, to inſtruct her in his philoſophy; and deſired him to reviſe and digeſt all his unpubliſhed writings, and to draw up from them a complete body of philoſophy. She purpoſed alſo to fix him in Sweden, by allowing him a revenue of 3000 crowns a year, with an eſtate which ſhould deſcend to his heirs and aſſigns for ever; and to eſtabliſh an academy, of which he was to be the director. But theſe deſigns were fruſtrated by his death, which happened the 11th of Feb. 1650, in the 54th year of his age. His body was interred at Stockholm: but 17 years after it was removed to Paris, where a magnificent monument was erected to him in the church of Genevieve du Mont.

As to the character of our author:

Dr. Barrow in his *Opuſcula* tells us, that Des Cartes was doubtleſs a very ingenious man, and a real philoſopher, and one who ſeems to have brought thoſe aſſiſtances to that part of philoſophy relating to matter and motion, which perhaps no one had done before: namely, a great ſkill in mathematics; a mind habituated, both by nature and cuſtom, to profound meditation; a judgment exempt from all prejudices and popular errors, and furniſhed with a good number of certain and ſelect experiments; a great deal of leiſure; an entire diſengagement, by his own choice, from the reading of uſeleſs books, and the avocations of life; with an incomparable acuteneſs of wit, and an excellent talent of thinking clearly and diſtinctly, and of expreſſing his thoughts with the utmoſt perſpicuity.

Dr. Halley, in a paper concerning Optics, communicated to Mr. Wotton, and publiſhed by the latter in his "Reflections upon Ancient and Modern Learning," writes as follows: As to dioptrics, though ſome of the ancients mention refraction, as a natural effect of tranſparent media; yet Des Cartes was the firſt, who in this age has diſcovered the laws of refraction, and brought dioptrics to a ſcience.

Dr. Keil, in the introduction to his "Examination of Dr. Burnet's Theory of the Earth," tells us, that Des Cartes was ſo far from applying geometry and obſervations to natural philoſophy, that his whole ſyſtem is but one continued blunder on account of his negligence in that point; which he could eaſily prove, by ſhewing that his theory of the vortices, upon which his ſyſtem is founded, is abſolutely falſe, for that Newton has ſhewn that the periodical times of all bodies, that ſwim in vortices, muſt be directly as the ſquares of their diſtances from the centre of them: but it is evident from obſervations, that the planets, in moving round the ſun, obſerve a law quite different from this; for the ſquares of their periodical times are always as the cubes of their diſtances: and therefore, ſince they do not obſerve that law, which of neceſſity they muſt if they ſwim in a vortex, it is a demonſtration that there are no vortices in which the planets are carried round the ſun.

"Nature, ſays Voltaire, had favoured Des Cartes with a ſtrong and clear imagination, whence he became a very ſingular perſon, both in private life, and in his manner of reaſoning. This imagination could not be concealed even in his philoſophical writings, which are every where adorned with very brilliant ingenious metaphors. Nature had almoſt made him a poet; and indeed he wrote a piece of poetry for the entertainment of Chriſtina queen of Sweden, which however was ſuppreſſed in honour of his memory. He extended the limits of geometry as far beyond the place where he found them, as Newton did after him; and firſt taught the method of expreſſing curves by equations. He applied this geometrical and inventive genius to dioptrics, which when treated by him became a new art; and if he was miſtaken in ſome things, the reaſon is, that a man who diſcovers a new tract of land, cannot at once know all the properties of the ſoil. Thoſe who come after him, and make theſe lands fruitful, are at leaſt obliged to him for the diſcovery." Voltaire acknowledges, that there are innumerable errors in the reſt of Des Cartes' works; but adds, that geometry was a guide which he himſelf had in ſome meaſure formed, and which would have ſafely conducted him through the ſeveral paths of natural philoſophy: nevertheleſs he had at laſt abandoned this guide, and gave entirely into the humour of framing hypotheſes; and then philoſophy was no more than an ingenious romance, fit only to amuſe the ignorant.

It has been pretty generally acknowledged, that he borrowed his improvements in Algebra from Harriot's *Artis Analyticæ Praxis*; which is highly probable, as he was in England about the time when Harriot's book was publiſhed, and as he follows the manner of Harriot, except in the method of noting the powers. Upon this head the following anecdote is told by Dr. Pell, in Wallis's Algebra, pa. 198. Sir Charles Cavendiſh, then reſident at Paris, diſcourſing there with M. Roberval, concerning Des Cartes's Geometry, then lately publiſhed: I admire, ſaid Roberval, that method in Des Cartes, of placing all the terms of the equation on one ſide, making the whole equal to nothing, and how he lighted upon it. The reaſon why you admire it, ſaid Sir Charles, is becauſe you are a Frenchman; for if you were an Engliſhman, you would not admire it. Why ſo? aſked Roberval. Becauſe, replied Sir Charles, we in England know whence he had it; namely from Harriot's Algebra. What book is that? ſays Roberval, I never ſaw it. Next time you come to my chamber, ſaith Sir Charles, I will ſhew it to you. Which a while after he did; and upon peruſal of it, Roberval exclaimed with admiration, *Il l'a vu! il l'a vu! He had ſeen it! he had ſeen it!* finding all that in Harriot which he had before admired in Des Cartes, and not doubting but that Des Cartes had it from thence. See alſo Montucla's Hiſtory of Mathematics.

The real improvements of Des Cartes in Algebra and Geometry, I have particularly treated of under the article Algebra; and his philoſophical doctrines are diſplayed in the article Cartesian *Philoſophy*, here following. He was never married, but had one natural daughter, who died when ſhe was but five years old. There have been ſeveral editions of his works, and commentaries upon them; particularly thoſe of Schooten on his Geometry.

CARTESIAN *Philoſophy*, or *Carteſianiſm*, the ſyſtem of philoſophy advanced by Des Cartes, and maintained by his followers, the Carteſians.

The Carteſian philoſophy is founded on two great principles, the one metaphyſical, the other phyſical. The metaphyſical one is this: *I think*, therefore *I am*, or *I exiſt*: the phyſical principle is, that *nothing exiſts but ſubſtances*. Subſtance he makes of two kinds; the one a ſubſtance that thinks, the other a ſubſtance extended: ſo that actual thought and actual extenſion make the eſſence of ſubſtance.

The eſſence of matter being thus fixed in extenſion, Des Cartes concludes that there is no vacuum, nor any poſſibility of it in nature; but that the univerſe is abſolutely full: by this principle, mere ſpace is quite excluded; for extenſion being implied in the idea of ſpace, matter is ſo too.

Des Cartes defines motion to be the tranſlation of a body from the neighbourhood of others that are in contact with it, and conſidered as at reſt, to the neighbourhood of other bodies: by which he deſtroys the diſtinction between motion that is abſolute or real, and that which is relative or apparent. He maintains that the ſame quantity of motion is always preſerved in the univerſe, becauſe God muſt be ſuppoſed to act in the moſt conſtant and immutable manner. And hence alſo he deduces his three laws of motion. See Motion.

Upon theſe principles Des Cartes explains mechanically how the world was formed, and how the preſent phenomena of nature came to ariſe. He ſuppoſes that God created matter of an indefinite extenſion, which he ſeparated into ſmall ſquare portions or maſſes, full of angles: that he impreſſed too motions on this matter; the one, by which each part revolved about its own centre; and another, by which an aſſemblage, or ſyſtem of them, turned round a common centre. From whence aroſe as many different vortices, or eddies, as there were different maſſes of matter, thus moving about common centres.

The conſequence of theſe motions in each vortex, according to Des Cartes, is as follows: The parts of matter could not thus move and revolve amongſt one another, without having their angles gradually broken; and this continual friction of parts and angles muſt produce three elements: the firſt of theſe, an infinitely fine duſt, formed of the angles broken off; the ſecond, the ſpheres remaining, after all the angular parts are thus removed; and thoſe particles not yet rendered ſmooth and ſpherical, but ſtill retaining ſome of their angles, and hamous parts, from the third element.

Now the firſt or ſubtileſt element, according to the laws of motion, muſt occupy the centre of each ſyſtem, or vortex, by reaſon of the ſmallneſs of its parts: and this is the matter which conſtitutes the ſun, and the fixed ſtars above, and the fire below. The ſecond element, made up of ſpheres, forms the atmoſphere, and all the matter between the earth and the fixed ſtars; in ſuch ſort, that the largeſt ſpheres are always next the circumference of the vortex, and the ſmalleſt next its centre. The third element, formed of the irregular particles, is the matter that compoſes the earth, and all terreſtrial bodies, together with comets, ſpots in the ſun, &c.

He accounts for the gravity of terreſtrial bodies from the centrifugal force of the ether revolving round the earth: and upon the ſame general principles he pretends to explain the phenomena of the magnet, and to account for all the other operations in nature.

CARY (Robert), a learned Engliſh chronologer and divine, was born at Cockington, in the county of Devon, about the year 1615. He took his degrees in arts, and LL.D. in Oxford. After returning from his travels he was preſented to the rectory of Portlemouth, near Kingſbridge in Devonſhire: but not long after he was drawn over by the preſbyterian miniſters to their party, and choſen moderator of that part of the ſecond diviſion of the county of Devon, which was appointed

pointed to meet at Kingſbridge. And yet, upon the reſtoration of Charles the 2d, he was one of the firſt to congratulate that prince upon his return, and ſoon after was preferred to the archdeaconry of Exeter; but from which he was however ſome time afterward ejected. He ſpent the reſt of his days at his rectory at Portlemouth, and died in 1688, at 73 years of age.—He publiſhed *Palælogia Chronica*, a chronological account of ancient time, in three parts. 1, Didactical; 2, Apodidactical; 3, Canonical: in 1677.

CASATI (Paul), a learned Jeſuit, born at Placentia in 1617. He entered early among the Jeſuits; and after having taught mathematics and divinity at Rome, he was ſent into Sweden to queen Chriſtina, whom he prevailed on to embrace the popiſh religion. His writings are as follow:

1. *Vacuum Proſcriptum.*—2. *Terra Machinis mota.*—3. *Mechanicorum, libri octo.*—4. *De Igne Diſſertationes.*—5. *De Angelis Diſputatio Theolog.*—6. *Hydroſtaticæ Diſſertationes.*—7. *Opticæ Diſputationes.* It is remarkable that he wrote this treatſe on optics at 88 years of age, and after he was blind. He was alſo author of ſeveral books in the Italian language.

CASCABEL, the knob or button of metal behind the breech of a cannon, as a kind of handle by which to elevate and direct the piece; to which ſome add the fillet and ogees as far as the baſe-ring.

CASEMATE, or Cazemate, in Fortification, a kind of vault or arch, of ſtone-work, in that part of the flank of a baſtion next the curtain; ſerving as a battery, to defend the face of the oppoſite baſtion, and the moat or ditch.

The caſemate ſometimes conſiſts of three platforms, one above another; the higheſt being on the rampart; though it is common to withdraw this within the baſtion.

The caſemate is alſo called the low place, and low flank, as being at the bottom of the wall next the ditch; and ſometimes the retired flank, as being the part of the flank neareſt the curtain, and the centre of the baſtion. It was formerly covered by an apaulement, or a maſſive body, either round or ſquare, which prevented the enemy from ſeeing within the batteries; whence it was alſo called *covered flank*.

It is now ſeldom uſed, becauſe the batteries of the enemy are apt to bury the artillery of the caſemate in the ruins of the vault: beſide, the great ſmoke made by the diſcharge of the cannon, renders it intolerable to the men. So that, inſtead of the ancient covered caſemates, later engineers have contrived open ones, only guarded by a parapet, &c.

Casemate is alſo uſed for a well with ſeveral ſubterraneous branches, dug in the paſſage of the baſtion, till the miner is heard at work, and air given to the mine.

CASERNS, or Cazerns, in Fortification, ſmall rooms, or huts, erected between the ramparts and the houſes of fortified towns, or even on the ramparts themſelves; to ſerve as lodgings for the ſoldiers on immediate duty, to eaſe the garriſon.

CASE-Shot, or Cannister-Shot, are a number of ſmall balls put into a round tin canniſter, and ſo ſhot out of great guns. Theſe have ſuperſeded, and been ſubſtituted inſtead of the grape-ſhot, which have been laid aſide.

CASSINI (John Dominic), an eminent aſtronomer, was born of noble parents, at a town in Piedmont in Italy, June 8, 1625. After laying a proper foundation in his ſtudies at home, he was ſent to continue them in a college of Jeſuits at Genoa. He had an uncommon turn for Latin poetry, which he exerciſed ſo very early, that ſome of his poems were publiſhed when he was but 11 years old. At length he met with books of aſtronomy, which he read, with great eagerneſs. Purſuing the bent of his inclinations in this way, in a ſhort time he made ſo amazing a progreſs, that in 1650 the ſenate of Bologna invited him to be their public mathematical profeſſor. Caſſini was but 25 years of age when he went to Bologna, where he taught mathematics, and made obſervations upon the heavens, with great care and aſſiduity. In 1652 a comet appeared, which he obſerved with great accuracy; and he diſcovered that comets were not bodies accidentally generated in the atmoſphere, as had been ſuppoſed, but of the ſame nature, and probably governed by the ſame laws, as the planets. The ſame year he reſolved an aſtronomical problem, which Kepler and Bulliald had given up as inſolvable; viz, to determine geometrically the apogee and eccentricity of a planet, from its true and mean place.—In 1653, when a church in Bologna was repaired and enlarged, he obtained leave of the ſenate to correct and ſettle a meridian line, which had been drawn by an aſtronomer in 1575.—In 1657 he attended, as an aſſiſtant, a nobleman, who was ſent to Rome to compoſe ſome differences, which had ariſen between Bologna and Ferrara, from the inundations of the Po; and he ſhewed ſo much ſkill and judgment in the management of the affair, that in 1663 the pope's brother appointed him inſpector general of the fortifications of the caſtle of Urbino: and he had afterward committed to him the care of all the rivers in the eccleſiaſtical ſtate.

Mean while he did not neglect his aſtronomical ſtudies, but cultivated them with great care. He made ſeveral diſcoveries relating to the planets Mars and Venus, particularly the revolution of Mars upon his own axis: but the point he had chiefly in view, was to ſettle an accurate theory of Jupiter's ſatellites; which, after much labour and obſervation, he happily effected, and publiſhed it at Rome, among other aſtronomical pieces, in 1666.

Picard, the French aſtronomer, getting Caſſini's tables of Jupiter's ſatellites, found them ſo very exact, that he conceived the higheſt opinion of his ſkill; and from that time his fame increaſed ſo faſt in France, that the government deſired to have him a member of the academy. Caſſini however could not leave his ſtation without leave of his ſuperiors; and therefore the king, Lewis the 14th, requeſted of the pope and the ſenate of Bologna, that Caſſini might be permitted to come into France. Leave was granted for 6 years; and he came to Paris in the beginning of 1669, where he was immediately made the king's aſtronomer. When this term of 6 years was near expiring, the pope and the ſenate of Bologna inſiſted upon his return, on pain of forfeiting his revenues and emoluments, which had hitherto been remitted to him: but the miniſter Colbert prevailed on him to ſtay, and he was naturalized in 1673; the ſame year alſo in which he was married.

The Royal Obſervatory of Paris had been finiſhed ſome time. The occaſion of its being built was this: In 1638, the celebrated Merſenne was the chief inſtitutor and promoter of a ſociety, where ſeveral ingenious and learned men met together to talk upon phyſical and aſtronomical ſubjects; among whom were Gaſſend, Deſcartes, Monmort, Thevenot, Bulliald, our countryman Hobbes, &c: and this ſociety was kept up by a ſucceſſion of learned men for many years. At length the government conſidering that a number of ſuch men, acting in a body, would ſucceed much better in the promotion of ſcience, than if they acted ſeparately, each in his particular art or province, eſtabliſhed under the direction of Colbert, in 1666, the Royal Academy of Sciences: and for the advancement of aſtronomy in particular, erected the Royal Obſervatory at Paris, and furniſhed it with all kinds of inſtruments that were neceſſary to make obſervations. The foundation of this noble pile was laid in 1667, and the building completed in 1670. Of this obſervatory, Caſſini was appointed to be the firſt inhabiter; which he took poſſeſſion of in Sept. 1671, when he ſet himſelf with freſh alacrity to attend the duties of his profeſſion. In 1672 he endeavoured to determine the parallax of Mars and the ſun: and in 1677 he proved that the diurnal rotation of Jupiter round his axis was performed in 9 hours 58 minutes, from the motion of a ſpot in one of his larger belts: alſo in 1684 he diſcovered four ſatellites of Saturn, beſides that which Huygens had found out. In 1693 he publiſhed a new edition of his "Tables of Jupiter's Satellites," corrected by later obſervations. In 1695 he took a journey to Bologna, to examine the meridian line, which he had fixed there in 1655; and he ſhewed, in the preſence of eminent mathematicians, that it had not varied in the leaſt, during that 40 years. In 1700 he continued the meridian line through France, which Picard had begun, to the very ſouthern limits of that country.

After our author had reſided at the royal obſervatory for more than 40 years, making many excellent and uſeful diſcoveries, which he publiſhed from time to time, he died September the 14th, 1712, at 87 years of age; and was ſucceeded by his only ſon James Caſſini. His publications were very numerous, far too much ſo, even to be enumerated in this place.

CASSINI (James), a celebrated French aſtronomer, and member of the ſeveral Academies of Sciences of France, England, Pruſſia, and Bologna, was born at Paris Feb. 18, 1677, being the younger ſon of John-Dominic Caſſini, above mentioned, whom he ſucceeded as aſtronomer at the royal obſervatory, the elder ſon having loſt his life at the battle of La Hogue.

After his firſt ſtudies in his father's houſe, in which it is not to be ſuppoſed that mathematics and aſtronomy were neglected, he was ſent to ſtudy philoſophy at the Mazarine college, where the celebrated Varignon was then profeſſor of mathematics; from whoſe aſſiſtance young Caſſini profited ſo well, that at 15 years of age he ſupported a mathematical theſis with great honour. At the age of 17 he was admitted a member of the Academy of Sciences; and the ſame year he accompanied his father in his journey to Italy, where he aſſiſted him in the verification of the meridian at Bologna, and other meaſurements. On his return he made other ſimilar operations in a journey into Holland, where he diſcovered ſome errors in the meaſure of the earth by Snell, the reſult of which was communicated to the Academy in 1702. He made alſo a viſit to England in 1696, where he was made a member of the Royal Society.—In 1712 he ſucceeded his father as aſtronomer royal at the obſervatory.—In 1717 he gave to the Academy his reſearches on the diſtance of the fixed ſtars, in which he ſhewed that the whole annual orbit, of near 200 million of miles diameter, is but as a point in compariſon of that diſtance. The ſame year he communicated alſo his diſcoveries concerning the inclination of the orbits of the ſatellites in general, and eſpecially of thoſe of Saturn's ſatellites and ring.—In 1725 he undertook to determine the cauſe of the moon's libration, by which ſhe ſhews ſometimes a little towards one ſide, and ſometimes a little on the other, of that half which is commonly behind or hid from our view.

In 1732 an important queſtion in aſtronomy exerciſed the ingenuity of our author. His father had determined, by his obſervations, that the planet Venus revolved about her axis in the ſpace of 23 hours: and M. Bianchini had publiſhed a work in 1729, in which he ſettled the period of the ſame revolution at 24 days 8 hours. From an examination of Bianchini's obſervations, which were upon the ſpots in Venus, he diſcovered that he had intermitted his obſervations for the ſpace of 3 hours, from which cauſe he had probably miſtaken new ſpots for the old ones, and ſo had been led into the miſtake. He ſoon afterwards determined the nature and quantity of the acceleration of the motion of Jupiter, at half a ſecond per year, and of that of the retardation of Saturn at two minutes per year; that theſe quantities would go on increaſing for 2000 years, and then would decreaſe again.—In 1740 he publiſhed his Aſtronomical Tables; and his Elements of Aſtronomy; very extenſive and accurate works.

Although aſtronomy was the principal object of our author's conſideration, he did not confine himſelf abſolutely to that branch, but made occaſional excurſions into other fields. We owe alſo to him, for example, Experiments on Electricity, or the light produced by bodies by friction. Experiments on the recoil of fire arms; Reſearches on the riſe of the mercury in the barometer at different heights above the level of the ſea; Reflections on the perfecting of burning-glaſſes; and other memoirs.

The French Academy had properly judged that one of its moſt important objects, was the meaſurement of the earth. In 1669 Picard meaſured a little more than a degree of latitude to the north of Paris; but as that extent appeared too ſmall from which to conclude the whole circumference with ſufficient accuracy, it was reſolved to continue that meaſurement on the meridian of Paris to the north and the ſouth, through the whole extent of the country. Accordingly, in 1683, the late M. de la Hire continued that on the north ſide of Paris, and the older Caſſini that on the ſouth ſide. The latter was aſſiſted in 1700 in the continuation of this operation by his ſon our author. The ſame work was farther continued by the ſame Academicians; and finally the part left unfiniſhed by de la Hire in the north, was finiſhed in 1718 by our author, with the late Maraldi, and de la Hire the younger.

Theſe operations produced a conſiderable degree of preciſion. It appeared alſo, from this meaſured extent of 6 degrees, that the degrees were of different lengths in different parts of the meridian; and in ſuch ſort that our author concluded, in the volume publiſhed for 1718, that they decreaſed more and more towards the pole, and that therefore the figure of the earth was that of an oblong ſpheroid, or having its axe longer than the equatorial diameter. He alſo meaſured the perpendicular to the ſame meridian, and compared the meaſured diſtance with the differences of longitude as before determined by the eclipſes of Jupiter's ſatellites; from whence he concluded that the length of the degrees of longitude was ſmaller than it would be on a ſphere, and that therefore again the figure of the earth was an oblong ſpheroid; contrary to the determination of Newton by the theory of gravity. In conſequence of theſe aſſertions of our author, the French government ſent two different ſets of meaſurers, the one to meaſure a degree at the equator, the other at the polar circle; and the compariſon of the whole determined the figure to be an oblate ſpheroid, contrary to Caſſini's determination.

After a long and laborious life, our author James Caſſini loſt his life by a fall in April 1756, in the 80th year of his age, and was ſucceeded in the Academy and Obſervatory by his ſecond ſon Ceſar-François de Thury. He publiſhed, A Treatiſe on the Magnitude and Figure of the Earth; as alſo The Elements or Theory of the Planets, with Tables; beſide an infinite number of papers in the Memoirs of the Academy, from the year 1699 to 1755.

CASSINI DE THURY (CESAR-FRANCOIS), a celebrated French aſtronomer, director of the obſervatory, penſioner aſtronomer, and member of moſt of the learned ſocieties of Europe, was born at Paris June 17, 1714, being the ſecond ſon of James Caſſini, whoſe occupations and talents our author inherited and ſupported, with great honour. He received his firſt leſſons in aſtronomy and mathematics from MM. Maraldi and Camus. He was hardly 10 years of age when he calculated the phaſes of the total eclipſe of the ſun of 1727. At the age of 18 he accompanied his father in his two journies undertaken for drawing the perpendicular to the obſervatory meridian from Straſbourg to Breſt. From that time a general chart of France was deviſed; for which purpoſe it was neceſſary to traverſe the country by ſeveral lines parallel and perpendicular to the meridian of Paris, and our author was charged with the conduct of this buſineſs. He did not content himſelf with the meaſure of a degree by Picard: ſuſpecting even that the meaſures which had been taken by his father and grandfather were not exempt from ſome errors, which the imperfections of their inſtruments at leaſt would be liable to, he again undertook to meaſure the meridian of Paris, by means of a new ſeries of triangles, of a ſmaller number, and more advantageouſly diſpoſed. This great work was publiſhed in 1740, with a chart ſhewing the new meridian of Paris, by two different ſeries of triangles, paſſing along the ſea coaſts, to Bayonne, traverſing the frontiers of Spain to the Mediterranean and Antibes, and thence along the eaſtern limits of France to Dunkirk, with parallel and perpendicular lines deſcribed at the diſtance of 6000 toiſes from one another, from ſide to ſide of the country.—In 1735, he had been received into the academy as adjoint ſupernumerary at 21 years of age.

A tour which our author made in Flanders, in company with the king, about 1741, gave riſe to the particular chart of France, at the inſtance of the king. Caſſini publiſhed different works relative to theſe charts, and a great number of the ſheets of the charts themſelves.

In 1761, Caſſini undertook an expedition into Germany; for the purpoſe of continuing to Vienna the perpendicular of the Paris meridian; to unite the triangles of the chart of France with the points taken in Germany; to prepare the means of extending into this country the ſame plan as in France; and thus to eſtabliſh ſucceſſively for all Europe a moſt uſeful uniformity. Our author was at Vienna the 6th of June 1761, the day of the tranſit of the planet Venus over the ſun, of which he obſerved as much as the ſtate of the weather would permit him to do, and publiſhed the account of it in his *Voyage en Allemagne*.

Finally, M. Caſſini, always meditating the perfection of his grand deſign, profited of the late peace to propoſe the joining of certain points taken upon the Engliſh coaſt with thoſe which had been determined on the coaſt of France, and thus to connect the general chart of the latter with that of the Britiſh iſles, like as he had before united it with thoſe of Flanders and Germany. The propoſal was favourably received by the Engliſh government, and preſently carried into effect, under the direction of the Royal Society, the execution being committed to the late General Roy; after whoſe death the buſineſs was for ſome time ſuſpended; but it has lately been revived under the auſpices of the duke of Richmond, Maſter General of the Ordnance, and the execution committed to the care of Col. Edward Williams and Capt. William Mudge, both reſpectable officers of the Artillery, and Mr. Iſaac Dalby, who had before accompanied and aſſiſted General Roy; from whoſe united ſkill and zeal the happieſt proſecution of this buſineſs may be expected.

M. Caſſini publiſhed in the volumes of Memoirs of the French Academy a prodigious number of pieces, chiefly aſtronomical, too numerous to particularize in this place, between the years 1735 and 1770; conſiſting of aſtronomical obſervations and queſtions; among which are obſervable, Reſearches concerning the Parallax of the ſun, the moon, Mars, and Venus; On aſtronomical refractions, and the effect cauſed in their quantity and laws by the weather; Numerous obſervations on the obliquity of the ecliptic, and on the law of its variations. In ſhort, he cultivated aſtronomy for 50 years, of the moſt important for that ſcience that ever elapſed, for the magnitude and variety of objects, in which he commonly ſuſtained a principal ſhare.

M. Caſſini was of a very ſtrong and vigorous conſtitution, which carried him through the many laborious operations in geography and aſtronomy which he conducted. An habitual retention of urine however rendered the laſt 12 years of his life very painful and diſtreſſing, till it was at length terminated by the ſmall-pox the 4th of September 1784, in the 71ſt year of his age; being ſucceeded in the academy, and as director of the obſervatory, by his only ſon the preſent count John-Dominic Caſſini; who is the 4th in order by direct deſcent in that honourable ſtation.

CASSIOPEIA, one of the 48 old conſtellations, placed near Cepheus, not far from the north pole. The Greeks probably received this figure, as they did the reſt, from the Egyptians, and in their fables added it to the family in the neighbouring part of the heavens, making her the wife of Cepheus, and mother of Andromeda. They pretend ſhe was placed in this ſituation, to behold the deſtruction of her favourite daughter Andromeda, who is chained juſt by her on the ſhore, to be devoured; and that as a puniſhment for her pride and vanity in preſuming to ſtand the compariſon of beauty with the Nereids.

In the year 1572 there burſt out all at once in this conſtellation a new ſtar, which at firſt ſurpaſſed Jupiter himſelf in magnitude and brightneſs; but it diminiſhed by degrees, till it quite diſappeared at the end of 18 months. This ſtar alarmed all the aſtronomers of that age, many of whom wrote diſſertations upon it; among the reſt Tycho Brahe, Kepler, Maurolycus, Lycetus, Gramineus, and others. Beza, the Landgrave of Heſſe, Roſa, and others, wrote to prove it a comet, and the ſame that appeared to the Magi at the birth of Chriſt, and that it came to declare his ſecond coming: theſe were anſwered by Tycho.

The ſtars in the conſtellation Caſſiopeia, are in Ptolomy's catalogue 13, in Hevelius's 37, in Tycho's 46, and in Flamſteed's 55.

CASTOR, a moiety of the conſtellation Gemini; called alſo Apollo. Alſo a ſtar in this conſtellation, whoſe latitude, for the year 1700, according to Hevelius, was 10° 4′ 20″ north; and its longitude ♋ 16° 4′ 14″.

CASTOR *and* POLLUX. See GEMINI.

CASTOR *and* POLLUX, in Meteorology, is a fiery meteor, which at ſea appears ſometimes adhering to a part of the ſhip, in the form of a ball, or even ſeveral balls. When one is ſeen alone, it is properly called Helena; but two are called Caſtor and Pollux, and ſometimes Tyndaridæ.

By the Spaniards, Caſtor and Pollux are called San Elmo; by the French, St. Elme, St. Nicholas, St. Clare, St. Helene; by the Italians, Hermo; and by the Dutch, Vree Vuuren.

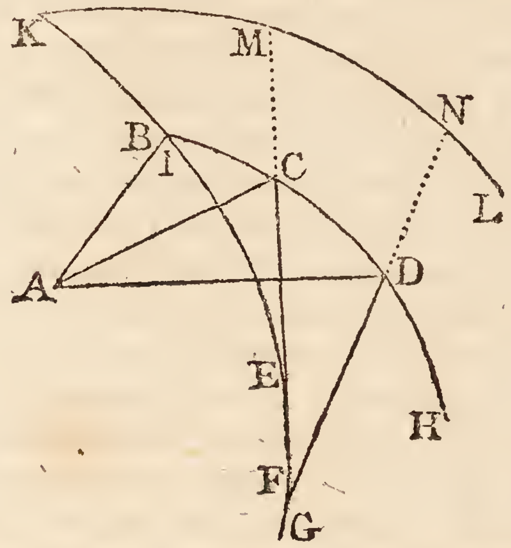
The meteor Caſtor and Pollux, it is commonly thought, denotes a ceſſation of the ſtorm, and a future calm; as it is rarely ſeen till the tempeſt is nigh ſpent. But Helena alone portends ill weather, and denotes the ſevereſt part of the ſtorm yet behind.

When the meteor adheres to the maſts, yards, &c, it is concluded, from the air not having motion enough to diſſipate this flame, that a profound calm is at hand; but if it flutter about, that it denotes a ſtorm.

CASTRAMETATION, the art, or act, of encamping an army.

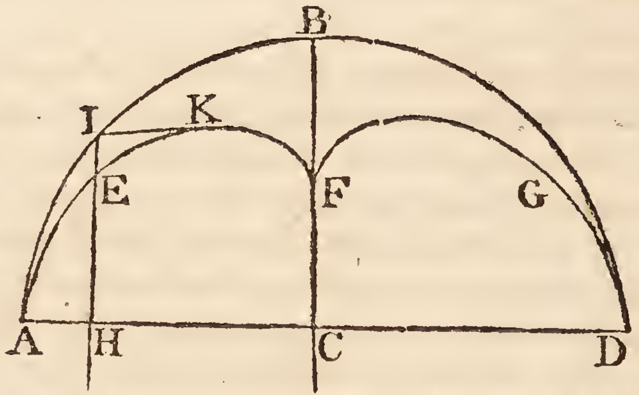
CATACAUSTICS, or *Catacauſtic Curves*, in the Higher Geometry, are the ſpecies of cauſtic curves formed by reflection.

Theſe curves are generated after the following manner: If there be an infinite number of rays A B, A C, A D, &c, proceeding from the radiating point A, and reflected at any given curve B C D H, ſo that the angles of incidence be ſtill equal to the angles of reflection; then the curve B E G, to which the reflected rays B I, C E, D F, &c, are always tangents, as at the points I, E, F, &c, is the catacauſtic, or cauſtic-by-reflection. Or it is the ſame thing as to ſay, that a cauſtic curve is that formed by joining the points of concurrence of the ſeveral reflected rays.



Some properties of theſe curves are as follow. If the reflected ray I B be produced to K, ſo that A B = B K, and the curve K L be the evolute of the cauſtic B E G, beginning at the point K; then the portion of the cauſtic B E is $= \overline{AC - AB} + \overline{CE - BI}$, that is, the difference of the two incident rays added to the difference of the two reflected rays.

When the given curve B C D is a geometrical one, the cauſtic will be ſo too, and will always be rectifiable. The cauſtic of the circle, is a cycloid, or epicycloid, formed by the revolution of a circle upon a circle.



Thus, A B D being a ſemicircle expoſed to parallel rays; then thoſe rays which fall near the axis C B will be reflected to F, the middle point of B C; and thoſe which fall at A, as they touch the curve only, will not be reflected at all; but any intermediate ray H I will be reflected to a point K, ſomewhere between A and F. And ſince every different incident ray will have a different focal point, therefore thoſe various focal points will form a curve line A E F in one quadrant, and F G D in the other, being the cycloid above-mentioned. And this figure may be beautifully exhibited experimentally by expoſing the inſide of a ſmooth bowl, or glaſs, to the ſun beams, or ſtrong candle light; for then this curve A E F G D will appear plainly delineated on any white ſurface placed horizontally within the ſame, or on the ſurface of milk contained in the bowl.

The cauſtic of the common cycloid, when the rays are parallel to its axis, is alſo a common cycloid, deſcribed by the revolution of a circle upon the ſame baſe. The cauſtic of the logarithmic ſpiral, is the ſame curve.

The principal writers on the cauſtics, are l'Hôpital, Carré, &c. See Memoires de l'Acad. an. 1666 & 1703.

CATACOUSTICS, or *Cataphonics*, is the ſcience of reflected ſounds; or that part of acouſtics which treats of the properties of echoes.

CATADIOPTRICAL *Teleſcope*, the ſame as Reflecting teleſcope; which ſee.

CATALOGUE *of the Stars*, is a liſt of the fixed ſtars, diſpoſed according to ſome order; in their ſeveral conſtellations; with the longitudes, latitudes, right-aſcenſions, &c, of each.

Catalogues of the ſtars have uſually been diſpoſed, either as collected into certain figures called conſtellations, or according to their right aſcenſions, that is the order of their paſſing over the meridian. All the catalogues, from the moſt ancient down to Flamſteed's incluſively, were of the firſt of theſe forms, or in conſtellations: but moſt of the others ſince that have been of the latter form, as being much more convenient for moſt purpoſes. Indeed one has lately been diſpoſed in claſſes according to zones or degrees of polar diſtance.

Hipparchus of Rhodes firſt undertook to make a catalogue of the ſtars, about 128 years before Chriſt; in which he made uſe of the obſervations of Timocharis and Ariſtyllus, for about 140 years before him. Ptolomy retained Hipparchus's catalogue, containing 1026 fixed ſtars in 48 conſtellations, though he himſelf made abundance of obſervations, with a view to a new catalogue, an. dom. 140. Albategni, a Syrian, brought the ſame down to his own time, viz, about the year of Chriſt 880. Anno 1437, Ulugh Beigh, or Beg, king of Parthia and India, made a new catalogue of 1022 fixed ſtars, or according to ſome 1016; ſince tranſlated out of Perſian into Latin by Dr. Hyde, in 1665. The third perſon who made a catalogue of ſtars from his own obſervations was Tycho Brahe, who determined the places of 777 ſtars for the end of the year 1600; which Kepler, from other obſervations of Tycho, afterwards increaſed to the number of 1000 in the Rudolphine tables; adding thoſe of Ptolomy and other authors, omitted by Tycho; ſo that his catalogue amounts to above 1160. About the ſame time, William, landgrave of Heſſe, with his mathematicians Byrgius and Rothman, determined the places of 400 ſtars from new obſervations, rectifying them for the year 1593; which Hevelius prefers to thoſe of Tycho. Ricciolus, in his Aſtronomia Reformata, determined the places of 101 ſtars for the year 1700, from his own obſervations: for the reſt he followed Tycho's catalogue; altering it where he thought fit. Anno 1667 Dr. Halley, in the iſland of St. Helena, obſerved 350 of the ſouthern ſtars, not viſible in our horizon. The ſame labour was repeated by Father Noel in 1710, who publiſhed a new catalogue of the ſame ſtars conſtructed for the year 1687. Alſo De la Caille, at the Cape of Good Hope, made accurate obſervations of about 10 thouſand ſtars near the ſouth pole, in the years 1751 and 1752; the catalogue of which was publiſhed in the Memoirs of the French Academy of Sciences for the year 1752, and in ſome of his own works, as more particularly noticed below.

Bayer, in his Uranometria, publiſhed in 1603 a catalogue of 1160 ſtars, compiled chiefly from Ptolomy and Tycho, in which every ſtar is marked with ſome letter of the Greek alphabet; the brighteſt or principal ſtar in any conſtellation being denoted by the firſt letter of the alphabet, the next ſtar in order by the 2d letter, and ſo on; and when the number of ſtars exceeds the Greek alphabet, the remaining ſtars are marked by the letters of the Roman alphabet; which letters are preſerved by Flamſteed in his catalogue, and by Senex on his globes, and indeed by moſt aſtronomers ſince that time.

In 1673, the celebrated John Hevelius, of Dantzick, publiſhed, in his *Machina Cœleſtis*, a catalogue of 1888 ſtars, of which 1553 were obſerved by himſelf; and their places ſet down for the end of the year 1660. But this catalogue, as it ſtands in Flamſteed's *Hiſtoria Cœleſtis* of 1725, contains only 1520 ſtars.

The moſt complete catalogue ever given from the labours of one man, was the Britannic catalogue, compiled from the obſervations of the accurate and indefatigable Mr. Flamſteed, the firſt Royal Aſtronomer at Greenwich; who for a long ſeries of years devoted himſelf wholly to that buſineſs. As there was nothing wanting either in the obſerver or apparatus, his may be conſidered as a perfect work, ſo far as it goes. It is however to be regretted that the edition had not paſſed through his own hands: that now extant was publiſhed by authority, but without the author's conſent, and contains 2734 ſtars. Another edition was publiſhed in 1725, purſuant to his teſtament, containing 3000 ſtars, with their places adapted to the beginning of the year 1689; to which is added Mr. Sharp's catalogue of the ſouthern ſtars not viſible in our hemiſphere, ſet down for the year 1726. See vol. 3 of his *Hiſtoria Cœleſtis*, in which are printed the catalogues of Ptolomy, Ulugh Beigh, Tycho, the Prince of Heſſe, and Hevelius; with an account of each of them in the Prolegomena.

The firſt catalogue we believe that was printed in the new or ſecond form, according to the order of the right aſcenſions, is that of De la Caille, given in his Ephemerides for the 10 years between 1755 and 1765, and printed in 1755. It contains the right aſcenſions and declinations of 307 ſtars, adapted to the beginning of the year 1750.—In 1757, De la Caille publiſhed his *Aſtronomiæ Fundamenta*, containing a catalogue of the right aſcenſions and declinations of 398 ſtars, likewiſe adapted to the beginning of 1750.—And in 1763, the year after his death, was publiſhed the *Cœlum Auſtrale Stelliferum* of the ſame author; containing a catalogue of the places of 1942 ſtars, all ſituated to the ſouthward of the tropic of Capricorn, and obſerved by him while he was at the Cape of Good Hope, in 1751 and 1752; their places being alſo adapted to the beginning of 1750.—In the ſame year was publiſhed his Ephemerides for the 10 years between 1765 and 1775; in the introduction to which are given the places of 515 zodiacal ſtars, all deduced from the obſervations of the ſame author; the places adapted to the beginning of the year 1765.

In the Nautical Almanac for 1773, is given a catalogue of 387 ſtars, in right aſcenſion, declination, longitude, and latitude, derived from the obſervations of the late celebrated Dr. Bradley, and adjuſted to the beginning of the year 1760. This ſmall catalogue, and the reſults of about 1200 obſervations of the moon, are all that the public have yet ſeen of the multiplied labours of this moſt accurate and indefatigable obſerver, although he has now (1794) been dead upwards of 32 years.

In 1775 was publiſhed a thin volume entitled *Opera Inedita*, containing ſeveral papers of the late Tobias Mayer, and among them a catalogue of the right aſcenſions and declinations of 998 ſtars, which may be occulted by the moon and planets; the places being adapted to the beginning of the year 1756.

At the end of the firſt volume of 'Aſtronomical Obſervations made at the Royal Obſervatory at Greenwich,' publiſhed in 1776, Dr. Maſkelyne, the preſent Aſtronomer Royal, has given a catalogue of the places of 34 principal ſtars, in right-aſcenſion and north-polar diſtance, adapted to the beginning of the year 1770. Theſe, being the reſult of ſeveral years' repeated obſervations, made with the utmoſt care, and the beſt inſtruments, it may be preſumed are exceedingly accurate.

In 1782, M. Bode, of Berlin, publiſhed a very extenſive catalogue of 5058 of the fixed ſtars, collected from the obſervations of Flamſteed, Bradley, Hevelius, Mayer, De la Caille, Meſſier, Monnier, D'Arquier, and other aſtronomers; all adapted to the beginning of the year 1780; and accompanied with a Celeſtial Atlas, or ſet of maps of the conſtellations, engraved in a moſt delicate and beautiful manner.

To theſe may be added, Dr. Herſchel's catalogue of double ſtars, printed in the Philoſ. Tranſ. for 1782 and 1783; Meſſier's nebulæ and cluſters of ſtars, publiſhed in the Connoiſſance des Temps for 1784; and Herſchel's catalogue of the ſame kind, given in the Philoſ. Tranſ. for 1786.

In 1789 Mr. Francis Wollaſton publiſhed 'A Specimen of a General Aſtronomical Catalogue, in Zones of North-polar Diſtance, and adapted to Jan. 1, 1790.' Theſe ſtars are collected from all the catalogues beforementioned, from that of Hevelius downwards. This work contains five diſtinct catalogues; viz, Dr. Maſkelyne's new catalogue of 36 principal ſtars; a general catalogue of all the ſtars, in zones of north-polar diſtance; an index to the general catalogue; a catalogue of all the ſtars, in the order in which they paſs the meridian; and a catalogue of zodiacal ſtars, in longitude and latitude.

Finally, in 1792, Dr. Zach publiſhed at Gotha, *Tabulæ Motuum Solis*, to which is annexed a new catalogue of the principal fixed ſtars, from his own obſervations made in the years 1787, 1788, 1789, 1790. This catalogue contains the right aſcenſions and declinations of 381 principal ſtars, adapted to the beginning of the year 1800.

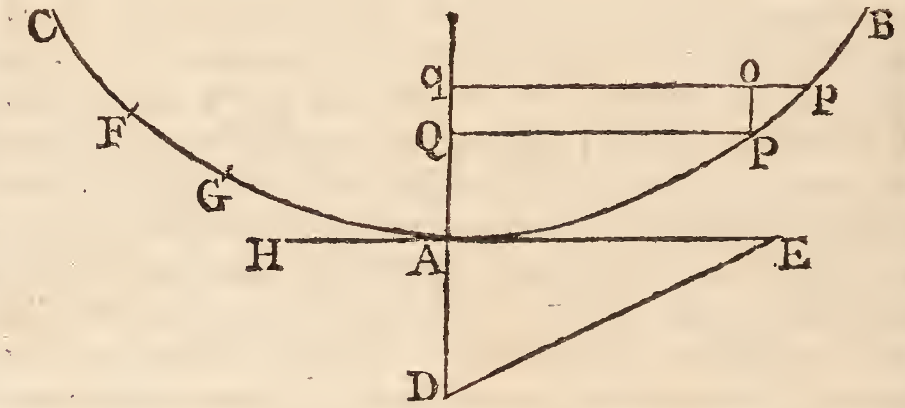
CATAPULT, *Catapulta*, a military engine, much uſed by the ancients for throwing huge ſtones, and ſometimes large darts and javelins, 12, 15, or even 18 feet long, on the enemy. It is ſometimes confounded with the Balliſta, which is more peculiarly adapted for throwing ſtones; ſome authors making them the ſame, and others different.

The catapulta, which it is ſaid was invented by the Syrians, conſiſted of two huge timbers, like maſts of ſhips, placed againſt each other, and bent by an engine for the purpoſe; theſe being ſuddenly unbent again by the ſtroke of a hammer, threw the javelins with prodigious force. Its ſtructure and the manner of working it are deſcribed by Vitruvius; and a figure of it is alſo given by Perrault. M. Folard aſſerts that the catapult made infinitely more diſorder in the ranks than our cannon charged with iron balls. See Vitruv. Archit. lib. x. cap. 15 and 18; and Perr. notes on the ſame; alſo Rivius, pa. 597.

Joſephus takes notice of the ſurpriſing effects of theſe engines, and ſays, that the ſtones thrown out of them, of a hundred weight or more, beat down the battlements, knocked the angles off the towers, and would level a whole file of men from one end to the other, were the phalanx ever ſo deep.

See plate V, fig. 3 and 4, for two forms of the catapult, the one for throwing darts and javelins, the other for ſtones.

CATENARY, a curve line which a chain, cord, or ſuch like, forms itſelf into, by hanging freely from two points of ſuſpenſion, whether theſe be in the ſame horizontal line or not; as the curve ACB, formed by a heavy flexible line ſuſpended by any two points A and B.



The nature of this curve was ſought after by Galileo, who thought it was the ſame with the parabola; but though Jungius detected this miſtake, its true nature was not diſcovered till the year 1691, in conſequence of M. John Bernoulli having publiſhed it as a problem in the Acta Eruditorum, to the mathematicians of Europe. In 1697 Dr. David Gregory publiſhed an inveſtigation of the properties before diſcovered by Bernoulli and Leibnitz; in which he pretends that an inverted catenary is the beſt figure for an arch of a bridge &c. See Philoſ. Tranſ. abr. vol. 1. pa. 39; alſo Bernoulli Opera, vol. 1. pa. 48, and vol. 3. pa. 491; and Cotes's Harmon. Menſur. pa. 108.

The catenary is a curve of the mechanical kind, and cannot be expreſſed by a finite algebraical equation, in ſimple terms of its abſciſs and ordinate; but is eaſily expreſſed by means of fluxions; thus if AQ be its axis perpendicular to the horizon, and PQ an ordinate parallel to the ſame, or perp. to AQ; alſo *pq* another ordinate indefinitely near the former, and *po* parallel to AQ; then, *a* being ſome given or conſtant quantity, the fundamental property of the curve is this, viz, $Po : op :: AP : a$, or $\dot{x} : \dot{y} :: z : a$, that is, the fluxion of the axis, is to the fluxion of the ordinate, as the length of the curve is to the given quantity *a*; where $x = AQ$, $y = PQ$, and $z = AP$. This, and the other properties of the curve, will eaſily appear from the following conſiderations: Firſt, ſuppoſing the curve hung up by its two points B and C againſt a perpendicular or upright wall: then, every lower part of the curve being kept in its poſition by the tenſion of that which is immediately above it, the lower parts of the curve will retain the ſame poſition unvaried, by whatever points it is ſuſpended above; thus, if it were fixed to the wall by the point F, or G, &c, the whole curve CAB will remain juſt as it was; for the tenſions at F and G have the ſame effect upon the other parts of the curve as when it is fixed by thoſe points: and hence it follows that the tenſion of the curve at the point A, in the horizontal direction, is a conſtant quantity, whether the two legs or branches of the curve, on both ſides of it, be longer or ſhorter: which conſtant tenſion at A let be denoted by the quantity a.

Now becauſe any portion of the curve, as AP, is kept in its poſition by three forces, viz, the tenſions at its extremities A and P, and its own weight, of which the tenſion at A acts in the direction AH or po, and the tenſion at P acts in the direction Pp, and the wt. of the line acts in the perpendicular direction oP; that is, the three forces which retain the curve AP in its poſition, act in the directions of the ſides of the elementary triangle opP; but, by the principles of mechanics, any three forces, keeping a body in equilibrio, are proportional to the three ſides of a triangle drawn in the directions in which thoſe forces act; therefore it follows that the forces keeping AP in its poſition, viz, the tenſion at A, the tenſion at P, and the wt. of AP, are reſpectively as op, pP, and oP,
that is, as $\dot{y}$, $\dot{z}$, and $\dot{x}$.
But the tenſion at A is the conſtant quantity a, and the wt. of the uniform curve AP may be expounded by its length z; therefore it follows that $\dot{x} : \dot{y} :: z : a$; which was to be proved.

Alſo from this laſt proportion, by proper analogy, or ſimilar combinations of the terms, there ariſes this other property, $\dot{x} : \sqrt{xx + yy}$ or $\dot{z} :: z : \sqrt{aa + zz}$, or $z\dot{z} = \dot{x}\sqrt{aa + zz}$, or $\dot{x} = \frac{z\dot{z}}{\sqrt{a^2 + z^2}}$; and the fluents of theſe give $x = \sqrt{a^2 + z^2}$. But at the vertex of the curve, where $x = 0$, and $z = 0$, this becomes $0 = \sqrt{a^2 + 0} = a$; and therefore by correction the true equation of the fluents is $x = \sqrt{a^2 + z^2} - a$, or $a + x = \sqrt{a^2 + z^2}$: and hence alſo $z = \sqrt{\overline{a + x}^2 - a^2} = \sqrt{2ax + x^2}$ and $a = \frac{z^2 - x^2}{2x}$.

Any of which is the equation of the curve in terms of the arch and its abſciſs; in which it appears that $a + x$ is the hypothenuſe of a right-angled triangle whoſe two legs are a and z. So that, if in QA and HA produced, there be taken AD $= a$, and AE $=$ the curve z or AP; then will the hypothenuſe DE be $=$ $a + x$ or DQ. And hence, any two of theſe three, a, x, z, being given, the third is given alſo.

Again, from the firſt ſimple property, viz, $\dot{x} : \dot{y} :: z : a$, or $a\dot{x} = z\dot{y}$, by ſubſtituting the value of z above found, it becomes $a\dot{x} = \dot{y}\sqrt{2ax + x^2}$, or $\dot{y} = \frac{a\dot{x}}{\sqrt{2ax + x^2}}$; and the fluent of this equation is $y = 2a \times$ hyp. log. of $\overline{\sqrt{x} + \sqrt{2a + x}}$. But at the vertex of the curve, where $x = 0$ and $y = 0$, this becomes $0 = 2a \times$ hyp. log. of $\sqrt{2a}$; therefore the correct equation of the fluents is $y = 2a \times$ hyp. log. of $\frac{\sqrt{x} + \sqrt{2a + x}}{\sqrt{2a}}$; an equation to the curve alſo, in terms of x and y, but not in ſimple algebraic terms. This laſt equation however may be brought to much ſimpler terms in different ways; as firſt by ſquaring the logarithmic quantity and dividing its coef. by 2, then $y = a \times$ hyp. log. of $\frac{a + x + \sqrt{2ax + x^2}}{a} = a \times$ hyp. log. $\frac{a + x + z}{a}$; and 2d by multiplying both numerator and denominator by $\sqrt{2a + x} - \sqrt{x}$, then ſquaring the product, and dividing the coef. by 2, which gives $y = a \times$ hyp. log. $\frac{z + x}{\sqrt{z^2 - x^2}} = a \times$ hyp. log. $\frac{\overline{z + x}^2}{z^2 - x^2} = a \times$ hyp. log. $\frac{z + x}{z - x}$.

CATHETUS, in Geometry, a name by which the perpendicular leg of a right-angled triangle is ſometimes called. Or it is in general any line or radius falling perpendicularly on another line, or ſurface.

CATHETUS *of Incidence*; in Catoptrics, is a right line drawn from a radiant point, or point of incidence, perpendicular to the reflecting line, or plane of the ſpeculum.

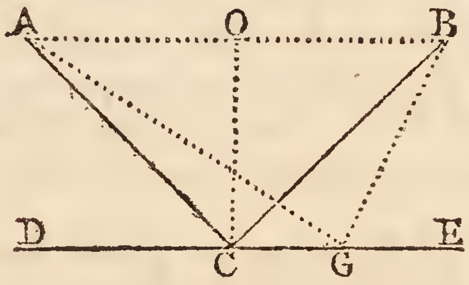
CATHETUS *of Reflection*, or *of the Eye*, a right line drawn from the eye, perpendicular to the plane of reflection.

CATHETUS, in Architecture, denotes the axis of a column &c. In the Ionic Capital, it denotes a line paſſing perpendicularly through the eye or centre of the volute.

CATOPTRICS, the ſcience of reflex viſion; or that part of optics which explains the laws and properties of light reflected from mirrors, or ſpecula.

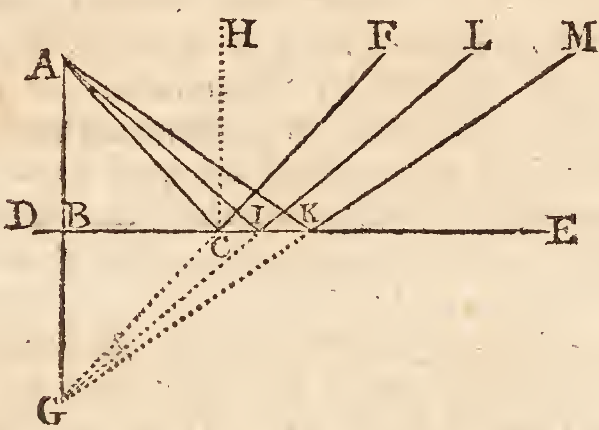
The firſt treatiſe extant on catoptrics, is that which was compoſed by Euclid: this was publiſhed in Latin in 1604 by John Pena; it is alſo contained in Herigon's Courſe of Mathematics, and in Gregory's edition of the works of Euclid: though it is ſuſpected by ſome that this piece was not the work of that great geometrician, notwithſtanding that it is aſcribed to him by Proclus in lib. 2, and by Marinus in his Preface to Euclid's Data. Alhazen, an Arabian author, compoſed a large volume of optics about the year 1100, in which he treats pretty fully of catoptrics: and after him Vitello, a Poliſh writer, compoſed another about the year 1270. Tacquet, in his Optics, has very well demonſtrated the chief propoſitions of plane and ſpherical ſpeculums. And the ſame is very ably done by Dr. Barrow in his Optical Lectures. There are alſo Trabe's Catoptrics; David Gregory's Elements of Catoptrics; Wolfius's Elements of Catoptrics; and thoſe of Dr. Smith, contained in his learned and very elaborate Treatiſe on Optics; and many others of leſs note.

As this ſubject is treated under the general term Optics, the leſs need be ſaid of it here. The whole doctrine of Catoptrics depends upon this ſimple principle, that the angle of incidence is equal to the angle of reflection, that is, that the angle in which a ray of light falls upon any ſurface, called the angle of incidence, is equal to the angle in which it quits it when reflected from it, called the angle of reflection; though it is ſometimes defined that the angles of incidence and reflection, are thoſe which the incident and reflected rays make, not with the reflecting ſurface itſelf, but with a perpendicular to that ſurface, at the point of contact, which are the complements to the others: but it matters not by what name theſe angles are called, as to the truth and principles of the ſcience; ſince, if the angles are equal, their complements are alſo equal. This principle of the equality between the angles of incidence and reflection, is mere matter of experience, being a phenomenon that has always been obſerved to take place, in every caſe that has fallen under obſervation, as near at leaſt as mechanical meaſurements can aſcertain; and hence it is inferred that it is a univerſal law of nature, and to be conſidered as matter of fact in all caſes. Thus, let AC be an incident ray falling upon the reflecting ſurface DE, and CB the reflected ray, alſo CO perpendicular to DE; then is the angle ACD = BCE, or the angle ACO = BCO.



Of this law in nature, viz, the equality between the angles of incidence and reflection, it is remarkable, that in this way, the length or rout AC + CB, in a ray paſſing from any point A to another given point B, by being reflected from any ſurface DE, is the ſhorteſt poſſible, namely AC + CB is ſhorter than the ſum AG + GB of any other two lines inflected at the line DE; and hence alſo the paſſage of the ray from A to B is performed in a ſhorter time than if it had paſſed by any other way.

From this ſimple principle, and the common properties of lineal geometry, the chief phenomena of catoptrics are eaſily deduced, and are as here following.



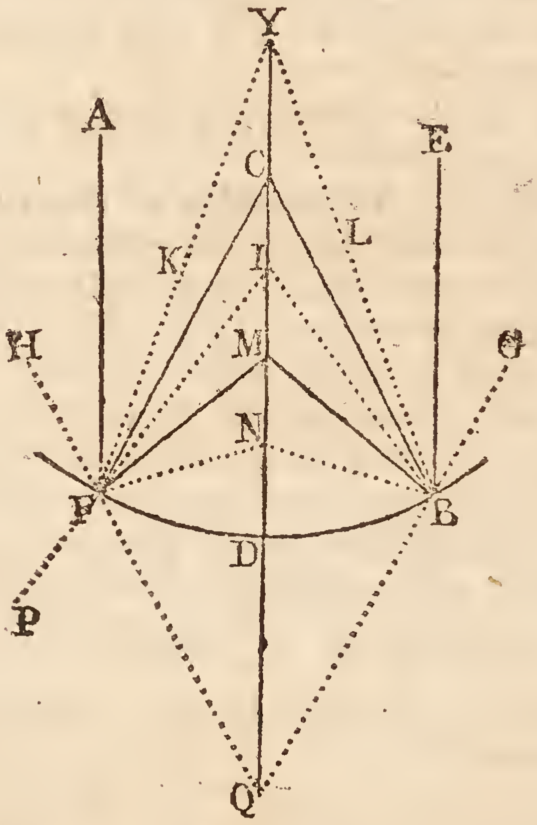
1. Rays of light reflected from a plane ſurface, have the ſame inclination to each other after reflection as they had before it. Thus, the rays AC, AI, AK, iſſuing from the radiant point A, and reflected by the ſurface DE into the lines CF, IL, KM; theſe latter lines will have the ſame inclination to each other as the former AC, AI, AK have. For draw ABG perpendicular to DE, and produce FC, LI, MK backwards to meet this perpendicular, ſo ſhall they all meet in the ſame point G, and AB will in every caſe be equal to BD: for the incident ∠ ACB is equal to the reflected ∠ FCE, which is equal to the oppoſite angle BCG; ſo that the two triangles ABC, GBC, have the angles at C equal, as alſo the right angles at B equal, and conſequently the 3d angles at A and G equal; and having alſo the ſide BC common, they are equal in all reſpects, and ſo AB = BG. And the ſame for the other rays. Conſequently the angles BGC, BGI, BGK are reſpectively equal to the angles BAC, BAI, BAK; that is, the reflected rays have the ſame inclinations as the incident ones have.

2. Hence it is that the image of an object, ſeen by reflection from a plain mirror, ſeems to proceed from a place G as far beyond, or on the other ſide of the reflecting plane DE, as the object A itſelf is before the plane. This is when the incident rays diverge from ſome point as A.

But if the caſe be reverſed, and FC, LI, MK be conſidered as incident rays, iſſuing from points F, L, M, and converging to ſome point G beyond the reflecting plane; then CA, IA, KA will become the reflected rays, and they will converge to the point A as far before the plane, as the point G is beyond it.

So that univerſally, when the incident rays diverge from a point A, the reflected rays will alſo diverge from a point G; and when the incident rays converge towards a point G, the reflected ones will alſo converge to a point A; and in both caſes theſe two points are at equal diſtances on the oppoſite ſides of the reflecting plane DE.

3. Parallel rays reflected from a concave ſpherical ſurface, converge after reflection. For, let AF, CD, EB be three parallel rays falling upon the concave ſurface FB, whoſe centre is C. To the centre draw the perpendiculars FC and BC; alſo draw FM making the reflected angle CFM equal to the incident angle CFA; and in like manner BM to make the angle CBM = the angle CBE; ſo ſhall the rays AF and EB be reflected into the converging rays FM and BM. As to the ray CD, being perpendicular to the ſurface, it is reflected back again in the ſame line DC.



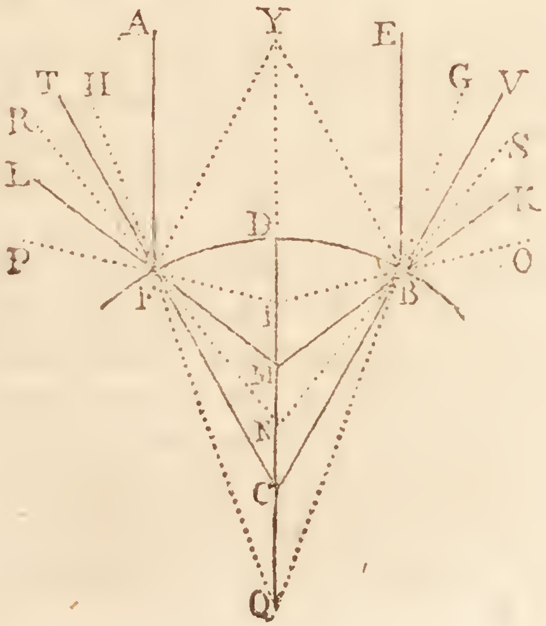
4. Converging rays falling upon the concave ſurface are made to converge more. Thus, let GB and HF be the incident rays: then becauſe the incident angle HFC is larger than the angle AFC, therefore the equal

reflected angle NFC is greater than the reflected angle MFC, and ſo the point N is below the point M, or the line FN below the line FM; and in like manner BN is below BM; that is, the reflected rays FN and BN are more converging in this caſe, than FM and BM in the other.

5. The focus to which all parallel rays, falling near the vertex D, are reflected, is in the middle of the radius M. For, becauſe the $\angle MFC = \angle AFC$ which is = the alternate $\angle FCM$, therefore the ſides oppoſite theſe angles are alſo equal, namely the ſide $FM = CM$; conſequently when the point F is very near the vertex D, then the ſum $CM + MF$ is nearly $= CD$, and ſo CM nearly $= MD$, or the focus of the parallel rays is nearly in the middle of the radius.—But the focus of other reflected rays is either above or below that of the parallel rays; namely, below when the incident rays are converging, and above when they are diverging; as is evident by inſpection; thus, N the reflected focus of the converging rays GB and HF, is below M; I that of the diverging rays YB and YF, is above M.

6. Incident and reflected rays are reciprocal, or ſo that if the reflected rays be returned back, or conſidered as incident ones, they will be reflected back into what were before their incident rays. And hence it follows that diverging rays, after reflection from a concave ſpherical ſurface, become either parallel or leſs diverging than before. Thus the incident rays MF and MB are reflected into the parallel rays FA and BE, and the rays NF and NB are reflected into FH and BG, which are leſs diverging; alſo the rays IF and IB are reflected into FK and BL, which converge.—And hence all the phenomena of concave mirrors will be evident.

7. Rays reflected from a convex ſpeculum, become quite contrary to thoſe reflected from a concave one; ſo that the parallel rays become diverging, and the diverging rays become ſtill more diverging; alſo converging rays will become either diverging, or parallel, or elſe leſs converging. Thus BDF being a ſpherical ſurface, whoſe centre is C, produce the radii CBV and CFT which are perpendicular to the ſurface; then it is evident that the parallel rays AF and EB will be reflected into the diverging ones FK and BL; and the diverging rays YB and YF become BO and FP which are more diverging; alſo the converging rays HF and GB become FR and BS which diverge, or elſe KF and LB become FA and BE which are parallel, or elſe laſtly PF and OB become FY and BY which are converging.



8. Hence, as in the concave ſpeculum, ſo alſo in the convex one, of parallel incident rays AF and EB, the imaginary focus M of their reflected rays FL and BK, is in the middle of the radius when the ſpeculum is a ſmall ſegment of a ſphere: but the reflected imaginary focus of other rays is either above or below the middle point M, viz N being that of the converging rays GB and HF, below M; but I, that of the diverging rays YB and YF, above M.

9. When the ſpeculum is the ſmall ſegment of a ſphere, either convex or concave, and the incident rays either converging or diverging, the diſtances of the foci, or points of concurrence, of the incident rays, and of the reflected rays, from the vertex of the ſpeculum, are directly proportional to the diſtances of the ſame from the centre of it;

that is $YD : ID :: YC : IC$,
and $QD : ND :: QC : NC$.

For becauſe the radius CF, or the ſame produced, biſects the angle YFI in the concave ſpeculum, or the external angle YFP in the convex one, therefore $YF : IF :: YC : IC$; but when F is very near to D, then YF and IF become nearly YD and ID; conſequently $YD : ID :: YC : IC$.

In like manner, becauſe CF biſects the angle QFN in the convex, or its external angle NFH in the concave ſpeculum, therefore $QF : FN :: QC : NC$; but when F is very near to D, then QF and FN become nearly QD and ND; and therefore $QD : ND :: QC : NC$.

For example, ſuppoſe it were required to find the focal diſtance of diverging rays incident upon a convex ſurface, the radius of the ſphere being 5 inches, and the diſtance of the radiant point from the ſurface 20 inches. Here then are given $YD = 20$, and $CD = 5$, to find ID: then

the theorem $YD : ID :: YC : IC$,
in numbers is $20 : ID :: 25 : 5 - ID$,
or by permutation $20 : 25 :: ID : 5 - ID$,
and by compoſition $45 : 20 :: 5 : ID = \frac{100}{45} = \frac{20}{9} =$
$2\frac{2}{9}$ the focal diſtance ſought.

And if it ſhould happen in any caſe that the value of ID in the calculation ſhould come out a negative quantity, the focal diſtance muſt then be taken on the contrary ſide of the ſurface.

From the foregoing principles may be deduced and collected the following practical maxims, for plane and ſpherical mirrors, viz,

I. *In a Plane Mirror,*

(1). The image will appear as far behind the mirror, as the object is before it.

(2). The image will appear of the ſame ſize, and in the ſame poſition as the object.

(3). Any plain mirror will reflect the image of an object of twice its own length and breadth.

II. *In a Spherical Convex Mirror,*

(1). The image will always appear behind the mirror, or within the ſphere.

(2). The image will be in the ſame poſition, but leſs than the object.

(3). The image will be curved, but not ſpherical, like the mirror.

(4). Parallel rays falling on this mirror, will have the image at half the diſtance of the centre from the mirror.

(5). In converging rays, the diſtance of the object muſt be equal to half the diſtance of the centre, to make the image appear behind the mirror.

(6). Diverging rays will have their image at leſs than half the diſtance of the centre.

III. *In a Spherical Concave Mirror,*

(1). Parallel rays have their focus, or the image, at half the diſtance of the centre.

(2). In the centre of the ſphere the image appears of the ſame dimenſions as the object.

(3). Converging rays form an image before the mirror.

(4). In diverging rays, if the object be at leſs than half the diſtance of the centre, the image will be behind the mirror, erect, curved, and magnified; but if the diſtance of the object be greater, the image will be before the mirror, inverted and diminiſhed.

(5). The ſolar rays, being parallel, will be collected in a focus at half the diſtance of its centre, where their heat will be augmented in proportion as the ſurface of the mirror exceeds that of the focal ſpot.

(6). If a luminous body be placed in the focus of a concave mirror, its rays, being reflected in parallel lines, will ſtrongly enlighten a ſpace of the ſame dimenſions with the mirror, at a great diſtance. If the luminous object be placed nearer than the focus, its rays will diverge, and ſo enlighten a larger ſpace, but not ſo ſtrongly. And upon this principle it is that reverberators are conſtructed.

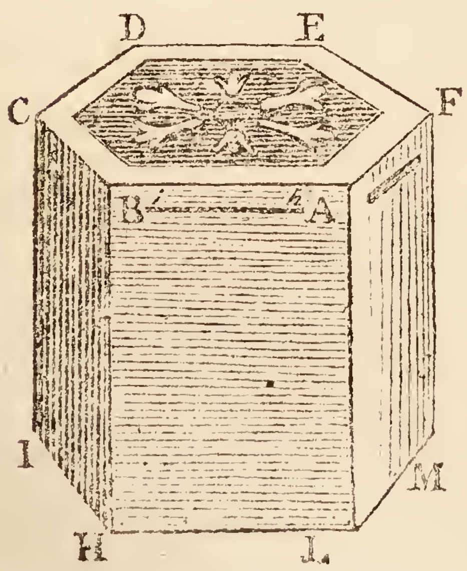
CATOPTRIC *Dial*, a dial that exhibits objects by reflected rays. See *Reflecting* DIAL.

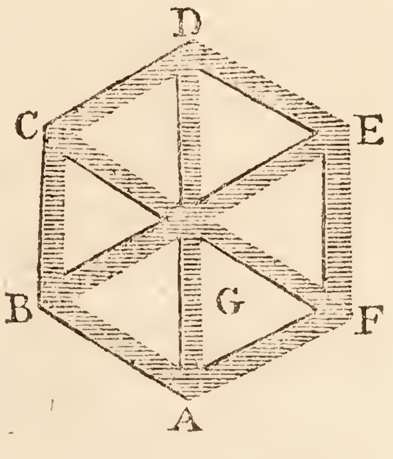
CATOPTRIC *Teleſcope*, a teleſcope that exhibits objects by reflection. See *Reflecting* TELESCOPE.

CATOPTRIC *Ciſtula*, a machine, or apparatus, by which ſmall bodies are repreſented extremely large, and near ones extremely wide, and diffuſed through a vaſt ſpace; with other very pleaſing phenomena, by means of mirrors, diſpoſed by the laws of catoptrics, in the concavity of a kind of cheſt.

There are various kinds of theſe machines, accommodated to the various intentions of the artificer: ſome multiply the objects, ſome magnify, ſome deform them, &c. The ſtructure of one or two of them will ſuffice to ſhew how many more may be made.

To make a Catoptric Ciſtula to repreſent ſeveral different ſcenes of objects, when viewed by different holes.





Provide a polygonal ciſtula, or box, like the multangular priſm ABCDEF, and divide its cavity by diagonal planes AD, BE, CF, interſecting in the centre, into as many triangular cells as the cheſt has ſides. Line thoſe diagonal partitions with plain mirrors; and in the ſides of the box make round holes, through which the eye may peep within the cells of it. Theſe holes are to be covered with plain glaſſes, ground within-ſide, but not poliſhed, to prevent the objects in the cells from appearing too diſtinctly. In each cell are to be placed the different objects whoſe images are to be exhibited; then covering up the top of the cheſt with a thin tranſparent membrane, or parchment, to admit the light, the machine is complete.

For, from the laws of reflection, it follows, that the images of objects, placed within the angles of mirrors, are multiplied, and appear ſome more remote than others; by which the objects in one cell will appear to take up more room than is contained in the whole box. Therefore by looking through one hole only, the objects in one cell will be ſeen, but thoſe multiplied, and diffuſed through a ſpace much larger than the whole box. Thus every hole will afford a new ſcene; and according to the different angles the mirrors make with each other, the repreſentations will be different: if they be at an angle greater than a right one, the images will be monſtrous, &c.

To make a Catoptric Ciſtula to repreſent the objects within it prodigiouſly multiplied, and diffuſed through a vaſt ſpace.

Make a polygonous ciſtula or box, as before, but without dividing the inner cavity into any apartments, or cells; line the inſides CBHI, BHLA, ALMF, &c, with plane mirrors, and at the holes pare off the tin and quickſilver, to look through; place any object in the bottom MI, as a bird in a cage, &c.

Now by looking through the aperture *hi*, each object placed at the bottom will be ſeen vaſtly multiplied, and the images removed at equal diſtances from one another, like a great multitude of birds, or a large aviary.

CAVALIER, in Fortification, a mount of earth raiſed in a fortreſs higher than the other works, on which to place cannon &c for ſcouring the field, or oppoſing a commanding work. Cavaliers are of different ſhapes; and are bordered with a parapet, to cover the cannon mounted upon them; their ſituation is alſo various, either in the curtain, baſtion, or gorge. The cavalier is ſometimes called a double baſtion, and its uſe is to overlook the enemy's batteries, and to ſcour their trenches.

CAVALIERI (BONAVENTURA), an eminent Italian mathematician in the 17th century. He was a native of Milan, and a friar of the order of the Jeſuati of St. Jerome. Cavalieri was a diſciple of Galileo, and the friend of Torricelli. He was a very eminent mathematician, and was profeſſor of that ſcience at Bologna; where ſeveral of his books were publiſhed, and where he died in the year 1647. His works that have been publiſhed, as far as I can find, are as follow:

1. *Directorium Generale Uranometricum; 4to, Bononiæ*, 1632.—In this work the author treats of Trigonometry; and Logarithms, their conſtruction, uſes, and applications. The work includes alſo tables of logarithms

of common numbers; with trigonometrical tables, of natural ſines, and logarithmic ſines, tangents, ſecants and verſed ſines.

2. *Lo Specchio Uſtorio, overo Trattato delle Settioni Coniche*: *4to, Bologna,* 1632.—An ingenious treatiſe of conic ſections.

3. *Geometria Indiviſibilibus continuorum nova quadam ratione promota*: *4to, Bononiæ,* 1635; and a 2d edition in 1653.—This is a curious original work in geometry, in which the author conceives the geometrical figures as reſolved into their very ſmall elements, or as made up of an infinite number of infinitely ſmall parts, and on account of which he paſſes in Italy for the inventor of the infiniteſimal calculus.

4. *Trigonometria Plana & Sphærica, Linearis, & Logarithmica*: *4to, Bononiæ,* 1643.—A very neat and ingenious treatiſe on Trigonometry; with the tables of ſines, tangents, and ſecants, both natural and logarithmical.

5. *Exercitationes Geometricæ Sex*: *4to, Bononiæ,* 1647. This work contains Exerciſes on the method of Indiviſibles; Anſwers to the objections of Guldini; The uſe of Indiviſibles in coſſic powers or algebra, and in conſiderations about gravity; with a miſcellaneous collection of problems.

CAUDA *Capricorni*, a fixed ſtar of the 4th magnitude, in the tail of Capricorn; called alſo by the Arabs, Dineb Algedi; and marked γ by Bayer.

CAUDA *Ceti*, a fixed ſtar of the 3d magnitude; called alſo by the Arabs, Dineb Kaetos; marked β by Bayer.

CAUDA *Cygni*, a fixed ſtar of the 2d magnitude in the Swan's tail; called by the Arabs, Dineb Adigege, or Eldegiagich; and marked α by Bayer.

CAUDA *Delphini*, a fixed ſtar of the 3d magnitude, in the tail of the Dolphin; marked ε by Bayer.

CAUDA *Draconis*, or Dragon's tail, the moon's ſouthern or deſcending node.

CAUDA *Leonis*, a fixed ſtar of the firſt magnitude in the Lion's tail; called alſo by the Arabs, Dineb Eleced; and marked β by Bayer. It is called alſo Lucida Cauda.

CAUDA *Urſæ Majoris*, a fixed ſtar of the 3d magnitude, in the tip of the Great Bear's tail; called alſo by the Arabs, Alalioth, and Benenath; and marked η by Bayer.

CAUDA *Urſæ Minoris*, a fixed ſtar of the 3d magnitude, at the end of the Leſſer Bear's tail; called alſo the Pole Star, and by the Arabs, Alrukabah; and marked α by Bayer.

CAVETTO, a hollow member or moulding, containing a quadrant of a circle, and having an effect juſt contrary to that of a quarter round. It is uſed as an ornament in cornices.

CAUSTIC CURVES. See *Catacauſtics*, and *Diacauſtics*.

CAZEMATE. See CASEMATE.

CAZERN. See CASERN.

CEGINUS, a fixed ſtar of the 3d magnitude, in the left ſhoulder of Bootes; and marked γ by Bayer.

CELERITY, is the velocity or ſwiftneſs of a body in motion; or that affection of a body in motion by which it can paſs over a certain ſpace in a certain time.

CELESTIAL GLOBE, &c. See GLOBE, &c.

CELLARIUS (CHRISTOPHER), a learned geographer and hiſtoriographer of the 17th century. He was born in 1638, at Smalcalde in Franconia, where his father was miniſter. Our author was ſucceſſively rector of the colleges at Weymar, Zeits, and Merſbourg, and profeſſor of eloquence and hiſtory in the univerſity founded by the king of Pruſſia at Hall in 1693, where he compoſed the greateſt part of his works.

His great application to ſtudy haſtened the infirmities of old age; for it has been ſaid, he would ſpend whole days and nights together over his books, without any attention to his health, or even the calls of nature. He died in 1707, at 69 years of age.

Cellar was author of an amazing number of books, upon various ſubjects; but thoſe on account of which he has a place here, are his geographical works, which are as follow:

1. *Notitia Orbis Antiqui*, 2 vols in 4to; and is eſteemed the beſt work extant on the ancient geography.

2. *Atlas Cœleſtis*; in folio.

3. *Hiſtoria Antiqua*, 2 vols in 12mo; being an abridgement of univerſal hiſtory.

CENTAURUS, the *Centaur*, one of the 48 old conſtellations, being a ſouthern one, and is in form half man and half horſe. It is fabled by the Greeks that it was Chiron the Centaur, who was the tutor of Achilles and Eſculapius. The ſtars of this conſtellation are, in Ptolomy's catalogue 37, in Tycho's 4, and in the Britannic catalogue, with Sharp's appendix, 35.

CENTER. See CENTRE.

CENTESM, the 100th part of any thing.

CENTRAL, ſomething relating to a centre. Thus we ſay central eclipſe, central forces, central rule, &c.

CENTRAL *Eclipſe*, is when the centres of the luminaries exactly coincide, and come in a line with the eye.

CENTRAL *Forces*, are forces having a tendency directly towards or from ſome point or centre; or forces which cauſe a moving body to tend towards, or recede from, the centre of motion. And accordingly they are divided into two kinds, in reſpect to their different relations to the centre, and hence are called centripetal, and centrifugal.

The doctrine of central forces makes a conſiderable branch of the Newtonian philoſophy, and has been greatly cultivated by mathematicians, on account of its extenſive uſe in the theory of gravity, and other phyſico-mathematical ſciences.

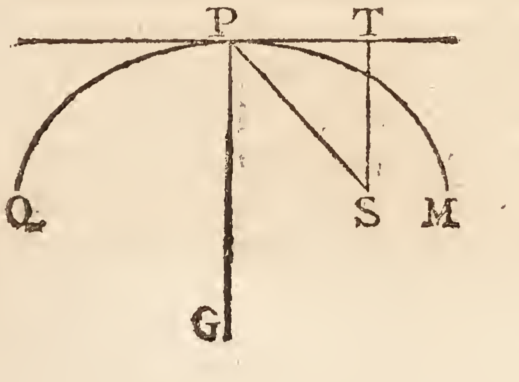
In this doctrine, it is ſuppoſed that matter is equally indifferent to motion or reſt; or that a body at reſt never moves itſelf, and that a body in motion never of itſelf changes either the velocity or the direction of its motion; but that every motion would continue uniformly, and its direction rectilinear, unleſs ſome external force or reſiſtance ſhould affect it, or act upon it. Hence, when a body at reſt always tends to move, or when the velocity of any rectilinear motion is continually accelerated or retarded, or when the direction of a motion is continually changed, and a curve line is thereby deſcribed, it is ſuppoſed that theſe circumſtances proceed from the influence of ſome power that acts inceſſantly; which power may be meaſured, in the firſt caſe, by the preſſure of the quieſcent body againſt the obſtacle which prevents it from moving, or by the velocity

locity gained or loſt in the ſecond caſe, or by the flexure of the curve deſcribed in the 3d caſe: due regard being had to the time in which theſe effects are produced, and other circumſtances, according to the principles of mechanics. Now the power or force of gravity produces effects of each of theſe kinds, which fall under our conſtant obſervation near the ſurface of the earth; for the ſame power which renders bodies heavy, while they are at reſt, accelerates their motion when they deſcend perpendicularly; and bends the track of the motion into a curve line, when they are projected in a direction oblique to that of their gravity. But we can judge of the forces or powers that act on the celeſtial bodies by effects of the laſt kind only. And hence it is, that the doctrine of central forces is of ſo much uſe in the theory of the planetary motions.

Sir I. Newton has treated of central forces in lib. 1 ſec. 2 of his Principia, and has demonſtrated this fundamental theorem of central forces, viz, that the areas which revolving bodies deſcribe by radii drawn to an immoveable centre, lie in the ſame immoveable planes, and are proportional to the times in which they are deſcribed. Prop. 1.

It is remarked by a late eminent mathematician, that this law, which was originally obſerved by Kepler, is the only general principle in the doctrine of centripetal forces; but ſince this law, as Newton himſelf has proved, cannot hold in caſes where a body has a tendency to any other than one and the ſame point, there ſeems to be wanting ſome law that may ſerve to explain the motions of the moon and ſatellites which gravitate towards two different centres: the law he lays down for this purpoſe is, That when a body is urged by two forces tending conſtantly to two fixed points, it will deſcribe, by lines drawn from the two fixed points, equal ſolids in equal times, about the line joining thoſe fixed points. See Machin, on the Laws of the Moon's Motion, in the Poſtſcript. See alſo a demonſtration of this law by Mr. William Jones, in the Philoſ. Tranſ. vol. 59. Very learned tracts have alſo been ſince given, when the motion reſpects, not two only, but ſeveral centres, by many ingenious authors, and practical rules deduced from them for computing the places &c of planets and ſatellites; as by La Grange, De la Place, Waring, &c, &c. See Berlin Memoirs; thoſe of the Academy of Sciences at Paris; and the Philoſ. Tranſ. of London.

M. De Moivre gave elegant general theorems relating to central forces, in the Philoſ. Tranſ. and in his Miſcel. Analyt. pa. 231.—Let MPQ be any given curve, in which a body moves: let P be the place of the body at any time; S the centre of force, or the point to which the central force acting on the body is always directed; PG the radius of curvature at the point P; and ST perpendicular to the tangent PT; then will the centripetal force be everywhere proportional to the quantity $\frac{SP}{GP \times ST^3}$. Vid. ut ſupra.



M. Varignon has alſo given two general theorems on this ſubject in the Memoirs of the Acad. an. 1700, 1701; and has ſhewn their application to the motions of the planets. See alſo the ſame Memoirs, an. 1706, 1710.

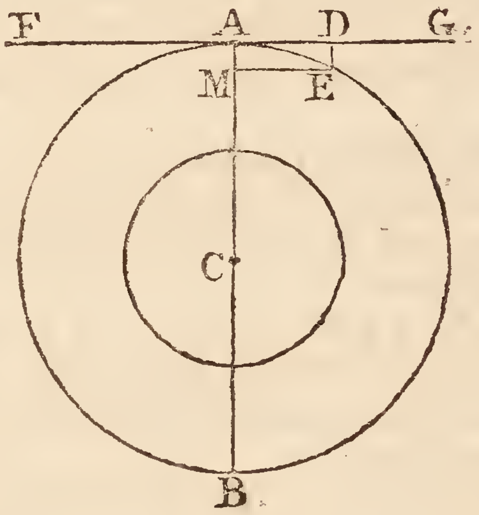
Mr. MacLaurin has alſo treated the ſubject of central forces very ably and fully, in his Treatiſe on Fluxions, art. 416 to 493; where he gives a great variety of expreſſions for theſe forces, and ſeveral elegant methods of inveſtigating them.

Laws of Central Forces.

1. The following is a very clear and comprehenſive rule, for which we are obliged to the marquis de l'Hôpital: Suppoſe a body of any determinate weight to revolve uniformly about a centre, with any given velocity; find from what height it muſt have fallen, by the force of gravity, to acquire that velocity; then, as the radius of the circle it deſcribes is to double that height, ſo is its weight to its centrifugal force. So that, if b be the body, or its weight or quantity of matter, v its velocity, and r the radius of the circle deſcribed, alſo $g = 16\frac{1}{12}$ feet; then, firſt $4g^2 : v^2 :: g : \frac{v^2}{4g}$ the height due to the velocity v; and as $r : \frac{v^2}{2g} :: b : \frac{v^2 b}{2gr} = f$ the centrifugal force. And hence, if the centrifugal force be equal to the gravity, the velocity is equal to that acquired by falling through half the radius.

2. The central force of a body moving in the periphery of a circle, is as the verſed ſine AM of the indefinitely ſmall arc AE; or it is as the ſquare of that arc AE directly, and as the diameter AB inverſely. For AM is the ſpace through which the body is drawn from the tangent in the given time, and 2AM is the proper meaſure of the central force. But, AE being very ſmall, and therefore nearly equal to its chord, by the nature of the circle

$$AB : AE :: AE : AM = \frac{AE^2}{AB}$$



3. If two bodies revolve uniformly in different circles; their central forces are in the duplicate ratio of their velocities directly, and the diameters or radii of the circles inverſely;

that is $F : f :: \frac{V^2}{D} : \frac{v^2}{d} :: \frac{V^2}{R} : \frac{v^2}{r}$

For the force, by the laſt article, is as

$\frac{AE^2}{AB}$ or $\frac{AE^2}{D}$; and the velocity v is as the ſpace AE uniformly deſcribed.

4. And hence, if the radii or diameters be reciprocally in the duplicate ratio of the velocities, the central forces will be reciprocally in the duplicate ratio of the radii, or directly as the 4th power of the velocities; that is, if $V^2 : v^2 :: r : R$, then $F : f :: r^2 : R^2 :: V^4 : v^4$.

5. The central forces are as the diameters of the circles directly, and ſquares of the periodic times inverſely. For if c be the circumference deſcribed in the time t, with the velocity v; then the ſpace $c = tv$, or $v = \frac{c}{t}$; hence, uſing this value of v in the 3d rule, it becomes

$$F : f :: \frac{C^2}{DF^2} : \frac{c^2}{dt^2} :: \frac{D}{T^2} : \frac{d}{t^2} :: \frac{R}{T^2} : \frac{r}{t^2};$$ ſince the diameter is as the circumference.

6. If two bodies, revolving in different circles, be acted on by the ſame central force; the periodic times are in the ſubduplicate ratio of the diameters or radii of the circles; for when $F = f$, then $\frac{D}{T^2} = \frac{d}{t^2}$, and $D : d :: T^2 : t^2$, or $T : t :: \sqrt{D} : \sqrt{d} :: \sqrt{R} : \sqrt{r}$.

7. If the velocities be reciprocally as the diſtances from the centre, the central forces will be reciprocally as the cubes of the ſame diſtances, or directly as the cubes of the velocities. That is, if $V : v :: r : R$, then is $F : f :: r^3 : R^3 :: V^3 : v^3$.

8. If the velocities be reciprocally in the ſubduplicate ratio of the central diſtances, the ſquares of the times will be as the cubes of the diſtances: for if $V^2 : v^2 :: r : R$, then is $T^2 : t^2 :: R^3 : r^3$.

9. Wherefore, if the forces be reciprocally as the ſquares of the central diſtances, the ſquares of the periodic times will be as the cubes of the diſtances; or when $F : f :: r^2 : R^2$, then is $T^2 : t^2 :: R^3 : r^3$.

Exam. From this, and ſome of the foregoing theorems, may be deduced the velocity and periodic time of a body revolving in a circle, at any given diſtance from the earth's centre, by means of its own gravity. Put $g = 16\frac{1}{12}$ feet, the ſpace deſcribed by gravity, at the ſurface, in the firſt ſecond of time, viz $=$ AM in the foregoing fig. and by rule 2; then, putting $r =$ the radius AC; it is $AE = \sqrt{AB \times AM} = \sqrt{2gr}$ the velocity in a circle at its ſurface, in one ſecond of time; and hence, putting $c = 3{\cdot}14159$ &c, the circumference of the earth being $2cr = 25{,}000$ miles, or 132,000,000 feet, it will be $\sqrt{2gr} : 2cr :: 1'' : c\sqrt{\frac{2r}{g}} = 5078$ ſeconds nearly, or $1^h\ 24^m\ 38^s$, the periodic time at the circumference: Alſo the velocity there, or $\sqrt{2gr}$ is $= 26000$ feet per ſecond nearly. Then, ſince the force of gravity varies in the inverſe duplicate ratio of the diſtance, by rules 8 and 9, it is $\sqrt{R} : \sqrt{r} :: v$ or $26000 : 26000\sqrt{\frac{r}{R}} = V$ the velocity of a body revolving about the earth at the diſtance R; and $\sqrt{r^3} : \sqrt{R^3} :: t$ or $5078'' : 5078\sqrt{\frac{R^3}{r^3}} = T$ the time of revolution in the ſame. So if, for inſtance, it be the moon revolving about the earth at the diſtance of 60 ſemidiameters; then $R = 60r$, and the above expreſſions become $V = 26000\sqrt{\frac{1}{60}} = 3357$ feet per ſecond, or $38\frac{1}{7}$ miles per minute, for the velocity of the moon in her orbit; and $T = 5078\sqrt{\frac{R^3}{r^3}} = 2360051$ ſeconds or $27\frac{3}{10}$ days nearly, for the periodic time of the moon in her orbit at that diſtance.

Thus alſo the ratio of the forces of gravitation of the moon towards the ſun and earth may be eſtimated. For, 1 year or $365\frac{1}{4}$ days being the periodic time of the earth and moon about the ſun, and $27\frac{3}{10}$ days the periodic time of the moon about the earth, alſo 60 being the diſtance of the moon from the earth, and 23920 the diſtance from the ſun, in ſemidiameters of the earth, by art. 5 it is

$$\frac{60}{27{\cdot}3^2} : \frac{23920}{365{\cdot}25^2} :: f \text{ or } 1 : \frac{23902}{60} \times \frac{27{\cdot}3^2}{365{\cdot}25^2} = 2\frac{2}{9}$$

that is, the proportion of the moon's gravitation towards the ſun, is to that towards the earth, as $2\frac{2}{9}$ to 1 nearly.

Again, we may hence compute the centrifugal force of a body at the equator, ariſing from the earth's rotation. For, the periodic time when the centrifugal force is equal to the force of gravity, it has been ſhewn above, is 5078 ſeconds, and 23 hours, 56 minutes, or 86160 ſeconds, is the period of the earth's rotation on its axis; therefore, by art. 5, as $86160^2 : 5078^2 :: 1 : \frac{1}{289}$, the centrifugal force required, which therefore is the 289th part of gravity at the earth's ſurface. Simpſon's Flux. pa. 240, &c.

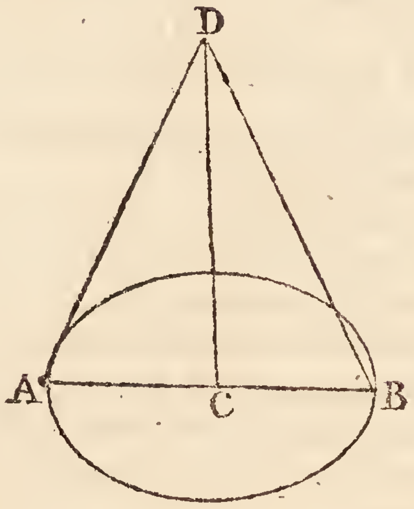
Alſo for another example, ſuppoſe A to be a ball of 1 ounce, which is whirled about the centre C, ſo as to deſcribe the circle ABE, each revolution being made in half a ſecond; and the length of the cord AC equal to 2 feet. Here then $t = \frac{1}{2}$, $r = 2$; and it having been found above that $c\sqrt{\frac{2R}{g}} = T$ is the periodic time at the circumference of the earth when the centrifugal force is equal to gravity; hence then, by art. 5, as $\frac{R}{T^2} : \frac{r}{t^2} :: F$ or $1 : f$, which proportion becomes

$$\frac{g}{2c^2} : \frac{r}{t^2} :: 1 : \frac{2c^2r}{gt^2} = \frac{16c^2}{g} = \frac{16 \times 3{\cdot}1416^2}{16\frac{1}{12}} = 9{\cdot}819$$

$=$ the centrifugal force, or that by which the ſtring is ſtretched, viz, nearly 10 ounces, or 10 times the weight of the ball.

Laſtly, ſuppoſe the ſtring and ball be ſuſpended from a point D, and deſcribes in its motion a conical ſurface ADB; then putting $DC = a$, $AC = r$, and $AD = b$; and putting $F = 1$ the force of gravity as before; then will the body A be affected by three forces, viz, gravity acting parallel to DC, a centrifugal force in the direction CA, and the tenſion of the ſtring, or force by which it is ſtretched, in the direction DA; hence theſe three powers will be as the three ſides of the triangle ADC reſpectively, and therefore as CD or a : AD or $b :: 1 : \frac{b}{a}$ the tenſion of the ſtring as compared with the weight of the body. Alſo CD or a : AC or $r :: 1 : \frac{2c^2r}{gt^2}$ the general expreſſion for the centrifugal force above-found;

hence, $gt^2 = 2ac^2$ and ſo $t = c\sqrt{\frac{2a}{g}} = 1{\cdot}108\sqrt{a} =$ the periodic time. And



10. When the force by which a body is urged towards a point is not always the ſame, but is either increaſed or decreaſed as ſome power of the diſtance; ſeveral curves will thence ariſe according to that power. If the force decreaſe as the ſquares of the diſtances increaſe, the body will deſcribe an ellipſis, and the force is directed towards one of its foci; ſo that in every revolution the body once approaches towards it, and once recedes from it: alſo the eccentricity of the ellipſe is greater or leſs, according to the projectile force; and the curve may ſometimes become a circle, when the eccentricity is nothing; the body may alſo deſcribe the other two conic ſections, the parabola and hyperbola, which do not return into themſelves, by ſuppoſing the velocity greater in certain proportions. Alſo if the force increaſe in the ſimple ratio as the diſtance increaſes, the body will ſtill deſcribe an ellipſe; but the force will in this caſe be directed to the centre of the ellipſe; and the body, in each revolution, will twice approach towards it, and again twice recede from that point.

Central Rule, is a rule or method diſcovered by Mr. Thomas Baker, rector of Nympton in Devonſhire, which he publiſhed in his Geometrical Key, in the year 1684, for determining the centre of a circle which ſhall cut a given parabola in as many points as a given equation, to be conſtructed, has real roots; which he has applied with good ſucceſs in the conſtruction of all equations as far as the 4th power incluſive.

The *Central Rule* is chiefly founded on this property of the parabola; that if a line be inſcribed in the curve perpendicular to any diameter, the rectangle of the ſegments of this line, is equal to the rectangle of the intercepted part of the diameter and the parameter of the axis.

The Central Rule has the advantage over the methods of conſtructing equations by Des Cartes and De Latteres, which are liable to the trouble of preparing the equations by taking away the ſecond term; whereas Baker's method effects the ſame thing without any previous preparation whatever. See alſo Philoſ. Tranſ. No 157.

CENTRE, or Center, in a general ſenſe, ſignifies a point equally remote from the extremes of a line, plane, or ſolid; or a middle point dividing them ſo that ſome certain effects are equal on all ſides of it.

Centre *of Attraction*, or *Gravitation*, is the point to which bodies tend by gravity; or that point to which a revolving planet or comet is impelled or attracted, by the force or impetus of gravity.

Centre *of a Baſtion*, is a point in the middle of the gorge, where the capital line commences, and which is uſually at the angle of the inner polygon of the figure. Or it is the point where the two adjacent curtains produced interſect each other.

Centre *of a Circle*, is the point in the middle of a circle, or circular figure, from which all lines drawn to the circumference are equal.

Centre *of a Conic Section*, is the middle point of any diameter, or the point in which all the diameters interſect and biſect one another.

In the ellipſe the centre is within the figure; but in the hyperbola it is without, or between the conjugate hyperbolas; and in the parabola it is at an infinite diſtance from the vertex.

Centre *of Converſion*, in Mechanics, a term firſt uſed by M. Parent, and may be thus conceived: Suppoſe a ſtick laid on ſtagnant water, and then drawn by a thread faſtened to it, ſo that the thread always makes the ſame angle with the ſtick, either a right angle or any other; then it will be found that the ſtick will turn about one point of it, which will be immoveable; and this point is termed the centre of converſion.

This effect ariſes from the reſiſtance of the fluid to the ſtick partly immerſed in it. And if, inſtead of the body thus floating on a fluid, the ſame be conceived to be laid on the ſurface of another body; then the reſiſtance of this plane to the ſtick will always have the ſame effect, and will determine the ſame centre of converſion. And this reſiſtance is preciſely what is called friction, ſo prejudicial to the effects of machines.

M. Parent has determined this centre in ſome certain caſes, with much laborious calculation. When the thread is faſtened to the extremity of the ſtick, he found that the diſtance of the centre from this extremity would be nearly $\frac{1}{10}$ of the whole length. But when it is a ſurface or a ſolid, there will be ſome change in the place of this centre, according to the nature of the figure. See Mem. of the Acad. of Sciences, vol. 1, pa. 191.

Centre *of a Curve*, of the higher kind, is the point where two diameters meet.—When all the diameters meet in the ſame point, it is called, by Sir Iſaac Newton, the *general centre*.

Centre *of a Dial*, is the point where its gnomon or ſtile, which is placed parallel to the axis of the earth, meets the plane of the dial; and from hence all the hour-lines are drawn, in ſuch dials as have centres, viz, all except that whoſe plane is parallel to the axis of the world; all the hour-lines of which are parallel to the ſtile, and to one another, the centre being as it were at an infinite diſtance.

Centre *of an Ellipſe*, is the middle of any diameter, or the point where all the diameters interſect.

Centre *of the Equant*, in the Old Aſtronomy, is a point in the line of the aphelion; being as far diſtant from the centre of the eccentric, towards the aphelion, as the ſun is from the ſame centre of the eccentric towards the perihelion.

Centre *of Equilibrium*, is the ſame with reſpect to bodies immerſed in a fluid, as the centre of gravity is to bodies in free ſpace; being a certain point, upon which if the body or bodies be ſuſpended, they will reſt in any poſition. To determine this centre, ſee Emmerſon's Mechanics, prop. 92, pa. 134.

Centre *of Friction*, is that point in the baſe of a body on which it revolves, into which if the whole ſurface of the baſe, and the maſs of the body were collected, and made to revolve about the centre of the baſe of the given body, the angular velocity deſtroyed by its friction would be equal to the angular velocity deſtroyed in the given body by its friction in the ſame time.—See Vince on the Motion of Bodies affected by friction, in the Philoſ. Tranſ. 1785.

Centre *of Gravity*, is that point about which all the parts of a body do in any ſituation exactly balance each other. Hence, by means of this property, if the body be ſupported or ſuſpended by this point, the body will reſt in any poſition into which it is put; as alſo that if a plane paſs through the ſame point, the ſegments on each ſide will equiponderate, neither of them being able to move the other.

The whole gravity, or the whole matter, of a body may be conceived united in the centre of gravity; and in demonſtrations it is uſual to conceive all the matter as really collected in that point.

Through the centre of gravity paſſes a right line, called the *diameter of gravity*; and therefore the interſection of two ſuch diameters determines the centre. Alſo the plane upon which the centre of gravity is placed, is called the *plane of gravity*; ſo that the common interſection of two ſuch planes determines the diameter of gravity.

In homogeneal bodies, which may be divided lengthways into ſimilar and equal parts, the centre of gravity is the ſame with the centre of magnitude. Hence therefore the centre of gravity of a line is in the middle point of it, or that point which biſects the line. Alſo the centre of gravity of a parallelogram, or cylinder, or any priſm whatever, is in the middle point of the axis. And the centre of gravity of a circle or any regular figure, is the ſame as the centre of magnitude.

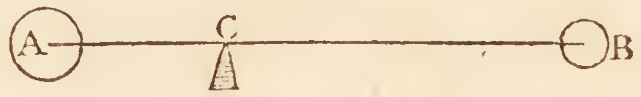
Alſo, if a line can be ſo drawn as to divide a plane into equal and ſimilar parts, that line will be a diameter of gravity, or will paſs through the centre of gravity; and it is the ſame as the axis of the plane. Thus the line drawn from the vertex and perpendicular to the baſe of the iſoſceles triangle, is a diameter of gravity; and thus alſo the axis of an ellipſe, or a parabola, &c, is a diameter of gravity. The centre of gravity of a ſegment or arc of a circle, is in the radius or line perpendicularly biſecting its chord or baſe.

Likewiſe, if a plane divide a ſolid in the ſame manner, making the parts on both ſides of it perfectly equal and ſimilar in all reſpects, it will be a plane of gravity, or will paſs through the centre of gravity. Thus, as the interſection of two ſuch planes determines the diameter of gravity, the centre of gravity of a right cone, or ſpherical ſegment, or conoid, &c, will be in the axis of the ſame.

Common Centre of Gravity of two or more bodies, or the different parts of the ſame body, is ſuch a point as that, if it be ſuſpended or ſupported, the ſyſtem of bodies will equiponderate, and reſt in any poſition. Thus, the point of ſuſpenſion in a common balance beam, or ſteelyard, is the centre of gravity of the ſame.

Laws and Determination of the Centre of Gravity.

1. In two equal bodies, or maſſes, the centre of gravity is equally diſtant from their two reſpective centres. For theſe are as two equal weights ſuſpended at equal diſtances from the point of ſuſpenſion; in which caſe they will equiponderate, and reſt in any poſition.

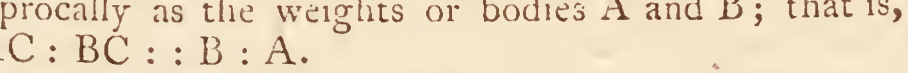


2. If the centres of gravity of two bodies A and B be connected by the right line AB, the diſtances AC and BC from the common centre of gravity C, are reciprocally as the weights or bodies A and B; that is, AC : BC : : B : A.

See this demonſtrated under the article Balance.

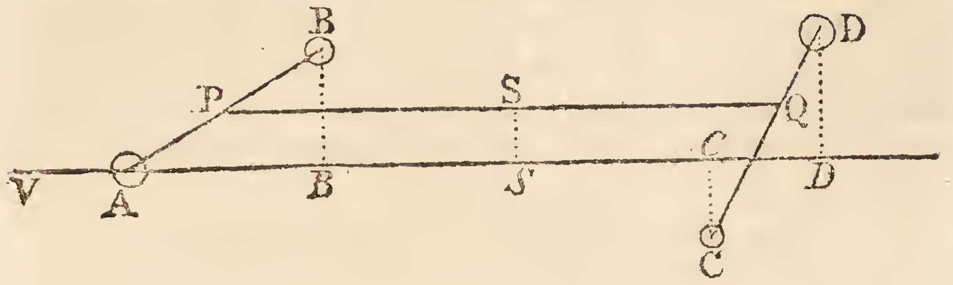
Hence, if the weights of the bodies A and B be equal, their common centre of gravity C will be in the middle of the right line AB, as in the foregoing article. Alſo ſince A : B : : BC : AC, therefore A × AC = B × BC; whence it appears that the powers of equiponderating bodies are to be eſtimated by the product of the maſs multiplied by the diſtance from the centre of gravity; which product is uſually called the *momentum* of the weights.

Further, from the foregoing proportion, by compoſition it will be A + B : A : : AB : BC, or A + B : B : : AB : AC. So that the common centre of gravity C of two bodies will be found, if the product of one weight by the whole diſtance between the two, be divided by the ſum of the two weights. Suppoſe, for example, that A = 12 pounds, B = 4lb, and AB = 36 inches; then 16 : 12 : : 36 : 27 = BC, and conſequently AC = 9, the two diſtances from the common centre of gravity.



3. *The Common Centre of Gravity of three or more given bodies or points* A, B, C, D, &c, will be thus determined.—If the given bodies lie all in the ſame ſtraight line AD; by the laſt article, find P the centre of gravity of the two A and B, and Q the centre of gravity of C and D; then, conſidering P as the place of a body equal to the ſum of A and B, and Q as the place of another body equal to both C and D, find S the common centre of gravity of theſe two ſums, viz A + B collected in P, and C + D united in Q; ſo ſhall S be the common centre of gravity of all the four bodies A, B, C, D. And the ſame for any other number of bodies, always conſidering the ſum of any number of them as united or placed in their common centre of gravity, when found.

Otherwiſe, thus. Take the diſtances of the given bodies from ſome fixed point as V, calling the diſtance $VA = a$, $VB = b$, $VC = c$, $VD = d$, and the diſtance of the centre of gravity $VS = x$; then $SA = x-a$, $SB = x-b$, $SC = c-x$, $SD = d-x$, and by the nature of the lever $A \cdot \overline{x-a} + B \cdot \overline{x-b} = C \cdot \overline{c-x} + D \cdot \overline{d-x}$; hence $Ax + Bx + Cx + Dx = Aa + Bb + Cc + Dd$, and $x = \frac{Aa + Bb + Cc + Dd}{A + B + C + D} = VS$ the diſtance ſought; which therefore is equal to the ſum of all the momenta, divided by the ſum of all the weights or bodies.

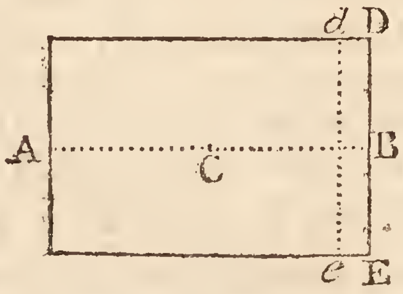


Or thus. When the bodies are not in the ſame ſtraight line, connect them with the lines AB, CD; then, as before, find P the common centre of A and B, and Q the common centre of C and D; then, conceiving A and B united in P, and C and D united in Q, find S the common centre of P and Q, which will again be the centre of gravity of the whole.

Or the bodies may be all reduced to any line V*AB* &c, drawn in any direction whatever, by perpendiculars B*B*, C*C*, &c, and then the common centre *S* in this line, found as before, will be at the ſame diſtance from V as the true centre S is; and conſequently the perpendicular from *S* will paſs through S the real centre.

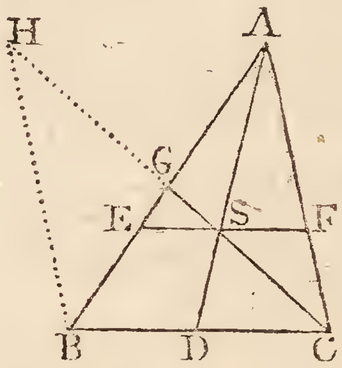
4. From the foregoing general expreſſion, viz, $x = \frac{Aa + Bb + Cc + \&c}{A + B + C + \&c}$, for the centre of gravity of any ſyſtem of bodies, may be derived a general method for finding that centre; for A, B, C, &c, may be conſidered as the elementary parts of any body, whoſe ſum or maſs is M = A + B + C &c, and A*a*, B*b*, C*c*, &c, are the ſeveral momenta of all theſe parts, viz, the product of each part multiplied by its diſtance from the fixed point V. Hence then, in any body, find a general expreſſion for the ſum of the momenta, and divide it by the content of the body, ſo ſhall the quotient be the diſtance of the centre of gravity from the vertex, or from any other fixed point, from which the momenta are eſtimated.

5. *Thus, in a right line* AB, all the particles which compoſe it may be conſidered as ſo many very ſmall weights, each equal to $\dot{x}$, which is therefore the fluxion of the weights, or of the line denoted by x. So that the ſmall weight $\dot{x}$ multiplied by its diſtance from A, viz x, is $x\dot{x}$ the momentum of that weight $\dot{x}$; that is, $x\dot{x}$ is the fluxion of all the momenta in the line AB or x; and therefore its fluent $\frac{1}{2}x^2$ is the ſum of all thoſe momenta; which being divided by x the ſum of all the weights, gives $\frac{1}{2}x$ or $\frac{1}{2}$AB for the diſtance of the centre of gravity C from the point A; that is, the centre is in the middle of the line.



6. *Alſo in the parallelogram,* whoſe axis or length AB = x, and its breadth DE = b; drawing de parallel and indefinitely near DE, the areola $d$DE$e$ = $b\dot{x}$ will be the fluxion of all the weights, which multipled by its diſtance x from the point A, gives $bx\dot{x}$ for the fluxion of all the momenta, and conſequently the fluent $\frac{1}{2}bx^2$ is the ſum of all thoſe momenta themſelves; which being divided by bx the ſum of all the weights, gives $\frac{1}{2}x = \frac{1}{2}$AB for the diſtance of the centre C from the extremity at A, and is therefore in the middle of the axis, as is known from other principles.

And the proceſs and concluſion will be exactly the ſame for a cylinder, or any priſm whatever, making b to denote the area of the end or of a tranſverſe ſection of the body.

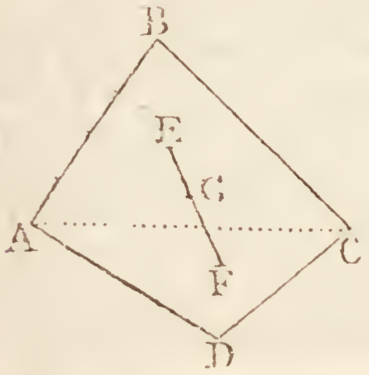


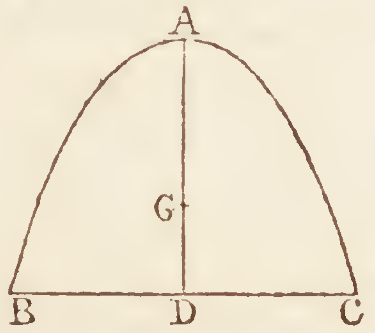
7. *In a Triangle* ABC; the line AD drawn from one angle to biſect the oppoſite ſide, will be a diameter of gravity, or will paſs through the centre of gravity; for if that line be ſupported, or conceived to be laid upon the edge of ſomething, the two halves of the triangle on both ſides of that line will juſt balance one another, ſince all the parallels EF &c to the baſe will be biſected, as well as the baſe itſelf, and ſo the two halves of each line will juſt balance each other. Therefore, putting the baſe BC = b, and the axis or biſecting line AD = a, the variable part AS = x; then, by ſimilar triangles AD : BC :: AS : EF, that is $a : b :: x : \frac{bx}{a}$ = EF; which, as a weight, multiplied by $\dot{x}$, gives $\frac{bx\dot{x}}{a}$ for the fluxion of the weights; and this again multiplied by x = AS, the diſtance from A, gives $\frac{bx^2\dot{x}}{a}$ for the fluxion of the momenta; the fluent of which, or $\frac{bx^3}{3a}$ divided by $\frac{bx^2}{a}$ the fluent for the weights, gives $\frac{2}{3}x = \frac{2}{3}$AS for the diſtance of the centre of gravity from the vertex A in the triangle AEF; and when x = AD, then $\frac{2}{3}$AD is the diſtance of the centre of gravity of the triangle ABC.

The Same Otherwiſe, without Fluxions.—Since a line drawn from any angle to the middle of the oppoſite ſide paſſes through the centre of gravity, therefore the interſection of any two of ſuch lines, will be that centre: thus then the centre of gravity is in the line AD; and it is alſo in the line CG biſecting AB; it is therefore in their interſection S. Now to determine the diſtance of S from any angle, as A, produce CG to meet BH parallel to AS in H; then the two triangles AGS, BGH are mutually equal and ſimilar; for the oppoſite angles at G are equal, as are the alternate angles at H and S, and at A and B, alſo the ſide AG = BG; therefore the other ſides BH, AS are equal. But the triangles CDS, CBH are ſimilar, and the ſide CB = 2CD, therefore BH or its equal AS = 2DS, that is AS = $\frac{2}{3}$AD, the ſame as was found before. And in like manner CS = $\frac{2}{3}$ CG.

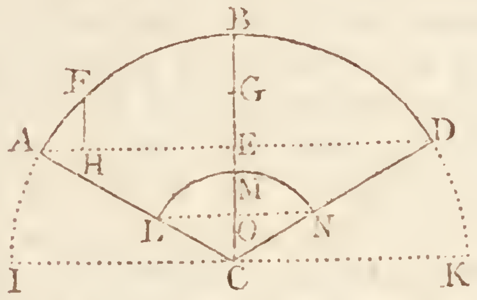
8. *In a Trapezium.* Divide the figure into two triangles by the diagonal AC, and find the centres of gravity E and F of theſe triangles; join EF, and find the common centre G of theſe two by this proportion, ABC : ADC : : FG : EG, or ABCD : ADC : : EF : EG.

In like manner, for any other figure, whatever be the number of ſides, divide it into ſeveral triangles, and find the centre of gravity of each; then connect two centres together, and find their common centre as above; then connect this and the centre of a third, and find the common centre of theſe; and ſo on, always connecting the laſt found common centre to another centre, till the whole are included in this proceſs; ſo ſhall the laſt common centre be that which is required.





9. *In the Parabola* BAC. Put AD $= x$, BD $= y$, and the parameter $= p$. Then, by the nature of the figure, $px = y^2$, and $2y = 2\sqrt{px}$; hence $2\dot{x}\sqrt{px}$ is the fluxion of the weights, and $2x\dot{x}\sqrt{px}$ is the fluxion of the momenta; then the fluent of the latter divided by that of the former, or $\frac{4}{5}x^{\frac{5}{2}}\sqrt{p}$ divided by $\frac{4}{3}x^{\frac{3}{2}}\sqrt{p}$, gives $\frac{3}{5}x = \frac{3}{5}$AD, for AG, the diſtance of the centre of gravity G from the vertex A of the parabola.

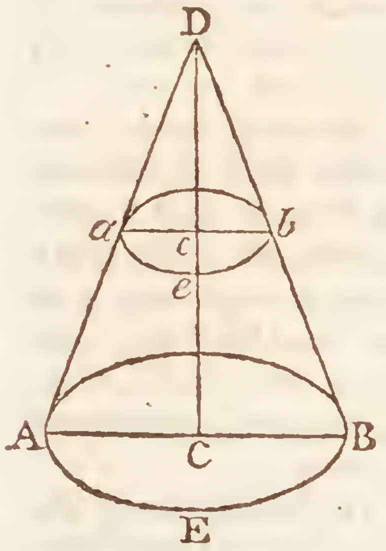


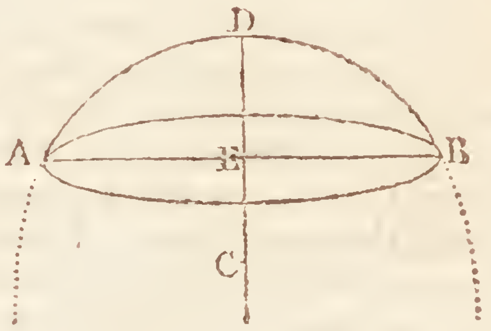
10. *In the Circular Arc* ABD, conſidered as a phyſical line having gravity. It is manifeſt that the centre of gravity G of the arc, will be ſomewhere in the axis, or middle radius BC, C being the centre of the circle, which is conſidered as the point of ſuſpenſion. Suppoſe F indefinitely near to A, and FH parallel to BC. Put the radius BC or AC $= r$, the ſemiarc AB $= z$, and the ſemichord AE $= x$; then is AH $= \dot{x}$, and AF $= \dot{z}$ the fluxion of the weights, and therefore CE $\times \dot{z}$ is the fluxion of the momenta. But, by ſimilar triangles, AC or r : CE : : AF or $\dot{z}$: AH or $\dot{x}$, therefore $r\dot{x} =$ CE $\times \dot{z}$, and ſo $r\dot{x}$ is alſo the fluxion of the momenta; the fluent of which is rx, and this divided by z the weight, gives $\frac{rx}{z} = \frac{\text{AC} \times \text{AE}}{\text{AB}} = \frac{\text{AC} \times \text{AED}}{\text{ABD}} =$ CG the diſtance of the centre of gravity from the centre C of the circle; being a 4th proportional to the given arc, its chord, and the radius of the circle.

Hence, when the arc becomes the ſemicircle ABK, the above expreſſion becomes $\frac{\text{IC}^2}{\text{IB}}$ or $\frac{r^2}{1{\cdot}5708r} = \frac{r}{1{\cdot}5708} = {\cdot}6366r$, viz a third proportional to a quadrant and the radius.

11. *In the Circular Sector* ABDC. Here alſo the centre of gravity will be in the axis or middle radius BC. Now with any ſmaller radius deſcribe the concentric arc LMN, and put the radius AC or BC $= r$, the arc ABD $= a$, its chord AED $= c$, and the variable radius CL or CM $= y$; then as $r : y : : a : \frac{ay}{r} =$ the arc LMN, and $r : y : : c : \frac{cy}{r} =$ the chord LON; alſo, by the laſt article, the diſtance of the centre of gravity of the arc LMN is $\frac{\text{CM} \times \text{LON}}{\text{LMN}} = \frac{\text{CM} \times \text{AED}}{\text{ABD}} = \frac{cy}{a}$; hence the arc LMN or $\frac{ay}{r}$ multiplied by $\dot{y}$ gives $\frac{ay\dot{y}}{r}$ the fluxion of the weights, and this multiplied by $\frac{cy}{a}$ the diſtance of the common centre of gravity, gives $\frac{cy^2\dot{y}}{r}$ the fluxion of the momenta; the fluent of which, viz $\frac{cy^3}{3r}$, divided by $\frac{ay^2}{2r}$, the fluent of the weights, gives $\frac{2cy}{3a}$ for the diſtance of the centre of gravity of the ſector CLEN from the centre C; and when $y = r$, it becomes $\frac{2cr}{3a} =$ CG for that of the ſector CABD propoſed; being $\frac{2}{3}$ of a 4th proportional to the arc of the ſector, its chord, and the radius of the circle.

Hence, when the ſector becomes a ſemicircle, the laſt expreſſion becomes $\frac{4r^2}{3a} = \frac{2\text{IC}^2}{3\text{IB}}$ or $\frac{2}{3}$ of a 3d proportional to a quadrantal arc and the radius. Or it is equal to $\frac{4r}{3p} = {\cdot}4244r$ from the centre C; where $p = 3{\cdot}1416$.





12. *In the Cone* ADB. Putting $a =$ DC, $b =$ area of the baſe AEB, and $x =$ Dc any variable altitude; then as $a^2 : x^2 : : b : \frac{bx^2}{a^2} =$ area aeb; hence the flux-

ion of the weights is $\frac{bx^2\dot{x}}{a^2}$, whoſe fluent, or the ſolid, is $\frac{bx^3}{3a^2}$; and the fluxion of the momenta is $\frac{bx^3\dot{x}}{a^2}$, whoſe fluent is $\frac{bx^4}{4a^2}$; then this fluent divided by the former fluent gives $\frac{3}{4}x$ or $\frac{3}{4}\mathrm{D}c$ for the diſtance of the centre of gravity of the cone D*ab*, or $\frac{3}{4}$DC for that of the cone DAB below the vertex D.

And the ſame is the diſtance in any other pyramid. So that all pyramids of the ſame altitude, have the ſame centre of gravity.

13. In like manner are we to proceed for the centre of gravity in other bodies. Thus, the altitude of the ſegment of a ſphere, or ſpheroid, or conoid, being *x*, *a* being the whole of that axis itſelf; then the diſtance of the centre of gravity in each of theſe bodies, from the vertex, will be as follows, viz,

$\frac{4a-3x}{6a-4x}x$ in the ſphere or ſpheroid,
$\frac{5}{8}x$ in the ſemiſphere or ſemiſpheroid,
$\frac{2}{3}x$ in the parabolic conoid,
$\frac{4a+3x}{6a+3x}x$ in the hyperbolic conoid.

14. *To determine the Centre of Gravity in any Body Mechanically.* Lay the body on the edge of any thing, as a triangular priſm, or ſuch like, moving it backward and forward till the parts on both ſides are in equilibrio; then is that line juſt in, or under the centre of gravity. Balance it again in another poſition, to find another line paſſing through the centre of gravity; then the interſection of theſe two lines will give the place of that centre itſelf.

The ſame may be done by laying the body on an horizontal table, as near the edge as poſſible without its falling, and that in two poſitions, as lengthwiſe and breadthwiſe: then the common interſection of the two lines contiguous to the edge, will be its centre of gravity. Or it may be done by placing the body on the point of a ſtyle, &c, till it reſt in equilibrio. It was by this method that Borelli found that the centre of gravity in a human body, is between the nates and pubis; ſo that the whole gravity of the body is collected into the place of the genitals; an inſtance of the wiſdom of the creator, in placing the membrum virile in the part, which is the moſt convenient for copulation.

The ſame otherwiſe thus. Hang the body up by any point; then a plumb line hung over the ſame point, will paſs through the centre of gravity; becauſe that centre will always deſcend to the loweſt point when the body comes to reſt, which it cannot do except when it falls in the plumb line. Therefore, marking that line upon it, and ſuſpending the body by another point, with the plummet, to find another ſuch line, the interſection of the two will give the centre of gravity.

Or thus. Hang the body by two ſtrings from the ſame tack, but fixed to different points of the body; then a plummet, hung by the ſame tack, will fall on the centre of gravity.

In the 4th volume of the New Acts of the Academy of Peterſburgh, is the demonſtration of a very general theorem concerning centres of gravity, by M. Lhuilier; a particular example only of the general propoſition, will be as follows: Let A, B, C, be the centres of gravity of three bodies; *a*, *b*, *c* their reſpective maſſes, and Q their common centre of gravity. Let right lines QA, QB, QC, be drawn from the common centre to that of each body, and the latter be connected by right lines AB, AC, and BC; then

$$\mathrm{QA}^2 \times a + \mathrm{QB}^2 \times b + \mathrm{QC}^2 \times c = \mathrm{AB}^2 \times \frac{ab}{a+b+c} + \mathrm{AC}^2 \times \frac{ac}{a+b+c} + \mathrm{BC}^2 \times \frac{bc}{a+b+c}.$$

Uſes of the Centre of Gravity. This point is of the greateſt uſe in mechanics, and many important concerns in life, becauſe the place of that centre is to be conſidered as the place of the body itſelf in computing mechanical effects; as in the oblique preſſures of bodies, banks of earth, arches of bridges, and ſuch like.

The ſame centre is even uſeful in finding the ſuperficial and ſolid contents of bodies; for it is a general rule, that the ſuperficies or ſolid generated by the rotation of a line or plane about any axis, is always equal to the product of the ſaid line or plane drawn into the circumference or path deſcribed by the centre of gravity. For example, it was found above at art. 11, that in a ſemicircle, the diſtance of the centre of gravity from the centre of the circle, is $\frac{4r}{3p}$; and therefore the path of that centre, or circumference deſcribed by it whilſt the ſemicircle revolves about its diameter, is $\frac{8}{3}r$; alſo the area of the ſemicircle is $\frac{1}{2}pr^2$; hence the product of the two is $\frac{4}{3}pr^3$, which, it is well known, is equal to the ſolidity of the ſphere generated by the revolution of the ſemicircle.

And hence alſo is obtained another method of finding mathematically the centre of gravity of a line or plane, from the contents of the ſuperficies or ſolid generated by it. For if the generated ſuperficies or ſolid be divided by the generating line or plane, the quotient will be the circumference deſcribed by the centre of gravity; and conſequently this divided by $2p$ gives the radius, or diſtance of that centre from the axis of rotation. So, in the ſemicircle, whoſe area is $\frac{1}{2}pr^2$, and the content of the ſphere generated by it $\frac{4}{3}pr^3$; here the latter divided by the former is $\frac{8}{3}r$, and this divided by $2p$ gives $\frac{4r}{3p}$ for the diſtance of the centre of gravity from the axis, or from the centre of the ſemicircle. The property laſt mentioned, relative to the relation between the centre of gravity and the figure generated by the revolution of any line or plane, is mentioned by Pappus, in the preface to his 7th book; and father Guldin has more fully demonſtrated it in his 2d and 3d books on the Centre of Gravity.

The principal writers on the centre of gravity are Archimedes, Pappus, Guldini, Wallis, Caſatus, Carré, Hays, Wolfius, &c.

Centre *of Gyration*, is that point in which if the whole maſs be collected, the ſame angular velocity will be generated in the ſame time, by a given force acting at any place, as in the body or ſyſtem itſelf. This point differs from the centre of oſcillation, in as much as in this latter caſe the motion of the body is produced by the gravity of its own particles, but in the caſe of the centre of gyration the body is put in motion by ſome other force acting at one place only.

To determine the Centre of Gyration, in any body, or ſyſtem of bodies compoſed of the parts A, B, C, &c, moving about the point S, when urged by a force f acting at any point P. Let R be that centre: then, by mechanics, the angular velocity generated in the ſyſtem by the force f, is as $\frac{f \cdot SP}{A \cdot SA^2 + B \cdot SB^2 + C \cdot SC^2 \,\&c}$, and, by the ſame, the angular velocity of the matter placed all in the point R, is $\frac{f \cdot AP}{\overline{A + B + C \,\&c} \times SR^2}$; then ſince theſe two are to be equal, their equation will give $SR = \sqrt{\frac{A \cdot SA^2 + B \cdot SB^2 + C \cdot SC^2 \,\&c}{A + B + C \,\&c}}$, for the diſtance of the centre of gyration ſought, below the axis of motion.

Now becauſe the quantity $A \cdot SA^2 + B \cdot SB^2 + \&c$ is $= SG \cdot SO \cdot b$, where G is the centre of gravity, O the centre of oſcillation, and b the whole body or ſum of A, B, C, &c; therefore it follows that $SR^2 = SG \cdot SO$; that is, the diſtance of the centre of gyration, is a mean proportional between thoſe of gravity and oſcillation.

And hence alſo, if p denote any particle of a body, placed at the diſtance d from the axis of motion; then is $SR = \frac{\text{ſum of all the } pd^2}{\text{body } b}$; from whence the point R may be determined in bodies by means of Fluxions.

CENTRE *of an Hyperbola*, is the middle of the axis, or of any other diameter, being the point without the figure in which all the diameters interſect one another; and it is common to all the four conjugate hyperbolas.

CENTRE *of Magnitude*, is the point which is equally diſtant from all the ſimilar external parts of a body. This is the ſame as the centre of gravity in homogeneal bodies that can be cut into like and equal parts according to their length, as in a cylinder or any other priſm.

CENTRE *of Motion*, is the point about which any body, or ſyſtem of bodies, moves, in a revolving motion.

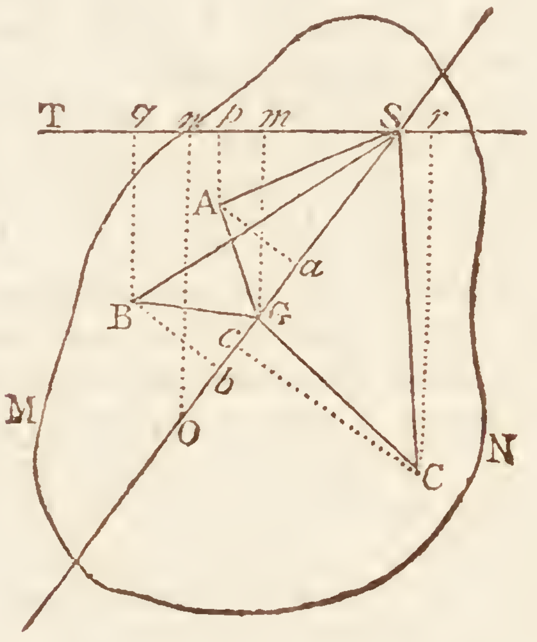
CENTRE *of Oſcillation*, is that point in the axis or line of ſuſpenſion of a vibrating body, or ſyſtem of bodies, in which if the whole matter or weight be collected, the vibrations will ſtill be performed in the ſame time, and with the ſame angular velocity, as before. Hence, in a compound pendulum, its diſtance from the point of ſuſpenſion is equal to the length of a ſimple pendulum whoſe oſcillations are iſochronal with thoſe of the compound one.

Mr. Huygens, in his Horologium Oſcillatorium, firſt ſhewed how to find the centre of oſcillation. At the beginning of his diſcourſe on this ſubject, he ſays, that Merſennus firſt propoſed the problem to him while he was yet very young, requiring him to reſolve it in the caſes of ſectors of circles ſuſpended by their angles, and by the middle of their baſes, both when they oſcillate ſideways and flatways; as alſo for triangles and the ſegments of circles, either ſuſpended from their vertex or the middle of their baſes. But, ſays he, not having immediately diſcovered any thing that would open a paſſage into this buſineſs, I was repulſed at firſt ſetting out, and ſtopped from a further proſecution of the thing; till being farther incited to it by adjuſting the motion of the pendulums of my clock, I ſurmounted all difficulties, going far beyond Deſcartes, Fabry, and others, who had done the thing in a few of the moſt eaſy caſes only, without any ſufficient demonſtration; and ſolving not only the problems propoſed by Merſennus, but many others that were much more difficult, and ſhewing a general way of determining this centre, in lines, ſuperficies, and ſolids.

In the Leipſic Acts for 1691 and 1714, this doctrine is handled by the two Bernoullis: and the ſame is alſo done by Herman, in his treatiſe De Motu Corporum Solidorum et Fluidorum.

It may alſo be ſeen in treatiſes on the Inverſe Method of Fluxions, where it is introduced as one of the examples of that method. See Hayes, Carré, Wolfius, &c.

To determine the Centre of Oſcillation, in any Compound Maſs or Body MN, or of any Syſtem of Bodies A, B, C, &c.



Let MN be the plane of vibration, to which plane conceive all the matter to be reduced by letting fall perpendiculars to this plane from every particle in the body; a ſuppoſition which will not alter the vibration of the body, becauſe the particles are ſtill at the ſame diſtance from the axis of motion. Let O be the centre of oſcillation, and G the centre of gravity; through the axis S draw SGO, and the horizontal line ST; then from every particle A, B, C, &c, let fall perpendiculars Aa and Ap, Bb and Bq, Cc and Cr, &c, to theſe two lines; and join SA, SB, SC; alſo draw Gm and On perpendicular to ST.

Now the forces of the weights A, B, C, to turn the body about the axis, are $A \cdot Sp$, $B \cdot Sq$, $-C \cdot Sr$; and, by mechanics, the forces oppoſing that motion are $A \cdot SA^2$, $B \cdot SB^2$, $C \cdot SC^2$; therefore the angular motion generated in the ſyſtem is $\frac{A \cdot Sp + B \cdot Sq - C \cdot Sr}{A \cdot SA^2 + B \cdot SB^2 + C \cdot SC^2}$; In like manner, the angular velocity which any body or particle p, ſituated in O, generates in the ſyſtem, by its weight, is $\frac{p \cdot Sn}{p \cdot SO^2}$, or $\frac{Sn}{SO^2}$, or $\frac{Sm}{SG \cdot SO}$ becauſe of the ſimilar triangles SGm, SOn. But, by the conditions of the problem, the vibrations are performed alike in both theſe caſes; therefore theſe two expreſſions muſt be equal to each other, that is $\frac{Sm}{SG \cdot SO} = \frac{A \cdot Sp + B \cdot Sq - C \cdot Sr}{A \cdot SA^2 + B \cdot SB^2 + C \cdot SC^2}$, and conſequently $SO = \frac{Sm}{SG} \times \frac{A \cdot SA^2 + B \cdot SB^2 + C \cdot SC^2}{A \cdot Sp + B \cdot Sq - C \cdot Sr}$. But, by mechanics again, the ſum of the forces $A \cdot Sp + B \cdot Sq - C \cdot Sr$ is equal $\overline{A + B + C} \cdot Sm$ the force of the ſame matter collected all into its centre of gravity G; and therefore $SO = \frac{A \cdot SA^2 + B \cdot SB^2 + C \cdot SC^2}{(A + B + C) \cdot SG}$, which is the diſtance of the centre of oſcillation O below the axis of ſuſpenſion.

Farther, becauſe it was found under the article *Centre of Gravity*, that $(A+B+C) \cdot SG = A \cdot Sa + B \cdot Sb + C \cdot Sc$, therefore $SO = \frac{A \cdot SA^2 + B \cdot SB^2 + C \cdot SC^2}{A \cdot Sa + B \cdot Sb + C \cdot Sc}$ is the ſame diſtance of the centre of oſcillation; where any of the products $A \cdot Sa$, $B \cdot Sb$, &c are to be taken negatively when the points a, b, &c lie above the point S, or where the axis paſſes through.

Again, becauſe, by Eucl. II 12 and 13,
it is $SA^2 = SG^2 + GA^2 - 2SG \cdot Ga$,
and $SB^2 = SG^2 + GB^2 + 2SG \cdot Gb$,
and $SC^2 = SG^2 + GC^2 + 2SG \cdot Gc$, &c;

and becauſe by Mechanics, the ſum of the laſt terms is nothing, namely $-2SG \cdot Ga + 2SG \cdot Gb + 2SG \cdot Gc$ &c $= 0$; therefore the ſum of the others, or $A \cdot SA^2 + B \cdot SB^2$ &c $= \overline{A + B \&c.} SG^2 + A \cdot GA^2 + B \cdot GB^2 + C \cdot GC^2$ &c, or $= b \cdot SG^2 + A \cdot GA^2 + B \cdot GB^2 + C \cdot GC^2$ &c; where b denotes the body, or ſum $A + B + C$ &c of all the parts: this value then being ſubſtituted in the numerator of the 2d value of SO above-found, it becomes

$$SO = \frac{B \cdot SG^2 + A \cdot GA^2 + B \cdot GB^2 \ \&c}{b \cdot SG}, \text{ or}$$

$$SO = SG + \frac{A \cdot GA^2 + B \cdot GB^2 \ \&c.}{b \cdot SG}$$

From which it appears that the centre of oſcillation is always below the centre of gravity, and that the difference or diſtance between them is

$$GO = \frac{A \cdot GA^2 + B \cdot GB^2 \ \&c}{b \cdot SG}.$$

It farther follows from hence, that $SG \cdot GO = \frac{A \cdot GA^2 + B . GB^2 \&c}{\text{the body } b}$; that is, the rectangle $SG \cdot GO$ is always the ſame conſtant quantity, wherever the point of ſuſpenſion S is placed, ſince the point G and the bodies A, B, &c, are conſtant. Or GO is always reciprocally as SG, that is GO is leſs as SG is greater; and the points G and O coincide when SG is infinite; but when S coincides with G, then GO is infinite, or O is at an infinite diſtance.

To find the Centre of Oſcillation by means of Fluxions. From the premiſes is derived this general method for the centre of oſcillation, viz, let x be the abſciſſa of an oſcillating body, and y its correſponding ordinate or ſection; then will the diſtance SO of the centre of oſcillation below the axis of ſuſpenſion S, be equal to the fluent of yx^2x divided by the fluent of $yx\dot{x}$. So that, if from the nature or equation of any given figure, the value of y be expreſſed in terms of x, or otherwiſe, and ſubſtituted in theſe two fluxions; then the fluents being duly found, and the one divided by the other, the quotient will be the diſtance to the centre of oſcillation in terms of the abſciſs x.

But when the body is ſuſpended by a very fine thread of a given length a, then the fluent of $\overline{a + x}|^2 \cdot y\dot{x}$ divided by the fluent of $\overline{a + x} \cdot y\dot{x}$ gives the diſtance of the ſame centre of oſcillation below the point of ſuſpenſion.

Ex. For example, in a right line, or rectangle or cylinder or any other priſm, whoſe conſtant ſection is y, or the conſtant quantity a; then $yx^2\dot{x}$ is $ax^2\dot{x}$, whoſe fluent is $\frac{1}{3}ax^3$; alſo $yx\dot{x}$ is $ax\dot{x}$, whoſe fluent is $\frac{1}{2}ax^2$; and the quotient of the former $\frac{1}{3}ax^3$ divided by the latter $\frac{1}{2}ax^2$, is $\frac{2}{3}x$ for the diſtance of the centre of oſcillation below the vertex in any ſuch figure, namely having every where the ſame breadth or ſection, that is, at two-thirds of its length.

In like manner the centre of oſcillation is found for various figures, vibrating flatways, and are as they are expreſſed below, viz,

Nature of the Figure.	*When ſuſpended by Vertex.*
Iſoſceles triangle	$\frac{3}{4}$ of its altitude
Common Parabola	$\frac{5}{7}$ of its altitude
Any Parabola	$\frac{2m+1}{3m+1} \times$ its altitude.

As to figures moved laterally or ſideways, or edgeways, that is about an axis perpendicular to the plane of the figure, the finding the centre of oſcillation is ſomewhat difficult; becauſe all the parts of the weight in the ſame horizontal plane, on account of their unequal diſtances from the point of ſuſpenſion, do not move with the ſame velocity; as is ſhewn by Huygens, in his Horol. Oſcil. He found, in this caſe, the diſtance of the centre of oſcillation below the axis, viz,

In a circle, - -	$\frac{3}{4}$ of the diameter:
In a rectangle, ſuſp. by one angle,	$\frac{2}{3}$ of the diagonal:
In a parabola ſuſp. by its vertex,	$\frac{5}{7}$ axis $+ \frac{1}{3}$ param.
The ſame ſuſp. by mid. of baſe,	$\frac{4}{7}$ axis $+ \frac{1}{2}$ param.
In a ſector of a circle -	$\frac{3 \text{ arc} \times \text{radius}}{4 \text{ chord}}$:
In a cone -	$\frac{4}{5}$ axis $+ \frac{\text{radius baſe}^2}{5 \text{ axis}}$:
In a ſphere -	$g + \frac{2r^2}{5g}$, where r is

the radius, and $g = a + r$ the rad. added to the length of the thread.

See alſo Simpſon's Fluxions, art. 183 &c.

To find the Centre of Oſcillation Mechanically or Experimentally. Make the body oſcillate about its point of ſuſpenſion; and hang up alſo a ſimple pendulum of ſuch a length that it may vibrate or juſt keep time with the other body: then the length of the ſimple pendulum is equal to the diſtance of the centre of oſcillation of the body below the point of ſuſpenſion.

Or it will be ſtill better found thus: Suſpend the body very freely by the given point, and make it vibrate in ſmall arcs, counting the vibrations it makes in any portion of time, as a minute, by a good ſtop watch; and let that number of oſcillations made in a minute be called n: then ſhall the diſtance of the centre of oſcillation be $SO = \frac{140850}{nn}$ inches. For, the length of the pendulum vibrating ſeconds, or 60 times in a minute, being $39\frac{1}{8}$ inches, and the lengths of pendulums being reciprocally as the ſquare of the number of vibrations made in the ſame time, therefore $n^2 : 60^2 :: 39\frac{1}{8} : \frac{140850}{nn}$ the length of the pendulum which vibrates n times in a minute, or the diſtance of the centre of oſcillation below the axis of motion.

Centre *of Percuſſion*, in a moving body, is that point where the percuſſion or ſtroke is the greateſt, in which the whole percutient force of the body is ſuppoſed to be collected; or about which the impetus of the parts is balanced on every ſide, ſo that it may be ſtopt by an immoveable obſtacle at this point, and reſt on it, without acting on the centre of ſuſpenſion.

1. When the percutient body revolves about a fixed point, the centre of percuſſion is the ſame with the centre of oſcillation; and is determined in the ſame manner, viz, by conſidering the impetus of the parts as ſo many weights applied to an inflexible right line void of gravity; namely, by dividing the ſum of the products of the forces of the parts multiplied by their diſtances from the point of ſuſpenſion, by the ſum of the forces. And therefore what has been above ſhewn of the centre of oſcillation, will hold alſo of the centre of percuſſion when the body revolves about a fixed point. For inſtance, that the centre of percuſſion in a cylinder is at $\frac{2}{3}$ of its length from the point of ſuſpenſion, or that a ſtick of a cylindrical figure, ſuppoſing the centre of motion at the hand, will ſtrike the greateſt blow at a point about two-thirds of its length from the hand.

2. But when the body moves with a parallel motion, or all its parts with the ſame celerity, then the centre of percuſſion is the ſame as the centre of gravity. For the momenta are the products of the weights and celerities; and to multiply equiponderating bodies by the ſame velocity, is the ſame thing as to take equimultiples; but the equimultiples of equiponderating bodies do alſo equiponderate; therefore equivalent momenta are diſpoſed about the centre of gravity, and conſequently in this caſe the two centres coincide, and what is ſhewn of the one will hold in the other.

Centre of Percuſſion in a fluid, is the ſame as out of it.

Centre *of a Parallelogram*, the point in which its diagonals interſect.

Centre *of Preſſure*, of a fluid againſt a plane, is that point againſt which a force being applied equal and contrary to the whole preſſure, it will juſt ſuſtain it, ſo as that the body preſſed on will not incline to either ſide.—This is the ſame as the centre of percuſſion, ſuppoſing the axis of motion to be at the interſection of this plane with the ſurface of the fluid; and the centre of preſſure upon a plane parallel to the horizon, or upon any plane where the preſſure is uniform, is the ſame as the centre of gravity of that plane. Emerſon's Mechanics, prop. 91.

Centre *of a Regular Polygon*, or *Regular Body*, is the ſame as that of the inſcribed, or circumſcribed circle or ſphere.

Centre *of a Sphere*, is the ſame as that of its generating ſemicircle, or the middle point of the ſphere, from whence all right lines drawn to the ſuperficies, are equal.

Centring *of an Optic Glaſs*, the grinding it ſo as that the thickeſt part be exactly in the middle.

Caſſini the younger has a diſcourſe expreſsly on the neceſſity of well centring the object glaſs of a large teleſcope, that is, of grinding it ſo as that the centre may fall exactly in the axis of the teleſcope. Mem. Acad. 1710.

Indeed one of the greateſt difficulties in grinding large optic glaſſes is, that in figures ſo little convex, the leaſt difference will throw the centre two or three inches out of the middle. And yet Dr. Hook remarks, that though it were better the thickeſt part of a long object glaſs were exactly in the middle, yet it may be a very good one when it is an inch or two out of it. Philoſ. Tranſ. N° 4.

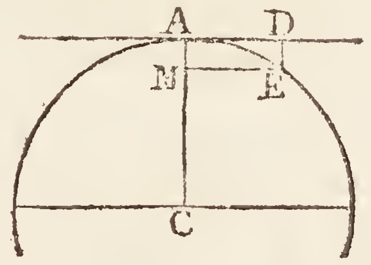
CENTRIFUGAL Force, is that by which a body revolving about a centre, or about another body, endeavours to recede from it. And

CENTRIPETAL Force, is that by which a moving body is perpetually urged towards a centre, and made to revolve in a curve, inſtead of a right line.

Hence, when a body revolves in a circle, theſe two forces, viz, the centrifugal and centripetal, are equal and contrary to each other, ſince neither of them gains upon the other, the body being in a manner equally balanced by them. But when, in revolving, the body recedes farther from the centre, then the centrifugal exceeds the centripetal force; as in a body revolving from the lower to the higher apſis, in an ellipſe, and reſpecting the focus as the centre. And when the revolving body approaches nearer to the centre, the centrifugal is leſs than the centripetal force; as while the body moves from the farther to the nearer extremity of the tranſverſe axis of the ellipſe: the two forces being equal to each other only at the very extremities of that axis.

It is one of the eſtabliſhed laws of nature, that all motion is of itſelf rectilinear, and that the moving body never recedes from its firſt right line, till ſome new impulſe be ſuperadded in a different direction: after that new impulſe the motion becomes compounded, but it is ſtill rectilinear, though not in the ſame line or direction as before. To move in a curve, it muſt receive a new impulſe in a different direction every moment; a curve not being reducible to any number of finite right lines. If then a body, continually drawn towards a centre, be projected in a line that does not paſs through that centre, it will deſcribe a curve; in each point of which, as A, it will endeavour to recede from the curve, and proceed in the tangent AD; and if nothing hindered, it would actually proceed in it; ſo as in the ſame time in which it deſcribes the arch AE, it would recede the length of the line DE, perpendicular to AD, by its centrifugal force: Or being projected in the direction AD, but being continually drawn out of its direction into a curve by a centripetal force.

force, ſo as to fall below the line of direction by the perpendicular ſpace DE: Then the centrifugal or centripetal force is as this line of deviation DE; ſuppoſing the arch AE indefinitely ſmall.



The doctrine of centrifugal forces was firſt mentioned by Huygens, at the end of his Horologium Oſcillatorium, publiſhed in 1673, and demonſtrated in the volume of his Poſthumous Works, as alſo by Guido Grando; where he has given a few eaſy caſes in bodies revolving in the circumference of circles. But Newton, in his Principia, was the firſt who fully handled this doctrine; at leaſt as far as regards the conic ſections. After him there have been ſeveral other writers upon this ſubject; as Leibnitz, Varignon in the Mem. de l'Acad. Keil in the Philoſ. Tranſ. and in his Phyſics, Bernoulli, Herman, Cotes in his Harmonia Menſurarum, Maclaurin in his Geometrica Organica, and in his Fluxions, and Euler in his book de Motu, where he conſiders the curves deſcribed by a body acted on by centripetal forces tending to ſeveral fixed points.

See alſo the art. *Central Forces*, where this doctrine is more fully explained.

CENTROBARICO, the ſame as centre of gravity.

CENTROBARIC-*Method*, is a method of determining the quantity of a ſurface or ſolid, by means of the generating line or plane, and its centre of gravity. The doctrine is chiefly comprized in this theorem:

Every figure, whether ſuperficial or ſolid, generated by the motion of a line or plane, is equal to the product of the generating magnitude and the path of its centre of gravity, or the line which its centre of gravity deſcribes.

See more of this ſubject under the article *Centre of Gravity*,

CENTRUM, in Geometry, Mechanics, &c. See CENTRE.

CENTRUM *Phonicum*, in Acouſtics, is the place where the ſpeaker ſtands in polyſyllabical and articulate echoes.

CENTRUM *Phonocampticum*, is the place or object that returns the voice in an echo.

CEPHEUS, a conſtellation of the northern hemiſphere, being one of the 48 old aſteriſms. The Greeks fable that Cepheus was a king of Ethiopia, and the father of Andromeda, the princeſs who was delivered up to be devoured by a ſea monſter, from which ſhe was reſcued by Perſeus.

The ſtars of this conſtellation, in Ptolomy's catalogue are 13, in Tycho's 11, in Hevelius's 51, and in the Britannic catalogue 35.

CERBERUS, one of the new conſtellations formed by Hevelius out of the unformed ſtars, and added to the 48 old aſteriſms. It contains only 4 ſtars, which are enumerated under Hercules in the Britannic catalogue.

CETUS, *the Whale*, a ſouthern conſtellation, and one of the 48 old aſteriſms. The Greeks pretend that it was the ſea-monſter ſent by Neptune to devour Andromeda, but was killed by Perſeus.

In the neck of the whale is a remarkable ſtar, Collo Ceti, which appears and diſappears periodically, or rather grows brighter and fainter by turns, owing it is ſuppoſed to the alternate turning of its bright and dark ſides towards us, as it revolves upon its axis, or elſe owing to the ſtar having a flattiſh form. The period of its changes is about 312 days. Bullialdus in Phil. Tranſ. vol. 2, Hevelius ibid. vol. 6, Herſchel ibid. vol. 70, Marald. in Mem. Acad. 1719.

The ſtars in the conſtellation Cetus, in Ptolomy's catalogue, are 22, in Tycho's 21, in Hevelius's 45, and in the Britannic catalogue 97.

CHAIN, in Surveying, is a lineal meaſure, conſiſting of a certain number of iron links, uſually 100: ſerving to take the dimenſions of fields &c.

At every 10th link is uſually faſtened a ſmall braſs plate, with a figure engraven upon it, or elſe cut into different ſhapes, to ſhew how many links it is from one end of the chain.

Chains are of various kinds and lengths; as

1. A chain of 100 feet long, each link one foot, for meaſuring of large diſtances only, when regard is not propoſed to be had to acres &c, in the ſuperficial content.

2. A chain of one pole or 16 feet and a half in length; eſpecially uſeful in meaſuring and laying out gardens and orchards, or the like, by the pole or rod meaſure.

3. A chain of 4 poles, or 66 feet, or 22 yards, in length, called Gunter's chain, and is peculiarly adapted to the buſineſs of Surveying or Land-meaſuring, becauſe that 10 ſquare chains juſt make an Engliſh acre of land; ſo that the dimenſions being taken in theſe chains, and thence the contents computed in ſquare chains, they are readily turned into acres by dividing by 10, or barely cutting off the laſt figure from the ſquare chains. But it is ſtill better in practice to proceed thus, viz, count the dimenſions, not in chains, but all in links; then the contents are in ſquare links; and five figures being cut off for decimals, the reſt are acres; that is four figures to bring the ſquare links to ſquare chains, and one more to bring the ſquare chains to acres.

In this chain, the links are each 7 inches and $\frac{92}{100}$, or 7·92 inches in length, which is very nearly $\frac{2}{3}$ of a foot. And hence any number of chains or links are eaſily brought to feet or inches, or the contrary: the beſt way of doing which is this: multiply the number of links by 66, then cut off two figures for decimals, and the reſt are feet: or multiply links by 22 for yards, cutting off two figures.

CHALDRON, of Coals, an Engliſh dry meaſure of capacity conſiſting of 36 buſhels heaped up.

The chaldron of coals is accounted to weigh about 2000 pounds.—On ſhip board, 21 chaldrons of coals are allowed to the ſcore.

CHAMBER *of a Mortar*, or *ſome cannon*, is a cell or cavity at the bottom of the bore, to receive the charge of powder.

It is not found by experience that chambers have any ſenſible effect on the velocity of the ſhot, unleſs in the largeſt ordnance, as mortars or very large cannon. Neither is it found that the form of them is very material ; a ſmall cylinder is as good as any ; though mathematical ſpeculations may ſhew a preference of one form over another. But in practice, the chief point to be obſerved, is to have the chamber of a ſize juſt to contain the charge of powder, and no more, that the ball may lie cloſe to the charge ; and that its entrance may point exactly to the centre of the ball.

CHAMBERS (Ehhraim), author of the dictionary of ſciences called the *Cyclopædia*. He was born at Milton in the county of Weſtmoreland, where he received the common education for qualifying a youth for trade and commerce. When he became of a proper age, he was put apprentice to Mr. Senex the globe-maker, a buſineſs which is connected with literature, eſpecially with geography and aſtronomy. It was during Mr. Chambers's reſidence with this ſkilful artiſt, that he acquired that taſte for literature which accompanied him through life, and directed all his purſuits. It was even at this time that he formed the deſign of his grand work, the Cyclopædia ; ſome of the firſt articles of which were written behind the counter. To have leiſure to purſue this work, he quitted Mr. Senex, and took chambers at Gray's-Inn, where he chiefly reſided during the reſt of his life. The firſt edition of the Cyclopædia, which was the reſult of many years intenſe application, appeared in 1728, in 2 vols. folio. The reputation that Mr. Chambers acquired by the execution of this work, procured him the honour of being elected F. R. S. Nov. 6, 1729. In leſs than ten years time, a ſecond edition became neceſſary ; which accordingly was printed, with corrections and additions, in 1738 ; and this was followed by a third edition the very next year.

Although the Cyclopædia was the chief buſineſs of Mr. Chambers's life, and may be regarded as almoſt the ſole foundation of his fame, his attention was not wholly confined to this undertaking. He was concerned in a periodical publication, called, *The Literary Magazine*, which was begun in 1735. In this work he wrote a variety of articles ; particularly a review of Morgan's *Moral Philoſopher*. He was alſo concerned with Mr. John Martyn, profeſſor of botany at Cambridge, in preparing for the preſs a tranſlation and abridgment of the *Philoſophical Hiſtory and Memoirs of the R. Acad. of Sciences at Paris* ; which work was not publiſhed till 1742, ſome time after our author's deceaſe, in 5 volumes 8vo. Mr. Chambers was alſo author of the tranſlation of the *Jeſuit's Perſpective*, from the French, in 4to ; which has gone through ſeveral editions.

Mr. Chambers's cloſe and unremitting attention to his ſtudies at length impaired his health, and obliged him occaſionally to take a country lodging, but without much benefit ; he afterwards viſited the ſouth of France, but ſtill with little effect ; he therefore returned to England, where he ſoon after died, at Iſlington, May 15, 1740, and was buried at Weſtminſter Abbey.

After the author's death, two more editions of his Cyclopædia were publiſhed. The proprietors afterwards procured a ſupplement to be compiled, by Mr. Scott and Dr. Hill, but chiefly by the latter, which extended to two volumes more ; and the whole has ſince been reduced into one alphabet in 4 volumes, by Dr. Rees, forming a very valuable body of the ſciences.

CHAMBRANLE, the border, frame, or ornament of ſtone or wood, ſurrounding the three ſides of doors, windows, and chimneys. This is different in the different orders : when it is plain, and without mouldings, it is called ſimply and properly, *band*, *caſe*, or *frame*. In an ordinary door, it is moſtly called *door-caſe* ; in a window, the *window-frame*.

The Chambranle conſiſts of three parts ; the two ſides, called aſcendants ; and the top, called the traverſe or ſupercilium.

CHAMFER, or Chamferet, an ornament, in architecture, conſiſting of half a ſcotia ; being a kind of ſmall furrow or gutter on a column.

CHAMFERING, is uſed for cutting the edge or the end of any thing bevel, or aſlope.

CHANCE, *the Doctrine and Laws of*, are the ſame as thoſe of expectation, or probability, &c ; which ſee. Chances, in play, conſiſt of the number of ways by which events may happen. Thus, if a halfpenny, or other piece of money, be toſſed up, there are two events, or chances, or ſides that may turn up, namely one chance for turning up a head, and one for the contrary ; that is, it is an equal chance to throw a head or not. And in throwing a common die, which has 6 faces, there are in all 6 chances, that is one chance for throwing an ace or any other ſingle point, and 5 chances againſt it ; or it is 5 to 1 that ſuch aſſigned point does not come up.

Upon this ſubject, ſee De Moivre, Simpſon, &c.

CHANDELIERS, in Fortification, a kind of wooden parapet, conſiſting of upright timbers ſupporting others laid acroſs the tops of them, 6 feet high, and fortified with faſcines &c. They are uſed to cover the workmen in approaches, galleries, and mines. And they differ from blinds only in this, that the former ſerve to cover the men before, and the latter over head.

CHANGES, the permutations or variations of any number of things, with regard to their poſition, order, &c ; as how many changes may be rung on a number of bells, or how many different ways any number of perſons may be placed, or how many ſeveral variations may be made of any number of letters, or any other things propoſed to be varied.

To find out ſuch number of changes, multiply continually together all the terms in a ſeries of arithmetical progreſſion, whoſe firſt term and common difference are each unity or 1, and the laſt term the number of things propoſed to be varied, thus 1 × 2 × 3 × 4 × 5 &c. till the laſt number be the propoſed number of things. For,

If there be only two things, as *a* and *b*, they admit of a double order or poſition only ; for they may be placed either thus *ab* or thus *ba*, viz, 1 × 2 = 2 ways. If there be three things, *a*, *b*, and *c*, they will admit of 6 variations = 1 × 2 ſince as in the margin, and no more ; three each of the three may be combined there different ways with each of the other two.

a b c
a c b
b a c
b c a
c a b
c b a.

And if there be 4 things, each of them may be combined 4 ways with each order of the other three, that is 4 times 6 ways, or 1 × 2 × 3 × 4 = 24 ways.

In like manner, the combinations
of 5 things are 1 × 2 × 3 × 4 × 5 = 120
of 6 things are 1 × 2 × 3 × 4 × 5 × 6 = 720
&c.

So that if it be propoſed to aſſign how many different ways a company of 6 perſons may be placed, at table for inſtance, the anſwer will be 720 ways. Alſo the number of changes that can be rung on 7 bells, are 1 × 2 × 3 × 4 × 5 × 6 × 7 or 720 × 7 = 5040 changes.

CHAPITERS, the crowns or upper parts of a pillar or column.

CHAPPE (JEAN D'AUTEROCHE), a French aſtronomer, was born at Mauriac, in Auvergne, March 2, 1728. A taſte for drawing and mathematics appeared in him at a very tender age; and he owed to Dom Germain a knowledge of the firſt elements of mathematics and aſtronomy. M. Caſſini, after aſſuring himſelf of the genius of this young man, undertook to improve it. He employed him upon the map of France, and the tranſlation of Halley's tables, to which he made conſiderable additions. The king charged him in 1753 with drawing the plan of the county of Bitche, in Lorraine, all the elements of which he determined geographically. He occupied himſelf greatly with the two comets of 1760; and the fruit of his labour was his Elementary treatiſe on the theory of thoſe comets, enriched with obſervations on the zodiacal light, and on the aurora borealis. He ſoon after went to Tobolſk, in Siberia, to obſerve the tranſit of Venus over the ſun; a journey which greatly impaired his health. After two years abſence he returned to France in 1762, where he occupied himſelf for ſome time in putting in order the great quantity of obſervations he had made. M. Chappe alſo went to obſerve the next tranſit of Venus, viz that of 1769, at California, on the weſt ſide of North America, where he died of a dangerous epidemic diſeaſe, the 1ſt of Auguſt 1769. He had been named Adjunct Aſtronomer to the Academy the 17th of January 1759.

The publiſhed works of M. Chappe, are,

1. The Aſtronomical Tables of Dr. Halley; with obſervations and additions: in 8vo, 1754.

2. Voyage to California to obſerve the tranſit of Venus over the ſun, the 3d of June 1769: in 4to, 1772.

3. He had a conſiderable number of papers inſerted in the Memoirs of the Academy, for the years 1760, 1761, 1764, 1765, 1766, 1767, and 1768; chiefly relating to aſtronomical matters.

CHAPTREL, the ſame with Impoſt.

CHARACTERISTIC, *of a Logarithm*, the ſame as Index, or Exponent. This term was firſt uſed by Briggs in the 4th ſection of his Arithmetica Logarithmica, where he treats particularly of it; meaning by it, the integral or firſt part of a logarithm towards the left hand, which expreſſes 1 leſs than the integer places or figures in the number anſwering to that logarithm, or how far the firſt figure of this number is removed from the place of units; namely, that 0 is the characteriſtic of all numbers from 1 to 10; and 1 the characteriſtic of all thoſe from 10 to 100; and 2 the characteriſtic of all thoſe from 100 to 1000; and ſo on.

CHARACTERS, are certain marks uſed by Aſtronomers, Mathematicians, &c, to denote certain things, whether for the ſake of brevity, or perſpicuity, in their operations.

1. ASTRONOMICAL CHARACTERS.

Planets &c.	*The twelve Signs or Conſtellations of the Zodiac.*
☉ The Sun	♈ Aries, the Ram
☾ The Moon	♉ Taurus, the Bull
⊕ The Earth	♊ Gemini, the Twins
☿ Mercury	♋ Cancer, the Crab
♀ Venus	♌ Leo, the Lion
♂ Mars	♍ Virgo, the Maid
♃ Jupiter	♎ Libra, the Balance
♄ Saturn	♏ Scorpio, the Scorpion
♅ Herſchel, or the Georgian Planet	♐ Sagittary, the Archer
☊ Aſcending Node	♑ Capricorn, the Goat
☋ Deſcending Node	♒ Aquarius, the Water-bearer
	♓ Piſces, the Fiſhes

The Aſpects, Time, Motion, &c.

☌ Conjunction	° Degrees
☍ Oppoſition	′ Minutes or Primes
⚹ Sextile	″ Seconds, &c.
□ Quartile	A. M. Ante merid. or in morn.
△ Trine	P. M. Poſt merid. or a aftern.
	h, m, s, Hours, min. ſec.

2. MATHEMATICAL &c. CHARACTERS.

Numeral Characters uſed by different Nations.

The moſt common numeral characters, are thoſe called Arabic or Indian, viz. 1, 2, 3, 4, 5, 6, 7, 8, 9, with 0 or cipher for nothing.

The Roman numeral characters are ſeven, viz, I one, V five, X ten, L fifty, C a hundred, D or IↃ five hundred, M or DCI or CIↃ a thouſand. Other combinations are as in the following ſynopſis of the Roman Notation.

1 = I
2 = II: As often as any character is repeated,
3 = III ſo many times its value is repeated.
4 = IIII or IV: A leſs character before a
5 = V greater diminiſhes its value.
6 = VI: A leſs character after a greater in-
7 = VII creaſes its value.
8 = VIII
9 = IX
10 = X
50 = L
100 = C
500 = D or IↃ: For every Ↄ added, this be-
comes 10 times as many.
1000 = M or CIↃ: For every C and Ↄ, ſet one at
2000 = MM [each end, it becomes 10 times as much.
5000 = IↃↃ or $\overline{\text{V}}$: A line over any number in-
6000 = $\overline{\text{VI}}$ creaſes it 1000 fold.
10000 = $\overline{\text{X}}$ or CCIↃↃ
50000 = IↃↃↃ
60000 = $\overline{\text{LX}}$
100000 = $\overline{\text{C}}$ or CCCIↃↃↃ
1000000 = $\overline{\text{M}}$ or CCCCIↃↃↃↃ
2000000 = $\overline{\text{MM}}$, &c.

Greek Numerals.

The Greeks had three ways of expreſſing numbers. Firſt, The moſt ſimple was, for every ſingle letter, according to its place in the alphabet, to denote a number from α 1 to ω 24; in which manner the books of Homer's Ilias are diſtinguiſhed. Secondly, Another way was by dividing the alphabet into *(firſt)* 8 units, α 1, β 2, &c; *(2nd)* 8 tens, ι 10, κ 20, &c; *(3d)* 8 hundreds, ρ 100, σ 200, &c: And thouſands they expreſſed by a point or accent under a letter, as α̦ 1000, β̦ 2000, &c. Thirdly, A third way was by ſix capital letters, thus, Ι (ια for μια) 1, Π (ϖεντε) 5, Δ (δεκα) 10, Η (Ηεκατον) 100, Χ (χιλια) 1000, Μ (μυρια) 10000: and when the letter Π incloſed any of theſe, except Ι, it ſhewed that the incloſed letter was five times its own value, as $\overline{|\Delta|}$ 50, $\overline{|\mathrm{H}|}$ 500, $\overline{|\mathrm{X}|}$ 5000, $\overline{|\mathrm{M}|}$ 50000.

Hebrew Numerals.

The Hebrew alphabet was divided into, Nine Units, as א 1, ב 2, &c; Nine Tens, as י 10, כ 20, &c; Nine Hundreds, as ק 100, ר 200, &c, ך 500, ם 600, ן 700, ף 800, ץ 900. Thouſands were ſometimes expreſſed by the units prefixed to hundreds, as אדלר 1534, &c; and even to tens, as אע 1070, &c. But more commonly thouſands were expreſſed by the word אלף 1000, אלפים 2000; and אלפים with the other numerals prefixed to ſignify the number of thouſands, as גאלפים 3000, &c.

Characters uſed in Arithmetic and Algebra.

The firſt letters of the alphabet, *a*, *b*, *c*, &c, denote given quantities; and the laſt letters *z*, *y*, *x*, &c, denote ſuch as are unknown or ſought. Stifelius firſt uſed the capitals A, B, C, &c, for the unknown or required quantities. After that, Vieta uſed the capital vowels A, E, I, O, U, Y for the unknown or required quantities, and the conſonants B, C, D, &c, for known or given numbers. Harriot changed Vieta's capitals into the ſmall letters, viz *a*, *e*, *i*, *o*, *u*, for unknown, and *b*, *c*, *d*, &c, for known quantities. And Deſcartes changed Harriot's vowels for the latter letters *z*, *y*, *x*, &c, and the conſonants for the leading letters *a*, *b*, *c*, *d*, &c.

Newton denotes the ſeveral orders of the fluxions of variable quantities by as many points over the latter letters;

as $\dot{x}$, $\dot{y}$, $\dot{z}$ are the 1ſt fluxions,
$\ddot{x}$, $\ddot{y}$, $\ddot{z}$ are the 2d fluxions,
$\dddot{x}$, $\dddot{y}$, $\dddot{z}$ are the 3d fluxions,

&c, of *x*, *y*, *z*. And Leibnitz denotes the differentials of the ſame quantities by prefixing *d* to each of them, thus *dx*, *dy*, *dz*.

Powers of quantities are denoted by placing the index or exponent after them, towards the upper part; thus a^2 is the 2d power, a^3 the third power, and a^n the *n* power of *a*. Diophantus marked the powers by their initials, thus δ^ν, κ^ν, δδ^ν, δκ^ν, κκ^ν, &c, for dynamis, cubus, dynamodynamis, &c, or the 2d, 3d, 4th, &c powers; and the ſame method has been uſed by ſeveral of the early writers, ſince the introduction of Algebra into Europe: but the firſt of them, as Paciolus, Cardan, &c, uſed no mark for powers, but the words themſelves. Stifel, and others about his time, uſed the initials or abbreviations, ℞, ʒ, cſ, ʒʒ, &c, of res or coſs, zenzus, cubus, zenzizenzus, &c, barbarous corruptions of the Italian coſa, cenſus, cubo, cenſi-cenſus, &c. But he uſed alſo numeral exponents, both poſitive and negative, to the general characters or roots A, B, C, &c. Bombelli uſed a half circle thus $\smile$ as a general character for the unknown or quantity required to be found in any queſtion, and the ſeveral powers of it he denoted by figures ſet above it; thus $\overset{1}{\smile}$, $\overset{2}{\smile}$, $\overset{3}{\smile}$, are the 1ſt, 2d, 3d powers of $\smile$; which powers he called dignities. Stevinus uſed a whole circle for the ſame unknown quantity, with the numeral index within it, and that both integral and fractional; thus ⓪, ①, ②, ③, are the 0, 1, 2, 3 powers of the general quantity ○; alſo $\textcircled{\tiny $\frac{1}{2}$}$, $\textcircled{\tiny $\frac{1}{3}$}$, $\textcircled{\tiny $\frac{1}{4}$}$, he uſes as the ſquare root, cubic root, 4th root of the ſame;
and $\textcircled{\tiny $\frac{2}{3}$}$, the cube root of the ſquare,
and $\textcircled{\tiny $\frac{3}{2}$}$, the ſquare root of the cube, and ſo on.
And theſe fractional exponents were adopted and farther uſed by his commentator Albert Girard. So that Stevinus ought to be eſteemed the firſt perſon who rendered general the notation of all powers and roots in the ſame way, the former by integral, and the latter by fractional exponents. Harriot denoted his powers by a repetition of the letters; thus *a*, *aa*, *aaa*, &c. And Deſcartes, inſtead of this, ſet the numeral index at the upper part of the letters, as at preſent thus a, a^2, a^3, &c. Though, I am informed, by ſuch as have ſeen Harriot's poſthumous papers, that he alſo there makes uſe of exponents.

The character √ is the ſign of radicality, or of a root, being derived from the initial R or *r*, which was uſed at firſt by Paciolus, Cardan, &c. This character √ I firſt find uſed by Stifel, in 1544, and by Robert Recorde in 1557. The character √ alone denotes the ſquare root only; but at firſt they uſed the initial of the name after it, to denote the ſeveral roots: as √*q* the quadrate or ſquare root, and √*c* the cubic root. But the numeral indices of the root were prefixed by Albert Girard, exactly the ſame as they are uſed at preſent, viz $\sqrt[2]{}$, $\sqrt[3]{}$, $\sqrt{}$, the 2d, 3d, or 4th root.

The character + denotes addition, and a poſitive quantity. At firſt the word itſelf was uſed, plus, piu, or the initial *p.* by Paciolus, Cardan, Tartalea, &c. And the character + for addition occurs in Stifelius.

The character − denotes ſubtraction, and a negative quantity; which alſo firſt occurs in the ſame author Stifelius. Before that, the word minus, mene, or the initial *m.* was uſed. Other characters have alſo been ſometimes uſed by other authors, for addition and ſubtraction; but they are now no longer in uſe.

× denotes multiplication, and was introduced by Oughtred.

÷ denoting diviſion, was introduced by Dr. Pell. Diviſion is alſo denoted like a fraction,

thus $\frac{a}{b}$ or $\frac{6}{3} = 2$.

= denotes equality, and was uſed by Robert Recorde. Deſcartes uſes ∝ for the ſame purpoſe.

The character : : for proportionality, or equality of ratios, was introduced by Oughtred; as was alſo the mark ∺ for continued proportion.

$>$ for greater, and $<$ for leſs, were uſed by Harriot.

And ⊐ and ⊐ were uſed by Oughtred for the ſame purpoſes.

Dr. Pell uſed ⊚ for involution, and ω for evolution.

∽ denotes a general difference between any two quantities, and was introduced by Dr. Wallis.

The Parentheſis ( ), as a vinculum, was invented by Albert Girard, and uſed in ſuch expreſſions as theſe, $\sqrt[3]{}(72 + \sqrt{}5120)$, and B (B$q$ + Cq), both for univerſal roots, and multiplication, &c.

The ſtraight-lined vinculum, $\overline{\quad\quad}$, was uſed by Vieta for the ſame purpoſe; thus $\overline{A - B}$ in $\overline{B + C}$.

Characters in Geometry and Trigonometry.

□	A Square	∠	An Angle
△	A Triangle	∟	A Rightangle
▭	A Rectangle	⊥	Perpendicular
○ or ⊙	A Circle	‖	Parallel.

CHARGE, in Electricity, in a ſtrict ſenſe, imports the accumulation of the electric matter on one ſurface of an electric, as the Leyden phial, a pane of glaſs, &c, whilſt an equal quantity paſſes off from the oppoſite ſurface. Or, more generally, electrics are ſaid to be charged, when the equilibrium of the electric matter on the oppoſite ſurface is deſtroyed, by communicating one kind of electricity to one ſide, and the contrary kind to the oppoſite ſide: nor can the equilibrium be reſtored till a communication be made by means of conducting ſubſtances between the two oppoſite ſurfaces: and when this is done, the electric is ſaid to be diſcharged. The charge properly refers to one ſide, in contradiſtinction from the other; ſince the whole quantity in the electric is the ſame before and after the operation of charging; and the operation cannot ſucceed, unleſs what is gained on one ſide is loſt by the other, by means of conductors applied to it, and communicating either with the earth, or with a ſufficient number of non-electrics. To facilitate the communication of electricity to an electric plate &c, the oppoſite ſurfaces are coated with ſome conducting ſubſtance, uſually with tin-foil, within ſome diſtance of the edge; in conſequence of which the electricity communicated to one part of the coating, is readily diffuſed through all parts of the ſurface of the electric in contact with it; and a diſcharge is eaſily made by forming a communication with any conductor from one coating to the other. If the oppoſite coatings approach too near each other, the electric matter forces a paſſage from one ſurface to the other before the charge is complete. And ſome kinds of glaſs have the property of conducting the electricity over the ſurface, ſo that they are altogether unfit for the operation of charging and diſcharging. If indeed the charge be too high, and the glaſs plate or phial too thin, the attraction between the two oppoſite electricities forces a paſſage through the glaſs, making a ſpontaneous diſcharge, and the glaſs becomes unfit for farther uſe. See *Conductors*, *Electrics*, *Leyden Phial*, &c.

Charge, in Gunnery, the load of a piece of ordnance, or the quantity of powder and ball, or ſhot, with which it is prepared for execution.

The charge of powder, for proving guns, is equal to the weight of the ball; but for ſervice, the charge is $\frac{1}{2}$ or $\frac{1}{3}$ the weight of the ball, or ſtill leſs; and indeed in moſt caſes of ſervice, the quantity of powder uſed is too great for the intended execution. In the Britiſh navy, the allowance for 32 pounders is but $\frac{7}{12}$ of the weight of the ball. But it is probable that, if the powder in all ſhip guns was reduced to $\frac{1}{3}$ the weight of the ball, or even leſs, it would be a conſiderable advantage, not only by ſaving ammunition, but by keeping the guns cooler and quieter, and at the ſame time more effectually injuring the veſſels of the enemy. With the preſent allowance of powder, the guns are heated, and their tackle and furniture ſtrained, and all this only to render the ball leſs efficacious: for a ball which can but juſt paſs through a piece of timber, and in the paſſage loſes almoſt all its motion, is found to rend and fracture it much more, than when it paſſes through with a much greater velocity. See Robins's Tracts, vol. 1. pa. 290, 291.

Again, the ſame author obſerves, that the charge is not to be determined by the greateſt velocity that may be produced; but that it ſhould be ſuch a quantity of powder as will produce the leaſt velocity neceſſary for the purpoſe in view; and if the windage be moderate, no field-piece ſhould ever be loaded with more than $\frac{1}{6}$, or at the utmoſt $\frac{1}{5}$ of the weight of its ball in powder; nor ſhould the charge of any battering piece exceed $\frac{1}{3}$ of the weight of its bullet. Ib. pa. 266.

Different charges of powder, with the ſame weight of ball, produce different velocities in the ball, which are in the ſubduplicate ratio of the weights of powder; and when the weight of powder is the ſame, and the ball varied, the velocity produced is in the reciprocal ſubduplicate ratio of the weight of the ball: which is agreeable both to theory and practice. See my paper on Gunpowder in the Philoſ. Tranſ. 1778, pa. 50; and my Tracts, vol. 1. pa. 266.

But this is on a ſuppoſition that the gun is of an indefinite length; whereas, on account of the limited length of guns, there is ſome variation from this law in practice, as well as in theory; in conſequence of which it appears that the velocity of the ball increaſes with the charge only to a certain point, which is peculiar to each gun, where the velocity is the greateſt; and that by farther increaſing the charge, the velocity gradually diminiſhes, till the bore is quite full of powder. By an eaſy fluxionary proceſs it appears that, calling the length of the bore of the gun b, the length of the charge producing the greateſt velocity, ought to be $\frac{b}{2{\cdot}718281828}$, or about $\frac{3}{8}$ of the length of the bore; where 2·718281828 is the number whoſe hyp. log. is 1. But, for ſeveral reaſons, in practice the length of the charge producing the greateſt velocity, falls ſhort of that above mentioned, and the more ſo as the gun is longer. From many experiments I have found the length of the chgre producing the greateſt velocity, in guns of various lengths of bore, from 15 to 40 calibres, as follows.

Length of Bore in Calibres.	Length of Charge for greateſt Veloc.
15	$\frac{3}{10}$
20	$\frac{3}{12}$
30	[illegible]
40	$\frac{3}{20}$.

CHARLES's WAIN, a name by which ſome of the aſtronomical writers, in our own language, have called Urſa Major, or the great bear; though ſome writers ſay the leſſer bear. Indeed both of the two bears have been called waggons or wains, and by the Latins, who have followed the Arabians, two biers, Feretrum majus & minus.

CHART, or Sea-Chart, a hydrographical or ſea-map, for the uſe of navigators; being a projection of ſome part of the ſea in plano, ſhewing the ſea coaſts, rocks, ſands, bearings, &c. Fournier aſcribes the invention of ſea-charts to Henry ſon of John king of Portugal. Theſe charts are of various kinds, the Plain chart, Mercator's or Wright's chart, the Globular chart, &c.

In the conſtruction of charts, great care ſhould be taken that the ſeveral parts of them preſerve their poſition to one another, in the ſame order as on the earth; and it is probable that the finding out of proper methods to do this, gave riſe to the various modes of projection.

There are many ways of conſtructing maps and charts; but they depend chiefly on two principles. Firſt, by conſidering the earth as a large extended flat ſurface; and the charts made on this ſuppoſition are uſually called Plain Charts. Secondly, by conſidering the earth as a ſphere; and the charts made on this principle are ſometimes called Globular Charts, or Mercator's Charts, or Reduced Charts, or Projected Charts.

Plain Charts have the meridians, as well as the parallels of latitude, drawn parallel to each other, and the degrees of longitude and latitude everywhere equal to thoſe at the equator. And therefore ſuch charts muſt be deficient in ſeveral reſpects. For, 1ſt, ſince in reality all the meridians meet in the poles, it is abſurd to repreſent them, eſpecially in large charts, by parallel right lines. 2dly, As plain charts ſhew the degrees of the ſeveral parallels as equal to thoſe of the equator, therefore the diſtances of places lying eaſt and weſt, muſt be repreſented much larger than they really are. And 3dly, In a plain chart, while the ſame rhumb is kept, the veſſel appears to ſail on a great circle, which is not really the caſe. Yet plain charts made for a ſmall extent, as a few degrees in length and breadth, may be tolerably exact, eſpecially for any part within the torrid zone; and even a plain chart made for the whole of this zone will differ but little from the truth.

Mercator's Chart, like the plain charts, has the meridians repreſented by parallel right lines, and the degrees of the parallels, or longitude, everywhere equal to thoſe at the equator, ſo that they are increaſed more and more, above their natural ſize, as they approach towards the pole; but then the degrees of the meridians, or of latitude, are increaſed in the ſame proportion at the ſame part; ſo that the ſame proportion is preſerved between them as on the globe itſelf. This chart has its name from that of the author, Girard Mercator, who firſt propoſed it for uſe in the year 1556, and made the firſt charts of this kind; though they were not altogether on true or exact principles, nor does it appear that he perfectly underſtood them. Neither indeed was the thought originally his own, viz. of lengthening the degrees of the meridian in ſome proportion s for this was hinted by Ptolemy near two thouſand year; ago. It was not perfected however till Mr. Wright firſt demonſtrated it about the year 1590, and ſhewed a ready way of conſtructing it, by enlarging the meridian line by the continual addition of the ſecants. See his Correction of Errors in Navigation, publiſhed in 1599.

Globular Chart, is a projection ſo called from the conformity it bears to the globe itſelf; and was propoſed by Meſſrs Senex, Wilſon, and Harris. This is a meridional projection, in which the parallels are equidiſtant circles, having the pole for their common centre, and the meridians curvilinear and inclined, ſo as all to meet in the pole, or common centre of the parallels. By which means the ſeveral parts of the earth have their proper proportion of magnitude, diſtance, and ſituation, nearly the ſame as on the globe itſelf; which renders it a good method for geographical maps.

Hydrographical Charts, are ſheets of large paper, on which ſeveral parts of the land and ſea are deſcribed, with their reſpective coaſts, harbours, ſounds, flats, rocks, ſhelves, ſands, &c, alſo the points of the compaſs, and the latitudes and longitudes of the places.

Selenographic Charts, are particular deſcriptions of the appearances, ſpots and maculæ of the moon.

Topographic Charts, are draughts of ſome ſmall parts only of the earth, or of ſome particular place, without regard to its relative ſituation, as London, York, &c.

For the Conſtruction of Charts, ſee Geography, Maps, Projection, &c.

CHASE, of a Gun, is its bore or cylinder.

CHAULNES (The Duke De), a peer of France, but more honourable and remarkable as an aſtronomer and mathematician. He was born at Paris Dec. 30, 1714. He ſoon diſcovered a ſingular taſte and genius for the ſciences; and in the tumults of armies and camps, he cultivated mathematics, aſtronomy, mechanics, &c. He was named honorary-academician the 27th of February 1743, and few members were more punctual in attending the meetings of that body; where he often brought different conſtructions and corrections of inſtruments of aſtronomy, of dioptrics, and achromatic teleſcopes. Theſe reſearches were followed with a new parallactic machine, more ſolid and convenient than thoſe that were in uſe; as alſo with many reflections on the manner of applying the micrometer to thoſe teleſcopes, and of meaſuring exactly the value of the parts of that inſtrument. The duke of Chaulnes propoſed many other works of the ſame kind, when death ſurpriſed him the 23d Sept. 1769.

He had ſeveral papers publiſhed in the volumes of Memoirs of the Academy of Sciences, as follow:

1. Obſervations on ſome Experiments in the 4th part of the 2d book of Newton's Optics: an. 1755.
2. Obſervations on the Platform for dividing mathematical inſtruments: 1765.
3. Determination of the diſtance of Arcturus from the Sun's limb, at the ſummer ſolſtice: 1765.
4. On ſome means of perfecting aſtronomical inſtruments: 1765.
5. O ſome experiments relative to dioptrics: 1767.
6. The art of dividing mathematical inſtruments: 1768.
7. Obſervations of the Tranſit of Venus, June 3, 1769: 1769.
8. New method of dividing mathematical and aſtronomical inſtruments.

CHAUSE TRAPPES, or *Caltrops*, or *Crowsfeet*, are iron inſtruments of ſpikes about 4 inches long, made like a ſtar, in ſuch a manner that whichever way they fall, one point ſtands always upwards, like a nail. They are uſually thrown and ſcattered into moats and breaches, to gall the horſes feet, and ſtop the haſty approach of the enemy.

CHAZELLES (JOHN MATTHEW), a French mathematician and engineer, was born at Lyons in 1657, and educated there in the college of Jeſuits, from whence he removed to Paris in 1675. He firſt became acquainted with Du Hamel, ſecretary to the Academy of Sciences, and through him with Caſſini, who employed him with himſelf at the Obſervatory, where Chazelles greatly improved himſelf, and alſo aſſiſted Caſſini in the meaſurement of the ſouthern part of the meridian of France. Having, in 1684, inſtructed the duke of Montemar in the mathematical ſciences, this nobleman procured him the appointment of hydrography-profeſſor to the galleys of Marſeilles. In diſcharging the duties of this department, he made numerous geometrical and aſtronomical obſervations, from which he drew a new map of the coaſt of Provence.—He alſo performed many other ſervices in that department, and as an engineer, along with the armies and naval expeditions. To make obſervations in Geography and Aſtronomy, he undertook alſo a voyage to the Levant, and among other things he meaſured the pyramids of Egypt, and found the four ſides of the largeſt of them exactly to face the four cardinal points of the compaſs. He made a report of his voyage, on his return, to the Academy of Sciences, upon which he was named a member of their body in 1695, and had many papers inſerted in the volumes of their Memoirs, from 1693 to 1708. Chazelles died at Marſeilles the 16th of January 1710.

CHEMIN *des Ronds*, in Fortification, the way of the rounds, or a ſpace between the rampart and the low parapet under it, for the rounds to go about it.

CHEMISE, a wall that lines a baſtion, or ditch, or the like, for its greater ſupport and ſtrength.

CHERSONESUS, a peninſula, or part of the land almoſt encompaſſed round with the ſea, only joining to the main land by a narrow neck or iſthmus. Varenius enumerates 14 of theſe.

CHEVAL *de Friſe*, pl. *Chevaux de Friſe*, or Friſeland horſe, ſo called becauſe it was firſt uſed in that country. It conſiſts of a joiſt or piece of timber, about a foot in diameter, and 10 or 12 long, pierced and tranſverſed with a great number of wooden ſpikes of 5 or 6 feet long, and armed or pointed with iron. It is ſometimes alſo called *turnpike*, or *tourniquet*. It is chiefly uſed to ſtop a breach, defend a paſſage, or make a retrenchment to ſtop the cavalry.

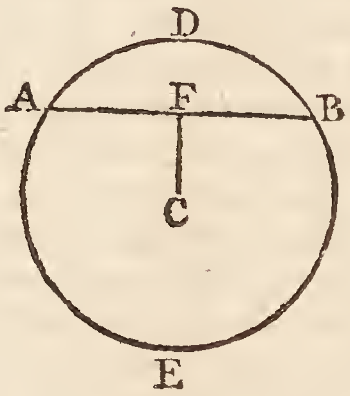
CHEVRETTE, in Artillery, an engine to raiſe guns or mortars into their carriage. It is formed of two pieces of wood of about 4 feet long, ſtanding upright upon a third, which is ſquare. The uprights are about a foot aſunder, and pierced with holes exactly oppoſite to one another, to receive a bolt of iron, which is put in either higher or lower at pleaſure, to ſerve as a ſupport to a handſpike by which the gun is raiſed up.

CHEYNE (GEORGE), a Britiſh phyſician, and mathematician, was born in Scotland, 1671, and educated at Edinburgh under Dr. Pitcairn. He paſſed his youth in cloſe ſtudy and great abſtemiouſneſs; but coming to London, when about 30 years of age, he ſuddenly changed his whole manner of living; which had ſuch an effect upon his conſtitution, that his body grew to a moſt enormous bulk, weighing it is ſaid about 448 pounds. From this load of oppreſſion however he was afterward in a great meaſure relieved, by means of a milk and vegetable diet, which reduced his weight to about one-third of what it had been, and reſtored him to a good ſtate of health; by which his life was prolonged to the 72d year of his age.

Dr. Cheyne was author of various medical and other tracts, and of a treatiſe on the Inverſe method of Fluxions, under the title of *Fluxionum Methodus Inverſa; ſive quantitatum fluentium leges generaliores*: in 4to, 1703. Upon this book De Moivre wrote ſome animadverſions in an 8vo vol. 1704; which were replied to by Cheyne in 1705.

CHILIAD, an aſſemblage of ſeveral things ranged by thouſands. It was particularly applied to tables of logarithms, becauſe they were at firſt divided into thouſands. Thus, in the year 1624, Mr. Briggs publiſhed a table of logarithms for 20 chiliads of abſolute numbers; afterward, he publiſhed 10 chiliads more; and laſtly, one more; making in all 31 chiliads.

CHILIAGON, a regular plane figure of a thouſand ſides and angles.



CHORD, a right line connecting the two extremes of an arch; ſo called from its reſemblance to the chord or ſtring of a bow; as AB, which is common to the two parts or arches ADB, AEB that make up the whole circle. The chords have ſeveral properties:

1. The Chord is biſected by a perpendicular CF drawn to it from the centre.

2. Chords of equal arcs, in the ſame or equal circles, are themſelves equal.

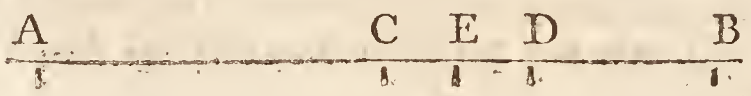
3. Unequal Chords have to one another a leſs ratio than that of their arcs.

4. The chord of an arc, is a mean proportional between the diameter and the verſed ſine of that arc.

Scale or Line of Chords. See *Plane Scale.*

CHORD, or CORD, in Muſic, denotes the ſtring or line by whoſe vibrations the ſenſation of ſound is excited; and by whoſe diviſions the ſeveral degrees of tune are determined.

To divide a Chord AB *in the moſt ſimple manner, ſo as to exhibit all the original concords.*



Divide the given line into two equal parts at C; then ſubdivide the part CB equally in two at D, and again the part CD into two equal parts at E. Here AC to AB is an octave; AC to AD a fifth; AD to AB a fourth; AC to AE a greater third, and AE to AD a leſs third; AE to EB a greater ſixth, and AE to AB a leſs ſixth. Malcolm's Treatiſe of Muſic, ch. 6. ſec. 3. See MONOCHORD.

To find the number of Vibrations made by a Muſical Chord or String in a given time; having given its weight, length, and tenſion. Let l be the length of the chord in feet, 1 its weight, or rather a ſmall weight fixed to the middle and equal to that of the whole chord, and w the tenſion, or a weight by which the chord is ſtretched. Then ſhall the time of one vibration be expreſſed by $\frac{11}{7}\sqrt{\frac{l}{32\frac{1}{6}w}}$, and conſequently the number of vibrations per ſecond is equal to $\frac{7}{11}\sqrt{\frac{32\frac{1}{6}w.}{l}}$

For example, ſuppoſe $w = 28800$, or the tenſion equal to 28800 times the weight of the chord, and the length of it 3 feet; then the laſt theorem gives 354 nearly for the number of vibrations made in each ſecond of time.

But if w were 14400, there would be made but 250 vibrations per ſecond; and if w were only 288, there would be no more than $35\frac{16}{75}$ vibrations per ſecond. See my Select Exerc. prob. 21. pag. 200.

CHORD, in Muſic, is uſed for the union of two or more ſounds uttered at the ſame time, and forming together a complete harmony.

Chords are divided into perfect and imperfect. The perfect chord is compoſed of the fundamental ſound below, of its third, its fifth, and its octave: they are likewiſe ſubdivided into major and minor, according as the thirds which enter into their compoſition are flat or ſharp. Imperfect chords are thoſe in which the ſixth, inſtead of the fifth, prevails; and in general all thoſe whoſe loweſt are not their fundamental ſounds.

Chords are again divided into conſonances and diſſonances. The conſonances are the perfect chord, and its derivatives. Every other chord is a diſſonance. A table of both, according to the ſyſtem of M. Rameau, may be ſeen in Rouſſeau's Muſical Dictionary, vol. 1, pa. 27.

CHOROGRAPHY, the art of delineating or deſcribing ſome particular country or province.

This differs from geography as the deſcription of a particular country differs from that of the whole earth: And from topography, as the deſcription of a country differs from that of a town or a diſtrict.

CHROMATIC, a ſpecies of muſic which proceeds by ſemitones and minor thirds. The word is derived from the Greek χρωμα, which ſignifies colour, and perhaps the ſhade or intermediate ſhades of colour, which mingle and connect colours, like as the ſmall intervals in this ſcale eaſily ſlide or run into each other.

Boethius and Zarlin aſcribe the invention of the chromatic genus to Timotheus, a Mileſian, in the time of Alexander the Great. The Spartans baniſhed it their city on account of its ſoftneſs. The character of this genus, according to Ariſtides Quintillianus, was ſweetneſs and pathos.

CHROMATICS, is that part of optics which explains the ſeveral properties of the colours of light, and of natural bodies.

Before the time of Sir I. Newton, the notions concerning colour were very vague and wild. The Pythagoreans called colour the ſuperficies of bodies: Plato ſaid that it was a flame iſſuing from them: According to Zeno, it is the firſt configuration of matter: And Ariſtotle ſaid it was that which made bodies actually tranſparent. Deſcartes accounted colour a modification of light, and he imagined that the difference of colour proceeds from the prevalence of the direct or rotatory motion of the particles of light. Grimaldi, Dechales, and many others, imagined that the differences of colour depended upon the quick or ſlow vibrations of a certain elaſtic medium with which the univerſe is filled. Rohault conceived, that the different colours were made by the rays of light entering the eye at different angles with reſpect to the optic axis. And Dr. Hooke imagined that colour is cauſed by the ſenſation of the oblique or uneven pulſe of light; which being capable of no more than two varieties, he concluded there could be no more than two primary colours.

Sir I. Newton, in the year 1666, began to inveſtigate this ſubject; when finding that the coloured image of the ſun, formed by a glaſs priſm, was of an oblong, and not of a circular form, as, according to the laws of equal refraction, it ought to be, he conjectured that light is not homogeneal; but that it conſiſts of rays of different colours, and endued with divers degrees of refrangibility. And, from a farther proſecution of his experiments, he concluded that the different colours of bodies ariſe from their reflecting this or that kind of rays moſt copiouſly. This method of accounting for the different colours of bodies ſoon became generally adopted, and ſtill continues to be the moſt prevailing opinion. It is hence agreed that the light of the ſun, which to us ſeems white and perfectly homogeneal, is compoſed of no fewer than ſeven different colours, viz red, orange, yellow, green, blue, purple, and violet or indigo: that a body which appears of a red colour, has the property of reflecting the red rays more plentifully than the reſt; and ſo of the other colours, the orange, yellow, green, &c: alſo that a body which appears black, inſtead of reflecting, abſorbs all or the moſt part of the rays that fall upon it; while, on the contrary, a body which appears white, reflects the greateſt part of all the rays indiſcriminately, without ſeparating them one from another.

The foundation of a rational theory of colours being thus laid, the next inquiry was, by what peculiar mechaniſm, in the ſtructure of each particular body, it was fitted to reflect one kind of rays more than another; and this is attributed, by Sir I. Newton, to the denſity of theſe bodies. Dr. Hooke had remarked, that thin tranſparent ſubſtances, particularly ſoap-water blown into bubbles, exhibited various colours, according to their thinneſs; and yet, when they have a conſiderable degree of thickneſs, they appear colourleſs. And Sir Iſaac himſelf had obſerved, that as he was compreſſing two priſms hard together, in order to make their ſides (which happened to be a little convex) to touch one another, in the place of contact they were both perfectly tranſparent, as if they had been but one continued piece of glaſs: but round the point of contact, where the glaſſes were a little ſeparated from each other, rings of different colours appeared. And when he afterwards, farther to elucidate this matter, employed two convex glaſſes of teleſcopes, preſſing their convex ſides upon one another, he obſerved ſeveral ſeries of circles or rings of ſuch colours, different, and of various intenſities, according to their diſtance from the common central pellucid point of contact.

As the colours were thus found to vary according to the different diſtances between the glaſs plates, Sir Iſaac conceived that they proceeded from the different thickneſs of the plate of air intercepted between the glaſſes; this plate of air being, by the mere circumſtance of thinneſs or thickneſs, diſpoſed to reflect or tranſmit the rays of this or that particular colour. Hence therefore he concluded, that the colours of all natural bodies depend on their denſity, or the magnitude of their component particles: and hence alſo he conſtructed a table, in which the thickneſs of a plate neceſſary to reflect any particular colour, was expreſſed in millionth parts of an inch.

From a great variety of ſuch experiments, and obſervations upon them, our author deduced his theory of colours. And hence it ſeems that every ſubſtance in nature is tranſparent, provided it be made ſufficiently thin; as gold, the denſeſt ſubſtance we know of, when reduced into thin leaves, tranſmits a bluiſh-green light through it. If we ſuppoſe any body therefore, as gold for inſtance, to be divided into a vaſt number of plates, ſo thin as to be almoſt perfectly tranſparent; it is evident that all, or the greateſt part of the rays, will paſs through the upper plates, and when they loſe their force will be reflected from the under ones. They will then have the ſame number of plates to paſs through which they had penetrated before; and thus, according to the number of thoſe plates through which they are obliged to paſs, the object appears of this or that colour, juſt as the rings of colours appeared different in the experiment of the two plates, according to their diſtance from one another, or the thickneſs of the plate of air between them.

This theory of the colours has been illuſtrated and confirmed by various experiments, made by other phyloſophers. Mr. E. H. Delaval produced ſimilar effects by the infuſions of flowers of different colours, and by the intimate mixture of the metals with the ſubſtance of glaſs, when they are reduced to very fine parts; the more denſe metals imparting to the glaſs the leſs refrangible colours, and the lighter ones thoſe colours that are more eaſily refrangible. Dr. Prieſtley and Mr. Canton alſo, by laying very thin leaves or ſlips of the metals upon glaſs, ivory, wood, or metal, and paſſing an electrical ſtroke through them, found that the ſame effect was produced, viz, that the ſubſtrated was tinged with different colours, according to the diſtance from the point of exploſion.

However, the Abbe Mazeas and M. du Tour contended, that the colours between the glaſſes are not to be aſcribed to the thin ſtratum of air, ſince they equally produced them by rubbing and preſſing together two flat glaſſes, which cohered ſo cloſely that it required the greateſt force to move or ſlide them over one another. See Prieſtley's Hiſtory of Viſion.

Of Newton's 8th Exper. in the 2d Book of Optics.

The event of this experiment, which has been contradicted by repetitions of the ſame by other philoſophers, having been the occaſion of much controverſy; and relating to a material part of the doctrine of chromatics, it will not be improper here to give an account of what has paſſed concerning it. Newton found, he ſays, that when light, by contrary refractions through different mediums, is ſo corrected, that it emerges in lines parallel to the incident rays, it continues ever after to be white. But that if the emergent rays be inclined to the incident ones, the whiteneſs of the emerging light will, by degrees, in paſſing on from the place of emergence, become tinged at its edges with colours. And theſe laws he inferred from experiments made by refracting light with priſms of glaſs, placed within a priſmatic veſſel of water.

By theorems deduced from this experiment he infers, that the refraction of the rays of every ſort, made out of any medium into air, are known by having the refraction of the rays of any one ſort: and alſo, that the refraction out of one medium into another is found, whenever we have the refractions out of them both, into any third medium.

Now the ſame experiment, when ſince performed by other perſons, turning out contrary to what is ſtated above, ſome rather free reflections have been thrown upon Newton concerning it; but which however have been very ſatisfactorily obviated by Mr. Peter Dollond, in a late pamphlet on this ſubject; as we ſhall ſhew below.

In the firſt place then, M. Klingenſtierna, a Swediſh philoſopher, having in the year 1755 conſidered the controverſy between Euler and Mr. John Dollond, relative to the refraction of light, formed a theorem of his own, from geometrical reaſoning, by which he was induced to believe that the reſult of Newton's experiment could not be as he had related it; except when the angles of the refracting mediums are ſmall. See the paper on this matter by Klingenſtierna in the pamphlet above cited by Mr. Peter Dollond.

This paper of Klingenſtierna being communicated to Mr. John Dollond by Mr. Mallet, to whom it was ſent for that purpoſe, made Dollond entertain doubts concerning Newton's report of the reſult of his experiment, and determined him to have recourſe to experiments of his own, which he did in the year 1757, as follows.

He cemented two glaſs planes together by their edges, ſo as to form a priſmatic veſſel when cloſed at the ends or baſes; and the edge being turned downward, he placed it in a glaſs priſm with one of its edges upward, filling up the vacancy with clear water; ſo that the refraction of the priſm was contrived to be contrary to that of the water, in order that a ray of light, tranſmitted through both theſe refracting mediums, might be affected by the difference only between the two refractions. As he found the water to refract more or leſs than the glaſs priſm, he diminiſhed or augmented the angle between the glaſs plates, till the two contrary refractions became equal, which he diſcovered by viewing an object through this double priſm. For when it appeared neither raiſed nor depreſſed, he was ſatisfied that the refractions were equal, and that the emergent rays were parallel to the incident ones.

Now, according to the prevailing opinion, he obſerves, that the object ought to have appeared through this double priſm in its natural colour; for if the difference of refrangibility had been in all reſpects equal, in the two equal refractions, they would have rectified each other. But this experiment fully proved the fallacy of the received opinion, by ſhewing that the divergency of the light by the glaſs priſm, was almoſt double of that by the water; for the image of the object, though not at all refracted, was yet as much infected with priſmatic colours, as if it had been ſeen through a glaſs wedge only, having its angle of near 30 degrees.

This experiment is the very ſame with that of Sir Iſaac Newton above-mentioned, notwithſtanding the reſult was ſo remarkably different. Mr. Dollond plainly ſaw however, that if the refracting angle of the water-veſſel could have admitted of a ſufficient increaſe, the divergency of the coloured rays would have been greatly diminiſhed, or entirely rectified; and that there would have been a very great refraction without colour, as he had already produced a great diſcolouring without refraction: but the inconveniency of ſo large an angle as that of the priſmatic veſſel muſt have been, to bring the light to an equal divergency with that of the glaſs priſm, whoſe angle was about 60 degrees, made it neceſſary to try ſome experiments of the ſame kind with ſmaller angles.

Accordingly he procured a wedge of plate-glaſs, whoſe angle was only 9 degrees; and, uſing it in the ſame circumſtances, he increaſed the angle of the water-wedge, in which it was placed, till the divergency of the light by the water was equal to that by the glaſs; that is, till the image of the object, though conſiderably refracted by the exceſs of the refraction of the water, appeared nevertheleſs quite free from any colours proceeding from the different refrangibility of the light.

Many conjectures were made as to the cauſe of ſo ſtriking a difference in the reſults of the ſame experiment; but none that gave any great ſatisfaction, till lately that it has been ſhewn to be probably owing to the nature of the glaſs then uſed by Newton. This conjecture is made by Mr. Peter Dollond, ſon of John, the inventor of the achromatic teleſcope, in a pamphlet by him lately publiſhed in defence of his father's invention, againſt the miſrepreſentations of ſome perſons who have unjuſtly attempted to give the invention to other philoſophers, who themſelves never imagined that they had any right to it. After a full and ſatisfactory vindication of his father, Mr. P. Dollond then adds,

“I now come to a more agreeable part of this paper, which is, to endeavour to reconcile the different reſults of the 8th experiment of the 2d part of the 1ſt book of Newton's Optics, as related by himſelf, and as it was found by Dollond, when he tried the ſame experiment, in the year 1757. Newton ſays, that light, as often as by contrary refractions it is ſo corrected, that it emergeth in lines parallel to the incident, continues ever after to be white. Now Dollond ſays, when he tried the ſame experiment, and made the mean refraction of the water equal to that of the glaſs priſm, ſo that the light emerged in lines parallel to the incident, he found the divergency of the light by the glaſs priſm to be nearly double to what it was by the water priſm. The light appeared to be ſo evidently coloured, that it was directly ſaid by ſome perſons, that if Newton had actually tried the experiment, he muſt have perceived it to have been ſo. Yet who could for a moment doubt the veracity of ſuch a character? Therefore different conjectures were made by different perſons. Mr. Murdoch in particular gave a paper to the Royal Society in defence of Newton; but it was ſuch as very little tended to clear up the matter. Philoſ. Tranſ. vol. 53. pa. 192.—Some have ſuppoſed that Newton made uſe of water ſtrongly impregnated with ſaccharum ſaturni, becauſe he mentions ſometimes uſing ſuch water, to increaſe the refraction, when he uſed water priſms inſtead of glaſs priſms. Newton's Opt. pa. 62.—And others have ſuppoſed, that he tried the experiment with ſo ſtrong a perſuaſion in his own mind that the divergency of the colours was always in the ſame proportion to the mean refraction, in all ſorts of refracting mediums, that he did not attend ſo much to that experiment as he ought to have done, or as he uſually did. None of theſe ſuppoſitions having appeared at all ſatisfactory, I have therefore endeavoured to find out the true cauſe of the difference, and thereby ſhew, how the experiment may be made to agree with Newton's deſcription of it, and to get rid of thoſe doubts, which have hitherto remained to be cleared up.

“It is well known, that in Newton's time the Engliſh were not the moſt famous for making optical inſtruments: Teleſcopes, opera-glaſſes, &c, were imported from Italy in great numbers, and particularly from Venice; where they manufactured a kind of glaſs which was much more proper for optical purpoſes than any made in England at that time. The glaſs made at Venice was nearly of the ſame refractive quality as our own crown-glaſs, but of a much better colour, being ſufficiently clear and tranſparent for the purpoſe of priſms. It is probable that Newton's priſms were made with this kind of glaſs; and it appears to be the more ſo, becauſe he mentions the ſpecific gravity of common glaſs to be to water as 2·58 to 1, Newton's Opt. pa. 247, which nearly anſwers to the ſpecific gravity we find the Venetian glaſs generally to have. Having a very thick plate of this kind of glaſs, which was preſented to me about 25 years ago by the late profeſſor Allemand, of Leyden, and which he then informed me had been made many years; I cut a piece from this plate of glaſs to form a priſm, which I conceived would be ſimilar to thoſe made uſe of by Newton himſelf. I have tried the Newtonian experiment with this priſm, and find it anſwers ſo nearly to what Newton relates, that the difference which remains may very eaſily be ſuppoſed to ariſe from any little difference which may and does often happen in the ſame kind of glaſs made at the ſame place at different times. Now the glaſs priſm made uſe of by Dollond to try the ſame experiment, was made of Engliſh flint-glaſs, the ſpecific gravity of which I have never known to be leſs than 3·22. This difference in the denſities of the priſms, uſed by Newton and Dollond, was ſufficient to cauſe all the difference which appeared to the two experimenters in trying the ſame experiment.

" From this it appears, that Newton was accurate in this experiment as in all others, and that his not having diſcovered that, which was diſcovered by Dollond ſo many years afterwards, was owing entirely to accident; for if his priſm had been made of glaſs of a greater or leſs denſity, he would certainly have then made the diſcovery, and refracting teleſcopes would not have remained ſo long in their original imperfect ſtate." See *Achromatic*, and *Teleſcope*.

Mr. Delaval's experiments on the colours of opaque bodies.—Beſide the experiments of this gentleman, before-mentioned, on the colours of tranſparent bodies, he has lately publiſhed an account of ſome made upon the permanent colours of opaque ſubſtances, the diſcovery of which muſt be of the utmoſt conſequence in the arts of colour-making and dyeing.

The changes of colour in permanently coloured bodies, our author obſerves, are produced by the ſame laws that take place in tranſparent colourleſs ſubſtances; and the experiments by which they are inveſtigated conſiſt chiefly of various methods of uniting the colouring particles into larger maſſes, or dividing them into ſmaller ones. Sir Iſaac Newton made his experiments chiefly on tranſparent ſubſtances; and in the few places where he treats of others, he acknowledges his want of experiments. He makes the following remark however on thoſe bodies which reflect one kind of light and tranſmit another, viz, that if theſe glaſſes or liquors were ſo thick and maſſy that no light could get through them, he queſtioned whether they would not, like other opaque bodies, appear of one and the ſame colour in all poſitions of the eye; though he could not yet affirm it from experience. Indeed it was the opinion of this great philoſopher, that all coloured matter reflects the rays of light, ſome reflecting the more refrangible rays moſt copiouſly, and others thoſe that are leſs ſo; and that this is at once the true and only reaſon of theſe colours. He was likewiſe of opinion that opaque bodies reflect the light from their anterior ſurface, by ſome power of the body evenly diffuſed over and external to it. With reſpect to tranſparent coloured bodies he thus expreſſes himſelf: " A tranſparent body which looks of any colour by tranſmitted light, may alſo look of the ſame colour by reflected light; the light of that colour being reflected by the farther ſurface of that body, or by the air beyond it: and then the reflected colour will be diminiſhed, and perhaps ceaſe, by making the body very thick, and pitching it on the back-ſide to diminiſh the reflection of its farther ſurface, ſo that the light reflected from the tinging particles may predominate. In ſuch caſes the colour of the reflected light will be apt to vary from that of the light tranſmitted."

To ſearch out the truth of theſe opinions, Mr. Delaval entered upon a courſe of experiments with tranſparent coloured liquors and glaſſes, as well as with opaque and ſemitranſparent bodies. And from theſe experiments he diſcovered ſeveral remarkable properties of the colouring matter; particularly, that in tranſparent coloured ſubſtances it does not reflect any light; and when, by intercepting the light which was tranſmitted, it is hindered from paſſing through ſuch ſubſtances, they do not vary from their former colour to any other, but become entirely black.

This incapacity of the colouring particles of tranſparent bodies to reflect light, being deduced from very numerous experiments, may therefore be taken as a general law. It will appear the more extenſive, if it be conſidered that, for the moſt part, the tinging particles of liquors, or other tranſparent ſubſtances, are extracted from opaque bodies; that the opaque bodies owe their colours to thoſe particles, in like manner as the tranſparent ſubſtances do; and that by the loſs of them they are deprived of their colours.

Notwithſtanding theſe and many other experiments, the theory of colours ſeems not yet determined with certainty; and it muſt be acknowledged that very ſtrong objections might be brought againſt every hypotheſis on this ſubject that has been invented. The diſcoveries of Sir Iſaac Newton however are ſufficient to juſtify the following Aphoriſms.

Aphoriſm 1. All the colours in nature ariſe from the rays of light.

2. There are ſeven primary colours, namely red, orange, yellow, green, blue, indigo, and violet.

3. Every ray of light may be ſeparated into theſe ſeven primary colours.

4. The rays of light, in paſſing through the ſame medium, have different degrees of refrangibility.

5. The difference in the colours of light ariſes from its different refrangibility: that which is the leaſt refrangible producing red; and that which is the moſt refrangible, violet.

6. By compounding any two of the primary, as red and yellow, or yellow and blue, the intermediate colour, orange or green, may be produced.

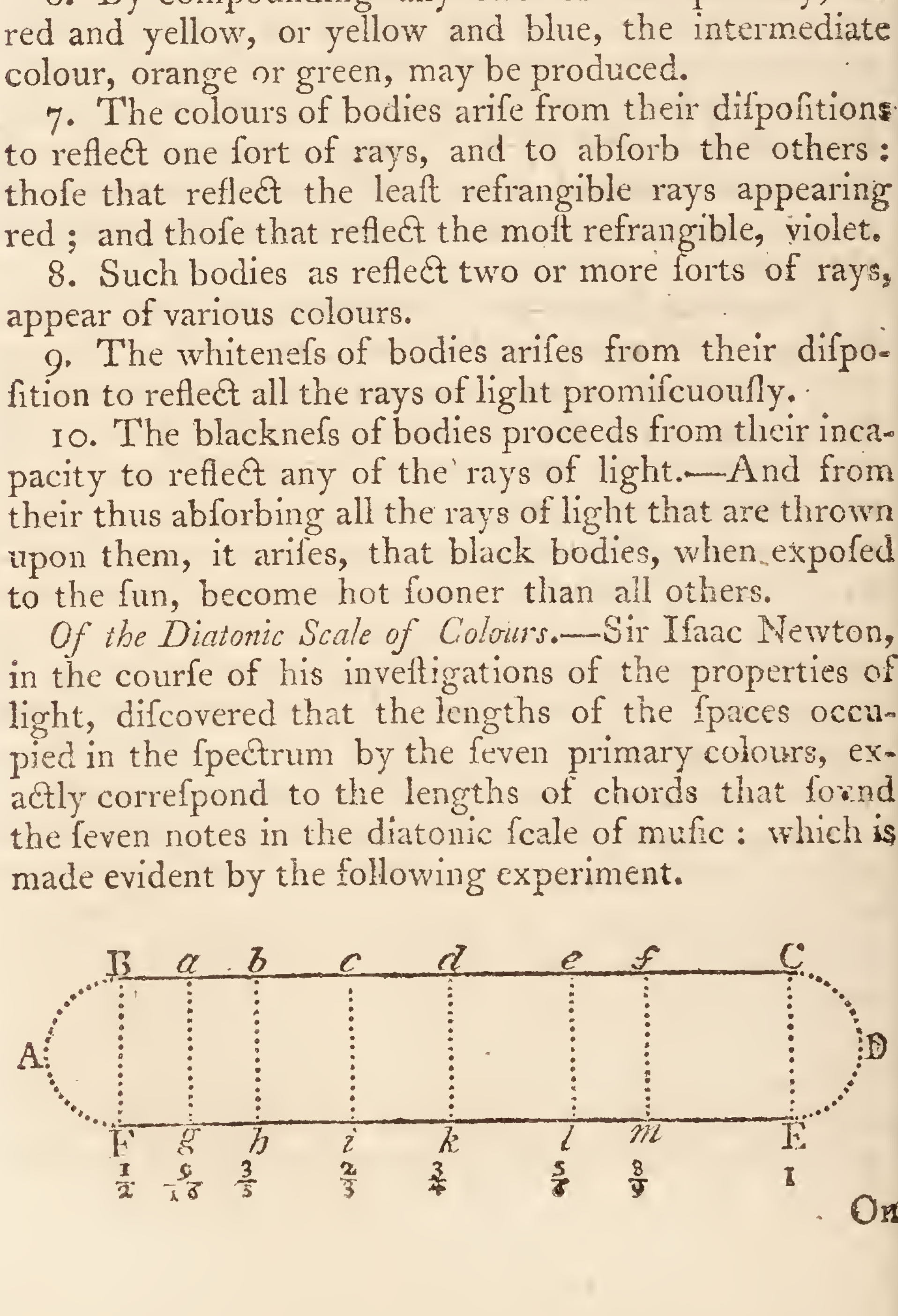
7. The colours of bodies ariſe from their diſpoſitions to reflect one ſort of rays, and to abſorb the others: thoſe that reflect the leaſt refrangible rays appearing red; and thoſe that reflect the moſt refrangible, violet.

8. Such bodies as reflect two or more ſorts of rays, appear of various colours.

9. The whiteneſs of bodies ariſes from their diſpoſition to reflect all the rays of light promiſcuouſly.

10. The blackneſs of bodies proceeds from their incapacity to reflect any of the rays of light.—And from their thus abſorbing all the rays of light that are thrown upon them, it ariſes, that black bodies, when expoſed to the ſun, become hot ſooner than all others.

Of the Diatonic Scale of Colours.—Sir Iſaac Newton, in the courſe of his inveſtigations of the properties of light, diſcovered that the lengths of the ſpaces occupied in the ſpectrum by the ſeven primary colours, exactly correſpond to the lengths of chords that ſound the ſeven notes in the diatonic ſcale of muſic: which is made evident by the following experiment.



On a paper, in a dark chamber, let a ray of light be largely refracted into the ſpectrum ABCDEF, marking upon it the preciſe boundaries of the ſeveral colours, as *a*, *b*, *c*, &c ; and acroſs the ſpectrum draw the perpendicular lines *ag*, *bh*, &c. Then it will be found that the ſpaces, by which the ſeveral colours are bounded, viz. B*ag*F containing the red, *abhg* containing the orange, *bcih* containing the yellow, &c, will be in exact proportion to the diviſions of a muſical chord for the notes of an octave ; that is, as the intervals of theſe numbers 1, $\frac{8}{9}$, $\frac{5}{6}$, $\frac{3}{4}$, $\frac{2}{3}$, $\frac{3}{5}$, $\frac{9}{16}$, $\frac{1}{2}$.

CHRONOLOGICAL, belonging to chronology.

Chronological Characters, are characters by which times are diſtinguiſhed. Of theſe, ſome are natural, or aſtronomical ; others are artificial or hiſtorical. The natural characters are ſuch as depend on the motions of the ſtars or luminaries ; as eclipſes, ſolſtices, equinoxes, the different aſpects of planets, &c. And the artificial characters are thoſe that have been invented and eſtabliſhed by men ; as the ſolar cycle, the lunar cycle, &c. Hiſtorical Chronological Characters are thoſe ſupported by the teſtimonies of hiſtorians, when they fix the dates of certain events to certain periods.

CHRONOLOGY, the art of meaſuring and diſtinguiſhing time ; with the doctrine of dates, epochs, eras, &c.

The meaſurement of time in the moſt early periods, was by means of the ſeaſons, or the revolutions of the ſun and moon. The ſucceſſion of Juno's prieſteſſes at Argos ſerved Hellanicus for the regulation of his narrative ; while Ephorus reckoned his matters by generations. Even in the hiſtories of Herodotus and Thucydides, there are no regular dates for the events recorded ; nor were there any endeavours to eſtabliſh a fixed era until the time of Ptolomy Philadelphus, who attempted it by comparing and correcting the dates of the olympiads, the kings of Sparta, and the ſucceſſion of the prieſteſſes of Juno at Argos. Eratoſthenes and Apollodorus digeſted the events related by them, according to the ſucceſſion of the olympiads and of the Spartan kings.

The chronology of the Latins is ſtill more uncertain. The records of the Romans were deſtroyed by the Gauls ; and Fabius Pictor, the moſt ancient of their hiſtorians, was obliged to borrow the chief part of his information from the Greeks. In other European nations the chronology is ſtill more imperfect, and of a later date ; and even in modern times a conſiderable degree of confuſion and inaccuracy has ariſen, from the want of attention in the hiſtorians to aſcertain the dates and epochs with preciſion.

Hence it is evident, how neceſſary a proper ſyſtem of chronology muſt be for the right underſtanding of hiſtory, and alſo how difficult it muſt be to eſtabliſh ſuch a ſyſtem. For this purpoſe, however, ſeveral learned men have ſpent much time, particularly Julius Africanus, Euſebius of Cæſarea, George Cyncelle, John of Antioch, Dennis, Petau, Clavius, Calviſius, Scaliger, Vieta, Newton, Uſher, Simſon, Marſham, Helvicus, Voſſius, Strauchius, Blair, and Playfair.

Such a ſyſtem is founded, 1ſt, On aſtronomical obſervations, eſpecially of the eclipſes of the ſun and moon, combined with calculations of the years and eras of different nations. 2d, The teſtimonies of credible authors. 3d, Such epochs in hiſtory as are ſo well atteſted and determined, that they have never been controverted. 4th, Ancient medals, coins, monuments, and inſcriptions.

The moſt obvious diviſion of time, as has been obſerved, is derived from the apparent or real revolutions of the luminaries, the ſun and moon. Thus, the apparent revolution of the ſun, or the real rotation of the earth on her axis cauſing the ſun to appear to riſe and ſet, conſtitutes the viciſſitudes of day and night, which muſt be evident to the moſt barbarous and ignorant nations. The moon, by her revolution about the earth, and her changes, as naturally and obviouſly forms months ; while the great annual courſe of the ſun through the ſeveral conſtellations of the zodiac, points out the larger diviſion of the year.

The Day is divided into hours, minutes, &c ; while the month is divided into weeks, and the year into months, having particular names, and a certain number of days.—See a particular account of each of theſe under the reſpective words.

Beſide the natural diviſions of time ariſing immediately from the revolutions of the heavenly bodies, there are others which are formed from ſome of the leſs obvious conſequences of theſe revolutions, and are called *cycles*, or circles. The moſt remarkable of theſe are, 1, The *Solar Cycle*, or cycle of the ſun, a period or revolution of 28 years, in which time the days of the months return to the ſame days of the week, the ſun's place to the ſame ſigns and degrees of the ecliptic on the ſame months and days, and the leap-years begin the ſame courſe over again with reſpect to the days of the week on which the days of the months fall. 2, The *Lunar Cycle*, or cycle of the moon, commonly called the Golden Number, is a revolution of 19 years ; in which time the conjunctions, oppoſitions, and other aſpects of the moon, are on the ſame days of the months as they were 19 years before, and within an hour and a half of the ſame time of the day.

The *Indiction*, or Roman Indiction, is a period of 15 years, uſed only by the Romans for indicating the times of certain payments made by the ſubjects to the republic.

The *Cycle of Eaſter*, called alſo the *Dionyſian Period*, is a revolution of 532 years, and is produced by multiplying the ſolar cycle 28, by the lunar cycle 19.

The *Julian Period*, is a revolution of 7980 years, and is produced from the continual multiplication of the three numbers 28, 19, 15, of the three former cycles, viz, the ſolar, lunar, and indiction.

As there are certain fixed points in the heavens, from which aſtronomers begin their computations, ſo there are certain points of time, from which hiſtorians begin to reckon ; and theſe points or roots of time are called *eras* or *epochs*. The moſt remarkable of theſe are, thoſe of the Creation, the Greek Olympiads, the building of Rome, the era of Nabonnaſſar, the death of Alexander, the birth of Chriſt, the Arabian Hegira, or flight of Mahomet, and the Perſian Jeſdegird. All which, with ſome others of leſs note, have their beginnings fixed by chronologers to the years of the Julian period, to the age of the world, and to the years before and after the birth of Chriſt.

The teſtimony of authors is the ſecond principal part of hiſtorical chronology. Though no man has a right to be conſidered as infallible, it would however be making a very unfair judgment of mankind, to treat them all as dupes or impoſtors; and it would be an injury offered to public integrity, to doubt the veracity of authors univerſally eſteemed, and facts that are truly worthy of belief. When the hiſtorian is allowed to be completely able to judge of an event, and to have no intent of deceiving by his relation, his teſtimony cannot be refuſed.

The *Epochs* form the 3d principal part of chronology; being thoſe fixed points in hiſtory that have never been conteſted, and of which there cannot reaſonably be any doubt. Notwithſtanding that chronologers fix upon the events which are to ſerve as epochs, in a manner quite arbitrary; yet this is of little conſequence, provided the dates of theſe epochs agree, and that there is no contradiction in the facts themſelves.

Medals, Monuments, and Inſcriptions, form the laſt of the four principal parts of chronology; and this ſtudy is but of very modern date, ſcarce more than 150 years having elapſed ſince cloſe application has been made to the ſtudy of theſe. To the celebrated Spanheim we owe the greateſt obligations, for the progreſs that is made in this method; and it is by the aid of medals that M. Vaillant has compoſed his judicious hiſtory of the kings of Syria, from the time of Alexander the Great to that of Pompey. Nor have they been of leſs ſervice in elucidating all ancient hiſtory, eſpecially that of the Romans; and even ſometimes that of the middle ages.

Beſides the foregoing general account, there are ſome few ſyſtems of chronology which may deſerve ſome more particular notice, as follows.

Sacred Chronology. There have been various ſyſtems relating to ſacred chronology; which is not to be wondered at, as the three chief copies of the Bible give a very different account of the firſt ages of the world. For while the Hebrew text reckons about 4000 years from the creation to the birth of Chriſt, and to the flood 1656 years; the Samaritan makes the former much longer, though it counts from the creation to the flood only 1307 years; and the Septuagint removes the creation of the world to 6000 years before Chriſt, and 2250 years before the flood. Many attempts have been made to reconcile theſe differences; though none of them are quite ſatisfactory. Walton and Voſſius give the preference to the accountof the Septuagint; while others have defended the Hebrew text. See an abſtract of the different opinions of learned men on this ſubject, in Strauchius's Brev. Chron. tranſlated by Sault, p. 166 and 176.

The Chineſe Chronology. No nation has boaſted more of its antiquity than the Chineſe: but though they be allowed to trace their origin as far back as the deluge, they have few or no authentic records of their hiſtory for ſo long a period as 500 years before the Chriſtian era. This indeed may be owing to the general deſtruction of ancient remains by the tyrant Tſin-chi-hoang, in the year 213, or ſome ſay 246, before Chriſt. From a chronology of the Chineſe hiſtory (for which we are obliged to an illuſtrious Tartar who was viceroy of Canton in the year 1724, and of which a Latin tranſlation was publiſhed at Rome in 1730), we learn that the moſt remote epoch of the Chineſe chronology does not ſurpaſs the firſt year of Guei-lie-wang, or 424 years before our vulgar era. And this opinion is confirmed by the practice of two of the moſt approved hiſtorians of China, who admit nothing into their hiſtories previous to this period.

The Chineſe, in their computation, make uſe of a cycle of 60 years, called kia-tſe, from the name given to the firſt year of it, which ſerves as the baſis of their whole chronology. Every year of this cycle is marked with two letters which diſtinguiſh it from the others; and all the years of the emperors, for upwards of 2000 years, have names in hiſtory common to them with the correſponding years of the cycle. Philoſ. Tranſ. Abridg. vol. 8, part 4, pag. 13.

According to M. Freret, in his Eſſays, the Chineſe date the epocha of Yao, one of their firſt emperors, about the year 2145, or 2057, before Chriſt; and they reckon that their firſt aſtronomical obſervations, and the compoſition of their calendar, preceded Yao 150 years: from whence it is inferred that the era of their aſtronomical obſervations coincides with that of the Chaldeans. But later authors date the riſe and progreſs of the ſciences in China from the grand dynaſty of Tcheou, about 1200 years before the Chriſtian era, and ſhew that all hiſtorical relations of events prior to the reign of Yao are fabulous. Mem. de l'Hiſtoire des Sciences &c. Chinois. vol. 1, Paris 1776.

Babylonian, Egyptian, and Chaldean Annals. Theſe, M. Gibert has attempted to reduce to our chronology, in a letter publiſhed at Amſterdam in 1743. He begins with ſhewing, by the authorities of Macrobius, Eudoxus, Varro, Diodorus Siculus, Pliny, Plutarch, St. Auguſtin, &c, that by a year, the ancients meant the revolution of any planet in the heavens; ſo that it might conſiſt ſometimes of only one day. Thus, according to him, the ſolar day was the aſtronomical year of the Chaldeans; and ſo the boaſted period of 473,000 years, aſſigned to their obſervations, is reduced to 1297 years 9 months; the number of years which, according to Euſebius, elapſed from the firſt diſcoveries of Atlas in aſtronomy, in the 384th year of Abraham, to the march of Alexander into Aſia in the year 1682 of the ſame era. And the 17,000 years added by Beroſus to the obſervations of the Chaldeans, reduced in the ſame manner, will give 46 years and 6 or 7 months; being the exact interval between Alexander's march, and the firſt year of the 123d Olympiad, or the time to which Beroſus carried his hiſtory.

Epigenius aſcribes 720,000 years to the obſervations preſerved at Babylon; but theſe, according to M. Gibert's ſyſtem, amount only to 1971 years 3 months; which differ from Calliſthenes's period of 1903 years, allotted to the ſame obſervations, only by 68 years, the period elapſed from the taking of Babylon by Alexander, which terminated the latter account, and to the time of Ptolomy Philadelphus, to which Epigenius extended his account.

The Newtonian Principles of Chronology.—Sir Iſaac Newton has ſhewn, that the chronology of ancient kingdoms is involved in the greateſt uncertainty: that the Europeans in particular had no chronology before the Perſian empire, which commenced 536 years before the birth of Chriſt, when Cyrus conquered Darius the Mede: that the antiquities of the Greeks are full of fables, becauſe their writings were in verſe only, till the conqueſt of Aſia by Cyrus the Perſian; about which time proſe was introduced by Pherecides Syrius and Cadmus Mileſius. After this time ſeveral of the Greek hiſtorians introduced the computation by generations. The chronology of the Latins was ſtill more uncertain: their old records having been burnt by the Gauls 120 years after the expulſion of their kings, or 64 years before the death of Alexander the Great, anſwering to 388 before the birth of Chriſt. The chronologers of Gaul, Spain, Germany, Scythia, Sweden, Britain, and Ireland, are of a ſtill later date. For Scythia, beyond the Danube, had no letters till Ulphilas, their biſhop, formed them, about the year 276. Germany had none till it received them from the weſtern empire of the Latins, about the year 400. The Huns had none in the days of Procopius, about the year 526. And Sweden and Norway received them ſtill later.

Sir Iſaac Newton, after a general account of the obſcurity and defects of the ancient chronology, obſerves that, though many of the ancients computed by ſucceſſions and generations, yet the Egyptians, Greeks, and Latins, reckoned the reigns of kings equal to generations of men, and three of them to a hundred, and ſometimes to 120 years; and this was the foundation of their technical chronology. He then proceeds, from the ordinary courſe of nature, and a detail of hiſtorical facts, to ſhew the difference between reigns and generations; and that, though a generation from father to ſon may at an average be reckoned about 33 years, or three of them equal to 100 years, yet, when they are taken by the eldeſt ſons, three of them cannot be eſtimated at more than about 75 or 80 years; and the reigns of kings are ſtill ſhorter; ſo that 18 or 20 years may be allowed a juſt medium. Sir Iſaac then fixes on four remarkable periods, viz, the return of the Heraclidæ into the Peloponneſus, the taking of Troy, the Argonautic expedition, and the return of Seſoſtris into Egypt, after his wars in Thrace; and he ſettles the epoch of each by the true value of a generation. To inſtance only his eſtimate of that of the Argonautic expedition: Having fixed the return of the Heraclidæ to about the 159th year after the death of Solomon, and the deſtruction of Troy to about the 76th year after the ſame period, he obſerves, that Hercules the Argonaut was the father of Hyllus, the father of Clerdius, the father of Ariſtomachus, the father of Ariſtodemus, who conducted the Heraclidæ into Peloponneſus; ſo that, reckoning by the chief of the family, their return was four generations later than the Argonautic expedition, which therefore happened about 43 years after the death of Solomon. This is farther confirmed by another argument: Æſculapius and Hercules were Argonauts: Hippocrates was the 18th incluſively from the former by the father's ſide, and the 19th from the latter by the mother's ſide: now, allowing 28 or 30 years to each of them, the 17 intervals by the father, and the 18 intervals by the mother, will on a medium give 507 years; and theſe, reckoning back from the commencement of the Peloponneſian war, or the 431ſt year before Chriſt, when Hippocrates began to flouriſh, will place the Argonautic expedition in the 43d year after the death of Solomon, or 937 years before Chriſt.

The other kind of reaſoning by which Newton endeavours to eſtabliſh this epoch, is purely aſtronomical. The ſphere was formed by Chiron and Muſæus for the uſe of the Argonautic expedition, as is plainly ſhewn by ſeveral of the aſteriſms referring to that event: and at the time of the expedition the cardinal points of the equinoxes and ſolſtices were placed in the middle of the conſtellations Aries, Cancer, Chelæ, and Capricorn. This point is eſtabliſhed by Newton from the conſideration of the ancient Greek calendar, which conſiſted of 12 lunar months, and each month of 30 days, which required an intercalary month. Of courſe this luniſolar year, with the intercalary month, began ſometimes a week or two before or after the equinox or ſolſtice; and hence the firſt aſtronomers were led to the before-mentioned diſpoſition of the equinoxes and ſolſtices: and that this was really the caſe, is confirmed by the teſtimonies of Eudoxus, Aratus, and Hipparchus. Upon theſe principles Sir Iſaac proceeds to argue in the following manner. The equinoctial colure in the end of the year 1689 cut the ecliptic in ♉ 6° 44'; and by this reckoning the equinox had then gone back 36° 44' ſince the time of the Argonautic expedition. But it recedes 50' in a year, or 1° in 72 years, and conſequently 36° 44' in 2645 years; and this, counted backwards from the beginning of 1690, will place this expedition about 25 years after the death of Solomon. But as there is no neceſſity for allowing that the middle of the conſtellations, according to the general account of the ancients, ſhould be preciſely the middle between the prima Arietis and ultima Caudæ, our author proceeds to "examine what were thoſe ſtars through which Eudoxus made the colures to paſs in the primitive ſphere, and in this way to fix the poſition of the cardinal points." Now from the mean of five places he finds, that the great circle, which in the primitive ſphere, deſcribed by Eudoxus, or which at the time of the Argonautic expedition was the equinoctial colure, did in the end of 1689 cut the ecliptic in ♉ 6° 29' 15''. In the ſame manner our author determines that the mean place of the ſolſtitial colure is ♌ 6° 28' 46'', and as it is at right angles with the other, he concludes that it is rightly drawn. And hence he infers that the cardinal points, in the interval between that expedition and the year 1689, have receded from thoſe colures 1^{s} 6° 29'; which, allowing 72 years to a degree, amounts to 2627 years; and theſe counted backwards, as above, will place the Argonautic expedition 43 years after the death of Solomon. Our author has, by other methods alſo of a ſimilar nature, eſtabliſhed this epoch, and reduced the age of the world 500 years.

This elaborate ſyſtem has not eſcaped cenſure; Meſſ. Freret and Souciet having both attacked it, and on much the ſame ground: but the former has confounded reigns and generations, which are carefully diſtinguiſhed in this ſyſtem. The aſtronomical objections of both have been anſwered by Sir Iſaac Newton himſelf, and by Dr. Halley. Philoſ. Tranſ. abr. vol. 8, part 4, pa. 4. Newton's Chronol. ch. 1.

CHRONOMETER, is any inſtrument or machine uſed in meaſuring time; ſuch as dials, clocks, watches, &c.

The term is however more particularly uſed for a kind of clock, ſo contrived as to meaſure a ſmall portion of time, even to the 16th, or the 40th part of a ſecond; one of this latter kind I have ſeen, made by an ingenious artiſt; but it could not be ſtopped to the 10th part of the propoſed degree of accuracy. There is a deſcription of one alſo in Deſaguliers's Experimental Philoſophy, invented by the late ingenious Mr. George Graham; which might be of great uſe for meaſuring ſmall portions of time in aſtronomical obſervations, the time of the fall of bodies, the velocity of running waters, &c. But long intervals of time cannot be meaſured by it with ſufficient exactneſs, unleſs its pendulum be made to vibrate in a cycloid; for otherwiſe it is liable to err conſiderably, as is the caſe of all clocks with ſhort pendulums that ſwing in large arches of the circle.

Various other contrivances, beſides clocks, have been uſed for meaſuring time for ſome particular purpoſes. See a muſical chronometer deſcribed in Malcolm's Treatiſe of Muſic, pa. 407.

CHRONOSCOPE, a word ſometimes uſed for a pendulum, or machine, to meaſure time.

CHRYSTALLINE. See CRYSTALLINE.

CIMA, or SIMA, in Architecture, a member, or moulding, called alſo ogee, and cimatium.

CINCTURE, in Architecture, is a ring or liſt around the ſhaft of a column, at its top and bottom.

CINTRE, in Building, the mould on which an arch is turned or built; popularly called centre, and ſometimes a cradle.

CIPHER, or Cypher, one of the numeral characters, or figures, thus formed 0. The word comes from the Hebrew *ſaphar*, to number.

The cipher of itſelf ſignifies nothing, or implies a privation of value; but when combined with other numeral characters, it alters their value in a tenfold proportion, for every cipher ſo annexed; viz, when ſet after a figure in common integral arithmetic, it increaſes its value in that proportion, though it has no effect when ſet before or to the left hand ſide of figures; but on the contrary, in decimal arithmetic, it decreaſes their value in that proportion when ſet before the figures, but has no effect when ſet after them.

Thus, 5 is five,
but 50 is fifty,
and 500 is five hundred;
whereas 05, or 005, &c, is ſtill but 5 or five.
Alſo ·5 is five tenths,
but ·05 is five hundredths,
and ·005 is five thouſandths;
whereas ·50, or ·500, &c, is ſtill but ·5 or five tenths.

The invention and uſe of the cipher, as in the common arithmetic and notation of numbers, is one of the happieſt devices that can be imagined; and is aſcribed to the Indians, by the Arabians, through whom it came into Europe, along with the revival of literature.

CIRCLE, a plane figure, bounded by a curve line which returns into itſelf, called its circumference, and which is every where equally diſtant from a point within, called its centre.

The circumference or periphery itſelf is called the circle, though improperly, as that name denotes the ſpace contained within the circumference.

A circle is deſcribed with a pair of compaſſes, fixing one foot in the centre, and turning the other round to trace out the circumference.

The circumference of every circle is ſuppoſed to be divided into 360 equal parts, called degrees, and marked °; each degree into 60 minutes or primes, marked ′; each minute into 60 ſeconds, marked ″; and ſo on. So 24° 12′ 15″ 20‴, is 24 degrees 12 minutes 15 ſeconds and 20 thirds.

Circles have many curious properties, ſome of the moſt important of which are theſe:

1. The circle is the moſt capacious of all plain figures, or contains the greateſt area within the ſame perimeter, or has the leaſt perimeter about the ſame area; being the limit and laſt of all regular polygons, having the number of its ſides infinite.

2. The area of a circle is always leſs than the area of any regular polygon circumſcribed about it, and its circumference always leſs than the perimeter of the polygon. But on the other hand, its area is always greater than that of its inſcribed polygon, and its circumference greater than the perimeter of the ſaid inſcribed polygon. However, the area and perimeter of the circle approach always nearer and nearer to thoſe of the two polygons, as the number of the ſides of theſe is greater; the circle being always limited between the two polygons.

3. The area of a circle is equal to that of a triangle whoſe baſe is equal to the circumference, and perpendicular equal to the radius. And therefore the area of the circle is found by drawing half the circumference into half the diameter, or the whole circumference into the whole diameter, and taking the 4th part of the product. Demonſtrated by Euclid.

4. Circles, like other ſimilar plane figures, are to one another, as the ſquares of their diameters. And the area of the circle is to the ſquare of the diameter, as 11 to 14 nearly, as proved by Archimedes; or as ·7854 to 1 more nearly; or ſtill more nearly as ·7853981633,9744830961,5660845819,8757210492, 9234984377,6455243736,1480769541,0157155224, 9657008706,3355292669,9553702162,8318076661, 7734611 + to 1; as it has been found by modern mathematicians.

In Wallis's Arithmetic of Infinites are contained the firſt infinite ſeries for expreſſing the ratio of a circle to the ſquare of its diameter: viz,

1ſt, The circle is to the ſquare of its diameter,

$$\left.\begin{array}{l} \text{as } 1 \text{ to } \dfrac{3\times3\times5\times5\times7\times7\&c}{2\times4\times4\times6\times6\times8\&c}, \\[2ex] \text{or } 1 \text{ to } \dfrac{9}{8}\times\dfrac{25}{24}\times\dfrac{49}{48}\ \&c, \end{array}\right\}\ \text{found out by Wallis himſelf;}$$

$$\text{or as } 1 \text{ to } 1+\cfrac{1}{2+\cfrac{9}{2+\cfrac{25}{2+\cfrac{49}{2+\&c}}}},\ \text{by Ld. Brounker;}$$

or as $1 - \frac{1}{2\times3} - \frac{1}{2\times4\times5} - \frac{1\times3}{2\times4\times6\times7} - \frac{1\times3\times5}{2\times4\times6\times8\times9}$ &c to 1, by Sir I. Newton;

or as $1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \frac{1}{11}$ &c to 1, by Gregory and Leibnitz; and a great many other forms of ſeries have been invented by different authors, to expreſs the ſame ratio between the circle and circumſcribed ſquare.

5. The circumferences of circles are to one another, as their diameters, or radii. And as the areas of circles are proportional to the rectangles of their radii and circumferences; therefore the quadrature of the circle will be effected by the rectification of its circumference; that is, if the true length of the circumference could be found, the true area could be found alſo. But whilſt ſeveral mathematicians have endeavoured to determine the true area and circumference, others have even doubted of the poſſibility of the ſame. Of this latter opinion is Dr. Iſaac Barrow: towards the end of his 15th Mathematical Lecture he ſays, he is of opinion, that the radius and circumference of a circle, are lines of ſuch a nature, as to be not only incommenſurable in length and ſquare, but even in length, ſquare, cube, biquadrate, and all other powers to infinity: for, continues he, the ſide of the inſcribed ſquare is incommenſurable to the radius, and the ſquare of the ſide of the inſcribed octagon is incommenſurable to the ſquare of the radius; and conſequently the ſquare of the octagonal perimeter is incommenſurable to the ſquare of the radius: and thus the ambits of all regular polygons, inſcribed in a circle, may have their ſuperior powers incommenſurate with the co-ordinate powers of the radius; from whence the laſt polygon, that is, the circle itſelf, ſeems to have its periphery incommenſurate with the radius. Which, if true, will put a final ſtop to the quadrature of the circle, ſince the ratio of the circumference to the radius is altogether inexplicable from the nature of the thing, and conſequently the problem requiring the explication of ſuch a ratio is impoſſible to be ſolved, or rather it requires that for its ſolution which is impoſſible to be apprehended. But, concludes he, this great myſtery cannot be explained in a few words: But if time and opportunity had permitted, I would have endeavoured to produce many things for the explication and confirmation of this conjecture. Sir Iſaac Newton too, in book 1 of his Principia, has attempted to demonſtrate the impoſſibility of the general quadrature of oval figures, by the deſcription of a ſpiral, and the impoſſibility of determining, by a finite equation, the interſections of that oval and ſpiral, which muſt be the caſe, if the oval be quadrable. And ſeveral other authors have attempted to demonſtrate the impoſſibility of the general quadrature of the circle by any means whatever. On the other hand, many authors not only believe in the poſſibility of the quadrature of the circle, but ſome have even pretended to have diſcovered the ſame, and have publiſhed to the world their pretended diſcoveries: of which no one has rendered himſelf more remarkable than our countryman Mr. Hobbes, though a great ſcholar, and of excellent underſtanding in other matters. See QUADRATURE.

The approximate quadrature of the circle however, or the determination of the ratio between the diameter and the circumference, is what the mathematicians of all ages have ſucceſsfully attempted, and with different degrees of accuracy, according to the improved ſtate of the ſcience. Archimedes, in his book *de Dimenſione Circuli*, firſt gave a near value of that ratio in ſmall numbers, being that of 7 to 22, which are ſtill uſed as very convenient numbers for this purpoſe in common meaſurements. Other and nearer ratios have ſince been ſucceſſively aſſigned, but in larger numbers,

as 106 to 333,
or 113 to 355,
or 1702 to 5347,
or 1815 to 5702, &c,

which are each more accurate than the foregoing. Vieta, in his *Univerſalium Inſpectionum ad Canonum Mathematicum*, publiſhed 1579, by means of the inſcribed and circumſcribed polygons of 393216 ſides, has carried the ratio to ten places of figures, ſhewing that if the diameter of a circle be 1000 &c, the circumference will be greater than 314,159,265,35,

but leſs than 314,159,265,37.

And Van Colen, in his book *de Circulo & Adſcriptis*, has, by the ſame means, carried that ratio to 36 places of figures; which were alſo recomputed and confirmed by Willebrord Snell. After theſe, that indefatigable computer, Mr. Abraham Sharp, extended the ratio to 72 places of figures, in a ſheet of paper, publiſhed about the year 1706, by means of the ſeries of Dr. Halley, from the tangent of an arc of 30 degrees. And the ingenious Mr. Machin carried the ſame to a hundred places, by other ſeries, depending on the differences of arcs whoſe tangents have certain relations to one another. See this method explained in my Menſuration, pa. 120 ſecond edit. And, finally, M. De Lagny, in the Memoirs de l'Acad. 1719, by means of the tangent of the arc of 30 degrees, has extended the ſame ratio to the amazing length of 128 places of figures; finding, that, if the diameter be 1000 &c, the circumference will be

31415,92653,58979,32384,62643,38327,95028,
84197,16939,93751,05820,97494,45923,07816,
40628,62089,98628,03482,53421,17067,98214,
80865,13272,30664,70938,446 + or 447 −

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12. 41

From ſuch methods as the foregoing, a variety of ſeries have been diſcovered for the length of the circumference of a circle, ſuch as the following, viz, If the diameter be 1, the circumference c will be variouſly expreſſed thus,

$$c = 4 \times (1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \frac{1}{11} + \frac{1}{13} - \frac{1}{15} \&c),$$

$$c = \sqrt{8} \times (1 + \frac{1}{3} - \frac{1}{5} - \frac{1}{7} + \frac{1}{9} + \frac{1}{11} - \frac{1}{13} - \frac{1}{15} \&c),$$

$$c = \sqrt{12} \times (1 - \frac{1}{3\cdot3} + \frac{1}{5\cdot3^2} - \frac{1}{7\cdot3^3} + \frac{1}{9\cdot3^4} \&c),$$

$$c = 8 \times (\frac{1}{1.1\cdot3} + \frac{1}{1\cdot3\cdot5} - \frac{1}{3\cdot5\cdot7} + \frac{1}{5\cdot7\cdot9} - \frac{1}{7\cdot9\cdot11} \&c),$$

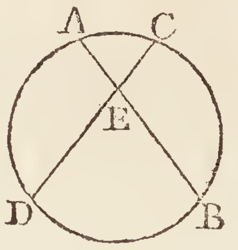
$$c = 8. \times (\frac{2}{3} - \frac{1}{5} - \frac{1}{4\cdot7} - \frac{1.3}{4.6.9} - \frac{1.3.5}{4.6.8.11} \&c),$$

$$c = 4\sqrt{2} \times (\frac{2}{3} - \frac{1}{5.2} - \frac{1}{4.7.2^2} - \frac{1.3}{4.6.9.2^3} - \frac{1.3.5}{4.6.8.11.2^4}$$

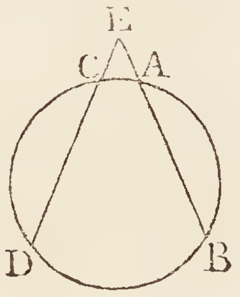
$$c = 4 \times (1 - \frac{1}{2.3} - \frac{1}{2.4.5} - \frac{1.3}{2.4.6.7} - \frac{1.3.5}{2.4.6.8.9} \&c).$$

And many other ſeries might here be added. See my Menſuration in ſeveral places; alſo my paper on ſuch ſeries in the Philoſ. Tranſ. 1776; Euler's *Introductio in Analyſin Infinitorum*; and many other authors.

6. Some of the more remarkable properties of the circle, are as follow.

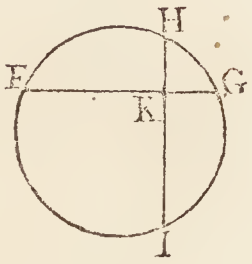


If two lines AB, CD cut the circle, and interſect within it, the angle of interſection E is meaſured by half the *ſum* of the intercepted arcs AC, DB.

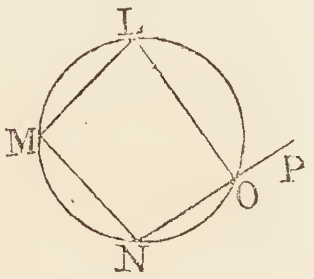


But if the lines interſect without the circle, the angle E is meaſured by half the *difference* of the intercepted arcs AC, DB.

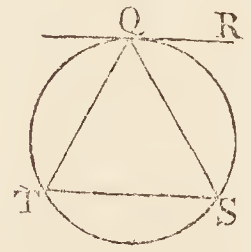
7. The angle at the centre of a circle is double the angle at the circumference, ſtanding on the ſame arc; and all angles in the ſame ſegment are equal. Alſo the angle at the centre is meaſured by the arc it ſtands upon, and the angle at the circumference by half the ſame arc.



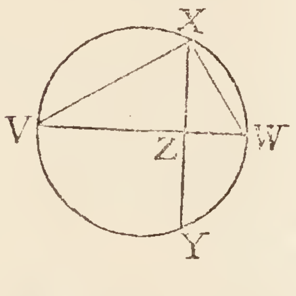
8. If the chords FG, HI croſs at right angles, the ſums of the oppoſite arcs are equal; viz FH + GI = FI + GH.



9. If one ſide NO of a trapezium inſcribed in a circle be continued out, the external angle, LOP will be equal to the oppoſite internal angle M.



10. An angle, as RQS, formed by a tangent QR and chord QS, is meaſured by half the arc of the chord QS, and is equal to any angle T formed in the oppoſite arc QTS.

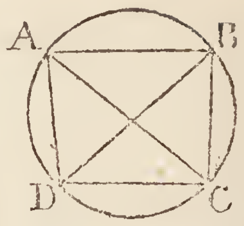


11. If VW be a diameter, and XYZ a chord perpendicular to it; then is XZ or ZY a mean proportional between the ſegments YZ, ZW. So that if d denote the diameter VW, x the abſciſs VZ, and y the ordinate ZX; then is $y^2 = \overline{d - x} \times x$ or $= dx - x^2$; which is called the equation to the circle.

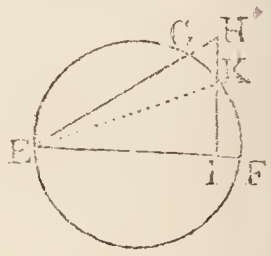
The chord VX is a mean proportional between the diameter VW and the verſed ſine VZ; and the chord WX is a mean proportional between the diameter and the verſed ſine WZ; alſo each verſed ſine is proportional to the ſquare of the correſponding chord; viz $VZ : WZ :: VX^2 : WX^2$.

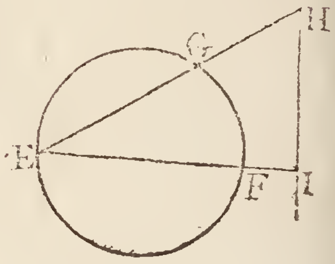
12. When two lines cut the circle, whether they interſect within the circle, or without it, as in the two figures to article 6, the ſegments between the common interſection and the two points where each line cuts the curve, are reciprocally proportional, and their rectangles are equal; viz, EA : EC :: ED : EB,

or $EA \times EB = EC \times ED$.

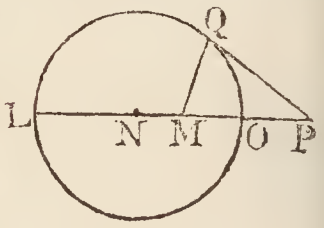


13. In a trapezium inſcribed in a circle, the rectangle of the two diagonals is equal to the ſum of the two rectangles of the two pairs of oppoſite ſides; viz, $AC \times BD = AB \times DC + AD \times BC$.

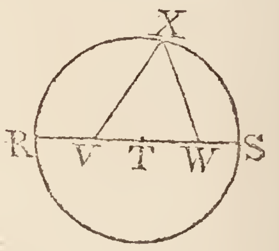




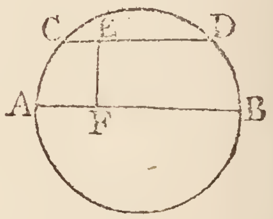
14. If any chords EF, EG, drawn from the ſame point E in the circumference, be cut by any other line HI, the rectangles will be all equal which are made of each chord and the part intercepted by this line, viz, $EF \times EI = EG \times EH = EK^2$.

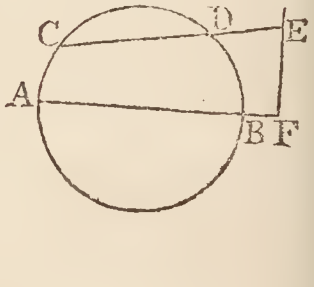


15. In a circle whoſe centre is N and radius NO, if two points M, P, in the radius produced, be ſo placed that the three NM, NO, MP, be in continued proportion; then if from the points M and P lines be drawn to any, or every point in the circumference, as Q; theſe lines will be always in the given ratio of MO to PO; viz, MQ : PQ :: MO : PO.

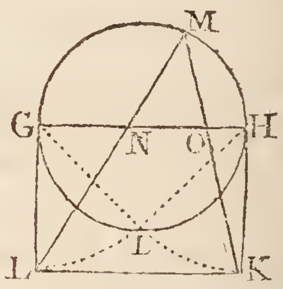


16. If V W, be two points in the diameter, equidiſtant from the centre T; and if two lines be drawn from theſe to any point X in the circumference; the ſum of their ſquares will be equal to the ſum of the ſquares of the ſegments of the diameter made by either point; viz, $VX^2 + WX^2 = RV^2 + VS^2$, or $= RW^2 + WS^2$, or $= 2RT^2 + 2VT^2$.





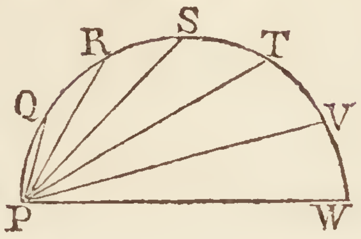
17. If a line FE perpendicular to the diameter AB, meet any other chord CD in the point E; then is $AF \times FB = CE \times ED + EF^2$.



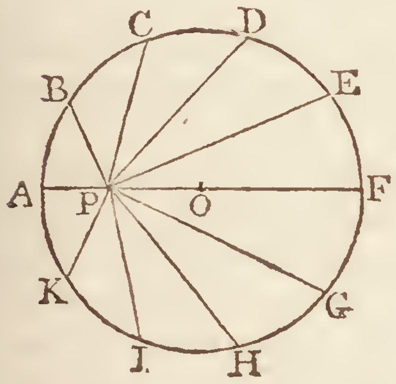
18. If upon the diameter GH of a circle there be formed a rectangle GHKI, whoſe breadth GI or HK is equal to GL or HL, the chord of a quadrant, or ſide of the inſcribed ſquare; then if from I and K lines be drawn to any point M in the circle GMH, they will cut the diameter GH in ſuch a manner that $GO^2 + HN^2 = GH^2$.

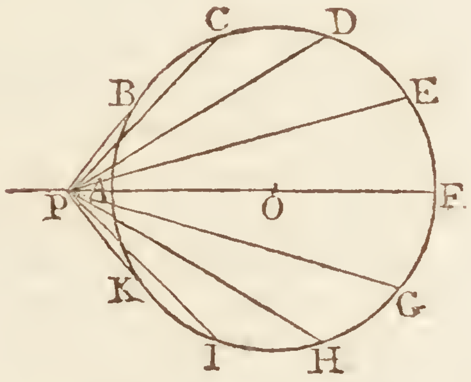
19. If the arcs PQ, QR, RS, &c, be equal, and there be drawn the chords PQ, PR, PS, PT, &c, then it

will be PQ : PR : : PR : PQ + PS : : PS : PR + PT : : PT : PS + PV, &c.

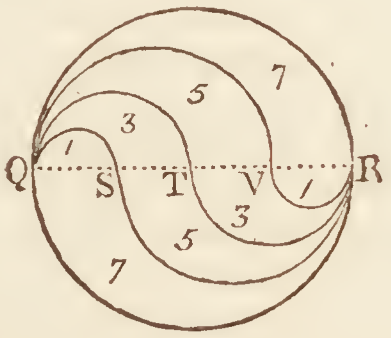


20. The centre of a circle being O, and P a point in the radius, or in the radius produced; if the circumference be divided into as many equal parts AB, BC, CD, &c, as there are units in 2n, and lines be drawn from P to all the points of diviſion; then ſhall the continual product of all the alternate lines viz PA × PC × PE &c be $= r^n - x^n$ when P is within the circle, or $= x^n - r^n$ when P is without the circle; and the product of the reſt of the lines, viz PB × PD + PF &c $= r^n + x^n$: where $r =$ AO the radius, and $x =$ OP the diſtance of P from the centre.





21. A circle may thus be divided into any number of parts that ſhall be equal to one another both in area and perimeter. Divide the diameter QR into the ſame number of equal parts at the points S, T, V, &c; then, on one ſide of the diameter deſcribe ſemicircles on the diameters QS, QT, QV, and on the other ſide of it deſcribe ſemicircles on RV, RT, RS; ſo ſhall the parts 1 7, 3 5, 5 3, 7 1 be all equal, both in area and perimeter. See my Tracts, pa. 93.



22. *To deſcribe a Circle either about or within a given Regular Polygon.* Biſect two of its angles, or two of its ſides, with perpendiculars, and the interſection of the biſecting lines will, in either caſe, be the centre of the circles.

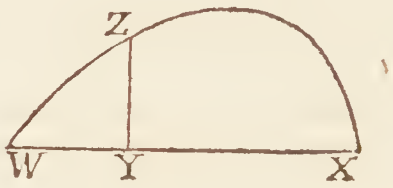
Parallel, or *Concentric* CIRCLES, are ſuch as are equally diſtant from each other in every point of their peripheries; or that have the ſame centre. As, on the other hand, thoſe are called the *eccentric* circles, that have not the ſame point for their centres.

The Quadrature of the CIRCLE, is the manner of deſcribing, or aſſigning, a ſquare, whoſe ſurface ſhall be perfectly equal to that of a circle. This problem has exerciſed the geometricians of all ages, but it is now generally given up as a problem impoſſible to be effected, by moſt perſons that have any juſt claim to that rank. Des Cartes inſiſts on the impoſſibility of it, for this reaſon, that a right line and a circle being of different natures, there can be no ſtrict proportion between them. Dr. Barrow ſhews the ſtrongeſt probability of the ſame thing; and not only that the diameter and circumference themſelves, but that all powers of them to infinity, are incommenſurate.

The Emperor Charles V offered a reward of 100,000 crowns to any perſon who ſhould reſolve this celebrated problem: and the States of Holland alſo propoſed a reward for the ſame thing. See *Quadrature*.

CIRCLES *of the Higher Orders*, are curves in which

$$WY^m : YZ^m :: YZ : YX,$$
$$\text{or } WY^m : YZ^m :: YZ_n : YX^n.$$



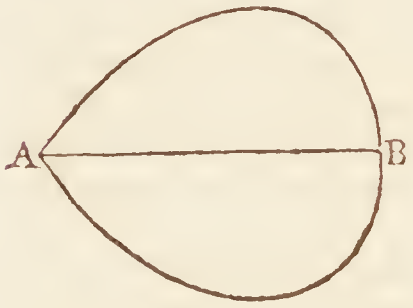
When m and n are each equal to 1, then WY : YZ : : YZ : YX, which is the property of the common circle.

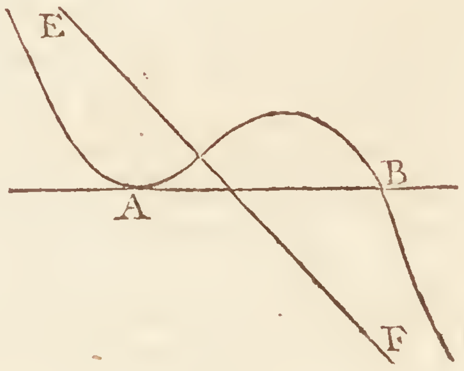
Put WY $= x$, YZ $= y$, WX $= a$; then is YX $- a - x$, and the above proportions become

$$x^m : y^m :: y : \overline{a-x}, \text{ or } y^{m+1} = x^m . \overline{a-x}, \text{ and}$$
$$x^m : y^m :: y^n : \overline{a-x}^n, \text{ or } y^{m+n} = x^m \cdot \overline{a-x}^n, \text{ the}$$

equations to curves of this kind.

Curves defined by this equation will be ovals when m is an odd number. Thus ſuppoſe $m = 1$, then the equation becomes $y^2 = x . \overline{a-x}$ or $ax - x^2$, the equation to the common circle. And if $m = 3$, it becomes $y^4 = x^3 . \overline{a-x}$ or $ax^3 - x^4$, which denotes a curve of this form AB.





But when m denotes an even number, the curve will have two infinite legs. So if $m = 2$, the equation becomes $y^3 = x^2 . \overline{a-x}$ or $ax^2 - x^3$, for a circle of the 2d order, and which defines one of Newton's defective hyperbolas, being his 37th ſpecies of curves, whoſe aſymptote is the right line EF, making an angle of 40 degrees with the abſciſs AB.

CIRCLE *of Curvature*, or circle of equi-curvature, is that circle which has the ſame curvature with a given curve at a certain point; or that circle whoſe radius is equal to the radius of curvature of the given curve at that point.

CIRCLES *of the Sphere*, are ſuch as cut the mundane ſphere, and have their circumference in its ſurface.

Theſe circles are either fixed or moveable.

The latter are thoſe whoſe peripheries are in the moveable or revolving ſurface; and which therefore move or turn with it; as the meridians, &c. The former, having their periphery in the immoveable ſurface, do not revolve; as the ecliptic, equator, and its parallels.

The circles of the ſphere are either great or little.

A Great Circle of the Sphere, is that which divides it into two equal parts or hemiſpheres, having the ſame centre and diameter with it. As the horizon, meridian, equator, ecliptic, the colures, and the azimuths.

A Little, or *Leſſer Circle* of the Sphere, divides the ſphere into two unequal parts, having neither the ſame centre nor diameter with the ſphere; its diameter being only ſome chord of the ſphere leſs than its axis. Such as the parallels of latitude, &c.

Circles *of Altitude*, or *Almucantars*, are little circles parallel to the horizon, having their common pole in the zenith, and ſtill diminiſhing as they approach it. They are ſo called from their uſe, which is to ſhew the altitude of a ſtar above the horizon.

Circles *of Declination*, are great circles interſecting each other in the poles of the world.

Circle *of Diſſipation*, in Optics. See the article Dissipation.

Diurnal Circles, are parallels to the equinoctial, ſuppoſed to be deſcribed by the ſeveral ſtars, and other points of the heavens, in their apparent diurnal rotation about the earth.

Circle *Equant*, in the Ptolomaic Aſtronomy, is a circle deſcribed on the centre of the equant. Its chief uſe is, to find the variation of the firſt inequality.

Circles *of Excurſion*, are little circles parallel to the ecliptic, and at ſuch a diſtance from it, as that the excurſions of the planets towards the poles of the ecliptic, may be included within them; being uſually fixed at about 10 degrees.

It may here be obſerved, that all the circles of the ſphere, deſcribed above, are transferred from the heavens to the earth; and ſo come to have a place in geography as well as in aſtronomy: all the points of each circle being conceived as let fall perpendicularly on the ſurface of the terreſtrial globe, and thus tracing out circles perfectly ſimilar to them. So, the terreſtrial equator is a circle conceived preciſely under the equinoctial line, which is in the heavens: and ſo of the reſt.

Horary Circles, in Dialling, are the lines which ſhew the hours on dials. Theſe are ſtraight lines on the dials, but called circles as being the projections of the meridians.

Horary Circle, or *Hour Circle*, on the globe, is a ſmall brazen circle fixed to the north pole, divided into 24 hours, and furniſhed with an index to point them out, thereby ſhewing the difference of meridians in time, and ſerving for the ſolution of many problems, on the artificial globes.

Circle *of Illumination*, is that imaginary circle on the ſurface of the earth, which ſeparates the illuminated ſide or hemiſphere of the earth from the dark ſide: and all lines paſſing from the ſun to the earth, being phyſically parallel, are perpendicular to the plane of this circle.

Circles *of Latitude*, or *Secondaries of the Ecliptic*, are great circles perpendicular to the plane of the ecliptic, interſecting one another in its poles, and paſſing through every ſtar and planet, &c.—Theſe are ſo called, becauſe they ſerve to meaſure the latitude of the ſtars, which is an arch of one of theſe circles, interſected between the ſtar and the ecliptic.

Circles *of Longitude*, are leſſer circles parallel to the ecliptic, diminiſhing more and more as they recede from it, or as they approach the pole of that circle.

They are ſo called, becauſe the longitudes of the ſtars are counted upon them.

Circle *of Perpetual Apparition*, one of the leſſer circles parallel to the equator, deſcribed by the moſt northern point of the horizon, as the ſphere revolves round by its diurnal motion.—All the ſtars included within this circle, are continually above the horizon, and ſo never ſet.

Circle *of Perpetual Occultation*, is another leſſer circle at a like diſtance from the equator, but on the other ſide of it, being deſcribed by the moſt ſouthern point of the horizon, and contains all thoſe ſtars which never appear in our hemiſphere, or which never riſe.

All other ſtars, being contained between theſe two circles, do alternately riſe and ſet, at certain moments of the diurnal rotation.

Polar Circles, are immoveable circles, parallel to the equator, and at ſuch a diſtance from the pole as is equal to the greateſt declination of the ecliptic, which now is 23° 28′. That next the northern pole is called the arctic, and that next the ſouthern one the antarctic.

Circles *of Poſition*, are circles paſſing through the common interſections of the horizon and meridian, and through any degree of the ecliptic, or the centre of any ſtar, or other point in the heavens; and are uſed for finding out the ſituation or poſition of any ſtar. Theſe are uſually ſix in number, cutting the equinoctial into 12 equal parts, which the aſtrologers call the Celeſtial Houſes, and hence they are ſometimes called Circles of the Celeſtial Houſes.

CIRCUIT, *Electrical*, denotes the courſe of the electric fluid from the charged ſurface of an electric body, to the oppoſite ſurface, into which the diſcharge is made. Some electricians at firſt apprehended, that the ſame particles of the electric fluid that were thrown on one ſide of the charged glaſs, actually made the whole circuit of the intervening conductors, and arrived at the oppoſite ſide: whereas Dr. Franklin's theory only requires, that the redundancy of electric matter on the charged ſurface ſhould paſs into thoſe bodies which form that part of the circuit which is contiguous to it, driving forward that part of the fluid which they naturally poſſeſs; and that the deficiency of the exhauſted ſurface ſhould be ſupplied by the neighbouring conductors, which form the laſt part of the circuit. On this ſuppoſition, a vibrating motion is ſucceſſively communicated through the whole length of the circuit.

Many attempts were made, both in France and England, at an early period in the practice of electricity, to aſcertain the diſtance to which the electric ſhock might be carried, and the velocity of its motion. The French philoſophers, at different times, cauſed it to paſs through circuits of 900 and even 2000 toiſes, or about 2 Engliſh miles and a half; and they diſcharged the Leyden phial through a baſon of water, whoſe ſurface was equal to about one acre. M. Monier found that, in paſſing through an iron wire of 950 toiſes in length, it did not ſpend a quarter of a ſecond; and that its motion was inſtantaneous through a wire of 1319 feet. In 1747, Dr. Watſon, and other Engliſh philoſophers, after many experiments of a ſimilar kind, conveyed the electric matter through a circuit of 4 miles; and, from two ſeveral trials, they concluded that its paſſage is inſtantaneous. By all which doubtleſs is meant, that its motion is too rapid to be meaſured. Prieſtley's Hiſt. of Elect. vol. 1, pa. 128, 8vo edit.

CIRCULAR, appertaining to a circle; as a circular form, circular motion, &c.

CIRCULAR *Lines*, a name given by ſome authors to ſuch ſtraight lines as are divided by means of the diviſions made in the arch of a circle. Such as the Sines, Tangents, Secants, &c.

CIRCULAR *Numbers*, are ſuch as have their powers ending in the roots themſelves. As the number 5, whoſe ſquare is 25, and its cube 125, &c.

Napier's CIRCULAR *Parts*, are five parts of a right-angled or a quadrantal ſpherical triangle; they are the two legs, the complement of the hypothenuſe, and the complements of the two oblique angles.

Concerning theſe circular parts, Napier gave a general rule in his *Logarithmorum Canonis Deſcriptio*, which is this; "*The rectangle under the radius and the ſine of the middle part, is equal to the rectangle under the tangents of the adjacent parts, and to the rectangle under the coſines of the oppoſite parts*. The right angle or quadrantal ſide being neglected, the two ſides and the complements of the other three natural parts are called the circular parts; as they follow each other as it were in a circular order. Of theſe, any one being fixed upon as the middle part, thoſe next it are the adjacent, and thoſe fartheſt from it the oppoſite parts." Lord Buchan's Life of Napier, pa. 98.

This rule contains within itſelf all the particular rules for the ſolution of right-angled ſpherical triangles, and they were thus brought into one general comprehenſive theorem, for the ſake of the memory; as thus, by charging the memory with this one rule alone: All the caſes of right-angled ſpherical triangles may be reſolved, and thoſe of oblique ones alſo, by letting fall a perpendicular, excepting the two caſes in which there are given either the three ſides, or the three angles.—And for theſe a ſimilar expedient has been deviſed by Lord Buchan and Dr. Minto. "M. Pingre, in the Memoires de Mathematique et de Phyſique for the year 1756, reduces the ſolution of all the caſes of ſpherical triangles to four analogies. Theſe four analogies are in fact, under another form, Napier's rule of the circular parts, and his ſecond or fundamental theorem, with its application to the ſupplemental triangle. Although it would be no difficult matter to get by heart the four analogies of M. Pingre, yet there are few perſons bleſſed with a memory capable of retaining them for any conſiderable time. For this reaſon, the rule for the circular parts ought to be kept under its preſent form. If the reader attends to the circumſtance of the ſecond letters of the words *tangents* and *coſines* being the ſame with the firſt of the words *adjacent* and *oppoſite*, he will find it almoſt impoſſible to forget the rule. And the rule for the ſolution of the two caſes of ſpherical triangles, for which the former of itſelf is inſufficient, may be thus expreſſed: *Of the circular parts of an oblique ſpherical triangle, the rectangle under the tangents of half the ſum and half the difference of the ſegments at the middle part* (formed by a perpendicular drawn from an angle to the oppoſite ſide), *is equal to the rectangle under the tangents of half the ſum and half the difference of the oppoſite parts*. By the circular parts of an oblique ſpherical triangle are meant its three ſides and the ſupplements of its three angles. Any of theſe ſix being aſſumed as a middle part, the oppoſite parts are thoſe two of the ſame denomination with it, that is, if the middle part is one of the ſides, the oppoſite parts are the other two, and, if the middle part is the ſupplement of one of the angles, the oppoſite parts are the ſupplements of the other two.—Since every plane triangle may be conſidered as deſcribed on the ſurface of a ſphere of an infinite radius, theſe two rules may be applied to plane triangles, provided the middle part be reſtricted to a ſide.

"Thus it appears that two ſimple rules ſuffice for the ſolution of all the poſſible caſes of plane and ſpherical triangles. Theſe rules, from their neatneſs and the manner in which they are expreſſed, cannot fail of engraving themſelves deeply on the memory of every one who is a little verſed in trigonometry. It is a circumſtance worthy of notice, that a perſon of a very weak memory may carry the whole art of trigonometry in his head." Napier's Life, pa. 102.

CIRCULAR *Sailing*, is that performed in the arch of a great circle.—It is chiefly on account of the ſhorteſt diſtance that this method of ſailing has been propoſed; and for the moſt part it is advantageous for a ſhip to reach her port by the ſhorteſt courſe.

As the ſolutions of the caſes in Mercator's ſailing are performed by plane triangles; ſo the caſes in great-circle ſailing are reſolved by the ſolution of ſpherical triangles. But, after all, the ſeveral caſes in this kind of ſailing ſerve rather for exerciſes in the ſolution of ſpherical triangles, than for any real uſe towards the navigating of a ſhip.

CIRCULAR *Spots* are made on pieces of metal by large electrical exploſions. See Philoſ. Tranſ. vol. 58, pa. 68; alſo Prieſtley's Hiſt. of Electricity, vol. 2, ſect. 9, edit. 8vo.

Theſe beautiful ſpots, produced by the moderate charge of a large battery, diſcharged between two ſmooth ſurfaces of metals, or ſemi-metals, lying at a ſmall diſtance from each other, conſiſt of one central ſpot, and ſeveral concentric circles, which are more or leſs diſtinct, and more or fewer in number, as the metal upon which they are marked is more eaſy or difficult of fuſion, and as a greater or leſs force is employed. They are compoſed of dots or cavities, which indicate a real fuſion. If the exploſion of a battery, iſſuing from a pointed body, be repeatedly taken on the plain ſurface of a piece of metal near the point, or be received from the ſurface on a point, the metal will be marked with a ſpot, conſiſting of all the priſmatic colours diſpoſed in circles, and formed by ſcales of the metal ſeparated by the force of the exploſion.

CIRCULAR *Velocity*, a term in aſtronomy ſignifying that velocity of a planet, or revolving body, which is meaſured by the arch of a circle.

Circulating *Decimals*, called alſo recurring or repeating decimals, are thoſe in which a figure or ſeveral figures are continually repeated. They are diſtinguiſhed into *ſingle* and *multiple*, and theſe again into *pure* and *mixed.*

A *pure ſingle* circulate, is that in which one figure only is repeated; as ·222 &c, and is marked thus $\cdot\dot{2}$.

A *pure multiple* circulate, is that in which ſeveral figures are continually repeated; as ·232323 &c, marked $\cdot\dot{2}\dot{3}$; and ·524524 &c, marked $\cdot\dot{5}2\dot{4}$.

A *mixed ſingle* circulate, is that which conſiſts of a terminate part, and a ſingle repeating figure; as 4·222 &c, or $4\cdot\dot{2}$. And

A *mixed multiple* circulate is that which contains a terminate part with ſeveral repeating figures; as $45\cdot\dot{5}2\dot{4}$.

That part of the circulate which repeats, is called the *repetend:* and the whole repetend, ſuppoſed infinitely continued, is equal to a vulgar fraction, whoſe numerator is the repeating number, or figures, and its denominator the ſame number of nines: ſo $\cdot\dot{2}$ is $= \frac{2}{9}$; and $\cdot\dot{2}\dot{3}$ is $= \frac{23}{99}$; and $\cdot\dot{5}2\dot{4}$ is $= \frac{524}{999}$.

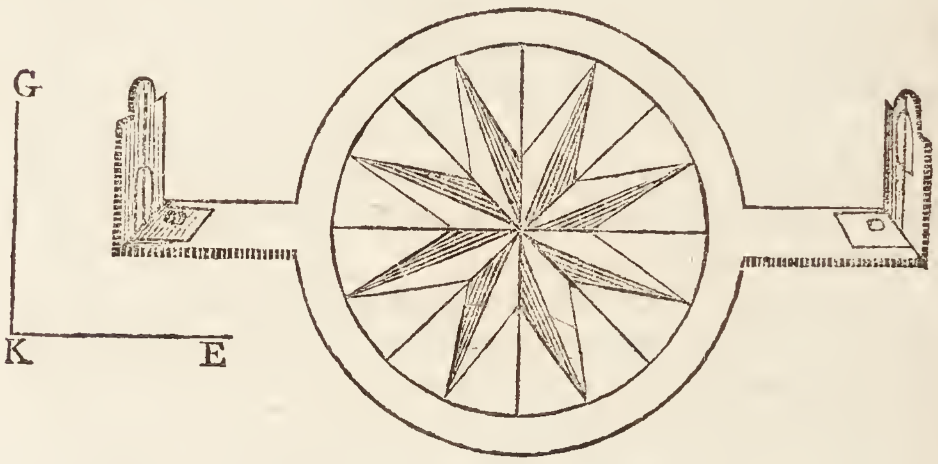
It ſeems it was Dr. Wallis who firſt diſtinctly conſidered, or treated of infinite circulating decimals, as he himſelf informs us in his Treatiſe of Infinites. Since his time many other authors have treated on this part of arithmetic; the principal of theſe however, to whom the art is moſtly indebted, are Meſſrs. Brown, Cunn, Martin, Emerſon, Malcolm, Donn, and Henry Clarke, in whoſe writings the nature and practice of this art may be fully ſeen, eſpecially in the laſt mentioned ingenious author.

CIRCUMFERENCE, in a general ſenſe denotes the line or lines bounding any figure. But it is commonly uſed in a more limited ſenſe, to denote the curve line which bounds a circle, and which is otherwiſe called the *periphery;* the boundary of a right-lined figure being expreſſed by the term *perimeter.*

The circumference of a circle is every where equidiſtant from the centre. And the circumferences of different circles are to one another as their radii or diameters, or the ratio of the diameter to the circumference is a conſtant ratio, in every circle, which is nearly as 7 to 22, as it was found by Archimedes, or, more nearly, as 1 to 3·1416. Under the article *Circle* may be ſeen various other approximations to that ratio, one of which is carried to 128 places of figures, viz by M. De Lagny.

The Circumference of every circle is ſuppoſed to be divided into 360 equal parts, called *degrees.*—Any part of a circumference is called an *arc* or *arch;* and a right line drawn from one end of an arc to the other, is called its *chord.*—The *angle* at the circumference is equal to half the angle at the centre, ſtanding on the ſame arc; and therefore it is meaſured by the half of that arc.

CIRCUMFERENTOR, a particular inſtrument uſed by ſurveyors for taking angles. It conſiſts of a braſs circle and index all of a piece; the diameter of the circle is commonly about 7 inches; the index about 14 inches long, and an inch and a half broad. On the circle is a card or compaſs, divided into 360 degrees; the meridian line of which anſwers to the middle of the breadth of the index. On the limb or circumference of the circle is ſoldered a braſs ring; which, with another fitted with a glaſs, forms a kind of box for the needle, which is ſuſpended on a pivot in the centre of the circle. There are alſo two ſights to ſcrew on, and ſlide up and down the index, as alſo a ſpangle and ſocket ſcrewed on the under ſide of the circle, to receive the head of the three-legged ſtaff.



To take, or obſerve the Quantity of an Angle by the Circumferentor. The angle propoſed being EKG; place the inſtrument at K, with the flower-de-luce of the card towards you; then direct the ſights to E, and obſerve what degrees are cut by the ſouth end of the needle, which let be 295; then, turning the inſtrument about on its ſtand, direct the ſights to G, noting again what degrees are cut by the ſouth end of the needle, which ſuppoſe are 213. This done, ſubtract the leſs number from the greater, viz, 213 from 295, and the remainder, or 82 degrees, is the quantity of the angle EKG ſought.

CIRCUMGYRATION, is the whirling motion of any body about a centre; as of the planets about the ſun, &c.

CIRCUM POLAR *Stars*, are thoſe ſtars which, by reaſon of their vicinity to the pole, move round it, without ſetting.

CIRCUMSCRIBED *Figure*, is a figure which is drawn about another, ſo that all its ſides or planes touch the latter or inſcribed figure.

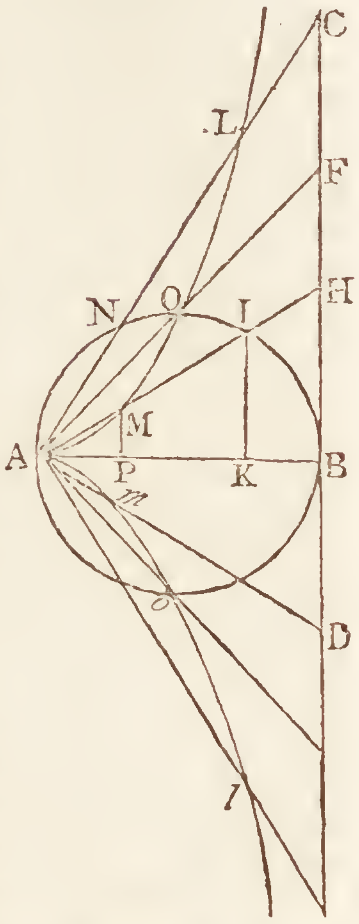
The area and perimeter of every polygon that can be circumſcribed about a circle, are greater than thoſe of the circle; and the area and perimeter of every inſcribed polygon, are leſs than thoſe of the circle; but they approach always nearer to equality as the number of ſides is more. And on theſe principles Archimedes, and ſome other authors ſince his time, attempted the quadrature of the circle; which is nothing elſe, in effect, but the meaſuring the area or capacity of a circle.

Circumscribed *Hyperbola*, is one of Newton's hyperbolas of the 2d order, that cuts its aſymptotes, and contains the parts cut off within its own ſpace.

CIRCUMVALLATION, or *Line of Circumvallation*, in the Art of War, is a trench, bordered with a parapet, thrown up around the beſieger's camp, as a ſecurity againſt any army that may attempt to relieve the place, as well as to prevent deſertion.

CIRCUMVOLUTION, in Architecture, the torus of the ſpiral line of the Ionic Order.

CISSOID, is a curve line of the ſecond order, invented by Diocles for the purpoſe of finding two continued mean proportionals between two other given lines. The generation or deſcription of this curve is as follows:



On the extremity B of the diameter AB of the circle AOB, erect the indefinite perpendicular CBD, to which from the other extremity A draw ſeveral lines, cutting the circle in I,O,N, &c; and upon theſe lines ſet off the correſponding equal diſtances, viz, HM = AI, and FO = AO, and CL = AN, &c; then the curve line drawn through all the points M, O, L, &c, is the ciſſoid of Diocles, who was an ancient Greek geometrician.

This curve is, by Newton, reckoned among the defective hyperbolas, being the 42d ſpecies in his *Enumeratio Linearum tertii ordinis*. And in his appendix *de Æquationum Conſtructione Lineari*, at the end of his *Arithmetica Univerſalis*, he gives another elegant method of deſcribing this curve by the continual motion of a ſquare ruler. Other methods have alſo been deviſed by different authors for the ſame thing.

The Properties of the Ciſſoid are the following:

1. The curve has two infinite legs AMOL, A*mol* meeting in a cuſp A, and tending continually towards the indefinite line CBD, which is their common aſymptote.

2. The curve paſſes through O and *o*, points in the circle equally diſtant from A and B; or it biſects each ſemicircle.

3. Letting fall perpendiculars MP, IK from any correſponding points I, M; then is AP = BK, and AM = HI, becauſe AI = MH.

4. AP : PB : : MP² : AP². So that, if the diameter AB be $= a$, the abſciſs AP $= x$, and the ordinate PM $= y$; then is $x : a - x : : y^2 : x^2$, or $x^3 = \overline{a - x} \times y^2$; which is the equation of the curve.

5. Sir Iſaac Newton, in his laſt letter to M. Leibnitz, has ſhewn how to find a right line equal to one of the legs of this curve, by means of the hyperbola; but he ſuppreſſed the inveſtigation, which however may be ſeen in his Fluxions.

6. The whole infinitely long ciſſoidal ſpace, contained between the infinite aſymptote BCD, and the curves LOA*ol* &c, of the ciſſoid, is equal to triple the generating circle AOB*o*A.

See more of this curve in Dr. Wallis, vol. 1, pa. 545.

CIVIL *Day*. See *Day*.

CIVIL *Month*. See *Month*.

CIVIL *Year*, is the legal year, or annual account of time, which every government appoints to be uſed within its own dominions.

It is ſo called in contradiſtinction to the natural year, which is meaſured exactly by the revolution of the heavenly bodies.

CLAIRAULT (ALEXIS-CLAUDE), a celebrated French mathematician and academician, was born at Paris the 13th of May 1713, and died the 17th of May 1765, at 52 years of age. His father, a teacher of mathematics at Paris, was his ſole inſtructor, teaching him even the letters of the alphabet on the figures of Euclid's Elements, by which he was able to read and write at 4 years of age. By a ſimilar ſtratagem it was that calculations were rendered familiar to him. At 9 years of age he put into his hands Guiſnée's Application of Algebra to Geometry; at 10 he ſtudied l'Hopital's Conic Sections; and between 12 and 13 he read a memoir to the Academy of Sciences concerning four new Geometrical curves of his own invention. About the ſame time he laid the firſt foundation of his work upon curves that have a double curvature, which he finiſhed in 1729, at 16 years of age. He was named Adjoint-Mechanician to the Academy in 1731, at the age of 18, Aſſociate in 1733, and Penſioner in 1738; during his connection with the Academy, he had a great multitude of learned and ingenious communications inſerted in their Memoirs, beſide ſeveral other works which he publiſhed ſeparately; the liſt of which is as follows.

1. On Curves of a Double Curvature; in 1730, 4to.
2. Elements of Geometry; 1741, 8vo.
3. Theory of the Figure of the Earth; 1743, 8vo.
4. Elements of Algebra; 1746, 8vo.
5. Tables of the Moon; 1754, 8vo.

His papers inſerted in the Memoirs of the Academy are too numerous to be particulariſed here; but they may be found from the year 1727, for almoſt every year till 1762; being upon a variety of ſubjects, aſtronomical, mathematical, optical, &c.

CLAVIUS (CHRISTOPHER), a German Jeſuit, was born at Bamberg in Germany, in 1537. He became a very ſtudious mathematician, and elaborate writer; his works, when collected, and cloſely printed, making 5 large folio volumes; being a complete body or courſe of the mathematics. They are moſtly elementary, and commentaries on Euclid and others; having very little of invention of his own. His talents and writings have been variouſly ſpoken of, and it muſt be acknowledged that they are heavy and elaborate. He was ſent for to Rome, to aſſiſt, with other learned men, in the reformation of the calendar, by pope Gregory; which he afterward undertook a defence of, againſt Scaliger, Vieta, and others, who attacked it. He died at Rome, the 6th of February, 1612, at 75 years of age, after more than 50 years cloſe application to the mathematical ſciences.

CLEFF. See *Cliff*.

CLERC (JOHN LE), a celebrated writer and univerſal ſcholar, was born at Geneva in 1657. After paſſing through the uſual courſe of ſtudy at Geneva, he went to France in 1678; but returning the year after, he took holy orders. In 1682 Le Clerc viſited England, to learn the language: but the ſmoky air of London not agreeing with his lungs, he ſoon returned to Holland, where he ſettled; and was appointed profeſſor of philoſophy, polite literature, and the Hebrew tongue, in the ſchool at Amſterdam. Here he long continued to read lectures; for which purpoſe he drew up and publiſhed his Logic, Ontology, Pneumatology, and Natural Philoſophy. He publiſhed alſo *Ars Critica*; a Commentary on the Old Teſtament; a Compendium of Univerſal Hiſtory; an Eccleſiaſtical Hiſtory of the two firſt centuries; a French tranſlation of the New Teſtament, and other works. In 1686, he began, jointly with M. de la Croſe, his *Bibliotheque Univerſelle et Hiſtorique*, in imitation of other literary journals; which was continued to the year 1693, making 26 volumes. In 1703 he began his *Bibliotheque Choiſie*, and continued it to 1714, when he commenced another work on the ſame plan, called *Bibliotheque Ancienne et Moderne*, which he continued to the year 1728; all of them juſtly eſteemed excellent ſtores of uſeful knowledge. He publiſhed alſo, in 1713, a neat little treatiſe on Practical Geometry, in 2 vols. ſmall 8vo, called, *Pratique de la Geometrie, ſur le papier et ſur le terrain*. In 1728 he was ſeized with a palſy and fever; and, after ſpending the laſt ſix years of his life with little or no underſtanding, he died in 1736, at 79 years of age.

CLEPSYDRA, a kind of water clock, or an hour-glaſs ſerving to meaſure time by the fall of a certain quantity, commonly out of one veſſel into another.—There have been alſo clepſydræ made with quickſilver; and the term is alſo uſed for hour-glaſſes of ſand.

By this inſtrument the Egyptians meaſured their time and the courſe of the ſun. Alſo Tycho Brahe, in modern times, made uſe of it to meaſure the motion of the ſtars, &c; and Dudley uſed the ſame contrivance in making all his maritime obſervations.

The uſe of Clepſydræ is very ancient. They were probably invented in Egypt under the Ptolemys; though ſome authors aſcribe the invention of them to the Greeks, and others to the Romans. Pliny informs us, that Scipio Naſica, about 150 years before Chriſt, gave the firſt hint for the conſtruction of them: and Pancirollus has particularly deſcribed them. According to his account, the clepſydra was a veſſel made of glaſs, with a ſmall hole in the bottom, edged with gold: in the upper part of this veſſel a line was drawn, and marked with the 12 hours: the veſſel was filled with water, and a cork with a pin fixed in it floated on the ſurface, pointing to the firſt hour; and as the water ſunk in the veſſel by iſſuing out of the ſmall hole, the pin indicated the other hours as it deſcended.

Clepſydræ were chiefly uſed in the winter; as ſun-dials ſerved for the ſummer. They had however two great defects; the one, that the water ran out more or leſs eaſily, as the air was more or leſs denſe; the other, that the water flowed more rapidly at the beginning, than towards the concluſion when its quantity and preſſure were much decreaſed. Amontons has invented a clepſydra which, it is ſaid, is free from both theſe inconveniences; and the ſame effect is produced by one deſcribed by Mr. Hamilton, in the Philoſ. Tranſ. vol. 44, pa. 171, or Abridg. vol. 10, pa. 248. Varignon too, in the *Memoires de l'Acad.* 1699, delivers a general geometrical method of making clepſydræ, or water-clocks, with any kind of veſſels, and with any given orifices for the water to run through.

Vitruvius, in lib. 9 of his Achitecture, treats of theſe inſtruments; and Pliny in chap. 60, lib. 7, ſays that Scipio Naſica was the firſt who meaſured time at Rome by clepſydræ, or water-clocks. Geſner, in his Pandects pa. 91, gives ſeveral contrivances for theſe inſtruments. Solomon de Caus alſo treats on this ſubject in his Reaſons of Moving Forces &c. So alſo does Ozanam, in his Mathematical Recreations, in which is contained a Treatiſe on Elementary Clocks, tranſlated from the Italian of Dominique Martinelli. There is likewiſe a treatiſe on Hour-Glaſſes by Arcangelo Maria Radi, called *Nova Sceinza de Horologi Polvere*. See alſo the *Technica Curioſa* of Gaſper Schottus; and Amonton's Remarques & Experiences Phyſiques ſur la Conſtruction d'une nouvelle Clepſydre, exempte des défauts des autres.

CLIFF, or CLEFF, a term in Muſic, for a certain mark, from the poſition of which the proper places of all other notes in a piece of muſic are known.

CLIMACTERIC, a critical year in a perſon's life. According to ſome, this is every 7th year: but others allow it only to thoſe years produced by multiplying 7 by the odd numbers 3, 5, 7, 9. Theſe years, ſay they, bring with them ſome remarkable change with reſpect to health, life, or fortune: the grand climacteric is the 63d year; but ſome add alſo the 81ſt to it: the other remarkable climacterics are the 7th, 21ſt, 35th, 49th, and 56th.

CLIMATE, or *Clime*, in Geography, a part of the ſurface of the earth, bounded by two leſſer circles parallel to the equator; and of ſuch a breadth, as that the longeſt day in the parallel nearer the pole exceeds the longeſt day in that next the equator, by ſome certain ſpace, as half an hour, or an hour, or a month.

The beginning of a climate, is a parallel circle in which the day is the ſhorteſt; and the end of the climate, is that in which the day is the longeſt. The climates therefore are reckoned from the equator to the pole; and are ſo many zones or bands, terminated by lines parallel to the equator: though, in ſtrictneſs, there are ſeveral climates, or different degrees of light or temperature, in the breadth of one zone. Each climate only differs from its contiguous ones, in that the longeſt day in ſummer is longer or ſhorter, by half an hour, for inſtance, in the one place than in the other.

As the climates commence at the equator, at the beginning of the firſt climate, that is at the equator, the day is juſt 12 hours long; but at the end of it, or at the beginning of the 2d climate, the longeſt day is 12 hours and a half long; and at the end of the 2d, or beginning of the 3d climate, the longeſt day is 13 hours long; and ſo of the reſt, as far as the polar circles, where the hour climates terminate, and month climates commence. And as an hour climate is a ſpace compriſed between two parallels of the equator, in the firſt of which the longeſt day exceeds that in the latter by half an hour; ſo the month climate is a ſpace contained between two circles parallel to the polar circles, and having its longeſt day longer or ſhorter than that of its contiguous one, by a month, or 30 days. But ſome authors, as Ricciolus, make the longeſt day of the contiguous climates to differ by half hours, to about the latitude of 45 degrees; then to differ by an hour, or ſometimes 2 hours, to the polar circle; and after that by a month each. See tables of climates in Varenius, chap. 25, prop. 13.

The ancients, who confined the climates to what they thought the habitable parts of the earth, reckoned only ſeven, the middles of which they made to paſs through ſome remarkable places; as the 1ſt through Meroe, the 2d through Sienna, the 3d through Alexandria, the 4th through Rhodes, the 5th through Rome, the 6th through Pontus, and the 7th through the mouth of the Boryſthenes. But the moderns, who have ſailed farther toward the poles, make 30 climates on each ſide.

Vulgarly the term Climate is beſtowed on any country or region differing from another either in reſpect of the ſeaſons, the quality of the ſoil, or even the manners of the inhabitants; without any regard to the length of the longeſt day. Abulfeda, an Arabic author, diſtinguiſhes the firſt kind of climates by the term *real climates*, and the latter by that of *apparent climates*.

CLOCK, a machine now conſtructed in ſuch a manner, and ſo regulated by the uniform motion of a pendulum, as to meaſure time, and all its ſubdiviſions, with great exactneſs. Before the invention of the pendulum, a balance, not unlike the fly of a kitchen-jack, was uſed inſtead of it.—They were at firſt called nocturnal dials to diſtinguiſh them from ſun-dials, which ſhewed the hour by the ſhadow of the ſun.

The invention of clocks with wheels is aſcribed to Pacificus, archdeacon of Verona, in the 9th century, on the credit of an epitaph quoted by Ughelli, and borrowed by him from Panvinius. Others attribute the invention to Boethius, about the year 510.

Mr. Derham, however, makes clock-work of a much older date; ranking Archimedes's ſphere, mentioned by Claudian, and that of Poſidonius, mentioned by Cicero, among machines of this kind: not that either their form or uſe were the ſame with thoſe of ours; but that they had their motion from ſome hidden weights, or ſprings, with wheels or pulleys, or ſome ſuch clock-work principle.

In the *Diſquiſitiones Monaſticæ* of Benedictus Haëften, publiſhed in the year 1644, he ſays, that clocks were invented by Silveſter the 4th, a monk of his order, about the year 998, as Dithmarus and Bozius have ſhewn; for before that time, they had nothing but ſun-dials and clepſydræ to ſhew the hour.—Conrade Geſner, in his Epitome, pa. 604, ſays, that Richard Wallingford, an Engliſh abbot of St. Albans, who flouriſhed in the year 1326, made a wonderful clock by a moſt excellent art, the like of which could not be produced by all Europe.—Moreri, under the word *Horologe* du Palais, ſays, that Charles the Fifth, called the wiſe king of France, ordered at Paris the firſt large clock to be made by Henry de Vie, whom he ſent for from Germany, and ſet it upon the tower of his palace, in the year 1372.—John Froiſſart, in his *Hiſtoire & Chronique*, vol. 2, ch. 28, ſays, the duke of Bourgogne had a clock, which ſounded the hour, taken away from the city of Courtray, in the year 1382: and the ſame thing is ſaid by Wm. Paradin, in his *Annales de Bourgogne*.

Clock-makers were firſt introduced into England in 1368, when Edward the 3d granted a licence for three artiſts to come over from Delft in Holland, and practiſe their occupation in this country.

The water-clocks, or clepſydræ, and ſun-dials, have both a much better claim to antiquity. The French annals mention one of the former kind, ſent by Aaron, king of Perſia, to Charlemagne, about the year 807, which it would ſeem bore ſome reſemblance to the modern clocks: it was of braſs, and ſhewed the hours by 12 little balls of the ſame metal, which at the end of each hour fell upon a bell, and made a ſound. There were alſo figures of 12 cavaliers, which at the end of each hour came out through certain apertures, or windows, in the ſide of the clock, and ſhut them again, &c.

The invention of pendulum clocks is owing to the happy induſtry of the laſt age; and the honour of that diſcovery is diſputed between Galileo and Huygens. The latter, who wrote an excellent volume on the ſubject, declares it was firſt put in practice in the year 1657, and the deſcription of it printed in 1658. Becher, *De Nova Temporis dimetiendi Theoria, anno* 1680, contends for Galileo; and relates, though at ſecond-hand, the whole hiſtory of the invention; adding that one Trefler, clock-maker to the father of the then grand duke of Tuſcany, made the firſt pendulum clock at Florence, under the direction of Galileo Galilei, a pattern of which was brought to Holland. And the Academy del Cimento ſay expreſsly, that the application of the pendulum to the movement of a clock was firſt propoſed by Galileo, and put in practice by his ſon Vincenzo Galilei, in 1649. But whoever may have been the inventor, it is certain that the invention never flouriſhed till it came into the hands of Huygens, who inſiſts on it, that if ever Galileo thought of ſuch a thing, he never brought it to any degree of perfection. The firſt pendulum clock made in England was in the year 1662, by one Fromantil, a Dutchman.

Among the modern clocks, thoſe of Straſburg and Lyons are very eminent for the richneſs of their furniture, and the variety of their motions and figures. In the former, a cock claps his wings, and proclaims the hour: the angel opens a door, and ſalutes the Virgin; and the holy ſpirit deſcends on her, &c. In the latter, two horſemen encounter, and beat the hour upon each other: a door opens, and there appears on the theatre the Virgin, with Jeſus Chriſt in her arms; the Magi, with their retinue, marching in order, and preſenting their gifts; two trumpeters ſounding all the while to proclaim the proceſſion.

Theſe, however, are far excelled by two that have lately been made by Engliſh artiſts, as a preſent from the Eaſt-India company to the emperor of China. Theſe two clocks are in the form of chariots, in each of which a lady is placed, in a fine attitude, leaning her right hand upon a part of the chariot, under which appears a clock of curious workmanſhip, little larger than a ſhilling, that ſtrikes and repeats, and goes for eight days. Upon the lady's finger ſits a bird, finely modelled, and ſet with diamonds and rubies, with its wings expanded in a flying poſture, and actually flutters for a conſiderable time, on touching a diamond button below it; the body of the bird, in which are contained part of the wheels that animate it as it were, is leſs than the 16th part of an inch. The lady holds in her left hand a golden tube little thicker than a large pin, on the top of which is a ſmall round box, to which is fixed a circular ornament not larger than a ſixpence, ſet with diamonds, which goes round in near three hours in a conſtant regular motion. Over the lady's head is a double umbrella, ſupported by a ſmall fluted pillar not thicker than a quill, and under the larger of which a bell is fixed at a conſiderable diſtance from the clock, with which it ſeems not to have any connection; but from which a communication is ſecretly conveyed to a hammer, that regularly ſtrikes the hour, and repeats the ſame at pleaſure, by touching a diamond button fixed to the clock below. At the feet of the lady is a golden dog; before which, from the point of the chariot, are two birds fixed on ſpiral ſprings, the wings and feathers of which are ſet with ſtones of various colours, and they appear as if flying away with the chariot, which, from another ſecret motion, is contrived to run in any direction, either ſtraight or circular, &c; whilſt a boy, that lays hold of the chariot behind, ſeems alſo to puſh it forwards. Above the umbrella are flowers and ornaments of precious ſtones; and it terminates with a flying dragon ſet in the ſame manner. The whole is of gold, moſt curiouſly executed, and embelliſhed with rubies and pearls.

The ingenious Dr. Franklin contrived a clock to ſhew the hours, minutes, and ſeconds, with only three wheels and two pinions in the whole movement. The dial-plate has the hours engraven upon it in ſpiral ſpaces along two diameters of a circle, containing four times 60 minutes. The index goes round in four hours, and counts the minutes from any hour by which it has paſſed to the next following hour. The ſmall hand, in an arch at top, goes round once in a minute, and ſhews the ſeconds. The clock is wound up by a line going over a pulley, on the axis of the great wheel, like a common 30 hour clock. Many of theſe very ſimple machines have ſince been conſtructed, that meaſure time exceedingly well. This clock is ſubject, however, to the inconvenience of requiring frequent winding, by drawing up the weight; as alſo to ſome uncertainty as to the particular hour ſhewn by the index.

Mr. Ferguſon has propoſed to remedy theſe inconveniences by another conſtruction, which is deſcribed in his Select Exerciſes, pa. 4. This clock will go a week without winding, and always ſhews the preciſe hour; but, as Mr. Ferguſon acknowledges, it has two diſadvantages which do not belong to Dr. Franklin's clock: when the minute hand is adjuſted, the hour plate muſt alſo be ſet right, by means of a pin; and the ſmallneſs of the teeth in the pendulum wheel will cauſe the pendulum ball to deſcribe but ſmall arcs in its vibrations; and therefore the momentum of the ball will be leſs, and the times of the vibrations will be more affected by any unequal impulſe of the pendulum wheel on the pallets. Beſides, the weight of the flat ring, on which the ſeconds are engraven, will load the pivots of the axis of the pendulum wheel with a great deal of friction, which ought by all poſſible means to be avoided. To remedy this inconvenience, the ſeconds plate might be omitted.

Mr. Ferguſon alſo contrived a clock, ſhewing the apparent diurnal motions of the ſun and moon, the age and phaſes of the moon, with the time of her coming to the meridian, and the times of high and low water; and all this by having only two wheels and a pinion added to the common movement. See his Select Exerciſes before mentioned. In this clock the figure of the ſun ſerves as an hour index, by going round the dial-plate in 24 hours; and a figure of the moon goes round in 24 h. 50½ min. the time of her going round in the heavens from any meridian to the ſame meridian again. A clock of this kind was adapted by Mr. Ferguſon to the movement of an old watch. See alſo a deſcription and drawing of an aſtronomical clock, ſhewing the apparent daily motions of the ſun, moon, and ſtars, with the times of their riſing, ſouthing, and ſetting; the places of the ſun and moon in the ecliptic, and the age and phaſes of the moon for every day of the year, in the ſame book, pa. 19.

There have been ſeveral treatiſes upon clocks; the principal of which are the following. Hieronymus Cardàn, de Varietate Rerum libri 17.—Conrade Daſypodius, Deſcriptio Horologii Aſtronomici Argentinenſis in ſumma Templi erecti.—Guido Pancirollus, Antiqua deperdita & nova reperta.—L'Uſage du Cadran, ou de l'Horloge Phyſique Univerſelle, par Galilée, Mathematicien du Duc de Florence.—Oughtred's Opuſcula Mathematica.—Huygens's Horologium Oſcillatorium.—Pendule perpetuelle, par l'Abbé de Hautefeuille.—J. J. Becheri Theoria & Experientia de nova Temporis dimetiendi Ratione & Horologiorum Conſtructione.—Clark's Oughtredus explicatus, ubi de Conſtructione Horologiorum.—Horological Diſquiſitions.—Huygens's Poſthumous Works.—Sully's Regle Artificielle du Temps, &c.—Serviere's Recueil d'Ouvrages Curieux.—Derham's Artificial Clock-maker.—Camus's Traités des Forces Mouvantes.—Alexandre's Traité Général des Horologies.—Alſo Treatiſes and Principles of Clock-making, by Hatton, Cuming, &c. &c.

CLOUD, a collection of vapours ſuſpended in the atmoſphere, and rendered viſible.

Although it be generally allowed that the clouds are formed from the aqueous vapours, which before were ſo cloſely united with the atmoſphere as to be inviſible: it is, however, not eaſy to account for the long continuance of ſome very opaque clouds without diſſolving; or to aſſign the reaſon why the vapours, when they have once begun to condenſe, do not continue to do ſo till they at laſt fall to the ground in the form of rain or ſnow, &c. It is now known that a ſeparation of the latent heat from the water of which vapour is compoſed is attended with a condenſation of that vapour in ſome degree; in ſuch caſe, it will firſt appear as a ſmoke, miſt, or fog; which, if interpoſed between the ſun and earth, will form a cloud; and the ſame cauſes continuing to operate, the cloud will produce rain or ſnow. It is however abundantly evident that ſome other cauſe beſides mere heat or cold is concerned in the formation of clouds, and the condenſation of atmoſpherical vapours. This cauſe is eſteemed in a great meaſure the electrical fluid; indeed electricity is now ſo generally admitted as an agent in all the great operations of nature, that it is no wonder to find the formation of clouds attributed to it; and this has accordingly been given by Beccaria as the cauſe of the formation of all clouds whatſoever, whether of thunder, rain, hail, or ſnow.

But whether the clouds are produced, that is, the atmoſpheric vapours rendered viſible, by means of electricity or not, it is certain that they do often contain the electric fluid in prodigious quantities, and many terrible and deſtructive accidents have been occaſioned by clouds very highly electrified. The moſt extraordinary inſtance of this kind perhaps on record happened in the iſland of Java, in the Eaſt-Indies, in Auguſt, 1772. On the 11th of that month, at midnight, a bright cloud was obſerved covering a mountain in the diſtrict called Cheribou, and ſeveral reports like thoſe of a gun were heard at the ſame time. The people who dwelt upon the upper parts of the mountain not being able to fly faſt enough, a great part of the cloud, eight or nine miles in circumference, detached itſelf under them, and was ſeen at a diſtance, riſing and falling like the waves of the ſea, and emitting globes of fire ſo luminous, that the night became as clear as day. The effects of it were aſtoniſhing; every thing was deſtroyed for 20 miles round; the houſes were demoliſhed; plantations were buried in the earth; and 2140 people loſt their lives, beſides 1500 head of cattle, and a vaſt number of horſes, goats, &c. Another remarkable inſtance of the dreadful effects of electric clouds, which happened at Malta the 29th of October 1757, is related in Brydone's Tour through Malta.

The height of the clouds is not uſually great: the ſummits of high mountains being commonly quite free from them, as many travellers have experienced in paſſing theſe mountains. It is found that the moſt highly electrified clouds deſcend loweſt, their height being often not more than 7 or 800 yards above the ground; and ſometimes thunder-clouds appear actually to touch the ground with one of their edges: but the generality of clouds are ſuſpended at the height of a mile, or little more, above the earth.

The motions of the clouds, though often directed by the wind, are not always ſo, eſpecially when thunder is about to enſue. In this caſe they are ſeen to move very ſlowly, or even to appear quite ſtationary for ſome time. The reaſon of this probably is, that they are impelled by two oppoſite ſtreams of air nearly of equal ſtrength; and in ſuch caſes it ſeems that both the aerial currents aſcend to a conſiderable height; for Meſſ. Charles and Robert, when endeavouring to avoid a thunder cloud, in one of their aerial voyages with a balloon, could find no alteration in the courſe of the current, though they aſcended to the height of 4000 feet above the earth. In ſome caſes the motions of the clouds evidently depend on their electricity, independent of any current of air whatever. Thus, in a calm and warm day, ſmall clouds are often ſeen meeting each other in oppoſite directions, and ſetting out from ſuch ſhort diſtances, that it cannot be ſuppoſed that any oppoſite winds are the cauſe. Such clouds, when they meet, inſtead of forming a larger one, become much ſmaller, and ſometimes quite vaniſh; a circumſtance moſt probably owing to the diſcharge of oppoſite electricities into each other. And this ſerves alſo to throw ſome light on the true cauſe of the formation of clouds; for if two clouds, the one electrified poſitively, and the other negatively, deſtroy each other on contact, it follows that any quantity of vapour ſuſpended in the atmoſphere, while it retains its natural quantity of electricity, remains inviſible, but becomes a cloud when electrified either plus or minus.

The ſhapes of the clouds are alſo probably owing to their electricity; for in thoſe ſeaſons in which a great commotion has been excited in the atmoſpherical electricity, the clouds are ſeen aſſuming ſtrange and whimſical ſhapes, that are continually varying. This, as well as the meeting of ſmall clouds in the air, and vaniſhing upon contact, is a ſure ſign of thunder.

The uſes of the clouds are evident, as from them proceeds the rain that refreſhes the earth, and without which, according to the preſent ſtate of nature, the whole ſurface of the earth muſt become a mere deſert. They are likewiſe uſeful as a ſcreen interpoſed between the earth and the ſcorching rays of the ſun, which are often ſo powerful as to deſtroy the graſs and other tender vegetables. In the more ſecret operations of nature too, where the electric fluid is concerned, the clouds bear a principal ſhare; and chiefly ſerve as a medium for conveying that fluid from the atmoſphere into the earth, and from the earth into the atmoſphere: in doing which, when electrified to a great degree, they ſometimes produce very terrible effects; an inſtance of which is related above.

CLOUTS, in Artillery, are thin plates of iron nailed on that part of the axle-tree of a gun-carriage which comes through the nave, and through which the linſpin goes.

CLUVIER, *or* CLUVERIUS, (PHILIP), a celebrated geographer, was born at Dantzic in 1580. After an education at home, he travelled into Poland, Germany, and the Netherlands, to improve himſelf in the knowledge of the law. But, when at Leyden, Joſeph Scaliger perſuaded him to give way to his genius for geography. In purſuance of this advice, Cluvier viſited the greateſt part of the European ſtates. He was well ſkilled in many languages, ſpeaking half a ſcore with facility, viz, Greek, Latin, German, French, Engliſh, Dutch, Italian, Hungarian, Poliſh, and Bohemian. On his return to Leyden, he taught there with great applauſe; and died in 1623, being only 43 years of age, juſtly eſteemed the firſt geographer who had put his reſearches in order, and reduced them to certain principles. He was author of ſeveral ingenious works in geography, viz:

1. *De Tribus Rheni Alveis.*
2. *Germania Antiqua.*
3. *Italia Antiqua, Sicilia, Sardinia, & Corſica.*
4. *Introductio in Univerſam Geographiam.*

COASTING, is that part of Navigation in which the places are not far aſunder, ſo that a ſhip may ſail in ſight of land, or within ſoundings between them.

COCHLEA, one of the five Mechanical powers, otherwiſe called the Screw; being ſo named from the reſemblance a ſcrew bears to the ſpiral ſhell of a ſnail, which the Latins call Cochlea. See SCREW, and MECHANICAL *Powers*.

COCK *of a Dial*, the pin, ſtyle, or gnomon.

COEFFICIENTS, in Algebra, are numbers, or given quantities, uſually prefixed to letters, or unknown quantities, by which it is ſuppoſed they are multiplied; and ſo, with ſuch letters, or quantities, making a product, or *coefficient* production; whence the name.

Thus, in $3a$ the coefficient is 3, in bx it is b, and in cx^2 it is c. If a quantity have no number prefixed, unity or 1 is underſtood; as a is the ſame as $1a$, and bc the ſame as $1bc$. The name *coefficient* was firſt given by Vieta.

In any equation ſo reduced as that its higheſt power or term has 1 for its coefficient; then the coefficient of the 2d term is equal to the ſum of all the roots, both poſitive and negative ; ſo that if the 2d term is wanting in an equation, the ſum of the poſitive roots of that equation is equal to the ſum of the negative roots, as they mutually balance and cancel each other. Alſo the coefficient of the 3d term of an equation is equal to the ſum of all the rectangles ariſing by the multiplication of every two of the roots, how many ways ſoever they can be combined by twos ; as once in the quadratic, 3 times in the cubic, 6 times in the biquadratic equation, &c. And the coefficient of the 4th term of an equation, is the ſum of all the ſolids made by the continual multiplication of every three of the roots, how often ſoever ſuch a ternary can be had ; as once in a cubic, 4 times in a biquadratic, 10 times in an equation of 5 dimenſions, &c. And thus it will go on infinitely.

COEFFICIENTS *of the ſame Order*, is a term ſometimes uſed for the coefficients prefixed to the ſame unknown quantities, in different equations.

Thus in the equations $\left\{\begin{array}{l} ax + by + cz = m, \\ dx + ey + fz = n, \\ gx + hy + kz = p, \end{array}\right.$

the coefficients a, d, g, are of the ſame order, being the coefficients of the ſame letter x ; alſo b, e, h are of the ſame order, being the coefficients of y ; and ſo on.

Oppoſite COEFFICIENTS, ſuch as are taken each from a different equation, and from a different order of coefficients. Thus, in the foregoing equations, a, e, k, or a, h, f, or d, b, k, &c, are oppoſite coefficients.

COELESTIAL. See CELESTIAL.

COFFER, in Architecture, a ſquare depreſſure or ſinking, in each interval between the modillions of the Corinthian cornice ; uſually filled up with a roſe ; ſometimes with a pomegranate, or other enrichment.

COFFER, in Fortification, denotes a hollow lodgment, athwart a dry moat, 6 or 7 feet deep, and 16 or 18 broad. The upper part of it is made of pieces of timber, raiſed 2 feet above the level of the moat ; the elevation having hurdles laden with earth for its covering, and ſerving as a parapet with embrazures.

The coffer is nearly the ſame with the caponiere, excepting that this laſt is ſometimes made beyond the counterſcarp on the glacis, and the coffer always in the moat, taking up its whole breadth, which the caponiere does not.

It differs from the traverſe and gallery, in that theſe are made by the beſiegers, and the coffer by the beſieged.

The beſieged commonly make uſe of coffers to repulſe the beſiegers, when they endeavour to paſs the ditch. And, on the other hand, the beſiegers, to ſave themſelves from the fire of theſe coffers, throw up the earth on that ſide towards the coffer.

COFFER-*Dams*, or *Batardeaux*, in Bridge-building, are incloſures formed for laying the foundation of piers, and for other works in water, to exclude the ſurrounding water, and ſo prevent it from interrupting the workmen.

Theſe incloſures are ſometimes ſingle, and ſometimes double, with clay rammed between them ; ſometimes they are made with piles driven cloſe by one another, and ſometimes the piles are notched or dove-tailed into one another ; but the moſt uſual method is to drive piles with grooves in them, at the diſtance of five or ſix feet from each other, and then boards are let down between them ; after which the water is pumped out.

COGGESHALL's *Sliding-Rule*, an inſtrument uſed in Gauging, and ſo called from its inventor. See the deſcription and uſe under SLIDING-*Rule*.

COHESION, one of the four ſpecies of attraction, denoting that force by which the parts of bodies adhere or ſtick together.

This power was firſt conſidered by Newton as one of the properties eſſential to all matter, and the cauſe of all that variety obſerved in the texture of different terreſtrial bodies. He did not, however, abſolutely determine that the power of coheſion was an immaterial one ; but that it might poſſibly ariſe, as well as that of gravitation, from the action of another. His doctrine of coheſion Newton delivers in theſe words : " The particles of all hard homogeneous bodies, which touch one another, cohere with a great force ; to account for which, ſome philoſophers have recourſe to a kind of hooked atoms, which in effect is nothing elſe but to beg the thing in queſtion. Others imagine that the particles of bodies are connected by reſt, i. e. in effect by nothing at all ; and others by conſpiring motions, i. e. by a relative reſt among themſelves. For myſelf, it rather appears to me, that the particles of bodies cohere by an attractive force, whereby they tend mutually toward each other : which force, in the very point of contact, is very great ; at little diſtances is leſs, and at a little farther diſtance is quite inſenſible."

It is uncertain in what proportion this force decreaſes as the diſtance increaſes : Deſaguliers conjectures, from ſome phenomena, that it decreaſes as the biquadratic or 4th power of the diſtance, ſo that at twice the diſtance it acts 16 times more weakly, &c.

" Now if compound bodies be ſo hard, as by experience we find ſome of them to be, and yet have a great many hidden pores within them, and conſiſt of parts only laid together ; no doubt thoſe ſimple particles which have no pores within them, and which were never divided into parts, muſt be vaſtly harder. For ſuch hard particles, gathered into a maſs, cannot poſſibly touch in more than a few points : and therefore much leſs force is required to ſever them, than to break a ſolid particle, whoſe parts touch throughout all their ſurfaces, without any intermediate pores or interſtices. But how ſuch hard particles, only laid together, and touching only in a few points, ſhould come to cohere ſo firmly, as in fact we find they do, is inconceivable ; unleſs there be ſome cauſe, whereby they are attracted and preſſed together. Now the ſmalleſt particles of matter may cohere by the ſtrongeſt attractions, and conſtitute larger, whoſe attracting force is feebler : and again, many of theſe larger particles cohering, may conſtitute others ſtill larger, whoſe attractive force is ſtill weaker ; and ſo on for ſeveral ſucceſſions, till the progreſſion end in the biggeſt particle, on which the operations in chemiſtry, and the colours of natural bodies, do depend ; and which by cohering compoſe bodies of a ſenſible magnitude."

Again, the opinion maintained by many is that which is ſo ſtrongly defended by J. Bernoulli, *De Gravitate Ætheris* ; who attributes the coheſion of the parts of matter to the uniform preſſure of the atmoſphere ; confirming this opinion by the known experiment of two poliſhed marble planes, which cohere very ſtrongly in the open air, but eaſily drop aſunder in an exhauſted receiver. However, if two plates of this kind be ſmeared with oil, to fill up the pores in their ſurfaces, and prevent the lodgment of air, and one of them be gently rubbed upon the other, they will adhere ſo ſtrongly, even when ſuſpended in an exhauſted receiver, that the weight of the lower plate will not be able to ſeparate it from the upper one. But although this theory might ſerve tolerably well to explain the cohesion of compoſitions, or greater collections of matter; yet it falls far ſhort of accounting for that firſt cohesion of the atoms, or primitive corpuſcles, of which the particles of hard bodies are compoſed.

Again, ſome philoſophers have poſitively aſſerted, that the powers, or means, are immaterial, by which matter coheres; and, in conſequence of this ſuppoſition, they have ſo refined upon attractions and repulſions, that their ſyſtems ſeem but little ſhort of ſcepticiſm, or denying the exiſtence of matter altogether. A ſyſtem of this kind is adopted by Dr. Prieſtley, from Meſſrs. Boſcovich and Michell, to ſolve ſome difficulties concerning the Newtonian doctrine of light. See his Hiſtory of Viſion, vol. I. pa. 392. "The eaſieſt method," ſays he, "of ſolving all difficulties, is to adopt the hypotheſis of Mr. Boſcovich, who ſuppoſes that matter is not impenetrable, as has been perhaps univerſally taken for granted; but that it conſiſts of phyſical points only, endued with powers of attraction and repulſion in the ſame manner as ſolid matter is generally ſuppoſed to be: provided therefore that any body move with a ſufficient degree of velocity, or have a ſufficient momentum to overcome any powers of repulſion that it may meet with, it will find no difficulty in making its way through any body whatever; for nothing elſe will penetrate one another but powers, ſuch as we know do in fact exiſt in the ſame place, and counterbalance or over-rule one another. The moſt obvious difficulty, and indeed almoſt the only one, that attends this hypotheſis, as it ſuppoſes the mutual penetrability of matter, ariſes from the idea of the nature of matter, and the difficulty we meet with in attempting to force two bodies into the ſame place. But it is demonſtrable, that the firſt obſtruction ariſes from no actual contact of matter, but from mere powers of repulſion. This difficulty we can overcome; and having got within one ſphere of repulſion, we fancy that we are now impeded by the ſolid matter itſelf. But the very ſame is the opinion of the generality of mankind with reſpect to the firſt obſtruction. Why, therefore, may not the next be only another ſphere of repulſion, which may only require a greater force than we can apply to overcome it, without diſordering the arrangement of the conſtituent particles; but which may be overcome by a body moving with the amazing velocity of light?"

Other philoſophers have ſuppoſed that the powers both of gravitation and cohesion are material; and that they are only different actions of the etherial fluid, or elementary fire. In proof of this doctrine, they allege the experiment with the Magdeburg hemiſpheres, as they are called. The preſſure of the aſmoſphere we ſee is, in this caſe, capable of producing a very ſtrong cohesion; and if there be in nature any fluid more penetrating, as well as more powerful in its effects, than the air we breath, it is poſſible that what is called the attraction of coheſion may in ſome meaſure be an effect of the action of that fluid. Such a fluid as this is the element of fire. Its activity is ſuch as to penetrate all bodies whatever; and in the ſtate in which it is commonly called fire, it acts according to the quantity of ſolid matter contained in the body. In this ſtate, it is capable of diſſolving the ſtrongeſt coheſions obſerved in nature. Fire, therefore, being able to diſſolve coheſions, muſt alſo be capable of cauſing them, provided its power be exerted for that purpoſe, which poſſibly it may be, when we conſider its various modes or appearances, viz, as fire or heat, in which ſtate it conſumes, deſtroys, and diſſolves; or as light, when it ſeems deprived of that deſtructive power; and as the electric fluid, when it attracts, repels, and moves bodies, in a great variety of ways. In the Philoſ. Tranſ. for 1777 this hypotheſis is noticed, and in ſome meaſure adopted by Mr. Henly. "Some gentlemen (ſays he) have ſuppoſed that the electric matter is the cauſe of the coheſion of the particles of bodies. If the electric matter be, as I ſuſpect, a real elementary fire inherent in all bodies, that opinion may probably be well founded; and perhaps the ſoldering of metals, and the cementation of iron, by fire, may be conſidered as ſtrong proofs of the truth of their hypotheſis."

But whatever the cauſe of coheſion may be, its effects are evident and certain. The different degrees of it conſtitute bodies of different forms and properties. Thus, Newton obſerves, the particles of fluids, which do not cohere too ſtrongly, and are ſmall enough to render them ſuſceptible of thoſe agitations which keep liquors in a fluor, are moſt eaſily ſeparated and rarefied into vapour, and make what the chemiſts call *volatile bodies*; being rarefied with an eaſy heat, and again condenſed with a moderate cold. Thoſe that have groſſer particles, and ſo are leſs ſuſceptible of agitation, or cohere by a ſtronger attraction, are not ſeparable without a greater degree of heat; and ſome of them not without fermentation: and theſe make what the chemiſts call *fixt bodies*.

Air, in its fixed ſtate, poſſeſſes the interſtices of ſolid ſubſtances, and probably ſerves as a bond of union to their conſtituent parts; for when theſe parts are ſeparated, the air is diſcharged, and recovers its elaſticity. And this kind of attraction is evinced by a variety of familiar experiments; as, by the union of two contiguous drops of mercury; by the mutual approach of two pieces of cork, floating near each other in a baſon of water; by the adheſion of two leaden balls, whoſe ſurfaces are ſcraped and joined together with a gentle twiſt, which is ſo conſiderable, that, if the ſurfaces are about a quarter of an inch in diameter, they will not be ſeparated by a weight of 100 lb; by the aſcent of oil or water between two glaſs planes, ſo as to form the hyperbolic curve, when they are made to touch on one ſide, and kept ſeparate at a ſmall diſtance on the other; by the depreſſion of mercury, and by the riſe of water in capillary tubes, and on the ſides of glaſs veſſels; alſo in ſugar, ſponge, and all porous ſubſtances. And where this coheſive attraction ends, a power of repulſion begins.

To determine the force of coheſion, in a variety of different ſubſtances, many experiments have been made, and particularly by profeſſor Muſchenbroek. The adheſion of poliſhed planes, about two inches in diameter, heated in boiling water, and ſmeared with greaſe, required the following weights to ſeparate them:

	Cold greaſe	Hot greaſe
Planes of Glaſs	130 lb	300 lb
Braſs	150	800
Copper	200	850
Marble	225	600
Silver	150	250
Iron	300	950

But when the Braſs planes were made to adhere by other ſorts of matter, the reſults were as in the following table:

With Water	12 oz
Oil	18
Venice Turpentine	24
Tallow Candle	800
Roſin	850
Pitch	1400

In eſtimating the *Abſolute Coheſion* of ſolid pieces of bodies, he applied weights to ſeparate them according to their length: his pieces of wood were long ſquare parallelopipedons, each ſide of which was ·26 of an inch, and they were drawn aſunder by the following weights:

Fir	600 lb
Elm	950
Alder	1000
Linden tree	1000
Oak	1150
Beech	1250
Aſh	1250

He tried alſo wires of metal, 1-10th of a Rhinland inch in diameter: the metals and weights were as follow:

Of Lead	29¼ lb
Tin	40¼
Copper	299¼
Yellow Braſs	360
Silver	370
Iron	450
Gold	500

He then tried the *Relative Coheſion*, or the force with which bodies reſiſt an action applied to them in a direction perpendicular to their length. For this purpoſe he fixed pieces of wood by one end into a ſquare hole in a metal plate, and hung weights towards the other end, till they broke at the hole: the weights and diſtances from the hole are exhibited in the following table.

	Diſtance	Weight
Pine	9½ inc	36½ oz
Fir	9	40
Beech	7	56½
Elm	9	44
Oak	8½	48
Alder	9¼	48

See his Elem. Nat. Philoſ. cap. 19.

COLD, the privation of heat, or the oppoſite to it.

As it is ſuppoſed that heat conſiſts in a particular motion of the parts of the hot body, hence the nature of cold, which is its oppoſite, is deduced; for it is found that cold extinguiſhes, or rather abates heat; hence it would ſeem to follow, that thoſe bodies are cold, which check and reſtrain the motion of the particles in which heat conſiſts.

In general, cold contracts moſt bodies, and heat expands them: though there are ſome inſtances to the contrary, eſpecially in the extreme caſes or ſtates of theſe qualities of bodies. Thus, though iron, in common with other bodies, expand with heat, yet, when melted, it is always found to expand in cooling again. So alſo, though water always is found to expand gradually as it is heated, and to contract as it cools, yet in the act of freezing, it ſuddenly expands again, and that with a moſt enormous force, capable of rending rocks, or burſting the very thick ſhells of metal, &c. A computation of the force of freezing water has been made by the Florentine Academicians, from the burſting of a very ſtrong braſs globe or ſhell, by freezing water in it; when, from the known thickneſs and tenacity of the metal, it was found that the expanſive power of a ſpherule of water only one inch in diameter, was ſufficient to overcome a reſiſtance of more than 27,000 pounds, or 13 tons and a half. See alſo experiments on burſting thick iron bomb-ſhells by freezing water in them by Major Edward Williams of the Royal Artillery, in the Edinb. Philoſ. Tranſ. vol. 2.

Such a prodigious power of expanſion, almoſt double that of the moſt powerful ſteam engines, and exerted in ſo ſmall a maſs, ſeemingly by the force of cold, was thought a very powerful argument in favour of thoſe who ſuppoſed that cold, like heat, is a poſitive ſubſtance. Dr. Black's diſcovery of latent heat, however, has now afforded a very eaſy and natural explication of this phenomenon. He has ſhewn, that, in the act of congelation, water is not cooled more than it was before, but rather grows warmer: that as much heat is diſcharged, and paſſes from a latent to a ſenſible ſtate, as, had it been applied to water in its fluid ſtate, would have heated it to 135°. In this proceſs, the expanſion is occaſioned by a great number of minute bubbles ſuddenly produced. Formerly theſe were ſuppoſed to be cold in the abſtract; and to be ſo ſubtle, that, inſinuating themſelves into the ſubſtances of the fluid, they augmented its bulk, at the ſame time that, by impeding the motion of its particles upon each other, they changed it from a fluid to a ſolid. But Dr. Black ſhews that theſe are only air extricated during the congelation; and to the extrication of this air he aſcribes the prodigious expanſive force exerted by freezing water. The only queſtion therefore now remaining, is, By what means this air comes to be extricated, and to take up more room than it naturally does in the fluid. To this it may be anſwered, that perhaps part of the heat which is diſcharged from the freezing water, combines with the air in its unelaſtic ſtate, and, by reſtoring its elaſticity, gives it that extraordinary force, as is ſeen alſo in the caſe of air ſuddenly extricated in the exploſion of gun-powder.

Cold alſo uſually tends to make bodies electric, which are not ſo naturally, and to increaſe the electric properties of ſuch as are ſo. And it is farther found that all ſubſtances do not tranſmit cold equally well; but that the beſt conductors of electricity, viz metals, are likewiſe the beſt conductors of cold. It may farther be added, that when the cold has been carried to ſuch an extremity as to render any body an electric, it then ceaſes to conduct the cold ſo well as before. This is exemplified in the practice of the Laplanders and Siberians; where, to exclude the extreme cold of the winters from their habitations the more effectually, and yet to admit a little light, they cut pieces of ice, which in the winter time muſt always be electric in thoſe countries, and put them into their windows; which they find to be much more effectual in keeping out the cold than any other ſubſtance.

Cold is the deſtroyer of all vegetable life, when increaſed to an exceſſive degree. It is found that many garden plants and flowers, which ſeem to be very ſtout and hardy, go off at a little increaſe of cold beyond the ordinary ſtandard. And in ſevere winters, nature has provided the beſt natural defence for the corn fields and gardens, namely, a covering of ſnow, which preſerves ſuch parts green and healthy as are under it, while ſuch as are uncovered by it are either killed or greatly injured.

Dr. Clarke is of opinion, that cold is owing to certain nitrous, and other ſaline particles, endued with particular figures proper to produce ſuch effects. Hence, ſal-ammoniac, ſaltpetre, or ſalt of urine, and many other volatile and alkalizate ſalts, mixed with water, very much increaſe its degree of cold. In the Philoſ. Tranſ. number 274, M. Geoffroy relates ſome remarkable experiments with regard to the production of cold. Four ounces of ſal-ammoniac diſſolved in a pint of water, made his thermometer deſcend 2 inches and ¾ in leſs than 15 minutes. An ounce of the ſame ſalt put into 4 or 5 ounces of diſtilled water, made the thermometer deſcend 2 inches and ¼. Half an ounce of ſal-ammoniac mixed with 3 ounces of ſpirit of nitre, made the thermometer deſcend 2 inches and 5/12; but, on uſing ſpirit of vitriol inſtead of nitre, it ſunk 2 inches and ½. In this laſt experiment it was remarked, that the vapours raiſed from the mixture had a conſiderable degree of heat, though the liquid itſelf was ſo extremely cold. Four ounces of ſaltpetre mixed with a pint of water, ſunk the thermometer an inch and ¼; but a like quantity of ſea ſalt ſunk it only ⅙ of an inch. Acids always produced heat, even common ſalt with its own ſpirit. Volatile alkaline ſalts produced cold in proportion to their purity, but fixed alkalies heat.

But the greateſt degree of cold produced by the mixture of ſalts and aqueous fluids, was that ſhewn by Homberg; who gives the following receipt for making the experiment: Take a pound of corroſive ſublimate, and as much ſal-ammoniac; powder them ſeparately, and mix the powders well; put the mixture into a vial, pouring upon it a pint and a half of diſtilled vinegar, ſhaking all well together. This compoſition grows ſo cold, that it can ſcarce be held in the hand in ſummer; and it happened, as M. Homberg was making the experiment, that the matter froze. The ſame thing once happened to M. Geoffroy, in making an experiment with ſal-ammoniac and water, but it never was in his power to make it ſucceed a ſecond time.

If, inſtead of making theſe experiments with fluid water, it be taken in its congealed ſtate of ice, or rather ſnow, degrees of cold will be produced greatly ſuperior to any that have yet been mentioned. A mixture of ſnow and common ſalt ſinks Fahrenheit's thermometer to 0; pot aſhes and pounded ice ſunk it 8 degrees farther; two affuſions of ſpirit of ſalt on pounded ice ſunk it 14½ below 0; and by repeated affuſions of ſpirit of nitre M. Fahrenheit ſunk it to 40° below 0. This is the ultimate degree of cold which the mercurial thermometer will meaſure; for the mercury itſelf begins then to congeal; and therefore recourſe muſt afterwards be had to ſpirit of wine, naptha, or ſome other fluid that will not congeal. The greateſt degree of cold hitherto produced by artificial means, has been 80° below 0; which was done at Hudſon's Bay by means of ſnow and vitriolic acid, the thermometer ſtanding naturally at 20° below 0. Indeed greater degrees of cold than this have been ſuppoſed: Mr. Martin, in his Treatiſe on Heat, relates, that at Kirenga in Siberia, the mercurial thermometer ſunk to 118° below 0; and profeſſor Brown at Peterſburg, when he made the firſt experiment of congealing quickſilver, fixed the point of congelation at 350° below 0; but from later experiments it has been more accurately determined, that 40° below 0 is the freezing point of quickſilver.

The moſt remarkable experiment however was made by Mr. Walker of Oxford, with ſpirit of nitre poured on Glauber's ſalt, the effect of which was found to be ſimilar to that of the ſame ſpirit poured on ice or ſnow; and the addition of ſal-ammoniac rendered the cold ſtill more intenſe. The proportions of theſe ingredients recommended by Mr. Walker, are concentrated nitrous acid two parts by weight, water one part; of this mixture, cooled to the temperature of the atmoſphere, 18 ounces; of Glauber's ſalt, a pound and a half avoirdupois; and of ſal-ammoniac, 12 ounces. On adding the Glauber's ſalt to the nitrous acid, the thermometer fell 52°, viz from 50 to −2; and on the addition of the ſal-ammoniac, it fell to − 9°. Thus Mr. Walker was able to freeze quickſilver without either ice or ſnow, when the thermometer ſtood at 45°; viz, by putting the ingredients in 4 different pans, and incloſing theſe within each other.

Exceſſive degrees of cold occur naturally in many parts of the globe in the winter ſeaſon.

Although the thermometer in this country hardly ever deſcends ſo low as 0, yet in the winter of 1780, Mr. Wilſon of Glaſgow obſerved, that a thermometer laid on the ſnow ſunk to 25° below 0; and Mr. Derham, in the year 1708, obſerved in England, that the mercury ſtood within one-tenth of an inch of its ſtation when plunged into a mixture of ſnow and ſalt. At Peterſburg, in 1732, the thermometer ſtood at 28° below 0; and when the French academicians wintered near the polar circle, the thermometer ſunk to 33° below 0; and in the Aſiatic and American continents, ſtill greater degrees of cold are often obſerved.

The effects of theſe extreme degrees of cold are very ſurpriſing. Trees are burſt, rocks rent, and rivers and lakes frozen ſeveral feet deep: metallic ſubſtances bliſter the ſkin like red-hot iron: the air, when drawn in by breathing, hurts the lungs, and excites a cough: even the effects of fire in a great meaſure ſeem to ceaſe; and it is obſerved, that though metals are kept for a conſiderable time before a ſtrong fire, they will ſtill freeze water when thrown upon them. When the French mathematicians wintered at Tornea in Lapland, the external air, when ſuddenly admitted into their rooms, converted the moiſture of the air into whirls of ſnow; their breaſts ſeemed to be rent when they breathed it, and the contact of it was intolerable to their bodies; and the ſpirit of wine, which had not been highly rectified, burſt ſome of their thermometers by the congelation of the aqueous part.

Extreme cold too often proves fatal to animals in thoſe countries where the winters are very ſevere; thus 7000 Swedes periſhed at once in attempting to paſs the mountains which divide Norway from Sweden. But it is not neceſſary that the cold, in order to prove fatal to human life, ſhould be ſo very intenſe as has juſt been mentioned; it is only requiſite to be a little below 32° of Fahrenheit, or the freezing point, accompanied with ſnow or hail, from which ſhelter cannot be obtained. The ſnow which falls upon the clothes, or the uncovered parts of the body, then melts, and by a continual evaporation carries off the animal heat to ſuch a degree, that a ſufficient quantity is not left for the ſupport of life. In ſuch caſes, the perſon firſt feels himſelf extremely chill and uneaſy; he turns liſtleſs, unwilling to walk or uſe exerciſe to keep himſelf warm, and at laſt turns drowſy, ſits down to refreſh himſelf with ſleep, but wakes no more.

COLLIMATION, *Line of*, in a teleſcope, is a line paſſing through the interſection of thoſe wires that are fixed in the focus of the object-glaſs, and the centre of the ſame glaſs.

COLLINS (John), an eminent accountant and mathematician, was born at Wood Eaton near Oxford, March 5, 1624. At 16 years of age he was put apprentice to a bookſeller at Oxford; but his genius appeared ſo remarkable for the ſtudy of the mechanical and mathematical ſciences, that he was taken under the tuition of Mr. Marr, who drew ſeveral curious dials, which were placed in different poſitions in the king's gardens; under whom Mr. Collins made no ſmall progreſs in the mathematics. In the courſe of the civil wars, he travelled abroad, to proſecute his favourite ſtudy; and on his return he took upon him the profeſſion of an accountant, and publiſhed, in the year 1652, a large work entitled, *An Introduction to Merchants Accompts;* which was followed by ſeveral other publications on different branches of accounts. In 1658, he publiſhed a treatiſe called *The Sector on a Quadrant;* containing the deſcription and uſe of four ſeveral quadrants, each accommodated to the making of ſun-dials, &c; to which he afterward added an appendix concerning reflected dialling, from a glaſs placed reclining.—In 1659, he publiſhed his *Geometrical Dialling;* and the ſame year alſo his *Mariner's Plain Scale new plained.*—Collins now became a fellow of the Royal Society in London, to which he made various communications; particularly ſome ingenious chronological rules for the calendar, printed in the Philoſ. Tranſ. number 46, for April 1669: alſo a curious diſſertation concerning the reſolution of equations in numbers, in number 69, for March 1671: an elegant conſtruction of the curious problem, having given the mutual diſtances of three objects in a plane, with the angles made by them at a fourth place in that plane, to find the diſtance of this place from each of the three former, vol. 6. pa. 2093: and thoughts about ſome defects in algebra, vol. 14. pa. 375.

Collins wrote alſo ſeveral commercial tracts, highly acceptable to the public; viz, A Plea for bringing over Iriſh cattle, and keeping out the fiſh caught by foreigners:—For the promotion of the Engliſh fiſhery:—For the working the Tin-mines:—A Diſcourſe of Salt and Fiſhery. He was frequently conſulted in nice and critical caſes of accounts, of commerce, and engineering. On one of theſe occaſions, being appointed to inſpect the ground for cutting a canal or river between the Iſis and the Avon, he contracted a diſorder by drinking cyder when he was too warm, which ended in his death, the 10th of November 1683, at 59 years of age.

Mr. Collins was a very uſeful man to the ſciences, keeping up a conſtant correſpondence with the moſt learned men, both at home and abroad, and promoting the publication of many valuable works, which, but for him, would never have been ſeen by the public; particularly Dr. Barrow's optical and geometrical lectures; his abridgment of the works of Archimedes, Apollonius, and Theodoſius; Branker's tranſlation of Rhronius's algebra, with Dr. Pell's additions, &c; which were procured by his frequent ſolicitations.

It was a conſiderable time after, that his papers were all delivered into the hands of the learned and ingenious Mr. William Jones, F. R. S. among which were found manuſcripts, upon mathematical ſubjects, of Briggs, Oughtred, Barrow, Newton, Pell, and many others. From a variety of letters from theſe, and many other celebrated mathematicians, it appears that Collins ſpared neither pains nor coſt to procure what tended to promote real ſcience: and even many of the late diſcoveries in phyſical knowledge owe their improvement to him; for while he excited ſome to make known every new and uſeful invention, he employed others to improve them. Sometimes he was peculiarly uſeful, by ſhewing where the defect was in any uſeful branch of ſcience, pointing out the difficulties attending the enquiry, and at other times ſetting forth the advantages, and keeping up a ſpirit and warm deſire for improvement. Mr. Collins was alſo as it were the regiſter of all the new improvements made in the mathematical ſciences; the magazine to which the curious had frequent recourſe: in ſo much that he acquired the appellation of the Engliſh Merſennus. If ſome of his correſpondents had not obliged him to conceal their communications, there could have been no diſpute about the priority of the invention of a method of analyſis, the honour of which evidently belongs to Newton; as appears undeniably from the papers printed in the *Commercium Epiſtolicum D. Joannis Collins & aliorum de Analyſi promota; juſſu Societatis Regiæ in lucem editum*, 1712; a work that was made out from the letters in the poſſeſſion of our author.

COLLINS's *Quadrant*. See QUADRANT.

COLLISION, is the friction, percuſſion, or ſtriking of bodies againſt one another.

Striking bodies are conſidered either as elaſtic, or non-elaſtic. They may alſo be either both in motion, or one of them in motion, and the other at reſt.

When non-elaſtic bodies ſtrike, they unite together as one maſs; which, after colliſion, either remains at reſt, or moves forward as one body. But when elaſtic bodies ſtrike, they always ſeparate after the ſtroke.

The principal theorems relating to the colliſion of bodies, are the following:

1. If any body impinge or act obliquely on a plane ſurface; the force or energy of the ſtroke, or action, is as the ſine of the angle of incidence. Or the force

upon the ſurface, is to the ſame when acting perpendicularly, as the ſine of incidence is to radius.

2. If one body act on another, in any direction, and by any kind of force; the action of that force on the ſecond body is made only in a direction perpendicular to the ſurface on which it acts.

3. If the plane, acted on, be not abſolutely fixed, it will move, after the ſtroke, in the direction perpendicular to its ſurface.

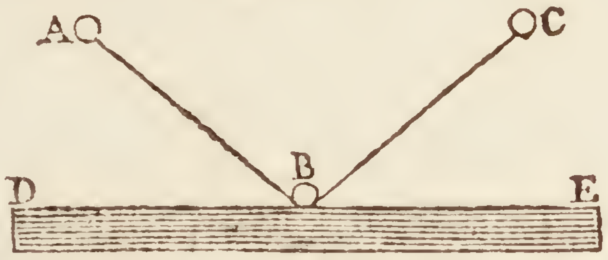
4. If a body A ſtrike another body B, which is either at reſt, or elſe in motion, either towards A or from it; then the momenta, or quantities of motion, of the two bodies, eſtimated in any one direction, will be the very ſame after the ſtroke that they were before it.

Thus, firſt, if A with a momentum of 10, ſtrike B at reſt, and communicate to it a momentum of 4, in the direction AB. Then there will remain in A only a momentum of 6 in that direction: which together with the momentum of B, viz 4, makes up ſtill the ſame momentum between them as before.—But if B were in motion before the ſtroke, with a momentum of 5, in the ſame direction, and receive from A an additional momentum of 2: then the motion of A after the ſtroke will be 8, and that of B, 7; which between them make up 15, the ſame as 10 and 5, the motions before the ſtroke.—Laſtly, if the bodies move in oppoſite directions, and meet one another, namely A with a motion of 10, and B, of 5; and A communicate to B a motion of 6 in the direction AB of its motion: then, before the ſtroke, the whole motion from both, in the direction AB, is 10 − 5, or 5: but after the ſtroke the motion of A is 4 in the direction AB, and the motion of B is 6 − 5, or 1 in the ſame direction AB; therefore the ſum 4 + 1, or 5, is ſtill the ſame motion from both as it was before.

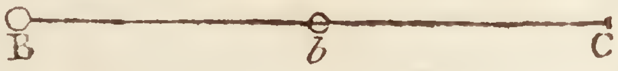
5. If a hard and fixed plane be ſtruck either by a ſoft or a hard unelaſtic body; the body will adhere to it. But if the plane be ſtruck by a perfectly elaſtic body, it will rebound from it with the ſame velocity with which it ſtruck the plane.

6. The effect of the blow of the elaſtic body, upon the plane, is double to that of the non-elaſtic one; the velocity and maſs being the ſame in both.

7. Hence, non-elaſtic bodies loſe, by their colliſion, only half the motion that is loſt by elaſtic bodies; the maſſes and velocities being equal.



8. If an elaſtic body A impinge upon a firm plane DE at the point B, it will rebound from it in an angle equal to that in which it ſtruck it; or the angle of incidence will be equal to the angle of reflection: namely, the angle ABD = CBE.



9. If the non-elaſtic body B, moving with the velocity V in the direction B*b*, and the body *b* with the velocity v, ſtrike each other, the direction of the motion being in the line BC; then they will move after the ſtroke with a common velocity, which will be more or leſs according as, before the ſtroke, *b* moved towards B, or from B, or was at reſt; and that common velocity, in each of theſe caſes, will be as follows: viz, it will be

$\frac{BV + bv}{B + b}$ when *b* moved from B,

$\frac{BV - bv}{B + b}$ when *b* moved towards B,

$\frac{BV}{B + b}$ when *b* was at reſt.

For example, if the bodies or weights, B and *b*, be 5lb and 3lb; and their velocities V and v, 60 feet and 40 feet per ſecond; then 300 and 120 will be their momenta BV and bv, and $18 = B + b$ the ſum of the weights. Conſequently the common velocity after the ſtroke, in the three caſes above mentioned, will be thus, viz,

$\frac{300 + 120}{18} = \frac{420}{18}$ or $23\frac{1}{4}$ in the firſt caſe,

$\frac{300 - 120}{18} = \frac{180}{18}$ or 10 in the ſecond caſe,

$\frac{300}{18}$ - - or $16\frac{2}{3}$ in the third caſe.

10. If two perfectly elaſtic bodies impinge on each other; their relative velocity is the ſame both before and after the impulſe; that is, they will recede from each other with the ſame velocity with which they approached and met.

It is not meant however by this theorem, that each body will have the very ſame velocity after the impulſe as it had before; but that the velocity of the one, after the ſtroke, will be as much increaſed, as that of the other is decreaſed, in one and the ſame direction. So, if the elaſtic body B move with the velocity V, and overtake the elaſtic body *b*, moving the ſame way, with the velocity v; then their relative velocity, or that with which they ſtrike, is only $V - v$; and it is with this ſame velocity that they ſeparate from each other after the ſtroke: but if they meet each other, or the body *b* move contrary to the body B; then they meet and ſtrike with the velocity $V + v$, and it is with the ſame velocity that they ſeparate again, and recede from each other after the ſtroke: in like manner, they would ſeparate with the velocity V of B, if *b* were at reſt before the ſtroke. Alſo the ſum of the velocities of the one body, is equal to the ſum of the others. But whether they move forwards or backwards after the impulſe, and with what particular velocities, are circumſtances that depend on the various maſſes and velocities of the bodies before the ſtroke, and are as ſpecified in the next theorem.

11. If the two elaſtic bodies B and *b* move directly towards each other, or directly from each other, the former with the velocity V, and the latter with the velocity v; then, after their meeting and impulſe, the reſpective velocities of B and *b* in the direction BC, in the three caſes of motion, will be as follow: viz,

$\frac{2bv + \overline{B - b} . V}{B + b}$ the velocity of B,

$\frac{2BV - \overline{B - b} . v}{B + b}$ the velocity of *b*,

when the bodies both moved towards C before the ſtroke; and

−2bv

$\frac{-2bv + \overline{B-b}.V}{B+b}$ the velocity of B,

$\frac{2BV + \overline{B-b}.v}{B+b}$ the velocity of b,

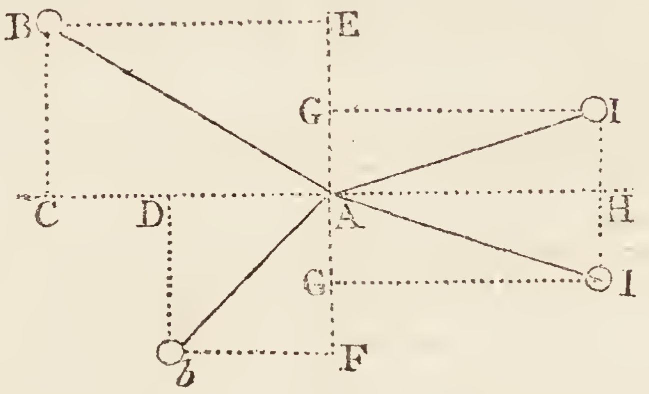
when B moved towards C, and b towards B before the ſtroke;

$\frac{B-b}{B+b} \times V$ the velocity of B,

$\frac{2B}{B+b} \times V$ the velocity of b,

when b was at reſt before the ſtroke.

12 The motions of bodies after impact, that ſtrike each other obliquely, are thus determined.



Let the two bodies B, b, move in the oblique directions BA, bA, and ſtrike each other at A with velocities which are in proportion to the lines BA, bA. Let CAH repreſent the plane in which the bodies touch in the point of concourſe; to which draw the perpendiculars BC, bD, and complete the rectangles CE, DF. Now the motion in BA is reſolved into the two BC, CA; and the motion in bA is reſolved into the two bD, DA; of which the antecedents BC, bD are the velocities with which they irectly meet, and the conſequents CA, DA are parallel, and therefore by theſe the bodies do not impinge on each other, and conſequently the motions according to theſe directions will not be changed by the impulſe; ſo that the velocities with which the bodies meet, are as BC or EA, and bD or FA. The motions therefore of the bodies B, b, directly ſtriking each other with the celerities EA, FA, will be determined by art. 11 or 9, according as the bodies are elaſtic or non-elaſtic; which being done, let AG be the velocity, ſo determined, of one of them, as A; and ſince there remains alſo in the ſame body a force of moving in the direction parallel to BE, with a velocity as BE, make AH equal to BE, and complete the rectangle GH: then the two motions in AH and AG, or HI, are compounded into the diagonal AI, which therefore will be the path and celerity of the body B after the ſtroke. And after the ſame manner is the motion of the other body b determined after the impact.

13. The ſtate of the common centre of gravity of bodies is not affected by the colliſion or other actions of thoſe bodies on one another. That is, if it were at reſt before their colliſion, ſo will it be alſo at reſt after colliſion: and if it were moving in any direction, and with any velocity, before colliſion; it will do the very ſame after it.

See more upon this ſubject under the article PERCUSSION.

COLONNADE, a Periſtyle of a circular figure; or a ſeries of columns diſpoſed in a circle, and inſulated within-ſide.

COLOUR, a property inherent in light, by which, according to the various ſizes of its parts, or from ſome other cauſe, it excites different vibrations in the optic nerve; which, propagated to the *ſenſorium*, affect the mind with different ſenſations. See the doctrine of colours fully explained under *Chromatics*. See alſo *Optics*, *Achromatic*, and *Teleſcope*.

COLUMBA *Noachi*, Noah's Dove, a ſmall conſtellation in the ſouthern hemiſphere, conſiſting of 10 ſtars.

COLUMN, in Architecture, a round pillar, made to ſupport or adorn a building.

The column is the principal part of an architectonical order, and is compoſed of three parts, the *baſe*, the *ſhaft*, and the *capital*; each of which is ſubdivided into a number of leſſer parts, called members, or mouldings.

Columns are different according to the different orders they are uſed in; and alſo according to their matter, conſtruction, form, diſpoſition, and uſe. The proportion of the length of each to its diameter, and the diminution of the diameter upwards, are diverſly ſtated by different authors. The medium of them is nearly as follows:

The *Tuſcan* is the ſimpleſt and ſhorteſt of all; its height $3\frac{1}{2}$ diameters, or 7 modules; and it diminiſhes $\frac{1}{4}$ part of its diameter.

The *Doric* is more delicate, and adorned with flutings; its height $7\frac{1}{2}$ or 8 diameters.

The *Ionic* is more delicate ſtill, being 9 diameters long. It is diſtinguiſhed from the reſt by the volutes, or curled ſcrolls in its capital, and by its baſe which is peculiar to it.

The *Corinthian* is the richeſt and moſt delicate of all the columns, being 10 diameters in length, and adorned with two rows of leaves, and ſtalks or ſtems, from whence ſpring out ſmall volutes.

The *Compoſite* Column is alſo 10 diameters long, its capital adorned with rows of leaves like the Corinthian, and with angular volutes like the Ionic.

COLURES, are two great circles imagined to interſect at right angles in the poles of the world, and to paſs, the one through the equinoctial points Aries and Libra, and the other through the ſolſtitial points Cancer and Capricorn; from whence they are called the Equinoctial and Solſtitial Colures. By thus dividing the ecliptic into four equal parts, they mark the four ſeaſons, or quarters of the year.

It is diſputed over what part of the back of Aries the equinoctial colure paſſed in the time of Hipparchus. Newton, in his Chronology, takes it to have been over the middle of the conſtellation. Father Souciet inſiſts that it paſſed over the dodecatemorion of Aries, or midway between the rump and firſt of the tail. There are ſome obſervations in the Philoſ. Tranſ. number 466, concerning the poſition of this colure in the ancient ſphere, from a draught of the conſtellation Aries, in the Aratæa publiſhed at Leyden and Amſterdam in 1652, which ſeem to confirm Newton's opinion; but the antiquity and authority of the original draught may ſtill remain in queſtion.

COMA BERENICES, *Berenice's Hair*, a modern conſtellation of the northern hemiſphere; compoſed of unformed ſtars between the Lion's tail and Bootes.

It is ſaid that this conſtellation was formed by Conon, an aſtronomer, to conſole the queen of Ptolomy Euergetes, for the loſs of a lock of her hair, which was ſtolen out of the temple of Venus, where ſhe had dedicated it on account of a victory obtained by her huſband.

The ſtars in this conſtellation are, in Tycho's catalogue 14, in Hevelius's 21, and in the Britannic catalogue 43.

COMBINATIONS, denote the alternations or variations of any number of quantities, letters, ſounds, or the like, in all poſſible ways.

Father Merſenne gives the combinations of all the notes and ſounds in muſic, as far as 64; the ſum of which amounts to a number expreſſed by 90 places of figures. And the number of poſſible combinations of the 24 letters of the alphabet, taken firſt two by two, then three by three, and ſo on, according to Preſtet's calculation, amounts to

1391724288887252999425128493402200.

Father Truchet, in Mem. de l'Acad. ſhews, that two ſquare pieces, each divided diagonally into two colours, may be arranged and combined 64 different ways, ſo as to form ſo many different kinds of chequer-work: a thing that may be of uſe to maſons, paviours, &c.

Doctrine of COMBINATIONS.

I. *Having given any number of things, with the number in each combination; to find the number of combinations.*

1. *When only two are combined together.*

One thing admits of no combination.

Two, *a* and *b*, admit of one only, viz *ab*.

Three, *a*, *b*, *c*, admit of three, viz *ab*, *ac*, *bc*.

Four admit of ſix, viz, *ab*, *ac*, *ad*, *bc*, *bd*, *cd*.

Five admit of 10, viz, *ab*, *ac*, *ad*, *ae*, *bc*, *bd*, *be*, *cd*, *ce*, *de*.

Whence it appears that the numbers of combinations, of two and two only, proceed according to the triangular numbers 1, 3, 6, 10, 15, 21, &c, which are produced by the continual addition of the ordinal ſeries 0, 1, 2, 3, 4, 5, &c. And if *n* be the number of things, then the general formula for expreſſing the ſum of all their combinations by twos, will be $\frac{n \cdot n-1}{1 \cdot 2}$.

Thus, if $n = 2$; this becomes $\frac{2 \cdot 1}{2} = 1$.

If $n = 3$; it is $\frac{3 \cdot 2}{2}$ - - $= 3$.

If $n = 4$; it is $\frac{4 \cdot 3}{2}$ - - $= 6$. &c.

2. *When three are combined together; then*

Three things admit of one order, *abc*.

Four admit of 4; viz *abc*, *abd*, *acd*, *bcd*.

Five admit of 10; viz *abc*, *abd*, *abe*, *acd*, *ace*, *ade*, *bcd*, *bce*, *bde*, *cde*. And ſo on according to the firſt pyramidal numbers 1, 4, 10, 20, &c, which are formed by the continual addition of the former, or triangular numbers 1, 3, 6, 10, &c. And the general formula for any number *n* of combinations, taken by threes, is $\frac{n \cdot n-1 \cdot n-2}{1 \cdot 2 \cdot 3}$.

So, if $n = 3$; it is $\frac{3 \cdot 2 \cdot 1}{1 \cdot 2 \cdot 3} = 1$.

If $n = 4$; it is $\frac{4 \cdot 3 \cdot 2}{6} = 4$.

If $n = 5$; it is $\frac{5 \cdot 4 \cdot 3}{2} = 6$. &c.

Proceeding thus, it is found that a general formula for any number *n* of things, combined by *m* at each time, is $s = \frac{n \cdot n-1 \cdot n-2 \cdot n-3 \text{ \&c}}{1 \cdot \quad 2 \cdot \quad 3 \cdot \quad 4 \cdot \text{ \&c}}$, continued to *m* factors, or terms, or till the laſt factor in the denominator be *m*.

So, in 6 things, combined by 4's, the number of combinations is $\frac{6 \cdot 5 \cdot 4 \cdot 3}{1 \cdot 2 \cdot 3 \cdot 4} = 15$.

3. By adding all theſe ſeries together, their ſum will be the whole number of poſſible combinations of *n* things combined both by twos, by threes, by fours, &c. And as the ſaid ſeries are evidently the coefficients of the power *n* of a binomial, wanting only the firſt two 1 and *n*; therefore the ſaid ſum, or whole number of all ſuch combinations, will be $\overline{1+1}|^{n} - n - 1$, or $2^{n} - n - 1$. Thus if the number of things be 5; then $2^{5} - 5 - 1 = 32 - 6 = 26$.

II. *To find the number of Changes and Alterations which any number of quantities can undergo, when combined in all poſſible varieties of ways, with themſelves and each other, both as to the things themſelves, and the Order or Poſition of them.*

One thing admits but of one order or poſition.

Two things may be varied four ways; thus, *aa*, *ab*, *ba*, *bb*.

Three quantities, taken by twos, may be varied nine ways; thus *aa*, *ab*, *ac*, *ba*, *ca*, *bb*, *bc*, *cb*, *cc*.

In like manner four things, taken by twos, may be varied 4^{2} or 16 ways; and 5 things, by twos, 5^{2} or 25 ways; and, in general, *n* things, taken by twos, may be changed or varied n^{2} different ways.

For the ſame reaſon, when taken by threes, the changes will be n^{3}; and when taken by fours, they will be n^{4}; and ſo generally, when taken by *n*'s, the changes will be n^{n}.

Hence, then, adding all theſe together, the whole number of changes, or combinations in *n* things, taken both by 2's, by 3's, by 4's, &c, to *n*'s, will be the ſum of the geometrical ſeries $n + n^{2} + n^{3} + n^{4} \text{----} n^{n}$, which ſum is $= \frac{n^{n} - 1}{n - 1} \times n$.

For example, if the number of things *n* be 4; this gives $\frac{4^{4} - 1}{4 - 1} \times 4 = \frac{255}{3} \times 4 = 340$.

And if *n* be 24, the number of letters in the alphabet; the theorem gives

$$\frac{24^{24} - 1}{24 - 1} \times 24 = \overline{24^{24} - 1} \times \frac{24}{23} =$$

1391724288887252999425128493402200. In ſo many different ways, therefore, may the 24 letters of the alphabet be varied or combined among themſelves, or ſo many different words may be made out of them.

COMBUST, or COMBUSTION, is ſaid of a planet when it is in conjunction with the ſun, or not diſtant from it above half their diſc.

But according to Argol, a planet is Combuſt, or in Combuſtion, when it is within eight degrees and a half of the ſun.

COMET, a heavenly body in the planetary region, appearing ſuddenly, and again diſappearing; and during the time of its appearance moving in a proper, though very eccentric orbit, like a planet.

Comets are vulgarly called *Blazing Stars*, and have this to diſtinguiſh them from other ſtars, that they are uſually attended with a long train of light, tending always oppoſite to the ſun, and being of a fainter luſtre the farther it is from the body of the comet. And hence ariſes a popular diviſion of comets, into three kinds; viz *bearded*, *tailed*, and *hairy* comets; though in reality, this diviſion rather relates to the ſeveral circumſtances of the ſame comet, than to the phenomena of ſeveral. Thus, when the comet is eaſtward of the ſun, and moves from him, it is ſaid to be *bearded*, becauſe the light precedes it in the manner of a beard: When the comet is weſtward of the ſun, and ſets after him, it is ſaid to be *tailed*, becauſe the train of light follows it in the manner of a tail: And laſtly, when the ſun and comet are diametrically oppoſite, the earth being between them, the train is hid behind the body of the comet, excepting the extremities, which, being broader than the body of the comet, appear as it were around it, like a border of hair, or *coma*, from which it is called *hairy*, and a comet.

But there have been comets whoſe diſc was as clear, round, and well defined, as that of Jupiter, without either tail, beard or coma.

Of the Nature of Comets.—Philoſophers and Aſtronomers, of all ages, have been much divided in their opinions as to the nature of comets. Their ſtrange appearance has in all ages been matter of terror to the vulgar, who have uniformly conſidered them as evil omens, and forerunners of war, peſtilence, &c. Diodorus Siculus and Appollinus Myndius, in Seneca, inform us, that many of the Chaldeans held them to be laſting bodies, having ſtated revolutions as well as the planets, but in orbits vaſtly more extenſive; on which account they are only viſible while near the earth, but diſappear again when they go into the higher regions. Others of them were of opinion, that the comets were only meteors raiſed very high in the air, which blaze for a while, and diſappear when the matter of which they conſiſt is conſumed or diſperſed.

Some of the Greeks, before Ariſtotle, ſuppoſed that a comet was a vaſt heap or aſſemblage of very ſmall ſtars meeting together, by reaſon of the inequality of their motions, and ſo uniting into a viſible maſs, by the union of all their ſmall lights; which muſt again diſappear, as thoſe ſtars ſeparated, and each proceeded in its courſe. Pythagoras, however, accounted them a kind of planets or wandering ſtars, diſappearing in the ſuperior parts of their orbits, and becoming viſible only in the lower parts of them.

But Ariſtotle held, that comets were only a kind of tranſient fires, or meteors, conſiſting of exhalations raiſed to the upper region of the air, and there ſet on fire; far below the courſe of the moon.

Seneca, who lived in the firſt century, and who had ſeen two or three comets himſelf, plainly intimates that he thought them above the moon; and argues ſtrongly againſt thoſe who ſuppoſed them to be meteors, or who held other abſurd opinions concerning them; declaring his belief that they were not fires ſuddenly kindled, but the eternal productions of nature. He points out alſo the only way to come at a certainty on this ſubject, viz, by collecting a number of obſervations concerning their appearance, in order to diſcover whether they return periodically or not. "For this purpoſe, ſays he, one age is not ſufficient; but the time will come when the nature of comets and their magnitudes will be demonſtrated, and the routes they take, ſo different from the planets, explained. Poſterity will then wonder, that the preceding ages ſhould be ignorant of matters ſo plain and eaſy to be known."

For a long time this prediction of Seneca ſeemed not likely to be fulfilled; and Tycho Brahe was the firſt among the moderns, who reſtored the comets to their true rank in the creation; for after diligently obſerving the comet of 1577, and finding that it had no ſenſible diurnal parallax, he aſſigned it its true place in the planetary regions. See his book De Cometa, anni 1577.

Before this however, there were various opinions concerning them. In the dark and ſuperſtitious ages, comets were held to be forerunners of every kind of calamity, and it was ſuppoſed they had different degrees of malignity, according to the ſhape they aſſumed; from whence alſo they were differently denominated. Thus, it was ſaid that ſome were bearded, ſome hairy; that ſome repreſented a beam, ſword, or ſpear; others a target, &c; whereas modern aſtronomers acknowledge only one ſpecies of comets, and account for their different appearances from their different ſituations with reſpect to the ſun and earth.

Kepler, in other reſpects a very great genius, indulged the moſt extravagant conjectures, not only concerning comets, but the whole ſyſtem of nature in general. The planets he imagined were huge animals ſwimming round the ſun; and the comets monſtrous and uncommon animals generated in the celeſtial ſpaces.

A ſtill more ridiculous opinion, if poſſible, was that of John Bodin, a learned Frenchman in the 16th century; who maintained that comets "are ſpirits, which having lived on the earth innumerable ages, and being at laſt arrived on the confines of death, celebrate their laſt triumph, or are recalled to the firmament like ſhining ſtars! This is followed by famine, plague, &c, becauſe the cities and people deſtroy the governors and chiefs who appeaſe the wrath of God."—Others again have denied even the exiſtence of comets, and maintained that they were only falſe appearances, occaſioned by the refraction or reflection of light.

Hevelius, from a great number of obſervations, propoſed it as his opinion, that the comets, like the ſolar maculæ or ſpots, are formed or condenſed out of the groſſer exhalations of his body; in which he differs but little from the opinion of Kepler.

James Bernoulli, in his Syſtema Cometarum, imagined that comets were no other than the ſatellites of ſome very diſtant planet, which was itſelf inviſible to us on account of its diſtance, as were alſo the ſatellites unleſs when in a certain part of their orbits.

Des Cartes advances another opinion: He conjectures that comets are only ſtars, formerly fixed, like the reſt, in the heavens; but which becoming gradually covered with maculæ or ſpots, and at length wholly deprived of their light, cannot keep their places, but are carried off by the vortices of the circumjacent ſtars; and in proportion to their magnitude and ſolidity, moved in ſuch a manner, as to be brought nearer the orb of Saturn; and thus coming within reach of the ſun's light, rendered viſible.

But the vanity of all theſe hypotheſes now abundantly appears from the obſerved phenomena of comets, and from the doctrine of Newton, which is as follows:

The comets, he ſays, are compact, ſolid, fixed, and durable bodies; in fact a kind of planets, which move in very oblique and eccentric orbits, every way with the greateſt freedom; perſevering in their motions, even againſt the courſe and direction of the planets: and their tail is a very thin and ſlender vapour, emitted by the head or nucleus of the comet, ignited or heated by the ſun. This theory of the comets at once ſolves their principal phenomena, which are as below.

The Principal Phenomena of the Comets.

1. Firſt then, thoſe comets which move according to the order of the ſigns, do all, a little before they diſappear, either advance ſlower than uſual, or elſe go retrograde, if the earth be between them and the ſun; but more ſwiftly, if the earth be placed in a contrary part. On the other hand, thoſe which proceed contrary to the order of the ſigns, move more ſwiftly than uſual, if the earth be between them and the ſun; and more ſlowly, or elſe retrograde, when the earth is in a contrary part.—For ſince this courſe is not among the fixed ſtars, but among the planets; as the motion of the earth either conſpires with them, or goes againſt them; their appearance, with reſpect to the earth, muſt be changed; and, like the planets, they muſt ſometimes appear to move ſwifter, ſometimes ſlower, and ſometimes retrograde.

2. So long as their velocity is increaſed, they nearly move in great circles; but towards the end of their courſe, they deviate from thoſe circles; and when the earth proceeds one way, they go the contrary way. Becauſe, in the end of their courſe, when they recede almoſt directly from the ſun, that part of the apparent motion which ariſes from the parallax, muſt bear a greater proportion to the whole apparent motion.

3. The comets move in ellipſes, having one of their foci in the centre of the ſun; and by radii drawn to the ſun, deſcribe areas proportional to the times. Becauſe they do not wander precariouſly from one fictitious vortex to another; but, making a part of the ſolar ſyſtem, return perpetually, and run a conſtant round. Hence, their elliptic orbits being very long and eccentric, they become inviſible when in that part which is moſt remote from the ſun. And from the curvity of the paths of comets, Newton concludes, that when they diſappear, they are much beyond the orbit of Jupiter; and that in their perihelion they frequently deſcend within the orbits of Mars and the inferior planets.

4. The light of their nuclei, or bodies, increaſes as they recede from the earth toward the ſun; and on the contrary, it decreaſes as they recede from the ſun. Becauſe, as they are in the regions of the planets, their acceſs towards the ſun bears a conſiderable proportion to their whole diſtance.

5. Their tails appear the largeſt and brighteſt, immediately after their tranſit through the region of the ſun, or after their perihelion. Becauſe then, their heads being the moſt heated, will emit the moſt vapours.—From the light of the nucleus we infer their vicinity to the earth, and that they are by no means in the region of the fixed ſtars, as ſome have imagined; ſince, in that caſe, their heads would be no more illuminated by the ſun, than the planets are by the fixed ſtars.

6. The tails always decline from a juſt oppoſition to the ſun towards thoſe parts which the nuclei or bodies paſs over, in their progreſs through their orbits. Becauſe all ſmoke, or vapour, emitted from a body in motion, tends upwards obliquely, ſtill receding from that part towards which the ſmoking body proceeds.

7. This declination, cæteris paribus, is the ſmalleſt when the nuclei approach neareſt the ſun: and it is alſo leſs near the nucleus, or head, than towards the extremity of the tail. Becauſe the vapour aſcends more ſwiftly near the head of the comet, than in the higher extremity of its tail; and alſo when the comet is nearer the ſun, than when it is farther off.

8. The tails are ſomewhat brighter, and more diſtinctly defined in their convex, than in their concave part. Becauſe the vapour in the convex part, which goes firſt, being ſomewhat nearer and denſer, reflects the light more copiouſly.

9. The tails always appear broader at their upper extremity, than near the centre of the comet. Becauſe the vapour in a free ſpace continually rareſies and dilates.

10. The tails are always tranſparent, and the ſmalleſt ſtars appear through them. Becauſe they conſiſt of infinitely thin vapour.

The Phaſes of Comets.—The nuclei, which are alſo called the heads, and bodies, of comets, viewed through a teleſcope, ſhew a face very different from thoſe of the fixed ſtars or planets. They are liable to apparent changes, which Newton aſcribes to changes in the atmoſphere of comets: and this opinion was confirmed by obſervations of the comet in 1744. Hiſt. Acad. Scienc. 1744. Sturmius ſays that, obſerving the comet of 1680 with a teleſcope, it appeared like a coal dimly glowing, or a rude maſs of matter illuminated with a duſky fumid light, leſs ſenſible to the extremes than in the middle; whereas a ſtar appears with a round diſc, and a vivid light.

Of the comet of 1661, Hevelius obſerves, that its body was of a yellowiſh colour, very bright and conſpicuous, but without any glittering light: in the middle was a denſe ruddy nucleus, almoſt equal to Jupiter, encompaſſed by a much fainter, thinner matter. February 5th, its head was ſomewhat larger and brighter, and of a gold colour; but its light more duſky than the ſtars: and here the nucleus appeared divided into ſeveral parts. Feb. 6th, the diſc was leſſened; the parts of the nucleus ſtill exiſted, though leſs than before: one of them, on the lower part of the diſc, on the left, much denſer and brighter than the reſt; its body round, and repreſenting a very lucid little ſtar: the nuclei ſtill encompaſſed with another kind of matter. Feb. 10th, the head ſomewhat more obſcure, and the nuclei more confuſed, but brighter at top than bottom. Feb. 13th, the head diminiſhed much both in ſize and ſplendor. March 2d, its roundneſs a little impaired, and its edges lacerated, &c. March 28th, very pale, and exceeding thin; its matter much diſperſed; and no diſtinct nucleus at all appearing.

Weigelius, who ſaw the comet of 1664, as alſo the moon, and a ſmall cloud in the horizon illuminated by the ſun at the ſame time, obſerved, that through the teleſcope the moon appeared of a continued luminous ſurface: but the comet very different; being exactly like the little cloud. And from theſe obſervations it was that Hevelius formed his opinion, that comets are like maculæ or ſpots formed out of the ſolar exhalations.

Of the Magnitude of Comets.—The eſtimates that have been given of the magnitude of comets by Tycho Brahe, Hevelius, and ſome others, are not very accurate; as it does not appear that they diſtinguiſhed between the nucleus and the ſurrounding atmoſphere. Thus Tycho computes that the true diameter of the comet in 1577 was in proportion to the diameter of the earth, as 3 is to 14; and Hevelius made the diameter of the comet of 1652 to that of the earth, as 52 to 100. But the diameter of the atmoſphere is often 10 or 15 times as great as that of the nucleus: the former, in the comet of 1682, was meaſured by Flamſteed, and found to be 2′, when the diameter of the nucleus alone was only 11 or 12″. Though ſome comets, eſtimated by a compariſon of their diſtance and apparent magnitude, have been judged much larger than the moon, and even equal to ſome of the primary planets. The diameter of that of 1744, when at the diſtance of the ſun from us, meaſured about 1′, which makes its diameter about three times that of the earth: at another time the diameter of its nucleus was nearly equal to that of the planet Jupiter.

Of the Tails of Comets.—There have been various conjectures about the nature of the tails of comets, the principal of which are thoſe of Newton, and the others that follow. Newton ſhews that the atmoſpheres of comets will furniſh vapour ſufficient to form their tails. This he argues from that wonderful rarefaction in our air at a diſtance from the earth; which is ſuch, that a cubic inch of common air, expanded to the rarity of that at the diſtance of half the earth's diameter, or 4000 miles, would fill a ſpace larger than the whole region of the ſtars. Since then the coma, or atmoſphere of a comet, is 10 times higher than the ſurface of the nucleus, from the centre; the tail, aſcending ſtill much higher, muſt neceſſarily be immenſely rare: ſo that it is no wonder the ſtars are viſible through it.

Now the aſcent of vapours into the tail of the comet, he ſuppoſes occaſioned by the rarefaction of the matter of the atmoſphere at the time of the perihelion. Smoke, it is obſerved, aſcends the chimney by the impulſe of the air in which it floats; and air, rarefied by heat, aſcends by the diminution of its ſpecific gravity, carrying up the ſmoke along with it: in the ſame manner then it may be ſuppoſed that the tail of a comet is raiſed by the ſun.

The tails therefore thus produced in the perihelions of comets, will go off along with their head into remote regions; and either return from thence, together with the comets, after a long ſeries of years; or rather be there loſt, and vaniſh by little and little, and the comet be left bare; till at its return, deſcending towards the ſun, ſome ſhort tails are again gradually produced from the head; which afterwards, in the perihelion, deſcending down into the ſun's atmoſphere, will be immenſely increaſed.

Newton farther obſerves, that the vapours, when thus dilated, rarefied, and diffuſed through all the celeſtial regions, may probably, by means of their own gravity, be gradually attracted down to the planets, and become intermingled with their atmoſpheres. He adds that this intermixture may be uſeful and neceſſary for the conſervation of the water and moiſture of the planets, dried up or conſumed in various ways. And I ſuſpect, adds our author, that the ſpirit, which makes the fineſt, ſubtileſt, and beſt part of our air, and which is abſolutely requiſite for the life and being of all things, comes principally from the comets.—On this principle there may ſeem to be ſome foundation for the popular opinion of preſages from comets; ſince the tail of a comet thus intermingled with our atmoſphere, may produce changes very ſenſible in animal and vegetable bodies.

It may here be added that another uſe which Newton conjectures comets may be deſigned to ſerve, is that of recruiting the ſun with freſh fuel, and repairing the conſumption of his light by the ſtreams continually ſent forth in every direction from that luminary. In ſupport of this conjecture he obſerves, that comets in their perihelion may ſuffer a diminution of their projectile force, by the reſiſtance of the ſolar atmoſphere; ſo that by degrees their gravitation towards the ſun may be ſo far increaſed, as to precipitate their fall into his body.

Other opinions on the tails of comets, are the following.

Apian, Tycho Brahe, and ſome others, think they were produced by the ſun's rays tranſmitted through the nucleus of the comet, which they ſuppoſed was tranſparent, and there refracted as in a glaſs lens, ſo as to form a beam of light behind the comet. Des Cartes accounted for the phenomenon of the tail by the refraction of light from the head of the comet to the ſpectator's eye. Mairan ſuppoſes that the tails are formed out of the luminous matter compoſing the ſun's atmoſphere: and M. De la Lande combines this hypotheſis with that of Newton recited above. But Mr. Rowning, not ſatisfied with Newton's opinion, accounts for the tails of comets in the following manner: It is well known, ſays he, that when the ſun's light paſſes through the atmoſphere of any body, as the earth, that which paſſes on one ſide, is by the refraction made to converge towards that which paſſes on the oppoſite ſide; and this convergency is not wholly effected either at the entrance of the light into the atmoſphere, or at its exit on going out; but beginning at its entrance, it increaſes in every point of its progreſs: It is alſo agreed that the atmoſpheres of the comets are very large and denſe: he therefore ſuppoſes that by ſuch time as the light of the ſun has paſſed through a conſiderable part of the atmoſphere of the comet, the rays are ſo far refracted towards each other, that they then begin ſenſibly to illuminate it, or rather the vapours floating in it; and ſo render that part they have yet to paſs through, viſible to us: and that this portion of the atmoſphere of a comet thus illuminated, appears to us in form of a beam of the ſun's light, and paſſes under the denomination of a comet's tail. Rowning's Nat. Philoſ. part 4. chap. 11.

M. Euler, Mem. Berlin tom. 2. pa. 117, thinks there is a great affinity between the tails of comets, the zodiacal light, and the aurora borealis, and that the common cauſe of all of them, is the action of the ſun's light on the atmoſpheres of the comets, of the ſun, and of the earth. He ſuppoſes that the impulſe of the rays of light on the atmoſphere of comets, may drive ſome of the finer particles of that atmoſphere far beyond its limits; and that this force of impulſe combined with that of gravity towards the comet, would produce a tail, which would always be in oppoſition to the ſun, if the comet did not move. But the motion of the comet in its orbit, and about an axis, muſt vary the poſition and figure of the tail, giving it a curvature, and deviation from a line joining the centres of the ſun and comet; and that this deviation will be greater, as the orbit of the comet has the greater curvature, and as the motion of the comet is more rapid. It may even happen, that the velocity of the comet, in its perihelion, may be ſo great, that the force of the ſun's rays may produce a new tail, before the old one can follow; in which caſe the comet might have two or more tails. The poſſibility of this is confirmed by the comet of 1744, which was obſerved to have ſeveral tails while it was in its perihelion.

Dr. Hamilton urges ſeveral objections againſt the Newtonian hypotheſis; and concludes that the tail of a comet is formed of matter which has not the power of refracting or reflecting the rays of light; but that it is a lucid or ſelf-ſhining ſubſtance: and from its ſimilarity to the Aurora borealis, that it is produced by the ſame cauſe, and is properly an electrical phenomenon. Dr. Halley too ſeemed inclined to this hypotheſis, when he ſaid, that the ſtreams of light in an Aurora borealis ſo much reſembled the long tails of comets, that at firſt ſight they might well be taken for ſuch: and that this light ſeems to have a greater affinity to that which the effluvia of electric bodies emit in the dark. Philoſ. Tranſ. N° 347. Hamilton's Philoſ. Eſſays, pa. 91.

The Motion of Comets.—If it be ſuppoſed that the paths of comets are perfectly parabolical, as ſome have imagined, it will follow that, being impelled towards the ſun by a centripetal force, they deſcend as from ſpaces infinitely diſtant; and that by their falls they acquire ſuch a velocity as will carry them off again into the remoteſt regions, never more to return. But the frequency of their appearance, and their degree of velocity, which does not exceed what they might acquire by their gravity towards the ſun, ſeem to put it paſt doubt that they move like the planets, in elliptic orbits, though exceedingly eccentric; and ſo return again after very long periods.

The apparent velocity of the comet of 1472, as obſerved by Regiomontanus, was ſuch as to carry it through 40° of a great circle in 24 hours: and it was obſerved that the comet of 1770 moved through more than 45° in the laſt 25 hours.

About the return of comets there have been different opinions. Newton, Flamſteed, Halley, and other Engliſh aſtronomers, ſeem ſatisfied of the return of comets: Caſſini and ſome of the French think it highly probable; but De la Hire and others oppoſe it. Thoſe on the affirmative ſide ſuppoſe that the comets deſcribe orbits prodigiouſly eccentric, inſomuch that we can ſee them only in a very ſmall part of their revolution: out of this, they are loſt in the immenſity of ſpace; hid not only from our eyes, but our teleſcopes: that little part of their orbit next us paſſing ſometimes within thoſe of all the inferior planets.

M. Caſſini gives the following reaſons in favour of the return of comets. 1. It is found that they move a conſiderable time in the arch of a great circle, when referred to the fixed ſtars, that is a circle whoſe plane paſſes through the centre of the earth; deviating but a little from it chiefly towards the end of their appearance; a deviation however common to them with the planets.—2. Comets, as well as planets, appear to move ſo much the faſter as they are nearer the earth; and when they are at equal diſtances from their perigee, their velocities are nearly the ſame. By ſubtracting from their motion the apparent inequality of velocity occaſioned by their different diſtance from the earth, their equal motion might be found: but we ſhould not ſtill be certain that this is their true motion; becauſe they might have conſiderable inequalities, not diſtinguiſhable in that ſmall part of their orbit viſible to us. It is rather probable that their real motion, as well as that of the planets, is unequal in itſelf; and hence we have a reaſon why the obſervations made during the appearance of a comet, cannot give the juſt period of their revolution.—3. There are no two different planets whoſe orbits cut the ecliptic in the ſame angle, whoſe nodes are in the ſame points of the ecliptic, and having the ſame apparent velocity in their perigee: conſequently, two comets ſeen at different times, yet agreeing in all thoſe three circumſtances, can only be one and the ſame comet. Not that this exact agreement, in theſe circumſtances, is abſolutely neceſſary to determine their identity: for the moon herſelf is irregular in all of them, ſo that it ſeems there may be caſes in which the ſame comet, at different periods of revolution, may diſagree in theſe points.

As to the objections againſt the return of comets, the principal is that of the rarity of their appearance, with regard to the number of revolutions aſſigned to them. In 1702 there was a comet, or rather the tail of one, ſeen at Rome, which M. Caſſini takes to be the ſame with that obſerved by Ariſtotle, and again lately in the year 1668; which would imply a period of 34 years: Now, it may ſeem ſtrange that a ſtar which has ſo ſhort a revolution, and of conſequence ſuch frequent returns, ſhould be ſo ſeldom ſeen. Again, in April of the ſame year 1702, a comet was obſerved by Meſſrs. Bianchini and Maraldi, which the latter ſuppoſed was the ſame with that of 1664, both on account of its motion, velocity, and direction. M. de la Hire thought it had ſome relation to another he had obſerved in 1698, which Caſſini refers to that of 1652; which would make it a period of 43 months, and the number of revolutions between 1652 and 1692, 14: now, it is hard to ſuppoſe, that in this age, when the heavens are ſo narrowly watched, a ſtar ſhould make 14 revolutions unperceived; eſpecially ſuch a ſtar as this, which might appear above a month together; and conſequently be often diſengaged from the crepuſcula. For this reaſon M. Caſſini is very reſerved in maintaining the hypotheſis of the return of comets, and only propoſes thoſe for planets where the motions are eaſy and ſimple, and are ſolved without ſtraining, or allowing any irregularities.

M. de la Hire propoſes one general difficulty againſt the whole ſyſtem of the return of comets, which would ſeem to prevent any comet from returning as a planet: which is this; that by the diſpoſition neceſſarily given to their courſes, they ought to appear as ſmall at firſt as at laſt; and always increaſe till they arrive at their neareſt proximity to the earth; or if they ſhould chance not to be obſerved, as ſoon as they are capable of being ſeen, it is yet hardly poſſible but they muſt often ſhew themſelves before they have arrived at their full magnitude and brightneſs: but, adds he, none were ever yet obſerved till they had arrived at it. However, the appearance of a comet in the month of October 1723, while at a great diſtance, ſo as to be too ſmall and dim to be viewed without a teleſcope, as well as the obſervations of ſeveral others ſince, may ſerve to remove this obſtacle, and ſet the comets ſtill on the ſame footing with the planets.

It is a conjecture of Newton, that as thoſe planets which are neareſt to the ſun, and revolve in the leaſt orbits, are the ſmalleſt; ſo among the comets, ſuch as in their perihelion come neareſt the ſun, are the ſmalleſt, and revolve in the leaſt orbits.

Of the Writings and Liſts of Comets.

There have been many writings upon the ſubject of comets, beſide the notices of hiſtorians as to the appearance of certain particular ones.

Regiomontanus firſt ſhewed how to find the magnitudes of comets, their diſtance from the earth, and their true place in the heavens. His 16 problems De Cometæ Magnitudine, Longitudine, ac Loco, are to be found in a book publiſhed in the year 1544, with the title of Scripta Joannis Regiomontani.

Peter Apian obſerved and wrote upon the comets of 1631, 1632, &c. Other writers are Tycho Brahe, in his Progymnaſmata Aſtronomiæ Inſtauratæ.—Kepler, of the comet in the year 1607, and de Cometis Libelli tres.—Ricciolus, in his Almageſtum Novum, publiſhed 1651, enumerates 154 comets cited by hiſtorians down to the year 1618.—Hevelius's Prodromus Cometicus, containing the hiſtory of the comet of the year 1664. Alſo his Cometographia.—Lubienietz, in a large folio work expreſsly on this ſubject, publiſhed 1667, extracts, with immenſe labour, from the paſſages of all hiſtorians, an account of 415 comets, ending with that of 1665.—Dr. Hook, in his Poſthumous works.—M. Caſſini's little Tract of Comets.—Sturmius's Diſſertatio de Cometarum Natura.—Newton, in his Principia, lib. 3; who firſt aſſigned their proper orbits, and by calculations compared the obſervations of the great comet of 1680 with his theory.—Dr. Halley, his Synopſis Cometica, in the Philoſ. Tranſ. number 218, &c; who computed the elements and orbits of 24 comets, and who firſt ventured to predict the return of one in 1759, which happened accordingly.—De la Lande, Théorie des Comètes, 1759; alſo, in his Aſtronomie, vol. 3.—Clairaut, Théorie du mouvement des Comètes, 1760.—D'Alembert, Opuſcules Mathématiques, vol. 2 pa. 97.—M. Albert Euler, 1762.—Séjour, Eſſai ſur les Comètes, 1775.—Beſides Boſcovich, De la Grange, De la Place, Friſi, Lexel, Barker, Hancocks, Cole, with many others.—And M. Pingré's Cométographie, in 2 vols. 4to, 1784; in which is contained the moſt ample liſt of ſuch comets as have been well obſerved, and their elements computed, to the number of 67. And accounts of a very few more that have been obſerved ſince that time, may be ſeen in the Mem. de l'Acad. and in the Philoſ. Tranſ.—And while this work is printing, there has juſt come out a very ingenious and ample work upon comets, by Sir Henry Englefield, entitled, "On the Determination of the Orbits of Comets."

The whole liſt of comets that have been noticed, on record, amount to upwards of 500, but the following is a complete liſt of all that have been properly obſerved, and their elements computed, the mean diſtance of the earth from the ſun being 100000.

TABLE OF THE ELEMENTS OF COMETS.

Year	Aſcending Node	Inclin. of Orbit	Perihelion	Perihel. Diſt.	Time of Perihelion	Motion	Calculated by
	s ° ′	° ′	s ° ′		d h		
837	6 26 33	11 abt.	9 19 3	58000	March 1 0	Ret.	Pingré
1231	0 13 30	6 5	4 14 48	94776	Jan. 30 7	Dir.	Pingré
1264	5 19 0	36 30	9 21 0	44500	July 6 8	Dir.	Duntherne
	5 28 45	30 25	9 5 45	41081	17 6	Dir.	Pingré
1299	3 17 8	68 57	0 3 20	31793	March 31 8	Ret.	Pingré
1301	0 15 about	70 abt.	9 about	45700	Oct. 22 about	Ret.	Pingré
1337	2 24 21	32 11	1 7 59	40666	June 2 7	Ret.	Halley
	2 6 22	32 11	0 20 0	64450	1 1	Ret.	Pingré
1456	1 18 30	17 56	10 1 0	58550	June 8 22	Ret.	Pingré
1472	9 11 46	5 20	1 15 34	54273	Feb. 28 23	Ret.	Halley

Year	Aſcending Node	Inclin. of Orbit	Perihelion	Perihel. Diſt.	Time of Perihelion	Motion	Calculated by
	s ° ′	° ′	s ° ′		d h		
1531	1 19 25	17 56	10 1 39	56700	Aug. 24 21	Ret.	Halley
1532	2 20 27	32 36	3 21 7	50910	Oct. 19 22	Dir.	Halley
1533	4 5 44	35 49	4 27 16	20280	June 16 20	Ret.	Douwes
1556	5 25 42	32 7	9 8 50	66390	April 21 20	Dir.	Halley
1577	0 25 52	74 33	4 9 22	18342	Oct. 26 19	Ret.	Halley
1580	0 18 57	64 40	3 19 6	59628	Nov. 28 15	Dir.	Halley
	0 19 8	64 52	3 19 12	59553	28 14	Dir.	Pingré
1582	7 21 7	61 28	8 5 23	22570	May 6 16	Ret.	Pingré
	7 4 43	59 29	9 11 27	4006	7 9	Ret.	Pingré
					NEW STYLE		
1585	1 7 43	6 4	0 8 51	109358	Oct. 7 20	Dir.	Halley
1590	5 15 31	29 41	7 6 55	57661	Feb. 8 4	Ret.	Halley
1593	5 14 15	87 58	5 26 19	8911	July 18 14	Dir.	La Caille
1596	10 12 13	55 12	7 18 16	51293	Aug. 10 20	Ret.	Halley
	10 15 37	52 10	7 28 31	54942	8 16	Ret.	Pingré
1607	1 20 21	17 2	10 2 16	58680	Oct. 16 4	Ret.	Halley
1618	9 23 25	21 28	10 18 20	51298	Aug. 17 3	Dir.	Pingré
1618	2 16 1	37 34	0 2 14	37975	Nov. 8 13	Dir.	Halley
1652	2 28 10	79 28	0 28 19	84750	Nov. 12 16	Dir.	Halley
1661	2 22 31	32 36	3 25 59	44851	Jan. 26 24	Dir.	Halley
1664	2 21 14	21 19	4 10 41	102575	Dec. 4 12	Ret.	Halley
1665	7 18 2	76 5	2 11 55	10649	April 24 5	Ret.	Halley
1672	9 27 31	83 22	1 17 0	69739	March 1 9	Dir.	Halley
1677	7 26 49	79 3	4 17 37	28059	May 6 1	Ret.	Halley
1678	5 11 40	3 4	10 27 46	123801	Aug. 26 14	Dir.	Douwes
1680	9 2 2	60 56	8 22 40	612½	Dec. 18 0	Dir.	Halley
	9 2 2	61 7	8 22 44	617	17 23	. . .	Halley
	9 2 59	58 40	8 23 27	656	17 21	. . .	Euler
	9 1 53	61 20	8 23 43	592	18 0	. . .	Newton
	9 1 57	61 23	8 22 40	603	18 0	. . .	Pingré
1682	1 21 17	17 56	10 2 53	58328	Sept. 14 8	Ret.	Halley
	1 20 48	17 42	10 1 36	58250	14 22	. . .	Halley
1683	5 23 23	83 11	2 25 30	56020	July 13 3	Ret.	Halley
1684	8 28 15	65 49	7 28 52	96015	June 8 10	Dir.	Halley
1686	11 20 35	31 22	2 17 1	32500	Sept. 16 15	Dir.	Halley
1689	10 23 45	69 17	8 23 45	1689	Dec. 1 15	Ret.	Pingré
1698	8 27 44	11 46	9 0 51	69129	Oct. 18 17	Ret.	Halley
1699	10 21 46	69 20	7 2 31	74400	Jan. 13 9	Ret.	La Caille
1702	6 9 25	4 30	4 18 41	64590	March 13 14	Dir.	La Caille
1706	0 13 12	55 14	2 12 29	42581	Jan. 30 5	Dir.	La Caille
	0 13 11	55 14	2 12 36	42686	30 5	. . .	Struyck
1707	1 22 8	88 50	2 17 4	86350		Dir.	Houtteryn
	1 22 47	88 36	2 19 55	85974	Dec. 12 0	. . .	La Caille
	1 22 50	88 38	2 19 58	85904	12 0	. . .	Struyck
1718	4 8 43	30 20	4 1 30	102655	Jan. 15 0	Ret.	La Caille
	4 7 55	31 13	4 1 27	102565	15 1	. . .	Douwes
1723	0 14 16	49 59	1 12 52	99865	Sept. 27 16	Ret.	Bradley
1729	10 10 35	77 2	10 22 17	406980	June 23 7	Dir.	Douwes
	10 10 33	76 58	10 22 40	426140	25 11	. . .	La Caille
	10 10 17	76 53	10 27 22	416927	23 0	. . .	Maraldi
	10 10 52	77 19	10 16 27	394927	May 22 11	. . .	Kies
	10 10 33	77 1	10 22 37	408165	June 25 9	. . .	De l'Iſle
1737	7 16 22	18 21	10 25 55	22282	Jan. 30 9	Dir.	Bradley
1739	6 27 18	55 53	3 12 34	67160	June 17 11	Ret.	Zanotti
	6 27 25	55 43	3 12 39	67358	17 10	. . .	La Caille
1742	6 5 35	67 4	7 7 33	76555	Feb. 8 5	Ret.	Struyck
	6 5 33		7 7 32	76550	8 4	. . .	Le Monnier
	6 5 38	66 59	7 7 35	76568	8 5	. . .	La Caille
	6 5 43	66 52	7 7 39	76530	8 8	. . .	Zanotti
	6 16 9	56 35	7 16 42	73766	1 22	. . .	Euler

Year	Aſcending Node			Inclin. of Orbit		Perihelion			Perihel. Diſt.	Time of Perihelion			Motion	Calculated by
	s	°	′	°	′	s	°	′			d	h		
1742	6	9	32	61	44	7	10	49	73668	Feb.	7	4	Ret.	Euler
	6	5	47	68	14	7	7	33	76890		7	22	. .	Wrigt
	6	5	29	67	11	7	7	26	76620		8	5	. . .	Klinkenberg
	6	5	42	66	51	7	7	38	76545		8	7	. . .	Houtteryn
1743	2	8	11	2	16	3	2	58	83811	Jan.	10	21	Dir.	Struyck
	2	8	21	2	20	3	2	42	83501		10	21	. . .	La Caille
1743	0	5	16	45	48	8	6	34	52057	Sept.	20	21	Ret.	Klinkenberg
1744	1	15	45	47	9	6	17	13	22206	March	1	8	Dir.	Bets and Bliſs
	1	15	47	47	4	6	17	6	22322		1	8	. . .	Maraldi
1744	1	15	46	47	5	6	17	10	22250	March	1	8	Dir.	La Caille
	1	15	51	47	18	6	17	18	22156		1	8	. . .	Zanotti
	1	16	5	47	50	6	17	19	22192		1	9	. . .	Chéſeaux
	1	15	46	47	11	6	17	12	22222		1	8	. . .	Euler
	1	15	48	47	8	6	17	13	22223		1	8	. . .	Pingré
	1	15	49	47	18	6	17	15	22200		1	8	. . .	Klinkenberg
	1	15	49	47	14	6	17	16	22176		1	9	. . .	Hiorter
1747	4	26	58	77	57	9	10	6	229388	Feb.	28	12	Ret.	Cheſeaux
	4	27	19	79	7	9	7	2	219859	March	3	10	. . .	Maraldi
	4	27	19	79	6	9	7	2	219851		3	7	. . .	La Caille
1748	7	22	52	85	27	7	5	1	84066	April	28	20	Ret.	Maraldi
	7	22	46	85	35	7	4	39	84150		29	1	. . .	Le Monnier
	7	22	52	85	28	7	5	23	84040		28	19	. . .	La Caille
1748	1	4	40	56	59	9	6	9	65525	June	18	2	Dir.	Struyck
1757	7	4	13	12	50	4	2	58	33754	Oct.	21	8	Dir.	Bradley
	7	4	6	12	39	4	2	39	33907		21	10	. . .	La Caille
	7	4	4	12	48	4	2	49	33797		21	10	. . .	Pingré
	7	4	7	12	41	4	2	36	33932		21	9	. . .	De Ratte
1758	7	20	50	68	19	8	27	38	21535	June	11	3	Dir.	Pingré
1759	1	23	48	17	38	10	3	14	58255	March	12	14	Ret.	Meſſier
	1	23	46	17	40	10	3	8	58490		12	14	. . .	De la Lande
	1	23	49	17	35	10	3	16	58360		12	13	. . .	Maraldi
	1	23	49	17	38	10	3	16	58380		12	$13\frac{1}{2}$	. . .	La Caille
	1	23	49	17	39	10	3	16	58350		12	$13\frac{2}{3}$	. . .	La Caille
	1	23	46	17	40	10	3	19	58298		12	13	. . .	Klinkenberg
	1	24	7	17	29	10	1	0	59708		13	10	. . .	Klinkenberg
	1	23	45	17	41	10	3	23	58234		12	13	. . .	Bailly
1759	4	19	40	79	7	1	23	34	80139	Nov.	27	0	Dir.	Pingré
	4	19	39	78	59	1	23	24	79851		27	2	. . .	La Caille
	4	19	40	79	3	1	23	38	80208		27	1	. . .	Chappe
1759	2	19	51	4	52	4	18	25	96599	Dec.	16	21	Ret.	La Caille
1759	2	19	20	4	42	4	19	4	96180	Dec.	16	13	Ret.	Chappe
1762	11	18	56	85	22	3	15	22	101415	May	29	0	Dir.	Maraldi
	11	19	20	84	45	3	15	15	101249		28	15	. . .	De la Lande
	11	18	58	85	12	3	14	24	101065		29	2	. . .	Bailly
	11	18	35	85	40	3	13	43	100686		28	2	. . .	Klinkenberg
	11	19	2	85	3	3	14	30	100986		28	7	. . .	Struyck
1763	11	26	23	72	41	2	24	52	49876	Nov.	1	20	Dir.	Pingré
1764	4	0	5	52	54	0	15	15	55522	Feb.	12	14	Ret.	Pingré
1766	8	4	11	40	50	4	23	15	50532	Feb.	17	9	Ret.	Pingré
1766	2	14	23	11	8	8	2	18	33275	April	22	21	Dir.	Pingré
1769	5	25	1	40	38	4	24	6	12376	Oct.	7	13	Dir.	De la Lande
	5	25	2	40	43	4	24	14	12287		7	12	. . .	Wallot
	5	25	3	40	47	4	24	11	12258		7	13	. . .	Caſſin, jun.
	5	25	7	40	49	4	24	11	12272		7	14	. . .	Proſperin
	5	25	3	40	41	4	24	9	12289		7	14	. . .	Audiffrédi
	5	25	11	41	1	4	24	33	12100		7	12	. . .	Slop
	5	19	41	29	41	4	13	15	15880		16	10	. . .	Zanotti
	5	25	5	40	41	4	24	7	12308		7	14	. . .	Aſclépi
	5	24	42	41	28	4	25	46	11640		7	11	. . .	Lambert
	5	25	14	40	43	4	24	22	12280		7	18	. . .	Widder

Year	Aſcending Node			Inclin. of Orbit		Perihelion			Perihel. Dist.	Time of Perihelion			Motion	Calculated by
	s	°	′	°	′	s	°	′			d	h		
1769	5	25	3	40	50	4	24	16	12264	Oct.	7	15	Dir.	Euler
	5	25	5	40	50	4	24	11	12269		7	16	. . .	Lexell
	5	25	6	40	47	4	24	16	12274		7	16	. . .	Pingré
1770	4	16	39	1	44	11	26	7	62959	Aug.	9	0	Dir.	Pingré
	4	13	39	1	41	11	25	5	65800		10	22	. . .	Pingré
	4	15	29	1	47	11	26	7	62955		9	0	. . .	Proſperin
	4	15	4	1	45	11	22	51	64456		8	9	. . .	Proſperin
	4	14	30	1	23	0	7	14	71717		25	2	. . .	Proſperin
	4	12	56	1	46	11	29	45	64946		12	21	. . .	Widder
1770	4	12	0	1	34	11	26	16	67438	Aug.	13	13	Dir.	Lexell
	4	12	17	1	35	11	26	26	67689		14	0	. . .	Pingré
	4	16	14	1	45	11	26	13	62872		9	1	. . .	Slop
	4	12	0	1	55	11	25	57	63100		9	4	. . .	Lambert
	4	14	22	1	49	11	26	19	62758		8	19	. . .	Rittenhouſe
1770	3	18	42	31	26	6	28	23	52824	Nov.	22	6	Ret.	Pingré
1771	0	27	51	11	15	3	13	28	90576	April	18	22	Dir.	Pingré
	0	27	50	11	17	3	13	48	90188		19	1	. . .	Proſperin
1772	8	12	43	19	0	3	18	6	101814	Feb.	18	21	Dir.	La Lande
1773	4	1	16	61	25	2	15	36	113390	Sept.	5	11	Dir.	Pingré
	4	3	15	62	33	2	21	40	123800		2	12	. . .	Lambert
	4	3	35	62	36	2	20	43	121550		2	19	. . .	Schultz
	4	1	12	61	21	2	15	16	113010		5	6	. . .	Lexell
	4	1	5	61	13	2	14	58	112530		5	9	. . .	Pingré
1774	6	0	57	82	48	10	16	28	142525	Aug.	14	4	Dir.	De Saron
	6	0	50	82	49	10	16	48	142525		14	18	. . .	De Saron
	6	1	22	82	21	10	17	26	142600		15	5	. . .	Boſcowich
	6	0	50	83	0	10	17	22	142860		15	11	. . .	Mechain
1779	0	25	3	32	26	2	27	14	71322	Jan.	4	3	Dir.	De Saron
	0	25	6	32	24	2	27	13	71313		4	2	. . .	Mechain
	0	25	4	32	26	2	27	14	71319		4	2	. . .	D'Angos
1780	4	4	0	53	56	8	6	30	9781	Sept.	30	20	Ret.	Lexell
	4	4	30	53	15	8	6	19	10047		30	16	. . .	Lexell
	4	4	9	53	48	8	6	21	9926		30	18	. . .	Mechain
1781	2	25	13	5	16	5	22	17	1027558	March	13	0	Dir.	Boſcowich
						5	28	13	944040	Jan.	27	6	. . .	La Place
1781	2	23	1	81	43	7	29	11	77586	July	7	5	Dir.	Mechain
1781	2	17	23	27	13	0	16	3	96101	Nov.	20	13	Ret.	Mechain

M. Facio has ſuggeſted, that ſome of the comets have their nodes ſo very near the annual orbit of the earth, that if the earth ſhould happen to be found in that part next the node at the time of a comet's paſſing by; as the apparent motion of the comet will be immenſely ſwift, ſo its parallax will become very ſenſible; and its proportion to that of the ſun will be given: whence, ſuch tranſits of comets will afford the beſt means of determining the diſtance between the earth and ſun.

The comet of 1472, for inſtance, had a parallax above 20 times greater than the ſun's: and if that of 1618 had come down in the beginning of March to its deſcending node, it would have been much nearer the earth, and its parallax much more notable. But hitherto none has threatened the earth with a nearer appulſe than that of 1680: for, Dr. Halley finds, by calculation, that Nov. 11th, at 1 h. 6 min. afternoon, that comet was not more than one ſemidiameter of the earth to the northward of the earth's path; at which time had the earth been in that part of its orbit, the comet would have had a parallax equal to that of the moon.—What might have been the conſequence of ſo near an appulſe, a contact, or laſtly, a ſhock of theſe bodies? Mr. Whiſton ſays, a deluge!

To determine the Place and Courſe of a Comet.—Obſerve the diſtance of the comet from two fixed ſtars, whoſe longitudes and latitudes are known: then from the diſtances thus known, calculate the place of the comet by ſpherical trigonometry.

Longomontanus ſhews an eaſy method of finding and tracing out the places of a comet mechanically, which is, to find two ſtars in the ſame line with the comet, by ſtretching a thread before the eye over all the three; then do the ſame by two other ſtars and the comet: this done, take a celeſtial globe, or a planiſphere, and draw a line upon it firſt through the former two ſtars, and then through the latter two; ſo ſhall the interſection of the two lines be the place of the comet at that time. If this be repeated from time to time, and all the points of interſection connected, it will ſhew the path of the comet in the heavens.

COMETARIUM, a machine adapted to give a repreſentation of the revolution of a comet about the ſun.

It is ſo contrived as, by elliptical wheels, to ſhew the unequal motion of a comet in every part of its orbit. The comet is repreſented by a ſmall braſs ball, carried by a radius vector, or wire, in an elliptical groove about the ſun in one of its foci; and the years of its period are ſhewn by an index moving with an equable motion over a graduated ſilvered circle. See a particular deſcription, with a figure of it, in Ferguſon's Aſtron. 8vo. pa. 400.

COMMANDINE (Frederick), a celebrated mathematician and linguiſt, was born at Urbino in Italy, in 1509; and died in 1575; conſequently at 66 years of age. He was famous for his learning and knowledge in the ſciences. To a great depth, and juſt taſte in the mathematics, he joined a critical ſkill in the Greek language; a happy conjunction which made him very well qualified for tranſlating and expounding the writings of the Greek mathematicians. And accordingly, with a moſt laudable zeal and induſtry, he tranſlated and publiſhed ſeveral of their works, to which no former writer had done that good office. On which account, Francis Moria, duke of Urbino, who was very converſant in thoſe ſciences, proved a very affectionate patron to him. He is greatly applauded by Bianchanus, and other writers; and he juſtly deſerved their encomiums.

Of his own works Commandine publiſhed the following:

1. Commentarius in Planiſphærium Ptolomæi: 1558, in 4to.
2. De Centro Gravitatis Solidorum: Bonon. 1565, in 4to.
3. Horologiorum Deſcriptio: Rom. 1562, in 4to.

He tranſlated and illuſtrated with notes the following works, moſt of them beautifully printed, in 4to. by the celebrated printer Aldus:

1. Archimedis Circuli Dimenſio; de Lineis Spiralibus; Quadratura Parabolæ; de Conoidibus & Sphæroidibus; de Arenæ Numero: 1558.
2. Ptolomæi Planiſphærium; & Planiſphærium Jordani: 1558.
3. Ptolomæi Analemma: 1562.
4. Archimedis de iis quæ vehuntur in aqua: 1565.
5. Apollonii Pergæi Conicorum libri quatuor, una cum Pappi Alexandrini Lemmatibus, & Commentariis Eutocii Aſcalonitæ, &c: 1566.
6. Machometes Bagdadinus de Superficierum Diviſionibus: 1570.
7. Elementa Euclidis: 1572.
8. Ariſtarchus de Magnitudinibus & Diſtantiis Solis & Lunæ: 1572.
9. Heronis Alexandrini Spiritualium liber: 1583.
10. Pappi Alexandrini Collectiones Mathematicæ: 1588.

COMMANDING *Ground*, in Fortification, an eminence, or riſing ground, overlooking any poſt or ſtrong place. This is of three ſorts. 1ſt, *A Front Commanding Ground*, or a height oppoſite to the face of the poſt, which plays upon its front. 2dly, *A Reverſe Commanding Ground*, or an eminence that can play upon the rear or back of the poſt. 3dly, *An Enfilade Commanding Ground*, or an eminence in flank which can, with its ſhot, ſcour all the length of a ſtraight line.

COMMENSURABLE *Quantities*, or *Magnitudes*, are ſuch as have ſome common aliquot part, or which may be meaſured or divided, without a remainder, by one and the ſame meaſure or diviſor, called their common meaſure. Thus, a foot and a yard are commenſurable, becauſe there is a third quantity that can meaſure each, viz an inch; which is 12 times contained in the foot, and 36 times in the yard.—Commenſurables are to each other, as one rational whole number is to another; but incommenſurables are not ſo: And therefore the ratio of commenſurables is rational; but that of incommenſurables, irrational: hence alſo the exponent of the ratio of commenſurables, is a rational number.

COMMENSURABLE *Numbers*, whether integers, or fractions, or ſurds, are ſuch as have ſome other number, which will meaſure or divide them exactly, or without a remainder. Thus, 6 and 8 are commenſurable, becauſe 2 meaſures or divides them both. And $\frac{2}{3}$ and $\frac{3}{4}$, or $\frac{8}{12}$ and $\frac{9}{12}$ are commenſurable fractions, becauſe the fraction $\frac{1}{6}$, or $\frac{1}{12}$, &c, will meaſure them both: and in this ſenſe, all fractions may be ſaid to be commenſurable. Alſo, the ſurds $2\sqrt{2}$ and $3\sqrt{2}$ are commenſurable, being meaſured by $\sqrt{2}$, or being to each other as 2 to 3.

Commensurable *in Power*. Euclid ſays, right lines are commenſurable in power, when their ſquares are meaſured by one and the ſame ſpace or ſuperficies.

COMMON, is applied to an angle, line, meaſure, or the like, that belongs to two or more figures, or other things. As, a common angle, a common ſide, a common baſe, a common meaſure, &c.

Common Measure, or diviſor, is that which meaſures two or more things without a remainder. So of 8 and 12, a common meaſure is 2, and ſo is 4.

The greateſt Common *Meaſure*, is the greateſt number that can meaſure two other numbers. So, of 8 and 12, the greateſt common meaſure is 4.

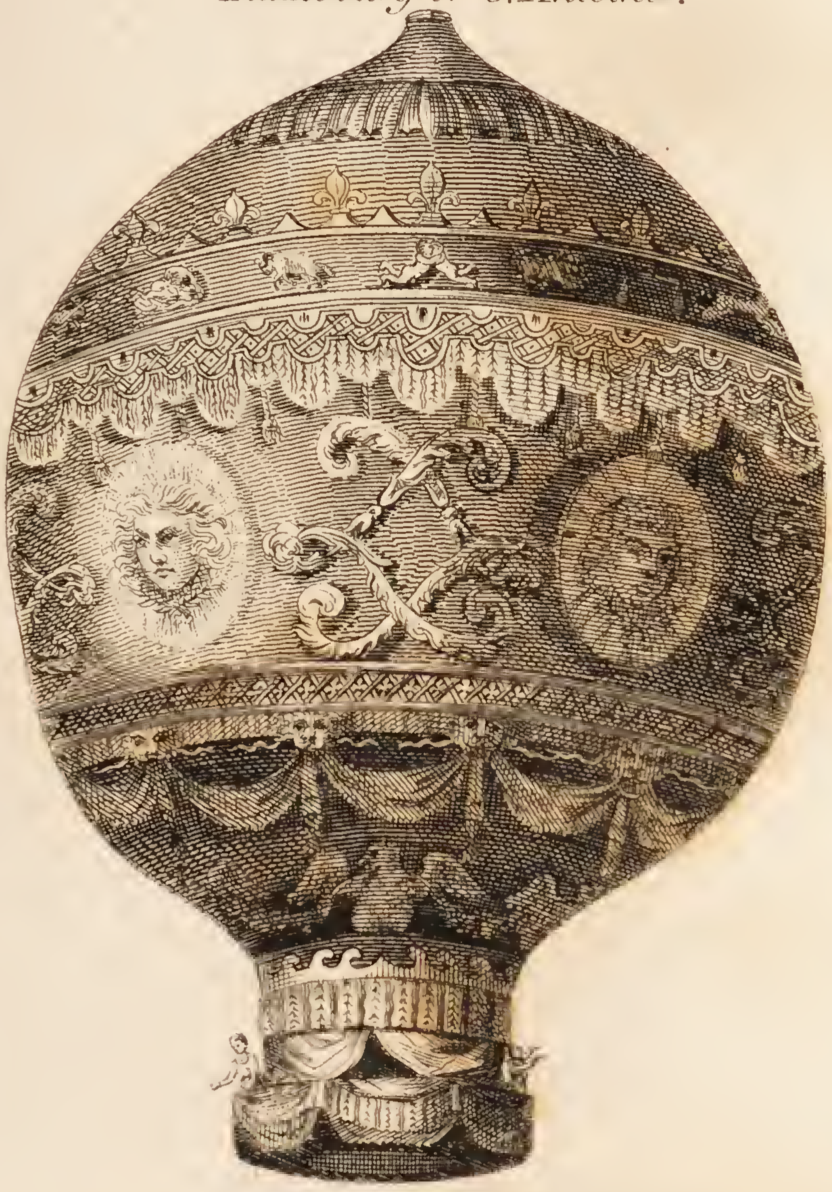
To find the greateſt common meaſure of two numbers. Divide the greater term by the leſs; then divide the diviſor by the remainder, if there be any; and ſo on continually, always dividing the laſt diviſor by the laſt remainder, till nothing remains; and then is the laſt diviſor the greateſt common meaſure ſought.

Thus, to find the greateſt common meaſure of 816 and 1488.

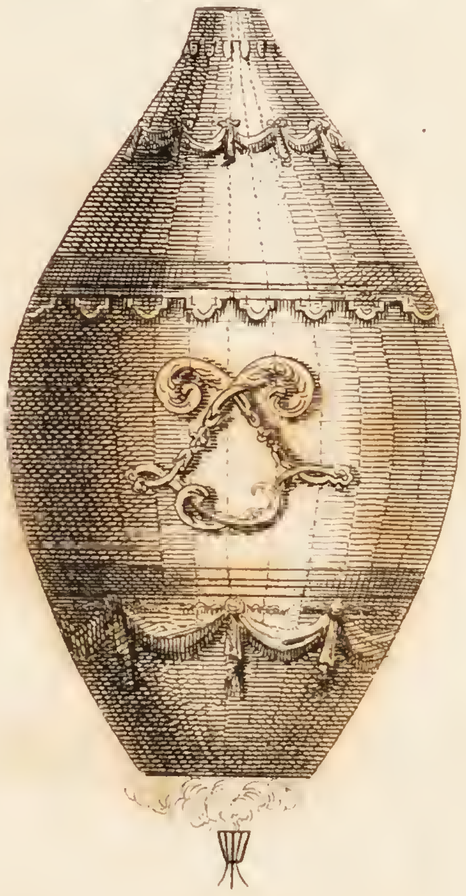
```
816)1488(1
     816
     ———
     672)816(1
         672
         ———
         144)672(4
             576
             ———
              96)144(1
                  96
                  ——
the common meaſ. 48)96(2
                    96
                    ——
                     0
                    ——
```

AIR-BALLOONS.

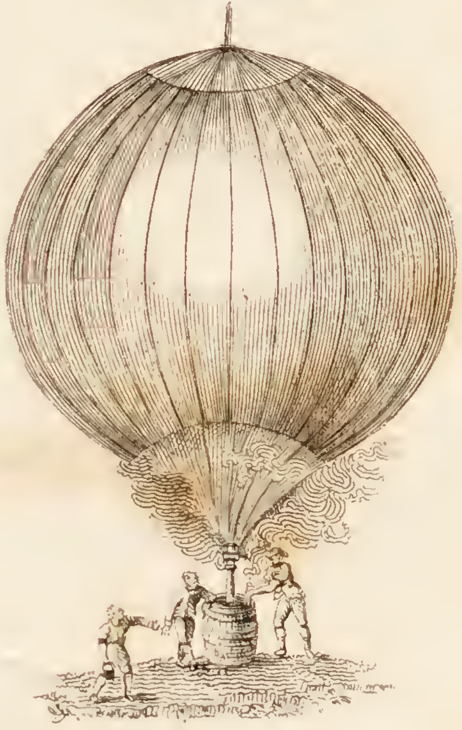
Montgolfier's.
Fauxbourg of S.t Antoine.



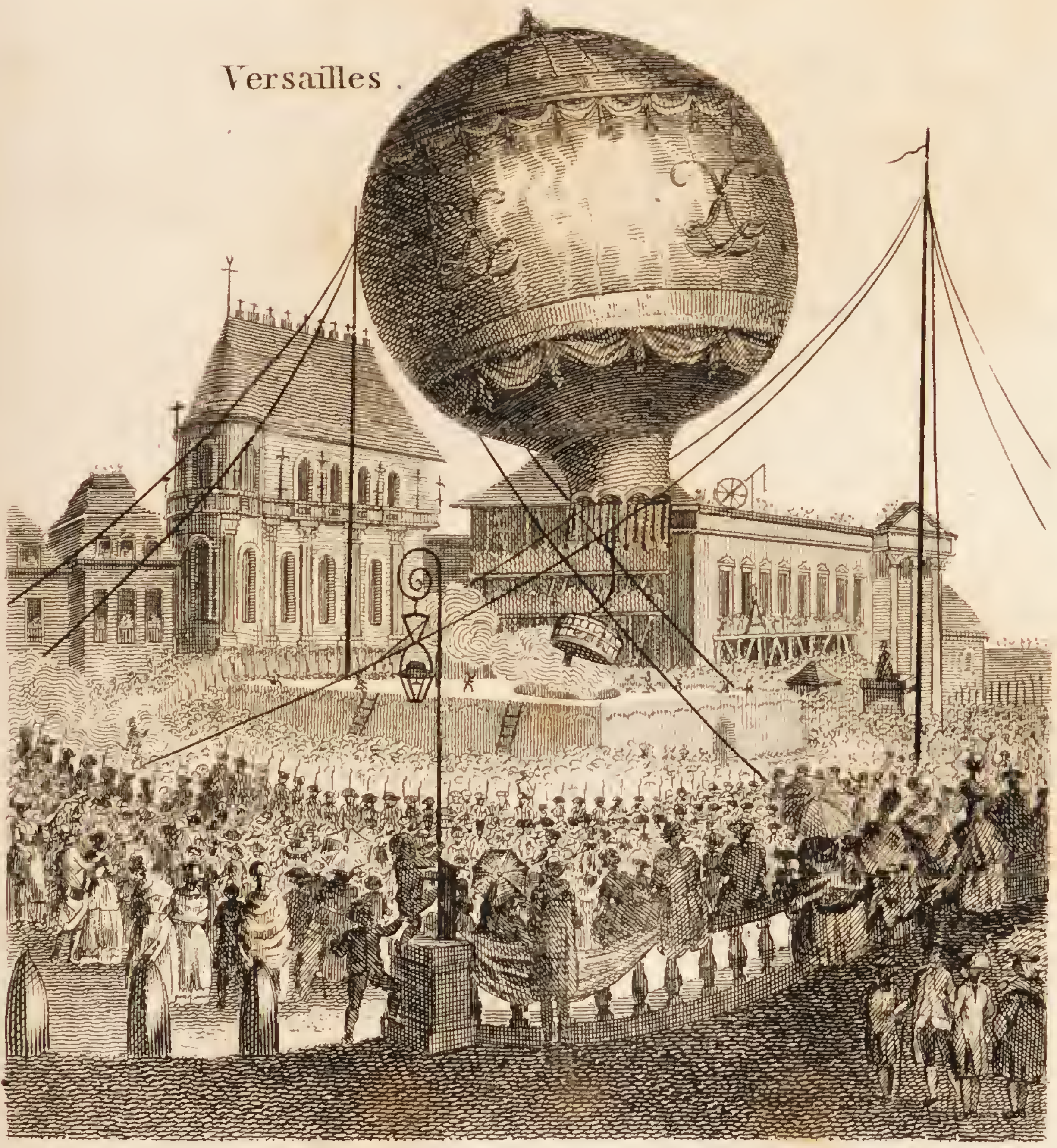
Montgolfier's.
Fauxbourg of S.t Germain.



Charles & Robert's.
Champ de Mars.



Versailles.



Blanchard's.

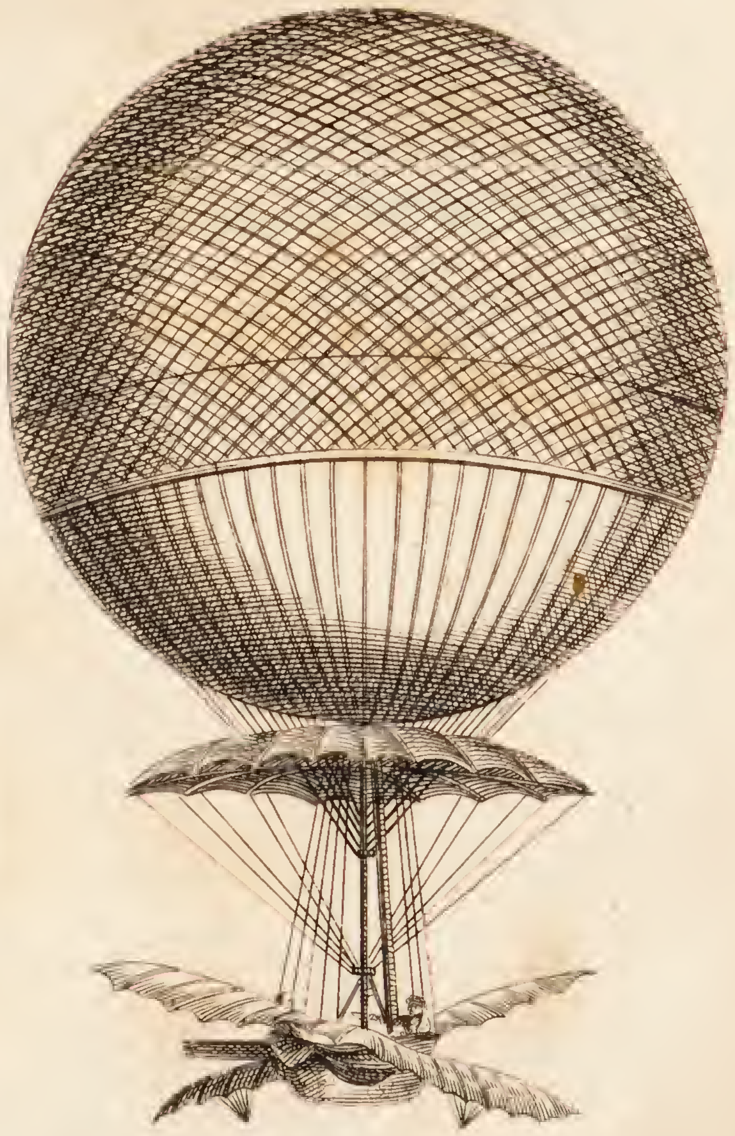


Fig. 1. AIR GUN.

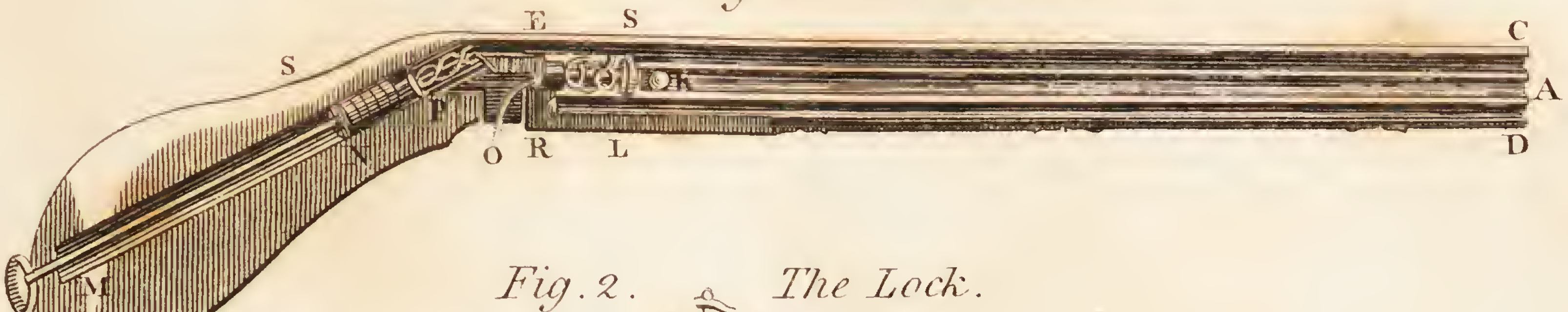


Fig. 2. The Lock.

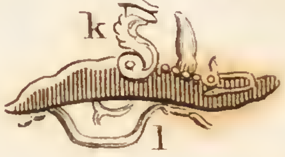


Fig. 3. Improved AIR GUN.

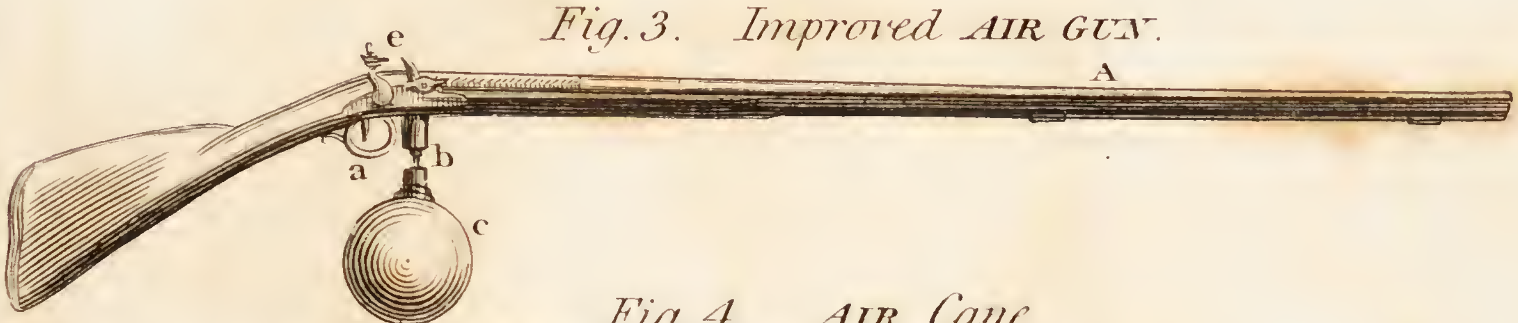


Fig. 4. AIR Cane.

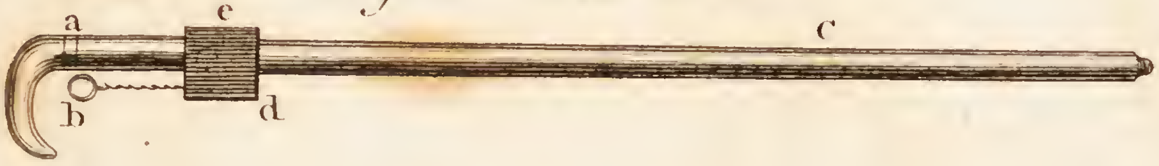
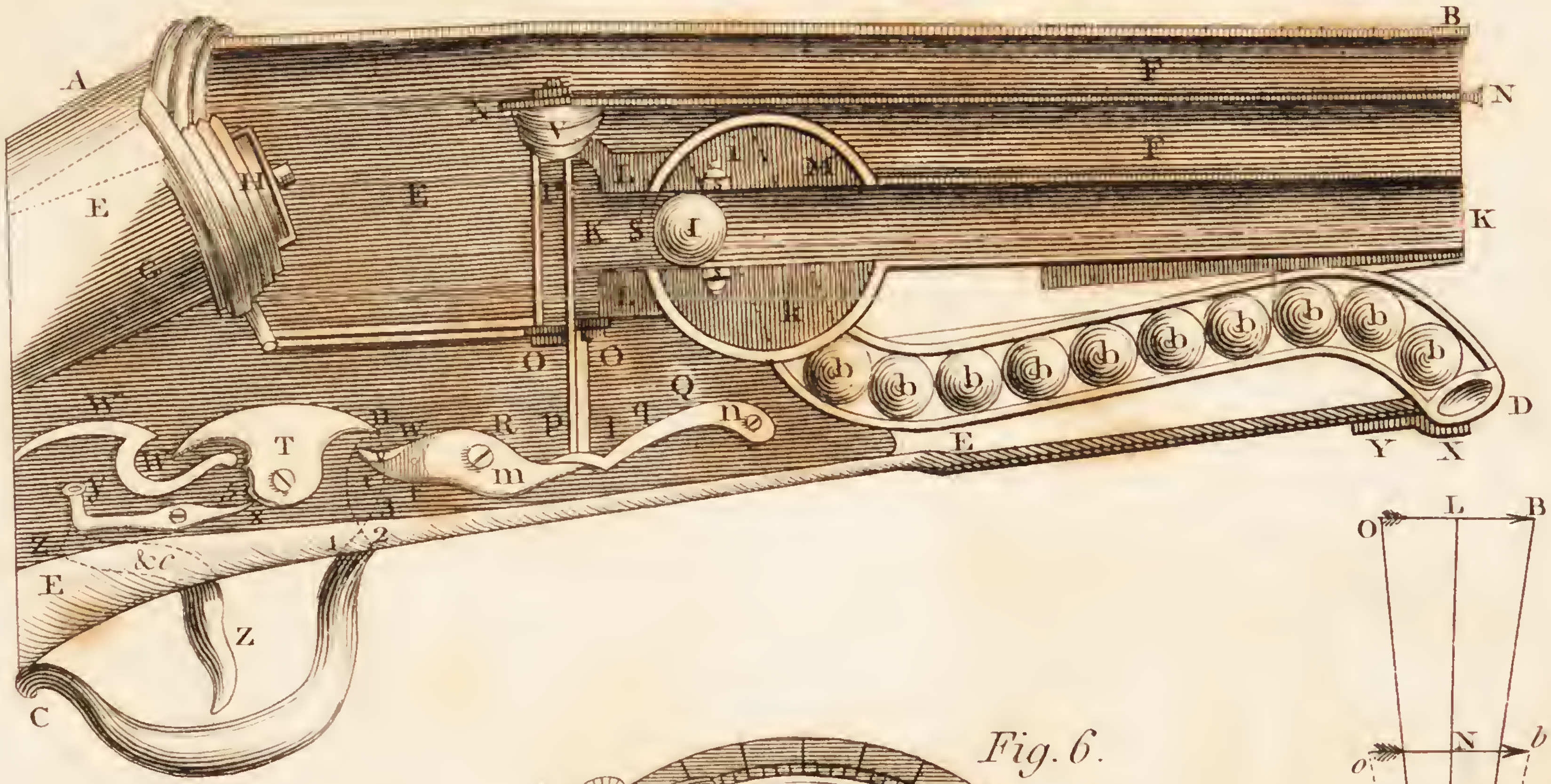


Fig. 5. Magazine AIR GUN.



ASTROLABE,

Fig. 7.

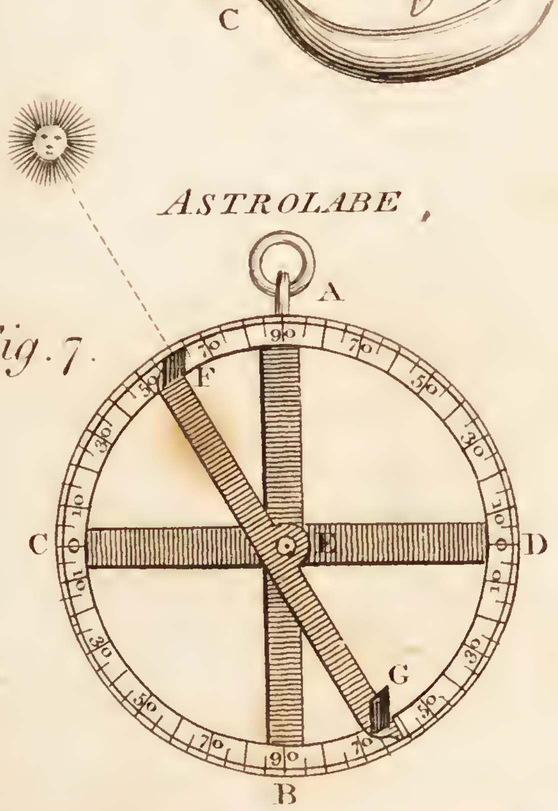


Fig. 6.

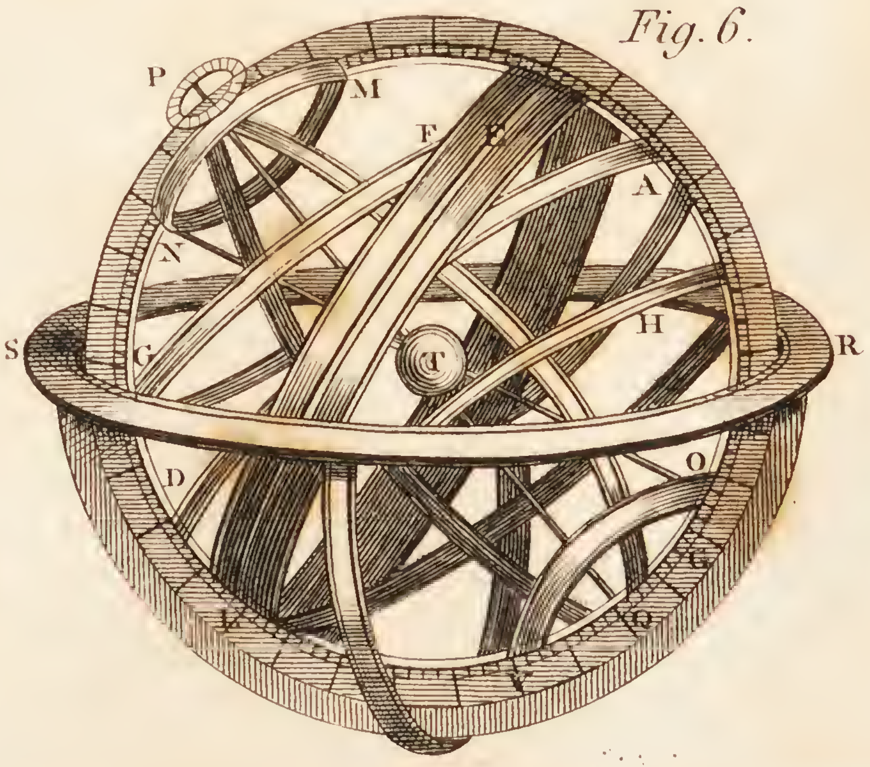
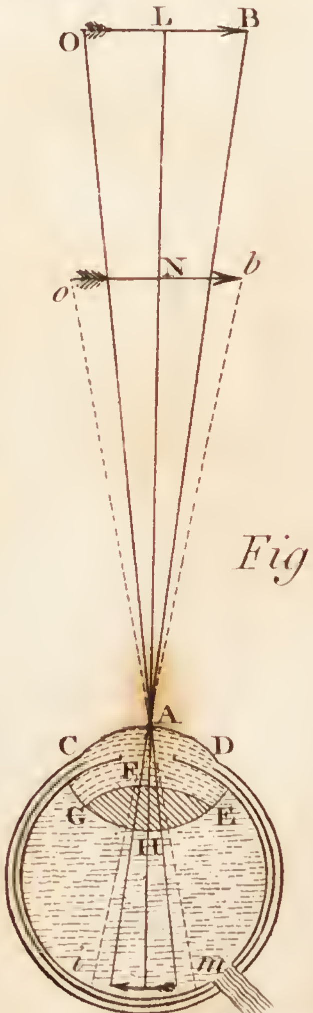


Fig. 8.



AIR PUMPS.

Fig. 1

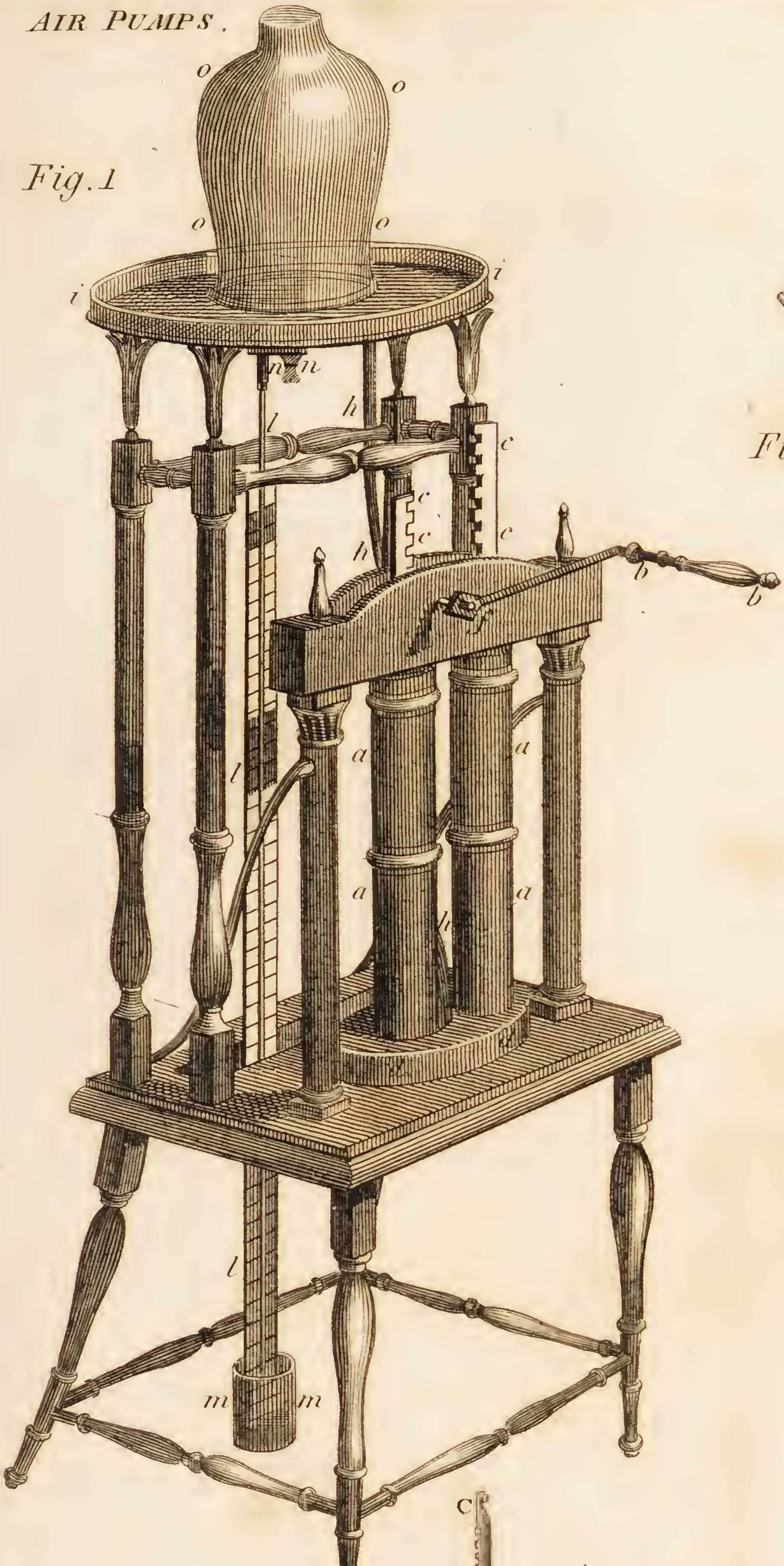
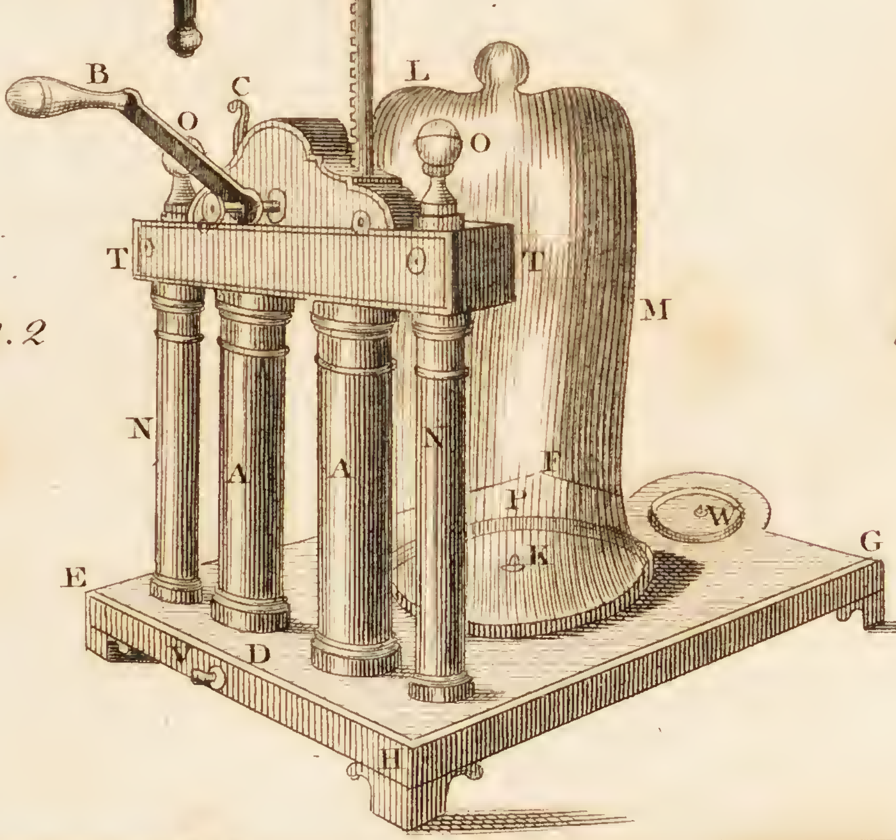


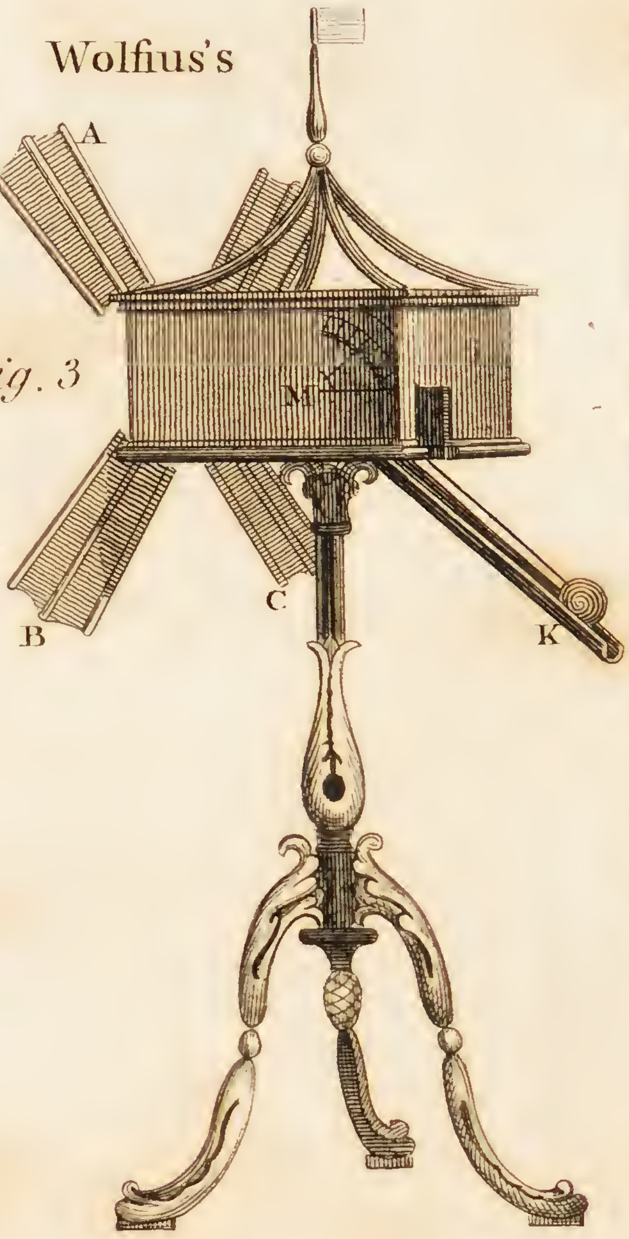
Fig. 2



ANEMOMETERS.

Wolfius's

Fig. 3



Lind's.

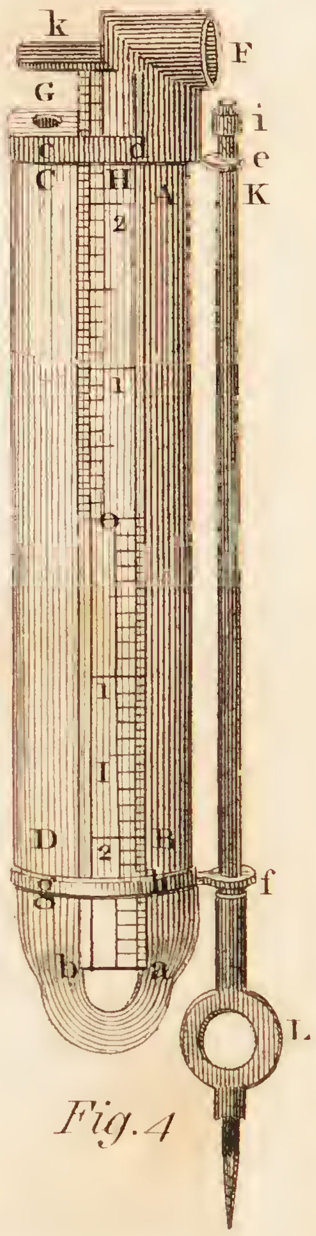
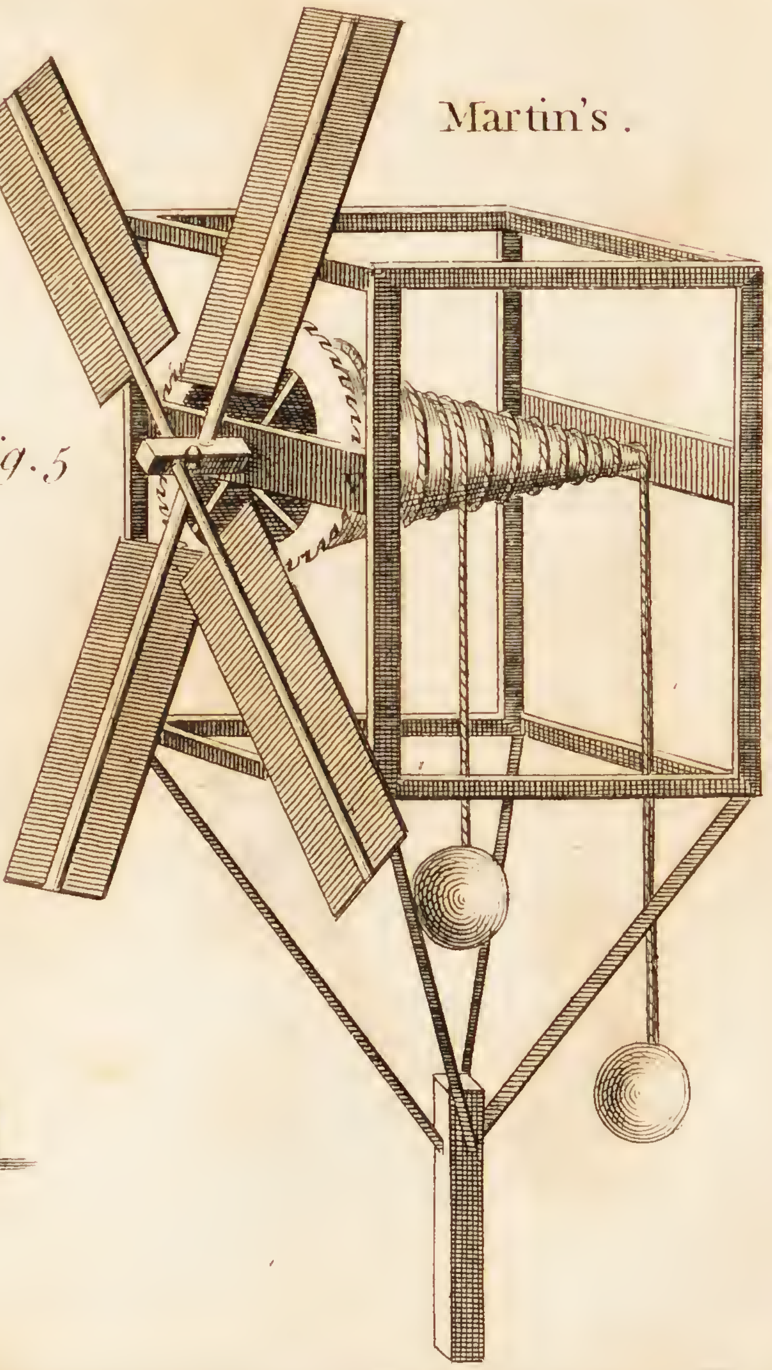


Fig. 4

Martin's.

Fig. 5



BAROMETERS.

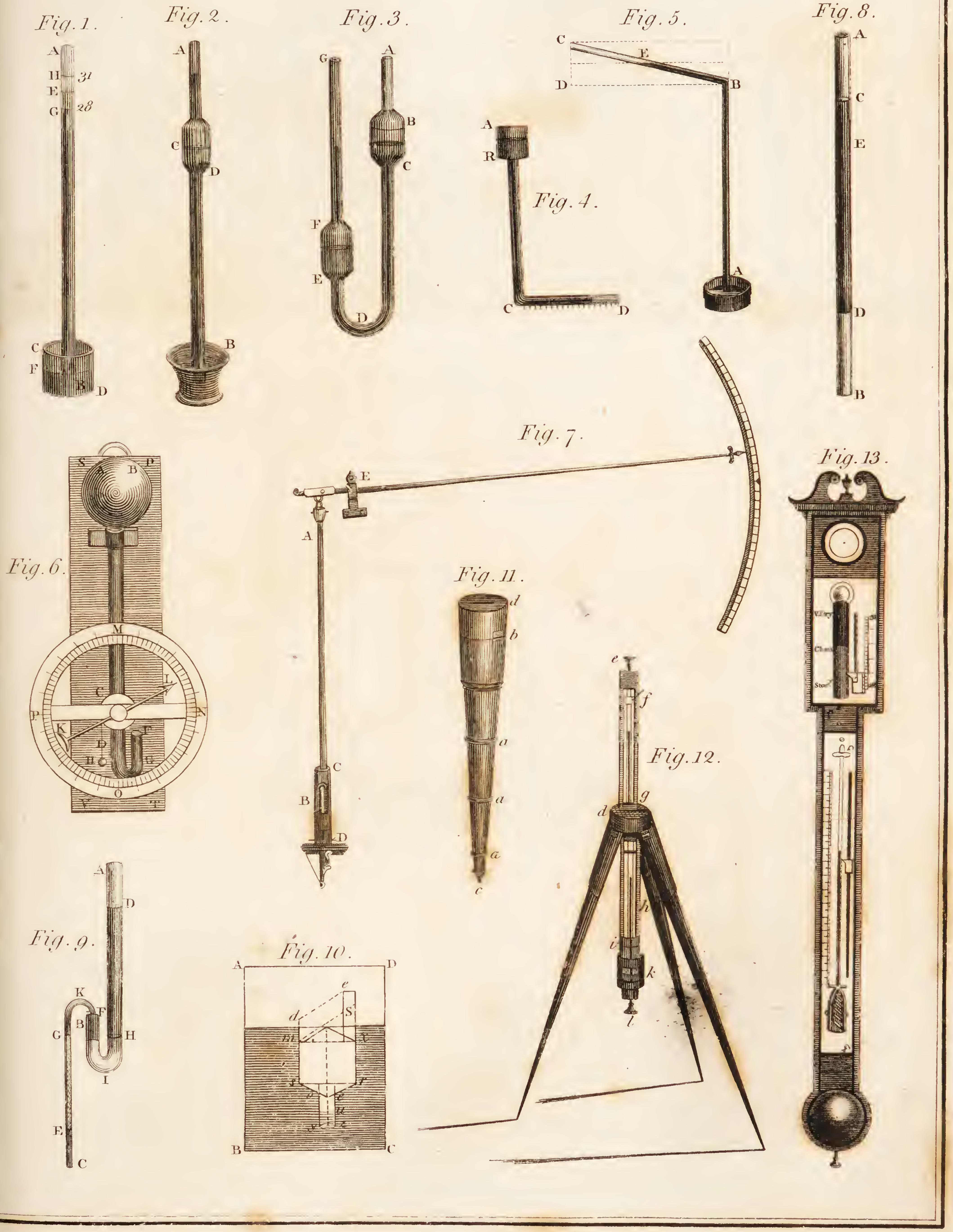


Fig. 1.
A
H
31
E
28
G
C
F
B
D
Fig. 2.
A
C
D
B
Fig. 3.
G
A
B
C
F
E
D
Fig. 5.
C
E
D
B
A
Fig. 4.
A
R
C
D
Fig. 8.
A
C
E
D
B
Fig. 6.
S
A
B
P
M
C
L
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N
K
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O
T
Fig. 7.
E
A
C
B
D
Fig. 13.
Fig. 11.
d
b
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c
Fig. 12.
e
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k
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Fig. 9.
A
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K
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Fig. 10.
A
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d
S
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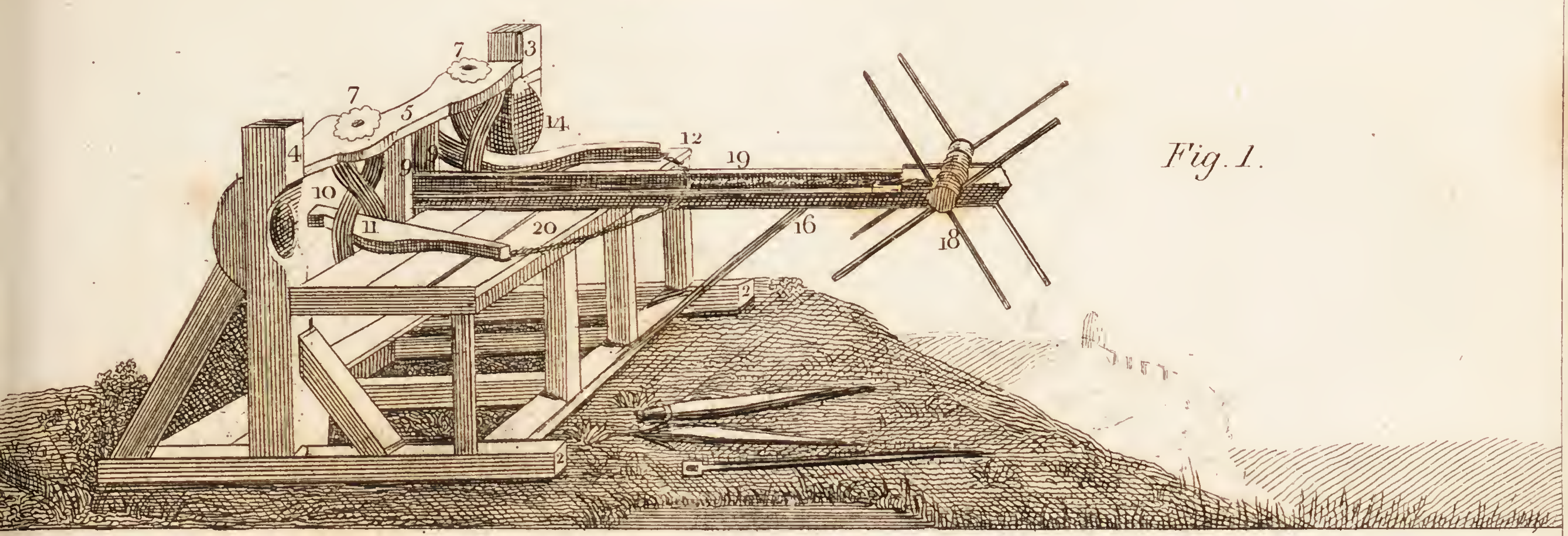
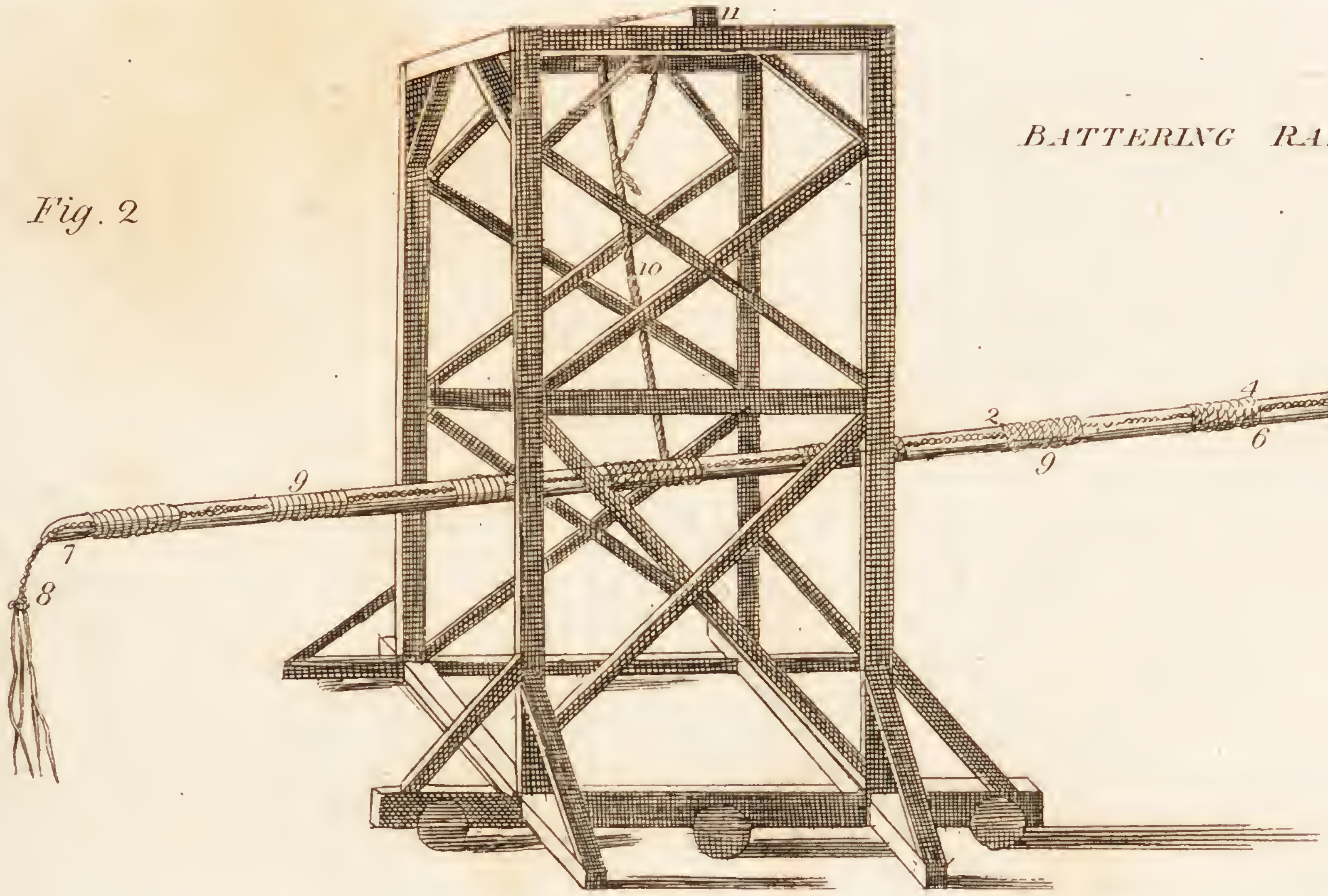
BALLISTA.

Fig. 1.



BATTERING RAM.

Fig. 2



CATAPULTS.

Fig. 3

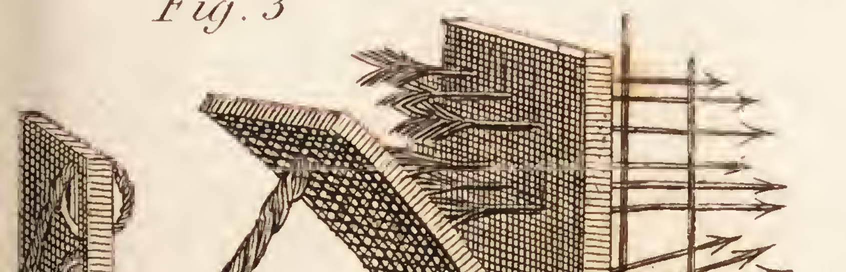
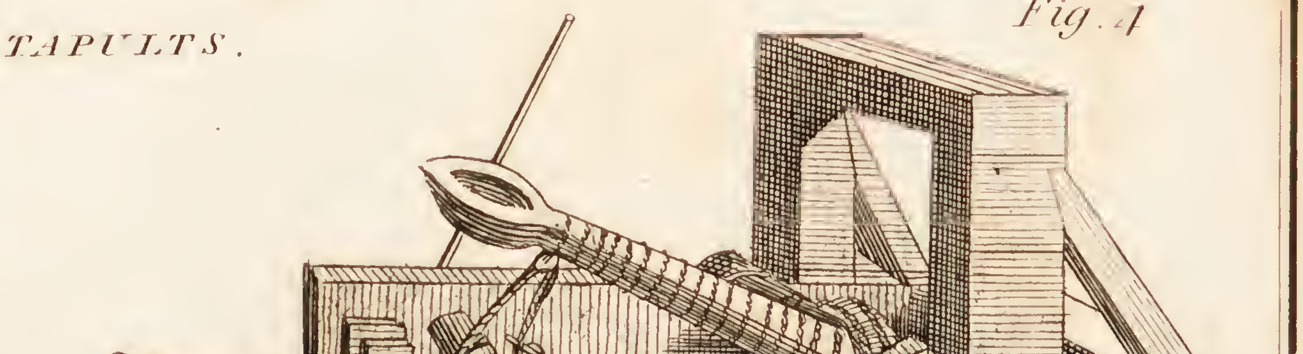
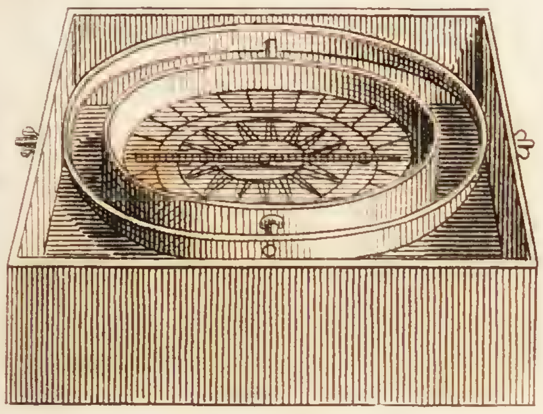


Fig. 4



COMPASS.

Fig. 1



Dr. Knight's COMPASSES.

Fig. 2

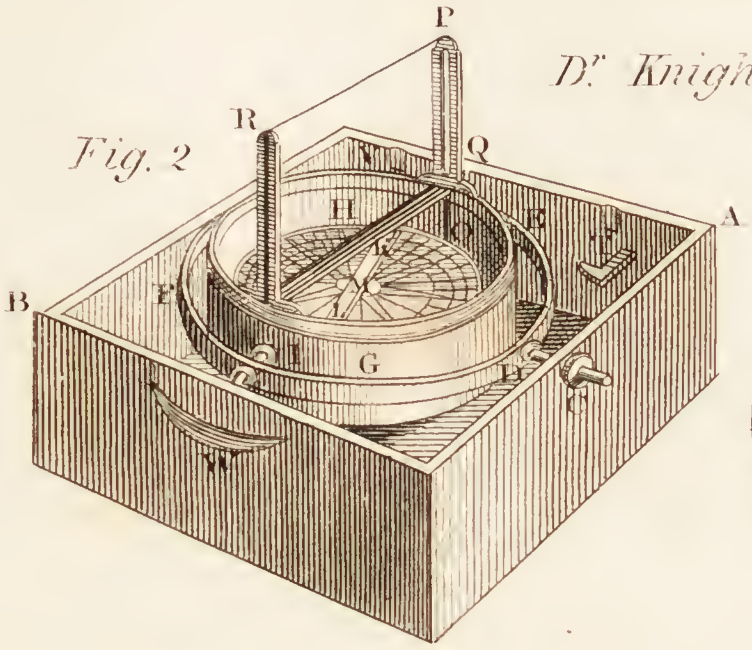
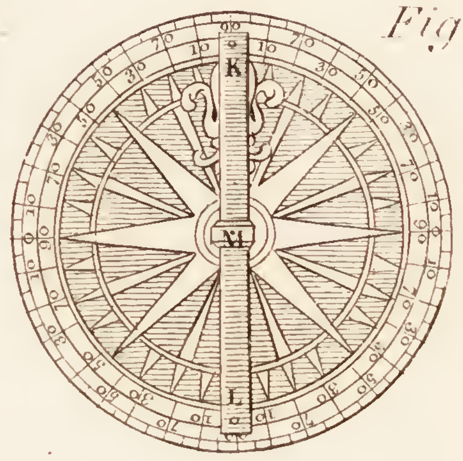
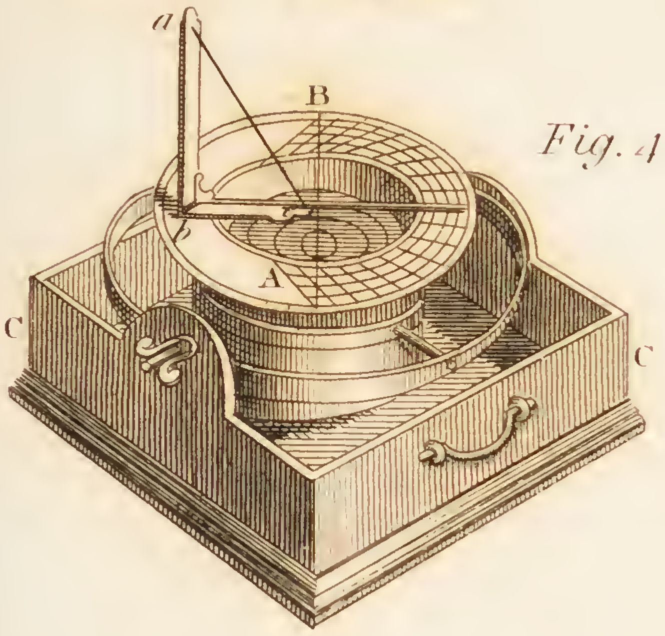


Fig. 3



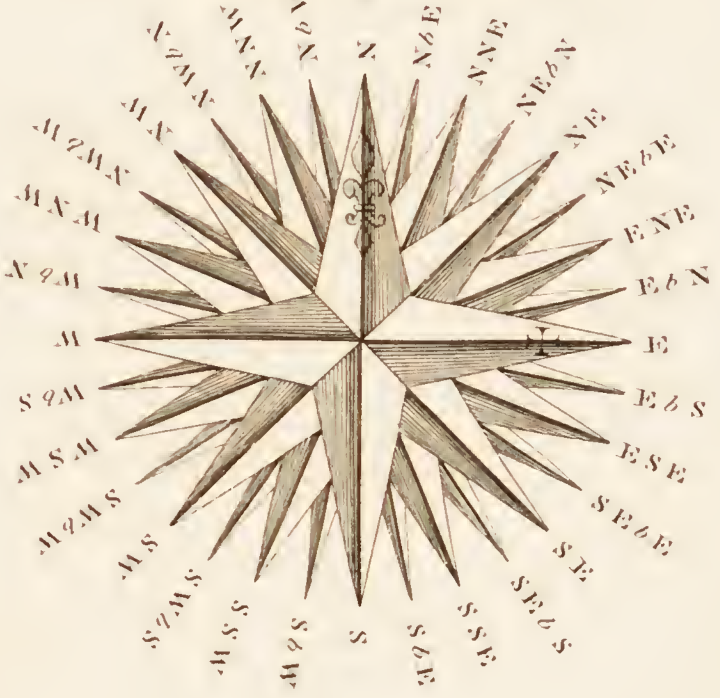
Azimuth COMPASS.

Fig. 4



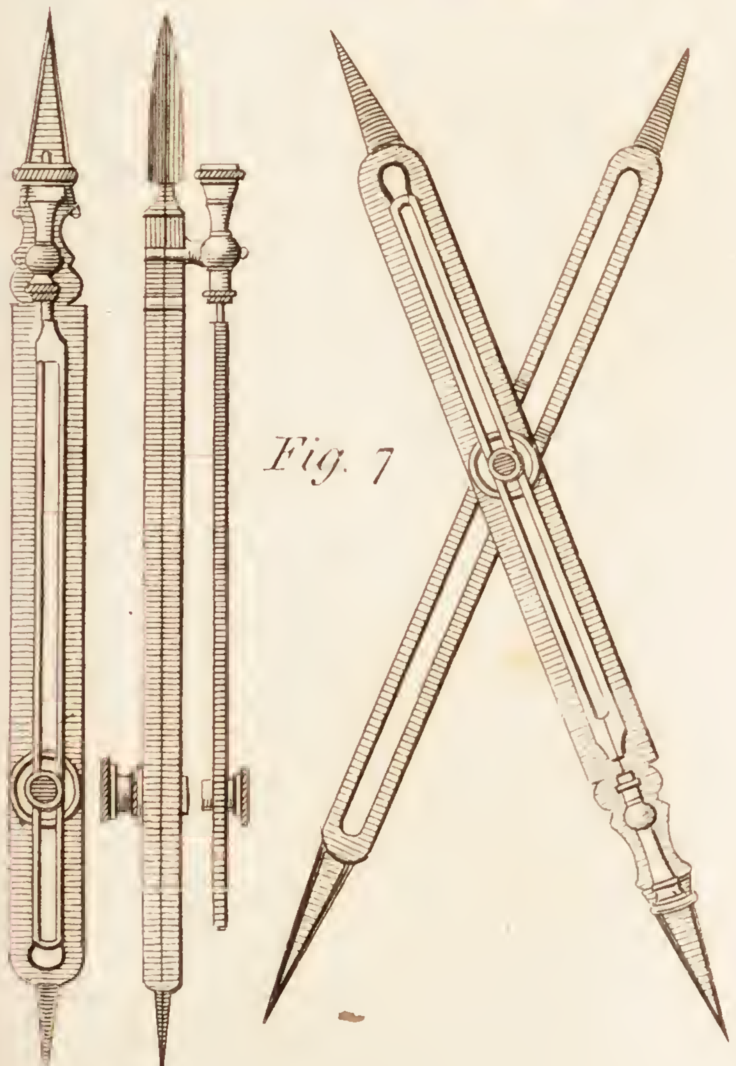
COMPASS Card.

Fig. 5



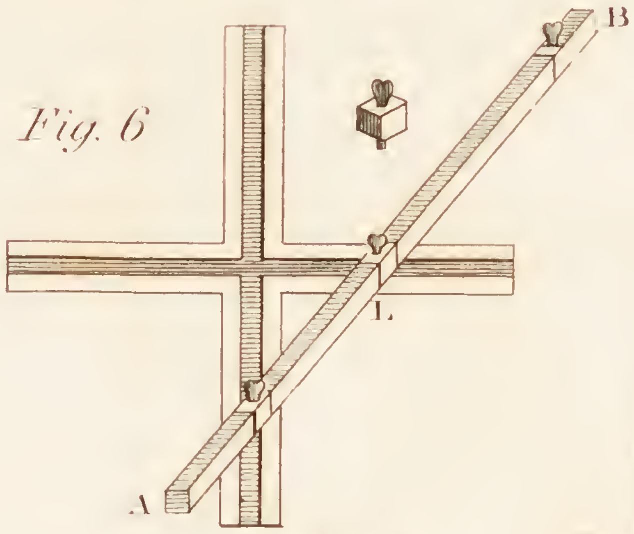
Proportional Compasses.

Fig. 7



Elliptic Compasses.

Fig. 6



FOUNTAINS.

Fig. 8

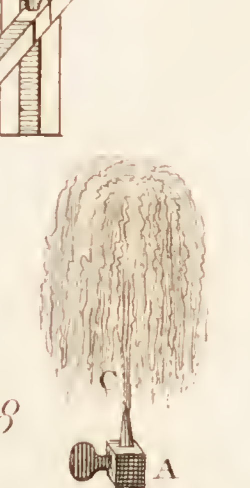
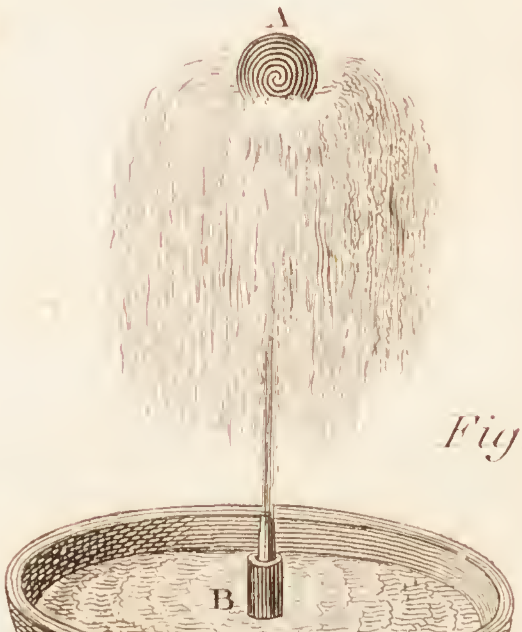


Fig. 9



Therefore 48 is the greateſt common meaſure of 816 and 1488, thus:

```
48)816(17        48)1488(31
   48               144
  ----              ----
   336                48
   336                48
  ----              ----
```

The common meaſure is uſeful in fractions, to reduce a fraction to its leaſt terms, by dividing thoſe that are given by their greateſt common meaſure. So $\frac{816}{1488}$ reduces to $\frac{17}{31}$, by dividing 816 and 1488 both by their greateſt common meaſure 48.

COMMUNICATION *of Motion*, that act of a moving body, by which it gives motion, or transfers its motion to another body.

Father Mallebranche conſiders the communication of motion, as ſomething metaphyſical; that is, as not neceſſarily ariſing from any phyſical principles, or any properties of bodies, but flowing from the immediate agency of God.

The communication of motion reſults from, and is an evidence of the impenetrability and inertia of matter, as ſuch; unleſs we admit the hypotheſis of the penetrability of matter, advanced by Boſcovich and Michell, and aſcribe to the power of repulſion thoſe effects which have been uſually aſcribed to its ſolidity and actual reſiſtance.

Newton ſhews that action and reaction are equal and oppoſite; ſo that one body ſtriking or acting againſt another, and thence cauſing a change in its motion, does itſelf undergo the very ſame change in its own motion, the contrary way. And hence, a moving body ſtriking directly another at reſt, it loſes juſt as much of its motion as it communicates to the other. For the laws and quantity of motion ſo communicated, either in elaſtic or nonelaſtic bodies, ſee COLLISION.

COMMUTATION, *Angle of*, is the diſtance between the ſun's true place ſeen from the earth, and the place of a planet reduced to the ecliptic: which therefore is found by taking the difference between the ſun's longitude and the heliocentric longitude of the planet.

COMPANY, *Rule of*, or *Rule of Fellowſhip*, in Arithmetic, is a rule by which are determined the true ſhares of profit or loſs, due to the ſeveral partners, or aſſociates, in any enterprize, or trade, in due proportion to the ſtock contributed by each, and the time it was employed. To do which properly, ſee the *Rule of Fellowſhip*.

COMPARTMENT, a deſign compoſed of ſeveral different figures, diſpoſed with ſymmetry; to adorn a parterre, a cieling, pannel of joinery, or the like.

COMPARTITION, the uſeful and graceful diſtribution of the whole ground-plot of an edifice, into rooms of office, and of reception, or entertainment.

COMPASS, or *Mariner's Compaſs*, is an inſtrument uſed at ſea by mariners to direct and aſcertain the courſe of their ſhips. It conſiſts of a circular braſs box, which contains a paper card with the 32 points of the compaſs, or winds, fixed on a magnetic needle that always turns to the north, excepting a ſmall deviation, which is variable at different places, and at the ſame place at different times. See VARIATION *of the Compaſs*.

The needle with the card turns on an upright pin fixed in the centre of the box. To the middle of the needle is fixed a braſs conical ſocket or cap, by which the card hanging on the pin turns freely round the centre.

The top of the box is covered with a glaſs, to prevent the wind from diſturbing the motion of the card. The whole is incloſed in another box of wood, where it is ſuſpended by braſs hoops or gimbals, to keep the card in a horizontal poſition during the motions of the ſhip. The whole is to be ſo placed in the ſhip, that the middle ſection of the box, parallel to its ſides, may be parallel to the middle ſection of the ſhip along its keel.

The invention of the compaſs is uſually aſcribed to Flavio Gioia, or Flavio of Malphi, about the year 1302; and hence it is that the territory of Principato, the part of the kingdom of Naples where he was born, has a compaſs for its arms. He divided his compaſs only into 8 points. Others aſcribe the invention to the Chineſe; and Gilbert, in his book *de Magnete*, affirms that Marcus Paulus, a Venetian, making a journey to China, brought back the invention with him in 1260. What ſtrengthens this conjecture is, that at firſt they uſed the compaſs, in the ſame manner as the Chineſe ſtill do, viz, letting it float on a ſmall piece of cork, inſtead of ſuſpending it on a pivot. It is added, that their emperor Chiningus, a celebrated aſtrologer, had a knowledge of it 1120 years before Chriſt. The Chineſe divide their compaſs only into 24 points. But Ludi Vertomanus affirms, that when he was in the Eaſt-Indies, about the year 1500, he ſaw a pilot of a ſhip direct his courſe by a compaſs, faſtened and framed as thoſe now commonly uſed. And Barlow, in his book called The Navigator's Supply, anno 1597, ſays, that in a perſonal conference with two Eaſt-Indians, they affirmed, that inſtead of our compaſs, they uſe a magnetical needle of 6 inches, and longer, upon a pin in a diſh of white China earth, filled with water; in the bottom of which they have two croſs lines for the 4 principal winds, the reſt of the diviſions being left to the ſkill of their pilots. Alſo in the ſame book he ſays that the Portugueſe, in their firſt diſcovery of the Eaſt-Indies, got a pilot of Mahinde, who brought them from thence in 33 days, within ſight of Calicut.

But Fauchette relates ſome verſes of Guoyot de Provence, who lived in France about the year 1200, which ſeem to make mention of the compaſs under the name of *marinette*, or *mariner's ſtone*; which ſhew it was uſed in France near 100 years before either the Malfite or Venetian one. The French even lay claim to the invention, from the fleur de lys with which moſt people diſtinguiſh the north point of the card. With as much reaſon Dr. Wallis aſcribes it to the Engliſh, from its name *compaſs*, by which name moſt nations call it, and which he obſerves is uſed in many parts of England to ſignify a circle.

The mariner's compaſs was long very rude and imperfect, but at length received great improvement from the invention and experiments of Dr. Knight, who diſcovered the uſeful practice of making artificial magnets; and the farther emendations of Mr. Smeaton, and

Mr. M'Culloch, by which the needles are larger and ſtronger than formerly, and inſtead of ſwinging in gimbals, the compaſs is ſupported in its very centre upon a prop, and the centres of motion, gravity, and magnetiſm are brought almoſt all to the ſame point.

After the diſcovery of that moſt uſeful property of the magnet, or loadſtone, viz, its giving a polarity to hardened iron or ſteel, the compaſs was many years in uſe before it was known in anywiſe to deviate from the poles of the world. About the middle of the 16th century, ſo confident were ſome perſons that the needle invariably pointed due north, that they treated with contempt the notion of the variation, which about that time began to be ſuſpected. However, careful obſervations ſoon diſcovered that in England, and its neighbourhood, the needle pointed to the eaſtward of the true north line; and the quantity of this deviation being known, mariners became as well ſatisfied as if the compaſs had none; becauſe the true courſe could be obtained by making allowance for the true variation.

From ſucceeding obſervations it was afterwards found, that the deviation of the needle from the north was not a conſtant quantity, but that it gradually diminiſhed, and at laſt, namely, about the year 1657, it was found that the needle pointed due north at London, and has ever ſince been going to the weſtward, till now the variation is upwards of two points of the compaſs: indeed it was 22° 41′ about the middle of the year 1781, as appears by the Philoſ. Tranſ. pa. 225, for that year, and is probably now ſomewhat more, which it would be of conſequence to know; but why ſuch uſeful obſervations and experiments, as thoſe of the variation and dip of the magnetic needle, have been ſo long diſcontinued, to the prejudice of ſcience, is beſt known to the learned Preſident of that Society, and his Council. So that in any one place it may be ſuſpected the variation has a kind of libratory motion, traverſing through the north, to unknown limits eaſtward and weſtward. But the ſettling of this point muſt be left to time and future experiments. See *Variation*, alſo *Inclination*, and *Dip*. Alſo for a farther deſcription of different compaſſes and their uſes, ſee that uſeful book, Robertſon's Navigation, vol. 2. p. 231.

The Azimuth Compass differs from the common ſea compaſs in this; that the circumference of the card or box is divided into degrees; and there is fitted to the box an index with two ſights, which are upright pieces of braſs, placed diametrically oppoſite to each other, having a ſlit down the middle of them, through which the ſun or ſtar is to be viewed at the time of obſervation.

The Uſe of the Azimuth Compaſs, is to take the bearing of any celeſtial object, when it is in, or above the horizon, that from the magnetical azimuth or amplitude, the variation of the needle may be known. See *Azimuth*, and *Amplitude*.

The figure of the compaſs card, with the names of the 32 points or winds, are as in fig. 5, plate vi; where other compaſſes are alſo exhibited.

As there are 32 whole points quite around the circle, which contains 360 degrees, therefore each point of the compaſs contains the 32d part of 360, that is 11¼ degrees, or 11° 15′; conſequently the half point is 5° 37′ 30″, and the quarter point 2° 48′ 45″.

The points of the compaſs are otherwiſe called Rhumbs; and the numbers of degrees, minutes and ſeconds made by every quarter point with the meridian, are exhibited in the following table.

A TABLE of Rhumbs, ſhewing the Degrees, Minutes, and Seconds, that every Point and Quarter-point of the Compaſs makes with the Meridian.

North		Pts.	qr.	°	′	″	Pts.	qr.	South	
		0	1	2	48	45	0	1		
		0	2	5	37	30	0	2		
		0	3	8	26	15	0	3		
N b E	N b W	1	0	11	15	0	1	0	S b E	S b W
		1	1	14	3	45	1	1		
		1	2	16	52	30	1	2		
		1	3	19	41	15	1	3		
N N E	N N W	2	0	22	30	0	2	0	S S E	S S W
		2	1	25	18	45	2	1		
		2	2	28	7	30	2	2		
		2	3	30	56	15	2	3		
NE b N	NW b N	3	0	33	45	0	3	0	SE b S	SW b S
		3	1	36	33	45	3	1		
		3	2	39	22	30	3	2		
		3	3	42	11	15	3	3		
N E	N W	4	0	45	0	0	4	0	S E	S W
		4	1	47	48	45	4	1		
		4	2	50	37	30	4	2		
		4	3	53	26	15	4	3		
NE b E	NWbW	5	0	56	15	0	5	0	SE b E	SW b W
		5	1	59	3	45	5	1		
		5	2	61	52	30	5	2		
		5	3	64	41	15	5	3		
N NE	WNW	6	0	67	30	0	6	0	E S E	W S W
		6	1	70	18	45	6	1		
		6	2	73	7	30	6	2		
		6	3	75	56	15	6	3		
E b N	W b N	7	0	78	45	0	7	0	E b S	W b S
		7	1	81	33	45	7	1		
		7	2	84	22	30	7	2		
		7	3	87	11	15	7	3		
Eaſt	Weſt	8	0	90	0	0	8	0	Eaſt	Weſt

Compass *Dials*, are ſmall dials, fitted in boxes, for the pocket, to ſhew the hour of the day by the direction of the needle that indicates how to place them right, by turning the dial about till the cock or ſtyle ſtand directly over the needle. But theſe can never be very exact, becauſe of the variation of the needle itſelf; unleſs that variation be allowed for, in making and placing the inſtrument.

COMPASSES, *or Pair of* Compasses, a mathematical inſtrument for deſcribing circles, meaſuring and dividing lines, or figures, &c.

The common compaſſes conſiſt of two ſharp-pointed branches or legs of iron, ſteel, braſs, or other metal, joined together at the top by a rivet, about which they move as on a centre. Thoſe compaſſes are of the beſt ſort in which the pin or axle, on which the joint turns, is made of ſteel, and alſo half the joint itſelf, as the oppoſite metals wear more equally: the points ſhould alſo be made of hard ſteel, well poliſhed; and the joint ſhould open and ſhut with a ſmooth, eaſy, and uniform motion. In ſome compaſſes, the points are both fixed; but in others, one is made to take out occaſionally, and a drawing-pen, or pencil, put in its place.

There are in uſe compaſſes of various kinds and contrivances, adapted to the various purpoſes they are intended for; as,

Compasses *of three Legs*, or Triangular Compaſſes; the conſtruction of which is like that of the common compaſſes, with the addition of a third leg or point, which has a motion every way. Their uſe is to take three points at once, and ſo to form triangles, and lay down three poſitions of a map to be copied at once.

Beam Compasses conſiſt of a long ſtraight beam or bar, carrying two braſs curſors; one of theſe being fixed at one end, the other ſliding along the beam, with a ſcrew to faſten it on occaſionally. To the curſors may be ſcrewed points of any kind, whether ſteel, pencils, or the like. To the fixed curſor is ſometimes applied an adjuſting or micrometer ſcrew, by which an extent is obtained to very great nicety. The beam compaſſes are uſed to draw large circles, to take great extents, or the like.

Bow Compasses, or Bows, are a ſmall ſort of compaſſes, that ſhut up in a hoop, which ſerves for a handle. Their uſe is to deſcribe arcs or circumferences with a very ſmall radius.

Caliber Compasses. See Caliber.

Clockmakers Compasses are jointed like the common compaſſes, with a quadrant or bow, like the ſpring compaſſes; only of different uſe, ſerving here to keep the inſtrument firm at any opening. They are made very ſtrong, with the points of their legs of well-tempered ſteel, as being uſed to draw or cut lines in paſteboard, or copper, &c.

Cylindrical and Spherical Compasses, conſiſt of four branches, joined in a centre, two of them being circular and two flat, a little bent at the ends. The uſe of them is to take the diameter, thickneſs, or caliber of round or cylindrical bodies; as cannons, balls, pipes, &c.

There are alſo ſpherical compaſſes, differing in nothing from the common ones, but that their legs are arched; ſerving to take the diameters of round bodies.

There is alſo another ſort of compaſſes lately invented, for meaſuring the diameter of round bodies, as balls, &c, which conſiſt of two flat pieces of metal ſet at right angles on a ſtraight bar or beam of the ſame; the one piece being fixed, and the other ſliding along it, ſo far as juſt to receive the round body between them; and then its diameter, or diſtance between the two pieces, is ſhewn by the diviſions marked on the beam.

Elliptical Compasses, are uſed to draw ellipſes or ovals of any kind. The inſtrument conſiſts of a beam AB (Plate vi. fig. 6.) about a foot long, bearing three curſors; to one of which may be ſcrewed points of any kind; and to the bottom of the other two are rivetted two ſliding dove-tails, adjuſted in grooves made in the croſs branches of the beam. The dove-tails having a motion every way, by turning about the long branch, they go backward and forward along the croſs; ſo that when the beam has gone half way round, one of theſe will have moved the whole length of one of the branches; and when the beam has gone quite round, the ſame dove-tail has gone back the whole length of the branch. Underſtand the ſame of the other dove-tail.

Note, the diſtance between the two ſliding dove-tails, is the diſtance between the two foci of the ellipſe; ſo that by changing that diſtance, the ellipſe will be rounder or flatter. Under the ends of the branches of the croſs, are placed four ſteel points to keep it faſt.

The uſe of this compaſs is eaſy: by turning round the long branch, the pen, pencil, or other points will draw the ellipſe required.

Its figure ſhews both its uſe and conſtruction.

German Compasses have their legs a little bent outwards, near the top; ſo that when ſhut, the points only meet.

Hair Compasses are ſo contrived within ſide by a ſmall adjuſting ſcrew to one of the legs, as to take an extent to a hair's breadth, or great exactneſs.

Proportional Compasses are thoſe whoſe joint lies, not at the end of the legs, but between the points terminating each leg. Theſe are either ſimple, or compound. In the former ſort the centre, or place of the joint is fixed; ſo that one pair of theſe ſerves only for one proportion.

Compound Proportional Compasses have the joint or centre moveable. They conſiſt of two parts or ſides of braſs, which lie upon each other ſo nicely as to ſeem but one when they are ſhut. Theſe ſides eaſily open, and move about the centre, which is itſelf moveable in a hollow canal cut through the greateſt part of their length. To this centre on each ſide is fixed a ſliding piece, of a ſmall length, with a fine line drawn on it ſerving as an index, to be ſet againſt other lines or diviſions placed upon the compaſſes on both ſides. Theſe lines are, 1, A line of lines; 2, a line of ſuperficies, areas, or planes, the numbers on which anſwer to the ſquares of thoſe on the line of lines; 3, a line of ſolids, the numbers on which anſwer to the cubes of thoſe on the line of lines; 4, a line of circles, or rather of polygons to be inſcribed in circles. Theſe lines are all unequally divided, the firſt three from 1 to 20, and the laſt from 6 to 20. The uſe of the firſt is to divide a line into any number of equal parts; by the 2d and 3d are found the ſides of like planes or ſolids in any given proportion; and by the 4th, circles are divided into any number of equal parts, or any polygons inſcribed in them. See Plate vi. fig. 7.

Spring Compasses, or *Dividers*, are made of hardened ſteel, with an arched head, which by its ſpring opens the legs; the opening being directed by a circular ſcrew faſtened to one of the legs, let through the other, and worked with a nut.

Triſecting Compasses, for the triſecting of angles geometrically, for which purpoſe they were invented by M. Tarragon.

The inſtrument conſiſts of two central rules, and an arch of a circle of 120 degrees, immoveable, with its radius: the radius is faſtened with one of the central rules, like the two legs of a ſector, that the central rule may be carried through all the points of the circumference of the arch. The radius and rule ſhould be as thin as poſſible; and the rule faſtened to the radius ſhould be hammered cold, to be more elaſtic; and the breadth of the other central rule muſt be triple the breadth of the radius: in this rule alſo is a groove, with a dove-tail faſtened on it, for its motion; there muſt alſo be a hole in the centre of each rule.

Turn-up Compasses, a late contrivance to ſave the trouble of changing the points: the body is like the common compaſſes; and towards the bottom of the legs without ſide, are added two other points, beſides the uſual ones; the one carrying a drawing pen point, and the other a port-crayon; both being adjuſted to turn up, to be uſed or not, as occaſion may require.

COMPLEMENT in general, is what is wanting, or neceſſary, to complete ſome certain quantity or thing. As, the

Complement *of an arch or angle,* as of 90° or a quadrant, is what any given arch or angle wants of it; ſo the complement of 50° is 40°, and the complement of 100 degrees is —10°, a negative quantity.—The complement to 180° is uſually called the ſupplement, to diſtinguiſh it from the complement to 90°, properly ſo called.—The ſine of the complement of an arc, is contracted into the word coſine; the tangent of the complement, into cotangent; &c.

Arithmetical Complement, is what a number or logarithm wants of unity or 1 with ſome number of ciphers. It is beſt found, by beginning at the left-hand ſide, and ſubtracting every figure from 9, except the laſt, or right-hand figure, which muſt be ſubtracted from 10. So, the arithmetical comp. of the log. 9·5329714, by ſubtracting from 9's, &c, is 0·4670286.

The arithmetical complements are much uſed in operations by logarithms, to change ſubtractions into additions, which are more conveniently performed, eſpecially when there are more than one of them in the operation.

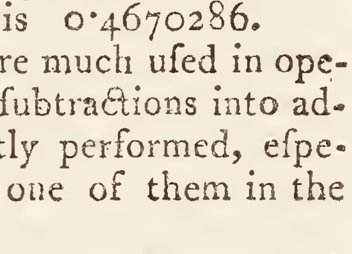
Complement, in Aſtronomy, is uſed for the diſtance of a ſtar from the zenith; or the arc contained between the zenith and the place of a ſtar which is above the horizon. It is the ſame as the complement of the altitude, or co-altitude, or the zenith diſtance.

Complement *of the Courſe,* in Navigation, is the quantity which the courſe wants of 90°, or 8 points, viz, a quarter of the compaſs.

Complement *of the Curtain,* in Fortification, is that part of the anterior ſide of the curtain, which makes the demigorge.

Complement *of the Line of Defence,* is the remainder of that line, after the angle of the flank is taken away.

Complements *of a Parallelogram,* or *in a Parallelogram,* are the two leſſer parallelograms, made by drawing two right lines parallel to each ſide of the given parallelogram, through the ſame point in the diagonal. So P and Q are the complements in the parallelogram ABCD.



In every caſe, theſe complements are always equal, viz, the parallelogram $P = Q$.

Complement *of Life,* a term much uſed, in the doctrine of Life Annuities, by De Moivre, and, according to him, it denotes the number of years which a given life wants of 86, this being the age which he conſidered as the utmoſt probable extent of life. So 56 is the complement of 30, and 30 is the complement of 56.

That author ſuppoſed an equal annual decrement of life through all its ſtages, till the age of 86. Thus, if there be 56 perſons living at 30 years of age, it is ſuppoſed that one will die every year, till they be all dead in 56 years. This hypotheſis in many caſes is very near the truth; and it agrees ſo nearly with Halley's table, formed from his obſervations of the mortuary bills of Breſlaw, that the value of lives deduced either from the hypotheſis, or the table, need not be diſtinguiſhed; hence it very much eaſes the labour of calculating them. See *Life* Annuities, alſo De Moivre's Treatiſe on Annuities, pa. 83, and Price on Reverſionary Payments, pa. 2.

COMPOSITE *Number,* is one that is compounded of, or made up by the multiplication of two other numbers, greater than 1, or which can be meaſured by ſome other number greater than 1. As 12, which is compoſed, or compounded of 2 and 6, or 3 and 4, viz by multiplying together 2 and 6, or 3 and 4, both products making the ſame number 12; which therefore is a compoſite number.

Compoſites are oppoſed to prime numbers, or primes, which cannot be exactly meaſured by any other number, and cannot be produced by multiplying together two other factors.

Compoſite Numbers between themſelves, are the ſame with commenſurable numbers, or ſuch as have a common meaſure or factor; as 15 and 12, which have the common term 3.

The doctrine of Prime and Compoſite numbers is pretty fully treated in the 7th and 8th books of Euclid's Elements.

Composite *Order,* in Architecture, is the laſt of the five orders of columns; and is ſo called becauſe its capital is compoſed out of thoſe of the other orders. Thus, it borrows a quarter-round from the Tuſcan and Doric; a double row of leaves from the Corinthian; and volutes from the Ionic. Its cornice has ſingle modillions, or dentils; and its column is 10 diameters in height.

This order is alſo called the Roman order, and Italic order, as having been invented by the Romans, like as the other orders are denominated from the people among whom they had their riſe.

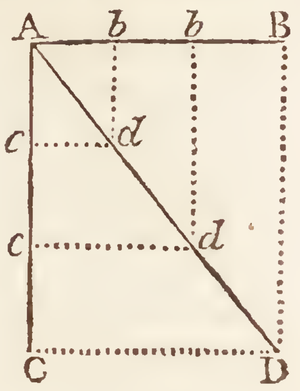
COMPOSITION, is a ſpecies of reaſoning by which we proceed from things that are known and given, ſtep by ſtep, till we arrive at ſuch as were before unknown or required; viz, proceeding upon principles ſelf-evident, on definitions, poſtulates, and axioms, with a previouſly demonſtrated ſeries of propoſitions, ſtep by ſtep, till it gives a clear knowledge of the thing to be known or demonſtrated. Compoſition, otherwiſe called the ſynthetical method, is oppoſed to Reſolution, or the analytical method, and is chiefly uſed by the ancients, Euclid, Apollonius, &c. See Pappus; alſo the term *Analyſis.*

Composition *of forces,* or *of motion,* is the union or aſſemblage of ſeveral forces or motions that are oblique to one another, into an equivalent one in another direction.

1. When ſeveral forces or motions are united, that act in the ſame line of direction, the combined force or motion will be in the ſame line of direction ſtill. But when oblique forces are united, the compounded force takes a new direction, different from both, and is either a right line or a curve, according to the nature of the forces compounded.

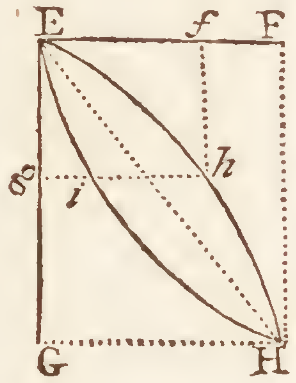
2. If two compounding motions be both equable ones, whether equal to each other or not, the line of the

compound motion will be a ſtraight line. Thus, if the one equable in the direction AB be ſufficient to carry a body over the ſpace AB in any time, and the other motion ſufficient to paſs over AC in the ſame time; then by the compound motion, or both acting on the body together, it would in the ſame time paſs over the diagonal AD of the parallelogram ABDC. For becauſe the motions are uniform, any ſpaces A*b*, A*c* paſſed over in the ſame time, are proportional to the velocities, or to AB and AC; and conſequently all the points A, *d*, *d*, D, of the path are in the ſame right line.

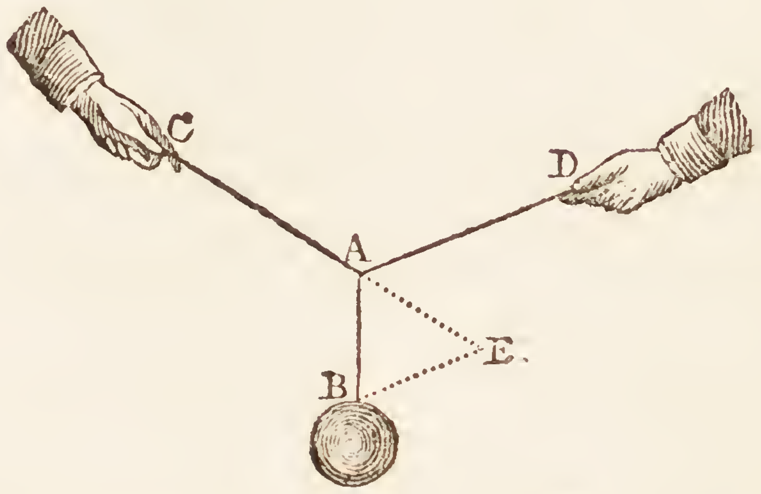


3. And though the compounding motions be not equable, but variable, either accelerated or retarded, provided they do but vary in a ſimilar manner, the compounded motion will ſtill be in a ſtraight line. Thus, ſuppoſe, for inſtance, that the motions both vary in ſuch a manner, as that the ſpaces paſſed over in the ſame time, whether they be equal to each other or not, are both as the ſame power *n* of the time; then $AB^n : AC^n : : Ab^n : Ac^n$, and hence $AB : AC : : Ab : Ac$, and therefore, as before, A*dd*D is ſtill a right line.

4. But if the compounding motions be not ſimilar to each other, as when the one is equable and the other variable, or when they are varied in a diſſimilar manner; then the compounded motion is in ſome curve line. So if the motion in the one direction EF be in a leſs proportion, with reſpect to the time, than that in the direction EG is, then the path will be a curve line E*i*H concave towards EF; but if the motion in EF be in a greater proportion than that in EG, then the path of the compound motion will be a curve E*h*H convex towards EF: that is, in general, the curvilineal path is convex towards that direction in which the motion is in the leſs proportion to the time. Hence, for a particular inſtance, if the motion in the direction EF be a motion of projection, which is an equable motion, and the motion in the direction EG that ariſing from gravity, which is a uniformly accelerated motion, or in proportion to the ſquares of the times; then is EG as GH^2, and E*g* as gh^2, that is $EG : Eg : : GH^2 : gh^2$, which is the property of the parabola; and therefore the path E*h*H of any body projected, is the common parabola.



5. If there be three forces united, or acting againſt the ſame point A at the ſame time, viz, the force or weight B in the direction AB, and the forces or tenſions in the directions AC, AD; and if theſe three forces mutually balance each other, ſo as to keep the common point A in equilibrio; then are theſe forces directly proportional to the reſpective ſides of a triangle formed by drawing lines parallel to the directions of theſe forces; or indeed perpendicular to thoſe directions, or making any one and the ſame angle with them. So that, if BE be drawn parallel, for inſtance, to AD, and meet CA produced in E, forming the triangle ABE; then are the three forces in the directions AB, AC, AD, reſpectively proportional to the ſides AB, AE, BE.



And this theorem, with its corollaries, Dr. Keil obſerves, is the foundation of all the new mechanics of M. Varignon: by help of which may the force of the muſcles be computed, and moſt of the mechanic theorems in Borelli, De Motu Animalium, may immediately be deduced.

See more of the Compoſition of Forces under the article COLLISION.

COMPOSITION *of Numbers and Quantities.* See COMBINATION.

COMPOSITION *of Proportion*, according to the 15th definition of the 5th book of Euclid's Elements, is when, of four proportionals, the ſum of the 1ſt and 2d is to the 2d, as the ſum of the 3d and 4th is to the 4th:

as if it be $a : b : : c : d$,
then by compoſition $a+b : b : : c+d : d$.
Or, in numbers, if $2 : 4 : : 9 : 18$,
then by compoſition $6 : 4 : : 27 : 18$.

COMPOSITION *of Ratios*, is the adding of ratios together: which is performed by multiplying together their correſponding terms, viz, the antecedents together, and the conſequents together, for the antecedent and conſequent of the compounded ratio; like as the addition of logarithms is the ſame thing as the multiplication of their correſponding numbers. Or, if the terms of the ratios be placed fraction-wiſe, then the addition or compoſition of the ratios, is performed by multiplying the fractions together.

Thus, the ratio of $a : b$, or of $2 : 4$,
added to the ratio of $c : d$, or of $6 : 8$,
makes the ratio of $ac : bd$, or of $12 : 32$; and ſo the ratio of *ac* to *bd* is ſaid to be compounded of the ratios of *a* to *b*, and *c* to *d*. So likewiſe, if it were required to compound together the three ratios, viz, of *a* to *b*, *c* to *d*, and *e* to *f*; then $\frac{a}{b}\times\frac{c}{d}\times\frac{e}{f}=\frac{ace}{bdf}$ are the terms of the compound ratio; or the ratio of *ace* to *bdf* is compounded, or made up of the ratios of *a* to *b*, *c* to *d*, and *e* to *f*.

Hence, if the conſequent of each ratio be the ſame as the antecedent of the preceding ratio, then is the ratio of the firſt term to the laſt, compounded, or made up of all the other ratios, viz, the ratio of *a* to *e*, equal to the ſum of all the ratios of *a* to *b*, of *b* to *c*, of *c* to *d*, and of *d* to *e*; for $\frac{a}{b}\times\frac{b}{c}\times\frac{c}{d}\times\frac{d}{e}=\frac{a}{e}$ the terms or exponents of the compounded ratio.

Hence alſo, in a ſeries of continual proportionals, the ratio of the firſt term to the third is double of the ratio of the firſt to the ſecond,
and the ratio of the 1ſt to the 4th is triple of it,
and the ratio of the 1ſt to the 5th is quadruple of it,
and ſo on; that is, the exponents are double, triple, quadruple, &c, of the firſt exponent: as in the ſeries $1, a, a^2, a^3, a^4$, &c; where the ratio 1 to a^2 is double, of 1 to a^3 triple, &c, of the ratio of 1 to a; or the exponent of a^2, a^3, a^4, &c, double, triple, quadruple, &c, of a.

COMPOUND Intereſt, called alſo *Intereſt upon Intereſt*, is that which is reckoned not only upon the principal, but upon the intereſt itſelf forborn, which thus becomes a ſort of ſecondary principal.

If r be the amount of 1 pound for 1 year, that is the ſum of the principal and intereſt together for one year; then is r^2 the amount for 2 years,
and r^3 the amount for 3 years,
and in general r^t the amount for t years; that is r^t is the ſum or total amount of all the principals and intereſts together of 1 l. for the whole time or number of years t; conſequently, if p be any other principal ſum, forborn for t years, then its amount in that time at compound intereſt, is $a = pr^t$.

The Rule therefore in words is this, to one pound add its intereſt for one year, or half year, or for the firſt time at which the intereſt is reckoned; raiſe the ſum r to the power denoted by the time or number of terms; then this power multiplied by the principal, or firſt ſum lent, will produce the whole amount.

For example, To find how much 50l. will amount to in 5 years at 5 per cent. per annum, compound intereſt.——Here the intereſt of 1 l. for 1 year is ·05, and therefore $r = 1·05$; hence the 5th power of it for 5 years, is $r^5 = 1·27628$ &c;
multiply this by p or - 50

gives the amount pr^t or 63·814l.
or 63l. 16s. 3¼d. for the amount ſought.

But Compound Intereſt is beſt computed by means of ſuch a table as the following, being the amounts of 1 pound for any number of years, and at ſeveral rates of compound intereſt.

As an example of the uſe of this table, ſuppoſe it were required to find the amount of 250l. for 35 years at 4 per cent. compound intereſt.

In the column of 4 per cent, and line of 35 years,
is - - - 3·94609,
which multiplied by the principal - - 250

gives - - - - - 986·52250
or - - - - - 986*l* 10*s* 5¼*d*,
which is the amount ſought.

Note, By a bare inſpection of this table, it appears how many years are required for any ſum of money to double itſelf, at any rate of compound intereſt; viz, by looking in the columns when the amount becomes the number 2. So it is found that at the ſeveral rates the reſpective times requiſite for doubling any ſum, are nearly thus: viz,

Rate	3	3½	4	4½	5	6
Years	23½	20¼	17¾	15¾	14¼	12

TABLE of the Amount of 1l. at Compound Intereſt for many Years and ſeveral Rates of Intereſt.

Yrs.	at 3 per cent	at 3½ per cent	at 4 per cent	at 4½ per cent	at 5 per cent	at 6 per cent
1	1·03000	1·03500	1·04000	1·04500	1·05000	1·06000
2	1·06090	1·07123	1·08160	1·09202	1·10250	1·12360
3	1·09273	1·10872	1·12486	1·14117	1·15763	1·19102
4	1·12551	1·14752	1·16986	1·19252	1·21551	1·26248
5	1·15927	1·18769	1·21665	1·24618	1·27628	1·33823
6	1·19405	1·22926	1·26532	1·30226	1·34010	1·41852
7	1·22987	1·27228	1·31593	1·36086	1·40710	1·50363
8	1·26677	1·31681	1·36857	1·42210	1·47746	1·59385
9	1·30477	1·36290	1·42331	1·48610	1·55133	1·68948
10	1·34392	1·41060	1·48024	1·55297	1·62890	1·79085
11	1·38423	1·45997	1·53945	1·62285	1·71034	1·89830
12	1·42576	1·51107	1·60103	1·69588	1·79586	2·01220
13	1·46853	1·56396	1·66507	1·77220	1·88565	2·13293
14	1·51259	1·61869	1·73168	1·85194	1·97993	2·26090
15	1·55797	1·67535	1·80094	1·93528	2·07893	2·39656
16	1·60471	1·73399	1·87298	2·02237	2·18287	2·54035
17	1·65285	1·79468	1·94790	2·11338	2·29202	2·69277
18	1·70243	1·85749	2·02582	2·20848	2·40662	2·85434
19	1·75351	1·92250	2·10685	2·30786	2·52695	3·02560
20	1·80611	1·98979	2·19112	2·41171	2·65330	3·20714
21	1·86029	2·05943	2·27877	2·52024	2·78596	3·39956
22	1·91610	2·13151	2·36992	2·63365	2·92526	3·60354
23	1·97359	2·20611	2·46472	2·75217	3·07152	3·81975
24	2·03279	2·28333	2·56330	2·87601	3·22510	4·04893
25	2·09378	2·36324	2·66584	3·00543	3·38635	4·29187
26	2·15659	2·44596	2·77247	3·14068	3·55567	4·54938
27	2·22129	2·53157	2·88337	3·28201	3·73346	4·82235
28	2·28793	2·62017	2·99870	3·42970	3·92013	5·11169
29	2·35657	2·71188	3·11865	3·58404	4·11614	5·41839
30	2·42726	2·80679	3·24340	3·74532	4·32194	5·74349
31	2·50008	2·90503	3·37313	3·91386	4·53804	6·08810
32	3·57508	3·00671	3·50806	4·08998	4·76494	6.45339
33	2·65234	3·11194	3·64838	4·27403	5·00319	6·84059
34	2·73191	3·22086	3·79432	4·46636	5·25335	7·25103
35	2·81386	3·33359	3·94609	4·66735	5·51602	7·68609
36	2·89828	3·45027	4·10393	4·87738	5·79182	8·14725
37	2·98523	3·57103	4·26809	5·09686	6·08141	8·63609
38	3·07478	3·69601	4·43881	5·32622	6·38548	9·15425
39	3·16703	3·82537	4·61637	5·56590	6·70475	9·70351
40	3·26204	3·95926	4·80102	5·81636	7·03999	10·28572
41	3·35990	4·09783	4·99306	6·07810	7·39199	10·90286
42	3·46070	4·24126	5·19278	6·35162	7·76159	11·55703
43	3·56452	4·38970	5·40050	6·63744	8·14967	12·25045
44	3·67145	4·54334	5·61652	6·93612	8·55715	12·98548
45	3·78160	4·70236	5·84118	7·24825	8·98501	13·76461
46	3·89504	4·86694	6·07482	7·57442	9·43426	14·59049
47	4·011190	5·03728	6·31782	7·91527	9·90597	15·46592
48	4·13225	5·21359	6·57053	8·27146	10·40127	16·39387
49	4·25622	5·39606	6·83335	8·64367	10·92133	17·37750
50	4·38391	5·58493	7·10668	9·03264	11·46740	18·42015

COMPOUND *Motion*, that motion which is the effect of ſeveral conſpiring powers or forces, viz, ſuch forces as are not directly oppoſite to each other: as when the radius of a circle is conſidered as revolving about a centre, and at the ſame time a point as moving ſtraight along it; which produces a kind of a ſpiral for the path of the point. And hence it is eaſily perceived, that all curvilinear motion is compound, or the effect of two or more forces; although every compound motion is not curvilinear.

It is a popular theorem in Mechanics, that in uniform compound motions, the velocity produced by the conſpiring powers or forces, is to that of either of the two compounding powers ſeparately, as the diagonal of a parallelogram, according to the direction of whoſe ſides they act ſeparately, is to either of the ſides. See COMPOSITION *of Motion*, and COLLISION.

COMPOUND *Numbers*, thoſe compoſed of the multiplication of two or more numbers; as 12, compoſed of 3 times 4. See COMPOSITE.

COMPOUND *Pendulum*, that which conſiſts of ſeveral weights conſtantly keeping the ſame diſtance, both from each other, and from the centre about which they oſcillate.

COMPOUND *Quantities*, are ſuch as are connected together by the ſigns + or −. Thus, $a + b$, or $a - c + d$, or $aa - 2a$, are compound quantities.

Compound quantities are diſtinguiſhed into binomials, trinomials, quadrinomials, &c, according to the number of terms in them; viz, the binomial having two terms; the trinomial, three; the quadrinomial, 4; &c. Alſo, thoſe that have more than two terms, are called by the general name of multinomials, as alſo polynomials.

COMPOUND *Ratio*, is that which is made by adding two or more ratios together; viz, by multiplying all their antecedents together for the antecedent, and all the conſequents together for the conſequent of the compound ratio. So 6 to 72 is a ratio compounded of the ratios of 2 to 6, and 3 to 12; becauſe $\frac{6}{72} = \frac{2}{6} \times \frac{3}{12}$: alſo ab to cd is a ratio compounded of the ratio of a to c, and b to d; for $\frac{ab}{cd} = \frac{a}{c} \times \frac{b}{d}$. See COMPOSITION *of Ratios*.

COMPRESSION, the act of preſſing, or ſqueezing ſomething, ſo as to bring its parts nearer together, and make it occupy leſs ſpace.

Compreſſion differs from condenſation as the cauſe from the effect, compreſſion being the action of any force on a body, without regarding its effects; whereas condenſation denotes the ſtate of a body that is actually reduced into a leſs bulk, and is an effect of compreſſion, though it may be effected alſo by other means. Nevertheleſs, compreſſion and condenſation are often confounded.

Pumps, which the ancients imagined to act by ſuction, do in reality act by compreſſion; the piſton, in working in the narrow pipe, compreſſes the incloſed air, ſo as to enable it, by the force of its increaſed elaſticity, to raiſe the valve, and make its eſcape; upon which, the balance being deſtroyed, the preſſure of the atmoſphere on the ſtagnant ſurface, forces up the water in the pipe, thus evacuated of its air.

It was long thought that water was not compreſſible into leſs bulk: and it was believed, till lately, that after the air had been purged out of it, no art or violence was able to preſs it into leſs ſpace. In an experiment made by the Academy del Cimento, water, when violently ſqueezed, made its way through the fine pores of a globe of gold, rather than yield to the compreſſion.

But the ingenious Mr. Canton, attentively conſidering this experiment, found that it was not ſufficiently accurate to juſtify the concluſion which had always been drawn from it; ſince the Florentine philoſophers had no method of determining that the alteration of figure in their globe of gold, occaſioned ſuch a diminution of its internal capacity, as was exactly equal to the quantity of water forced into its pores. To bring this matter therefore to a more accurate and deciſive trial, he procured a ſmall glaſs tube of about two feet long, with a ball at one end, of an inch and a quarter in diameter. Having filled the ball and part of the tube with mercury, and brought it exactly to the heat of 50° of Fahrenheit's thermometer, he marked the place where the mercury ſtood in the tube, which was about ſix inches and a half above the ball; he then raiſed the mercury by heat to the top of the tube, and there ſealed the tube hermetically; then upon reducing the mercury to the ſame degree of heat as before, it ſtood in the tube $\frac{32}{100}$ of an inch higher than the mark. The ſame experiment was repeated with water exhauſted of air, inſtead of mercury, and the water ſtood in the tube $\frac{43}{100}$ of an inch above the mark. Since the weight of the atmoſphere on the outſide of the ball, without any counterbalance from within, will compreſs the ball, and equally raiſe both the mercury and water, it appears that the water expands $\frac{11}{100}$ of an inch more than the mercury by removing the weight of the atmoſphere. Having thus determined that water is really compreſſible, he proceeded to eſtimate the degree of compreſſion correſponding to any given weight. For this purpoſe he prepared another ball, with a tube joined to it; and finding that the mercury in $\frac{23}{100}$ of an inch of the tube was the hundred thouſandth part of that contained in the ball, he divided the tube accordingly. He then filled the ball and part of the tube with water exhauſted of air; and leaving the tube open, placed this apparatus under the receiver of an air-pump, and obſerved the degree of expanſion of the water anſwering to any degree of rarefaction of the air: and again by putting it into the glaſs receiver of a condenſing engine, he noted the degree of compreſſion of the water correſponding to any degree of condenſation of the air. He thus found, by repeated trials, that, in a temperature of 50°, and when the mercury has been at its mean height in the barometer, the water expands one part in 21740; and is as much compreſſed by the weight of an additional atmoſphere; or the compreſſion of water by twice the weight of the atmoſphere, is one part in 10870 of its whole bulk. Should it be objected, that the compreſſibility of the water was owing to any air which it might be ſuppoſed to contain, he anſwers, that more air would make it more compreſſible; he therefore let into the ball a bubble of air, and found that the water was not more compreſſed by the ſame weight than before.

In ſome farther experiments of the ſame kind, Mr. Canton found that water is more compreſſible in winter than in ſummer; but he obſerved the contrary in ſpirit of wine, and oil of olives.

The following table was formed, when the barometer was at 29 inches and a half, and the thermometer at 50 degrees.

Compreſſion of	Millionth parts.	Spec. grav.
Spirit of wine	66	846
Oil of olives	48	918
Rain water	46	1000
Sea water	40	1028
Mercury	3	13595

He infers that theſe fluids are not only compreſſible, but elaſtic; and that the compreſſions of them, by the ſame weight, are not in the inverſe ratio of their denſities, or ſpecific gravities, as might be ſuppoſed. Phil. Tranſ. vol. lii. 1762. art. 103. and vol. liv. 1764. art. 47.

The compreſſion of the air, by its own weight, is ſurpriſingly great: but the air may be ſtill further compreſſed by art. See *Elaſticity of* Air.

This immenſe compreſſion and dilatation, Newton obſerves, cannot be accounted for in any other way, but by a repelling force, with which the particles of air are endued; by virtue of which, when at liberty, they mutually fly each other.

This repelling power, he adds, is ſtronger and more ſenſible in air, than in other bodies; becauſe air is generated out of very fixed bodies, but not without great difficulty, and by the help of fermentation: now thoſe particles always recede from each other with the greateſt violence, and are compreſſed with the greateſt difficulty, which, when contiguous, cohere the moſt ſtrongly. See Air, Attraction, Cohesion, Dilatation, and Repulsion.

COMPUTATION, the manner of accounting and eſtimating time, weights, meaſures, money, &c. See Calculation, which it is alſo uſed for.

CONCAVE, an appellation uſed in ſpeaking of the inner ſurface of hollow bodies, more eſpecially of ſpherical or circular ones.

Concave glaſſes, lenſes, and mirrors, have either one ſide or both ſides concave.

The property of all concave lenſes is, that the rays of light, in paſſing through them, are deflected, or made to recede from one another; as in convex lenſes they are inflected towards each other; and that the more as the concavity or convexity has a ſmaller radius. Hence parallel rays, as thoſe of the ſun, by paſſing through a concave lens, become diverging; diverging rays are made to diverge more; and converging rays are made either to converge leſs, or to become parallel, or go out diverging. And hence it is, that objects viewed through concave lenſes, appear diminiſhed; and the more ſo, as they are portions of leſs ſpheres. See Lens.

Concave mirrors have the contrary effect to lenſes: they reflect the rays which fall on them, ſo as to make them approach more to, or recede from each other, than before, according to the ſituation of the object; and that the more as the concavity is greater, or as the radius of concavity is leſs. Hence it is that concave mirrors magnifying objects that are preſented to them; and that in a greater proportion, as they are portions of greater ſpheres. And hence alſo concave mirrors have the effect of burning glaſſes. See Mirror, and Burning Glass.

CONCAVITY, that ſide of a figure or body which is hollow.

An arch of a curve has its concavity turned all one way, when the right lines that join any two of its points are all on the ſame ſide of the arch.

Archimedes, intending to include in his definition ſuch lines as have rectilinear parts, ſays, a line has its concavity turned one way, when the right lines that join any two of its points, are either all upon one ſide of it, or while ſome fall upon the line itſelf, none fall upon the oppoſite ſide. Archim. de Sphær. et Cyl. Def. 2, and Maclaurin's Fluxions, art. 180.

When two lines that have their concavity turned the ſame way, have the ſame extremes, and the one includes the other, or has its concavity towards it, the perimeter of that which includes, is greater than that which is included. Archim. ib. ax. 2.

CONCENTRIC, having the ſame centre. It is oppoſed to excentric, or having different centres.

The word is chiefly uſed in ſpeaking of round bodies and figures, ſuch as circular, and elliptic ones; but it may likewiſe be uſed for polygons that are drawn parallel to each other, from the ſame centre.

Nonnius's method of graduating inſtruments conſiſts in deſcribing with the ſame quadrant 45 concentric arches, dividing the outermoſt into 90 equal parts, the next into 89, and ſo on.

CONCHOID, or Conchites, the name of a curve invented by Nicomedes. It was much uſed by the ancients in the conſtruction of ſolid problems, as appears by what Pappus ſays.

It is thus conſtructed: AP and BD being two lines interſecting at right angles; from P draw a number of other lines PFDE, &c, on which take always DE = DF = AB or BC; ſo ſhall the curve line drawn through all the points E, E, E, be the firſt conchoid, or that of Nicomedes; and the curve drawn through all the other points F, F, F, is called the ſecond conchoid; though in reality, they are both but parts of the ſame curve, having the ſame pole P, and four infinite legs, to which the line DBD is a common aſymptote.

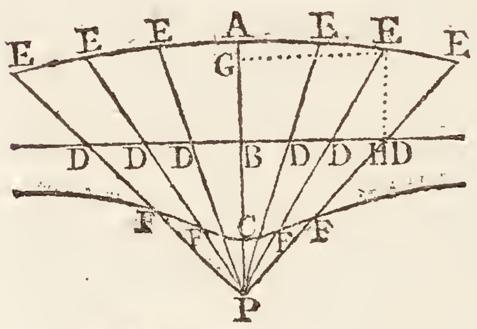


Fig. 1

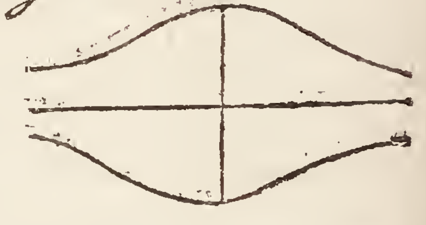


Fig. 2

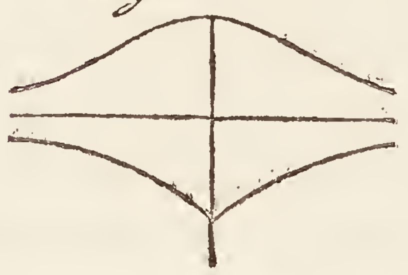
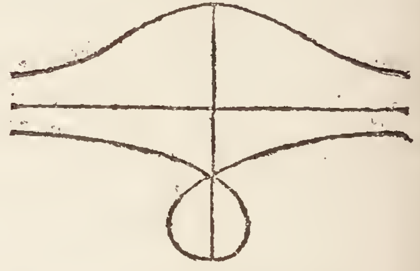


Fig. 3



The inventor, Nicomedes, contrived an inſtrument for deſcribing his conchoid by a mechanical motion: thus, in the rule AD is a channel or groove cut, ſo that a ſmooth nail, firmly fixed in the moveable rule CB, in the point F, may ſlide freely within it: into the rule EG is fixed another nail at K, for the moveable rule CB to ſlide upon. If therefore the rule BC be ſo moved, as that the nail F paſſes along the canal AD; the ſtyle, or point in C, will deſcribe the firſt conchoid.

To determine the equation of the curve: put $AB = BC = DE = DF = a$, $PB = b$, $BG = EH = x$, and $GE = BH = y$; then the equation to the firſt con-

choid will be $x^2 \times \overline{b+x}|^2 + x^2y^2 = a^2 \times \overline{b+x}|^2$, or $x^4 + 2bx^3 + b^2x^2 + x^2y^2 = a^2b^2 + 2a^2bx + a^2x^2$; and, changing only the ſign of x, as being negative in the other curve, the equation to the 2d conchoid will be $x^2 \times \overline{b-x}|^2 + x^2y^2 = a^2 \times \overline{b-x}|^2$, or
$x^4 - 2bx^3 + b^2x^2 + x^2y^2 = a^2b^2 - 2a^2bx + a^2x^2$.

Of the whole conchoid, expreſſed by theſe two equations, or rather one equation only, with different ſigns, there are three caſes or ſpecies; as firſt,
when BC is leſs than BP, the conchoid will be as in fig. 1;
when BC is equal to BP, the conchoid will be as in fig. 2;
and when BC is greater than BP, the conchoid will be as in fig. 3.

Newton approves of the uſe of the conchoid for triſecting angles, or finding two mean proportionals, or for conſtructing other ſolid problems. Thus, in the Linear Conſtruction of equations, towards the end of his Univerſal Arithmetic, he ſays, "The antients at firſt endeavoured in vain at the triſection of an angle, and the finding of two mean proportionals by a right line and a circle. Afterwards they began to conſider ſeveral other lines, as the conchoid, the ciſſoid, and the conic ſections, and by ſome of theſe to ſolve theſe problems." Again, "Either therefore the trochoid is not to be admitted at all into geometry, or elſe, in the conſtruction of problems, it is to be preferred to all lines of a more difficult deſcription: and there is the ſame reaſon for other curves; for which reaſon we approve of the triſections of an angle by a conchoid, which Archimedes in his Lemmas, and Pappus in his Collections, have preferred to the inventions of all others in this caſe; becauſe we ought either to exclude all lines, beſides the circle and right line, out of geometry, or admit them according to the ſimplicity of their deſcriptions, in which caſe the conchoid yields to none, except the circle." Laſtly, "That is *arithmetically* more ſimple which is determined by the more ſimple equations, but that is *geometrically* more ſimple which is determined by the more ſimple drawing of lines; and in geometry, that ought to be reckoned beſt which is geometrically moſt ſimple: wherefore I ought not to be blamed, if, with that prince of mathematicians, Archimedes, and other antients, I make uſe of the conchoid for the conſtruction of ſolid problems."

CONCRETE *Numbers* are thoſe that are applied to expreſs or denote any particular ſubject; as 3 men, 2 pounds, &c. Whereas, if nothing be connected with a number, it is taken abſtractedly or univerſally: thus, 4 ſignifies only an aggregate of 4 units, without any regard to a particular ſubject, whether men or pounds, or any thing elſe.

CONCURRING, or CONGRUENT *Figures*, in Geometry, are ſuch as, being laid upon one another, do exactly correſpond to, and cover one another, and conſequently muſt be equal among themſelves. Thus, triangles having two ſides and the contained angle equal, each to each, are equal to each other in all reſpects.

CONDENSATION, is the compreſſing or reducing of a body into a leſs bulk or ſpace; by which means it is rendered more denſe and compact.

Wolfius, and ſome other writers, reſtrain the uſe of the word *condenſation* to the action of cold: that which is done by external application, they call compreſſion.

Condenſation however, in general, conſiſts in bringing the parts cloſer to each other, and increaſing their contact, whatever be the means by which it is effected: in oppoſition to rarefaction, which renders the body lighter and looſer, by ſetting the parts farther aſunder, and diminiſhing their contact, and of conſequence their coheſion.

Air eaſily condenſes, either by cold, or by preſſure, but much more by the latter; but moſt of all by chemical proceſs. Water condenſes alſo both by cold and by preſſure; but it ſuddenly expands by congelation: indeed almoſt all matter, both ſolids and fluids, has the ſame property of condenſation by thoſe means. See COMPRESSION. So alſo vapour is condenſed, or converted into water, by diſtillation, or naturally in the clouds. The way in which vapour commonly condenſes, is by the application of ſome cold ſubſtance. On touching it, the vapour parts with its heat which it had before abſorbed: and on doing ſo, it immediately loſes the proper characteriſtics of vapour, and becomes water. But though this be the moſt common and uſual way in which we obſerve vapour to be condenſed, nature certainly proceeds after another manner; ſince we often obſerve the vapours moſt plentifully condenſed when the weather is really warmer than at other times.

CONDENSER, a pneumatic engine, or ſyringe, by which an extraordinary quantity of air may be crowded or puſhed into a given ſpace; ſo that frequently ten atmoſpheres, or ten times as much air as the ſpace naturally contains, without the engine, may be thrown in by means of it, and its egreſs prevented by valves properly diſpoſed.

The condenſer is made either of metal, or glaſs, and either in a cylindrical or globular form, into which the air is thrown with an injecting ſyringe.

The receiver, or veſſel containing the condenſed air, ſhould be made very ſtrong, to bear the force of the air's elaſticity thus increaſed; for which reaſon it is commonly made of braſs. When glaſs is uſed, it will not bear ſo great a condenſation of air; but then the experiment will be more entertaining, as the effect may be viewed of the condenſed air upon any ſubject put within it.

CONDUCTOR, in Electricity, a term firſt introduced in this ſcience by Dr. Deſaguliers, and uſed to denote thoſe ſubſtances which are capable of receiving and tranſmitting electricity; in oppoſition to electrics, in which the matter or virtue of electricity may be excited and accumulated, or retained. The former are alſo called *non-electrics*, and the latter *non-conductors*. And all bodies are ranked under one or other of theſe two claſſes, though none of them are *perfect electrics*, nor *perfect conductors*, ſo as wholly to retain, or freely and without reſiſtance to tranſmit the electric fluid.

To the claſs of conductors, belong all metals and ſemi-metals, ores, and all fluids (except air and oils), together with the ſubſtances containing them, the effluvia of flaming bodies, ice (unleſs very hard frozen), and ſnow, moſt ſaline and ſtony ſubſtances, charcoals, of which the beſt are thoſe that have been expoſed to the greateſt heat, ſmoke, and the vapour of hot water.

It ſeems probable that the electric fluid paſſes through the ſubſtance, and not merely over the ſurfaces of metallic conductors; becauſe, if a wire of any kind of metal be covered with ſome electric ſubſtance, as reſin, ſealing-wax, &c, and a jar be diſcharged through it, the charge will be conducted as well as without the electric coating.

It has alſo been alleged, that electricity will pervade a vacuum, and be tranſmitted through it almoſt as freely as through the ſubſtance of the beſt conductor: but Mr. Walſh found, that the electric ſpark or ſhock would no more paſs through a perfect vacuum, than through a ſtick of ſolid glaſs. In other inſtances however, when the vacuum has been made with all poſſible care, the experiment has not ſucceeded.

It may alſo be obſerved, that many of the forementioned ſubſtances are capable of being electrified, and that their conducting power may be deſtroyed and recovered by different proceſſes: for example, green wood is a conductor; but baked, it becomes a non-conductor: again its conducting power is reſtored by charring it; and laſtly it is deſtroyed by reducing this to aſhes.

Again, many electric ſubſtances, as glaſs, reſin, air, &c, become conductors by being made very hot: however, air heated by glaſs muſt be excepted.

See, on this ſubject, Prieſtley's Hiſt. of Electricity, vol. 1; Franklin's Letters &c, pa. 96 and 262 edit. 1769; Cavallo's Complete Treat. of Electr. chap. 2; Henley's Exper. and Obſer. in Electr. alſo Philoſ. Tranſ. vol. 67 pa. 122; and elſewhere in the different volumes of the Tranſactions.

Prime CONDUCTOR, is an inſulated conductor, ſo connected with the electrical machine, as to receive the electricity immediately from the excited electric.

Mr. Grey firſt employed metallic conductors in this way, in 1734; and theſe were ſeveral pieces of metal ſuſpended on ſilken ſtrings, which he charged with electricity. Mr. Du Fay faſtened to the end of an iron bar, which he uſed as his prime conductor, a bundle of linen threads, to which he applied the excited tube: but theſe were afterwards changed for ſmall wires ſuſpended from a common gun-barrel, or other metallic rod.

In the preſent advanced ſtate of the ſcience, this part of the electrical apparatus has been conſiderably improved. The prime conductor is made of hollow braſs, and uſually of a cylindrical form. Care ſhould be taken, that it be perfectly ſmooth and round, without points and ſharp edges. The ends of the conductor are ſpherical; and it is neceſſary, that the part moſt remote from the electric ſhould be made round and much larger than the reſt, the better to prevent the electric matter from eſcaping, which it always endeavours moſt to do at the greateſt diſtance from the electric: and the other end ſhould be furniſhed with ſeveral pointed wires or needles, either ſuſpended from, or fixed to an open metallic ring, and pointing to the globe or cylinder, or plate, to collect the fire. It is beſt ſupported by pillars of ſolid glaſs, covered with ſealing-wax or good varniſh. Prime conductors of a large ſize are uſually made of paſte-board, covered with tin-foil or gilt paper; theſe being uſeful for throwing off a longer and denſer ſpark than thoſe of a ſmaller ſize: they ſhould terminate in a ſmaller knob or obtuſe edge, at which the ſparks ſhould be ſolicited. Mr. Nairne prepared a conductor 6 feet in length, and 1 foot in diameter, from which he drew electrical ſparks at the diſtance of 16, 17, or 18 inches; and Dr. Van Marum ſtill far exceeded this, with a conductor of 8 inches diameter, and upwards of 20 feet long, formed of different pieces, and applied to the large electrical machine in Teyler's Muſeum at Harlem, the moſt powerful machine of the kind ever yet conſtructed. But the ſize of the conductor is always limited by that of the electric, there being a maximum which the ſize of the former ſhould not exceed; for it may be ſo large, that the diſſipation of the electricity from its ſurface may be greater than that which the electric is capable of ſupplying.

Dr. Prieſtley recommends a prime conductor of poliſhed copper, in the form of a pear, ſupported by a pillar and a firm baſis of baked wood: this receives its fire by a long arched wire of ſoft braſs, which may be eaſily bent, and raiſed or lowered to the globe: it is terminated by an open ring, in which ſome ſharp-pointed wires are hung. In the body of this conductor are holes for the inſertion of metalline rods. This, he ſays, collects the fire perfectly well, and retains it equally everywhere. Philoſ. Tranſ. vol. 64, art. 7. Hiſt. Elect. vol. 2, § 2.

Mr. Henly has contrived a new kind of prime conductor, which, from its uſe, is called the *luminous* conductor. It conſiſts of a glaſs tube 18 inches long, and 2 inches diameter. The tube is furniſhed at both ends with braſs caps and ferules about 2 inches long, cemented and made air-tight, and terminated by braſs balls. In one of theſe caps is drilled a ſmall hole, which is covered by a ſtrong valve, and ſerves for exhauſting the tube of its air. Within the tube at each end there is a knobbed wire, projecting to the diſtance of 2 inches and a half from the braſs caps. To one of the balls is annexed a fine-pointed wire for receiving and collecting the electricity, and to the other a wire with a knob or ball for diſcharging it. The conductor, thus prepared, is ſupported on pillars of ſealing-wax or glaſs. Beſide the common purpoſes of a prime conductor to an electrical machine, this apparatus ſerves to exhibit and aſcertain the direction of the electric matter in its paſſage through it. See a figure of this conductor in the Philoſ. Tranſ. with a deſcription of experiments, &c. with it, vol. 64, pa. 403.

CONDUCTORS *of Lightning*, are pointed metallic rods fixed to the upper parts of buildings, to ſecure them from ſtrokes of lightning. Theſe were invented and propoſed by Dr. Franklin for this purpoſe, ſoon after the identity of electricity and lightning was aſcertained; and they exhibit a very important and uſeful application of modern diſcoveries in this ſcience. This ingenious philoſopher, having found that pointed bodies are better fitted for receiving and throwing off the electric fire, than ſuch as are terminated by blunt ends or flat ſurfaces, and that metals are the readieſt and beſt conductors, ſoon diſcovered that lightning and electricity reſembled each other in this and other diſtinguiſhing properties: he therefore recommended a pointed metalline rod, to be raiſed ſome feet above the higheſt part of a building, and to be continued down into the ground, or the neareſt water. The lightning, ſhould it ever come within a certain diſtance of this rod or wire, would be attracted by it, and paſs through it preferably to any other part of the building, and be conveyed into the earth or water, and there diſſipated, without doing any damage to the building. Many facts have occurred to evince the utility of this ſimple and ſeemingly trifling apparatus. And yet ſome electricians, of whom Mr. Wilſon was the chief, have objected to the pointed termination of this conductor; preferring rather a blunt end: becauſe, they pretend, a point invites the electricity from the clouds, and attracts it at a greater diſtance than a blunt conductor. Philoſ. Tranſ. vol. 54, pa. 234; vol. 63, pa. 49; and vol. 68, pa. 232.

This ſubject has indeed been very accurately examined and diſcuſſed; and pointed conductors are almoſt univerſally, and for the beſt reaſons, recommended as the moſt proper and eligible. A ſharp-pointed conductor, as it attracts the electric fire of a cloud at a greater diſtance than the other, draws it off gradually: and by conveying it away gently, and in a continued ſtream, prevents an accumulation and a ſtroke; whereas a conductor with a blunt termination receives the whole diſcharge of a cloud at once, and is much more likely to be exploded, whenever a cloud comes within a ſtriking diſtance. To this may be added experience; for buildings guarded by either natural or artificial conductors terminating in a point, have very ſeldom been ſtruck by lightning; but others, having flat or blunt terminations, have often been ſtruck and damaged by it.

The beſt conductor for this purpoſe, is a rod of iron, or rather of copper, as being a better conductor of electricity, and leſs liable to ruſt, about 3 quarters of an inch thick, which is either to be faſtened to the walls of a building by wooden cramps, or ſupported by wooden poſts, at the diſtance of a foot or two from the wall; though leſs may do: the upper end of it ſhould terminate in a pyramidal form, with a ſharp point and edges; and, when made of iron, gilt or painted near the top, or elſe pointed with copper; and be elevated 5 or 6 feet above the higheſt part of the building, or chimneys, to which it may be faſtened. The lower end ſhould be driven 5 or 6 feet into the ground, and directed away from the foundations of the building, or continued till it communicates with the neareſt water: and if this part be made of lead, it will be leſs apt to decay. When the conductor is formed of different pieces of metal, care ſhould be taken that they are well joined: and it is farther recommended, that a communication be made from the conductor by plates of lead, 8 or 10 inches broad, with the lead on the ridges and gutters, and with the pipes that carry down the rain water, which ſhould be continued to the bottom of the building, and be made to communicate either with water or moiſt earth, or with the main pipe which ſerves the houſe with water. If the building be large, two, three, or more conductors ſhould be applied to different parts of it, in proportion to its extent. Philoſ. Tranſ. vol. 64, pa. 403.

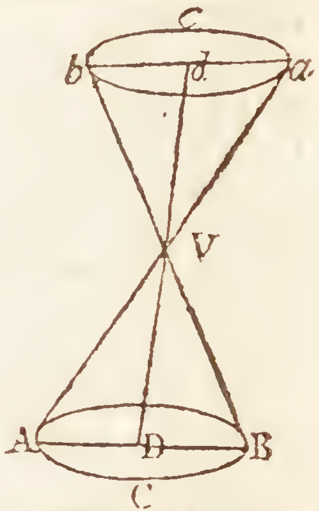
Chains have been uſed as conductors for preſerving ſhips; but as the electric matter does not paſs readily through the links of it, copper wires, a little thicker than a gooſe quill, have been preferred, and are now generally uſed. They ſhould reach 2 or 3 feet above the higheſt maſt, and be continued down in any convenient direction, ſo as always to touch the ſea water. Philoſ. Tranſ. vol. 52, pa. 633. See alſo Franklin's Letters &c 1769, pa. 65, 124, 479, &c; and Cavallo's Electr. chap. 9.

For the *Conſtruction* and management of *Electrical Kites*, and *Conductors* or *Machines* for drawing electricity from the clouds, ſee Prieſtley's Hiſt. of Electr. vol. 2, pa. 103 edit. 1775.

CONE, a kind of round pyramid, or a ſolid body having a circle for its baſe, and its ſides formed by right lines drawn from the circumference of the baſe to a point at top, being the vertex or apex of the cone.

Euclid defines a cone to be a ſolid figure, whoſe baſe is a circle, and is produced by the entire revolution of a right-angled triangle about its perpendicular leg, called the axis of the cone. If this leg, or axis, be greater than the baſe of the triangle, or radius of the circular baſe of the cone, then the cone is *acute-angled*, that is, the angle at its vertex is an acute angle; but if the axis be leſs than the radius of the baſe, it is an *obtuſe-angled* cone; and if they are equal, it is a *right-angled* cone.

But Euclid's definition only extends to a *right cone*, that is, to a cone whoſe axis is perpendicular or at right-angles to its baſe; and not to oblique ones, in which the axis is oblique to the baſe, the general definition, or deſcription of which may be this: If a line VA continually paſs through the point V, turning upon that point as a joint, and the lower part of it be carried round the circumference ABC of a circle; then the ſpace incloſed between that circle and the path of the line, is a cone. The circle ABC is the baſe of the cone; V is its vertex; and the line VD, from the vertex to the centre of the baſe, is the axis of the cone. Alſo the other part of the revolving line, produced above V, will deſcribe another cone V*acb*, called the oppoſite cone, and having the ſame common axis produced DV*d*, and vertex V.



Properties of the Cone.—1. The area or ſurface of every right cone, excluſive of its baſe, is equal to a triangle whoſe baſe is the periphery, and its height the ſlant ſide of the cone. Or, the curve ſuperficies of a right cone, is to the area of its circular baſe, as the ſlant ſide is to the radius of the baſe. And therefore, the ſame curve ſurface of the cone is equal to the ſector of a circle whoſe radius is the ſlant ſide, and its arch equal to the circumference of the baſe of the cone.

2. Every cone, whether right or oblique, is equal to one-third part of a cylinder of equal baſe and altitude; and therefore the ſolid content is found by multiplying the baſe by the altitude, and taking $\frac{1}{3}$ of the product; and hence alſo all cones of the ſame or equal baſe and altitude, are equal.

3. Although the ſolidity of an oblique cone be obtained in the ſame manner with that of a right one, it is otherwiſe with regard to the ſurface, ſince this cannot be reduced to the meaſure of a ſector of a circle, becauſe all the lines drawn from the vertex to the baſe are not equal. See a Memoir on this ſubject, by M. Euler, in the Nouv. Mem. de Peterſburgh vol. 1. Dr. Barrow has demonſtrated, in his Lectiones Geometricæ, that the ſolidity of a cone with an elliptic baſe, forming part of a right cone, is equal to the product of its ſurface by a third part of one of the perpendiculars

drawn from the point in which the axis of the right cone interſects the ellipſe; and that it is alſo equal to $\frac{1}{3}$ of the height of the cone multiplied by the elliptic baſe: conſequently that the perpendicular is to the height of the cone, as the elliptic baſe is to the curve ſurface. For the curve ſurface of all the oblique parts of a cone, ſee my Menſur. pa. 234 &c.

4. *To find the Curve Surface of the Fruſtum of a Cone.* Multiply half the ſum of the circumferences of the two ends, by the ſlant ſide, or diſtance between theſe circumferences.

5. *For the Solidity of the Fruſtum of a Cone,* add into one ſum the areas of the two ends and the mean proportional between them, multiply that ſum by the perpendicular height, and $\frac{1}{3}$ of the product will be the ſolidity. See alſo my Menſuration, pa. 189.

6. The Centre of Gravity of a cone is $\frac{3}{4}$ of the axis diſtant from the vertex.

Cones *of the Higher Kinds,* are thoſe whoſe baſes are circles of the higher kinds; and are generated, like the common cone, by conceiving a line turning on a point or vertex on high, and revolving round the circle of the higher kind.

Cone *of Rays,* in Optics, includes all the ſeveral rays which fall from any point of a radiant object, on the ſurface of a glaſs.

Double Cone, or *Spindle,* in Mechanics, is a ſolid formed of two equal cones joined at their baſes. If this be laid on the lower part of two rulers, making an angle with each other, and elevated in a certain degree above the horizontal plane, the cones will roll up towards the raiſed ends, and ſeem to aſcend, though in reality its centre of gravity deſcends perpendicularly lower.

CONFIGURATION, the exterior ſurface or ſhape that bounds bodies, and gives them their particular figure.

Configuration *of the planets,* in Aſtrology, is a certain diſtance or ſituation of the planets in the zodiac, by which it is ſuppoſed that they aſſiſt or oppoſe each other.

CONFUSED *Viſion.* See Vision.

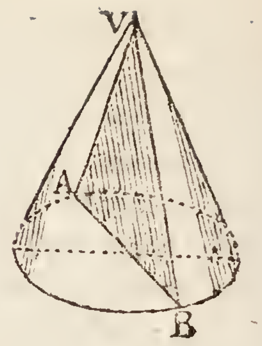
CONGELATION, or Freezing, the act of fixing the fluidity of any liquid, by cold, or the application of cold bodies: in which it differs from coagulation, which is produced by other cauſes. See Freezing, Frost, and Ice.

CONGRUITY, in Geometry, is applied to lines and figures, which exactly correſpond when laid over one another; as having the ſame terms, or bounds. It is aſſumed, as an axiom, that thoſe things are equal and ſimilar, between which there is a congruity. Euclid, and moſt geometricians after him, demonſtrate great part of their elements from the principle of congruity: though Leibnitz and Wolfius ſubſtitute the notion of Similitude inſtead of that of congruity.

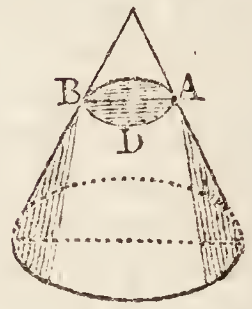
CONIC Sections, are the figures made by cutting a cone by a plane.

2. According to the different poſitions of the cutting plane, there ariſe five different figures or ſections, viz, a *triangle,* a *circle,* an *ellipſe,* a *parabola,* and an *hyperbola:* the laſt three of which only are peculiarly called conic ſections.

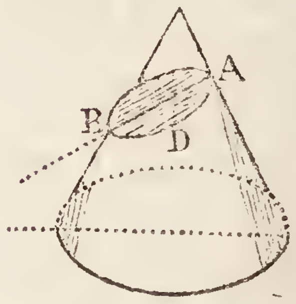
3. If the cutting plane paſs through the vertex of the cone, and any part of the baſe, the ſection will evidently be a *triangle;* as VAB.



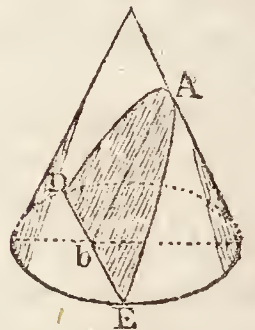
4. If the plane cut the cone parallel to the baſe, or make no angle with it, the ſection will be a *circle,* as ABD.



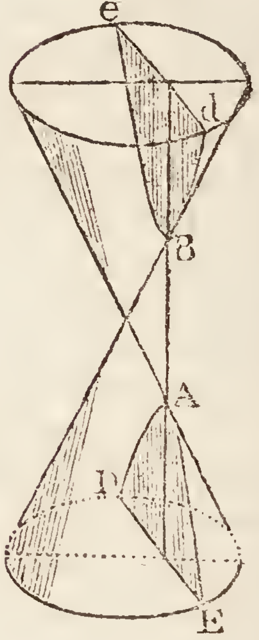
5. The ſection DAB is an *ellipſe,* when the cone is cut obliquely through both ſides, or when the plane is inclined to the baſe in a leſs angle than the ſide of the cone is.



6. The ſection is a *parabola,* when the cone is cut by a plane parallel to the ſide, or when the cutting plane and the ſide of the cone make equal angles with the baſe.



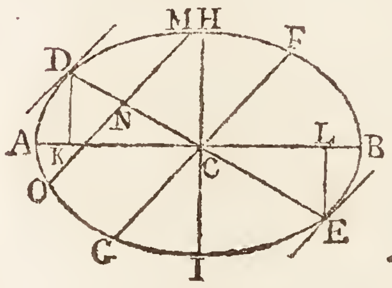
7. The ſection is an *hyperbola,* when the cutting plane makes a greater angle with the baſe than the ſide of the cone makes. And if the plane be continued to cut the oppoſite cone, this latter ſection is called the oppoſite hyperbola to the former; as *dBe.*



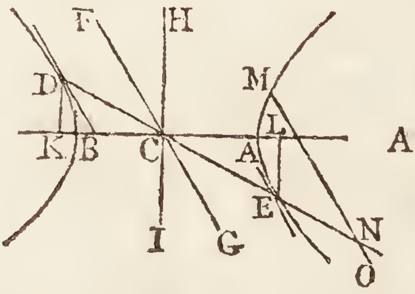
8. The *vertices* of any ſection, are the points where the cutting plane meets the oppoſite ſides of the cone, or the ſides of the vertical triangular ſection; as A and B. —Hence, the ellipſe and the oppoſite hyperbolas have each two vertices; but the parabola only one; unleſs we conſider the other as at an infinite diſtance.

9. The *axis,* or *tranſverſe diameter* of a conic ſection, is the line or diſtance AB between the vertices.—Hence the axis of a parabola is infinite in length.

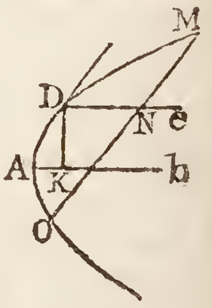
Ellipſe.



Oppoſ. Hyperb.



Parabola.



10. The *centre* C is the middle of the axis.—Hence the centre of a parabola is infinitely diſtant from the vertex. And of an ellipſe, the axis and centre lie within the curve; but of an hyperbola, without.

11. A *Diameter* is any right line, as AB or DE, drawn through the centre, and terminated on each ſide by the curve: and the extremities of the diameter, or its interſections with the curve, are its *vertices.*—Hence all the diameters of a parabola are parallel to the axis, and infinite in length; becauſe drawn through the centre, a point at an infinite diſtance. And hence alſo every diameter of the ellipſe and hyperbola have two vertices; but of the parabola, only one; unleſs we conſider the other as at an infinite diſtance.

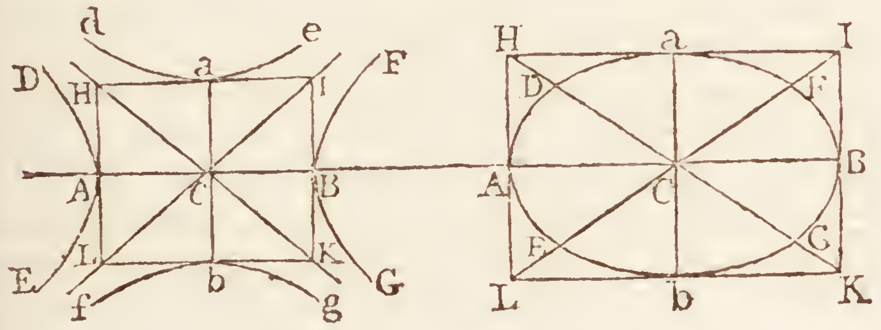
12. The *conjugate* to any diameter, is the line drawn through the centre, and parallel to the tangent of the curve at the vertex of the diameter. So FG, parallel to the tangent at D, is the conjugate to DE; and HI, parallel to the tangent at A, is the conjugate to AB. —Hence the conjugate HI, of the axis AB, is perpendicular to it; but the conjugates of other diameters are oblique to them.

13. An *ordinate* to any diameter, is a line parallel to its conjugate, or to the tangent at its vertex, and terminated by the diameter and curve. So DK and EL are ordinates to the axis AB; and MN and NO ordinates to the diameter DE.—Hence the ordinates of the axis are perpendicular to it; but of other diameters, the ordinates are oblique to them.

14. An *abſciſs* is a part of any diameter, contained between its vertex and an ordinate to it; as AK or BK, and DN or EN.—Hence, in the ellipſe and hyperbola, every ordinate has two abſciſſes; but in the parabola, only one; the other vertex of the diameter being infinitely diſtant.

15. The *parameter* of any diameter, is a third proportional to that diameter and its conjugate.

16. The *focus* is the point in the axis where the ordinate is equal to half the parameter: as K and L, where DK or EL is equal to the ſemiparameter.—— Hence, the ellipſe and hyperbola have each two foci; but the parabola only one.——The foci, or burning points, were ſo called, becauſe all rays are united or reflected into one of them, which proceed from the other focus, and are reflected from the curve.



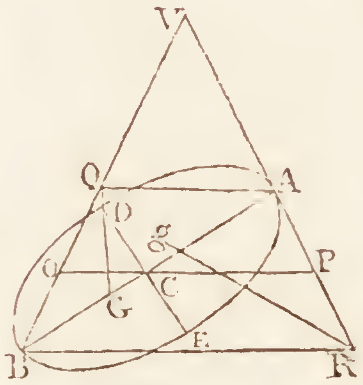
17. If DAE, FBG be two oppoſite hyperbolas, having AB for their firſt or tranſverſe axis, and *ab* for their ſecond or conjugate axis; and if *dae*, *fbg* be two other oppoſite hyperbolas, having the ſame axis, but in the contrary order, viz, *ab* their firſt axis, and AB their ſecond; then theſe two latter curves *dae*, *fbg*, are called the *conjugate hyperbolas* to the two former DAE, FBG; and each pair of oppoſite curves mutually conjugate to the other.

18. And if tangents be drawn to the four vertices of the curves, or extremities of the axis, forming the inſcribed rectangle HIKL; the diagonals HCK and ICL, of this rectangle, are called the *aſymptotes* of the curves.

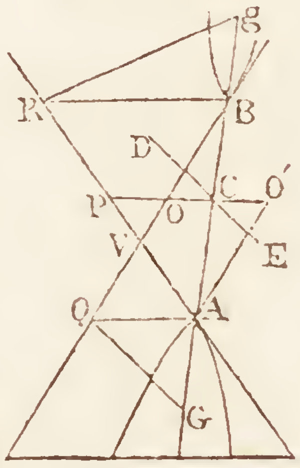
19. *Scholium.* The rectangle inſcribed between the four conjugate hyperbolas, is ſimilar to a rectangle circumſcribed about an ellipſe, by drawing tangents, in like manner, to the four extremities of the two axes; alſo the aſymptotes or diagonals in the hyperbola, are analogous to thoſe in the ellipſe, cutting this curve in ſimilar points, and making the pair of equal conjugate diameters. Moreover, the whole figure, formed by the four hyperbolas, is, as it were, an ellipſe turned inſide out, cut open at the extremities D, E, F, G, of the ſaid equal conjugate diameters, and thoſe four points drawn out to an infinite diſtance, the curvature being turned the contrary way, but the axes, and the rectangle paſſing through their extremities, remaining fixed, or unaltered.

From the foregoing definitions are eaſily derived the following general corollaries to the ſections.

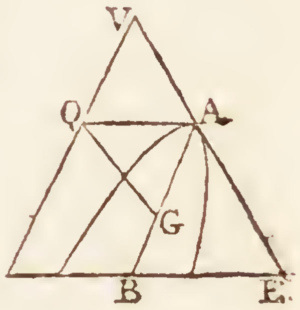
Ellipſe.



Hyperbola.



Parabola.



20. *Corol.* 1. In the ellipſe, the ſemiconjugate axis, CD or CE, is a mean proportional between CO and CP, the parts of the diameter OP of a circular ſection of the cone, drawn through the centre C of the ellipſe, and parallel to the baſe of the cone. For DE is a double ordinate in this circle, being perpendicular to OP as well as to AB.

21. In like manner, in the hyperbola, the length of the ſemiconjugate axis, CD or CE, is a mean proportional between CO and CP, drawn parallel to the baſe, and meeting the ſides of the cone in O and P. Or, if AO′ be drawn parallel to the ſide VB, and meet PC produced in O′, making $CO' = CO$; and on this diameter O′P a circle be drawn parallel to the baſe: then the ſemiconjugate CD or CE will be an ordinate of this circle, being perpendicular to O′P as well as to AB.

Or, in both figures, the whole conjugate axis DE is a mean proportional between QA and BR, parallel to the baſe of the cone. See my Conic Sections, pa. 6.

In the parabola, both the tranſverſe and conjugate are infinite; for AB and BR are both infinite.

22. *Corol.* 2. In all the ſections, AG will be equal to the parameter of the axis, if QG be drawn making the angle AQG equal to the angle BAR. In like manner B*g* will be equal to the ſame parameter, if R*g* be drawn to make the angle B*Rg* = the angle ABQ.

23. *Corol.* 3. Hence the upper hyperbolic ſection, or ſection of the oppoſite cone, is equal and ſimilar to the lower one. For the two ſections have the ſame tranſverſe or firſt axis AB, and the ſame conjugate or ſecond axis DE, which is the mean proportional between AQ and RB; and they have alſo equal parameters.

ters AG, Bg. So that the two oppoſite ſections make, as it were, but the two oppoſite ends of one entire ſection or hyperbola, the two being every where mutually equal and ſimilar. Like the two halves of an ellipſe, with their ends turned the contrary way.

24. *Corol.* 4. And hence, although both the tranſverſe and conjugate axis in the parabola be infinite, yet the former is infinitely greater than the latter, or has an infinite ratio to it. For the tranſverſe has the ſame ratio to the conjugate, as the conjugate has to the parameter, that is, as an infinite to a finite quantity, which is an infinite ratio.

The peculiar properties of each particular curve, will be beſt referred to the particular words Ellipse, Hyperbola, Parabola; and therefore it will only be proper here to lay down a few of the properties that are common to all the conic ſections.

Some other General Properties.

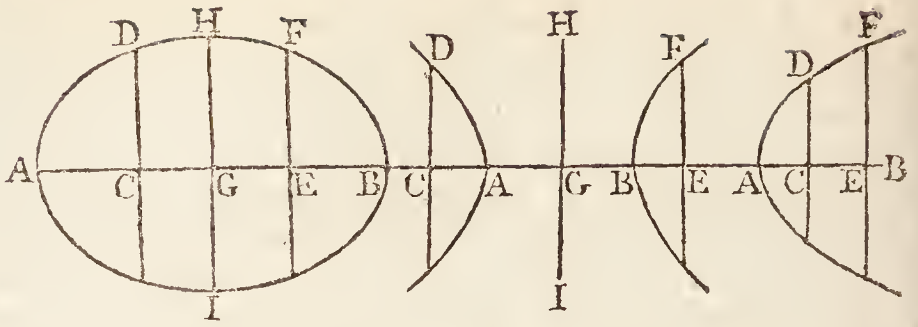
25. From the foregoing definitions, &c, it appears, that the conic ſections are in themſelves a ſyſtem of regular curves, naturally allied to each other; and that one is changed into another perpetually, when it is either increaſed, or diminiſhed, in infinitum. Thus, the curvature of a circle being ever ſo little increaſed or diminiſhed, paſſes into an ellipſe; and again, the centre of the ellipſe going off infinitely, and the curvature being thereby diminiſhed, is changed into a parabola; and laſtly, the curvature of a parabola being ever ſo little changed, there ariſeth the firſt of the hyperbolas; the innumerable ſpecies of which will all of them ariſe orderly by a gradual diminution of the curvature; till this quite vaniſhing, the laſt hyperbola ends in a right line. From whence it is manifeſt, that every regular curvature, like that of a circle, from the circle itſelf to a right line, is a conical curvature, and is diſtinguiſhed with its peculiar name, according to the divers degrees of that curvature.

26. That all diameters in a circle and ellipſe interſect one another in the centre of the figure within the ſection: that in the parabola they are all parallel among themſelves, and to the axis: but in the hyperbola, they interſect one another, without the figure, in the common centre of the oppoſite and conjugate ſections.

27. In the circle, the *latus rectum*, or parameter, is double the diſtance from the vertex to the focus, which is alſo the centre. But in ellipſes, the parameters are in all proportions to that diſtance, between the double and quadruple, according to their different ſpecies. While, in the parabola, the parameter is juſt quadruple that diſtance. And, laſtly in hyperbolas, the parameters are in all proportions beyond the quadruple, according to their various kinds.

28. The firſt general property of the conic ſections, with regard to the abſciſſes and ordinates of any diameter, is, that the rectangles of the abſciſſes are to each other, as the ſquares of their correſponding ordinates. Or, which is the ſame thing, that the ſquare of any diameter is to the ſquare of its conjugate, as the rectangle of two abſciſſes of that diameter, to the ſquare of the ordinate which divides them. That is, in all the figures,

the rect. AC . CB : rect. AE . EB : : $CD^2 : EF^2$:



But as, in the parabola the infinites CB and EB are in a ratio of equality, for this curve the ſame property becomes AC : AE : : $CD^2 : EF^2$, that is, in the parabola, the abſciſſes are as the ſquares of their ordinates.

Or, when one of the ordinates is the ſemiconjugate GH, dividing the diameter equally in the centre, the ſame general property becomes,

AG . GB or AG^2 : AC . CB : : $GH^2 : CD^2$,
or $AB^2 : HI^2$: : AC . CB : CD^2.

29. From hence is derived the equation of the curves of the conic ſections; thus, putting the diameter AB $= d$, its conjugate HI $= c$, abſciſs AC $= x$, and its ordinate CD $= y$; then is the other abſciſs CB $= d - x$ in the ellipſe, or $d + x$ in the hyperbola, or d in the parabola; and hence the laſt analogy above, becomes

$d^2 : c^2 :: x . \overline{d \mp x}$ or $dx \mp x^2 : y^2$,

or $dy^2y^2 = c^2 . \overline{dx \mp x^2}$ is the general equation for all the conic ſections; and, in particular, it is

$d^2y^2 = c^2 . \overline{dx - x^2}$ in the ellipſe,

$d^2y^2 = c^2 . \overline{dx + x^2}$ in the hyperbola, and

$d^2y^2 = c^2dx$, or $dy^2 = c^2x$ in the parabola: Or the three equations may be otherwiſe expreſſed thus:

$y^2 = \frac{c^2}{d^2} . \overline{dx - x^2}$ in the ellipſe,

$y^2 = \frac{c^2}{d^2} . \overline{dx + x^2}$ in the hyperbola, and

$y^2 = \frac{c^2}{d} x$ or $= px$ in the parabola, where the parameter $p = \frac{c^2}{d}$ the third proportional to the diameter and its conjugate, by the definition of it.

And from this one general propoſition alone, which is eaſily derived from the ſection in the ſolid cone itſelf, together with the definitions only, as laid down above, all the other properties of all the ſections may eaſily be derived, without any farther reference to the cone, and without mechanical deſcriptions of the curves in plano; as is done in my Treatiſe on Conic Sections, for the uſe of the Royal Mil. Acad.; in which alſo all the ſimilar propoſitions in the ellipſe and hyperbola are carried on word for word in them both.

The more ancient mathematicians, before the time of Apollonius Pergæus, admitted only the right cone into their geometry, and they ſuppoſed the ſection of it to be made by a plane perpendicular to one of its ſides; and as the vertical angle of a right cone may be either right, acute, or obtuſe, the ſame method of cutting theſe ſeveral cones, viz, by a plane perpendicular to one ſide, produced all the three conic ſections. The parabola was called the ſection of a right-angled cone; the ellipſe, the ſection of the acute-angled cone; and the hyperbola, the ſection of the obtuſe-angled cone. But Apollonius, who, on account of his writings on this ſubject, obtained the appellation of *Magnus Geometra*, the Great Geometrician, obſerved, that theſe three ſections might be obtained in every cone, both oblique and right, and that they depended on the different inclinations of the plane of the ſection to the cone itſelf. Apollon. Con. Halley's edit. lib. 1, p. 9.

Inſtead of conſidering theſe curves as ſections cut from the ſolid cone, which is the true genuine way of all the ancients, and of the moſt elegant writers among the moderns, Deſcartes, and ſome others of the moderns, have given arbitrary conſtructions of curves on a plane, from which conſtructions they have demonſtrated the properties of theſe, and have afterwards proved that ſome principal property of them belongs to ſuch curves or ſections as are cut from a cone; and hence it is inferred by them that thoſe curves, ſo deſcribed on a plane, are the ſame with the conic ſections.

The doctrine of the conic ſections is of great uſe in phyſical and geometrical aſtronomy, as well as in the phyſico-mathematical ſciences. The doctrine has been much cultivated by both ancient and modern geometricians, who have left many good treatiſes on the ſubject. The moſt ancient of theſe is that of Apollonius Pergæus, containing 8 books, the firſt 4 of which have often been publiſhed; but Dr. Halley's edition has all the eight. Pappus, in his Collect. Mathem. lib. 7, ſays that the firſt four of theſe were written by Euclid, though perfected by Apollonius, who added the other 4 to them. Among the moderns, the chief writers are Mydorgius de Sectionibus Conicis; Gregory St. Vincent's Quadratura Circuli & Sectionum Coni; De la Hire de Sectionibus Conicis; Trevigar Elem. Section. Con.; De Witt's Elementa Curvarum; Dr. Wallis's Conic Sections; De l'Hoſpital's Anal. Treat. of Conic Sections; Dr. Simſon's Section. Con.; Milne's Elementa Section. Conicarum; Muller's Conic Sections; Steel's Conic Sections; Dr. Hamilton's elegant treatiſe; my own treatiſe, above cited; and at the writing of this, my friend Mr. Abram Robertſon of Oxford is preparing a curious work on this ſubject, containing at the ſame time a treatiſe on the ſcience, and a hiſtory of the writings relating to it.

CONICS, that part of the higher geometry, or geometry of curves, which conſiders the cone, and the ſeveral curve lines ariſing from the ſections of it.

CONJUGATE *Axis*, or *Diameter*, in the Conic Sections, is the axis, or a diameter parallel to a tangent to the curve at the vertex of another axis, or diameter, to which that is a conjugate. Indeed the two are mutually conjugates to each other, and each is parallel to the tangent at the vertex of the other.

Conjugate Hyperbolas, alſo called *Adjacent Hyperbolas*, are ſuch as have the ſame axes, but in the contrary order, the firſt or principal axis of the one being the 2d axis of the other, and the 2d axis of the former, the 1ſt axis of the latter. See art. 17 of Conic Sections.

CONJUNCTION, in Aſtronomy, is the meeting of the ſtars and planets in the ſame point or place in the heavens; and is either true or apparent.

True Conjunction is when the line drawn through the centres of the two ſtars paſſes alſo through the centre of the earth. And *Apparent* Conjunction is when that line does not paſs through the earth's centre.

CONOID, is a figure reſembling a cone, except that the ſlant ſides from the baſe to the vertex are not ſtraight lines as in the cone, but curved. It is generated by the revolution of a conic ſection about its axis; and it is therefore threefold, anſwering to the three ſections of the cone, viz, the *Elliptical Conoid*, or ſpheroid, the *Hyperbolic Conoid*, and the *Parabolic Conoid*.

If a conoid be cut by a plane in any poſition, the ſection will be of the figure of ſome one of the conic ſections; and all parallel ſections, of the ſame conoid, are like and ſimilar figures. When the ſection of the ſolid returns into itſelf, it is an ellipſe; which is always the caſe in the ſections of the ſpheroid, except when it is perpendicular to the axis; which poſition is alſo to be excepted in the other ſolids, the ſection being always a circle in that poſition. In the parabolic conoid, the ſection is always an ellipſe, except when it is parallel to the axis. And in the hyperbolic conoid, the ſection is an ellipſe, when its axis makes with the axis of the ſolid, an angle greater than that made by the ſaid axe of the ſolid and the aſymptote of the generating hyperbola; the ſection being an hyperbola in all other caſes, but when thoſe angles are equal, and then it is a parabola.

But when the ſection is parallel to the fixed axis, it is of the ſame kind with, and ſimilar to the generating plane itſelf; that is, the ſection parallel to the axis, in the ſpheroid, is an ellipſe ſimilar to the generating ellipſe; in the parabolic conoid it is a parabola, ſimilar to the generating one; and in the hyperbolic conoid, it is an hyperbola ſimilar to the generating one.

The ſection through the axis, which is the generating plane, is, in the ſpheroid the greateſt of the parallel ſections, but in the hyperboloid it is the leaſt, and in the paraboloid thoſe parallel ſections are all equal.

The analogy of the ſections of the hyperboloid to thoſe of the cone, are very remarkable, all the three conic ſections being formed by cutting an hyperboloid in the ſame poſitions as the cone is cut. Thus, let an hyperbola and its aſymptote be revolved together about the tranſverſe axis, the former deſcribing an hyperboloid, and the latter a cone circumſcribing it: then let it be ſuppoſed that they are both cut by one plane in any poſition; ſo ſhall the two ſections be like, ſimilar, and concentric figures: that is, if the plane cut both the ſides of each, the ſections will be concentric and ſimilar ellipſes; but if the cutting plane be parallel to the aſymptote, or to the ſide of the cone, the ſections will be parabolas; and in all other poſitions, the ſections will be ſimilar and concentric hyperbolas.

And this analogy of the ſections will not ſeem ſtrange, when it is conſidered that a cone is a ſpecies of the hyperboloid; or a triangle a ſpecies of the hyperbola, the axes being infinitely ſmall. See my Menſuration, prop. 1, part 3, ſect. 4, pag. 265 edit. 8vo.

CONON (*of* Samos), a reſpectable mathematician and philoſopher, who flouriſhed about the 130th olympiad, being a contemporary and friend of Archimedes, to whom Conon communicated his writings, and ſent him ſome problems, which Archimedes received with approbation, ſaying they ought to be publiſhed while Conon was living, for he comprehends them with eaſe, and can give a proper demonſtration of them.

At another time he laments the loſs of Conon, thus admiring his genius. “ How many theorems in geometry, ſays he, which at firſt ſeemed impoſſible, would in time have been brought to perfection ! Alas ! Conon, though he invented many, with which he enriched geometry, had not time to perfect them, but left many in the dark, being prevented by death.” He had an uncommon ſkill in mathematics, joined to an extraordinary patience and application. This is farther confirmed by a letter ſent to Archimedes by a friend of Conon's. “ Having heard of Conon's death, with whoſe friendſhip I was honoured, and with whom you kept an intimate correſpondence ; as he was thoroughly verſed in geometry, I greatly lament the loſs of a ſincere friend, and a perſon of ſurpriſing knowledge in mathematics. I then determined to ſend to you, as I had before done to him, a theorem in geometry, hitherto obſerved by no one.”

Conon had ſome diſputes with Nicoteles, who wrote againſt him, and treated him with too much contempt. Apollonius confeſſes it ; though he acknowledges that Conon was not fortunate in his demonſtrations.

Conon invented a kind of volute, or ſpiral, different from that of Dynoſtratus ; but becauſe Archimedes explained the properties of it more clearly, the name of the inventor was forgotten, and it was hence called Archimedes's volute or ſpiral.

As to Conon's aſtrological or aſtronomical knowledge, it may in ſome meaſure be gathered from the poem of Catullus, who deſcribes it in the beginning of his verſes on the hair of Berenice, the ſiſter and wife of Ptolomy Euergetes, upon the occaſion of Conon having given out that it was changed into a conſtellation among the ſtars, to conſole the queen for the loſs, when it was ſtolen out of the temple, where ſhe had conſecrated it to the gods.

CONSECTARY, or *Corollary*, a conſequence deduced from ſome foregoing principles.

CONSEQUENT, is the latter of the two terms of a ratio ; or that to which the antecedent is referred and compared. Thus, in the ratio $a : b$, or a to b, the latter term b is the conſequent, and a is the antecedent.

CONSISTENT *Bodies*, is a term much uſed by Mr. Boyle, for ſuch as are uſually called *firm*, or *fixed bodies ;* in oppoſition to *fluid* ones.

CONSOLE, in Architecture, is an ornament cut upon the key of an arch, having a projecture or jetting, and occaſionally ſerving to ſupport ſmall cornices, buſts, and baſes.

CONSONANCE, in Muſic, is commonly uſed in the ſame ſenſe with *concord*, viz, for the union or agreement of two ſounds produced at the ſame time, the one grave, the other acute, which is compounded together by ſuch a proportion of each, as proves agreeable to the ear.

An uniſon is the firſt conſonance, an eighth is the 2d, a fifth is the 3d ; and then follow the fourth, with the third and ſixths, major and minor.

CONSTANT *Quantities* are ſuch as remain invariably the ſame, while others increaſe or decreaſe. Thus, the diameter of a circle is a conſtant quantity ; for it remains the ſame while the abſciſſes and ordinates, or the ſines, tangents, &c, are variable.

Theſe are ſometimes called *given*, or *invariable* or *permanent* quantities ; and in algebra it is now uſual to repreſent them by the leading letters of the alphabet, a, b, c, &c ; while the variable ones are denoted by the laſt latters, z, y, x, &c.

CONSTELLATIONS, certain imaginary figures of birds, beaſts, fiſhes, and other things in the heavens, within which are arranged certain ſtars. Theſe aſſemblages are alſo ſometimes called aſteriſms.

The ancients portioned out the firmament into ſeveral parts, or conſtellations ; reducing a certain number of ſtars under the repreſentation of certain images, to aſſiſt the imagination and memory, to conceive or retain their number, order, and diſpoſition, or even to diſtinguiſh the virtues they attributed to them.

The diviſion of the heavens into conſtellations is very ancient ; being known to the moſt early authors, whether ſacred or profane. In the book of Job the names of ſome of them are mentioned ; witneſs that ſublime expoſtulation, *Canſt thou reſtrain the ſweet influence of the* Pleiades, *or looſen the bands of* Orion ? And the ſame may be obſerved of the oldeſt among the heathen writers, Heſiod and Homer.

The diviſion of the ancients took in only the viſible firmament, or ſo much as came under their notice, as viſible to the naked eye. The firſt or earlieſt of theſe, is contained in the catalogue of Ptolomy, given in the 7th book of his Almageſt, prepared, as he aſſures us, from his own obſervations, compared with thoſe of Hipparchus, and the other ancient aſtronomers. In this catalogue Ptolomy has formed 48 conſtellations. Of theſe, 12 are about the ecliptic, commonly called the 12 ſigns ; 21 to the north of it ; and 15 to the ſouth. The northern conſtellations are, the Little Bear, the Great Bear, the Dragon, Cepheus, Bootes, the Northern Crown, Hercules, the Harp, the Swan, Caſſiopeia, Perſeus, Auriga, Ophiucus or Serpentary, the Serpent, the Arrow, the Eagle, the Dolphin, the Horſe, Pegaſus, Andromeda, and the Triangle.

The conſtellations about the ecliptic are Aries, Taurus, Gemini, Cancer, Leo, Virgo, Libra, Scorpio, Sagittarius, Capricorn, Aquarius, and Piſces : or according to the Engliſh names, the Ram, the Bull, the Twins, the Crab, the Lion, the Virgin, the Balance, the Scorpion, the Archer, the Goat, the Water bearer, and the Fiſhes.

The Southern conſtellations are, the Whale, Orion, the Eridanus, the Hare, the Great Dog, the Little Dog, the Ship, the Hydra, the Cup, the Raven, the Centaur, the Wolf, the Altar, the Southern Crown, and the Southern Fiſh.

The other ſtars not comprehended under theſe conſtellations, yet viſible to the naked eye, the ancients called *informes*, or *ſporades*, ſome of which the modern aſtronomers have ſince reduced into new figures, or conſtellations. Ptolomy has ſet down the longitude and latitude of all theſe ſtars to about the year of Chriſt 137, amounting to the number of 1022, viz,

in the northern conſtellations - -	360
in the zodiacal conſtellations - -	346
in the ſouthern conſtellations - -	316
in all of Ptolomy's catalogue - -	1022

Among the modern aſtronomers, Tycho Brahe is the firſt who determined, with exactneſs, and in conſequence of his own obſervations, the long. and lat. of the fixed ſtars, out of which he formed 45 conſtellations; of theſe, 43 were of the old ones deſcribed by Ptolomy, to which Tycho added the Coma Berenices, and Antinous; but he omits 5 of the old ſouthern conſtellations, viz, the Centaur, the Wolf, the Altar, the Southern Crown, and Southern Fiſh; which he could not obſerve, becauſe of the high northern latitude of Uranibourg.

After Tycho, Bayer gave the figures of 60 conſtellations, very exactly repreſented, and with tables annexed, having added, to the 48 old ones of Ptolomy, the following 12 about the ſouth pole, viz, the Peacock, the Toucan, the Crane, the Phœnix, the Dorado, the Flying Fiſh, the Hydra, the Chameleon, the Bee, the Bird of Paradiſe, the Triangle, and the Indian. Beſides accurately diſtinguiſhing the relative ſize and the ſituation of every ſtar, Bayer marks the ſtars in each conſtellation with the letters of the Greek and Roman alphabets, ſetting the firſt letter α to the firſt or principal ſtar in each conſtellation, β to the 2d in order, γ to the 3d, and ſo on; a very uſeful method of noting and deſcribing the ſtars, which has been uſed by all aſtronomers ſince, and who have farther enlarged this method, by adding the ordinal numbers 1, 2, 3, &c, to the other ſtars diſcovered ſince his time, when any conſtellation contains more than can be marked by the two alphabets. The number and order of the ſtars, as mentioned by Bayer, are,

of the 1ſt magnitude - -	17
of the 2d magnitude - -	63
of the 3d magnitude .	196
of the 4th magnitude - -	415
of the 5th magnitude - -	348
of the 6th magnitude - -	341
of the unformed ſtars - -	326
in all -	1706

After Bayer, a catalogue, with new conſtellations, was publiſhed by Schiller, in 1627, in a work called Coelum Stellatum Chriſtianum, the Chriſtian Starry Heaven, in which he ſubſtitutes, very improperly, other figures of the conſtellations, and names, taken from the ſacred ſcriptures, inſtead of the old ones.

In the year 1665, Riccioli publiſhed his Aſtronomy Reformed, containing a catalogue of the ſtars in 62 conſtellations, viz, the 60 of Bayer, with the Coma Berenices and Antinous of Tycho. He diſtributes the ſtars in all the conſtellations into four claſſes. In the firſt of theſe claſſes are contained thoſe ſtars determined by his own obſervations, and thoſe of Grimaldi. In the ſecond are thoſe ſtars which had been aſcertained by Tycho Brahe and Kepler. In the 3d are the ſtars determined by Hipparchus and Ptolomy. And the 4th claſs conſiſts of thoſe of the ſouthern hemiſphere diſcovered by Navigators, who have aſcertained their places in a more or leſs accurate manner; in which he has marked the longitudes and latitudes for the year 1700, the period to which he has reduced all his obſervations. This catalogue was followed by a number of celeſtial ſchemes and maps of the heavens, publiſhed in 1673 by Pardies, who has repreſented very carefully all the conſtellations, with the ſtars they contain. After this, Vitalis publiſhed a catalogue of the fixed ſtars in his Tables of the Primum Mobile, in which their longitudes and latitudes, with the right-aſcenſions and declinations are ſet down for the year 1675.

Some time after this, Royer publiſhed maps of the heavens, reduced into 4 tables, with a catalogue of the fixed ſtars for the year 1700. To the ſtars marked by Bayer, he adds a number of ſtars not before ſeen, with others taken from the tables of Riccioli, and not mentioned by Bayer: he alſo forms, out of the unformed ſtars, eleven other conſtellations. Five of theſe are to the north, and are called the Giraffe, the River Jordan, the River Tigris, the Sceptre, and the Flower-de-luce; with 6 on the ſouth part, which are the Dove, the Unicorn, the Croſs, the Great Cloud, the Little Cloud, and the Rhomboide. To this work Royer has joined the catalogue of the ſouthern ſtars obſerved by Dr. Halley at the iſland of St. Helena.

Hevelius has alſo improved upon the labours of thoſe who went before him, and collected together ſeveral ſtars of the before unformed claſs into ſome new conſtellations. Theſe are, the Unicorn, the Cameloparda-lis, deſcribed by Bartſchius, the Sextant of Urania, the Dogs, the Little Lion, the Lynx, the Fox and Gooſe, the Sobieſki's Crown, the Lizard, the Little Triangle, and the Cerberus; to which Gregory has added the Ring and the Armilla. Some of theſe new conſtellations however anſwer to thoſe of Royer, as the Camelopardal to the Giraffe, the Dogs to the River Jordan, and the Fox to the River Tigris. The latitudes and longitudes are added for the year 1700.

Finally, Flamſteed has given a catalogue of the fixed ſtars, not only much more correct, but much larger than thoſe of all that went before him. He has ſet down the longitude, latitude, right aſcenſion, and polar diſtance of 2934 ſtars, as they were at the beginning of 1690, all determined from his own obſervations. He diſtinguiſhes all the ſtars into ſeven claſſes, or orders of magnitude, diſtinguiſhing thoſe of Bayer by his letters, and marking their variation in right aſcenſion, for ſhewing their ſituation in the ſucceeding years. See the term Catalogue.

This catalogue was followed by an Atlas Cœleſtis, publiſhed at London in the year 1729, deſcribing, in ſeveral ſchemes, the figures of the conſtellations ſeen in our hemiſphere, with the exact poſition of the fixed ſtars, with reſpect to the circles of the ſphere, as reſulting from the laſt catalogue corrected by Flamſteed. And ſtill later obſervations, made with farther improved teleſcopes, have greatly enlarged the number and accuracy of the ſtars; but the number of the conſtellations remains the ſame as above deſcribed, except that an attempt has lately been made by Dr. Hill to add to the liſt 14 new ones, formed out of more of the cluſters of unformed ſtars.

Beſide the literal marks of the ſtars introduced by Bayer, it is uſual alſo to diſtinguiſh them by that part of the conſtellation in which they are placed; and many of them again have their peculiar names; as Arcturus, between the knees of Bootes; Gemina, or Lucida, in the Corona Septentrionalis, or Northern Crown; Palilitium, or Aldebaran, in the Bull's eye, Pleiades in his neck, and Hyades in his forehead; Caſtor and Pollux in the heads of Gemini; Capella, with the Hœdi in the ſhoulder of Auriga; Regulus, or Cor Leonis, the Lion's Heart; Spica Virginis in the hand, and Vindemiatrix in the ſhoulder of Virgo; Antares, or Cor Scorpionis, the Scorpion's Heart; Fomalhaut, in the mouth of Piſcis Auſtralis, or Southern Fiſh; Regel, in the foot of Orion; Sirius, in the mouth of Canis Major, the Great Dog; Procyon, in the back of Canis Minor, the Little Dog; and the Pole Star, the laſt in the tail of Urſa Minor, the Little Bear.

The Greek and Roman poets, from the ancient theology, give wild and romantic fables about the origin of the conſtellations, probably derived from the hieroglyphics of the Egyptians, and tranſmitted, with ſome alterations, from them to the Greeks, who probably obſcured them greatly with their own fables. See Hyginus's Poeticon Aſtron.; Riccioli Almageſt. lib. 6. cap. 3, 4, 5; Shelburne's Notes upon Manilius; Bailly's Antient Aſtronomy; and Gebelin's Monde Primitif, vol. 4: from the whole of which it is made probable, that the invention of the ſigns of the zodiac, and probably of moſt of the other conſtellations of the ſphere, is to be aſcribed to ſome very ancient nation, inhabiting the northern temperate zone, probably what is now called Tartary, or the parts to the northward of Perſia and China; and from thence tranſmitted through China, India, Babylon, Arabia, Egypt, Greece, &c.

It is a very probable conjecture, that the figures of the ſigns in the zodiac, are deſcriptive of the ſeaſons of the year, or months, in the ſun's path: thus, the firſt ſign Aries, denotes, that about the time when the ſun enters that part of the ecliptic, the lambs begin to follow the ſheep; that on the ſun's approach to the 2d conſtellation, Taurus, the Bull, is about the time of the cows bringing forth their young. The third ſign, now Gemini, was originally two kids, and ſignified the time of the goats bringing forth their young, which are uſually two at a birth, while the former, the ſheep and cow, commonly produce only one. The 4th ſign, Cancer, the Crab, an animal that goes ſide-ways and backwards, was placed at the northern ſolſtice, the point where the ſun begins to return back again from the north to the ſouthward. The 5th ſign, Leo, the Lion, as being a very furious animal, was thought to denote the heat and fury of the burning ſun, when he has left Cancer, and entered the next ſign Leo. The ſucceeding conſtellation, the 6th in order, received the ſun at the time of ripening corn and approaching harveſt; which was aptly expreſſed by one of the female reapers, with an ear of corn in her hand; viz, Virgo the maid. The ancients gave to the next ſign Scorpio, two of the 12 diviſions of the zodiac: Autumn, which affords fruits in great abundance, affords the means and cauſes of diſeaſes, and the ſucceeding time is the moſt unhealthy of the year; expreſſed by this venemous animal, here ſpreading out his long claws into the one ſign, as threatening miſchief, and in the other brandiſhing his tail to denote the completion of it. The fall of the leaf was the ſeaſon of the ancient hunting; for which reaſon the ſtars which marked the ſun's place at this ſeaſon, into the conſtellation Sagittary, a huntſman with his arrows and his club, the weapons of deſtruction for the large creatures he purſued. The reaſon of the Wild Goat's being choſen to mark the ſouthern ſolſtice, when the ſun has attained his extreme limit that way, and begins to return and mount again to the northward, is obvious enough; the character of that animal being, that it is moſtly climbing, and aſcending ſome mountain as it browzes. There yet remain two of the ſigns of the zodiac to be conſidered with regard to their origin, viz, Aquarius and Piſces. As to the former, it is to be conſidered that the winter is a wet and uncomfortable ſeaſon; this therefore was expreſſed by Aquarius, the figure of a man pouring out water from an urn. The laſt of the zodiacal conſtellations was Piſces, a couple of fiſhes, tied together, that had been caught: The leſſon was, the ſevere ſeaſon is over, your flocks do not yet yield their ſtore; but the ſeas and rivers are open, and there you may take fiſh in abundance.

Through a vain and blind zeal, rather than through any love for the ſcience, ſome perſons have been moved to alter either the figures of the conſtellations, or their names. Thus, venerable Bede, inſtead of the profane names and figures of the twelve zodiacal conſtellations, ſubſtituted thoſe of the 12 apoſtles; which example was followed by Schiller, who completed the reformation, and gave ſcripture names to all the conſtellations in the heavens. Thus, Aries, or the Ram, was changed into Peter; Taurus, or the Bull, into St. Andrew; Andromeda, into the Sepulchre of Chriſt; Lyra, into the Manger of Chriſt; Hercules, into the Magi coming from the Eaſt; the Great Dog, into David; and ſo on. And Weigelius, profeſſor of Mathematics in the univerſity of Jena, made a new order of conſtellations; changing the firmament into a Cœlum Heraldicum; and introducing the arms of all the princes in Europe, by way of conſtellations. Thus Urſa major, the Great Bear, he transformed into the elephant of the kingdom of Denmark; the Swan, into the Ruta with ſwords of the Houſe of Saxony; Ophiuchus, into the Croſs of Cologne; the Triangle, into Compaſſes, which he calls the Symbol of Artificers; and the Pleiades into the Abacus Pythagoricus, which he calls that of merchants; &c.

But the more judicious among aſtronomers never approved of ſuch innovations; as they only tend to introduce confuſion into aſtronomy. The old conſtellations are therefore ſtill retained; both becauſe better could not be ſubſtituted, and likewiſe to keep up the greater correſpondence and uniformity between the old aſtronomy and the new. See CATALOGUE.

CONSTRUCTION, in Geometry, the art or manner of drawing or deſcribing a figure, ſcheme, the lines of a problem, or ſuch like.

CONSTRUCTION *of Equations*, in Algebra, is the finding the roots or unknown quantities of an equation, by geometrical conſtruction of right lines or curves; or the reducing given equations into geometrical figures. And this is effected by lines or curves according to the order or rank of the equation.

The roots of any equation may be determined, that is, the equation may be conſtructed, by the interſections of a ſtraight line with another line or curve of the ſame dimenſions as the equation to be conſtructed: for the roots of the equation are the ordinates of the curve at the points of interſection with the right line; and it is well known that a curve may be cut by a right line in as many points as its dimenſions amount to. Thus, then, a ſimple equation will be conſtructed by the interſection of one right line with another: a quadratic equation, or an affected equation of the 2d rank, by the interſections of a right line with a circle, or any of the conic ſections, which are all lines of the 2d order; and which may be cut, by the right line, in two points, thereby giving the two roots of the quadratic equation. A cubic equation may be conſtructed by the interſection of the right line with a line of the 3d order: and ſo on.

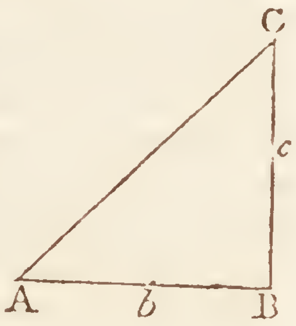
But if, inſtead of the right line, ſome other line of a higher order be uſed; then the 2d line, whoſe interſections with the former are to determine the roots of the equation, may be taken as many dimenſions lower, as the former is taken higher. And, in general, an equation of any height will be conſtructed by the interſections of two lines whoſe dimenſions, multiplied together, produce the dimenſion of the given equation. Thus, the interſections of a circle with the conic ſections, or of theſe with each other, will conſtruct the biquadratic equations, or thoſe of the 4th power, becauſe 2 × 2 = 4; and the interſections of the circle or conic ſections with a line of the 3d order, will conſtruct the equations of the 5th and 6th power; and ſo on.—For example,

To conſtruct a Simple Equation. This is done by reſolving the given ſimple equation into a proportion, or finding a third or 4th proportional, &c. Thus, 1. If the equation be $ax = bc$; then $a : b :: c : x = \frac{bc}{a}$, the fourth proportional to a, b, c.

2. If $ax = b^2$; then $a : b :: b : x = \frac{b^2}{a}$, a third proportional to a and b.

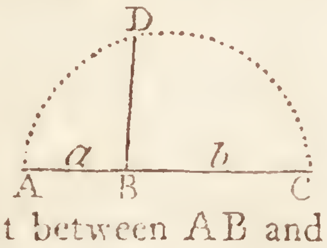
3. If $ax = b^2 - c^2$; then, ſince $b^2 - c^2 = \overline{b+c} \times \overline{b-c}$, it will be $a : b + c :: b - c : x = \frac{\overline{b+c} \times \overline{b-c}}{a}$, a fourth proportional to a, $b + c$ and $b - c$.

4. If $ax = b^2 + c^2$; then conſtruct the right-angled triangle ABC, whoſe baſe is b, and perpendicular is c, ſo ſhall the ſquare of the hypothenuſe be $b^2 + c^2$, which call h^2; then the equation is $ax = h^2$, and $x = \frac{h^2}{a}$ a third proportional to a and h.



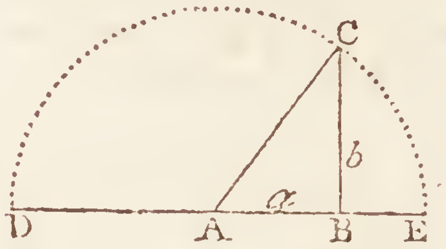
To conſtruct a Quadratic Equation.

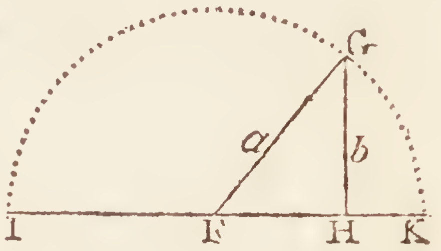
1. If it be a ſimple quadratic, it may be reduced to this form $x^2 = ab$; and hence $a : x :: x : b$, or $x = \sqrt{ab}$ a mean proportional between a and b. Therefore upon a ſtraight line take AB = a, and BC = b; then upon the diameter AC deſcribe a ſemicircle, and raiſe the perpendicular BD to meet it in D; ſo ſhall BD be = x the mean proportional ſought between AB and BC, or between a and b.



2. If the quadratic be affected, let it firſt be $x^2 + 2ax = b^2$; then form the right-angled triangle whoſe baſe AB is a, and perpendicular BC is b; and with the centre A and radius AC deſcribe the ſemicircle DCE; ſo ſhall DB and BE be the two roots of the given quadratic equation $x^2 + 2ax = b^2$.

3. If the quadratic be $x^2 - 2ax = b^2$, then the conſtruction will be the very ſame as of the preceding one $x^2 + 2ax = b^2$.





4. But if the form be $2ax - x^2 = b^2$: form a right-angled triangle whoſe hypothenuſe FG is a, and perpendicular GH is b; then with the radius FG and centre F deſcribe a ſemi-circle IGK; ſo ſhall IH and HK be the two roots of the given equation $2ax - x^2 = b^2$, or $x^2 - 2ax = -b^2$. See Maclaurin's Algebra, part 3, cap. 2, and Simpſon's Algebra, pa. 267.

To conſtruct Cubic and Biquadratic Equations.—Theſe are conſtructed by the interſections of two conic ſections; for the equation will riſe to 4 dimenſions, by which are determined the ordinates from the 4 points in which theſe conic ſections may cut one another; and the conic ſections may be aſſumed in ſuch a manner, as to make this equation coincide with any propoſed biquadratic: ſo that the ordinates from theſe 4 interſections will be equal to the roots of the propoſed biquadratic. When one of the interſections of the conic ſection falls upon the axis, then one of the ordinates vaniſhes, and the equation, by which theſe ordinates are determined, will then be of 3 dimenſions only, or a cubic; to which any propoſed cubic equation may be accommodated. So that the three remaining ordinates will be the roots of that propoſed cubic. The conic ſections for this purpoſe ſhould be ſuch as are moſt eaſily deſcribed; the circle may be one, and the parabola is uſually aſſumed for the other.

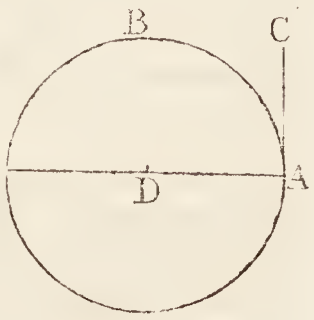
Vieta, in his Canonica Recenſione Effectionum Geometricarum, and Ghetaldus, in his Opus Poſthumum de Reſolutione & Compoſitione Mathematica, as alſo Des Cartes, in his Geometria, have ſhewn how to conſtruct ſimple and quadratic Equations. Des Cartes has alſo ſhewn how to conſtruct cubic and biquadratic equations, by the interſection of a circle and a parabola: And the ſame has been done more generally by Baker in his Clavis Geometrica, or Geometrical Key. But the genuine foundation of all theſe conſtructions was firſt laid and explained by Sluſius in his Meſolabium, part 2. This doctrine is alſo pretty well handled by De la Hire, in a ſmall treatiſe, called La Conſtruction des Equations Analytiques, annexed to his Conic Sections. Newton, at the end of his Algebra, has given the conſtruction of cubic and biquadratic equations mechanically; as alſo by the conchoid and ciſſoid, as well as the conic ſections. See alſo Dr. Halley's Conſtruction of Cubic and Biquadratic Equations; Colſon's, in the Philoſ. Tranſ.; the Marquis de l'Hoſpital's Traité Analytique des Sections Coniques; Maclaurin's Algebra, part 3, c. 3 &c.

CONTACT, the relative ſtate of two things that touch each other, but without cutting or entering; or whoſe ſurfaces join to each other without any interſtice.

The contact of curve lines or ſurfaces, with either ſtraight or curved ones, is only in points ; and yet theſe points have different proportions to one another, as is ſhewn by Mr. Robartes, in the Philoſ. Tranſ. vol. 27 pa. 470 ; or Abr. vol. 4. pa. 1.

Becauſe few or no ſurfaces are capable of touching in all points, and the coheſion of bodies is in proportion to their contact, thoſe bodies will adhere faſteſt together, that are capable of the greateſt contact.

Angle of Contact is the opening between a curve line and a tangent to it, particularly the circle and its tangent ; as the angle formed at A between BA and AC, at the point of contact A.



It is demonſtrated by Euclid, that the line CA ſtanding perpendicular to the radius DA, touches the circle only in one point : and that no other right line can be drawn between the tangent and the circle.

Hence, the angle of contact is leſs than any rectilinear angle ; and the angle of the ſemi-circle between the radius DA and the arch AB, is greater than any rectilinear acute angle.

This ſeeming paradox of Euclid has exerciſed the wits of mathematicians : it was the ſubject of a long controverſy between Peletarius and Clavius; the former of whom maintained that the angle of contact is heterogeneous to a rectilinear one ; as a line is to a ſurface ; the latter maintained the contrary.

Dr. Wallis has a formal treatiſe on the angle of contact, and of the ſemi-circle ; where, with other great mathematicians, he approves of the opinion of Peletarius.

CONTENT, a term often uſed for the meaſurement of bodies and ſurfaces, whether ſolid or ſuperficial ; or the capacity of a veſſel and the area of a ſpace ; being the quantity either of matter or ſpace included within certain bounds or limits.

CONTIGUITY, the relation of bodies touching one another.

CONTIGUOUS, a relative term, underſtood of things diſpoſed ſo near each other, that they join their ſurfaces, or touch.

Contiguous *Angles*, are ſuch as have one leg or ſide common to each angle; and are otherwiſe called *adjoining angles* ; in contradiſtinction to thoſe made by continuing their legs through the point of contact, which are called *oppoſite* or *vertical angles*.

CONTINENT, a terra firma, main-land, or a large extent of country, not interrupted by ſeas : ſo called, in oppoſition to iſland, peninſula, &c.

The world is uſually divided into two grand continents, the old and the new : the old continent comprehends Europe, Aſia, and Africa ; the new, North and South America. Since the diſcovery of New Holland and New South Wales, it is a doubt with many whether to call that vaſt country an iſland or a continent.

CONTINGENT *Line*, the ſame with tangent line in Dialling, being the interſection of the planes of the dial and equinoctial, and at right angles to the ſubſtilar line.

CONTINUAL Proportionals, are a ſeries of three or more quantities compared together, ſo that the ratio is the ſame between every two adjacent terms, viz between the 1ſt and 2d, the 2d and 3d, the 3d and 4th, &c. As 1, 2, 4, 8, 16, &c, where the terms continually increaſe in a double ratio ; or 12, 4, $\frac{4}{3}$, $\frac{4}{9}$, where the terms decreaſe in a triple ratio.

A ſeries of continual or continued proportionals, is otherwiſe called a *progreſſion*.

CONTINUED *Quantity*, or *Body*, is that whoſe parts are joined and united together.

CONTINUED *Proportion*, is that in which the conſequent of the firſt ratio is the ſame with the antecedent of the ſecond ; as in theſe, 3 : 6 : : 6 : 12. See Continual *Proportion*.

On the contrary, if the conſequent of the firſt ratio be different from the antecedent of the ſecond, the proportion is called *diſcrete :* as 3 : 6 : : 4 : 8.

CONTRA-Harmonical *Proportion*, that relation of three terms, in which the difference of the firſt and ſecond is to the difference of the 2d and 3d, as the 3d is to the firſt. Thus, for inſtance, 3, 5, and 6, are numbers contra-harmonically proportional ; for 2 : 1 : : 6 : 3.

CONTRA-Mure, in Fortification, is a little wall built before another partition wall, to ſtrengthen it, ſo that it may receive no damage from the adjacent buildings.

CONTRATE-Wheel, is that wheel in watches which is next to the crown, whoſe teeth and hoop lie contrary to thoſe of the other wheels ; from whence comes its name.

CONTRAVALLATION, *Line of*, in Fortification, is a trench, guarded with a parapet ; being made by the beſiegers, between them and the place beſieged, to ſecure themſelves on that ſide, and ſtop the ſallies of the garriſon. It is made without muſket-ſhot of the town ; ſometimes going quite around it, and ſometimes not, as occaſion may require. The beſiegers lie between the lines of circumvallation and contravallation : but it is now ſeldom uſed.

CONVERGING *Curves*. See Curve.

Converging, or Convergent *Lines*, in Geometry, are thoſe that continually approximate, or whoſe diſtance becomes continually leſs and leſs the farther they are continued, till they meet : in oppoſition to *divergent* lines, whoſe diſtance becomes continually greater.

Lines that converge the one way, diverge the other.

Converging *Rays*, in Optics, are ſuch as incline towards one another in their paſſage, and in Dioptrics, are thoſe rays which, in their paſſage out of one medium into another of a different denſity, are refracted towards one another ; ſo that, if far enough continued, they will meet in a point or focus.

Converging *Series*, a ſeries of terms or quantities, that always decreaſe the farther they proceed, or which tend to a certain magnitude or limit : in oppoſition to diverging ſeries, or ſuch as become larger and larger continually. See Series.

CONVERSE. A propoſition is ſaid to be the converſe of another, when, after drawing a concluſion from ſomething firſt ſuppoſed, we return again, and, making a ſuppoſition of what had before been concluded, draw from thence as a concluſion what before was made the ſuppoſition. Thus, when it is ſuppoſed that the two ſides of a triangle are equal, and thence demonſtrate or conclude that the two angles oppoſite to thoſe ſides are equal alſo; then the converſe is to ſuppoſe that the two angles of a triangle are equal, and thence to prove or conclude that the ſides oppoſite to thoſe angles are alſo equal.

Converse-*Direction*, in Aſtrology, is uſed in oppoſition to *direct* direction; that is, by the latter the promoter is carried to the ſignificator, according to the order of the ſigns: whereas by the other it is carried from eaſt to weſt, contrary to the order of the ſigns.

CONVERSION, or Convertendo, is when there are four proportionals, and it is inferred, that the firſt is to its exceſs above the 2d, as the third to its exceſs above the 4th: according to Euclid, lib. 5, def. 17.

Thus, if it be - - - $8 : 6 :: 4 : 3$,
then convertendo, or by converſion, $8 : 2 :: 4 : 1$.
Or if there be - - $a : b :: c : d$,
then convertendo, or by converſion, $a : a - b :: c : c - d$.

CONVEX, round or curved and protuberant outwards, as the outſide of a globular body.

Convex *Lens*, *Mirror*, &c. See Lens, Mirror, &c.

CONVEXITY, the exterior or outward ſurface of a convex or round body.

COPERNICAN, ſomething relating to Copernicus. As, the

Copernican *Sphere*. See Sphere.

Copernican *Syſtem*, is that ſyſtem of the world, in which it is ſuppoſed that the ſun is at reſt in the centre, and the earth and planets all moving around him in their own orbits.

Here it is ſuppoſed, that the heavens and ſtars are at reſt; and the diurnal motion which they appear to have, from eaſt to weſt, is imputed to the earth's diurnal motion from weſt to eaſt.

This ſyſtem was maintained by many of the ancients; particularly Ecphantus, Seleucus, Ariſtarchus, Philolaus, Cleanthes Samius, Nicetas, Heraclides Ponticus, Plato, and Pythagoras; from the laſt of whom it was anciently called the Pythagoric, or Pythagorean Syſtem.

This ſyſtem was alſo held by Archimedes, in his book of the number of the Grains of Sand; but after him it became neglected, and even forgotten, for many ages; till about 300 years ſince, when Copernicus revived it; from whom it took the new name of the Copernican Syſtem. See the next article.

COPERNICUS (Nicholas), an eminent aſtronomer, was born at Thorn in Pruſſia, January 19, 1473. He was inſtructed in the Latin and Greek languages at home; and afterward ſent to Cracow, where he ſtudied philoſophy, mathematics, and medicine: though his genius was naturally turned to mathematics, which he chiefly ſtudied, and purſued through all its various branches.

He ſet out for Italy at 23 years of age; ſtopping at Bologna, that he might converſe with the celebrated aſtronomer of that place, Dominic Maria, whom he aſſiſted for ſome time in making his obſervations. From hence he paſſed to Rome, where he was preſently conſidered as not inferior to the famous Regiomontanus. Here he ſoon acquired ſo great a reputation, that he was choſen profeſſor of mathematics, which he taught there for a long time with the greateſt applauſe; and here alſo he made ſome aſtronomical obſervations about the year 1500.

Afterward, returning to his own country, he began to apply his fund of obſervations and mathematical knowledge, to correcting the ſyſtem of aſtronomy which then prevailed. He ſet about collecting all the books that had been written by philoſophers and aſtronomers, and to examine all the various hypotheſes they had invented for the ſolution of the celeſtial phenomena; to try if a more ſymmetrical order and conſtitution of the parts of the world could not be diſcovered, and a more juſt and exquiſite harmony in its motions eſtabliſhed, than what the aſtronomers of thoſe times ſo eaſily admitted. But of all their hypotheſes, none pleaſed him ſo well as the Pythagorean, which made the ſun to be the centre of the ſyſtem, and ſuppoſed the earth to move both round the ſun, and alſo round its own axis. He thought he diſcerned much beautiful order and proportion in this; and that all the embarraſſment and perplexity, from epicycles and excentrics, which attended the Ptolemaic hypotheſes, would here be entirely removed.

This ſyſtem he began to conſider, and to write upon, when he was about 35 years of age. He carefully contemplated the phenomena; made mathematical calculations; examined the obſervations of the antients, and made new ones of his own; till, after more than 20 years chiefly ſpent in this manner, he brought his ſcheme to perfection, eſtabliſhing that ſyſtem of the world which goes by his name, and is now univerſally received by all philoſophers.

This ſyſtem however was at firſt looked upon as a moſt dangerous hereſy, and his work had long been finiſhed and perfected, before he could be prevailed upon to give it to the world, being ſtrongly urged to it by his friends. At length yielding to their intreaties, it was printed, and he had but juſt received a perfect copy, when he died the 24th of May 1543, at 70 years of age; by which it is probable he was happily relieved from the violent fanatical perſecutions of the church, which were but too likely to follow the publication of his aſtronomical opinions; and which indeed was afterward the fate of Galileo, for adopting and defending them.

The above work of Copernicus, firſt printed at Norimberg in folio, 1543, and of which there have been other editions ſince, is intitled *De Revolutionibus Orbium Cæleſtium*, being a large body of aſtronomy, in 6 books.

When Rheticus, the diſciple of our author, returned out of Pruſſia, he brought with him a tract of Copernicus, on plane and ſpherical trigonometry, which he had printed at Norimberg, and which contained a table of ſines. It was afterward printed at the end of the firſt book of the Revolutions. An edition of our author's great work was alſo publiſhed in 4to at Amſterdam in 1617, under the title of *Aſtronomia Inſtaurata*, illuſtrated with notes by Nicolas Muler of Groningen.

COPERNICUS, the name of an aſtronomical inſtrument, invented by Whiſton, to ſhew the motion and phenomena of the planets, both primary and ſecondary. It is founded upon the Copernican ſyſtem, and therefore called by his name.

COR Caroli, Charles's Heart, an extra-conſtellated ſtar of the 2d magnitude in the northern hemiſphere, between the Coma Berenices and Urſa Major; ſo called by Sir Charles Scarborough, in honour of king Charles I.

Cor Hydræ, the Hydra's Heart, a ſtar of the 2d magnitude, in the Heart of the conſtellation Hydra.

Cor Leonis, Lion's Heart, or Regulus, a ſtar of the firſt magnitude in the conſtellation Leo.

Cor Scorpii. See Antares.

CORBEILS, in Fortification, are little baſkets about a foot and a half high, 8 inches broad at the bottom, and 12 at the top; which being filled with earth, are ſet againſt one another on the parapet, or elſewhere, leaving certain port-holes, from whence to fire under cover upon the enemy.

CORBEL, in Architecture, the repreſentation of a baſket, ſometimes ſeen on the heads of caryatides.

Corbel, or Corbil, is alſo uſed, in Building, for a ſhort piece of timber placed in a wall, with its end projecting out 6 or 8 inches, as occaſion ſerves, in the manner of a ſhouldering-piece.

CORBET, the ſame as Corbel.

CORDON, in Fortification, a row of ſtones jutting out between the rampart and the baſis of the parapet, like the tore of a column. The cordon ranges round the whole fortreſs, and ſerves to join the rampart, which is aſlope, and the parapet, which is perpendicular, more agreeably together.

In fortifications raiſed of earth, this ſpace is filled up with pointed ſtakes, inſtead of a cordon.

CORDS, in Muſic, are the ſounds produced by an inſtrument or the voice.

CORIDOR, or Corridor, in Fortification, is the covert-way lying round about the whole compaſs of the works of a place, between the outſide of the moat and the palliſadoes, being about 20 yards broad.

Coridor is alſo uſed, in Architecture, for a gallery, or long aile, around a building, leading to ſeveral chambers at a diſtance from each other, ſometimes wholly incloſed, and ſometimes open on one ſide.

CORINTHIAN *Order*, of Architecture, is the 4th in order, or the 5th and laſt according to Scamozzi and Le Clerc.

This order was invented by an Athenian Architect, and is the richeſt and moſt delicate of them all; its capital being adorned with rows of leaves, and of 8 volutas, which ſupport the abacus. The height of its column is 10 diameters, and its cornice is ſupported by modillions.

CORNEA *Tunica*, the ſecond coat of the eye; ſo called from its ſubſtance reſembling the horn of a lantern. This is ſituated in the fore-part; and is ſurrounded by the ſclerotica. It has a greater convexity than the reſt of the eye, and is a portion of a ſmall ſphere, or rather ſpheroid, and conſolidates the whole eye.

CORNICE, Corniche, or Cornish, the third and uppermoſt part of the entablature of a column, or the uppermoſt ornament of any wainſcotting, &c.

COROLLARY, or Consectary, a conſequence drawn from ſome propoſition or principles already advanced or demonſtrated, and without the aid of any other propoſition: as if from this theorem, *That a triangle which has two equal ſides, has alſo two equal angles*, this conſequence ſhould be drawn, *that a triangle which hath the three ſides equal, has alſo its three angles equal.*

CORONA, *Crown* or *Crowning*, in Architecture, the flat and moſt advanced part of the cornice; ſo called, becauſe it crowns the cornice and entablature: by the workmen it is called the *drip*, as ſerving by its projecture to ſcreen the reſt of the building from the rain.

CORONA, in Optics, a luminous circle, uſually coloured, round the ſun, moon, or largeſt planets. See Halo.

Corona *Borealis*, or *Septentrionalis*, the *Northern Crown* or *Garland*, a conſtellation of the northern hemiſphere, being one of the 48 old ones. It contains 8 ſtars according to the catalogue of Ptolomy, Tycho, and Hevelius; but according to the Britannic Catalogue, 21.

Corona *Auſtralis*, or *Meridianalis*, the *Southern Crown*, a conſtellation of the ſouthern hemiſphere, whoſe ſtars in Ptolomy's catalogue are 13; in the Britiſh Catalogue, 12.

CORPUSCLE the diminutive of corpus, uſed to expreſs the minute parts, or particles, that conſtitute natural bodies; meaning much the ſame as *atoms*.

Newton ſhews a method of determining the ſizes of the corpuſcles of bodies, from their colours.

CORPUSCULAR *Philoſophy*, that ſcheme or ſyſtem of phyſics, in which the phenomena of bodies are accounted for, from the motion, reſt, poſition, &c, of the corpuſcles or atoms of which bodies conſiſt.

The Corpuſcular philoſophy, which now flouriſhes under the name of the mechanical philoſophy, is very ancient. Leucippus and Democritus taught it in Greece; from them Epicurus received it, and improved it; and from him it was called the *Epicurean Philoſophy*.

Leucippus, it is ſaid, received it from one Mochus, a Phenician phiſiologiſt, before the time of the Trojan war, and the firſt who philoſophized about atoms: which Mochus is, according to the opinion of ſome, the Moſes of the Scriptures.

After Epicurus, the corpuſcular philoſophy gave way to the peripatetic, which became the popular ſyſtem. Thus, inſtead of atoms, were introduced ſpecific and ſubſtantial forms, qualities, ſympathies, &c, which amuſed the world, till Gaſſendus, Charleton, Deſcartes, Boyle, Newton, and others, retrieved the corpuſcularian hypotheſes; which is now become the baſis of the mechanical and experimental philoſophy.

Boyle reduces the principles of the corpuſcular philoſophy to the 4 following heads.

1. That there is but one univerſal kind of matter, which is an extended, impenetrable, and diviſible ſubſtance, common to all bodies, and capable of all forms. —On this head, Newton finely remarks thus: "All things conſidered, it appears probable to me, that God in the beginning created matter in ſolid, hard, impenetrable, moveable particles; of ſuch ſizes and figures, and with ſuch other properties, as moſt conduced to the end for which he formed them: and that theſe primitive particles, being ſolids, are incomparably harder than any of the ſenſible porous bodies compounded of them; even ſo hard as never to wear, or break in pieces: no other power being able to divide what God made one in the firſt creation. While theſe corpuſcles remain entire, they may compoſe bodies of one

and the ſame nature and texture in all ages: but ſhould they wear away, or break in pieces, the nature of things depending on them would be changed: water and earth, compoſed of old worn particles, of fragments of particles, would not be of the ſame nature and texture now, with water and earth compoſed of entire particles at the beginning. And therefore, that nature may be laſting, the changes of corporeal things are to be placed only in the various ſeparations, and new aſſociations, of theſe permanent corpuſcles."

2. That this matter, in order to form the vaſt variety of natural bodies, muſt have motion in ſome, or all its aſſignable parts; and that this motion was given to matter by God, the creator of all things; and has all manner of directions and tendencies.—"Theſe corpuſcles, ſays Newton, have not only a vis inertiæ, accompanied with ſuch paſſive laws of motion as naturally reſult from that force; but alſo are moved by certain active principles; ſuch as that of gravity, and that which cauſes fermentation, and the coheſion of bodies."

3. That matter muſt alſo be actually divided into parts; and each of theſe primitive particles, fragments, or atoms of matter, muſt have its proper magnitude, figure, and ſhape.

4. That theſe differently ſized and ſhaped particles, have different orders, poſitions, ſituations, and poſtures, from whence all the variety of compound bodies ariſes.

CORRIDOR. See CORIDOR.

CORVUS, the *Raven*, a ſouthern conſtellation, fabled by the Greeks, as taken up to heaven by Apollo, to whom it tattled that the beautiful maid Coronis, the daughter of Phlegeos, and mother of Eſculapius by Apollo, played the deity falſe with Iſchys, under a tree upon which the animal happened to be perched.

The ſtars in this conſtellation, in Ptolomy's and Tycho's catalogues are 7; but in the Britannic catalogue, 9.

COSECANT, COSINE, COTANGENT, COVERSED SINE, are the ſecant, ſine, tangent, and verſed ſine of the complement of an arch or angle; *Co* being, in this caſe, a contraction of the word complement, and was firſt introduced by Gunter.

COSMICAL ASPECT, among aſtrologers, is the aſpect of a planet with reſpect to the earth.

COSMICAL *Riſing*, or *Setting*, is ſaid of a ſtar when it riſes or ſets at the ſame time when the ſun riſes.

But, according to Kepler, to riſe or ſet coſmically, is only ſimply to riſe or ſet, that is, to aſcend above, or deſcend below, the horizon; as much as to ſay, to riſe or ſet to the world.

COSMOGONY, the ſcience of the formation of the univerſe; as diſtinguiſhed from coſmography, which is the ſcience of the parts of the univerſe, ſuppoſing it formed, and in the ſtate as we behold it; and from coſmology, which reaſons on the actual and permanent ſtate of the world as it now is; whereas coſmogony reaſons on the variable ſtate of the world at the time of its formation.

COSMOGRAPHY, the deſcription of the world; or the art that teaches the conſtruction, figure, diſpoſition, and relation of all the parts of the world, with the manner of repreſenting them on a plane. It conſiſts chiefly of two parts; viz, *Aſtronomy*, which ſhews the ſtructure of the heavens, with the diſpoſition of the ſtars; and *Geography*, which ſhews thoſe of the earth.

COSMOLOGY, the ſcience of the world in general.

COSS, *Rule of*, meant the ſame as Algebra, by which name it was for ſome time called, when firſt introduced into Europe through the Italians, who named it *Regola de Coſa*, *the Rule of the thing*; the unknown quantity, or that which was required in any queſtion, being called *coſa*, the *thing*; from whence we have Coſs, and Coſſic numbers, &c.

COTES (ROGER), a very eminent mathematician, philoſopher, and aſtronomer, was born July 10, 1682, at Burbach in Leiceſterſhire, where his father Robert was rector. He was firſt placed at Leiceſter ſchool; where, at 12 years of age, he diſcovered a ſtrong inclination to the mathematics. This being obſerved by his uncle, the Rev. Mr. John Smith, he gave him all the encouragement he could; and prevailed on his father to ſend him for ſome time to his houſe in Lincolnſhire, that he might aſſiſt him in thoſe ſtudies: and here he laid the foundation of that deep and extenſive knowledge in that ſcience, for which he was afterward ſo deſervedly famous. He was hence removed to St. Paul's ſchool, London, where he made a great progreſs in claſſical learning; and yet he found ſo much leiſure as to ſupport a conſtant correſpondence with his uncle, not only in mathematics, but alſo in metaphyſics, philoſophy, and divinity. His next remove was to Trinity College Cambridge, where he took his degrees, and became fellow.

Jan. 1706, he was appointed profeſſor of aſtronomy and experimental philoſophy, upon the foundation of Dr Thomas Plume, archdeacon of Rocheſter; being the firſt that enjoyed that office, to which he was unanimouſly choſen, on account of his high reputation and merits. He entered into orders in 1713; and the ſame year, at the deſire of Dr. Bentley, he publiſhed at Cambridge the ſecond edition of Newton's Mathematica Principia; inſerting all the improvements which the author had made to that time. To this edition he prefixed a moſt admirable preface, in which he pointed out the true method of philoſophiſing, ſhewing the foundation on which the Newtonian philoſophy was raiſed, and refuting the objections of the Carteſians and all other philoſophers againſt it.

The publication of this edition of Newton's Principia added greatly to his reputation; nor was the high opinion the public now conceived of him in the leaſt diminiſhed, but rather much increaſed, by ſeveral productions of his own, which afterward appeared. He gave in the Philoſ. Tranſactions, two papers, viz, 1, Logometria, in vol. 29; and a Deſcription of the great fiery meteor that was ſeen March 6, 1716, in vol. 31.

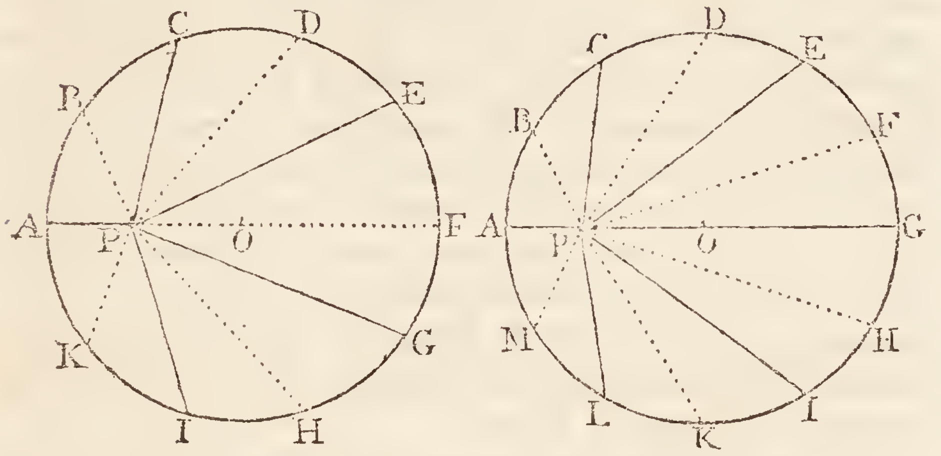
This extraordinary genius in the mathematics died, to the great regret of the univerſity, and all the lovers of the ſciences, June 5, 1716, in the very prime of his life, being not quite 34 years of age.

Mr. Cotes left behind him ſome very ingenious, and indeed admirable tracts, part of which, with the Logometria above mentioned, were publiſhed, in 1722, by Dr. Robert Smith, his couſin and ſucceſſor in his profeſſorſhip, afterward maſter of Trinity College, under the title of *Harmonia Menſurarum*, which contains a number of very ingenious and learned works: ſee the Introduction to my Logarithms. He wrote alſo a *Compendium of Arithmetic;* of the *Reſolution of Equations;* of *Dioptrics;* and of the *Nature of Curves*. Beſide theſe pieces, he drew up, in the time of his lectures, a courſe of *Hydroſtatical and Pneumatical Lectures*, in Engliſh, which were publiſhed alſo by Dr. Smith in 8vo, 1737, and are held in great eſtimation.

So high an opinion had Sir Iſaac Newton of our author's genius, that he uſed to ſay, "If Cotes had lived, we had known ſomething."

COTESIAN *theorem*, in Geometry, an appellation uſed for an elegant property of the circle diſcovered by Mr. Cotes. The theorem is this:

If the factors of the binomial $a^c \underset{\backsim}{+} x^c$ be required, the index c being an integer number. With the centre O, and radius AO $= a$, deſcribe a circle, and divide its circumferance into as many equal parts as there are units in $2c$, at the points A, B, C, D, &c; then in the radius, produced if neceſſary, take OP $= x$, and from the point P, to all the points of diviſion in the circumference, draw the lines PA, PB, PC, &c; ſo ſhall theſe lines taken alternately be the factors ſought; viz,



$$PB \times PD \times PF \;\&c = a^c + x^c,$$

and $PA \times PC \times PE \;\&c = a^c \backsim x^c$, viz, $a^c - x^c$ or $x^c - a^c$, according as the point P is within or without the circle.

For inſtance, if $c = 5$, divide the circumference into 10 equal parts, and the point P being within the circle, then will $OA^5 + OP^5 = BP \times DP \times FP \times HP \times KP$,

and $OA^5 - OP^5 = AP \times CP \times EP \times GP \times IP$.

In like manner, if $c = 6$, having divided the circumference into 12 equal parts, then will

$$OA^6 + OP^6 = BP \times DP \times FP \times HP \times KP \times MP,$$
$$OA^6 - OP^6 = AP \times CP \times EP \times GP \times IP \times LP.$$

The demonſtration of this theorem may be ſeen in Dr. Pemberton's Epiſt. de Coteſii inventis. See alſo Dr. Smith's Theoremata Logometrica and Trigonometrica, added to Cotes's Harm. Menſ. pa. 114; De Moivre Miſcel. Analyt. pa. 17; and Waring's Letter to Dr. Powell, pa. 39.

By means of this theorem, the acute and elegant author was enabled to make a farther progreſs in the inverſe method of Fluxions, than had been done before. But in the application of his diſcovery there ſtill remained a limitation, which was removed by Mr. De Moivre. Vide ut ſupra.

COVERT-WAY, in fortification, a ſpace of ground level with the adjoining country, on the outer edge of the ditch, ranging quite round all the works. This is otherwiſe called the *corridor*, and has a parapet with its banquette and glacis, which form the height of the parapet. It is ſometimes alſo called the counterſcarp, becauſe it is on the edge of the ſcarp.

One of the greateſt difficulties in a ſiege, is to make a lodgment on the covert-way; becauſe it is uſual for the beſieged to paliſade it along the middle, and undermine it on all ſides.

COVING, in Building, si when houſes are built projecting over the ground plot, and the turned projecture formed into an arch.

COVING *Cornice*, is one that has a large caſemate or hollow in it.

COUNT-WHEEL, is a wheel in the ſtriking part of a clock, moving round once in 12 or 24 hours. It is ſometimes called the *locking-wheel*, becauſe it has uſually 11 notches in it at unequal diſtances from one another, to make the clock ſtrike.

COUNTER-APPROACHES, in Fortification, lines or trenches made by the beſieged, where they come out to attack the lines of the beſiegers in form.

COUNTER-BATTERY, a battery raiſed to play on another, to diſmount the guns, &c.

COUNTER-BREAST-WORK, the ſame as *Fauſſe-Braye*.

COUNTER-FORTS, *Buttreſſes*, or *Spurs*, are pillars of maſonry ſerving to prop or ſuſtain walls, or terraces, ſubject to bulge, or be thrown down.

COUNTER-FUGUE, in Muſic, is when fugues proceed contrary to one another

COUNTER-GUARD, in Fortification, a work commonly ſerving to cover a baſtion. It is compoſed of two faces, forming a ſalient angle before the flanked angle of a baſtion.

COUNTER-HARMONICAL. See CONTRA-HARMONICAL.

COUNTER-MINE, a ſubterraneous paſſage, made by the beſieged, in ſearch of the enemy's mine, to give air to it, to take away the powder; or by any other means to fruſtrate the effect of it.

COUNTER-PART, a term in Muſic, only denoting that one part is oppoſite to another, ſo, the baſs and treble are counterparts to each other.

COUNTER-POINT, in Muſic, the art of compoſing harmony; or diſpoſing and concerting ſeveral parts ſo together, as that they may make an agreeable whole.

COUNTER-POISE, any thing ſerving to weigh againſt another; particularly a piece of metal, uſually of braſs or iron, making an appendage to the Roman *ſtatera*, or ſteel-yard. It is contrived to ſlide along the beam; and from the diviſion at which it keeps the balance in equilibrio, the weight of the body is determined. It is ſometimes called the *pear*, on account of its figure; and *maſs*, by reaſon of its weight.

Rope-dancers make uſe of a pole by way of counterpoiſe, to keep their bodies in equilibrio.

COUNTERSCARP, is that ſide of the ditch that is next the country; or properly the talus that ſupports the earth of the covert-way: though by this word is often underſtood the whole covert-way, with its parapet and glacis. And ſo it muſt be underſtood when it is ſaid, The enemy lodged themſelves on the counterſcarp.

Counter-swallows-tail, is an outwork in Fortification, in form of a ſingle tenaille, wider towards the place, or at the gorge, than at the head, or next the country.

Counter-tenor, one of the mean or middle parts of muſic: ſo called, as being oppoſite to the tenor.

COURSE, in Navigation, the point of the compaſs, or horizon, which a ſhip ſteers on; or the angle which the rhumb line on which it ſails makes with the meridian; being ſometimes reckoned in degrees, and ſometimes in points of the compaſs.

When a ſhip ſails either due north or ſouth, ſhe ſails on a meridian, makes no departure, and her diſtance and difference of latitude are the ſame.

When ſhe ſails due eaſt or weſt, her courſe makes right-angles with the meridian, and ſhe ſails either upon the equator, or a parallel to it; in which caſe ſhe makes no difference of latitude, but her diſtance and departure are the ſame.

But when the ſhip ſails between the cardinal points, on a courſe making always the ſame oblique angle with the meridians, her path is then the loxodromic curve, being a ſpiral cutting all the meridians in the ſame angle, and terminating in the pole.

COURTAIN. See Curtin.

CRAB, in Mechanics, an engine uſed for mounting guns on their carriages. See Gin.

Crab, on ſhip-board, is a wooden pillar, whoſe lower end is let down through the ſhip's decks, and reſts upon a ſocket like the capſtan: in its upper end are three or four holes, at different heights, through the middle of it, above one another, to receive long bars, againſt which men act by puſhing or thruſting.—It is employed to wind in the cable, and for other purpoſes requiring a great mechanical power.

The Crab with three claws is uſed to launch ſhips, and to heave them into the dock, or off the key.

CRANE, a machine uſed in building, and in commerce, for raiſing large ſtones and other weights.

M. Perrault, in his notes on Vitruvius, makes the crane the ſame with the corvus, or raven, of the ancients.

The modern crane conſiſts of ſeveral members or pieces, the principal of which is a ſtrong upright beam, or arbor, firmly fixed in the ground, and ſuſtained by eight arms, coming from the extremities of four pieces of wood laid acroſs, through the middle of which the foot of the beam paſſes. About the middle of the arbor the arms meet, and are mortiſed into it: its top ends in an iron pivot, on which is borne a tranſverſe piece, advancing out to a good diſtance like a crane's neck; whence the name. The middle and extremities of this are again ſuſtained by arms from the middle of the arbor: and over it comes a rope, or cable, to one end of which the weight is fixed; the other is wound about the ſpindle of a wheel, which, turned, draws the rope, and that heaves up the weight; to be afterwards applied to any ſide or quarter, by the mobility of the tranſverſe piece on the pivot.

Several improvements of this uſeful machine are mentioned in Deſaguliers's Exper. Philoſ. pa. 178 & ſeq. particularly how to prevent the inconveniences ariſing from ſudden jerks, as well as to increaſe its force by uſing a double axis in peritrochio, and two handles.

The Crane is of two kinds; in the firſt kind, called the rat-tailed crane, the whole machine, with the load, turns upon a ſtrong axis: in the ſecond kind, the gibbet alone moves on its axis. See Deſaguliers, as above, for a particular account of the different cranes, and of the gradual improvements they have received. See alſo the Supplement to Ferguſon's Lectures, pa. 3, &c, or Philoſ. Tranſ. vol. 54, pa. 24, for a deſcription of a new and ſafe crane, with four different powers adapted to different weights.

Crane is the name of a ſouthern conſtellation. See Grus.

Crane is alſo a popular name for a ſyphon.

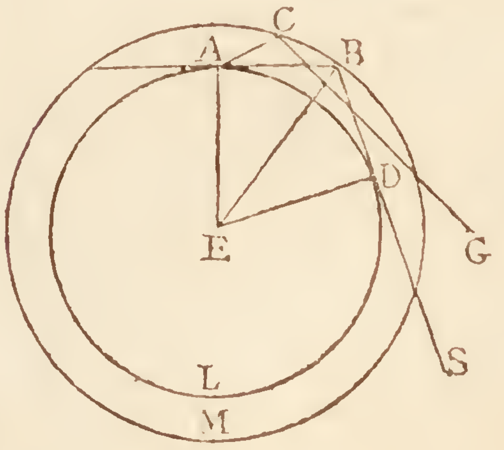
CRANK, a contrivance in machines, in manner of an elbow, only of a ſquare form; projecting out from an axis, or ſpindle; and ſerving, by its rotation, to raiſe and fall the piſtons of engines for raiſing water, or the like.

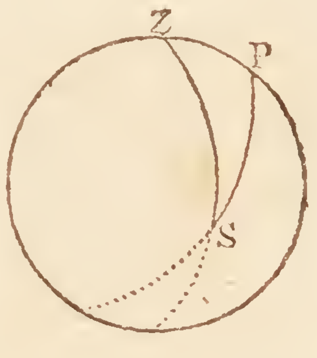
CRATER, the Cup, a conſtellation in the ſouthern hemiſphere; whoſe ſtars, in Ptolomy's catalogue, are 7; in Tycho's, 8; in Hevelius's, 10; and in the Britannic catalogue, 31.

CREEK, a part of a haven, where any thing is landed from the ſea. It is alſo ſaid to be a ſhore or bank on which the water beats, running in a ſmall channel from any part of the ſea.

CREPUSCULUM, *Twilight;* the time from the firſt dawn or appearance of the morning, to the riſing of the ſun; and again, between the ſetting of the ſun, and the laſt remains of day.

The Crepuſculum, or twilight, it is ſuppoſed, uſually begins and ends when the ſun is about 18 degrees below the horizon; for then the ſtars of the 6th magnitude diſappear in the morning, and appear in the evening. It is of longer duration in the ſolſtices than in the equinoxes, and longer in an oblique ſphere, than in a right one; becauſe, in thoſe caſes the ſun, by the obliquity of his path, is longer in aſcending through 18 degrees of altitude.





Twilight is occaſioned by the ſun's rays refracted in our atmoſphere, and reflected from the particles of it to the eye. For let A be the place of an obſerver on the earth ADL, AB the ſenſible horizon, meeting in B the circle CBM bounding that part of the atmoſphere which is capable of refracting and reflecting light to the eye. It is plain that when the ſun is under this horizon, no direct rays can come to the eye at A: but the ſun being in the refracted line CG, the particle C will be illuminated by the direct rays of the ſun; and that particle may reflect thoſe rays to A, where they enter the eye of the ſpectator. And thus the ſun's light illuminating an innumerable multitude of particles, may be all reflected to the ſpectator at A.—From B draw BD touching the circle ADL in D; and let the ſun be in the line BD at S: Then the ray SB will be reflected into BA, and will enter the eye, becauſe the angle of incidence DBE is equal to the angle of reflection ABE: And that will be the firſt ray that reaches the eye in the morning, when the dawning begins; or the laſt that falls upon the eye at night, when the twilight ends: for when the ſun goes lower down, the particles at B can be no longer illuminated.

Kepler indeed aſſigns another cauſe of the crepuſculum, viz, the luminous matter or atmoſphere about the ſun; which, ariſing near the horizon, in a circular figure, exhibits the crepuſculum; in no wiſe, he thinks, owing to the refraction of the atmoſphere.—The ſun's luminous atmoſphere indeed, though neither the ſole nor principal cauſe of twilight, may lengthen its duration, by illuminating our air, when the ſun is too low to reach it with his own light. Gregor. Aſtr. lib. 2, prop. 8.

The depth of the ſun below the horizon, at the beginning of the morning, or end of the evening twilight, is determined in the ſame manner as the arch of viſion; viz, by obſerving the moment when the air firſt begins to ſhine in the morning, or ceaſes to ſhine in the evening; then finding the ſun's place for that moment, and thence the time till his riſing in the horizon, or from his ſetting in it in the evening. It is now generally agreed that this depth is about 18 degrees upon an average.—Alhazen found it to be 19°; Tycho, 17°; Rothmann, 24°; Stevenius, 18°; Caſſini, 15°; Riccioli, in the equinox in the morning 16°, in the evening 20° 30′; in the ſummer ſolſtice in the morning 21° 25′, in the winter ſolſtice in the morning 17° 25′.

Nor is this difference among the determinations of aſtronomers to be wondered at; the cauſe of the crepuſculum being inconſtant: for, if the exhalations in the atmoſphere be either more copious, or higher, than ordinary; the morning twilight will begin ſooner, and the evening hold longer than ordinary: for the more copious the exhalations are, the more rays will they reflect, conſequently the more will they ſhine; and the higher they are, the ſooner will they be illuminated by the ſun. On this account too, the evening twilight is longer than the morning, at the ſame time of the year in the ſame place. To this it may be added, that in a denſer air, the refraction is greater; and that not only the brightneſs of the atmoſphere is variable, but alſo its height from the earth: and therefore the twilight is longer in hot weather than in cold, in ſummer than in winter, and alſo in hot countries than in cold, other circumſtances being the ſame. But the chief differences are owing to the different ſituations of places upon the earth, or to the difference of the ſun's place in the heavens. Thus, the twilight is longeſt in a parallel ſphere, and ſhorteſt in a right ſphere, and longer to places in an oblique ſphere in proportion as they are nearer to one of the poles; a circumſtance which affords relief to the inhabitants of the more northern countries, in their long winter nights. And the twilights are longeſt in all places of north latitude, when the ſun is in the tropic of cancer; and to thoſe in ſouth latitude, when he is in the tropic of capricorn. The time of the ſhorteſt twilight is alſo different in different latitudes; in England, it is about the beginning of October and of March, when the ſun is in the ſigns ♎ and ♓. For the method of determining it by trigonometry, ſee Gregor. Aſtron. lib. 2, prob. 41. See alſo Robertſon's Navigation, book 5, prob. 12.—Hence, when the difference between the ſun's declination and the depth of the equator is leſs than 18°, ſo that the ſun does not deſcend more than 18° below the horizon; the crepuſculum will continue the whole night, as is the caſe in England from about the 22d of May to the 22d of July.

Given the latitude of the place, and the ſun's declination; to find the beginning of the morning, and end of the evening twilight.—In the oblique-angled ſpherical triangle ZPS, are given ZP the colatitude, PS the codeclination, and ZS = 108°, being the ſum of 90° the quadrant and 18° the depreſſion at the extremity of the twilight. Then, by ſpherical trigonometry, calculate the angle ZPS the hour-angle from noon; which changed into time, at the rate of 15° to the hour, gives the time from noon at the beginning or end of twilight. See Robertſon, ubi ſupra.

Of the Height of the ſenſible Atmoſphere, as determined from the duration of twilight, ſee Keil's Aſtron. Lect. lect. 20, pa. 235, ed. 1721; or Long's Aſtron. vol. 1, pa. 260; where it is determined that the height where the atmoſphere is denſe enough to reflect the rays of light, is about 42 miles.

CRESCENT, the new moon, which, as it begins to recede from the ſun, ſhews a ſmall rim of light, terminating in horns or points, which are ſtill increaſing, till it becomes full, and round in the oppoſition.

The term is ſometimes alſo uſed for the ſame figure of the moon in her wane, or decreaſe, but improperly; both becauſe the horns are then turned towards the weſt, and becauſe the figure is on the decreaſe; the *creſcent* properly ſignifying increaſe, from *creſco, I grow*.

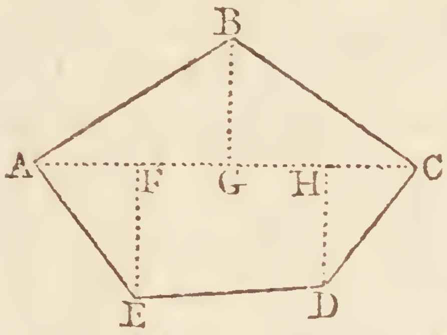
CRONICAL. See ACRONICAL.

CRONOS, a name given to Saturn by ſome of the old aſtronomical writers.

CROSIER, four ſtars, in form of a croſs; by the help of which, thoſe that ſail in the ſouthern hemiſphere find the antarctic pole.

CROSS, in Surveying, is an inſtrument conſiſting of a braſs circle, divided into 4 equal parts, by two lines croſſing each other in the centre. At each extremity of theſe lines is fixed a perpendicular ſight, with ſmall holes below each ſlit, for the better diſcovering of diſtant objects. The croſs is mounted on a ſtaff, or ſtand, to fix it in the ground, and is very uſeful for meaſuring ſmall pieces of land, and taking offſets, &c.

Ex. Suppoſe it be required to ſurvey the field ABCDE with the Croſs. Meaſure along the diagonal line AC, and obſerve, with the Croſs, when you are perpendicularly oppoſite to the corners, as at F, G, H, and from thence meaſure the perpendiculars FE, GB, HD. When you think you are nearly oppoſite a corner, ſet up the croſs, with one of the bars or croſs lines in the direction AC; then look through the ſights of the other croſs bar for the corner, as B; if it be ſeen through them, the croſs is fixed in the right place; if not, take it up and move it backward or forward in the line AC, till the point B be ſeen through thoſe ſights; and then you have the true place of the perpendicular.



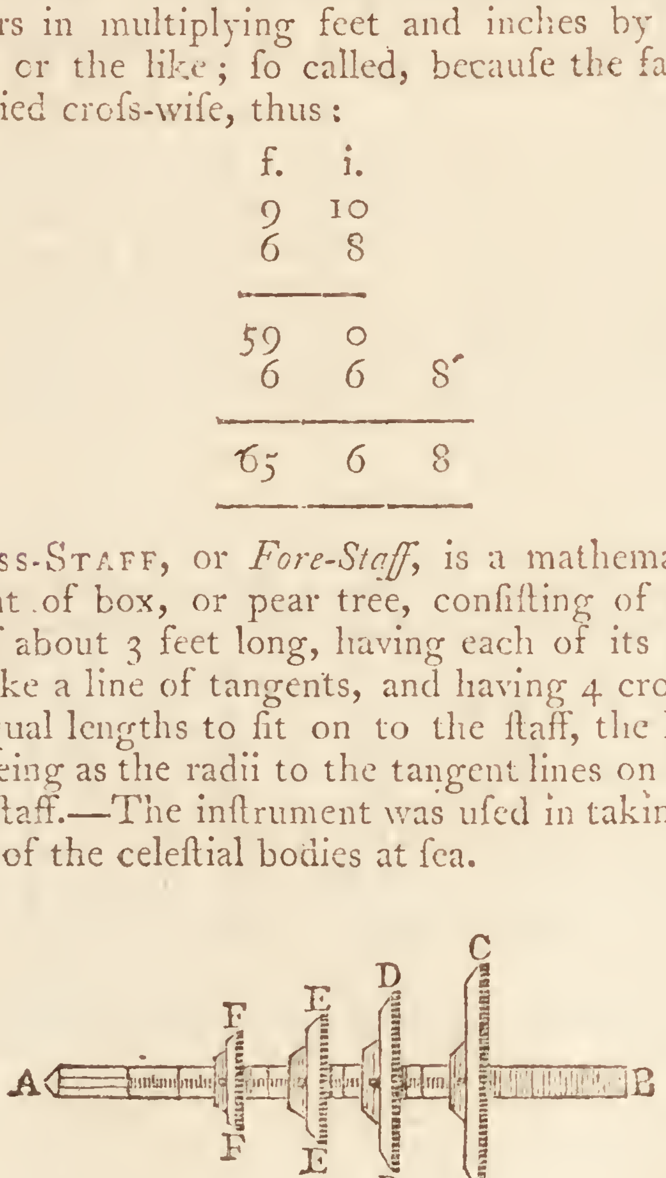
Invention of the Cross, *Inventio Crucis*, an ancient feaſt, which is ſtill retained in our calendar, and ſolemnized on the 3d of May, in memory of the finding of the true Croſs of Chriſt, deep in the ground, on Mount Calvary, by St. Helena, the mother of Conſtantine; where ſhe erected a church for the preſervation of part of it: the reſt being brought to Rome, and depoſited in the church of the Holy Croſs of Jeruſalem.

Exaltation of the Cross, an ancient feaſt, held on the 14th of September, in memory of this, that Heraclitus reſtored to Mount Calvary the true croſs, in 642, which had been carried off, 14 years before, by Coſroes king of Perſia, upon his taking Jeruſalem from the emperor Phocas. This feaſt is ſtill retained in our calendar, on Sept. 14, under the denomination of *Holy Rood*, or *Holy Croſs*.

Cross-Multiplication, a method uſed chiefly by artificers in multiplying feet and inches by feet and inches, or the like; ſo called, becauſe the factors are multiplied croſs-wiſe, thus:

f.	i.	
9	10	
6	8	
59	0	
6	6	8'
65	6	8

Cross-Staff, or *Fore-Staff*, is a mathematical inſtrument of box, or pear tree, conſiſting of a ſquare ſtaff, of about 3 feet long, having each of its faces divided like a line of tangents, and having 4 croſs pieces of unequal lengths to fit on to the ſtaff, the halves of theſe being as the radii to the tangent lines on the faces of the ſtaff.—The inſtrument was uſed in taking the altitudes of the celeſtial bodies at ſea.



CROUSAZ (John Peter de), a learned philoſopher and mathematician, was born at Lauſanne in Switzerland, April 13, 1663; where he died in 1748, at 85 years of age. Having made great progreſs in mathematics and the philoſophy of Des Cartes, he travelled into Geneva, Holland, and France. He was ſucceſſively profeſſor in ſeveral univerſities; and at length was choſen governor to Prince Frederick of Heſſe-Caſſel, nephew to the king of Sweden.

Crouſaz was author of many works, in various branches; belles-lettres, logic, philoſophy, divinity, &c, &c; but the moſt eſteemed of them are, 1. His Logic; the beſt edition of which is that of 1741, in 6 vols. 8vo.—2. A Treatiſe on Beauty.—3. A Treatiſe on Education, 2 vols, 12mo.—4. A Treatiſe on the Human Underſtanding.—5. Several Treatiſes on Philoſophy and Mathematics; as a Treatiſe on Motion, &c. with ſeveral papers inſerted in the Memoirs of the French Academy of Sciences.

CROW, in Mechanics, an iron lever, made with a ſharp point at one end, and two claws at the other; being uſed in heaving and purchaſing great weights, &c.

CROWN, in Aſtronomy, a name given to two conſtellations, the ſouthern and the northern.

Crown, in Geometry, a plane ring included between two parallel or concentric peripheries, of unequal circles.

The area of this is had, by multiplying its breadth by the length of a middle periphery, which is an arithmetical mean between the two peripheries that bound it; or by multiplying half the ſum of the circumferences by half the difference of the diameters; or laſtly by multiplying the ſum of the diameters by the difference of the diameters, and this laſt product by ·7854. See my Menſuration, pa. 148, 2d ed.

Crown-*Poſt*, is a poſt in ſome buildings ſtanding upright in the middle, between two principal rafters; and from which proceed ſtruts or braces to the middle of each rafter. It is otherwiſe called a *king-poſt*, or *king's-piece*, or *joggle-piece*.

Crown-*Wheel*, of a Watch, is the upper wheel next the balance, or that which drives the balance.

Crown-*Work*, in Fortification, is an out-work running into the field; deſigned to keep off the enemy, gain ſome hill, or advantageous poſt, and cover the other works of the place. It conſiſts of two demi-baſtions at the extremities, and an entire baſtion in the middle, with curtains.

Crowned *Horn-work*, is a Horn-work with a crown-work before it.

CRYSTALLINE *Humour*, is a thick compact humour of the eye, in form of a flattiſh convex lens, placed in the middle of the eye, and ſerving to make that refraction of the rays of light which is neceſſary to have them meet in the retina, and form an image there, by which viſion may be performed.

Crystalline *Heavens*, in the Old Aſtronomy, two orbs imagined between the primum mobile and the firmament, in the Ptolomaic ſyſtem, which ſuppoſed the heavens ſolid, and only ſuſceptible of a ſingle motion.

King Alphonſus of Arragon, it is ſaid, introduced the Cryſtallines, to explain what they called the *motion of trepidation*, or *titubation*.

The firſt Cryſtalline, according to Regiomontanus,

&c, ſerves to account for the ſlow motion of the fixed ſtars; by which they advance a degree in about 70 years, according to the order of the ſigns, or from weſt to eaſt; which occaſions a preceſſion of the equinox. The 2d ſerves to account for the motion of libration, or trepidation; by which the celeſtial ſphere librates from one pole towards the other, cauſing a difference in the ſun's greateſt declination.

CUBATURE, or Cubation, of a ſolid, is the meaſuring the ſpace contained in it, or finding the ſolid content of it, or finding a cube equal to it.

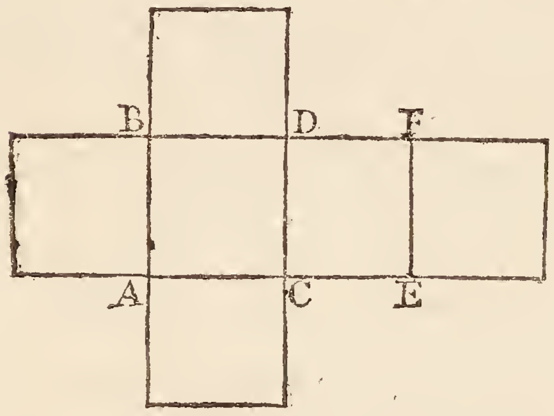
The cubature regards the content of a body, as the quadrature does the ſuperficies or area of a figure.

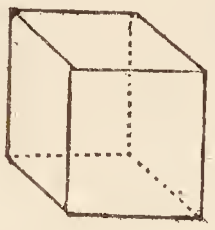
CUBE, a regular or ſolid body, conſiſting of ſix equal ſides or faces, which are ſquares.—A die is a ſmall cube.

It is alſo called a *hexaedron*, becauſe of its ſix ſides, and is the 2d of the five Platonic or Regular bodies.

The cube is ſuppoſed to be generated by the motion of a ſquare plane, along a line equal and perpendicular to one of its ſides.

To deſcribe a Rete, or Net, for forming a cube, or with which it may be covered.—Deſcribe ſix ſquares as in the annexed figure, upon card paper, paſte-board, or the like, of the ſize of the faces of the propoſed cube; and cut it half through by the lines AB, CD, EF, AC, BD; then fold up the ſeveral ſquares till their edges meet, and ſo form the cube, or a covering over one, as in the figure annexed.





To determine the Surface and Solidity of a Cube.—Multiply one ſide by itſelf, which will give one ſquare or face; then this multiplied by 6, the number of faces, will give the whole ſurface. Alſo multiply one ſide twice by itſelf, that is, cube it, and that will be the ſolid content.

Duplication of a Cube. See Duplication.

Cubes, or Cubic *Numbers*, are formed by multiplying any numbers twice by themſelves. So the cubes of 1, 2, 3, 4, 5, 6, &c,
are 1, 8, 27, 64, 125, 216, &c.

The third differences of the cubes of the natural numbers are all equal to each other, being the conſtant number 6. For, let m^3, n^3, p^3 be any three adjacent cubes in the natural ſeries as above, that is, whoſe roots m, n, p have the common difference 1; then becauſe
$n = m + 1$, theref. $n^3 = m^3 + 3m^2 + 3m + 1$,
$p = n + 1$, theref. $p^3 = n^3 + 3n^2 + 3n + 1$; ſo that the difference between the 1ſt and 2d, and between the 2d and 3d cubes, are

$$\left.\begin{array}{l} n^3 - m^3 = 3m^2 + 3m + 1, \\ p^3 - n^3 = 3n^2 + 3n + 1, \end{array}\right\} \text{the 1ſt differences;}$$

and the dif. of theſe differences, is

$$3 \cdot \overline{n^2 - m^2} + 3 \cdot \overline{n - m} = 3 \cdot \overline{n + m + 1} = 6 \cdot \overline{m + 1},$$

the 2d difference.

In like manner the next 2d dif. is $6 \cdot \overline{n + 1}$: hence the dif. of theſe two 2d diffs. is $6 \cdot \overline{n - m} = 6$, which is therefore the conſtant 3d difference of all the ſeries of cubes. And hence that ſeries of cubes will be formed by addition only, viz, adding always the 3d dif. 6 to find the column or ſeries of 2d diffs, then theſe added always for the 1ſt diffs, and laſtly theſe always added for the cubes themſelves, as below:

3d Difs.	2d Difs.	1ſt Difs.	Cubes.
6	6	1	0
6	12	7	1
6	18	19	8
6	24	37	27
6	30	61	64
6	36	91	125
6	42	127	216
6	48	169	343

Peletarius, among various ſpeculations concerning ſquare and cubic numbers, ſhews that the continual ſums of the cubic numbers, whoſe roots are 1, 2, 3, &c, form the ſeries of ſquares whoſe roots are 1, 3, 6, 10, 15, 21, &c.

$$\begin{array}{rcl} \text{Thus,}\quad 1 &=& 1 = 1^2, \\ 1 + 8 &=& 9 = 3^2, \\ 1 + 8 + 27 &=& 36 = 6^2, \\ 1 + 8 + 27 + 64 &=& 100 = 10^2, \text{ \&c.} \end{array}$$

Or, in general,

$$1^3 + 2^3 + 3^3 + 4^3 \text{ \&c. to } n^3 = \overline{1 + 2 + 3 + 4 \cdots n}|^2 = \tfrac{1}{2} n \cdot n + 1.$$

It is alſo a pretty property, that any number, and the cube of it, being divided by 6, leave the ſame remainder; the ſeries of remainders being 0, 1, 2, 3, 4, 5, continually repeated. Or that the differences between the numbers and their cubes, divided by 6, leave always 0 remaining; and the quotients, with their ſucceſſive differences, form the ſeveral orders of figurate numbers. Thus,

Num.	Cubes.	Difs.	Quot.	1 Dif.	2 Dif.
1	1	0	0	0	0
2	8	6	1	1	1
3	27	24	4	3	2
4	64	60	10	6	3
5	125	120	20	10	4
6	216	210	35	15	5
7	343	336	56	21	6

The following is a Table of the firſt 1000 cubic numbers.

TABLE OF CUBES.

Num	Cubes	Num.	Cubes	Num.	Cubes	Num.	Cubes	Num.	Cubes	Num.	Cubes
1	1	60	216000	119	1685159	178	5639752	237	13312053	296	25934336
2	8	61	226981	120	1728000	179	5735339	238	13481272	297	26198073
3	27	62	238328	121	1771561	180	5832000	239	13651919	298	26463592
4	64	63	250047	122	1815848	181	5929741	240	13824000	299	26730899
5	125	64	262144	123	1860867	182	6028568	241	13997521	300	27000000
6	216	65	274625	124	1906624	183	6128487	242	14172488	301	27270901
7	343	66	287496	125	1953125	184	6229504	243	14348907	302	27543608
8	512	67	300763	126	2000376	185	6331625	244	14526784	303	27818127
9	729	68	314432	127	2048383	186	6434856	245	14706125	304	28094464
10	1000	69	328509	128	2097152	187	6539203	246	14886936	305	28372625
11	1331	70	343000	129	2146689	188	6644672	247	15069223	306	28652616
12	1728	71	357911	130	2197000	189	6751269	248	15252992	307	28934443
13	2197	72	373248	131	2248091	190	6859000	249	15438249	308	29218112
14	2744	73	389017	132	2299968	191	6967871	250	15625000	309	29503629
15	3375	74	405224	133	2352637	192	7077888	251	15813251	310	29791000
16	4096	75	421875	134	2406104	193	7189057	252	16003008	311	30080231
17	4913	76	438976	135	2460375	194	7301384	253	16194277	312	30371328
18	5832	77	456533	136	2515456	195	7414875	254	16387064	313	30664297
19	6859	78	474552	137	2571353	196	7529536	255	16581375	314	30959144
20	8000	79	493039	138	2628072	197	7645373	256	16777216	315	31255875
21	9261	80	512000	139	2685619	198	7762392	257	16974593	316	31554496
22	10648	81	531441	140	2744000	199	7880599	258	17173512	317	31855013
23	12167	82	551368	141	2803221	200	8000000	259	17373979	318	32157432
24	13824	83	571787	142	2863288	201	8120601	260	17576000	319	32461759
25	15625	84	592704	143	2924207	202	8242408	261	17779581	320	32768000
26	17576	85	614125	144	2985984	203	8365427	262	17984728	321	33076161
27	19683	86	636056	145	3048625	204	8489664	263	18191447	322	33386248
28	21952	87	658503	146	3112136	205	8615125	264	18399744	323	33698267
29	24389	88	681472	147	3176523	206	8741816	265	18609625	324	34012224
30	27000	89	704969	148	3241792	207	8869743	266	18821096	325	34328125
31	29791	90	729000	149	3307949	208	8998912	267	19034163	326	34645976
32	32768	91	753571	150	3375000	209	9123329	268	19248832	327	34965783
33	35937	92	778688	151	3442951	210	9261000	269	19465109	328	35287552
34	39304	93	804357	152	3511808	211	9393931	270	19683000	329	35611289
35	42875	94	830584	153	3581577	212	9528128	271	19902511	330	35937000
36	46656	95	857375	154	3652264	213	9663597	272	20123648	331	36264691
37	50653	96	884736	155	3723875	214	9800344	273	20346417	332	36594368
38	54872	97	912673	156	3796416	215	9938375	274	20570824	333	36926037
39	59319	98	941192	157	3869893	216	10077696	275	20796875	334	37259704
40	64000	99	970299	158	3944312	217	10218313	276	21024576	335	37595375
41	68921	100	1000000	159	4019679	218	10360232	277	21253933	336	37933056
42	74088	101	1030301	160	4096000	219	10503459	278	21484952	337	38272753
43	79507	102	1061208	161	4173281	220	10648000	279	21717639	338	38614472
44	85184	103	1092727	162	4251528	221	10793861	280	21952000	339	38958219
45	91125	104	1124864	163	4330747	222	10941048	281	22188041	340	39304000
46	97336	105	1157625	164	4410944	223	11089567	282	22425768	341	39651821
47	103823	106	1191016	165	4492125	224	11239424	283	22665187	342	40001688
48	110592	107	1225043	166	4574296	225	11390625	284	22906304	343	40353607
49	117649	108	1259712	167	4657463	226	11543176	285	23149125	344	40707584
50	125000	109	1295029	168	4741632	227	11697083	286	23393656	345	41063625
51	132651	110	1331000	169	4826809	228	11852352	287	23639903	346	41421736
52	140608	111	1367631	170	4913000	229	12008989	288	23887872	347	41781923
53	148877	112	1404928	171	5000211	230	12167000	289	24137569	348	42144192
54	157464	113	1442897	172	5088448	231	12326391	290	24389000	349	42508549
55	166375	114	1481544	173	5177717	232	12487168	291	24642171	350	42875000
56	175616	115	1520875	174	5268024	233	12649337	292	24897088	351	43243551
57	185193	116	1560896	175	5359375	234	12812904	293	25153757	352	43614208
58	195112	117	1601613	176	5451776	235	12977875	294	25412184	353	43986977
59	205379	118	1643032	177	5545233	236	13144256	295	25672375	354	44361864

Num.	Cubes	Num.	Cubes	Num.	Cubes	Num	Cubes	Num.	Cubes	Num.	Cubes
355	44738875	417	72511713	479	109902239	541	158340421	603	219256227	665	294079625
356	45118016	418	73034632	480	110592000	542	159220088	604	220348864	666	295408296
357	45499293	419	73560059	481	111284641	543	160103007	605	221445125	667	296740963
358	45882712	420	74088000	482	111980168	544	160989184	606	222545016	668	298077632
359	46268279	421	74618461	483	112678587	545	161878625	607	223648543	669	299418309
560	46656000	422	75151448	484	113379904	546	162771336	608	224755712	670	300763000
361	47045881	423	75686967	485	114084125	547	163667323	609	225866529	671	302111711
362	47437928	424	76225024	486	114791256	548	164566592	610	226981000	672	303464448
363	47832147	425	76765625	487	115501303	549	165469149	611	228099131	673	304821217
364	48228544	426	77308776	488	116214272	550	166375000	612	229220928	674	306182024
365	48627125	427	77854483	489	116930169	551	167284151	613	230346397	675	307546875
366	49027896	428	78402752	490	117649000	552	168196608	614	231475544	676	308915776
367	49430863	429	78953589	491	118370771	553	169112377	615	232608375	677	310288733
368	49836032	430	79507000	492	119095488	554	170031464	616	233744896	678	311665752
369	50243409	431	80062991	493	119823157	555	170953875	617	234885113	679	313046839
370	50653000	432	80621568	494	120553784	556	171879616	618	236029032	680	314432000
371	51064811	433	81182737	495	121287375	557	172808693	619	237176659	681	315821241
372	51478848	434	81746504	496	122023936	558	173741112	620	238328000	682	317214568
373	51895117	435	82312875	497	122763473	559	174676879	621	239483061	683	318611987
374	52313624	436	82881856	498	123505992	560	175616000	622	240641848	684	320013504
375	52734375	437	83453453	499	124251499	561	176558481	623	241804367	685	321419125
376	53157376	438	84027672	500	125000000	562	177504328	624	242970624	686	322828856
377	53582633	439	84604519	501	125751501	563	178453547	625	244140625	687	324242703
378	54010152	440	85184000	502	126506008	564	179406144	626	245314376	688	325660672
379	54439939	441	85766121	503	127263527	565	180362125	627	246491883	689	327082769
380	54872000	442	86350888	504	128024064	566	181321496	628	247673152	690	328509000
381	55306341	443	86938307	505	128787625	567	182284263	629	248858189	691	329939371
382	55742968	444	87528384	506	129554216	568	183250432	630	250047000	692	331373888
383	56181887	445	88121125	507	130323843	569	184220009	631	251239591	693	332812557
384	56623104	446	88716536	508	131096512	570	185193000	632	252435968	694	334255384
385	57066625	447	89314623	509	131872229	571	186169411	633	253636137	695	335702375
386	57512456	448	89915392	510	132651000	572	187149248	634	254840104	696	337153536
387	57960603	449	90518849	511	133432831	573	188132517	635	256047875	697	338608873
388	58411072	450	91125000	512	134217728	574	189119224	636	257259456	698	340068392
389	58863869	451	91733851	513	135005697	575	190109375	637	258474853	699	341532099
390	59319000	452	92345408	514	135796744	576	191102976	638	259694072	700	343000000
391	59776471	453	92959677	515	136590875	577	192100033	639	260917119	701	344472101
392	60236288	454	93576664	516	137388096	578	193100552	640	262144000	702	345948008
393	60698457	455	94196375	517	138188413	579	194104539	641	263374721	703	347428927
394	61162984	456	94818816	518	138991832	580	195112000	642	264609288	704	348913664
395	61629875	457	95443993	519	139798359	581	196122941	643	265847707	705	350402625
396	62099136	458	96071912	520	140608000	582	197137368	644	267089984	706	351895816
397	62570773	459	96702579	521	141420761	583	198155287	645	268336125	707	353393243
398	63044792	460	97336000	522	142236648	584	199176704	646	269586136	708	354894912
399	63521199	461	97972181	523	143055667	585	200201625	647	270840023	709	356400829
400	64000000	462	98611128	524	143877824	586	201230056	648	272097792	710	357911000
401	64481201	463	99252847	525	144703125	587	202262003	649	273359449	711	359425431
402	64964808	464	99897344	526	145531576	588	203297472	650	274625000	712	360944128
403	65450827	465	100544625	527	146363183	589	204336469	651	275894451	713	362467097
404	65939264	466	101194696	528	147197952	590	205379000	652	277167808	714	363994344
405	66430125	467	101847563	529	148035889	591	206425071	653	278445077	715	365525875
406	66923416	468	102503232	530	148877000	592	207474688	654	279726264	716	367061696
407	67419143	469	103161709	531	149721291	593	208527857	655	281011375	717	368601813
408	67911312	470	103823000	532	150568768	594	209584584	656	282300416	718	370146232
409	68417929	471	104487111	533	151419437	595	210644875	657	283593393	719	371694959
410	68921000	472	105154048	534	152273304	596	211708736	658	284890312	720	373248000
411	69426531	473	105823817	535	153130375	597	212776173	659	286191179	721	374805361
412	69934528	474	106496424	536	153990656	598	213847192	660	287496000	722	376367048
413	70444997	475	107171875	537	154854153	599	214921799	661	288804781	723	377933067
414	70957944	476	107850176	538	155720872	600	216000000	662	290117528	724	379503424
415	71473375	477	108531333	539	156590819	601	217081801	663	291434247	725	381078125
416	71991296	478	109215352	540	157464000	602	218167208	664	292754944	726	382657176

Num	Cubes.	Num.	Cubes.	Num.	Cubes.	Num.	Cubes.	Num.	Cubes.
727	384240583	782	478211768	837	586376253	892	709732288	947	849278123
728	385828352	783	480048687	838	588480472	893	712121957	948	851971392
729	387420489	784	481890304	839	590589719	894	714516984	949	854670349
730	389017000	785	483736625	840	592704000	895	716917375	950	857375000
731	390617891	786	485587656	841	594823321	896	719323136	951	860085351
732	392223168	787	487443403	842	596947688	897	721734273	952	862801408
733	393832837	788	489303872	843	599077107	898	724150792	953	865523177
734	395446904	789	491169069	844	601211584	899	726572699	954	868250664
735	397065375	790	493039000	845	603351125	900	729000000	955	870983875
736	398688256	791	494913671	846	605495736	901	731432701	956	873722816
737	400315553	792	496793088	847	607645423	902	733870808	957	876467493
738	401947272	793	498677257	848	609800192	903	736314327	958	879217912
739	403583419	794	500566184	849	611960049	904	738763264	959	881974079
740	405224000	795	502459875	850	614125000	905	741217625	960	884736000
741	406869021	796	504358336	851	616295051	906	743677416	961	887503681
742	408518488	797	506261573	852	618470208	907	746142643	962	890277128
743	410172407	798	508169592	853	620650477	908	748613312	963	893056347
744	411830784	799	510082399	854	622835864	909	751089429	964	895841344
745	413493625	800	512000000	855	625026375	910	753571000	965	898632125
746	415160936	801	513922401	856	627222016	911	756058031	966	901428696
747	416832723	802	515849608	857	629422793	912	758550528	967	904231063
748	418508992	803	517781627	858	631628712	913	761048497	968	907039232
749	420189749	804	519718464	859	633839779	914	763551944	969	909853209
750	421875000	805	521660125	860	636056000	915	766060875	970	912673000
751	423564751	806	523606616	861	638277381	916	768575296	971	915498611
752	425259008	807	525557943	862	640503928	917	771095213	972	918330048
753	426957777	808	527514112	863	642735647	918	773620632	973	921167317
754	428661064	809	529475129	864	644972544	919	776151559	974	924010424
755	430368875	810	531441000	865	647214625	920	778688000	975	926859375
756	432081216	811	533411731	866	649461896	921	781229961	976	929714176
757	433798093	812	535387328	867	651714363	922	783777448	977	932574833
758	435519512	813	537366797	868	653972032	923	786330467	978	935441352
759	437245479	814	539353144	869	656234909	924	788889024	979	938313739
760	438976000	815	541343375	870	658503000	925	791453125	980	941192000
761	440711081	816	543338496	871	660776311	926	794022776	981	944076141
762	442450728	817	545338513	872	663054848	927	796597983	982	946966168
763	444194947	818	547343432	873	665338617	928	799178752	983	949862087
764	445943744	819	549353259	874	667627624	929	801765089	984	952763904
765	447697125	820	551368000	875	669921875	930	804357000	985	955671625
766	449455096	821	553387661	876	672221376	931	806954491	986	958585256
767	451217663	822	555412248	877	674526133	932	809557568	987	961504803
768	452984832	823	557441767	878	676836152	933	812166237	988	964430272
769	454756609	824	559476224	879	679151439	934	814780504	989	967361669
770	456533000	825	561515625	880	681472000	935	817400375	990	970299000
771	458314011	826	563559976	881	683797841	936	820025856	991	973242271
772	460099648	827	565609283	882	686128968	937	822656953	992	976191488
773	461889917	828	567663552	883	688465387	938	825293672	993	979146657
774	463684824	829	569722789	884	690807104	939	827936019	994	982107784
775	465484375	830	571787000	885	693154125	940	830584000	995	985074875
776	467288576	831	573856191	886	695506456	941	833237621	996	988047936
777	469097433	832	575930368	887	697864103	942	835896888	997	991026973
778	470910952	833	578009537	888	700227072	943	838561807	998	994011992
779	472729139	834	580093704	889	702595369	944	841232384	999	997002999
780	474552000	835	582182875	890	704969000	945	843908625	1000	1000000000
781	476379541	836	584277056	891	707347971	946	846590536		

The Cube of a Binomial, is equal to the cubes of the two parts or members, together with triple of the two parallelopipedons under each part and the ſquare of the other; viz, $\overline{a+b}|^3 = a^3 + 3a^2b + 3ab^2 + b^3$. And hence the common method of extracting the cube root.

Cubic *Equations*, are thoſe in which the unknown quantities riſe to three dimenſions; as $x^3 = a$, or $x^3 + ax^2 = b$, or $x^3 + ax^2 + bx = c$, &c.

All cubic equations may be reduced to this form, $x^3 + px = q$; viz, by taking away the 2d term.

All cubic equations have three roots; which are either all real, or elſe one only is real, and the other two imaginary; for all roots become imaginary by pairs.

But the nature of the roots as to real and imaginary, is known partly from the ſign of the co-efficient p, and partly from the relation between p and q: for the equation has always two imaginary roots when p is poſitive; it has alſo two imaginary roots when p is negative, provided $\overline{\tfrac{1}{3}p}|^3$ is leſs $\overline{\tfrac{1}{2}q}|^2$, or $4p^3$ leſs than $27q^2$; otherwiſe the roots are all real, namely, whenever p is negative, and $4p^3$ either equal to, or greater than $27q^2$.

Every cubic equation of the above form, viz, wanting the 2d term, has both poſitive and negative roots, and the greateſt root is always equal to the ſum of the two leſs roots; viz, either one poſitive root equal to the ſum of the two negative ones, or elſe one negative root equal to the ſum of two ſmaller and poſitive ones. And the ſign of the greateſt, or ſingle root, is poſitive or negative, according as q is poſitive or negative when it ſtands on the right-hand ſide of the equation, thus $x^3 + px = q$; and the two ſmaller roots have always the contrary ſign to q.

So that, in general, the ſign of p determines the nature of the roots, as to real and imaginary; and the ſign of q determines the affection of the roots, as to poſitive and negative. See my Tract on Cubic Equations in the Philoſ. Tranſ. for 1780.

To find the Values of the Roots of Cubic Equations. Having reduced the equation to this form $x^3 + px = q$, its root may be found in various ways; the firſt of theſe, is that which is called Cardan's Rule, by whom it was firſt publiſhed, but invented by Ferreus and Tartalea. See Algebra. The rule is this: Put $a = \tfrac{1}{3}p$, and $b = \tfrac{1}{2}q$; then is Cardan's root

$$x = \sqrt[3]{b + \sqrt{b^2 + a^3}} + \sqrt[3]{b - \sqrt{b^2 + a^3}};$$ or if there be put $s = \sqrt[3]{b + \sqrt{b^2 + a^3}}$, and $d = \sqrt[3]{b - \sqrt{b^2 + a^3}}$; then $s + d = x$, the 1ſt or Cardan's root,

alſo $-\dfrac{s+d}{2} + \dfrac{s-d}{2}\sqrt{-3}$ is the 2d root,

and $-\dfrac{s+d}{2} - \dfrac{s-d}{2}\sqrt{-3}$ is the 3d root.

Now the firſt of theſe, or Cardan's root, is always a real root, though it is not always the greateſt root, as it has been commonly miſtaken for. And yet this rule always exhibits the root in the form of an imaginary quantity when the equation has no imaginary roots at all; but in the form of a real quantity when the equation has two imaginary roots. See the reaſon of this explained in my Tract above cited, pa. 407. As to the other two roots, viz $-\dfrac{s+d}{2} \pm \dfrac{s-d}{2}\sqrt{-3}$, though, in their general form, they have an imaginary appearance; yet, by ſubſtituting certain particular numbers, they come out in a real form in all ſuch caſes as they ought to be ſo.

But, after the firſt root is found, by Cardan's rule, the other two roots may be found, or exhibited, in ſeveral other different ways; ſome of which are as follow:

Let r denote the 1ſt, or Cardan's root,
and v and w the other two roots:
then is $v + w = -r$, and $vwr = q$;
and the reſolution of theſe two equations will give the other two roots v and w.

Or reſolve the quadratic equation $x^2 + rx + r^2 + p = 0$, and its two roots will be thoſe ſought.

Or the ſame two roots will be

either $-\tfrac{1}{2}r + \tfrac{1}{2}\sqrt{-p - \dfrac{3q}{r}}$ and $-\tfrac{1}{2}r - \tfrac{1}{2}\sqrt{-p - \dfrac{3q}{r}}$,

or $-\tfrac{1}{2}r + \tfrac{1}{2}\sqrt{r^2 - \dfrac{4q}{r}}$ and $-\tfrac{1}{2}r - \tfrac{1}{2}\sqrt{r^2 - \dfrac{4q}{r}}$,

or $-\tfrac{1}{2}r + \tfrac{1}{2}\sqrt{1 - \dfrac{4q}{q - pr}}$ and $-\tfrac{1}{2}r - \tfrac{1}{2}r\sqrt{1 - \dfrac{4q}{q - pr}}$.

Ex. 1. If the equation be $x^3 - 15x = 4$: here $p = -15$, $q = 4$, $a = -5$, $b = 2$; hence $\sqrt{b^2 + a^3} = \sqrt{4 - 125} = \sqrt{-121} = 11\sqrt{-1}$, $s = \sqrt[3]{2 + \sqrt{-121}} = 2 + \sqrt{-1}$, and $d = \sqrt[3]{2 - \sqrt{-121}} = 2 - \sqrt{-1}$: therefore $r = c + d = 4$, the 1ſt root; and $-\dfrac{s+d}{2} \pm \dfrac{s-d}{2}\sqrt{-3} = -2 \mp \sqrt{3}$, the other two roots.

Ex. 2. If $x^3 - 6x = 4$: here $a = -2$, and $b = 2$; therefore $\sqrt{b^2 + a^3} = \sqrt{4 - 8} = \sqrt{-4} = 2\sqrt{-1}$, $s = \sqrt[3]{2 + 2\sqrt{-1}} = -1 + \sqrt{-1}$, & $d = \sqrt[3]{2 - 2\sqrt{-1}} = -1 - \sqrt{-1}$: hence then $r = s + d = -2$, the firſt root; and $1 \pm \sqrt{3}$ the other two roots.

Ex. 3. If $x^3 + 18x = 6$: here $a = 6$, and $b = 3$; then $\sqrt{b^2 + a^3} = \sqrt{9 + 216} = \sqrt{225} = 15$, $s = \sqrt[3]{3 + 15} = \sqrt[3]{18}$, and $d = \sqrt[3]{3 - 15} = \sqrt[3]{-12} = -\sqrt[3]{12}$: therefore $r = s + d = \sqrt[3]{18} - \sqrt[3]{12} = \cdot 331313$, the 1ſt root, and $-\dfrac{\sqrt[3]{18} - \sqrt[3]{12}}{2} \pm \dfrac{\sqrt[3]{18} + \sqrt[3]{12}}{2}\sqrt{-3}$ are the two other roots.

2. Another method for the roots of the equation $x^3 + px = q$, is by means of infinite ſeries, as ſhewn at pa. 415 and ſeq. of my Tract above cited; whence it appears that the roots are exhibited in various forms of ſeries as follow: viz,

$$2\sqrt[3]{b} \times \quad : 1 - \frac{2c^2}{3.6b^2} - \frac{2.5.8c^4}{3.6.9.12b^4} \ \&c$$

for the 1ſt root, and

$$-\sqrt[3]{b} \times \quad : 1 - \frac{2c^2}{3.6b^2} - \frac{2.5.8c^4}{3.6.9.12b^4} \ \&c$$

$$\pm \frac{c\sqrt{-3}}{\sqrt[3]{b^2}} \times : \tfrac{1}{3} + \frac{2.5c^2}{3.6.9b^2} + \frac{2.5.8.11c^4}{3.6.9.12.15b^4} \ \&c$$

for the two other roots: where $a = \tfrac{1}{3}p$, $b = \tfrac{1}{2}q$, and $c = \sqrt{b^2 + a^3}$.

And various other ſeries for the ſame purpoſe may alſo be ſeen in my Tract, ſo often before cited.

3. A third method for the roots of cubic equations, is by angular ſections, and the table of ſines. It was firſt hinted by Bombelli, in his Algebra, that angles are triſected by the reſolution of cubic equations. Afterwards, Vieta gave the reſolution of cubics, and the higher equations, by angular ſections. Next, Albert Girard, in his Invention Nouvelle en l'Algebre, ſhews how to reſolve the irreducible caſe in cubics by a table of ſines: and he alſo conſtructs the ſame, or finds the roots, by the interſection of the hyperbola and circle. Halley and De Moivre alſo gave rules and examples of the ſame ſort of reſolutions by a table of ſines. And, laſtly, Mr. Anthony Thacker invented, and Mr. William Brown computed, a large ſet of ſimilar tables, for reſolving affected quadratic and cubic equations, with their application to the reſolution of biquadratic ones.

4. Laſtly, the ſeveral methods of approach, or approximation, for the roots of all affected equations, which have been uſed in various ways by Stevin, Vieta, Newton, Halley, Raphſon, and others.

To theſe may be added the method of Trial-and-error, or of Double Poſition, one of the eaſieſt and beſt of any. Of this method, let there be taken the laſt example, viz, $x^3 + 18x = 6$, in which it is evident that x is very nearly equal to $\frac{1}{3}$, but a little leſs; take it therefore $x = \cdot 33$; then $x^3 + 18x = 5\cdot 975937$, but ſhould be 6, and therefore the error is ·024063 in defect.

Again ſuppoſe $x = \cdot 34$; then $x^3 + 18x = 6\cdot 159304$, which is ·159304 in exceſs.

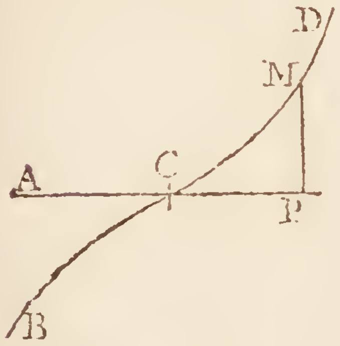
Therefore $\frac{\cdot 34 \times \cdot 024063 + \cdot 33 \times \cdot 159304}{\cdot 024063 + \cdot 159304} = \frac{60752}{183367} =$ ·3313, the root as before very nearly.

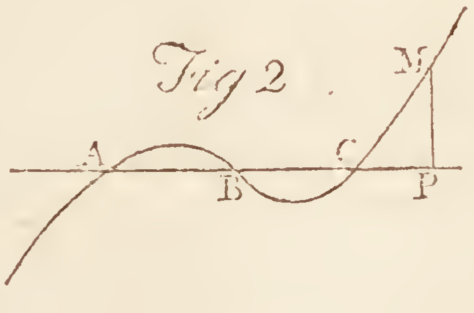
For the conſtruction of cubic equations, ſee CONSTRUCTION.

CUBIC *Foot*, of any thing, is ſo much of it as is contained in a cube whoſe ſide is one foot.

CUBIC *Hyperbola*, is a figure expreſſed by the equation $xy^2 = a$, having two aſymptotes, and conſiſting of two hyperbolas, lying in the adjoining angles of the aſymptotes, and not in the oppoſite angles, like the Apollonian hyperbola; being otherwiſe called by Newton, in his Enumeratio Linearum Tertii Ordinis, an hyperboliſmus of a parabola; and is the 65th ſpecies of thoſe lines according to him.

CUBIC *Numbers*. See CUBES.



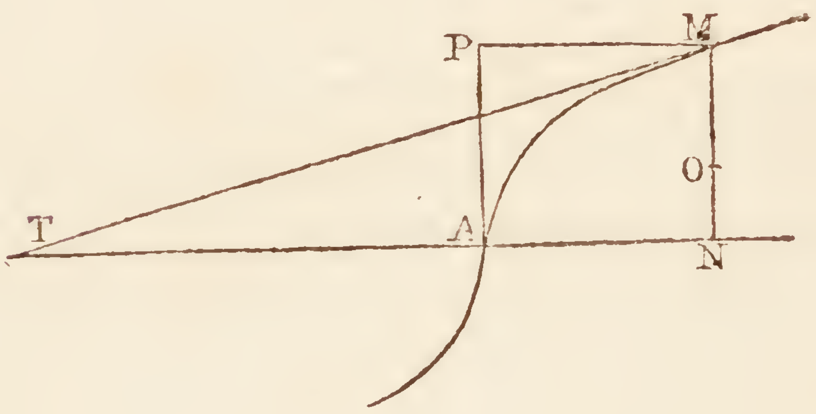


CUBIC *Parabola*, a curve, as BCD, of the 2d order, having two infinite legs CB, CD, tending contrary ways. And if the abſciſs, AP or x, touch the curve in C, the relation between the abſciſs and ordinate, viz, $AP = x$, and $PM = y$, is expreſſed by the equation $y = ax^3 + bx^2 + cx + d$; or when A coincides with C, by the equation $y = ax^3$, which is the ſimpleſt form of the equation of this curve.

If the right line AP (fig. 2) cut the cubical parabola in three points A, B, C; and from any point P there be drawn the right line or ordinate PM, cutting the curve in one point M only: then will PM be always as the ſolid $AP \times BP \times CP$; which is an eſſential property of this curve.

And hence it is eaſy to conſtruct a cubic equation, as $x^3 + a^2x = b^3$, by the interſection of this curve and a right line. See the Conſtruction of a cubic equation by means of the cubic parabola and a right line, by Dr. Wallis, in his Algebra: As alſo the Conſtruction of equations of 6 dimenſions, by means of the cubic parabola and a circle, by Dr. Halley, in a lecture formerly read at Oxford.

The curve of this parabola cannot be rectified, not even by means of the conic ſections. But a circle may be found equal to the *Curve Surface*, generated by the rotation of the curve AM about the tangent AP to the principal vertex A. Let MN be an ordinate, and MT a tangent at the point M; and let PM be parallel to AN. Divide MN in the point O, ſo that MO be to ON as TM to MN. Then a mean proportional between TM + ON and $\frac{1}{3}$ of AN will be the radius of a circle, whoſe area is equal to the ſuperficies deſcribed by that rotation, viz, of AM about AP.



The Area of a Cubic Parabola is $\frac{3}{4}$ of its circumſcribing parallelogram.

CUBIC *Root*, of any number, or quantity, is ſuch a quantity as being cubed, or twice multiplied by itſelf, ſhall produce that which was given. So, the cubic root of 8 is 2, becauſe 2^3 or $2 \times 2 \times 2$ is equal to 8.

The common method of extracting the cube root, founded on the property given above, viz, $\overline{a+b}|^3 = a^3 + 3a^2b + 3ab^2 + b^3$, is found in every book of common arithmetic, and is as old at leaſt as Lucas de Burgo, where it is firſt met with in print. Other methods for the cube root may be ſeen under the article EXTRACTION *of Roots*, particularly this one, viz, the cube root of n, or $\sqrt[3]{n} = \frac{2n + a^3}{n + 2a^3} \times a$, very nearly, or $2a^3 + n : 2n + a^3 :: a : \sqrt[3]{n}$ the cube root of n nearly; where n

is any number given whoſe cube root is ſought, and a^3 is the neareſt complete cube to n, whether greater or leſs.

For example, ſuppoſe it were propoſed to double the cube, or, which comes to the ſame thing, to extract the cube root of the number 2. Herc the neareſt cube is 1, whoſe cube root is 1 alſo, that is, $a^3 = 1$, and $a = 1$, alſo $n = 2$; therefore

$$\begin{array}{lclcl} 2a^3+n & & 2n+a^3 & & a \\ 4 & : & 5 & :: 1 : & \frac{5}{4} = 1{\cdot}25 = \sqrt[3]{2} \text{ nearly.} \end{array}$$

But, for a nearer value, aſſume now $a = \frac{5}{4}$, or $a^3 = \frac{125}{64}$; then is $2a^3 + n = \frac{250}{64} + 2 = \frac{378}{64}$, and $2n + a^3 = 4 + \frac{125}{64} = \frac{381}{64}$; hence $378 : 381 :: \frac{5}{4} : \frac{635}{504}$ or $1{\cdot}259921 = \sqrt[3]{2}$, or the cube root of 2, which is true in the laſt place of decimals.

And this is the ſimpleſt and eaſieſt method for the cube root of any number. See its inveſtigation in my Tracts, vol. 1, pa. 49.

Every number or quantity has three cubic roots, one that is real, and two imaginary: So, the cube root of 1 is either 1, or $-\frac{1+\sqrt{-3}}{2}$, or $-\frac{1-\sqrt{-3}}{2}$; and if r be the real root of any cube r^3, the two imaginary cubic roots of it will be

$-\frac{1+\sqrt{-3}}{2}r$, and $-\frac{1-\sqrt{-3}}{2}r$: for any one of theſe being cubed, gives the ſame cube r^3.

CUBING *of a Solid.* See CUBATURE.

CUBO-CUBE, the 6th power.

CUBO-CUBO-CUBE, the 9th power.

CULMINATION, the paſſage of a ſtar or planet over the meridian, or that point of its orbit which it is in at its greateſt altitude. Hence a ſtar is ſaid to culminate, when it paſſes the meridian.

To find the time of a Star's culminating. Eſtimate the time nearly; and find the right aſcenſion both of the ſun and ſtar corrected for this eſtimated time; then the difference between theſe right aſcenſions, converted into ſolar time, at the rate of 15 degrees to the hour, gives the time of ſouthing. See an example of this calculation every year in White's Ephemeris, pa. 45.

CULVERIN, was the name of a piece of ordnance; but is now diſuſed.

CUNETTE. See CUVETTE.

CUNEUS. See WEDGE.

CUNITIA (MARIA), a lady of Sileſia, who was famous for her extenſive knowledge in many branches of learning, but more particularly in mathematics and aſtronomy, upon which ſhe wrote ſeveral ingenious treatiſes; particularly one, entitled *Urania Propitia*, printed in 1650, in Latin and German, and dedicated to Ferdinand the third, emperor of Germany. In this work are contained aſtronomical tables of great eaſe and accuracy, founded upon Kepler's hypotheſis. But notwithſtanding her merit ſhines with ſuch peculiar luſtre as to reflect honour on her ſex, hiſtory does not inform us of the time of her birth or death.

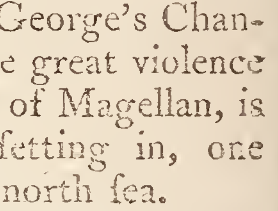
CURRENT, a ſtream or flux of water in any direction. The *ſetting* of the current, is that point of the compaſs towards which the waters run; and the *drift* of a current is the rate it runs an hour.

Currents in the ſea, are either natural and general, as ariſing from the diurnal rotation of the earth on its axis, or the tides, &c; or accidental and particular, cauſed by the waters being driven againſt promontories, or into gulphs and ſtreights; from whence they are forced back, and thus diſturb the natural flux of the ſea.

The currents are ſo violent under the equator, where the motion of the earth is greateſt, that they hurry veſſels very ſpeedily from Africa to America; but abſolutely prevent their return the ſame way: ſo that ſhips are forced to run as far as the 40th or 45th degree of north latitude, to fall into the return of the current again, to bring them home to Europe. It is ſhewn by Governor Pownal, that this current performs a continual circulation, ſetting out from the Guinea coaſt in Africa, for example, from thence croſſing ſtraight over the Atlantic ocean, and ſo ſetting into the Gulph of Mexico by the ſouth ſide of it; then ſweeping round by the bottom of the Gulph, it iſſues out by the north ſide of it, and thence takes a direction north-eaſterly along the coaſt of North America, till it arrives near Newfoundland, where it is turned by a rounding motion backward acroſs the Atlantic again, upon the coaſts of Europe, and from thence ſouthward again to the coaſt of Africa, from whence it ſet out.

In the ſtreights of Gibraltar, the currents ſet in by the ſouth ſide, ſweep along the coaſt of Africa to Egypt, by Paleſtine, and ſo return by the northern ſide, or European coaſts, and iſſue out again by the northern ſide of the ſtreights. In St. George's Channel too they uſually ſet eaſtward. The great violence and danger of the ſea in the Streights of Magellan, is attributed to two contrary currents ſetting in, one from the ſouth, and the other from the north ſea.

Currents are of ſome conſideration in the art of navigation, as a ſhip is by them either accelerated or retarded in her courſe, according as the ſet is with or againſt the ſhip's motion. If a ſhip ſail along the direction of a current, it is evident that the velocity of the current muſt be added to that of the veſſel: but if her courſe be directly againſt the current, it muſt be ſubtracted: and if ſhe ſail athwart it, her motion will be compounded with that of the current; and her velocity augmented or retarded, according to the angle of her direction with that of the current; that is, ſhe will proceed in the diagonal of the parallelogram formed according to the two lines of direction, and will deſcribe or paſs over that diagonal in the ſame time in which ſhe would have deſcribed either of the ſides, by the ſeparate velocities.



For ſuppoſe ABDC be a parallelogram, the diagonal of which is AD. Now if the wind alone would drive the ſhip from A to B, in the ſame time as the current alone would drive it from A to C: Then, as the wind neither helps nor hinders the ſhip from coming towards the line CD, the current will bring it there in the ſame time as if the wind did not act: And as the current neither helps nor hinders the ſhip from coming towards the line BD, the wind will bring it there in the ſame time as if the current did not act: Therefore the ſhip muſt, at the end of that time, be found in both thoſe lines, that is, in their meeting D: Conſequently the ſhip muſt have paſſed from A to D in the diagonal AD. See Composition *of Forces*.

See the Sailing in Currents largely exemplified in Robertſon's Navigation, vol. 2, book 7, ſect. 8.

CURSOR, a ſmall piece of braſs &c that ſlides; as, the piece in an equinoctial ring dial that ſlides to the day of the month; or the little ruler or label of braſs ſliding in a groove along the middle of another label, repreſenting the horizon in the analemma; or the point that ſlides along the beam compaſs; &c.

CURTATE *Diſtance*, is the diſtance of a planet's place from the ſun, reduced to the ecliptic; or, the interval between the ſun and that point where a perpendicular, let fall from the planet, meets the ecliptic.

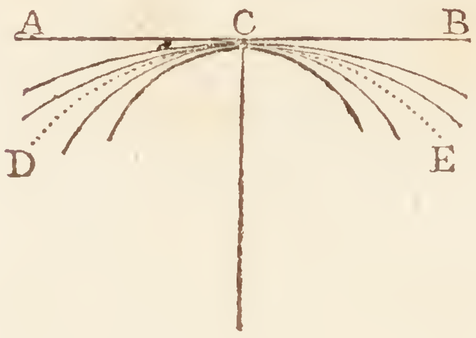
CURTATION, the interval between a planet's diſtance from the ſun, and the curtate diſtance.

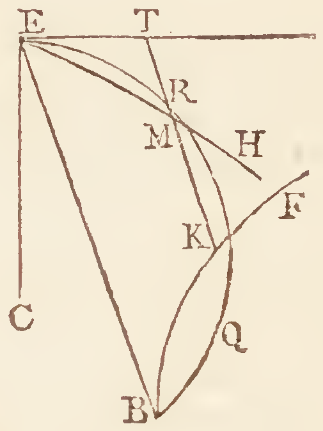
From the foregoing article it is eaſy to find the curtate diſtance; whence the manner of conſtructing tables of curtation is obvious; the quantity of inclination, reduction, and curtation of a planet, depending on the argument of latitude. Kepler, in his Rodolphine Tables, reduces the tables of them all into one, under the title of *Tabulæ Latitudinariæ*.

CURTIN, Curtain, or Courtine, in Fortification, that part of a wall or rampart that joins two baſtions, or lying between the flank of one and that of another.—The curtain is uſually bordered with a parapet 5 feet high; behind which the ſoldiers ſtand to fire upon the covert-way, and into the moat.

CURVATURE *of a Line*, is its bending, or flexure; by which it becomes a curve, of any peculiar form and properties. Thus, the nature of the curvature of a circle is ſuch, as that every point in the periphery is equally diſtant from a point within, called the centre; and ſo the curvature of the ſame circle is every where the ſame; but the curvature in all other curves is continually varying.—The curvature of a circle is ſo much the more, as its radius is leſs, being always reciprocally as the radius; and the curvature of other curves is meaſured by the reciprocal of the radius of a circle having the ſame degree of curvature as any curve has, at ſome certain point.

Every curve is bent from its tangent by its curvature, the meaſure of which is the ſame as that of the angle of contact formed by the curve and tangent. Now the ſame tangent AB is common to an infinite number of circles, or other curves, all touching it and each other in the ſame point of contact C. So that any curve DCE may be touched by an infinite number of different circles at the ſame point C; and ſome of theſe circles fall wholly within it, being more curved, or having a greater curvature than that curve; while others fall without it near the point of contact, or between the curve and tangent at that point, and ſo, being leſs deflected from the tangent than the curve is, they have a leſs degree of curvature there. Conſequently there is one, of this infinite number of circles, which neither falls below it nor above it, but, being equally deflected from the tangent, coincides with it moſt intimately of all the circles; and the radius of this circle is called the *radius of curvature* of the curve; alſo the circle itſelf is called the *circle of curvature*, or the *oſculatory* circle of that curve, becauſe it touches it ſo cloſely that no other circle can be drawn between it and the curve.





As a curve is ſeparated from its tangent by its flexure or curvature, ſo it is ſeparated from the oſculatory circle by the increaſe or decreaſe of its curvature; and as its curvature is greater or leſs, according as it is more or leſs deflected from the tangent, ſo the variation of curvature is greater or leſs, according as it is more or leſs ſeparated from the circle of curvature.

It appears, however, from the demonſtration of geometricians, that circles may touch curve lines in ſuch a manner, that there may be indefinite degrees of more or leſs intimate contact between the curve and its oſculatory circle; and that a conic ſection may be deſcribed that ſhall have the ſame curvature with a given line at a given point, and the ſame variation of curvature, or a contact of the ſame kind with the circle of curvature.

If we conceive the tangent of any propoſed curve to be a baſe, and that a new line or curve be deſcribed, whoſe ordinate, upon the ſame baſe or abſciſs, is a 3d proportional to the ordinate and baſe of the firſt; this new curve will determine the chord of the circle of curvature, by its interſection with the ordinate at the point of contact; and it will alſo meaſure the variation of curvature, by means of the tangent of the angle in which it cuts that circle: the leſs this angle is, the cloſer is the contact of the curve and circle of curvature; and of this contact there may be indefinite degrees.

For example, let EMH be any curve, to which ET is a tangent at the point E; then let there be always taken MT : ET :: ET : TK, and through all the points K draw the curve BKF; then from the point of contact E draw EB parallel to the ordinate TK, meeting the laſt curve in B; and finally, deſcribe a circle ERQB through the point B and touching ET in E; and it ſhall be the oſculatory circle to the given curve EMH. And the contact of EM and ER is always the cloſer, the leſs the angle KBQ is. See Maclaurin's Fluxions, art. 366.

Hence it follows, that the contact of the curve EMH and its oſculatory circle is cloſeſt, when the curve BK touches the arch BQ in B, the angle BET being given; and it is fartheſt from this, or moſt open, when BK touches the right line BE in B.

Hence alſo there may be indefinite degrees of more and more intimate contact between a circle and a curve. The firſt degree is when the ſame right line touches them both in the ſame point; and a contact of this ſort may take place between any circle and any arch of a curve. The 2d is when the curve EMH and circle ERB have the ſame curvature, and the tangents of the curve BKF and circle BQE interſect each other at B in any aſſignable angle. The contact of the curve EM and circle of curvature ER at E, is of the 3d degree, or order, and their oſculation is of the 2d, when the curve BKF touches the circle BQE at B, but ſo as not to have the ſame curvature with it. The contact is of the 4th degree, or order, and their oſculation of the 3d, when the curve BKF has the ſame curvature with the circle BQE at B, but ſo as that their contact is only of the 2d degree. And this gradation of more and more intimate contact, or of approximation towards coincidence, may be continued indefinitely, the contact of EM and ER at E being always of an order two degrees cloſer than that of BK and BQ at B. There is alſo an indefinite variety comprehended under each order: thus, when EM and ER have the ſame curvature, the angle formed by the tangents of BK and BQ admits of indefinite variety, and the contact of EM and ER is the cloſer the leſs that angle is. And when that angle is of the ſame magnitude, the contact of EM and ER is the cloſer, the greater the circle of curvature is. When BK and BQ touch at B, they may touch on the ſame or on different ſides of their common tangent; and the angle of contact KBQ may admit of the ſame variety with the angle of contact MER; but as there is ſeldom occaſion for conſidering thoſe higher degrees of more intimate contact of the curve EMH and circle of curvature ERB, Mr. Maclaurin calls the contact or oſculation of the ſame kind, when, the chord EB and angle BET being given, the angle contained by the tangents of BK and BQ is of the ſame magnitude.

When the curvature of EMH increaſes from E towards H, and conſequently correſponds to that of a circle gradually leſs and leſs, the arch EM falls within ER, the arch of the oſculatory circle, and BK is within BQ. The contrary happens when the curvature of EM decreaſes from E towards H, and conſequently correſponds to that of a circle which is gradually greater and greater, the arch EM falls without ER, and BK is without BQ. And according as the curvature of EM varies more or leſs, it is more or leſs unlike to the uniform curvature of a circle; the arch of the curve EMH ſeparates more or leſs from the arch of the oſculatory circle ERB, and the angle contained by the tangents of BKF and BQE at B, is greater or leſs. Thus the quality of curvature, as it is called by Newton, depends on the angle contained by the tangents of BK and BQ at B; and the meaſure of the inequability or variation of curvature, is as the tangent of this angle, the radius being given, and the angle BET being a right one.

The radii of curvature of ſimilar arcs in ſimilar figures, are in the ſame ratio as any homologous lines of theſe figures; and the variation of curvature is the ſame. See Maclaurin, art. 370.

When the propoſed curve EMH is a conic ſection, the new line BKF is alſo a conic ſection; and it is a right line when EMH is a parabola, to the axis of which the ordinates TK are parallel. BKF is alſo a right line when EMH is an hyperbola, to one aſymptote of which the ordinate TK is parallel.

When the ordinate EB, at the point of contact E, inſtead of meeting the new curve BK, is an aſymptote to it, the curvature of EM will be leſs than in any circle; and this is the caſe in which it is ſaid to be infinitely little, or that the radius of curvature is infinitely great. And of this kind is the curvature at the points of contrary flexure in lines of the 3d order.

When the curve BK paſſes through the point of contact E, the curvature is greater than in any circle, or the radius of curvature vaniſhes; and in this caſe the curvature is ſaid to be infinitely great. Of this kind is the curvature at the cuſps of the lines of the 3d order.

As to the degree of curvature in lines of the 3d and higher orders, ſee Maclaurin, art. 379; alſo art. 386, when the propoſed curve is mechanical.

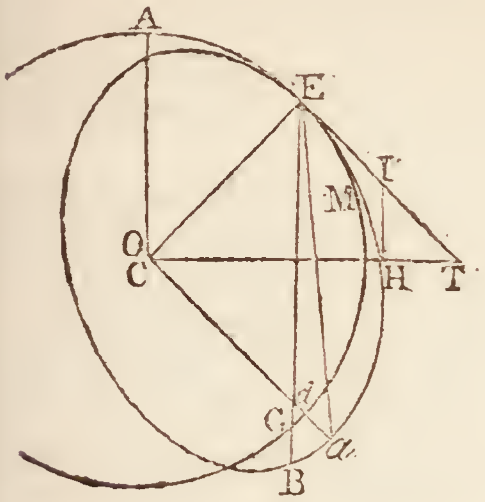
As curves which paſs through the ſame point have the ſame tangent when the firſt fluxions of the ordinates are equal, ſo they have the ſame curvature when the 2d fluxions of the ordinate are likewiſe equal; and half the chord of the oſculatory circle that is intercepted between the points where it interſects the ordinate, is a 3d proportional to the right lines that meaſure the 2d fluxion of the ordinate and firſt fluxion of the curve, the baſe being ſuppoſed to flow uniformly. When a ray revolving about a given point, and terminated by the curve, becomes perpendicular to it, the firſt fluxion of the radius vaniſhes; and if its 2d fluxion vaniſh at the ſame time, that point muſt be the centre of curvature. The ſame may be ſaid, when the angular motion of the ray about that point is equal to the angular motion of the tangent of the curve; as the angular motion of the radius of a circle about its centre is always equal to the angular motion of the tangent of the circle. Hence the various properties of the circle may ſuggeſt ſeveral theorems for determining the centre of curvature.

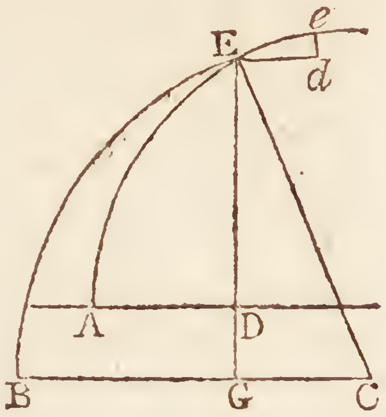
See art. 396 of the ſaid book, alſo the following, concerning the curvature of lines that are deſcribed by means of right lines revolving about given poles, or of angles that either revolve about ſuch poles, or are carried along fixed lines.

It is to be obſerved that, as when a right line interſects an arc of a curve in two points, if by varying the poſition of that line the two interſections unite in one point, it then becomes the tangent of the arc; ſo when a circle touches a curve in one point, and interſects it in another, if, by varying the centre, this interſection joins the point of contact, the circle has then the cloſeſt contact with the arc, and becomes the circle of curvature; but it ſtill continues to interſect the curve at the ſame point where it touches it, that is, where the ſame right line is their common tangent, unleſs another interſection join that point at the ſame time. In general, the circle of curvature interſects the curve at the point of oſculation, only when the number of the ſucceſſive orders of fluxions of the radius of curvature, that vaniſhes when this radius comes to the point of oſculation, is an even number.

It has been ſuppoſed by ſome, that two points of contact, or four interſections of the curve and circle of curvature, muſt join to form an oſculation. But Mr.

James Bernoulli juſtly inſiſted, that the coalition of one point of contact and one interſection, or of three interſections, was ſufficient. In which caſe, and in general, when an odd number of interſections only join each other, the point where they coincide continues to be an interſection of the curve and circle of curvature, as well as a point of their mutual contact and oſculation. See Maclaurin's Flux. art. 493.





From theſe principles may be determined the circle of curvature at any point of a conic ſection. Thus, ſuppoſe AEMHG be any conic ſection, to the point E of which the circle or radius of curvature is to be found. Let ET be a tangent at E, and draw EGB and the tangent HI parallel to the chords of the circle of curvature; then take EB to EG as EI^2 to HI^2; or, when the ſection has a centre O, as in the ellipſe and hyperbola, as the ſquare of the ſemi-diameter O*a* parallel to ET, is to the ſquare of the ſemi-diameter OA parallel to EB; and a circle EB deſcribed upon the chord EB that touches ET, will be the circle of curvature ſought.

When BET is a right angle, or EB is the diameter of the circle of curvature, EG will be the axis of the conic ſection, and EB will be the parameter of this axis; alſo when the point G, where the conic ſection cuts EB, and the point B, are on the ſame ſide of E, then EMG will be an ellipſis, and EG the greater or leſs axis, according as EG is greater or leſs than EB.

The propoſitions relating to the curvature of the conic ſections, commonly given by authors, follow without much difficulty from this conſtruction.

1. When the chord of curvature, thus found, paſſes through the centre of the conic ſection, it will then be equal to the parameter of the diameter that paſſes through the point of contact.

2. The ſquare of the ſemi-diameter O*a*, is to the rectangle of half the tranſverſe and half the conjugate axis, as the radius of curvature CE is to O*a*. And therefore the cube of the ſemi-diameter O*a*, parallel to the tangent ET, is equal to the ſolid contained by the radius of curvature CE, and the rectangle of the two axes.

3. The perpendicular to either axis biſects the angle made by the chord of curvature, and the common tangent of the conic ſection and circle of curvature.

4. The chord of the oſculatory circle that paſſes through the focus, the diameter conjugate to that which paſſes through the point of contact, and the tranſverſe axis of the figure, are in continued proportion.

5. When the ſection is an ellipſe, if the circle of curvature at E meet O*a* in *d*, the ſquare of E*d* will be equal to twice the ſquare of O*a*. Hence E*d* : O*a* :: $\surd 2 : 1$. Which gives an eaſy method of determining the circle of curvature to any point E, when the ſemi-diameter O*a* is given in magnitude and poſition.

Several other properties of the circle of curvature, and methods of determining it when the ſection is given; or vice verſa, of determining the ſection when the circle of curvature is given, may be ſeen in Maclaurin's Flux. art. 375. See alſo the Appendix to Maclaurin's Algebra, ſect. 1.

To determine the Radius and Circle of Curvature by the Method of Fluxions. Let AE*e* be any curve, concave towards its axis AD; draw an ordinate DE to the point E where the curve is required to be found; and ſuppoſe EC perpendicular to the curve, and equal to the radius of the circle BE*e* of curvature ſought; laſtly, draw E*d* parallel to AD, and *de* parallel and indefinitely near to DE; thereby making E*d* the fluxion or increment of the abſciſs AD, alſo *de* the fluxion of the ordinate DE, and E*e* that of the curve AE. Now put $x =$ AD, $y =$ DE, $z =$ AE; and $r =$ CE the radius of curvature; then is ED $= \dot{x}$, *de* $= \dot{y}$, and E*e* $= \dot{z}$.

Now, by ſim. tri. the 3 lines - - E*d*, *de*, E*e*,
or - $\dot{x}$, $\dot{y}$, $\dot{z}$,
are reſpectively as the three - - GE, GC, CE;
therefore - - - $GC \cdot \dot{x} = GE\, \dot{y}$;
and the flux. of this equa. is

$$GC \cdot \ddot{x} + \dot{GC} \cdot \dot{x} = GE \cdot \ddot{y} + \dot{GE}\, \dot{y},$$

or becauſe $\dot{GC} = -\dot{BG}$, it is

$$GC \cdot \ddot{x} - \dot{BG} \cdot \dot{x} = GE \cdot \ddot{y} + \dot{GE} \cdot \dot{y}.$$

But, ſince the two curves AE and BE have the ſame curvature at the point E, their abſciſſes and ordinates have the ſame fluxions at that point, that is E*d* or $\dot{x}$ is the fluxion both of AD and BG, and *de* or $\dot{y}$ is the fluxion both of DE and GE. In the above equation therefore ſubſtitute $\dot{x}$ for $\dot{BG}$, and $\dot{y}$ for $\dot{GE}$, and it becomes - $GC \cdot \ddot{x} - \dot{x}\dot{x} = GE \cdot \ddot{y} + \dot{y}\dot{y}$,
or - $GC \cdot \ddot{x} - GE \cdot \ddot{y} = \dot{x}^2 + \dot{y}^2$ or $\dot{z}^2$.

Now mult. the three terms of this equation reſpectively by theſe three quantities, $\frac{\dot{y}}{GC}, \frac{\dot{x}}{GE}, \frac{\dot{z}}{CE}$, which are equal, and it becomes $\dot{y}\ddot{x} - \dot{x}\ddot{y} = \frac{\dot{z}^3}{CE}$, or $\frac{\dot{z}^3}{}$;

and hence $r = \frac{\dot{z}^3}{\dot{y}\ddot{x} - \dot{x}\ddot{y}}$, which is the general value of the radius of curvature for all curves whatever, in terms of the fluxions of the abſciſs and ordinate.

Farther, as in any caſe either x or y may be ſuppoſed to flow equably, that is, either $\dot{x}$ or $\dot{y}$ conſtant quantities, or $\ddot{x}$ or $\ddot{y}$ = to nothing, by this ſuppoſition either of the terms in the denominator of the value of r may be made to vaniſh. So that when $\dot{x}$ is conſtant, the value of r is $\frac{\dot{z}^3}{-\dot{x}\ddot{y}}$, but r is $= \frac{\dot{z}^3}{\dot{y}\ddot{x}}$ when $\dot{y}$ is conſtant.

For example, ſuppoſe it were required to find the radius or circle of curvature to any point of a parabola, its vertex being A, and axis AD.—Now the equation.

tion of the curve is $ax = y^2$; hence $a\dot{x} = 2y\dot{y}$, and $a\ddot{x} = 2\dot{y}^2$, ſuppoſing $\dot{y}$ conſtant, alſo $a^2\dot{x}^2 = 4y^2\dot{y}^2$; hence r or

$$\frac{\dot{z}^3}{\dot{y}\,\ddot{x}} \text{ or } \frac{\overline{\dot{x}^2+\dot{y}^2}|^{\frac{3}{2}}}{\dot{y}\,\ddot{x}} \text{ is } \frac{\overline{a^2+4y^2}|^{\frac{3}{2}}}{2a^2} \text{ or } \frac{\overline{a+4x}|^{\frac{3}{2}}}{2\sqrt{a}},$$

the general value of the radius of curvature for any point E, the ordinate to which cuts offthe abſciſs AD $= x$.

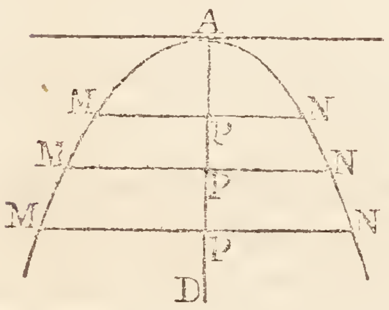
Hence, when x or the abſciſs is nothing, the laſt expreſſion becomes barely $\frac{a^{\frac{3}{2}}}{2\sqrt{a}} = \frac{1}{2}a = r$ for the radius of curvature at the vertex of the parabola; that is, the diameter of the circle of curvature at the vertex of a parabola, is equal to a the parameter of axis.

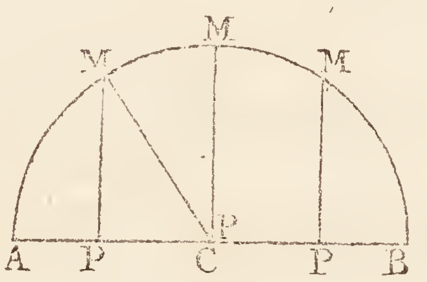
Variation of CURVATURE. See VARIATION.

Double CURVATURE, is uſed for the curvature of a line, which twiſts ſo that all the parts of it do not lie in the ſame plane.

CURVE, a line whoſe ſeveral parts proceed bowing, or tend different ways; in oppoſition to a ſtraight line, all whoſe parts have the ſame courſe or direction.

The doctrine of curves, and of the figures and ſolids generated from them, conſtitute what is called the higher geometry.





In a curve, the line AD, which biſects all the parallel lines MN, is called a *diameter;* and the point A, where the diameter meets the curve, is called the *vertex:* if AD biſect all the parallels at right angles, it is called the *axis*. The parallel lines MN are called *ordinates*, or *applicates;* and their halves, PM, or PN, *ſemi-ordinates*. The portion of the diameter AP, between the vertex, or any other fixed point, and an ordinate, is called the abſciſs; alſo the concourſe of all the diameters, if they meet all in one point, is the *centre*. This definition of the diameter, as biſecting the parallel ordinates, reſpects only the conic ſections, or ſuch curves as are cut only in two points by the ordinates; but in the lines of the 3d order, which may be cut in three points by the ordinates, then the diameter is that line which cuts the ordinates ſo, that the ſum of the two parts that lie on one ſide of it, ſhall be equal ~~to the~~ part on the other ſide: and ſo on for curves of higher orders, the ſum of the parts of the ordinates on one ſide of the diameter, being always equal to the ſum of the parts on the other ſide of it.

Curve lines are diſtinguiſhed into *algebraical* or *geometrical*, and *tranſcendental* or *mechanical*.

Algebraical or *Geometrical* CURVES, are thoſe in which the relation of the abſciſſes AP, to the ordinates PM, can be expreſſed by a common algebraic equation

And *Tranſcendental* or *Mechanical Curves*, are ſuch as cannot be ſo defined or expreſſed by an algebraical equation. See TRANSCENDENTAL *Curve*.

Thus, ſuppoſe, for inſtance, the curve be the circle; and that the radius AC $= r$, the abſciſs AP $= x$, and the ordinate PM $= y$; then, becauſe the nature of the circle is ſuch, that the rectangle AP × PB is always $=$ PM², therefore the equation is $x\,.\,\overline{2r-x} = y^2$, or $2rx - x^2 = y^2$, defining this curve, which is therefore an algebraical or geometrical line. Or, ſuppoſe CP $= x$; then is CM² − CP² $=$ PM², that is $r^2 - x^2 = y^2$; which is another form of the equation of the curve.

The doctrine of curve lines in general, as expreſſed by algebraical equations, was firſt introduced by Des Cartes, who called algebraical curves geometrical ones; as admitting none elſe into the conſtruction of problems, nor conſequently into geometry. But Newton, and after him Leibnitz and Wolfius, are of another opinion; and think, that in the conſtruction of a problem, one curve is not to be preferred to another for its being defined by a more ſimple equation, but for its being more eaſily deſcribed.

Algebraical or geometrical lines are beſt diſtinguiſhed into orders according to the number of dimenſions of the equation expreſſing the relation between its ordinates and abſciſſes, or, which is the ſame thing, according to the number of points in which they may be cut by a right line. And curves of the ſame kind or order, are thoſe whoſe equations riſe to the ſame dimenſion. Hence, of the firſt order, there is the right line only; of the 2d order of lines, or the firſt order of curves, are the circle and conic ſections, being 4 ſpecies only, viz, $dx - x^2 = y^2$ the circle, $\frac{c^2}{d^2}\cdot\overline{dx - x^2} = y^2$ the ellipſe, $\frac{c^2}{d^2}\cdot dx + x^2 = y^2$ the hyperbola, and $dx = y^2$ the parabola: the lines of the 3d order, or curves of the 2d order, are expreſſed by an equation of the 3d degree, having three roots; and ſo on. Of theſe lines of the 3d order, Newton wrote an expreſs treatiſe, under the title of Enumeratio Linearum Tertii Ordinis, ſhewing their diſtinctive characters and properties, to the number of 72 different ſpecies of curves: but Mr. Stirling afterwards added four more to that number; and Mr. Nic. Bernoulli and Mr. Stone added two more.

Curves of the 2d and other higher kinds, Newton obſerves, have parts and properties ſimilar to thoſe of the 1ſt kind: Thus, as the conic ſections have diameters and axes; the lines biſected by theſe are ordinates; and the interſection of the curve and diameter, the vertex: ſo, in curves of the 2d kind, any two parallel right lines being drawn to meet the curve in 3 points; a right line cutting theſe parallels ſo, as that the ſum of the two parts between the ſecant and the curve on one ſide, is equal to the 3d part terminated by the curve on the other ſide, will cut, in the ſame manner, all other right lines parallel to theſe, and that meet the curve in three points, that is, ſo as that the ſum of the two parts on one ſide, will ſtill be equal to the 3d part on the other ſide. Theſe three parts therefore thus equal, may be called *ordinates*, or *applicates;* the cutting line, the *diameter;*

meter ; and where it cuts the ordinates at right angles, the *axis ;* the interſection of the diameter and the curve, the *vertex ;* and the concourſe of two diameters, the *centre ;* alſo the concourſe of all the diameters, the *common* or *general centre.*

Again, as an hyperbola of the firſt kind has two aſymptotes; that of the 2d has 3 ; that of the 3d has 4 ; &c : and as the parts of any right line between the conic hyperbola and its two aſymptotes, are equal on either ſide ; ſo, in hyperbolas of the 2d kind, any right line cutting the curve and its three aſymptotes in three points ; the ſum of the two parts of that right line, extended from any two aſymptotes, the ſame way, to two points of the curve, will be equal to the 3d part extended from the 3d aſymptote, the contrary way, to the 3d point of the curve.

Again, as in the conic ſections that are not parabolical, the ſquare of an ordinate, i. e. the rectangle of the ordinates drawn on the contrary ſides of the diameter, is to the rectangle of the parts of the diameter terminated at the vertices of an ellipſe or hyperbola; in the ſame proportion as a given line called the latus rectum, is to that part of the diameter which lies between the vertices, and called the latus tranſverſum: ſo, in curves of the 2d kind, not parabolical, the parallelopiped under three ordinates, is to the parallelopiped under the parts of the diameter cut off at the ordinates and the three vertices of the figure, in a given ratio: in which, if there be taken three right lines ſituate at the three parts of the diameter between the vertices of the figure, each to each ; then theſe three right lines may be called the *latera recta* of the figure ; and the parts of the diameter between the vertices, the *latera tranſverſa.*

And, as in a conic parabola, which has only one vertex to one and the ſame diameter, the rectangle under the ordinates is equal to the rectangle under the part of the diameter cut off at the ordinates and vertex, and a given right line called the latus rectum : ſo, in curves of the 2d kind, which have only two vertices to the ſame diameter, the parallelopiped under three ordinates, is equal to the parallelopiped under two parts of the diameter cut off at the ordinates and the two vertices, and a given right line, which may therefore be called the *latus tranſverſum.*

Further, as in the conic ſections, where two parallels, terminated on each ſide by a curve, are cut by two other parallels terminated on each ſide by a curve, the 1ſt by the 3d, and the 2d by the 4th ; the rectangle of the parts of the 1ſt is to the rectangle of the parts of the 3d, as that of the 2d is to that of the 4th : ſo, when four ſuch right lines occur in a curve of the 2d kind, each in three points ; the parallelopiped of the parts of the 1ſt, will be to that of the parts of the 3d, as that of the 2d to that of the 4th.

Laſtly, the legs of curves, both of the 1ſt, 2d, and higher kinds, are either of the parabolic or hyperbolic kind : an hyperbolic leg being that which approaches infinitely towards ſome aſymptote ; and a parabolic one, that which has no aſymptote.

Theſe legs are beſt diſtinguiſhed by their tangents ; for, if the point of contact go off to an infinite diſtance, the tangent of the hyperbolic leg will coincide with the aſymptote ; and that of the parabolic leg, recede infinitely, and vaniſh. Therefore the aſymptote of any leg is found, by ſeeking the tangent of that leg to a point infinitely diſtant ; and the direction of an infinite leg is found, by ſeeking the poſition of a right line parallel to the tangent, where the point of contact is infinitely remote, for this line tends that way towards which the infinite leg is directed.

Reduction of CURVES *of the 2d kind.*

Newton reduces all curves of the 2d kind to four caſes of equations, expreſſing the relation between the ordinate and abſciſs, viz,
in the 1ſt caſe, $xy^2 + ey = ax^3 + bx^2 + cx + d$;
in the 2d, - $xy = ax^3 + bx^2 + cx + d$;
in the 3d, . $y^2 = ax^3 + bx^2 + cx + d$;
in the 4th, - $y = ax^3 + bx^2 + cx + d$.
See Newton's Enumeratio, ſect. 3 ; and Stirling's Lineæ, &c, pa. 83.

Enumeration of the CURVES *of the 2d kind.*

Under theſe four caſes, the author brings a great number of different forms of curves, to which he gives different names. An hyperbola lying wholly within the angle of the aſymptotes, like a conic hyperbola, he calls an *inſcribed hyperbola ;* that which cuts the aſymptotes, and contains the parts cut off within its own periphery, a *circumſcribed hyperbola ;* that which has one of its infinite legs inſcribed and the other circumſcribed, he calls *ambigenal ;* that whoſe legs look towards each other, and are directed the ſame way, *converging ;* that where they look contrary ways, *diverging ;* that where they are convex different ways, *croſs-legged ;* that applied to its aſymptote with a concave vertex, and diverging legs, *conchoidal ;* that which cuts its aſymptote with contrary flexures, and is produced each way into contrary legs, *anguineous,* or *ſnake-like ;* that which cuts its conjugate acroſs, *cruciform ;* that which returning around cuts itſelf, *nodated ;* that whoſe parts concur in the angle of contact, and there terminate, *cuſpidated ;* that whoſe conjugate is oval, and infinitely ſmall, i. e. a point, *pointed ;* that which, from the impoſſibility of two roots, is without either oval, node, cuſp, or point, *pure.* And in the ſame manner he denominates a parabola *converging, diverging, cruciform,* &c. Alſo when the number of hyperbolic legs exceeds that of the conic hyperbola, that is more than two, he calls the hyperbola *redundant.*

Under thoſe 4 caſes the author enumerates 72 different curves: of theſe, 9 are redundant hyperbolas, without diameters, having three aſymptotes including a triangle ; the firſt conſiſting of three hyperbolas, one inſcribed, another circumſcribed, and the third ambigenal, with an oval ; the 2d, nodated ; the 3d, cuſpidated ; the 4th, pointed ; the 5th and 6th, pure ; the 7th and 8th, cruciform ; the 9th or laſt, anguineal. There are 12 redundant hyperbolas, having only one diameter : the 1ſt, oval ; the 2d, nodated ; the 3d, cuſpidated ; the 4th, pointed ; 5th, 6th, 7th, and 8th, pure ; the 9th and 10th, cruciform ; the 11th and 12th, conchoidal. And to this claſs Stirling adds 2 more. There are 2 redundant hyperbolas, with three diameters. There are 9 redundant hyperbolas, with three aſymptotes converging to a common point ; the 1ſt being formed of the 5th and 6th redundant parabolas, whoſe aſymptotes include a triangle; the 2d formed of the 7th and 8th; the 3d and 4th, of the 9th; the 5th is formed of the 5th and 7th of the redundant hyperbolas, with one diameter; the 6th, of the 6th and 7th; the 7th, of the 8th and 9th; the 8th, of the 10th and 11th; the 9th, of the 12th and 13th: all which converſions are effected, by diminiſhing the triangle comprehended between the aſymptotes, till it vaniſh into a point.

Six are defective parabolas, having no diameters: the 1ſt, oval; the 2d, nodated; the 3d, cuſpidated; the 4th, pointed; the 5th, pure; &c.

Seven are defective hyperbolas, having diameters; the 1ſt and 2d, conchoidal, with an oval; the 3d, nodated; the 4th, cuſpidated, which is the ciſſoid of the ancients; the 5th and 6th, pointed; the 7th, pure.

Seven are parabolic hyperbolas, having diameters: the 1ſt, oval; the 2d, nodated; the 3d, cuſpidated; the 4th, pointed; the 5th, pure; the 6th, cruciform; the 7th, anguineous.

Four are parabolic hyperbolas: four are hyperboliſms of the hyperbola: three, hyperbolas of the ellipſis: two, hyperboliſms of the parabola.

Six are diverging parabolas; one, a trident; the 2d, oval; the 3d, nodated; the 4th, pointed; the 5th, cuſpidated (which is Neil's parabola, uſually called the ſemi-cubical parabola); the 6th, pure.

Laſtly, one, commonly called the cubical parabola.

Mr. Stirling and Mr. Stone have ſhewn that this enumeration is imperfect, the former having added four new ſpecies of curves to the number, and the latter two, or rather theſe two were firſt noticed by Mr. Nic. Bernoulli. Alſo Mr. Murdoch and Mr. Geo. Sanderſon have found ſome new ſpecies; though ſome perſons diſpute the reality of them. See the Geneſis Curvarum per umbras, and the Ladies' Diary 1788 and 1789, the prize queſtion.

Organical Deſcription of CURVES.—Sir Iſaac Newton ſhews that curves may be generated by ſhadows. He ſays, if upon an infinite plane, illuminated from a lucid point, the ſhadows of figures be projected; the ſhadows of the conic ſections will always be conic ſections; thoſe of the curves of the 2d kind, will always be curves of the 2d kind; thoſe of the curves of the 3d kind, will always be curves of the 3d kind; and ſo on ad infinitum

And, like as the projected ſhadow of a circle generates all the conic ſections, ſo the 5 diverging parabolas, by their ſhadows, will generate and exhibit all the reſt of the curves of the 2d kind: and thus ſome of the moſt ſimple curves of the other kinds may be found, which will form, by their ſhadows upon a plane, projected from a lucid point, all the other curves of that ſame kind. And in the French Memoirs may be ſeen a demonſtration of this projection, with a ſpecimen of a few of the curves of the 2d order, which may be generated by a plane cutting a ſolid formed from the motion of an infinite right line along a diverging parabola, having an oval, always paſſing through a given or fixed point above the plane of that parabola. The above method of Newton has alſo been purſued and illuſtrated with great elegance by Mr. Murdoch, in his treatiſe entitled *Newtoni Geneſis Curvarum per umbras, ſeu Perſpectivæ Univerſalis Elementa.*

Mr. Maclaurin, in his *Geometria Organica*, ſhews how to deſcribe ſeveral of the ſpecies of curves of the 2d order, eſpecially thoſe having a double point, by the motion of right lines and angles: but a good commodious deſcription by a continued motion of thoſe curves which have no double point, is ranked by Newton among the moſt difficult problems. Newton gives alſo other methods of deſcription, by lines or angles revolving above given poles; and Mr. Brackenridge has given a general method of deſcribing curves, by the interſection of right lines moving about points in a given plane. See Philoſ. Tranſ. No. 437, or Abr vol. 8, pa. 58; and ſome particular caſes are demonſtrated in his *Exerc. Geometrica de Curvarum Deſcriptione.*

CURVES *above the 2d Order.* The number of ſpecies in the higher orders of curves increaſe amazingly, thoſe of the 3d order only it is thought amounting to ſome thouſands, all comprehended under the following ten particular equations,

viz, 1. $y^4 + fx^2y^2 + gxy^3 + hx^2y + iy^2 + hxy + ly,$
or 2. $y^4 + fxy^3 + gx^2y + hxy^2 + ixy + ky,$
or 3. $x^2y^2 + fy^3 + gx^2y + hy^3 + ky,$
or 4. $x^2y^2 + fy^3 + gy^2 + hxy + iy,$
or 5. $y^3 + fxy^2 + gx^2y + hy,$
or 6. $y^3 + fxy^2 + gxy + hy$
$= ax^4 + bx^3 + cx^2 + dx + e;$
or 7. $y^4 + ex^3y + fxy^3 + gxy^2 + hy^2 + ixy + ky,$
or 8. $x^3y + exy^3 + fx^2y + gy^2 + hxy + iy,$
or 9. $x^3y + ey^3 + fxy^2 + gxy + hy,$
or 10. $x^3y + cy^3 + fy^2 + gxy + hy,$
$= ax^3 + bx^2 + cx + d.$

Thoſe who wiſh to ſee how far this doctrine has been advanced, with regard to curves of the higher orders, as well as thoſe of the 1ſt and 2d orders, may conſult Mr. Maclaurin's Geometria Organica, and Brackenridge's Exerc. Geom.

All geometrical lines of the odd orders, viz, the 3d, 5th, 7th, &c, have at leaſt one leg running on infinitely; becauſe all equations of the odd dimenſions have at leaſt one real root. But vaſt numbers of the lines of the even orders are only ovals; among which there are ſeveral having very pretty figures, ſome being like ſingle hearts, ſome double ones, ſome reſembling fiddles, and others again ſingle knots, double knots, &c.

Two geometrical lines of any order will cut one another in as many points, as are denoted by the product of the two numbers expreſſing thoſe orders.

The theory of curves forms a conſiderable branch of the mathematical ſciences. Thoſe who are curious of advancing beyond the knowledge of the circle and the conic ſections, and to conſider geometrical curves of a higher nature, and in a general view, will do well to ſtudy Cramer's Introduction à l'Analyſe des Lignes Courbes Algebraiques, which the learned and ingenious author compoſed for the uſe of beginners. There is an excellent poſthumous piece too of Maclaurin's, printed as an Appendix to his Algebra, and entitled De Linearum Geometricarum Proprietatibus Generalibus. The ſame author, at a very early age, gave a remarkable ſpecimen of his genius and knowledge in his Geometria Organica; and he carried theſe ſpeculations farther afterwards, as may be ſeen in the theorems he has given in the Philoſ. Tranſ. See Abr. vol. 8, pa. 62. Other writings on this ſubject, beſide the Treatiſes on the Conic Sections, are Archimedes de Spiralibus; Des Cartes Geometria; Dr. Barrow's Lectiones Geometricæ; Newton's Enumeratio Linearum Tertii Ordinis; Stirling's Illuſtratio Tractatûs Newtoni de Lineis Tertii Ordinis; Maclaurin's Geometria Organica; Brackenridge's Deſcriptio Linearum Curvarum; M. De Gua's Uſages de l'Analyſe de Des Cartes; beſide many other Tracts on Curves in the Memoirs of ſeveral Academies &c.

Uſe of CURVES *in the Conſtruction of Equations.* One great uſe of curves in Geometry is, by means of their interſections, to give the ſolution of problems. See CONSTRUCTION.

Suppoſe, *ex. gr.* it were required to conſtruct the following equation of 9 dimenſions,

$x^9+bx^7+cx^6+dx^5+ex^4+\overline{m+f}.x^3+gx^2+hx+k=0$: aſſume the equation to a cubic parabola $x^3=y$; then, by writing y for x^3, the given equation will become $y^3+bxy^2+cy^2+dx^2y+exy+my+fx^3+gx^2+hx+k=0$; an equation to another curve of the 2d kind, where m or f may be aſſumed $=0$ or any thing elſe: and by the deſcriptions and interſections of theſe curves will be given the roots of the equation to be conſtructed. It is ſufficient to deſcribe the cubic parabola once. When the equation to be conſtructed, by omitting the two laſt terms hx and k, is reduced to 7 dimenſions; the other curve, by expunging m, will have the double point in the beginning of the abſciſs, and may be eaſily deſcribed as above: If it be reduced to 6 dimenſions, by omitting the laſt three terms, gx^2+hx+k; the other curve, by expunging f, will become a conic ſection. And if, by omitting the laſt three terms, the equation be reduced to three dimenſions, we ſhall fall upon Wallis's conſtruction by the cubic parabola and right line.

Rectification, Inflection, Quadrature, &c of CURVES. See the reſpective terms.

CURVE *of a Double Curvature*, is ſuch a curve as has not all its parts in the ſame plane.

M. Clairaut has publiſhed an ingenious treatiſe on curves of a double curvature. See his Recherches ſur les Courbes à Double Courbure. Mr. Euler has alſo treated this ſubject in the Appendix to his Analyſis Infinitorum, vol. 2, pa. 323.

Family of CURVES, is an aſſemblage of ſeveral curves of different kinds, all defined by the ſame equation of an indeterminate degree; but differently, according to the diverſity of their kind. For example, Suppoſe an equation of an indeterminate degree, $a^{m-1}x=y^m$: if $m=2$, then will $ax=y^2$; if $m=3$, then will $a^2x=y^3$; if $m=4$, then is $a^3x=y^4$; &c: all which curves are ſaid to be of the ſame family or tribe.

The equations by which the families of curves are defined, are not to be confounded with the tranſcendental ones: for though with regard to the whole family, they be of an indeterminate degree; yet with reſpect to each ſeveral curve of the family, they are determinate; whereas tranſcendental equations are of an indefinite degree with reſpect to the ſame curve.

All Algebraical curves therefore compoſe a certain family, conſiſting of innumerable others, each of which comprehends infinite kinds. For the equations by which curves are defined involve only products, either of powers of the abſciſſes and ordinates by conſtant coefficients; or of powers of the abſciſſes by powers of the ordinates; or of conſtant, pure, and ſimple quantities by one another. Moreover, every equation to a curve may have 0 for one member or ſide of it; for example, $ax=y^2$, by tranſpoſition becomes $ax-y^2=0$. Therefore the equation for all algebraic curves will be

$$\left.\begin{array}{llll} ay^m & +\ bxy^{m-1} & +\ nx^2y^{m-2}\ \&c \text{ - - - } & fy^m \\ & +\ fy^{m-1} & +\ kxy^{m-2} & \\ & & +\ qy^{m-2} & \end{array}\right\}=0.$$

Catacauſtic, and Diacauſtic CURVES. See CATACAUSTIC, and DIACAUSTIC.

Exponential CURVE, is that which is defined by an exponential equation, as $ax^z=y$, &c.

CURVES *by the Light*, or COURBES *a la Lumiere*, a name given to certain curves by M. Kurdwanowſki, a Poliſh gentleman. He obſerved that any line, ſtraight or curved, expoſed to the action of a luminous point, received the light differently in its different parts, according to their diſtance from the light. Theſe different effects of the light upon each point of the line, may be repreſented by the ordinates of ſome curve, which will vary preciſely with theſe effects. Prieſtley's Hiſt. of Viſion, pa. 752.

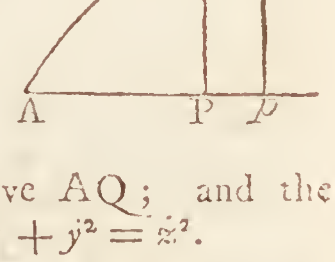
Logarithmic CURVE. See LOGARITHMIC *Curve.*

CURVE *Reflectoire*, ſo called becauſe it is the appearance of the plane bottom of a baſon covered with water, to an eye perpendicularly over it. In this poſition, the bottom of the baſon will appear to riſe upwards, from the centre outwards; but the curvature will be leſs and leſs, and at laſt the ſurface of the water will be an aſymptote to it. M. Mairan, who firſt conceived this idea from the phenomena of light, found alſo ſeveral kinds of theſe curves; and he gives a geometrical deduction of their properties, ſhewing their analogy to cauſtics by refraction. Mem. Ac. 1740; Prieſtley's Hiſt. of Viſion, pa. 752.

Radical CURVES, a name given by ſome authors to curves of the ſpiral kind, whoſe ordinates, if they may be ſo called, do all terminate in the centre of the including circle; and appear like ſo many radii of that circle: whence the name.

Regular CURVES, are ſuch as have their curvature turning regularly and continually the ſame way; in oppoſition to ſuch as bend contrary ways, by having points of contrary flexure, which are called irregular curves.

Characteriſtic Triangle of a CURVE, is the differential or elementary right-angled triangle whoſe three ſides are, the fluxions of the abſciſs, ordinate, and curve; the fluxion of the curve being the hypothenuſe. So, if pq be parallel to, and indefinitely near to the ordinate PQ, and Qr parallel to the abſciſs AP; then Qr is the fluxion of the abſciſs AP, and qr the fluxion of the ordinate PQ, and Qq the fluxion of the curve AQ; hence the elementary triangle Qqr is the characteriſtic triangle of the curve AQ; and the three ſides are $\dot{x}$, $\dot{y}$, $\dot{z}$; in which $\dot{x}^2+\dot{y}^2=\dot{z}^2$.



CURVILINEAR *Angle, Figure, Superficies, &c,*

are ſuch as are formed or bounded by curves; in oppoſition to rectilinear ones, which are formed by ſtraight lines or planes.

CUSP, in Aſtronomy, is uſed to expreſs the points or horns of the moon, or other luminary.

Cusp, in Aſtrology, is uſed for the 1ſt point of each of the twelve houſes, in a figure or ſcheme of the heavens. See House.

Cusp, in the Higher Geometry, is uſed for the point or corner formed by two parts of a curve meeting and terminating there. See Curve.

Cuspidated *Hyperbola*, &c. See Curve.

CUT-Bastion. See Bastion.

CUVETTE, or Cunette, in Fortification, is a kind of ditch within a ditch, being a pretty deep trench, about four fathoms broad, ſunk and running along the middle of the great dry ditch, to hold water; ſerving both to keep off the enemy, and prevent him from mining.

CYCLE, a certain period or ſeries of numbers proceeding orderly from firſt to laſt, then returning again to the firſt, and ſo circulating perpetually.

Cycles have chiefly ariſen from the incommenſurability of the revolutions of the earth and celeſtial bodies to one another. The apparent revolution of the ſun about the earth, has been arbitrarily divided into 24 hours, which is the baſis or foundation of all our menſuration of time, whether days, years, &c. But neither the annual motion of the ſun, nor that of the other heavenly bodies, can be meaſured exactly, and without any remainder, by hours, or their multiples. That of the ſun, for example, is 365 days 5 hours 49 minutes nearly; that of the moon, 29 days 12 hours 44 minutes nearly.

Hence, to ſwallow up theſe fractions in whole numbers, and yet in numbers which only expreſs days and years, cycles have been invented; which, comprehending ſeveral revolutions of the ſame body, replace it, after a certain number of years, in the ſame points of the heaven whence it firſt departed; or, which is the ſame thing, in the ſame place of the civil calendar.

There are various cycles; as, the cycle of Indiction, the cycle of the moon, the cycle of the ſun, &c.

Cycle *of Indiction*, is a ſeries of 15 years, returning conſtantly around like the other cycles; and commenced from the third year before Chriſt; whence it happens that if 3 be added to any given year of Chriſt, and the ſum be divided by 15, what remains is the year of the indiction. See Indiction.

Cycle *of the Moon*, or the *Lunar Cycle*, is a period of 19 years; in which time the new and full moons return to the ſame day of the Julian year. See Calippic.

This cycle is alſo called the *Metonic period* or *cycle*, from its inventor Meton, the Athenian; and alſo the *Golden Number*, from its excellent uſe in the calendar: though, properly ſpeaking, the golden number is rather the particular number which ſhews the year of the lunar cycle, which any given year is in. This cycle of the moon only holds true for $310\frac{7}{10}$ years: for, though the new moons do return to the ſame day after 19 years; yet not to the ſame time of the day, but near an hour and a half ſooner; an error which in $310\frac{7}{10}$ years amounts to an entire day. Yet thoſe employed in reforming the calendar went on a ſuppoſition that the lunations return preciſely from 19 years to 19 years, for ever.

The uſe of this cycle, in the ancient calendar, is to ſhew the new moon of each year, and the time of Eaſter. In the new one, it only ſerves to find the Epacts; which ſhew, in either calendar, that the new moon falls 11 days too late.

As the Orientals began the uſe of this cycle at the time of the Council of Nice in 325, they aſſumed, that the firſt year of the cycle the paſchal new moon fell on the 13th of March: on which account the lunar cycle 3 fell on the 1ſt of January in the third year.

The Occidentals, on the contrary, placed the number 1 to the 1ſt of January, which occaſioned a conſiderable difference in the time of Eaſter. Hence, Dionyſius Exiguus, on framing a new calendar, perſuaded the Chriſtians of the weſt to ſalve the difference, and come into the practice of the church of Alexandria.

To find the Year of the Lunar Cycle, is to find the golden number. See Golden-*Number*.

Cycle *of the Sun*, or *Solar Cycle*, is a period or revolution of 28 years; beginning with 1, and ending with 28; which elapſed, the Dominical or Sunday-letters, and thoſe that expreſs the other feaſts, &c, return into their former place, and proceed in the ſame order as before. The days of the month return again to the ſame days of the week; the ſun's place to the ſame ſigns and degrees of the ecliptic on the ſame months and days, ſo as not to differ one degree in a hundred years; and the leap years begin the ſame courſe with reſpect to the days of the week on which the days of the month fall.

This is called the cycle of the *ſun*, or the *ſolar* cycle, not from any regard to the ſun's courſe, which has no concern in it; but from *Sunday*, anciently called *dies ſolis*, the *ſun's day*; as the dominical or ſunday letter is chiefly ſought for from this revolution.

The reformation of the calendar under pope Gregory the 13th, occaſioned a conſiderable alteration of this cycle: In the Gregorian calendar, the ſolar cycle is not conſtant and perpetual; becauſe every 4th ſecular year is common; whereas, in the Julian, it is biſſextile. The epoch, or beginning of the ſolar cycle, both Julian and Gregorian, is the 9th year before Chriſt. And therefore,

To find the Cycle of the Sun for any given year: add 9 to the number given, and divide the ſum by 28; the remainder will be the number of the cycle, and the quotient the number of revolutions ſince Chriſt. If there be no remainder, it will be the 28th or laſt year of the cycle.

CYCLE of the Sun, with the correſpondent Sunday letters, in Julian Years.

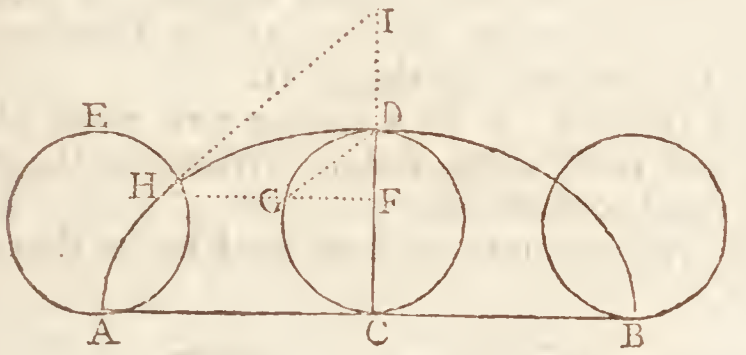
1	GF	5	BA	9	DC	13	FE	17	AG	21	CB	25	ED
2	E	6	G	10	B	14	D	18	F	22	A	26	C
3	D	7	F	11	A	15	C	19	E	23	G	27	B
4	C	8	E	12	G	16	B	20	D	24	F	28	A

CYCLE of the Sun, and Sunday Letters, from the Gregorian year 1700 to the year 1800.

1	DC	5	FE	9	AG	13	CB	17	ED	21	GF	25	BA
2	B	6	D	10	F	14	A	18	C	22	E	26	G
3	A	7	C	11	E	15	G	19	B	23	D	27	F
4	G	8	B	12	D	16	F	20	A	24	C	28	E

Great Paſcal CYCLE, is another name for the Victorian or Dionyſian Period. Which ſee.

CYCLOID, or TROCHOID, a mechanical or tranſcendental curve, which is thus generated: Suppoſe a wheel, or a circle, AE, to roll along a ſtraight line AB, beginning at the point A, and ending at B, where it has completed juſt one revolution, thereby meaſuring out a right line AB exactly equal to the circumference of the generating circle AE, whilſt a nail or point A in the circumference of the wheel, or circle, traces out or deſcribes a curvilineal path ADB; then this curve ADB is the cycloid, or trochoid.



Schooten, in his Commentary on Des Cartes, ſays that Des Cartes firſt conceived the notion of this elegant curve, and after him it was firſt publiſhed by Father Merſenne, in the year 1615. But Torricelli, in the Appendix de Dimenſione Cycloidis, at the end of his treatiſe De Dimenſione Parabolæ, publiſhed 1644, ſays that this curve was conſidered and named a cycloid, by his predeceſſors, and particularly by Galileo about 45 years before, i. e. about 1599. And Dr. Wallis ſhews that it is of a much older ſtanding, having been known to Bovilli about the year 1500, and even conſidered by cardinal Cuſanus much earlier, viz, before the year 1451. Philoſ. Tranſ. Abr. vol. 1, pa. 116. It would ſeem however that Torricelli's was the firſt regular treatiſe on the Cycloid; though ſeveral particular properties of it might be known prior to his work. He firſt ſhewed, that the cycloidal ſpace is equal to triple the generating circle, (though Paſcal contends that Roberval ſhewed this): alſo that the ſolid generated by the rotation of that ſpace about its baſe, is to the circumſcribing cylinder, as 5 to 8: about the tangent parallel to the baſe, as 7 to 8: about the tangent parallel to the axis, as 3 to 4: &c.

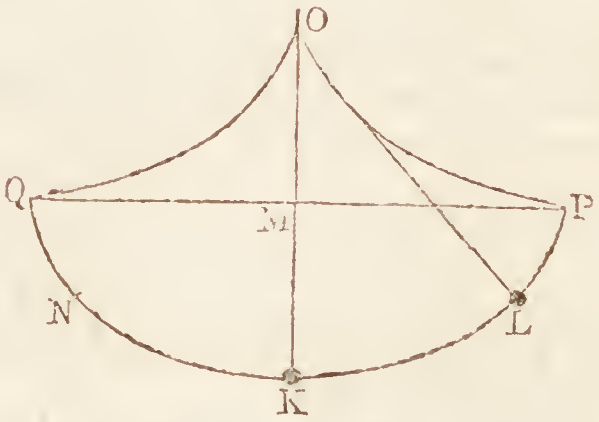
Honoratus Fabri, in his Synopſis Geom. has a ſhort treatiſe on the cycloid, containing demonſtrations of the above, and many other theorems concerning the centres of gravity of the cycloidal ſpace, &c; which he ſays he found out before the year 1658.

From the preface to Dr. Wallis's treatiſe on the cycloid we learn, that, in the year 1658, M. Paſcal publicly propoſed at Paris, under the name of D'Ettonville, the two following problems as a challenge, to be ſolved by the mathematicians of Europe, with a reward of 20 piſtoles for the ſolution: viz, to find the area of any ſegment of the cycloid, cut off by a right line parallel to the baſe; alſo the content of the ſolid generated by the rotation of the ſame about the axis, and about the baſe of that ſegment. This challenge ſet the Doctor upon writing that treatiſe upon the cycloid, which is a much better and compleater piece than had been given before upon this curve. He here gives the curve ſurfaces of the ſolids generated by the rotation of the cycloidal ſpace about its axis, and about its baſe, with determinations of the centres of gravity, &c. He here aſſerts too, that Sir Chriſtopher Wren, in 1658, was the firſt who found out a right line equal to the curve of the cycloid; and Mr. Huygens, in his Herolog. Oſcillat. ſays that he himſelf was the firſt inventor of the ſegment of a cycloidal ſpace, cut off by a right line parallel to the baſe at the diſtance of $\frac{1}{4}$ the axis of the curve from the centre, being equal to a rectilinear ſpace, viz, to a regular hexagon inſcribed in the generating circle; the demonſtration of which may be ſeen in Wallis's treatiſe.

Several other authors have ſpoken or treated of the cycloid: as Paſcal, in his treatiſe, under the name of D'Ettonville: Schooten in his Commentary on Des Cartes's Geometry, near the end of the 2d book; M. Reinau, in his Analyſe Démontrée, tom. 2, pa. 595: alſo Newton, Leibnitz, de la Loubere, Roberval, Des Cartes, Wren, Fabri, the Bernoulli's, De la Hire, Cotes, &c, &c.

Properties of the CYCLOID.—The circle AE, by whoſe revolution the cycloid is traced out, is called the *generating circle*; the line AB, which is equal to the circumference of the circle, is the *baſe* of the cycloid; and the perpendicular DC on the middle of the baſe, is its *axis*. The properties of the cycloid are among the moſt beautiful and uſeful of all curve lines: ſome of the moſt remarkable of which are as follow:

1. The circular arc DG = the line GH parallel to AB.
2. The ſemicircumf. DGC = the ſemibaſe AC.
3. The arc DH = double the chord DG.
4. The arc DA = double the diam. DC.
5. The tang. HI is parallel to the chord DG.
6. The ſpace ADBA = triple the circle AE or CGD &c.
7. The ſpace ADGCA = the ſame circle AE, &c.



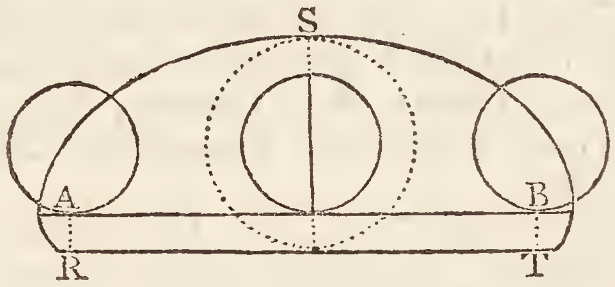
8. A body falls through any arc KL of a cycloid reverſed, in the ſame time, whether that arc be great or ſmall; that is, from any point L, to the loweſt point K, which is the vertex reverſed: and that time

is to the time of falling perpendicularly through the axis MK, as the ſemicircumference of a circle is to its diameter, or as 3·1416 to 2. And hence it follows that, if a pendulum be made to vibrate in the arc LKN of a cycloid, all the vibrations will be performed in the ſame time.

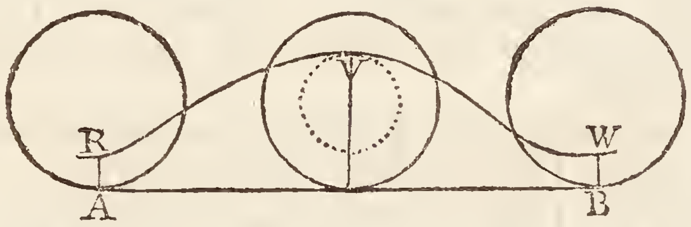
9. The evolute of a cycloid, is another equal cycloid. So that if two equal ſemicycloids OP, OQ, be joined at O, ſo that OM be = MK the diameter of the generating circle, and the ſtring of a pendulum hung up at O, having its length = OK or = the curve OP; then, by plying the ſtring round the curve OP, to which it is equal, and then the ball let go, it will deſcribe, and vibrate in the other cycloid PKQ.

10. The cycloid is the curve of ſwifteſt deſcent: or a heavy body will fall from one given point to another, by the way of the arc of a cycloid paſſing through thoſe two points, in a leſs time, than by any other rout. See the Works of James and John Bernoulli for many other curious properties concerning the deſcents in cycloids, &c.

Cycloids are alſo either *curtate* or *prolate*.



Cycloid, *Curtate*, or *contracted*, is the path deſcribed by ſome point without the circle, while the circumference rolls along a ſtraight line; and a Cycloid, *Prolate*, or *Inflected*, is in like manner the path of ſome point taken within the generating circle.

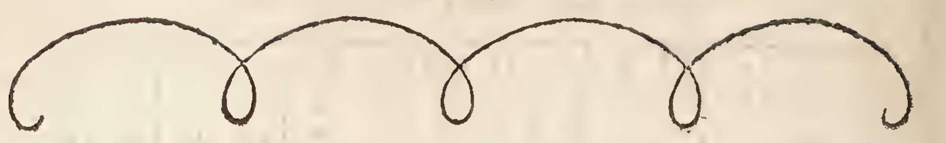


Thus, if, while the circle rolls along the line AB, the point R be taken without the circle, it will deſcribe or trace out the curtate or contracted cycloid RST; but the point being taken within the circle, it will deſcribe the prolate or inflected cycloid RVW.

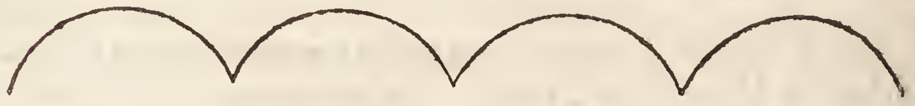
Theſe two curves were both noticed by Torricelli and Schooten, and more fully treated of by Wallis, in his Treatiſe on the Cycloid, printed at Oxford in 1659; where he ſhews that theſe have properties ſimilar to the firſt or primary cycloid; only the laſt of theſe is a curve having a point of inflection, and the other croſſing itſelf, and forming a node.

By continuing the motion of the wheel, or circle, ſo as to deſcribe a right line equal to the generating circumference ſeveral times repeated, there will be produced as many repetitions of the cycloids, which ſo united together will appear as in theſe figures following:

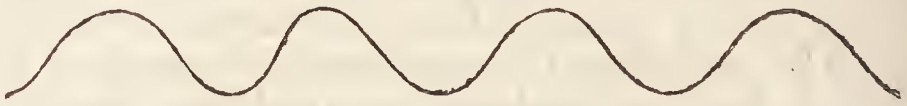
Curtate Cycloid.



Common Cycloid.



Inflected Cycloid.

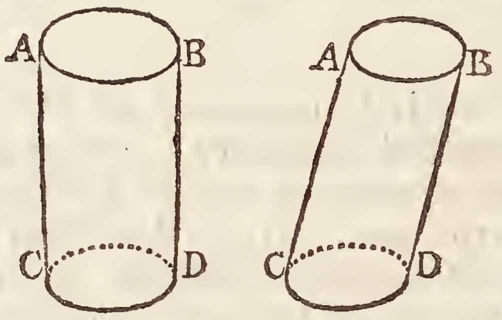


CYGNUS, the *Swan*, a conſtellation of the northern hemiſphere, being one of the 48 old ones, and fabled by the Greeks to be the ſwan, under the form of which Jupiter deceived Leda or Nemeſis, from which embrace ſprung the beauteous Helen.

The ſtars in the conſtellation Cygnus, in Ptolomy's catalogue are 19, in Tycho's 18, in Hevelius's 47, and in the Britannic catalogue 81.

CYLINDER, a ſolid having two equal circular ends, and every plane ſection parallel to the ends a circle equal to them alſo.

The cylinder may be conceived to be thus generated:



Suppoſe two parallel circles AB and CD, and a right line carried continually round them, always parallel to itſelf; this line will deſcribe the curve ſurface of a cylinder, ABDC, of which the two parallel circles AB and CD form the two ends. When the line, or ſides is perpendicular to the ends, the cylinder is a right or perpendicular one; otherwiſe it is oblique.

Or the right cylinder may be conceived to be generated by the rotation of a rectangle about one of its ſides. The *axis* of the cylinder is the line connecting the centres of its two parallel circular ends; and is equal to the altitude of the cylinder when this is a right one, but exceeds the altitude in the oblique cylinder, in the proportion of radius to the ſine of the angle of its inclination to the baſe.

The convex ſurface of a cylinder is equal to the product of the axis multiplied by the circumference of its baſe.

The ſolidity of a cylinder is equal to the area of its baſe multiplied by its perpendicular altitude.

Cylinders of equal baſes and altitudes, are equal.

Cylinders are to each other, as the product of their baſes and altitudes. And equal cylinders have their baſes reciprocally as their altitudes.

A cylinder is to its inſcribed ſphere, or ſpheroid, as 3 to 2; and to its inſcribed cone as 3 to 1.

The oblique plane ſections of a cylinder, are ellipſes; but all the ſections parallel to the ends, are circles.

For the ſurfaces and ſolidities of the ungulas, or oblique ſlices, of a cylinder, ſee my Menſuration, pa. 218, 2d edition.

CYLINDRICAL, pertaining to a cylinder.

CYLINDROID, a ſolid reſembling the figure of a cylinder; but differing from it as having ellipſes for its ends or baſes, inſtead of circles, in the cylinder.

In the cylindroid, the ſolidity and curve ſuperficies are found the ſame way as thoſe of the cylinder; viz, by multiplying the circumference of the baſe by the length or axis, for the ſurface; and the area of the baſe by the altitude, for the ſolidity.

CYMATIUM, Cimatium, or Cima, in Architecture, a member, or moulding of the cornice, whoſe profile is waved; i. e. concave at top, and convex at bottom.

CYNOSURA, a name given by the Greeks, to urſa minor, or little bear, otherwiſe called Charles's wain; the ſtar in the extremity of the tail being called the pole ſtar.

CYPHER, or nought. See Cipher.

D.

DACTYLONOMY, the art of counting or numbering by the fingers.—The rule is this: the left thumb is reckoned 1, the index or fore-finger 2, and ſo on to the right thumb, which is the tenth or laſt, and conſequently is denoted by the cipher 0.

DADO, that part in the middle of the pedeſtal of a column &c, between its baſe and cornice.

DAILY, in Aſtronomy. See Diurnal.

DARCY (Count), an ingenious philoſopher and mathematician, was born in Ireland in 1725; but his friends being attached to the Stuart family, he was ſent to France, at 14 years of age, where he ſpent the reſt of his life. Being put under the care of the celebrated Clairaut, he improved ſo rapidly in the mathematics, that at 17 years of age he gave a new ſolution of the problem concerning the curve of equal preſſure in a reſiſting medium. This was followed the year after by a determination of the curve deſcribed by a heavy body, ſliding by its own weight along a moveable plane, at the ſame time that the preſſure of the body cauſes a horizontal motion in the plane. Darcy ſerved in the war of 1744, and was taken priſoner by the Engliſh: and yet, during the courſe of the war he gave two memoirs to the academy; the firſt of theſe contained a general principle in mechanics, that of *the preſervation of the rotatory motion;* a principle which he again brought forward in 1750, by the name of *the principle of the preſervation of action.*

In 1760, Darcy publiſhed *An Eſſay on Artillery*, containing ſome curious experiments on the charges of gunpowder, &c, &c, and improvements on thoſe of the ingenious Robins; a kind of experiments which our author carried on occaſionally to the end of his life.

In 1765, he publiſhed his *Memoir on the Duration of the Senſation of Sight*, the moſt ingenious of his works: the reſult of theſe reſearches was, that a body may ſometimes paſs by our eyes without being ſeen, or marking its preſence, otherwiſe than by weakening the brightneſs of the object it covers.

All Darcy's works bear the character which reſults from the union of genius and philoſophy; but as he meaſured every thing upon the largeſt ſcale, and required extreme accuracy in experiment, neither his time, fortune, nor avocations, allowed him to execute more than a very ſmall part of what he projected. In his diſpoſition, he was amiable, ſpirited, lively, and a lover of independence, a paſſion to which he nobly ſacrificed, even in the midſt of literary ſociety.—He died of a cholera morbus in 1779, at 54 years of age.

Darcy was admitted of the French academy in 1749, and was made penſioner-geometrician in 1770.—His eſſays, printed in the Memoirs of the Academy of Sciences, are various and very ingenious, and are contained in the volumes for the years 1742, 1747, 1749, 1750, 1751, 1752, 1753, 1754, 1758, 1759, 1760, 1765, and in tom. 1, of the Savans Etrangers.

DARK *Chamber*. See Camera *Obſcura.*

DARK *Tent*, a portable camera obſcura, made ſomewhat like a deſk, and fitted with optic glaſſes, to take proſpects of landſcapes, buildings, &c.

DATA, in General Mathematics, denote certain things or quantities, ſuppoſed given or known, from which other quantities are diſcovered that were unknown, or ſought. A problem or queſtion uſually conſiſts of two parts, *data* and *quæſita.*

Euclid has an expreſs and excellent treatiſe of *Data;* in which he uſes the word for ſuch ſpaces, lines, angles, &c, as are given; or to which others can be found equal.

Euclid's *Data* is the firſt in order of the books that have been written by the ancient geometricians, to facilitate and promote the method of reſolution or analyſis. In general a thing is ſaid to be given which is either actually exhibited, or can be found out, that is, which is either known by hypotheſis, or that can be demonſtrated to be known: and the propoſitions in the book of Euclid's Data ſhew what things can be found out or known, from thoſe that by hypotheſis are already known: ſo that in the analyſis or inveſtigation of a problem, from the things that are laid down as given or known, by the help of theſe propoſitions, it is demonſtrated that other things are given, and from theſe laſt that others again are given, and ſo on, till it is demonſtrated that that which was propoſed to be found out in the problem is given; and when this is done, the problem is ſolved, and its compoſition is made and derived from the compoſitions of the Data which were employed in the analyſis. And thus the Data of Euclid are of the moſt general and neceſſary uſe in the ſolution of problems of every kind.

Marinus, at the end of his Preface to the Data, is miſtaken in aſſerting that Euclid has not uſed the ſynthetical, but the analytical method in delivering them: for, though in the analyſis of a theorem, the thing to be demonſtrated is aſſumed in the analyſis; yet, in the demonſtrations of the Data, the thing to be demonſtrated, which is, that ſomething is given, is never once aſſumed in the demonſtration; from which it is manifeſt that every one of them is demonſtrated ſynthetically: though indeed if a propoſition of the Data be turned into a problem, the demonſtration of the propoſition becomes the analyſis of the problem. See Simſon's edition of Euclid's Data, which is eſteemed the beſt.

DAVIS's *Quadrant*, the common ſea quadrant, or back-ſtaff. See Back-Staff. See alſo Robertſon's Navigation, book 9, ſect. 7.

DAY, a diviſion of time ariſing from the appearance and diſappearance of the ſun.

Day is either *natural* or *artificial*.

Artificial Day is that which is primarily meant by the word Day, and is the time of its being light, or the time while the ſun is above the horizon. Though ſometimes the twilight is included in the term day-light; in oppoſition to night or darkneſs, being the time from the end of twilight to the beginning of day-light.

Natural Day is the portion of time in which the ſun performs one revolution round the earth; or rather the time in which the earth makes a rotation on its axis. And this is either aſtronomical or civil.

Aſtronomical Day begins at noon, or when the ſun's centre is on the meridian, and is counted 24 hours to the following noon.

Civil Day is the time allotted for day in civil purpoſes, and begins differently in different nations, but ſtill including one whole rotation of the earth on its axis; beginning either at ſun-riſe, ſun-ſet, noon, or midnight.

1ſt, At ſun-riſing, among the ancient Babylonians, Perſians, Syrians, and moſt other eaſtern nations, with the preſent inhabitants of the Balearic iſlands, the Greeks, &c. 2dly, At ſun-ſetting, among the ancient Athenians and Jews, with the Auſtrians, Bohemians, Marcomanni, Sileſians, modern Italians, and Chineſe. 3dly, At noon, with aſtronomers, and the ancient Umbri and Arabians. And 4thly, at midnight, among the ancient Egyptians and Romans, with the modern Engliſh, French, Dutch, Germans, Spaniards, and Portugueſe.

The day is divided into hours; and a certain number of days makes a week, a month, or a year.

The different length of the natural day in different climates, has been matter of controverſy, viz, whether the natural days be all equally long throughout the year; and if not, what their difference is? A profeſſor of mathematics at Seville, in the Philoſ. Tranſ. vol. 10, pa. 425, aſſerts, from a continued ſeries of obſervations for three years, that they are all equal. But Mr. Flamſteed, in the ſame Tranſ. pa. 429, refutes the opinion; and ſhews that one day, when the ſun is in the equinoctial, is ſhorter than when he is in the tropics, by 40 ſeconds; and that 14 tropical days are longer than ſo many equinoctial ones, by 10 minutes. This inequality of the days flows from two ſeveral principles: the one, the eccentricity of the earth's orbit; the other, the obliquity of the ecliptic with regard to the equator, which is the meaſure of time. As theſe two cauſes happen to be differently combined, the length of the day is varied. See Equation *of time*.

Day's-*Work*, in Navigation, denotes the reckoning or account of the ſhip's courſe, during 24 hours, or between noon and noon.

DECAGON, a plane geometrical figure of ten ſides and ten angles. When all the ſides and angles are equal, it is a *regular* decagon, and may be inſcribed in a circle; otherwiſe, not.

If the radius of a circle, or the ſide of the inſcribed hexagon, be divided in extreme and mean proportion, the greater ſegment will be the ſide of a decagon inſcribed in the ſame circle. And therefore, as the ſide of the decagon is to the radius, ſo is the radius to the ſum of the two. Whence, if the radius of the circle be r, the ſide of the inſcribed decagon will be $\frac{\sqrt{5}-1}{2} \times r$.

If the ſide of a regular decagon be 1, its area will be $\frac{5}{2}\sqrt{5+2\sqrt{5}} = 7\cdot6942088$; therefore as 1 is to 7·6942088, ſo is the ſquare of the ſide of any regular decagon, to the area of the ſame: ſo that, if s be the ſide of ſuch a decagon, its area will be equal to $7\cdot6942088s^2$. See Regular *Figure*.

To inſcribe a decagon in a circle geometrically. See my Menſuration, prob. 35, pa. 25, 2d edit.

DECEMBER, the laſt month of the year; in which the ſun enters the tropic of Capricorn, making the winter ſolſtice.

In the time of Romulus, December was the 10th month; whence the name, viz, from decem, ten; for the Romans began their year in March, from which December is the 10th month.

The month of December was under the protection of Veſta. Romulus aſſigned it 30 days; Numa reduced it to 29; which Julius Cæſar increaſed to 31.

At the latter part of this month they had the *Juveniles Ludi*, and the country people kept the feaſt of the goddeſs Vacuna in the fields, having then gathered in their fruits, and ſown their corn; whence it ſeems is derived our popular feſtival called Harveſt-home.

DECHALES (Claud-Francis-Milliet), an excellent mathematician, mechaniſt, and aſtronomer, was born at Chambery, the capital of Savoy, in 1611. He chiefly excelled in a juſt knowledge of the mathematical and mechanical ſciences: not that he was bent upon new diſcoveries, or happy in making them; as his talent rather lay in explaining thoſe ſciences with eaſe and accuracy; which perhaps rendered him equally uſeful and deſerving of eſteem. Indeed it was generally allowed that he made the beſt uſe of the productions of other men, and that he drew the ſeveral parts of the mathematical ſciences together with great judgment and perſpicuity. It is alſo ſaid of him, that his probity was not inferior to his learning; and that both theſe qualities made him generally admired and beloved at Paris, where for four years together he read public mathematical lectures in the college of Clermont.—— From hence he removed to Marſeilles, where he taught the art of navigation and the practical mathematical ſciences.—He afterward became profeſſor of mathematics in the univerſity of Turin, where he died March 28, 1678, at 67 years of age.

Among other works which do honour to his memory, are,

1. An edition of Euclid's Elements; in which he has omitted the leſs important propoſitions, and explained the uſes of thoſe he has retained.

2. A Diſcourſe on Fortification; and another on Navigation.

3. Theſe performances, with ſome others, were collected in 3 volumes folio, under the title of *Mundus Mathematicus*, being indeed a complete courſe of mathematics. And the ſame was afterward much enlarged, and publiſhed at Lyons, 1690, in 4 large volumes, folio.

DECIL, *Decilis*, an aſpect or poſition of two planets, when they are diſtant from each other a 10th part of the zodiac, or 36 degrees; and is one of the new aſpects invented by Kepler.

DECIMALS, any thing proceeding by tens; as Decimal arithmetic, Decimal fractions, Decimal ſcales, &c.

DECIMAL *Arithmetic*, in a general ſenſe, may be conſidered as the common arithmetical computation in uſe, in which the decimal ſcale of numbers is uſed, or in which the places of the figures change their value in a tenfold proportion, being 10 times as much for every place more towards the left hand, or 10 times leſs for every place more towards the right hand; the places being ſuppoſed indefinitely continued, both to the right and left. In this ſenſe, the word includes both the arithmetic of integers, and decimal fractions. In a more reſtrained ſenſe however, it means only

DECIMAL *Fractions*, which are fractions whoſe denominator is always a 1 with ſome number of ciphers annexed, more or fewer according to the value of the fraction, the numerator of which may be any number whatever; as $\frac{1}{10}$, $\frac{3}{100}$, $\frac{7}{1000}$, &c.

As the denominator of a decimal is always one of the numbers 10, 100, 1000, &c, the inconvenience of writing theſe denominators down may be ſaved, by placing a proper diſtinction before the figures of the numerator only, to diſtinguiſh them from integers, for the value of each place of figures will be known in decimals, as well as in integers, by their diſtance from the 1ſt or unit's place of integers, having ſimilar names at equal diſtances, as appears by the following ſcale of places, both in decimals and integers:

&c	8	8	8	8	8	8	8	·	8	8	8	8	8	8	&c
	millions	hund. of thouſands	tens of thouſands	thouſands	hundreds	tens	units		tenths	hundredths	thouſandths	ten thouſandths	hund. thouſandths	millionths	

The mark of diſtinction for decimals, called the *ſeparatrix*, has been various at different times, according to the fancy of different authors; ſometimes a ſemiparentheſis, or a ſemicrotchet, or a perpendicular bar, or the ſame with a line drawn under the figures, or ſimply this line itſelf, &c; but it is uſual now to write either a comma or a full point near the bottom of the figures; I place the point near the upper part of the figures, as was done alſo by Newton; a method which prevents the ſeparatrix from being confounded with mere marks of punctuation.

In ſetting down a decimal fraction without its denominator, the numerator muſt conſiſt of as many places as there are ciphers in the denominator; and if it has not ſo many figures, the defect muſt be ſupplied by ſetting before them as many ciphers as will make them up ſo many: thus $\frac{3}{10}$ is ·3; and $\frac{14}{100}$ is ·14; and $\frac{14}{1000}$ is ·014; and $\frac{3}{1000}$ is ·003; &c.

So that, as ciphers on the right-hand ſide of integers *increaſe* their value decimally, or in a tenfold proportion, as 2, 20, 200, &c; ſo, when ſet on the left-hand of decimal fractions, they *decreaſe* the value decimally, or in a tenfold proportion, as ·2, ·02, ·002, &c. But ciphers ſet on the other ſides of theſe numbers, make no alteration in their value, neither of increaſe nor decreaſe, viz, on the left-hand of integers, or on the right-hand of decimals; ſo 2, or 02, or 002, &c, are all the ſame; as are alſo ·2, or ·20, or ·200, &c.

Decimal fractions may be conſidered as having been introduced by Regiomontanus, about the year 1464, viz, when he transformed the tables of ſines from a ſexageſimal to a decimal ſcale. They were alſo uſed by Ramus, in his Arithmetic, written in 1550; and before his time by our countrymen Buckley and Recorde. But it was Stevinus who firſt wrote an expreſs treatiſe on decimals, viz, about the year 1582, in *La Practique d'Arithmetique*; ſince which time, this has commonly made a part in moſt treatiſes on arithmetic.

To reduce any Vulgar fraction, or parts of any thing, as ſuppoſe $\frac{3}{8}$, to a decimal fraction of the ſame value; add ciphers at pleaſure to the numerator, and divide by the denominator: thus,

$$8)3{\cdot}000$$

$$\underline{}$$

$$\cdot375 = \tfrac{3}{8};$$

and therefore ·375 or $\frac{375}{1000}$ is a decimal of the ſame value with the propoſed vulgar fraction $\frac{3}{8}$.

Some vulgar fractions can never be reduced into decimals without defect; as $\frac{1}{3}$, which by diviſion is ·33333 &c infinitely.

Such numbers are very properly called circulating decimals, and repetends, becauſe of the continual return of the ſame figures. See REPETENDS and CIRCULATES.

The common arithmetical operations are performed the ſame way in decimals, as they are in integers; regard being had only to the particular notation, to diſtinguiſh the fractional from the integral part of a ſum. Thus,

In Addition and Subtraction, all figures of the ſame place or denomination are ſet ſtraight under each other, the ſeparatrix, or decimal points, forming a ſtraight column.

In Multiplication, ſet down the numbers, and multiply them as integers; and point off from the product as many places of decimals as there are in both factors; prefixing ciphers if there be any defect of figures.

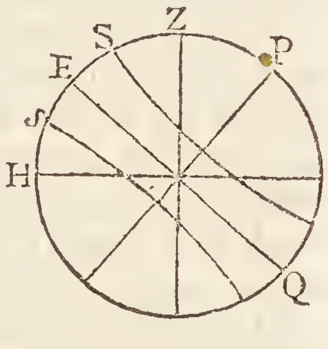
In Diviſion, ſet down the numbers and divide alſo as in integers; making as many decimals in the quotient, as thoſe in the dividend are more than thoſe in the diviſor.

Examples are numerous and common in moſt books of arithmetic.

DECIMAL *Scales*, are any ſcales divided decimally, or by tens.

DECLINATION, in Aſtronomy, is the diſtance of the ſun, ſtar, planet, &c, from the equinoctial, either northward or ſouthward; being the ſame with latitude in geography, or diſtance from the equator.

Declination is either *real* or *apparent*, according as the real or apparent place of the point or object is conſidered.



The declination of any point S is an arch of the meridian SE, contained between the given point and the equinoctical EQ. The declination of a ſtar &c, is found by knowing or obſerving the latitude of the place, i. e. the height of the pole, and then the meridian altitude of the ſtar, &c; hence the difference between the co-latitude and the altitude of the ſtar &c, is the declination, viz, the difference between EH the co-latitude, and SH the altitude, is ES the declination. For ex. Tycho found at Uranibourg the meridian altitude of Cauda Leonis, viz, HS = 50° 59′ 00″

the co-latitude is - - HE = 34 5 45

rem. declin. north - - ES = 16 53 15

To find the Sun's Declination at any time; having given his place in the ecliptic; the rule is, as radius is to the ſine of the ſun's longitude, ſo is the ſine of the greateſt declination, or obliquity of the ecliptic, to his preſent declination ſought.

In conſtructing tables of declination of the ſun, planets, and ſtars, regard ſhould be had to refraction, aberration, nutation, and parallax.

By comparing ancient obſervations with the modern, it appears that the declination of the fixed ſtars is variable; and that differently in different ſtars; for in ſome it increaſes, and in others decreaſes, and that in different quantities.

Circles of DECLINATION, are great circles of the ſphere paſſing through the poles of the world, on which the declination is meaſured; and conſequently are the ſame as meridians in geography.

Parallax of DECLINATION, is an arch of the circle of declination, by which the parallax in altitude increaſes or diminiſhes the declination of a ſtar.

Parallels of DECLINATION, are leſſer circles parallel to the equinoctial. The tropic of Cancer is a parallel of declination at 23° 28′ diſtance from the equinoctial northward; and the tropic of Capricorn is the parallel of declination as far diſtant ſouthward.

Refraction of the DECLINATION, an arch of the circle of declination, by which the declination of a ſtar is increaſed or diminiſhed by means of the refraction.

DECLINATION *of the Compaſs*, or *Needle*, is its deviation from the true meridian. See VARIATION.

DECLINATION *of a Vertical Plane*, or *Wall*, in Dialling, is an arch of the horizon, comprehended either between the plane and the prime vertical circle, when it is counted from the eaſt or weſt; or between the plane and the meridian, if it be accounted from the north or ſouth.

DECLINATOR, or DECLINATORY, an inſtrument in dialling, by which the declination, inclination, and reclination of planes are determined.

DECLINERS, or DECLINING *Dials*, are thoſe which cut obliquely, either the plane of the prime vertical circle, or the plane of the horizon.

The uſe of declining vertical dials is very frequent; becauſe the erect walls of houſes, on which dials are commonly drawn, moſtly decline from the cardinal points. But incliners and recliners are very rare.

DECLIVITY, a ſloping or oblique deſcent.

DECREMENT, *Equal, of Life.* See COMPLEMENT *of life*.

DECREMENTS are the ſmall parts by which a variable and decreaſing quantity becomes leſs and leſs. The indefinitely ſmall decrements are proportional to the fluxions, which in this caſe are negative. See FLUXIONS, alſo INCREMENTS.

DECUPLE, a term of relation or proportion in arithmetic, implying a tenfold change or ſcale of variation, or one thing 10 times as much as another.

DECUSSATION, a term in geometry and optics, ſignifying the croſſing of any two lines or rays &c: or the action itſelf of croſſing.

The rays of light decuſſate in the chryſtalline, before they reach the retina.

Many of the lines of the 3d order decuſſate themſelves. See Newton's Enumeratio, &c.

DEE (JOHN), a famous mathematician and aſtrologer, was born at London 1527. In 1542 he was ſent to St. John's College, Cambridge. After five years cloſe application to ſtudy, chiefly in the mathematical and aſtronomical ſciences, he went over to Holland, to viſit ſome mathematicians on the Continent; whence, after a year's abſence, he returned to Cambridge, and was there elected one of the Fellows of Trinity College, then firſt erected by King Henry the 8th. In 1548 he left England a ſecond time, his ſtay at home being rendered uneaſy to him, by the ſuſpicions that were entertained of his being a conjurer, ariſing chiefly from his application to aſtronomy, and from ſome mechanical inventions of his.

He now viſited the univerſity of Louvain; where he was much careſſed, and viſited by ſeveral perſons of high rank. After two years he went into France, and read lectures, in the college of Rheims, upon Euclid's Elements. In 1551, he returned to England, and was introduced to King Edward, who aſſigned him a penſion of 100 crowns, which he afterward relinquiſhed for the rectory of Upton upon Severn. But ſoon after the acceſſion of Queen Mary, having ſome correſpondence with her ſiſter Elizabeth, he was accuſed of practiſing againſt the queen's life by enchantment: on which account he ſuffered a tedious confinement, and was ſeveral times examined; till, in the year 1555, he obtained his liberty by an order of council.

When Queen Elizabeth aſcended the throne, Dee was conſulted concerning a propitious day for the coronation: on which occaſion he was introduced to the queen, who made him great promiſes, which were but ill performed. In 1564, he made another voyage to the continent, to preſent a book which he had dedicated to the Emperor Maximilian. He returned to England the ſame year; but in 1571 we find him in Lorrain; where, being dangerouſly ill, the queen ſent over two phyſicians to his relief. Having once more returned to his native country, he ſettled at Mortlake in Surry, where he continued his ſtudies with much ardour, and collected a great library of printed books and manuſcripts, with a number of inſtruments; moſt of which were afterward deſtroyed by the mob, as belonging to one who dealt with the devil.

In 1578, the queen being much indiſpoſed, Mr. Dee was ſent abroad to conſult with German phyſicians and philoſophers (aſtrologers no doubt) on the occaſion; though ſome have ſaid ſhe employed him as a ſpy; probably he acted in a double capacity. We next find him again in England, where he was ſoon after employed in a more rational ſervice. The queen, deſirous to be informed concerning her title to thoſe countries which had been diſcovered by Engliſhmen, ordered Dee to conſult the ancient records, and to furniſh her with proper geographical deſcriptions. Accordingly, in a ſhort time, he preſented to the queen, at Richmond, two large rolls, in which the diſcovered countries were geographically deſcribed and hiſtorically illuſtrated. His next employment was the reformation of the calendar, on which ſubject he wrote a rational and learned treatiſe, preſerved in the Aſhmolean library at Oxford.

Hitherto the extravangancies of our eccentrical philoſopher ſeem to have been tempered with a tolerable proportion of reaſon and ſcience; but henceforward he is to be conſidered as a mere necromancer and credulous alchymiſt. In the year 1581 he became acquainted with one Edward Kelly, by whoſe aſſiſtance he performed divers incantations, and maintained a frequent imaginary intercourſe with ſpirits and angels; one of whom made him a preſent of a black ſpeculum (a poliſhed piece of cannel-coal), in which theſe appeared to him as often as he had occaſion for them, anſwering his queſtions, &c. Hence Butler ſays,

> Kelly did all his feats upon
> The devil's looking-glaſs, a ſtone.
>
> HUDIBRAS.

In 1583 they became acquainted with a certain Poliſh nobleman, then in England, named Albert Laſki, a perſon equally addicted to the ſame ridiculous purſuits: he was ſo charmed with Dee and Kelly, that he perſuaded them to accompany him to his native country; by whoſe means they were introduced to Rodolph king of Bohemia; who, though a credulous man, was ſoon diſguſted with their nonſenſe. They were afterward introduced to the king of Poland, but with no better ſucceſs. Soon after this they were entertained at the caſtle of a rich Bohemian nobleman, where they lived for ſome time in great affluence; owing, as they aſſerted, to their art of tranſmutation by means of a certain powder in the poſſeſſion of Kelly.

Dee, now quarrelling with his companion, quitted Bohemia, and returned to England, where he was once more graciouſly received by the queen; who, in 1595, made him warden of Mancheſter college, in which town he reſided ſeveral years. In 1604 he returned to his houſe at Mortlake, where he died in 1608, at 81 years of age; leaving a large family and many works behind him.

The books that were printed and publiſhed by Dee, are, 1. *Propædumata Aphoriſtica, &c.* in 1558, in 12mo.—2. *Monas Hieroglyphica ad Regem Romanorum Maximilianum*; 1564.—3. *Epiſtola ad eximium ducis Urbini mathematicum, Fredericum Commandinum, prefixa libello Machomeii Bagdadini de Superficierum Diviſionibus &c*; 1570.—4. *The Britiſh Monarchy, otherwiſe called, The Petty Navy Royal*; 1576.—5. *Preface Mathematical to the Engliſh Euclid*, publiſhed by Henry Billingſley, 1570: certainly a very curious and elaborate compoſition, and where he ſays, many more arts are wholly invented by name, definition, property, and uſe, than either the Grecian or Roman mathematicians have left to our knowledge.—6. *Divers and many annotations and inventions diſperſed and added after the* 10*th book of Engliſh Euclid*; 1570.—7. *Epiſtola prefixa Ephemeridibus Joannis Feldi à* 1557, *cui rationem declaraverat Ephemerides conſcribendi.*—8. *Parallaticæ Commentationis Paxeoſque Nucleus quidam*; 1573.

This catalogue of Dee's printed and publiſhed works is to be found in his *Compendious Rehearſal &c*, as well as in his letter to Abp. Whitgift: and from the ſame places might be tranſcribed more than 40 titles of books unpubliſhed, that were written by him.

DEFENCE, in Sieges, is uſed for any thing that ſerves to preſerve or ſcreen the ſoldiers, or the place. So the parapets, flanks, caſemates, ravelins, and outworks, that cover the place, are called the defences, or covers of the place: and when the cannon have beaten down or ruined theſe works, ſo that the men cannot fight under cover, the defences of the place are ſaid to be demoliſhed.

Line of DEFENCE, is that which flanks a baſtion, being drawn from the flank oppoſite to it.

The line of defence ſhould not exceed a muſket ſhot, i. e. 120 fathoms: indeed Melder allows 130, Scheiter 140, Vauban and Pagan 150.

Line of DEFENCE, *greater*, or *fichant*, is a line drawn from the point of the baſtion to the concourſe of the oppoſite flank and curtin.

Line of DEFENCE, *leſſer*, or *raſant*, or *flanquant*, is the face of the baſtion continued to the curtin.

DEFERENT, or DEFERENS, in the ancient aſtronomy, an imaginary circle, which, as it were, carries about the body of a planet, and is the ſame with the eccentric; being invented to account for the eccentricity, perigee, and apogee of the planets.

DEFICIENT *Hyperbola*, is a curve having only one aſymptote, though two hyperbolic legs running out infinitely by the ſide of the aſymptote, but contrary ways. See CURVE.

This name was given to the curves by Newton, in his Enumeratio Linearum tertii Ordinis. There are 6 different ſpecies of them, which have no diameters, expreſſed by the equation $xyy + ey = -ax^3 + bx^2 + cx + d$, the term ax^3 being negative. When the equation ax^4

$= bx^3 + cx^2 + dx + \frac{1}{4}ee$ has all its roots real and unequal, the curve has an oval joined to it. When the two middle roots are equal, the oval joins to the legs, which then cut one another in ſhape of a nooſe. When three roots are equal, the nodus is changed into a very acute cuſp or point. When, of three roots with the ſame ſign, the two greateſt are equal, the oval vaniſhes into a point. When any two roots are imaginary, there is only a pure ſerpentine hyperbola, without any oval, decuſſation, cuſp, or conjugate point; and when the terms b and d are wanting, it is of the 6th ſpecies.

There are alſo 7 different ſpecies of theſe curves, having each one diameter, expreſſed by the above equation when the term ey is wanting: according to the various conditions of the roots of the equation $ax^3 = bx^2 + cx + d$, as to their reality, equality, their having the ſame ſigns, or two of them being imaginary.

Deficient *Numbers*, are thoſe whoſe aliquot parts added together, make a ſum leſs than the whole number: as 8, whoſe parts 1, 2, 4, make only 7; or the number 16, whoſe parts 1, 2, 4, 8 make only 15.

DEFILE, *in Fortification*, a narrow line or paſſage through which troops can paſs only in file, making a ſmall front, ſo that the enemy may eaſily ſtop their march, and charge them with the more advantage, as the front and rear cannot come to the relief of one another.

DEFINITION, an enumeration, or ſpecification of the chief ſimple ideas of which a compound idea conſiſts, in order to aſcertain or explain its nature and character.

Definitions are of two kinds; the one *nominal*, or of the *name;* the other *real*, or of the *thing*.

Nominal Definition, is an enumeration of ſuch known characters as are ſufficient for diſtinguiſhing any propoſed thing from others; as is that of a ſquare, when it is ſaid that it is a quadrilateral, equilateral, rectangular figure.

Real Definition, a diſtinct notion, explaining the geneſis of a thing; that is, how the thing is made or done: as is this definition of a circle, viz, that it is a figure deſcribed by the motion of a right line about a fixed point.

DEFLECTION, the turning any thing aſide from its former courſe, by ſome adventitious or external cauſe.

The word is often applied to the tendency of a ſhip from her true courſe, by reaſon of currents, &c, which turn her out of her right way.

Deflection *of the Rays of Light*, is a property which Dr. Hook obſerved in 1675. He found it different both from reflection and refraction; and that it was made perpendicularly towards the ſurface of the opacous body.

This is the ſame property which Newton calls *inflection*. And by others it is called *diffraction*.

DEGREE, *in Algebra*, is uſed in ſpeaking of equations, when they are ſaid to be of ſuch a degree according to the higheſt power of the unknown quantity. If the index of that power be 2, the equation is of the 2d degree; if 3, it is of the 3d degree, and ſo on.

Degree, in Geometry or Trigonometry, is the 360th part of the circumference of any circle; for every circle is conſidered as divided into 360 parts, called degrees; which are marked by a ſmall ° near the top of the figure; thus 45° is 45 degrees.

The degree is ſubdivided into 60 ſmaller parts, called minutes, meaning firſt minutes; the minute into 60 others, called ſeconds; the ſecond into 60 thirds; &c. Thus 45° 12′ 20″ are 45 degrees, 12 minutes, 20 ſeconds.

The magnitude or quantity of angles is accounted in degrees; for becauſe of the uniform curvature of a circle in all its parts, equal angles at the centre are ſubtended by equal arcs, and by ſimilar arcs in peripheries of different diameters; and an angle is ſaid to be of ſo many degrees, as are contained in the arc of any circle comprehended between the legs of the angle, and having the angular point for its centre. Thus we ſay an angle of 90°, or of 45° 24′, or of 12° 20′ 30″. It is alſo uſual to ſay, ſuch a ſtar is mounted ſo many degrees above the horizon, or declines ſo many degrees from the equator; or ſuch a town is ſituate in ſo many degrees of latitude or longitude.—A ſign of the ecliptic, or zodiac, contains 30 degrees.

The diviſion of the circle into 360 degrees is uſually aſcribed to the Egyptians, probably from the circle of the ſun's annual courſe, or according to their number of days in the year, allotting a degree to each day. It is a convenient number too, as admitting of a great many aliquot parts, as 2, 3, 4, 5, 6, 8, 9, &c. The ſexageſimal ſubdiviſion, however, has often been condemned as improper, by many eminent mathematicians, as Stevinus, Oughtred, Wallis, Briggs, Gellibrand, Newton, &c; who adviſe a decimal diviſion inſtead of it, or elſe that of centeſms; as the degree into 100 parts, and each of theſe into 100 parts again, and ſo on. Stevinus even holds, that this diviſion of the circle which he contends for, obtained in the wiſe age, *in ſæculo ſapienti*. Stev. Coſmog. lib. 1, def. 6. And ſeveral large tables of ſines &c have been conſtructed according to that plan, and publiſhed, by Briggs, Newton, and others. And I myſelf have carried the idea ſtill much farther, in a memoir publiſhed in the Philoſ. Tranſ. of 1783, containing a propoſal for a new diviſion of the quadrant, viz, into equal decimal parts of the radius; by which means the degrees or diviſions of the arch would be the real lengths of the arcs, in terms of the radius: and I have ſince computed thoſe lengths of the arcs, with their ſines, &c, to a great extent and accuracy.

Degree *of Latitude*, is the ſpace or diſtance on the meridian through which an obſerver muſt move, to vary his latitude by one degree, or to increaſe or diminiſh the diſtance of a ſtar from the zenith by one degree; and which, on the ſuppoſition of the perfect ſphericity of the earth, is the 360th part of the meridian.

The quantity of a degree of a meridian, or other great circle, on the ſurface of the earth, is variouſly determined by different obſervers: and the methods made uſe of are alſo various.

Eratoſthenes, 250 years before Chriſt, firſt determined the magnitude of a degree of the meridian, between Alexandria and Syene on the borders of Ethiopia, by meaſuring the diſtance between thoſe places, and comparing it with the difference of a ſtar's zenith diſtances at thoſe places; and found it $694\frac{4}{9}$ ſtadia.

Poſidonius, in the time of Pompey the Great, by means of the different altitudes of a ſtar near the horizon, taken at different places under the ſame meridian, compared in like manner with the diſtance between thoſe places, determined the length of a degree only 600 ſtadia.

Ptolomy fixes the degree at $68\frac{2}{3}$ Arabic miles, counting $7\frac{1}{2}$ ſtadia to a mile. The Arabs themſelves, who made a computation of the diameter of the earth, by meaſuring the diſtance of two places under the ſame meridian, in the plains of Sennar, by order of Almamon, make it only 56 miles. Kepler, determining the diameter of the earth by the diſtance of two mountains, makes a degree 13 German miles; but his method is far from being accurate. Snell, ſeeking the diameter of the earth from the diſtance between two parallels of the equator, finds the quantity of a degree,

by one method 57064 Paris toiſes, or 342384 feet;
by another meth. 57057 - - toiſes, or 342342 feet.

The mean between which two numbers, M. Picard found by menſuration, in 1669, from Amiens to Malvoiſin, the moſt certain, and he makes the quantity of a degree 57060 toiſes, or 342360 feet. However, M. Caſſini, at the king's command, in the year 1700, repeated the ſame labour, and meaſuring the ſpace of 6° 18′, from the obſervatory at Paris, along the meridian, to the city of Collioure in Rouſſillon, that the greatneſs of the interval might diminiſh the error, found the length of the degree equal to 57292 toiſes, or 343742 Paris feet, amounting to 365184 Engliſh feet.

And with this account nearly agrees that of our countryman Norwood, who, about the year 1635, meaſured the diſtance between London and York, and found that diſtance 905751 Engliſh feet; the difference of latitude being 2° 28′, hence, he determined the quantity of one degree at 367196 Engliſh feet, or 57300 Paris toiſes, or 69 miles, 288 yards. See Newt. Princ. Phil. prop. 19; and Hiſt. Acad. Scienc. anno 1700, pa. 153.

M. Caſſini, the ſon, completed the work of meaſuring the whole arc of the meridian through France, in 1718. For this purpoſe he divided the meridian of France into two arcs, which he meaſured ſeparately.

The one from Paris to Collioure gave him 57097 toiſes;
the other from Paris to Dunkirk - - 56960; and
the whole arc from Dunkirk to Collioure 57060,
the ſame as M. Picard's.

M. Muſchenbroek, in 1700, reſolving to correct the errors of Snell, found by particular obſervations, that the degree between Alemaer and Bergen-op-zoom contained 57033 toiſes.

Meſſieurs Maupertuis, Clairaut, Camus, Monnier, and Outheir of France, were ſent on a northern expedition, and began their operations, aſſiſted by M. Celſus, an eminent aſtronomer of Sweden, in Swediſh Lapland, in July 1736, and finiſhed them by the end of May following. They obtained the meaſure of that degree, whoſe middle point was in lat. 66° 20′ north, and found it 57439 toiſes, when reduced to the level of the ſea. About the ſame time another company of philoſophers was ſent to South America, viz, Meſſieurs Godin, Bouguer, and Condamine of France, to whom were joined Don Jorge Juan, and Don Antonio de Ulloa of Spain. They left Europe in 1735, and began their operations in the province of Quito in Peru, about October 1736, and finiſhed them, after many interruptions, about 8 years after. The Spaniſh gentlemen publiſhed a ſeparate account, and aſſign for the meaſure of a degree of the meridian at the equator 56768 toiſes. M. Bouguer makes it 56753 toiſes, when reduced to the level of the ſea; and M. Condamine ſtates it at 56749 toiſes.

M. Caille, being at the Cape of Good Hope in 1752, found the length of a degree of the meridian in lat. 33° 18′ 30″ ſouth, to be 57037 toiſes. In 1755, father Boſcovich found the length of a degree in lat. 43° north to be 56972 toiſes, as meaſured between Rome and Rimini in Italy. In the year 1740, Meſſrs Caſſini and La Caille again examined the former meaſures in France, and, after making all the neceſſary corrections, found the meaſure of a degree, whoſe middle point is in lat. 49° 22′ north, to be 57074 toiſes; and in the lat. of 45°, it was 57050 toiſes.

In 1764, F. Beccaria completed the meaſurement of a portion of the meridian near Turin; from which it is deduced that the length of a degree, whoſe middle lat. is 44° 44′ north, is 57024 Paris toiſes.

At Vienna, 3 degrees of the meridian were meaſured; and the medium, for the latitude of 47° 40′ north may be taken at 57091 Paris toiſes. See an account of this meaſurement, by father Joſeph Lieſganig, in the Philoſ. Tranſ. 1768, pa. 15.

Finally, in the ſame vol. too is an account of the meaſurement of a part of the meridian in Maryland and Penſilvania, North America, 1766, by Meſſrs Maſon and Dixon; from which it follows that the length of a degree whoſe middle point is 39° 12′ north, was 363763 Engliſh feet, or $56904\frac{1}{2}$ Paris toiſes.

Hence, from the whole we may collect the following table of the principal meaſures of a degree in different parts of the earth, as meaſured by different perſons, viz,

Mean Latitude.	Length of a Degree in Paris toiſes.	Names of the Meaſurers.	Years of Meaſurement.
66° 20′ N	57422	Maupertuis &c	1736 & 1737
49 23 N	57074	Maupertuis &c and Caſſini	1739 & 1740
47 40 N	57091	Lieſganig	1766
45 0 N	57028	Caſſini	1739 & 1740
44 44 N	57069	Beccaria	1760 to 1764
43 0 N	56979	Boſcovich and Le Maire	1752
39 12 N	56888	Maſon & Dixon	1764 to 1768
0 0	56750	Bouguer and Condamine	1736 to 1744
33 18 S	57037	La Caille	1752

The method of obtaining the length of a degree of the terreſtrial meridian, is to meaſure a certain diſtance upon it by a ſeries of triangles, whoſe angles may be found by actual obſervation, connected with a baſe, whoſe length may be taken by an actual ſurvey, or otherwiſe; and then to obſerve the different altitudes of ſome ſtar at the two extremities of that diſtance, which gives the difference of latitude between them: then, by proportion, as this difference of latitude is to one degree, ſo is the meaſured length to the length of one degree of the meridian ſought. This method was firſt practiſed by Eratoſthenes, in Egypt. See Geography, and the beginning of this article.

Degree *of longitude*, is the ſpace between two meridians that make an angle of 1° with each other at the poles; the quantity or length of which is variable, according to the latitude, being every where as the coſine of the latitude; viz, as the coſine of one lat. is to the coſine of another, ſo is the length of a degree in the former lat. to that in the latter; and from this theorem is computed the following Table of the length of a degree of long. indifferent latitudes, ſuppoſing the earth to be a globe.

Degr. lat.	Engliſh miles.	Degr. lat.	Engliſh miles.	Degr. lat.	Engliſh miles.
0	69·07	31	59·13	61	33·45
1	69·06	32	58·51	62	32·40
2	69·03	33	57·87	63	31·33
3	68·97	34	57·20	64	30·24
4	68·90	35	56·51	65	29·15
5	68·81	36	55·81	66	28·06
6	68·62	37	55·10	67	26·96
7	68·48	38	54·37	68	25·85
8	68·31	39	53·62	69	24·73
9	68·15	40	52·85	70	23·60
10	67·95	41	52·07	71	22·47
11	67·73	42	51·27	72	21·32
12	67·48	43	50·46	73	20·17
13	67·21	44	49·63	74	19·02
14	66·95	45	48·78	75	17·86
15	66·65	46	47·93	76	16·70
16	66·31	47	47·06	77	15·52
17	65·98	48	46·16	78	14·35
18	65·62	49	45·26	79	13·17
19	65·24	50	44·35	80	11·98
20	64·84	51	43·42	81	10·79
21	64·42	52	42·48	82	9·59
22	63·97	53	41·53	83	8·41
23	63·51	54	40·56	84	7·21
24	63·03	55	39·58	85	6·00
25	62·53	56	38·58	86	4·81
26	62·02	57	37·58	87	3·61
27	61·48	58	36·57	88	2·41
28	60·93	59	35·54	89	1·21
29	60·35	60	34·50	90	0·00
30	59·75				

Note, This table is computed on the ſuppoſition that the length of the degrees of the equator are equal to thoſe of the meridian at the medium latitude of 45°, which length is $69\frac{1}{15}$ Engliſh miles.

The expreſſions *Latitude* and *Longitude*, are borrowed from the ancients, who happened to be acquainted with a much larger extent of the earth in the direction eaſt and weſt, than in that of north and ſouth; the former of which therefore paſſed, with them, for the length of the earth, or longitude, and the latter for the breadth or ſhorter dimenſion, viz, the latitude.

DEJECTION, in Aſtrology, is applied to the planets when in their detriment, as aſtrologers ſpeak, i. e. when they have loſt their force, or influence, as is pretended, by reaſon of their being in oppoſition to ſome others, which check and counteract them.

Or, it is uſed when a planet is in a ſign oppoſite to that in which it has its greateſt effect, or influence, which is called its exaltation. Thus, the ſign Aries being the exaltation of the Sun, the oppoſite ſign Libra is its dejection.

DEINCLINERS, or Deinclining *Dials*, are ſuch as both decline and incline, or recline, at the ſame time. Suppoſe, for inſtance, a plane cutting the prime vertical circle at an angle of 30 degrees, and the horizontal plane at an angle of 24 degrees, the latitude of the place being 52 degrees; a dial drawn on this plane, is called a deincliner.

DELIACAL *Problem*, a celebrated problem among the ancients, concerning the duplication of the cube.

DELPHINUS, the *Dolphin*, a conſtellation of the northern hemiſphere; whoſe ſtars, in Ptolomy's catalogue, are 10; in Tycho's the ſame; in Hevelius's 14; and in Flamſteed's 18.

DEMETRIUS, a Cynic philoſopher, and diſciple of Apollonius Thyaneus, in the age of Caligula. That emperor wiſhing to gain the philoſopher to his intereſt by a large preſent, he refuſed it with indignation, ſaying, If Caligula wiſhes to bribe me, let him ſend me his crown. Veſpaſian was diſpleaſed with his inſolence, and baniſhed him to an iſland. The cynic derided the puniſhment, and bitterly inveighed againſt the emperor.

Demetrius lived to a very great age. And Seneca obſerves, that "nature had brought him forth to ſhew mankind that an exalted genius can live ſecurely without being corrupted by the vice of the ſurrounding world."

DEMI-*Baſtion*, in Fortification, one that has only one face and one flank.

Demi-*Cannon*, and Demi-*Culverin*, names of certain ſpecies of cannon, now no longer uſed.

Demi-*Croſs*, an inſtrument uſed by the Dutch to take the altitude of the ſun or a ſtar at ſea; inſtead of which we uſe the croſs ſtaff, or foreſtaff.

Demi-*Gorge*, is half the gorge or entrance into the baſtion; not taken directly from angle to angle, where the baſtion joins to the curtin, but from the angle of the flank to the centre of the baſtion; or the angle the two curtins would make, were they thus protracted to meet in the baſtion.

Demi-*Lune*, or *Half-moon*, an outwork conſiſting of two faces, and two little flanks. It is often built before the angle of a baſtion, and ſometimes alſo before the curtin; though now it is very ſeldom uſed.

DEMOCRITUS, one of the greateſt philoſophers of antiquity, was born at Abdera, a town of Thrace, about the 80th olympiad, or about 400 years before Chriſt. His father, ſays Valerius Maximus, was able to entertain the army of Xerxes; and Diogenes Laertius adds, upon the teſtimony of Herodotus, that the king, in requital, preſented him with ſome Magi and Chaldeans. From theſe, it ſeems, Democritus received the firſt part of his education; and from them, whilſt yet a boy, he learned theology and aſtronomy. He next applied to Leucippus, from whom he learned the ſyſtem of atoms and a vacuum. His father dying, he and his two brothers divided his effects. Democritus made choice of that part which conſiſted in money, as being, though the leaſt ſhare, the moſt convenient for travelling; and it is ſaid that his portion amounted to more than 100 talents, which is near 20 thouſand pounds ſterling. His extraordinary inclination for knowledge and the ſciences, induced him to travel into all parts of the world where he might find learned men. He went to viſit the prieſts of Egypt, from whom he learned geometry: He conſulted the Chaldean and Perſian philoſophers: and it is ſaid that he penetrated even into India and Ethiopia, to confer with the Gymnoſophiſts. In theſe travels he waſted his ſubſtance; after which, at his return he was obliged for ſome time to be maintained by his brother. Settling himſelf at Abdera, he there governed in the moſt abſolute manner, by virtue of his conſummate wiſdom. The magiſtrates of that city made him a preſent of 500 talents, and erected ſtatues to him, even in his lifetime: but being naturally more inclined to contemplation than delighted with public honours and employments, he withdrew into ſolitude and retirement.

Democritus always laughed at human life, as a continued farce, which made the people think he was mad; on which they ſent for Hippocrates to cure him: but that celebrated phyſician having diſcourſed with the philoſopher, told the people that he had a great veneration for Democritus; and that, in his opinion, thoſe who eſteemed themſelves the moſt healthy, were the moſt diſtempered.

It is ſaid, though with little probability, that Democritus put out his own eyes, that he might meditate more profoundly on philoſophical ſubjects. He died, according to Diogenes Laertius, in the 361ſt year before the Chriſtian era, at 109 years of age. He was the author of many books, which are loſt; from which Epicurus borrowed his philoſophy.

DEMOIVRE (Abraham), a celebrated mathematician, of French original, but who ſpent moſt of his life in England. He was born at Vitri in Champagne 1667. The revocation of the edict of Nantz, in 1685, determined him, with many others, to take ſhelter in England; where he perfected his mathematical ſtudies, the foundation of which he had laid in his own country. A mediocrity of fortune obliged him to employ his talent in this way in giving leſſons, and reading public lectures, for his better ſupport: in the latter part of his life too, he chiefly ſubſiſted by giving anſwers to queſtions in chances, play, annuities, &c, and it is ſaid moſt of theſe reſponſes were delivered at a Coffee-houſe in St. Martin's-lane, where he ſpent moſt of his time. The Principia Mathematica of Newton, which chance is ſaid to have thrown in his way, ſoon convinced Demoivre how little he had advanced in the ſcience he profeſſed. This induced him to redouble his application; which was attended by a conſiderable degree of ſucceſs; and he ſoon became connected with, and celebrated among, the firſt rate mathematicians. His eminence and abilities in this line, opened him an entrance into the Royal Society of London, and into the academies of Berlin and Paris. By the former his merit was ſo well known and eſteemed, that they judged him a fit perſon to decide the famous conteſt between Newton and Leibnitz, concerning the invention of Fluxions.

The collection of the Academy of Paris contains no memoir of this author, who died at London Nov. 1754, at 87 years of age, ſoon after his admiſſion into it. But the Philoſophical Tranſactions of London have ſeveral, and all of them intereſting, viz, in the volumes 19, 20, 22, 23, 25, 27, 29, 30, 32, 40, 41, 43.

He publiſhed alſo ſome very reſpectable works, viz,

1. *Miſcellanea Analytica, de Seriebus & Quadraturis &c;* 1730, in 4to. But perhaps he has been more generally known by his

2. *Doctrine of Chances; or, Method of Calculating the Probabilities of Events at Play.* This work was firſt printed, 1718, in 4to, and dedicated to Sir Iſaac Newton: it was reprinted in 1738, with great alterations and improvements; and a third edition was afterwards printed.

3. *Annuities on Lives;* firſt printed 1724, in 8vo.—In 1742 the ingenious Thomas Simpſon (then only 33 years of age) publiſhed his *Doctrine of Annuities and Reverſions;* in which he paid ſome handſome compliments to our author. Notwithſtanding which, Demoivre preſently brought out a ſecond edition of his Annuities, in the preface to which he paſſed ſome harſh reflections upon Simpſon. To theſe the latter gave a handſome and effectual anſwer, 1743, in *An Appendix, containing ſome Remarks on a late book on the ſame ſubject, with anſwers to ſome perſonal and malignant miſrepreſentations, in the preface thereof.* At the end of this anſwer, Mr. Simpſon concludes, "Laſtly, I appeal to all mankind, whether, in his treatment of me, he has not diſcovered an air of ſelf-ſufficiency, ill-nature and inveteracy, unbecoming a gentleman." Here it would ſeem the controverſy dropped: Mr. Demoivre publiſhed the 3d edition of his book in 1750, without any farther notice of Simpſon, but omitted the offenſive reflections that had been in the preface.

DEMONSTRATION, a certain or convincing proof of ſome propoſition: ſuch as the demonſtrations of the propoſitions in Euclid's Elements.

The method of demonſtrating in mathematics, is the ſame with that of drawing concluſions from principles in logic. Indeed, the demonſtrations of mathematicians are no other than ſeries of enthymemes; every thing is concluded by force of ſyllogiſm, only omitting the premiſes, which either occur of their own accord, or are recollected by means of quotations.

DENDROMETER, an inſtrument lately invented by Meſſrs Duncombe and Whittel; ſo called, from its uſe in meaſuring trees.

DENEB, an Arabic term, ſignifying tail; uſed by aſtronomers as a name to ſome of the fixed ſtars, but eſpecially for the bright ſtar in the Lion's tail.

DENOMINATOR, *of a fraction*, is the number or quantity placed below the line, which ſhews the whole integer, or into how many parts the integer is ſuppoſed to be divided by the fraction; as that which gives denomination or name to the parts of the fraction. Thus, in the fraction $\frac{5}{12}$, five-twelfths, the number 12 is the denominator, and ſhews that the integer is here divided into 12 parts, or that it conſiſts of 12 of thoſe parts of which the numerator contains 5. Alſo b is the denominator of the fraction $\frac{a}{b}$.

DENOMINATOR *of a ratio*, is the quotient ariſing from the diviſion of the antecedent by the conſequent. Thus, 6 is the denominator of the ratio 30 to 5, becauſe 30 divided by 5 gives 6. It is otherwiſe called the *exponent* of the ratio.

DENSITY, that property of bodies, by which they contain a certain quantity of matter, under a certain bulk or magnitude. Accordingly a body that contains more matter than another, under the ſame bulk, is ſaid to be denſer than the other, and that in proportion to the quantity of matter; or if the quantity of matter be the ſame, but under a leſs bulk, it is ſaid to be denſer, and ſo much the more ſo as the bulk is leſs. So that, in general, the denſity is directly proportional to the maſs or quantity of matter, and reciprocally or inverſely proportional to the bulk or magnitude under which it is contained.

The quantities of matter in bodies, or at leaſt the proportions of them, are known by their gravity or weight; every equal particle of matter being endowed with an equal gravity, it is inferred that equal maſſes or quantities of matter have an equal weight or gravity; and unequal maſſes have proportionally unequal weights. So that, when body, or maſs, or quantity of matter is ſpoken of, we are to underſtand their weight or gravity.

From the foregoing general proportion of the denſity of bodies, viz, that it is as the maſs directly, and as the bulk inverſely, may be inferred the proportion of the maſſes, or of the magnitudes; viz, that the maſs or quantity of matter, is in the compound ratio of the bulk and denſity; and that the bulk or magnitude, is as the maſs directly, and the denſity inverſely. Hence, if B, b be two bodies, or maſſes, or weights;

and D, d their reſpective denſities;

alſo M, m their magnitudes, or bulks:

Then the theorems above are thus expreſſed,

viz, $D \propto \frac{B}{M}$, and $B \propto DM$, and $M \propto \frac{B}{D}$;

or $D : d :: \frac{B}{M} : \frac{b}{m}$, and $B : b :: DM : dm$, &c;

or $\frac{D}{d} = \frac{Bm}{bM}$, and $\frac{B}{b} = \frac{DM}{dm}$, and $\frac{M}{m} = \frac{Bd}{bD}$.

No body is abſolutely or perfectly denſe; or no ſpace is perfectly full of matter, ſo as to have no vacuity or interſtices; on the contrary, it is the opinion of Newton, that even the denſeſt bodies, as gold &c, contain but a ſmall portion of matter, and a very great portion of vacuity; or that it contains a great deal more of pores or empty ſpace, than of real ſubſtance.

It has been obſerved above, that the relative denſity of bodies may be known by their weight or gravity; and hence the moſt general way of knowing thoſe denſities, is by actually weighing an equal bulk or magnitude of the bodies, whether ſolid or fluid; if ſolid, by ſhaping them to the ſame figure and dimenſions; if fluid, by filling the ſame veſſel with them, and weighing it.

For fluids, there are alſo other methods of finding their denſity: as 1ſt, by making an equilibrium between them in tubes that communicate; for, the diameters of the tubes being equal, and the weights or quantities of matter alſo equal, the denſities will be inverſely as the altitudes of the liquids in them, that is inverſely as the bulk.

2dly, The denſities of fluids are alſo compared together by immerging a ſolid in them; for if the ſolid be lighter than the liquids, the part immerged by its own weight, will be inverſely as the denſity of the fluid; or if it be heavier, and ſink in the liquids, by weighing it in them; then the weights loſt by the body will be directly proportional to the denſities of the fluids.

DENSITY *of the Air*, is a property that has much employed the later philoſophers, ſince the diſcovery of the Torricellian experiment, and the air-pump. By means of the barometer it is demonſtrated that the air is of the ſame denſity at all places at the ſame diſtance from the level of the ſea; provided the temperature, or degree of heat, be the ſame. Alſo the denſity of the air always increaſes in proportion to the compreſſion, or the compreſſing forces. And hence the lower parts of the atmoſphere are always denſer than the upper: yet the denſity of the lower air is not exactly proportional to the weight of the atmoſphere, by reaſon of heat and cold, which make conſiderable alterations as to rarity and denſity; ſo that the barometer meaſures the elaſticity of the air, rather than its denſity. If the height of the barometer be conſidered as the meaſure both of the denſity and elaſticity of the air, when the thermometer is at 31°, and b be any other height of the barometer, when the thermometer is at t degrees; then in this caſe, b is the meaſure of the elaſticity,

and $\frac{466 - t}{435} b$ is the meaſure of the denſity of the air.

DENSITY *of the Planets*. In homogeneous, unequal, ſpherical bodies, the gravities on their ſurfaces, are as their diameters when the denſities are equal, or the gravities are as the denſities when the bulks are equal; therefore, in ſpheres of unequal magnitude and denſity, the gravity is in the compound ratio of the diameters and denſities, or the denſities are as the gravities divided by the diameters. Knowing therefore the diameters of the planets by obſervation and compariſon, and the gravities at their ſurface by means of the revolution of the ſatellites, the relation of their denſities becomes known. And as I have found the mean denſity of the earth to be about $4\frac{1}{2}$ times that of water, Philoſ. Tranſ. 1778; hence the denſities of the planets, with reſpect to water, become known, and are as below:

	Denſities.
Water - -	1
The Sun - -	$1\frac{2}{15}$
Mercury - -	$9\frac{1}{4}$
Venus - -	$5\frac{11}{15}$
The Earth - -	$4\frac{1}{2}$
Mars - -	$3\frac{2}{7}$
The Moon - -	$3\frac{1}{11}$
Jupiter - -	$1\frac{1}{24}$
Saturn - -	$0\frac{13}{32}$
Georgian Planet -	$0\frac{99}{100}$

As it is not likely that any of theſe bodies are homogeneal,

geneal, the denſities here determined are ſuppoſed to be the mean denſities, or ſuch as the bodies would have if they were homogeneal, and of the ſame maſs of matter and magnitude.

DENTICLES, or Dentils, are ornaments in a cornice, cut after the manner of teeth. Theſe are moſtly affected in the Ionic and Corinthian orders; and of late alſo in the Doric. The ſquare member on which they are cut, is called the Denticule.

DEPARTURE, in Navigation, is the eaſting or weſting of a ſhip, with regard to the meridian ſhe *departed* or ſailed from. Or, it is the difference in longitude, either eaſt or weſt, between the preſent meridian the ſhip is under, and that where the laſt reckoning or obſervation was made. This departure, any where but under the equator, muſt be accounted according to the number of miles in a degree proper to the parallel the ſhip is in.

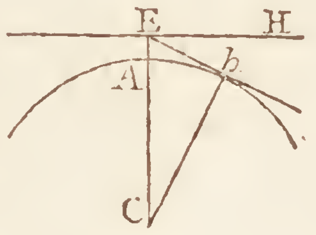
The Departure, in Plane and Mercator's Sailing, is always repreſented by the baſe of a right-angled plane triangle, where the courſe is the angle oppoſite to it, and the diſtance ſailed is the hypothenuſe, the perpendicular or other leg being the difference of latitude. And then the theorem for finding it, is always this: As radius is to the ſine of the courſe, ſo is the diſtance ſailed, to the departure ſought.

DEPRESSION *of the Pole.* So many degrees &c as you ſail or travel from the pole towards the equator, ſo many it is ſaid you depreſs the pole, becauſe it becomes ſo much lower, or nearer the horizon.

Depression *of a Star*, or *of the Sun*, is its diſtance below the horizon; and is meaſured by an arc of a vertical circle, intercepted between the horizon and the place of the ſtar.

Depression *of the Viſible Horizon*, or *Dip of the Horizon*, denotes its ſinking or dipping below the true horizontal plane, by the obſerver's eye being raiſed above the ſurface of the ſea; in conſequence of which, the obſerved altitude of an object is by ſo much too great.

Thus, the eye being at E, the height AE above the ſurface of the earth, whoſe centre is C; then EH is the real horizon, and E*h* the viſible one, below the former by the angle HE*h*, by reaſon of the elevation AE of the eye.



To compute the Depreſſion or Dip of the Horizon.

In the right-angled triangle CE*h*, are given C*h* the earth's radius = 21000000 feet, and the hypothenuſe CE = the radius increaſed by the height AE of the eye; to find the angle C which is = the angle HE*h*, or depreſſion ſought;

viz, as C*h* : CE : : radius : ſec. ∠ C,
or as CE : C*h* : : radius : coſin. ∠ C.

By either of theſe theorems are computed the numbers in the following table, which ſhews the depreſſion or dip of the horizon of the ſea for different heights of the eye, from 1 foot to 100 feet.

Height of the eye	Dip of the horizon	Height of the eye	Dip of the horizon	Height of the eye	Dip of the horizon
feet	′ ″	feet	′ ″	feet	′ ″
1	0 57	13	3 26	26	4 52
2	1 21	14	3 34	28	5 3
3	1 39	15	3 42	30	5 14
4	1 55	16	3 49	35	5 39
5	2 8	17	3 56	40	6 2
6	2 20	18	4 3	45	6 24
7	2 31	19	4 10	50	6 44
8	2 42	20	4 16	60	7 23
9	2 52	21	4 22	70	7 59
10	3 1	22	4 28	80	8 32
11	3 10	23	4 34	90	9 3
12	3 18	24	4 40	100	9 33

See Robertſon's Navigation, book 9 appendix; and Tables requiſite to be uſed with the Nautical Ephemeris, pa. 1. See alſo Levelling.

DEPTH, the oppoſite of Height, and one of the dimenſions of bodies, or of ſpace. See Height, Altitude, Elevation, &c.

DERHAM (Doctor William), an eminent Engliſh philoſopher and divine, was born at Stowton, near Worceſter, 1657, and educated in Trinity College, Oxford. In 1682, he was preſented to the vicarage of Wargrave in Berkſhire; and, in 1689, to the valuable rectory of Upminſter in Eſſex; which, lying at a convenient diſtance from London, afforded him an opportunity of converſing and correſponding with the principal literary geniuſes of the nation. Applying himſelf there with great eagerneſs to the purſuit of his ſtudies in natural and experimental philoſophy, he ſoon became a diſtinguiſhed and uſeful member of the Royal Society, whoſe Philoſophical Tranſactions contain a great variety of curious and valuable pieces, the fruits of his laudable induſtry, in all or moſt of the volumes, from the 20th to the 39th, both incluſive; the principal of which are:

1. Experiments on the Motion of Pendulums in vacuo.
2. A Deſcription of an inſtrument for finding the Meridian.
3. Experiments and Obſervations on the Motion of Sound.
4. On the Migration of Birds.
5. Hiſtory of the Spots in the Sun, from 1703 to 1711.
6. Obſervations on the Northern Lights, Oct. 8, 1726, and Oct. 13, 1728.
7. Tables of the Eclipſes of Jupiter's Satellites.
8. The difference of Time in the meridian of different places.
9. Of the meteor called Ignis Fatuus.
10. The Hiſtory of the Death Watch.
11. Meteorological Diaries for ſeveral years.

In his younger days he publiſhed his Artificial Clockmaker, a very uſeful little work, that has gone through ſeveral editions. In 1711, 1712, 1714, he preached thoſe ſermons at Boyle's lecture, which he afterward digeſted under the well-known titles of *Phyſico-Theology* and *Aſtro-Theology*, or Demonſtrations of the being and attributes of God, from his works of creation, and a ſurvey of the heavens.

In 1716 he was made a canon of Windſor, being at that time chaplain to the Prince of Wales; and in 1730 received, from the univerſity of Oxford, the degree of Doctor of Divinity. He reviſed the *Miſcellanea Curioſa*, in 3 vols 8vo, containing many curious papers of Dr. Halley and ſeveral other ingenious philoſophers. To him alſo the world is indebted for the publication of the Philoſophical Experiments of the late eminent Dr. Hooke, and other ingenious men of his time; as well as notes and illuſtrations of ſeveral other works.

Dr. Derham was very well ſkilled in medical as well as in phyſical knowledge; and was conſtantly a phyſician to the bodies as well as the ſouls of his pariſhioners. This great and good man, after ſpending his life in the moſt agreeable and improving ſtudy of nature, and the diligent and pious diſcharge of his duty, died at Upminſter in 1735, at 78 years of age.

DESAGULIERS (JOHN THEOPHILUS), an eminent experimental philoſopher, was the ſon of the Rev. John Deſaguliers, a French Proteſtant refugee, and born at Rochelle in 1683. His father brought him to England an infant; and having taught him the claſſics himſelf, he ſent him at a proper age to Chriſt-church College, Oxford; where in 1702 he ſucceeded Dr. Keil in reading lectures on experimental philoſophy at Hart Hall. In 1712 he married, and ſettled in London, when he firſt of any introduced the reading of lectures in experimental philoſophy in the metropolis, which he continued during the reſt of his life with the greateſt applauſe, having ſeveral times the honour of reading his lectures before the king and royal family. In 1714 he was elected F. R. S. and proved a very uſeful member, as appears from the great number of his papers that are printed in their Philoſ. Tranſ. on the ſubjects of optics, mechanics, and meteorology. The magnificent duke of Chandos made Dr. Deſaguliers his chaplain, and preſented him to the living of Edgware, near his ſeat at Cannons; and he became afterward chaplain to Frederick prince of Wales. In the latter part of his life, he removed to lodgings over the Great Piazza in Covent Garden, where he carried on his lectures with great ſucceſs till the time of his death in 1749, at 66 years of age.

He was a member of ſeveral foreign academies, and correſponding member of the Royal Academy of Sciences at Paris; from which academy he obtained the prize, propoſed by them for the beſt account of electricity. He communicated a multitude of curious and valuable papers to the Royal Society, for the year 1714 to 1743, or from vol. 29 to vol. 42.

Beſide thoſe numerous communications, he publiſhed a valuable *Courſe of Experimental Philoſophy*, 1734, in 2 large vols. 4to; and gave an edition of *Gregory's Elements of Catoptrics and Dioptrics*, with an Appendix on Reflecting Teleſcopes, 8vo, 1735. This appendix contains ſome Original Letters that paſſed between Sir Iſaac Newton and Mr. James Gregory, relating to thoſe teleſcopes.

DESCENDING, a going or moving from above, downwards.

There are aſcending and deſcending ſtars, and aſcending and deſcending degrees, &c.

DESCENDING *Latitude*, is the latitude of a planet in its return from the nodes to the equator.

DESCENSION, in Aſtronomy, is either *right*, or *oblique*.

Right DESCENSION is a point, or arch, of the equator, which deſcends with a ſtar, or ſign, below the horizon, in a *right* ſphere, and

Oblique DESCENSION is a point, or arch, of the equator, which deſcends at the ſame time with a ſtar, or ſign, below the horizon, in an oblique ſphere.

Deſcenſions, both right and oblique, are counted from the firſt point of Aries, or the vernal interſection, according to the order of the ſigns, i. e. from weſt to eaſt. And, as they are unequal, when it happens that they anſwer to equal arcs of the ecliptic, as for example to the 12 ſigns of the zodiac, it follows, that ſometimes a greater part of the equator riſes or deſcends with a ſign, in which caſe the ſign is ſaid to aſcend or deſcend rightly: and ſometimes again, a leſs part of the equator riſes or ſets with the ſame ſign, in which caſe it is ſaid to aſcend or deſcend obliquely. See ASCENSION.

Refraction of the DESCENSION. See REFRACTION.

DESCENSIONAL *Difference*, is the difference between the right and oblique deſcenſion of the ſame ſtar, or point of the heavens.

DESCENT, or *Fall*, in mechanics, &c, is the motion, or tendency, of a body towards the centre of the earth, either directly or obliquely.

The deſcent of bodies may be conſidered either as freely, like as in a vacuum, or as clogged or reſiſted by ſome external force, as an oppoſing body, or a fluid medium, &c.

1ſt, If the body b deſcend freely, and perpendicularly, by the force of gravity; then the motive force urging it downwards, is equal to its whole weight b; and the quantity of matter being b alſo, the accelerative force will be $\frac{b}{b}$ or 1.

2dly, If the body b deſcending, be oppoſed by ſome mechanical power, ſuppoſe a wedge or inclined plane, that is, inſtead of purſuing the perpendicular line of gravity, it is made to deſcend in a ſloping direction down the inclined plane: then if the ſine of the angle the plane makes with the horizon be s, to the radius 1, the motive force urging the body down the plane will be bs; and therefore the accelerative force is $\frac{bs}{b}$ or s; which is leſs than in the former caſe in the proportion of s to 1.

3dly, In a medium, a body ſuſpended loſes as much of its weight, as is the weight of a like bulk of the medium; and when deſcending, it loſes the ſame, beſide the obſtruction ariſing from the coheſion of the parts of the medium, and the oppoſing force of the particles ſtruck, which laſt produces a greater or leſs reſiſtance, according to the velocity of the motion. But, the weight of the body being b, and that of a like bulk of the fluid medium m, the motive force urging the body to deſcend, is only $b-m$; that is, the body only falls by the exceſs of its weight above that of an equal bulk of the medium.

Hence, the power that ſuſtains a body in a medium, is equal to the exceſs of the abſolute weight of the body above an equal bulk of the medium. Thus, a piece of copper weighing $47\frac{1}{3}$lb, loſes $5\frac{1}{3}$lb of its weight in water: and therefore a power of 42lb will ſuſtain it in the water.

4thly, If two bodies have the ſame ſpecific gravity, the leſs the bulk of the deſcending body is, the more of its gravity does it loſe, and the ſlower does it deſcend, in the ſame medium. For, though the proportion of the ſpecific gravity of the body to that of the fluid be ſtill the ſame, whether the bulk be greater or leſs, yet the ſmaller the body, the more the ſurface is, in proportion to the maſs; and the more the ſurface, the more the reſiſtance of the parts of the fluid, in proportion.

5thly, If the ſpecific gravities of two bodies be different; that which has the greateſt ſpecific gravity will deſcend with greater velocity in the air, or reſiſting medium, than the other body. Thus, a ball of lead deſcends ſwifter than wood or cork, becauſe it loſes leſs of its weight, though in a vacuum they both fall equally ſwift.

The cauſe of this deſcent, or tendency downwards, has been greatly controverted. Two oppoſite hypotheſes have been advanced; the one, that it proceeds from an internal principle, and the other from an external one: the firſt is maintained by the Peripatetics, Epicureans, and the Newtonians; and the latter by the Carteſians and Gaſſendiſts. See alſo ACCELERATION.

Laws of DESCENT *of Bodies.*

1ſt, Heavy bodies, in an unreſiſting medium, fall with an uniformly accelerated motion. For, it is the nature of all conſtant and uniform forces, ſuch as that of gravity at the ſame diſtance from the centre of the earth, to generate or produce equal additions of velocity in equal times. So that, if in one ſecond of time there be produced 1 degree of velocity, in 2 ſeconds there will be 2 degrees of velocity, in 3 ſeconds 3 degrees, and ſo on, the degree or quantity of velocity being always proportional to the length of the time.

2nd, The ſpace deſcended by an uniform gravity, in any time, is juſt the half of the ſpace that might be uniformly deſcribed in the ſame time by the laſt velocity acquired at the end of that time, if uniformly continued. For, as the velocity increaſes uniformly in an arithmetic progreſſion, the whole ſpace deſcended by the variable velocity, will be equal to the ſpace that would be deſcribed with the middle velocity uniformly continued for the ſame time; and this again will be only half the ſpace that would be deſcribed with the laſt velocity, alſo uniformly continued for the ſame time, becauſe the laſt velocity is double of the middle velocity, being produced in a double time.

3d, The ſpaces deſcended by an uniform gravity, in different times, are proportional to the ſquares of the times, or to the ſquares of the velocities. For the whole ſpace deſcended in any number of particles of time, conſiſts of the ſums of all the particular ſpaces, or velocities, which are in arithmetical progreſſion; but the ſum of ſuch an arithmetical progreſſion, beginning at 0, and having the laſt term and the number of terms the ſame quantity, is equal to half the ſquare of the laſt term, or of the number of terms; therefore the whole ſums are as the ſquares of the times, or of the velocities.

This theory of the deſcents by gravity was firſt diſcovered and taught by Galileo, who afterwards confirmed the ſame by experiments; which have often been repeated in various ways by many other perſons ſince his time, as Grimaldi, Riccioli, Huygens, Newton, and many others, all confirming the ſame laws.

The experiments of Grimaldi and Riccioli were made by dropping a number of balls, of half a pound weight, from the tops of ſeveral towers, and meaſuring the times of falling by a pendulum. Ricciol. Almag. Nov. tom. 1 lib. 2, cap. 21, prop. 4. An abſtract of their experiments is exhibited here below:

Vibrations of the pendulum	The time		Space at the end of the time	Space deſcended each time
	″	‴	Rom. feet	Rom. feet
5	0	50	10	10
10	1	40	40	30
15	2	30	90	50
20	3	20	160	70
25	4	10	250	90
6	1	0	15	15
12	2	0	60	45
18	3	0	135	75
24	4	0	240	105

The ſpace deſcended by a heavy body in any given time, being determined by experiment, is ſufficient, in connection with the preceding theorems, for determining every inquiry concerning the times, velocities, and ſpaces deſcended, depending on an uniform force of gravity. From many accurate experiments made in England, it has been found that a heavy body deſcends freely through 16 feet 1 inch, or $16\frac{1}{12}$ feet, in the firſt ſecond of time; and conſequently, by theorem 2, the velocity gained at the end of 1 ſecond, is $32\frac{1}{6}$ feet per ſecond. Hence, by the ſame, and theorem 3, the velocity gained in any other time t is $32\frac{1}{6}t$, and the ſpace deſcended is $16\frac{1}{12}t^2$. So that, if v denote the velocity, and s the ſpace due to the time t, and there be put $g = 16\frac{1}{12}$; then

$$\text{is } v = 2gt = 2\sqrt{gs} = \frac{2s}{t}.$$

$$s = gt^2 = \frac{v^2}{4g} = \tfrac{1}{2}tv.$$

$$t = \frac{v}{2g} = \sqrt{\frac{s}{g}} = \frac{2s}{v}.$$

The experiments with pendulums give alſo the ſame ſpace for the deſcent of a heavy body in a ſecond of time. Thus, in the latitude of London, it is found by experiment, that the length of a pendulum vibrating ſeconds is juſt $39\frac{1}{8}$ inches; and it being known that the circumference of a circle is to its diameter, as the time of one vibration of any pendulum, is to the time in which a heavy body will fall through half the length of the pendulum; therefore as $3{\cdot}1416 : 1 :: 1 : \frac{1}{3{\cdot}1416}$ which is the time of deſcending through $19\frac{9}{16}$ inches, or half the length of the pendulum; then, ſpaces being as the ſquares of the times, as $\frac{1}{3\cdot1416^2} : 1^2 :: 19\frac{9}{16} :$ 193 inches, or 16 feet 1 inch, which therefore is the ſpace a heavy body will deſcend through in one ſecond; the very ſame as before.

4th, For any other conſtant force, inſtead of the perpendicular free deſcent by gravity, find by experiment, or otherwiſe, the ſpace deſcended in one ſecond by that force, and ſubſtitute that inſtead of $16\frac{1}{12}$ for the value of g in theſe formulæ: or, if the proportion of the force to the force of gravity be known, let the value of g be altered in the ſame proportion, and the ſame formulæ will ſtill hold good. So, if the deſcent be on an inclined plane, making, for inſtance, an angle of 30° with the horizon; then, the force of deſcent upon the plane being always as the ſine of the angle it makes with the horizon, in the preſent caſe it will be as the ſine of 30°, that is, as $\frac{1}{2}$ the radius; therefore in this caſe the value of g will be but half the former, $8\frac{1}{24}$, in all the foregoing formulæ.

Or, if one body deſcending perpendicularly draw another after it, by means of a cord ſliding over a pulley; then it will be, as the ſum of the two bodies is to the deſcending body, ſo is $16\frac{1}{12}$ to the value of g in this caſe; which value of it being uſed in the ſaid formulæ, they will ſtill hold good. And in like manner for any other conſtant forces whatever.

5th, The time of the oblique deſcent down any chord of a circle, drawn either from the uppermoſt point or lowermoſt point of the circle, is equal to the perpendicular deſcent through the diameter of the circle.

6th. The deſcent, or vibration, through all arcs of the ſame cycloid are equal, whether great or ſmall.

7th. But the deſcent, or vibration, through unequal arcs of a circle, are unequal; the times being greater in the greater arcs, and leſs in the leſs.

8th, For Deſcents by Forces that are variable, ſee FORCES, &c. See alſo INCLINED PLANE, CYCLOID, PENDULUM, &c.

Line of Swifteſt DESCENT, is that which a body, falling by the action of gravity, deſcribes in the ſhorteſt time poſſible, from one given point to another. And this line, it is proved by philoſophers, is the arc of a cycloid, when the one point is not perpendicularly over the other. See CYCLOID.

DESCRIBENT, a term in Geometry, ſignifying a line or ſuperficies, by the motion of which a ſuperficies or ſolid is deſcribed.

DETENTS, in a clock, are thoſe ſtops which, by being lifted up or let down, lock and unlock the clock in ſtriking.

DETENT-*Wheel*, or HOOP-*Wheel*, that wheel in a clock which has a hoop almoſt round it, in which there is a vacancy, where the clock locks.

DETERMINATE *Number*. See NUMBER.

DETERMINATE *Problem*, is that which has but one ſolution, or a certain limited number of ſolutions; in contradiſtinction to an indeterminate problem, which admits of infinite ſolutions.

Such, for inſtance, is the problem, To form an iſoſceles triangle on a given line, ſo that each of the angles at the baſe ſhall be double of that at the vertex; which has only one ſolution: or this, To find an iſoſceles triangle whoſe area and perimeter are given; which admits of two ſolutions.

DETERMINATE *Section*, the name of a Tract, or General Problem, written by the ancient geometrician Apollonius. None of this work has come down to us, excepting ſome extracts and an account of it by Pappus, in the preface to the 7th book of his Mathematical Collections. He there ſays that the general problem was, "To cut an infinite right line in one point ſo, that, of the ſegments contained between the point of ſection ſought, and given points in the ſaid line, either the ſquare on one of them, or the rectangle contained by two of them, may have a given ratio, either to the rectangle contained by one of them and a given line, or to the rectangle contained by two of them."

Pappus farther informs us, that this Tract of Apollonius was divided into two books; that the firſt book contained 6 problems, and the ſecond 3; that the 6 problems of the firſt book contained 16 epitagmas, or caſes, reſpecting the diſpoſitions of the points; and the ſecond book 9. Farther, that of the epitagmas of the 6 problems of the firſt book, 4 were maxima, and one a minimum: that the maxima are at the 2d epitagma of the 2d problem, at the 3d of the 4th, the 3d of the 5th, and the 3d of the 6th; but that the minimum was at the 3d epitagma of the 3d problem. Alſo, that the ſecond book contained three determinations; of which the 3d epitagma of the 1ſt problem, and the 3d of the 2d were minima, and the 3d of the 3d a maximum. Moreover, that the firſt book had 27 lemmas, and the ſecond book 24; and laſtly, that both books contained 83 theorems.

From ſuch account of the contents of this Tract, and the lemmas alſo given by Pappus, ſeveral perſons have attempted to reſtore, or recompoſe what they thought might be nearly the form of Apollonius's tract, or the ſubject of each problem, caſe, determination, &c; among whom are, Snellius, an eminent Dutch mathematician of the laſt century; a tranſlation of whoſe work was publiſhed in Engliſh by Mr. John Lawſon, in 1772, together with a new reſtoration of the whole work by his friend Mr. William Wales.

DEW, a thin light inſenſible miſt, or rain, aſcending with a ſlow motion, and falling while the ſun is below the horizon.

To us it appears to differ from rain, as leſs from more. Its origin and matter are doubtleſs from the vapours and exhalations that riſe from the earth and water. See EXHALATION. Some define it a vapour liquefied, and let fall in drops. M. Huet, in one of his letters, ſhews that dew does not fall, but riſes; and others have adopted the ſame opinion.

M. du Fay made ſeveral experiments, firſt with glaſſes, then with pieces of cloth ſtretched horizontally at different heights; and he found that the lower bodies, with their under ſurfaces, were wetted before thoſe that were placed higher, or their upper ſurfaces. And Du Fay and Muſchenbroek both found, that different ſubſtances, and even different colours, receive the dew differently, and ſome little or not at all.

From the principles laid down under the article EVAPORATION, the ſeveral phenomena of dews are eaſily accounted for. Such as, for inſtance, that dews are more copious in the ſpring, than in the other ſeaſons of the year; there being then a greater ſtock of vapour in readineſs, than at other times, by reaſon of the ſmall expence of it in the winter's cold and froſt. Hence it is too, that Egypt, and ſome other hot countries, abound with dews throughout all the heats of ſummer; for the air there being too hot to conſtipate the vapours in the day-time, they never gather into clouds; and hence they have no rain: but in climates that are exceſſively hot, the nights are remarkably cold; ſo that the vapours raiſed after ſun-ſet, are readily condenſed into dews.

It is natural to conclude, from the different ſubſtances which are combined with dew, that it muſt be either ſalutary or injurious, both to plants and animals.

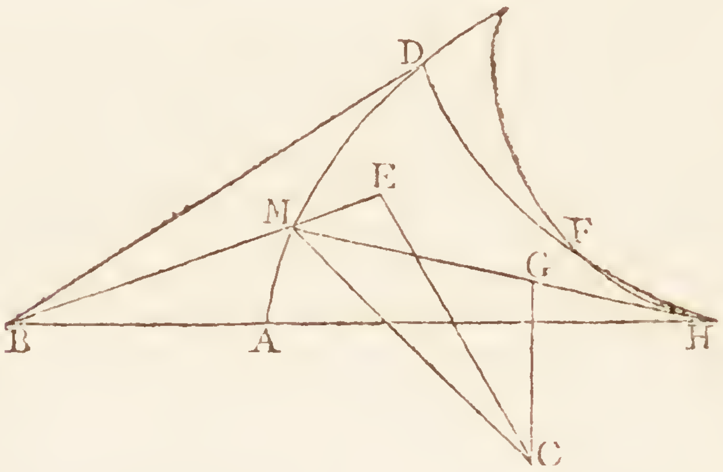
It is not eaſy to aſcertain the quantity of dew that riſes every night, or in the whole year, becauſe of the winds which diſperſe it, the rains which carry it down, and other inconveniences: but it is known that it riſes in greater abundance after rain than after dry weather, and in warm countries than in cold ones. There are ſome places in which dew is obſerved only to aſcend, and not to fall: and others again in which it is carried upwards in greater plenty than downwards, being diſperſed by the winds.

Dr. Hales made ſome experiments, to determine the quantity of dew that falls in the night. For this purpoſe, on the 15th of Auguſt, at 7 in the evening, he filled two glazed earthen pans with moiſt earth; the dimenſions of the pans being, 3 inches deep, and 12 inches diameter: and he obſerves, that the moiſter the earth, the more dew falls on it in a night; and that more than a double quantity of dew falls on a ſurface of water, than on an equal ſurface of moiſt earth. Theſe pans increaſed in weight by the night's dew, 180 grains; and decreaſed in weight by the evaporation of the day, 1 oz 282grs: ſo that 540 grains more are evaporated from the earth every 24 hours in ſummer, than the dew that falls in the night: i. e. in 21 days near 26 ounces from a circular area of a foot diameter. Now if 180 grains of dew, falling in one night on ſuch an area, which is equal to 113 ſquare inches, be equally ſpread on the ſurface, its depth will be the 159th part of an inch. He likewiſe found that the depth of dew in a winter's night was the 90th part of an inch. If therefore we allow 159 nights for the extent of the ſummer's dew, it will in that time amount to one inch in depth; and reckoning the remaining 206 nights for the extent of the winter's dew, it will produce 2·28 inches depth; and the dew of the whole year will amount to 3·28 inches depth. But the quantity which evaporated in a fair ſummer's day from the ſame ſurface, being 1 oz and 282 grs, gives the 40th part of an inch deep for evaporation, which is 4 times as much as fell at night. Dr. Hales obſerves that the evaporation of a winter's day is nearly the ſame as in a ſummer's day; the earth's greater moiſture in winter compenſating for the ſun's greater heat in ſummer. Hales's Vegetable Statics, vol. 1, pa. 52 of 4th edit. See EVAPORATION.

Signor Beccaria made ſeveral experiments to demonſtrate the exiſtence of the electricity that is produced by dew. He obſerves in general, that ſuch electricity took place in clear and dry weather, during which no ſtrong wind prevailed; and that it depends on the quantity of the dew, as the electricity of the rain depends on the quantity of the rain. He ſometimes found that it began before ſun-ſet; at other times not till 11 o'clock at night. *Artificial Electricity*, Appendix, letter 3.

DE WIT (JOHN), the famous Dutch penſionary, was born at Dort, in 1625; where he proſecuted his ſtudies ſo diligently, that at 23 years of age, he publiſhed *Elementa Curvarum Linearum*, one of the deepeſt books in mathematics at that time. After taking his degrees, and travelling, he, in 1650, became penſionary of Dort, and diſtinguiſhed himſelf very early in the management of public affairs, which ſoon after raiſed him to the rank of penſionary of Holland. After rendering the greateſt benefits to his country in many important inſtances, and ſerving it in ſeveral high capacities, with the greateſt ability, diligence, and integrity, by ſome intrigues of the court, it is ſaid, he and his brother were thrown into priſon, from whence they were dragged by the mob, and butchered with the moſt cruel and ſavage barbarity.

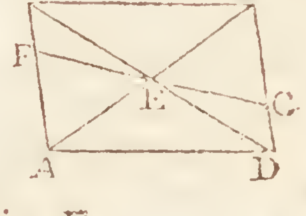
DIACAUSTIC *Curve*, or the *Cauſtic by Refraction*, is a ſpecies of cauſtic curves, the geneſis of which is in the following manner. Imagine an infinite number of rays BA, BM, BD, &c, iſſuing from the ſame luminous point B, refracted to or from the perpendicular MC, by the given curve AMD; and ſo, that CE the ſines of the angles of incidence CME, be always to CG the ſines of the refracted angles CMG, in a given ratio: then the curve HFN that touches all the refracted rays AH, MF, DN, &c, is called the **Diacauſtic**, or Cauſtic by Refraction.



DIACOUSTICS, or DIAPHONICS, the conſideration of the properties of ſound refracted in paſſing through different mediums; that is, out of a denſer into a more ſubtile, or out of a more ſubtile into a denſer medium. See SOUND.

DIADROME, a term ſometimes uſed for the vibration, motion, or ſwing of a pendulum.

DIAGONAL, is a right line drawn acroſs a figure, from one angle to another; and is ſometimes called a diameter. It is uſed chiefly in quadrilateral figures, viz, in parallelograms and trapeziums.



1. Every diagonal, as AC, divides a parallelogram into two equal parts or triangles ABC, ADC.

2. Two diagonals, AC, BD, drawn in a parallelogram, do mutually biſect each other; as in the point E.

3. Any line, as FG, drawn through the middle of the diagonal of a parallelogram, is biſected by it at the point E; and it divides the parallelogram into two equal parts, BFGC and AFGD.

4. The diagonal of a ſquare is incommenſurable with its ſide.

5. In any parallelogram, the ſum of the ſquares of the four ſides is equal to the ſum of the ſquares of the two diagonals.

6. In any trapezium, the ſum of the ſquares of the four ſides is equal to the ſum of the ſquares of the two diagonals together with 4 times the ſquare of the diſtance between the middle points of the diagonals.

7. In any trapezium, the ſum of the ſquares of the two diagonals is double the ſum of the ſquares of two lines biſecting the two pairs of oppoſite ſides.

8. In any quadrilateral inſcribed in a circle, the rectangle of the two diagonals is equal to the ſum of the two rectangles under the two pairs of oppoſite ſides.

DIAGONAL *Scale*. See SCALES.

DIAGRAM, is a ſcheme for the explanation or demonſtration of any figure, or of its properties.

DIAL, or SUN-DIAL, an inſtrument for meaſuring time by means of the ſun's ſhadow. Or, it is a draught or deſcription of certain lines on the ſurface of a body, ſo that the ſhadow of a ſtyle, or ray of the ſun through a hole, ſhould touch certain marks at certain hours.

Sun-Dials are doubtleſs of great antiquity. But the firſt upon record is, it ſeems, the dial of Ahaz, who began to reign 400 years before Alexander, and within 12 years of the Building of Rome: it is mentioned in Iſaiah, chap. 38, ver. 8.

Several of the antients are ſpoken of, as makers of dials; as Anaximenes Mileſius, Thales. Vitruvius mentions one made by the ancient Chaldee hiſtorian Beroſus, on a reclining plane, almoſt parallel to the equator. Ariſtarchus Samius invented the hemiſpherical dial. And there were at the ſame time ſome ſpherical ones, with a needle for a gnomon. The diſcus of Ariſtarchus was an horizontal dial, with its rim raiſed up all around, to prevent the ſhadow from ſtretching too far.

It was late before the Romans became acquainted with dials. The firſt ſun-dial at Rome was ſet up by Papyrius Curſor, about the 460th year of the city; before which time, Pliny ſays there is no mention of any account of time but by the ſun's riſing and ſetting: the firſt dial was ſet up near the temple of Quirinus; but being inaccurate, about 30 years after, another was brought out of Sicily by the conſul M. Valerius Meſſala, which he placed on a pillar near the Roſtrum; but neither did this ſhew time truly, becauſe not made for that latitude; and, after uſing it 99 years, Martius Philippus ſet up another more exact.

The diverſity of ſun-dials ariſes from the different ſituation of the planes, and from the different figure of the ſurfaces upon which they are deſcribed; whence they become denominated *equinoctial*, *horizontal*, *vertical*, *polar*, *direct*, *erect*, *declining*, *inclining*, *reclining*, *cylindrical*, &c. For the general principles of their conſtruction, ſee DIALLING.

Dials are ſometimes diſtinguiſhed into *primary* and *ſecondary*.

Primary DIALS are ſuch as are drawn either on the plane of the horizon, and thence called *horizontal* dials; or perpendicular to it, and called *vertical* dials; or elſe drawn on the polar and equinoctial planes, though neither horizontal nor vertical. And

Secondary DIALS are all thoſe that are drawn on the planes of other circles, beſide thoſe laſt mentioned; or thoſe which either decline, incline, recline, or deincline.

Each of theſe again is divided into ſeveral others, as follow:

Equinoctial DIAL, is that which is deſcribed on an equinoctial plane, or one parallel to it.

Horizontal DIAL, is deſcribed on an horizontal plane, or a plane parallel to the horizon.—This dial ſhews the hours from ſun-riſe to ſun-ſet.

South DIAL, or an *Erect, direct South Dial*, is that deſcribed on the ſurface of the prime vertical circle looking towards the ſouth.—This dial ſhews the time from 6 in the morning till 6 at night.

North DIAL, or an *Erect, direct North Dial*, is that which is deſcribed on the ſurface of the prime vertical looking northward. This dial only ſhews the hours before 6 in the morning, and after 6 in the evening.

Eaſt DIAL, or *Erect, direct Eaſt Dial*, is that drawn on the plane of the meridian, looking to the eaſt.—This can only ſhew the hours till 12 o'clock.

Weſt Dial, or *Erect, direct Weſt Dial*, is that deſcribed on the weſtern ſide of the meridian.—This can only ſhew the hours after noon. Conſequently this, and the laſt preceding one, will ſhew all the hours of the day between them.

Polar DIAL, is that which is deſcribed on a plane paſſing through the poles of the world, and the eaſt and weſt points of the horizon. It is of two kinds; the firſt looking up towards the zenith, and called the *upper*; the latter, down towards the nadir, called the *lower*. The polar dial therefore is inclined to the horizon in an angle equal to the elevation of the pole.—The upper polar dial ſhews the hours from 6 in the morning till 6 at night, and the lower one ſhews the hours before 6 in the morning, and after 6 in the evening, viz. from ſun-riſe and till ſun-ſet.

Declining DIALS, are erect or vertical dials which decline from any of the cardinal points; or they are ſuch as cut either the plane of the prime vertical, or of the horizon, at oblique angles.

Declining dials are of very frequent uſe; as the walls of houſes, on which dials are moſtly drawn, commonly deviate from the cardinal points.

Of declining dials there are ſeveral kinds, which are denominated from the cardinal points which they are neareſt to; as decliners from the ſouth, and from the north, and even from the zenith.

Inclined DIALS, are ſuch as are drawn on planes not erect, but inclining, or leaning forward towards the ſouth, or ſouthern ſide of the horizon, in an angle, either greater or leſs than the equinoctial plane.

Reclining DIALS, are thoſe drawn on planes not erect, but reclined, or leaning backwards from the zenith towards the north, in an angle greater or leſs than the polar plane.

Deinclined DIALS, are ſuch as both decline and incline, or recline.—Theſe laſt three ſorts of dials are very rare.

Dials *without Centres*, are thoſe whoſe hour lines converge ſo ſlowly, that the centre, or point of their concourſe, cannot be expreſſed on the given plane.

Quadrantal Dial. See *Horodictical* Quadrant.

Reflecting Dial. See Reflecting *Dial.*

Cylindrical Dial, is one drawn on the curve ſurface of a cylinder. This may firſt be drawn on a paper plane, and then paſted round a cylinder of wood, &c. It will ſhew the time of the day, the ſun's place in the ecliptic, and his altitude at any time of obſervation.

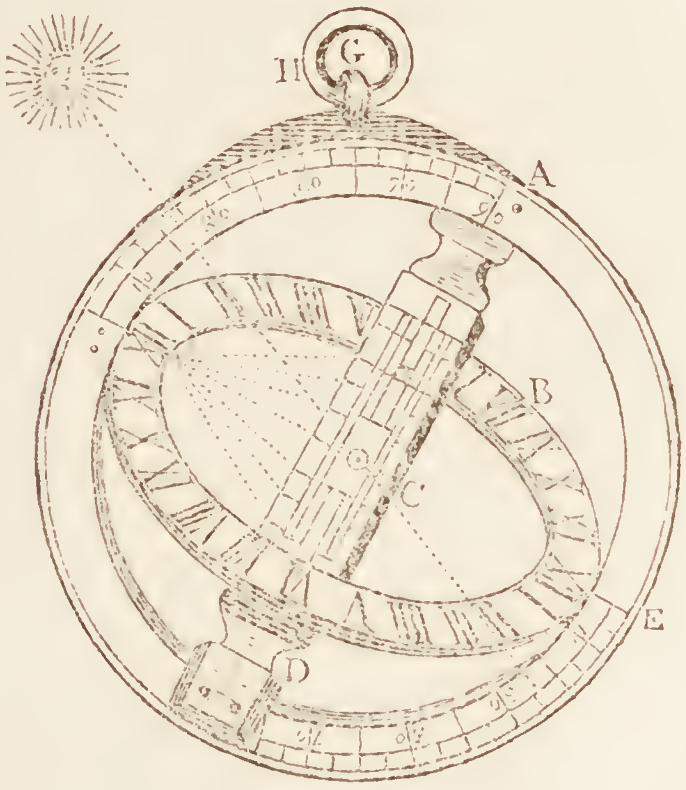
There are alſo *Portable* Dials, or *on a Card*, and *Univerſal* Dials *on a Plain Croſs*, &c.

Refracted Dials, are ſuch as ſhew the hour by means of ſome refracting tranſparent fluid.

Ring Dial, is a ſmall portable dial, conſiſting of a braſs ring or rim, about 2 inches in diameter, and one-third of an inch in breadth. In a point of this rim there is a hole, through which the ſun beams paſs, and form a bright ſpeck in the concavity of the oppoſite ſemi-circle, which gives the hour of the day in the diviſions marked within it.

When the hole is fixed, the dial only ſhews true about the time of the equinox. But to have it perform throughout the whole year, the hole is made moveable, the ſigns of the zodiac, or the days of the month, being marked on the convex ſide of the ring; hence, in uſing it, the moveable hole is ſet to the day of the month, or the degree of the zodiac the ſun is in; then ſuſpending the dial by the little ring, turn it towards the ſun, and his rays through the hole will ſhew the hour on the diviſions within ſide.

Univerſal, or *Aſtronomical Ring* Dial, is a ring dial which ſhews the hour of the day in any part of the earth; whereas the former is confined to a certain latitude. Its figure ſee repreſented below.



It conſiſts of two rings or flat circles, from 2 to 6 inches in diameter, and of a proportionable breadth &c. The outward ring A repreſents the meridian of any place you are at, and contains two diviſions of 90 degrees each, diametrically oppoſite to one another, the one ſerving from the equator to the north pole, the other to the ſouth pole. The inner ring repreſents the equator, and turns exactly within the outer, by means of two pivots in each ring at the hour of 12.

Acroſs the two circles goes a thin reglet or bridge, with a curſor C, ſliding along the middle of the bridge, and having a ſmall hole for the ſun to ſhine through. The middle of this bridge is conceived as the axis of the world, and the extremities as the poles: on the one ſide are drawn the ſigns of the zodiac, and on the other the days of the month. On the edge of the meridian ſlides a piece, to which is fitted a ſmall ring to ſuſpend the inſtrument by.

In this dial, the diviſions on the axis are the tangents of the angles of the ſun's declination, adapted to the ſemi-diameter of the equator as radius, and placed on either ſide of the centre: but inſtead of laying them down from a line of tangents, a ſcale of equal parts may be made, of which 1000 ſhall anſwer exactly to the length of the ſemi-axis, from the centre to the inſide of the equinoctial ring; and then 434 of theſe parts may be laid down from the centre towards each end, which will limit all diviſions on the axis, becauſe 434 is the natural tangent of 23° 28′. And thus, by a nonius fixed to the ſliding piece, and taking the ſun's declination from an ephemeris, and the tangent of that declination from the table of natural tangents, the ſlider might be always ſet true within 2 minutes of a degree. This ſcale of 434 equal parts might be placed right againſt the 23° 28′ of the ſun's declination, on the axis, inſtead of the ſun's place, which is there of little uſe. For then the ſlider might be ſet in the uſual way, to the day of the month, for common uſe; or to the natural tangent of the declination, when great accuracy is required.

To uſe this Dial: Place the line *a* (on the middle of the ſliding piece) over the degree of latitude of the place, as for inſtance 51½ degrees for London: put the line which croſſes the hole of the curſor to the degree of the ſign, or day of the month. Open the inſtrument ſo as that the two rings be at right angles to each other, and ſuſpend it by the ring H, that the axis of the dial, repreſented by the middle of the bridge, may be parallel to the axis of the world. Then turn the flat ſide of the bridge towards the ſun, ſo that his rays, ſtriking through the ſmall hole in the middle of the curſor, may fall exactly on a line drawn round the middle of the concave ſurface of the inner ring; in which caſe the bright ſpot ſhews the hour of the day in the ſaid concave ſurface of the ring.

Nocturnal or *Night*-Dial, is that which ſhews the hour of the night, by the light, or ſhadow projected from the moon or ſtars.

Lunar or Moon Dials may be either purpoſely deſcribed and adapted to the moon's motion; or the hour may be found on a ſun-dial by the moon ſhining upon it, thus: Obſerve the hour which the ſhadow of the index points at by moon light; find the days of the moon's age in the calendar, and take 3-4ths of that number, for the hours to be added to the hour ſhewn by the ſhadow, to give the hour of the night. The reaſon of which is, that the moon comes to the ſame horary circle later than the ſun by about three quarters of an hour every day; and at the time of new moon the ſolar and lunar hour coincide.

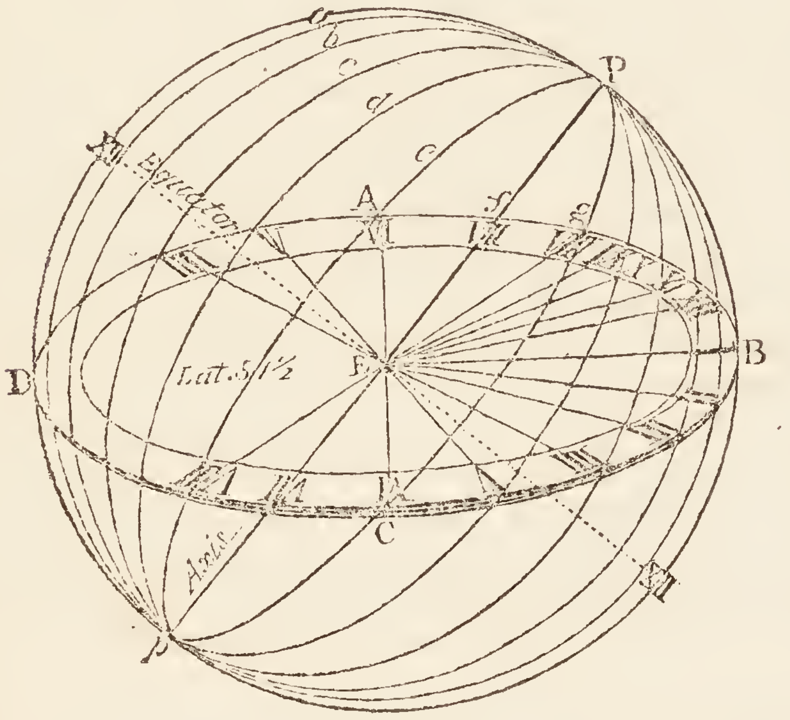
Dial *Planes*, are the plane ſuperficies upon which the hour lines of dials are drawn.

Tide Dial. See Tide *Dial.*

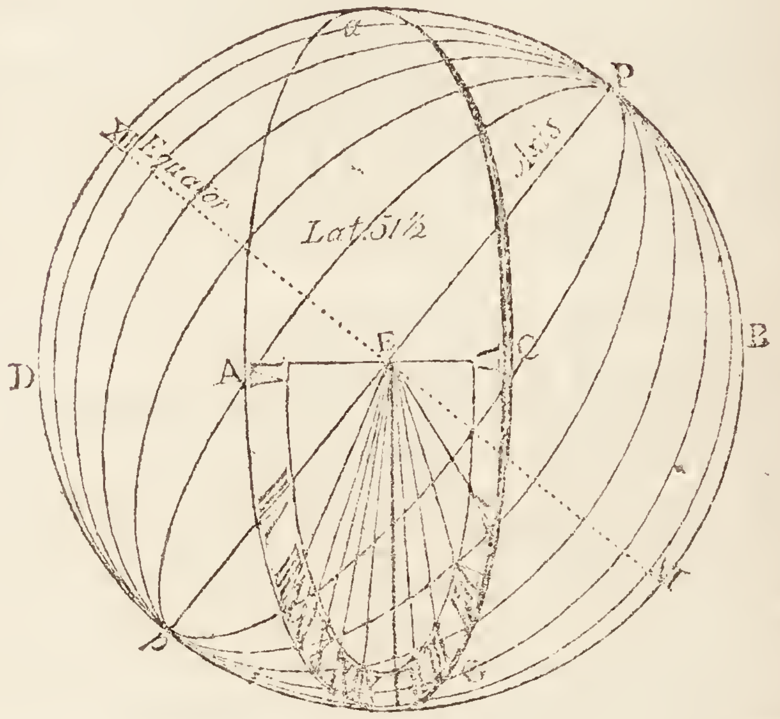
DIALLING, the art of drawing ſun, moon, and ſtar-dials on any ſort of ſurface, whether plane or curved.

Dialling is wholly founded on the firſt motion of the heavenly bodies, and chiefly the ſun; or rather on the diurnal rotation of the earth: ſo that the elements of ſpherics, and ſpherical trigonometry, ſhould be underſtood, before a perſon advances to the doctrine of dialling.

The principles of dialling may be aptly deduced from, and illuſtrated by, the phenomena of a hollow or tranſparent ſphere, as of glaſs. Thus, ſuppoſe *aPcp* to repreſent the earth as tranſparent; and its equator as divided into 24 equal parts by ſo many meridian ſemicircles *a*, *b*, *c*, *d*, *e*, &c, one of which is the geographical meridian of any given place, as London, which it is ſuppoſed is at the point *a*; and if the hour of 12 were marked at the equator, both upon that meridian and the oppoſite one, and all the reſt of the hours in order on the other meridians, thoſe meridians would be the hour circles of London: becauſe, as the ſun appears to move round the earth, which is in the centre of the viſible heavens, in 24 hours, he will paſs from one meridian to another in an hour. Then, if the ſphere had an opake axis, as PE*p*, terminating in the poles P and *p*, the ſhadow of the axis, which is in the ſame plane with the ſun and with each meridian, would fall upon every particular meridian and hour, when the ſun came to the plane of the oppoſite meridian, and would conſequently ſhew the time at London, and at all other places on the ſame meridian. If this ſphere were cut through the middle by a ſolid plane ABCD in the rational horizon of London, one half of the axis EP would be above the plane, and the other half below it; and if ſtraight lines were drawn from the centre of the plane to thoſe points where its circumference is cut by the hour circles of the ſphere, thoſe lines would be the hour lines of an horizontal dial for London; for the ſhadow of the axis would fall upon each particular hour line of the dial, when it fell upon the like hour circle of the ſphere.



If the plane which cuts the ſphere be upright, as AFCG, touching the given place, for ex. London, at F, and directly facing the meridian of London, it will then become the plane of an erect direct ſouth dial; and if right lines be drawn from its centre E, to thoſe points of its circumference where the hour circles of the ſphere cut it, theſe will be the hour lines of a vertical or direct ſouth dial for London, to which the hours are to be ſet in the figure, contrary to thoſe on an horizontal dial; and the lower half E*p* of the axis will caſt a ſhadow on the hour of the day in this dial, at the ſame time that it would fall upon the like hour circle of the ſphere, if the dial plane was not in the way.



If the plane, ſtill facing the meridian, be made to incline, or recline, any number of degrees, the hour circles of the ſphere will ſtill cut the edge of the plane in thoſe points to which the hour lines muſt be drawn ſtraight from the centre; and the axis of the ſphere will caſt a ſhadow on theſe lines at the reſpective hours. The like will ſtill hold, if the plane be made to decline by any number of degrees from the meridian towards the eaſt or weſt; provided the declination be leſs than 90 degrees, or the reclination be leſs than the co-latitude of the place; and the axis of the ſphere will be the gnomon: otherwiſe, the axis will have no elevation above the plane of the dial, and cannot be a gnomon.

Thus it appears that the plane of every dial repreſents the plane of ſome great circle on the earth, and the gnomon the earth's axis; the vertex of a right gnomon the centre of the earth or viſible heavens; and the plane of the dial is juſt as far from this centre as from the vertex of this ſtile. The earth itſelf, compared with its diſtance from the ſun, is conſidered as a point; and therefore, if a ſmall ſphere of glaſs be placed upon any part of the earth's ſurface, ſo that its axis be parallel to the axis of the earth, and the ſphere have ſuch lines upon it, and ſuch planes within it, as above deſcribed; it will ſhew the hours of the day as truly as if it were placed at the earth's centre, and the ſhell of the earth were as tranſparent as glaſs. Ferguſon, lect. 10.

The principal writers on Dials, and Dialling, are the following: Vitruvius, in his Architecture, cap. 4

and 7, lib. 9: Sebaſtian Munſter, his Horolographia: John Dryander de Horologiorum varia Compoſitione: Conrade Geſner's Pandectæ: Andrew Schoner's Gnomonicæ: Fred. Commandine de Horologiorum Deſcriptione: Joan. Bapt. Benedictus de Gnomonum Umbrarumque Solarium Uſu: Joannes Georgius Schomberg, Exegeſis Fundamentorum Gnomonicorum: Solomon de Caus, Traité des Horologes Solaires: Joan. Bapt. Trolta, Praxis Horologiorum: Deſargues, Maniere Univerſelle pour poſer l'Eſſieu & placer les Heures & autres Choſes aux Cadrans Solaires: Ath. Kircher, Ars magna Lucis & Umbræ: Hallum, Explicatio Horologii in Horto Regio Londini: Tractatus Horologiorum Joannis Mark: Clavius, Gnomonices de Horologiis; in which he demonſtrates both the theory and the operations after the rigid manner of the ancient mathematicians: Dechales, Ozanam, and Schottus, gave much eaſier treatiſes on this ſubject; as did alſo Wolfius in his Elementa: M. Picard gave a new method of making large dials, by calculating the hour lines; and M. De la Hire, in his Dialling, printed in 1683, gave a geometrical method of drawing hour lines from certain points, determined by obſervation. Everhard Walper, in 1625, publiſhed his Dialling, in which he lays down a method of drawing the primary dials on a very eaſy foundation; and the ſame foundation is alſo deſcribed at length by Sebaſtian Munſter, in his Rudimenta Mathematica, publiſhed in 1651. In 1672, Sturmius publiſhed a new edition of Walper's Dialling, with the addition of a whole ſecond part, concerning inclining and declining dials, &c. In 1708, the ſame work, with Sturmius's additions, was re-publiſhed, with the addition of a 4th part, containing Picard's and De la Hire's methods of drawing large dials, which makes much the beſt and fulleſt book on the ſubject. Peterſon, Michael, and Muller, have each written on Dialling, in the German language: Coetſius, in his Horologiographia Plana, printed in 1689: Gauppen, in his Gnomonica Mechanica: Leybourn, in his Dialling: Bion, in his Uſe of Mathematical Inſtruments: Wells, in his Art of Shadows. There is alſo a treatiſe by M. Deparceux, 1740. Mr. Ferguſon has alſo written on the ſame ſubject in his Lectures on Mechanics; beſides Emerſon, in his Dialling; and Mr. W. Jones, in his Inſtrumental Dialling.

Univerſal DIALLING *Cylinder*, is repreſented in fig. 2, plate vii, where ABCD is a glaſs cylindrical tube, cloſed at both ends with braſs plates, in the centres of which a wire or axis EFG is fixed. The tube is either fixed to an horizontal board H, ſo that its axis may make an angle with the board equal to that which the earth's axis makes with the horizon of any given place, and be parallel to the axis of the world; or it may be made to move on a joint, and elevated for any particular latitude. The twenty-four hour lines are drawn with a diamond on the outſide of the glaſs, equidiſtant from each other, and parallel to the axis. The XII next B ſtands for midnight, and the XII next the board H for noon. When the axis of this inſtrument is elevated according to the latitude, and the board ſet level, with the line HN in the plane of the meridian, and the end towards the north; the axis EFG will ſerve as a gnomon or ſtile, and caſt a ſhadow on the hour of the day among the parallel hour lines, when the ſun ſhines on the inſtrument. As the plate AD at the top is parallel to the equator, and the axis EFG perpendicular to it, right lines drawn from the centre to the extremities of the parallels will be the hour lines of an equinoctial dial, and the axis will be the ſtile. An horizontal plate *ef* put down into the tube, with lines drawn from the centre to the ſeveral parallels, cutting its edge, will be an horizontal dial for the given latitude; and a vertical plate *gc*, fronting the meridian, and touching the tube with its edge, with lines drawn from its centre to the parallels, will be a vertical ſouth dial: the axis of the inſtrument ſerving in both caſes for the ſtile of the dial: and if a plate be placed within the tube, ſo as to decline, incline, or recline, by any given number of degrees, and lines be drawn, as above, a declining, inclining, or reclining dial will be formed for the given latitude. If the axis with the ſeveral plates fixed to it be drawn out of the tube, and ſet up in ſunſhine in the ſame poſition as they were in the tube, AD will be an equinoctial dial, *ef* an horizontal dial, and *gc* a vertical ſouth dial; and the time of the day will be ſhewn by the axis EFG. If the cylinder were wood, inſtead of glaſs, and the parallel lines drawn upon it in the ſame manner, it would ſerve to facilitate the operation of making theſe ſeveral dials. The upper plate with lines drawn to the ſeveral interſections of the parallels, which appears obliquely in fig. 2, would be an equinoctial dial as in fig. 3, and the axis perpendicular to it be its ſtile. An horizontal dial for the latitude of the elevation of the axis might be made, by drawing out the axis and cutting the cylinder, as at *efgh*, parallel to the horizontal board H; the ſection would be elliptic as in fig. 4. A circle might be deſcribed on the centre, and lines drawn to the diviſions of the ellipſe would be the hour lines; and the wire put in its place again, as E, would be the ſtile. If this cylinder were cut by a plane perpendicular to the horizontal board H, or to the line SHN, beginning at *g*, the plane of the ſection would be elliptical as in fig. 5, and lines drawn to the points of interſection of the parallels on its edge would be the hour lines of a vertical direct ſouth dial, which might be made of any ſhape, either circular or ſquare, and F the axis of the cylinder would be its ſtile. Thus alſo inclining, declining, or reclining dials might be eaſily conſtructed, for any given latitude. Ferguſon, ubi ſupra.

DIALLING *Globe*, is an inſtrument made of braſs, or wood, with a plane fitted to the horizon, and an index; particularly contrived to draw all ſorts of dials, and to give a clear exhibition of the principles of that art.

DIALLING *Lines*, or *Scales*, are graduated lines, placed on rules or the edges of quadrants, and other inſtruments, to expedite the conſtruction of dials. The principal of theſe lines are, 1. A ſcale of ſix hours, which is only a double tangent, or two lines of tangents each of 45 degrees, joined together in the middle, and equal to the whole line of ſines, with the declination ſet againſt the meridian altitudes in the latitude of London, ſuppoſe, or any place for which it is made: the radius of which line of ſines is equal to the dialling ſcale of ſix hours. 2. A line of latitudes, which is fitted to the hour ſcale, and is made by this canon: as the radius is to the chord of 90 degrees; ſo are the tangents of each reſpective degree of the line of latitudes, to the tangents of other arches: and then the natural ſines of thoſe arches are the numbers, which, taken from a diagonal ſcale of equal parts, will graduate the diviſions of the line of latitude to any radius. The line of hours and latitudes is generally for pricking down all dials with centres. For the method of conſtructing theſe ſcales, ſee Scale.

Dialling *Sphere*, is an inſtrument made of braſs, with ſeveral ſemicircles ſliding over one another, on a moving horizon, to demonſtrate the nature of the doctrine of ſpherical triangles, and to give a true idea of the drawing of dials on all manner of planes.

DIAMETER, *of a circle*, is a right line paſſing through the centre, and terminated at the circumference on both ſides.

The diameter divides the circumference, and the area of the circle, into two equal parts. And half the diameter, or the ſemi-diameter, is called the radius.

For the proportion between the diameter and the circumference of a circle, ſee Circle and Circumference.

Diameter *of a Conic Section*, or *Tranſverſe Diameter*, is a right-line paſſing through the centre of the ſection, or the middle of the axis.—The diameter biſects all ordinates, or lines drawn parallel to the tangent at its vertex. See Conic *Sections*.

Conjugate Diameter, is a diameter, in Conic Sections, parallel to the ordinates of another diameter, called the tranſverſe; or parallel to the tangent at the vertex of this other.

Diameter, *of any Curve*, is a right line which divides two other parallel right lines, in ſuch manner that, in each of them, all the ſegments or ordinates on one ſide, between the diameter and different points of the curve, are equal to all thoſe on the other ſide. This is Newton's ſenſe of a Diameter.

But, according to ſome, a diameter is that line, whether right or curved, which biſects all the parallels drawn from one point to another of a curve. So that in this way every curve will have a diameter; and hence the curves of the 2d order, have, all of them, either a right-lined diameter, or elſe the curves of ſome one of the conic ſections for diameters. And many geometrical curves of the higher orders, may alſo have for diameters, curves of more inferior orders.

Diameter *of Gravity*, is a right line paſſing through the centre of gravity.

Diameter *in Aſtronomy*. The diameters of the heavenly bodies are either *apparent*, i. e. ſuch as they appear to the eye; or *real*, i. e. ſuch as they are in themſelves.

The apparent diameters are beſt meaſured with a micrometer, and are eſtimated by the meaſure of the angle they ſubtend at the eye. Theſe are different in different circumſtances and parts of the orbits, or according to the various diſtances of the luminary; being in the inverſe ratio of the diſtance.

The ſun's vertical diameter is found by taking the height of the upper and lower edge of his diſk, when he is in the meridian, or near it; correcting the altitude of each edge on account of refraction and parallax; then the difference between the true altitudes of the two, is the true apparent diameter ſought. Or the apparent diameter may be determined by obſerving, with a good clock, the time which the ſun's diſc takes in paſſing over the meridian: and here, when the ſun is in or near the equator, the following proportion may be uſed; viz, as the time between the ſun's leaving the meridian and returning to it again, is to 360 degrees, ſo is the time of the ſun's paſſing over the meridian, to the number of minutes and ſeconds of a degree contained in his apparent diameter: but when the ſun is in a parallel at ſome diſtance from the equator, his diameter meaſures a greater number of minutes and ſeconds in that parallel than it would do in a great circle, and takes up proportionally more time in paſſing over the meridian; in which caſe ſay, as radius is to the coſine of the ſun's declination, ſo is the time of the ſun's paſſing the meridian reduced to minutes and ſeconds of a degree, to the arc of a great circle which meaſures the ſun's apparent horizontal diameter. See Transit.

The ſun's apparent diameter may alſo be taken by the projection of his image in a dark room.

There are ſeveral ways of finding the apparent diameters of the planets: but the moſt certain method is that with the micrometer.

The following is a table of the apparent diameters of the ſun and planets, in different circumſtances, and as determined by different aſtronomers.

Table of Apparent Diameters.

1. *Of the Sun, according to*	Greateſt			Mean			Leaſt		
	′	″	‴	′	″	‴	′	″	‴
Ariſtarchus and Archimedes	30	0	0	30	0	0	30	0	0
Ptolomy	33	20	0	32	18	0	31	20	0
Albategnius	33	40	0	32	28	0	31	20	0
Regiomontanus	34	0	0	32	27	0	31	0	0
Copernicus	33	54	0	32	44	0	31	40	0
Tycho	32	0	0	31	0	0	30	0	0
Kepler	31	4	0	30	30	0	30	0	0
Riccioli	32	8	0	31	40	0	31	0	0
J. D. Caſſini	32	46	0	32	13	0	31	40	0
Gaſcoigne	32	50	0	-	-	-	31	40	0
Flamſteed	32	48	0	-	-	-	31	30	0
Mouton	32	32	0	-	-	-	30	29	0
De la Hire	32	44	0	32	11	0	31	38	0
Louville	32	37	7	32	4	36	31	32	50
M. Caſſini, jun.	32	37	30	32	5	0	31	32	30
Monnier	-	-	-	32	5	0	-	-	-
Short	32	33	0	-	-	-	31	28	0
2. *Of the Moon.*									
Ptolomy	35	30	0	-	-	-	31	20	0
Tycho, in Conjunc.	28	48	0	-	-	-	25	36	0
Tycho, in Oppoſit.	36	0	0	-	-	-	32	0	0
Kepler	32	44	0	-	-	-	30	0	0
De La Hire	33	30	0	-	-	-	29	30	0
Newton, in Syzygy	-	-	-	31	30	0	-	-	-
Newton, in Quadrat.	-	-	-	31	3	0	-	-	-
Mouton, Full, in Perigee	33	29	0	-	-	-	-	-	-
Monnier, in Syzygy	-	-	-	31	30	0	-	-	-
Monnier, in Quadrat	-	-	-	31	0	0	-	-	-

	Greateſt			Mean			Leaſt		
3. Of Mercury.	′	″	‴	′	″	‴	′	″	‴
Albategnius	-	-	-	2	5	10	-	-	-
Alfraganus	-	-	-	1	15	12	-	-	-
Tycho	3	57	0	2	10	0	1	29	0
Hevelius	0	11	0	0	6	0	0	4	0
Hortenſius	0	28	0	0	19	0	0	10	0
Riccioli	0	25	12	0	13	48	0	9	20
Bradley	0	10	45	-	-	-	-	-	-
Monnier	-	-	-	-	-	-	0	10	0
4. Of Venus.									
Albategnius	3	8	0	-	-	-	-	-	-
Alfraganus	1	34	0	-	-	-	-	-	-
Tycho	4	40	0	3	15	0	1	52	0
Hevelius	1	5	0	0	16	0	0	9	0
Hortenſius	1	40	0	0	53	0	0	15	20
Kepler	7	6	0	-	-	-	-	-	-
Riccioli	4	8	0	1	4	12	0	33	30
Huygens	-	-	-	-	-	-	1	25	0
Flamſteed	1	12	0	-	-	-	-	-	-
Horrox	1	18	30	-	-	-	-	-	-
Crabtree	1	9	0	-	-	-	-	-	-
Monnier	-	-	-	1	17	0	-	-	-
Tranſit of 1761	-	-	-	0	58	0	-	-	-
Tranſit of 1769	-	-	-	0	59	0	-	-	-
5. Of Mars.									
Albateg. and Alfrag.	-	-	-	1	34	0	-	-	-
Tycho	2	46	0	1	40	0	0	57	0
Hevelius	0	20	0	0	5	0	0	2	0
Hortenſius	1	4	0	0	36	0	0	9	0
Kepler	6	30	0	-	-	-	-	-	-
Riccioli	1	32	0	0	22	0	0	10	6
Huygens	-	-	-	-	-	-	0	30	0
Flamſteed	0	33	0	-	-	-	-	-	-
Monnier	-	-	-	-	-	-	0	26	0
Herſchel, Polar Diam.	-	-	-	-	-	-	0	21	29
Herſchel, Equat. Diam.	-	-	-	-	-	-	0	22	25
6. Of Jupiter.									
Albateg. and Alfrag.	-	-	-	2	36	40	-	-	-
Tycho	3	59	0	2	45	0	2	14	0
Hevelius	0	24	0	0	18	0	0	14	0
Hortenſius	1	1	40	0	50	0	0	38	30
Kepler	0	50	0	-	-	-	-	-	-
Riccioli	1	8	46	0	49	46	0	38	18
Huygens	-	-	-	-	-	-	1	4	0
Flamſteed	0	54	0	-	-	-	-	-	-
Newton, from Pound's Obſ.	-	-	-	0	37	15	-	-	-
Monnier	-	-	-	0	37	0	-	-	-
7. Of Saturn.									
Albateg and Alfrag.	-	-	-	1	44	28	-	-	-
Tycho	2	12	0	1	50	0	1	34	0
Hevelius	0	19	0	0	16	0	0	14	0
Hortenſius	0	42	40	0	37	0	0	31	0
Kepler	0	30	0	-	-	-	-	-	-
Of Saturn	′	″		′	″	‴	′	″	‴
Riccioli	1	12	0	0	57	0	0	46	0
Huygens	-	-	-	-	-	-	0	30	0
Flamſteed	0	25	0	-	-	-	-	-	-
Newton, from Pound's Obſerv.	-	-	-	0	16	0	-	-	-
Monnier	-	-	-	0	16	0	-	-	-
Huygens, ♄'s Ring	-	-	-	1	4	0	1	8	0
Newton, from Pound's Obſ.	-	-	-	0	40	0	-	-	-
Monnier	-	-	-	0	42	0	-	-	-
8. *New Planet.*									
Herſchel	-	-	-	0	3	54	-	-	-

The Mean Apparent diameters of the planets, as ſeen from the ſun, are as follow:

Mercury,	Venus,	Earth,	Moon,	Mars,	Jup.	Sat.	Herſch.
20″	30″	17″	6″	11″	37″	16″	4″

For the true diameters of the ſun and planets, and their proportions to each other, ſee Planets, Semidiameter, and Solar System.

Diameter *of a Column*, is its thickneſs juſt above the baſe. From this the module is taken, which meaſures all the other parts of the column.

Diameter *of the Diminution*, is that taken at the top of the ſhaft.

Diameter *of the Swelling*, is that taken at the height of one third from the baſe.

DIAPASON, a muſical interval, otherwiſe called an octave, or eighth: ſo called becauſe it contains all the poſſible diverſities of ſound.

If the lengths of two ſtrings be to each other as 1 to 2, the tenſions being equal, their tones will produce an octave.

DIAPENTE, is the perfect fifth or ſecond of the concords, making an octave with the Diateſſaron. The lengths of the chords are as 3 to 2.

DIAPHONOUS Body or Medium, one that is tranſlucent, or through which the rays of light eaſily paſs; as water, air, glaſs, talc, fine porcelain, &c. See Transparency.

DIAPHONICS, is ſometimes uſed for the ſcience of refracted ſound, as it paſſes through different mediums.

DIASTYLE, a ſort of edifice, in which the pillars ſtand at ſuch a diſtance from one another, that three diameters of their thickneſs are allowed for the intercolumnation.

DIATESSARON, is the perfect fourth; or a muſical interval, conſiſting of one greater tone, one leſſer, and one greater ſemitone. The lengths of ſtrings to ſound the diateſſaron, are as 3 to 4.

DIATONIC, a term ſignifying the ordinary ſort of muſic, which proceeds by tones or degrees, both aſcending and deſcending. It contains or admits only the greater and leſſer tone, and the greater ſemitone.

DIESIS, a diviſion of a tone, leſs than a ſemitone; or an interval conſiſting of a leſſer, or imperfect ſemitone. The Dieſis is the ſmalleſt and ſofteſt change, or inflexion, of the voice imaginable. It is alſo called a *feint*, and is expreſſed by a St. Andrew's Croſs, or ſaltier,

DIFFERENCE, is the exceſs by which one magnitude or quantity exceeds another. When a leſs quantity is ſubtracted from a greater, the remainder is otherwiſe called difference.

Aſcenſional DIFFERENCE. See ASCENSIONAL.

DIFFERENCE *of Longitude*, of two places, is an arch of the equator contained between the meridians of thoſe two places, or the meaſure of the angle formed by their meridians.

DIFFERENTIAL, an indefinitely ſmall quantity, part, or difference. By ſome, the Differential is conſidered as infinitely ſmall, or leſs than any aſſignable quantity; and alſo as of the ſame import as fluxion.

It is called a Differential, or Differential Quantity, becauſe often conſidered as the difference between two quantities; and as ſuch it is the foundation of the Differential Calculus. Newton uſed the term *moment* in a like ſenſe, as being the momentary increaſe or decreaſe of a variable quantity. M. Leibnitz and others call it alſo an infiniteſimal.

DIFFERENTIAL *of the 1ſt, 2d, 3d, &c degree*. See *Differentio*-DIFFERENTIAL.

DIFFERENTIAL *Calculus*, or *Method*, is a method of differencing quantities. See DIFFERENTIAL *Method*, CALCULUS, and FLUXIONS.

DIFFERENTIO-DIFFERENTIAL *Calculus*, is a method of differencing differential quantities.

As the ſign of a differential is the letter *d* prefixed to the quantity, as dx the differential of x; ſo that of a differential of dx is ddx, and the differential of ddx is $dddx$, &c; ſimilar to the fluxions $\dot{x}$, $\ddot{x}$, $\dddot{x}$, &c.

Thus we have degrees of differentials. The differential of an ordinary quantity, is a differential of the firſt order or degree, as dx; that of the 2d degree is ddx; that of the 3d degree, $dddx$, &c. The rules for differentials, are the very ſame as thoſe for fluxions. See FLUXIONS.

DIFFERENTIAL, in Logarithms. Kepler calls the logarithms of tangents, *differentiales*; which we uſually call *artificial tangents*.

DIFFERENTIAL *Equation*, is an equation involving or containing differential quantities; as the equation $3x^2dx - 2axdx + aydx + axdy = 0$. Some mathematicians, as Stirling, &c, have alſo applied the term differential equation in another ſenſe, to certain equations defining the nature of ſeries.

DIFFERENTIAL *Method*, a method of finding quantities by means of their ſucceſſive differences.

This method is of very general uſe and application, but eſpecially in the conſtruction of tables, and the ſummation of ſeries, &c. This method was firſt uſed, and the rules of it laid down, by Briggs, in his Conſtruction of Logarithms and other Numbers, much the ſame as they were afterwards taught by Cotes, in his Conſtructio Tabularum per Differentias; as I have ſhewn in the Introduction to my Logs. pa. 69 & ſeq. See Briggs's Arithmetica Logarithmica, cap. 12 and 13, and his Trigonometria Britannica.

The method was next treated in another form by Newton in the 5th Lemma of the 3d book of his Principia, and in his Methodus Differentialis, publiſhed by Jones in 1711, with the other tracts of Newton. This author here treats it as a method of deſcribing a curve of the parabolic kind, through any given number of points. He diſtinguiſhes two caſes of this problem; the firſt, when the ordinates drawn from the given points to any line given in poſition, are at equal diſtances from one another; and the 2d, when theſe ordinates are not at equal diſtances. He has given a ſolution of both caſes, at firſt without demonſtration, which was afterwards ſupplied by himſelf and others: ſee his Methodus Differentialis above mentioned; and Stirling's Explanation of the Newtonian Differential Method, in the Philoſ. Tranſ. Nº 362; Cotes, De Methodo Differentiali Newtoniana, publiſhed with his Harmonia Menſurarum; Herman's Phoronomia; and Le Seur & Jacquier, in their Commentary on Newton's Principia. It may be obſerved, that the methods there demonſtrated by ſome of theſe authors extend to the deſcription of any algebraic curve through a given number of points, which Newton, writing to Leibnitz, mentions as a problem of the greateſt uſe.

By this method, ſome terms of a ſeries being given, and conceived as placed at given intervals, any intermediate term may be found nearly; which therefore gives a method for interpolations. Briggs's Arith. Log. ubi ſupra; Newton Meth. Differ. prop. 5; Stirling, Methodus Differentialis.

Thus alſo may any curvilinear figure be ſquared nearly, having ſome few of its ordinates. Newton, ibid. prop 6; Cotes De Method. Differ.; Simpſon's Mathematical Differt. pa. 115. And thus may mathematical tables be conſtructed by interpolation: Briggs, ibid. Cotes Canonotechnia.

The ſucceſſive differences of the ordinates of parabolic curves, becoming ultimately equal, and the intermediate ordinate required, being determined by theſe differences of the ordinates, is the reaſon for the name Differential Method.

To be a little more particular.—The firſt caſe of Newton's problem amounts to this: A ſeries of numbers, placed at equal intervals, being given, to find any intermediate number of that ſeries, when its interval or diſtance from the firſt term of the ſeries is given.—— Subtract each term of the ſeries from the next following term, and call the remainders firſt differences; then ſubtract in like manner each of theſe differences from the next following one, calling theſe remainders 2d differences; again, ſubtract each 2d difference from the next following, for the 3d differences; and ſo on: then if A be the 1ſt term of the ſeries,

d' the firſt of the 1ſt differences,
d'' the firſt of the 2d differences,
d''' the firſt of the 3d differences, &c;

and if x be the interval or diſtance between the firſt term of the ſeries and any term ſought, T, that is, let the number of terms from A to T, both included, be $= x + 1$; then will the term ſought, T, be =

$$A + \frac{x}{1}d' + \frac{x}{1} \cdot \frac{x-1}{2}d'' + \frac{x}{1} \cdot \frac{x-1}{2} \cdot \frac{x-2}{3}d''' \text{ \&c.}$$

Hence, if the differences of any order become equal, that is, if any of the diffs. d'', d''', &c, become $= 0$, the above ſeries will give a finite expreſſion for T the term ſought; it being evident, that the ſeries muſt terminate when any of the diffs. d'', d''', &c, become $= 0$.

It is alſo evident that the co-efficients $\frac{x}{1}, \frac{x}{1}\cdot\frac{x-1}{2}$, &c, of the differences, are the ſame as to the terms of the binomial theorem.

For ex. Suppoſe it were required to find the log. tangent of $5' 1'' 12''' 24''''$, or $5' 1''\frac{62}{300}$, or $5' 1''\cdot 2066$ &c.

Take out the log. tangents to ſeveral minutes and ſeconds, and take their firſt and ſecond differences, as below:

	Tang.	d′	d″	
5′ 0″ - - -	7·1626964			
		14453		
5 1 - - -	7·1641417		—49	
		14404		—48
5 2 - - -	7·1655821		—47	
		14357		
5 3 - - -	7·1670178			

Here $A = 7\cdot 1641417$; $x = \frac{62}{300}$; $d' = 14404$; and the mean 2d difference $d'' = -48$. Hence

A - - - - - -	7·1641417
xd' - - - - -	2977
$\frac{x}{1}\cdot\frac{x-1}{1}d''$ -	4
Theref. the tang. of $5' 1'' 12''' 24''''$ is	7·1644398

Hence may be deduced a method of finding the ſums of the terms of ſuch a ſeries, calling its terms A, B, C, D, &c. For, conceive a new ſeries having its 1ſt term = 0, its 2d = A, its 3d = A + B, its 4th = A + B + C, its 5th = A + B + C + D, and ſo on; then it is plain that aſſigning one term of this ſeries, is finding the ſum of all the terms A, B, C, D, &c. Now ſince theſe terms are the differences of the ſums, 0, A, A + B, A + B + C, &c; and as ſome of the differences of A, B, C, &c, are = 0 by ſuppoſition; it follows that ſome of the differences of the ſums will be = 0; and ſince in the ſeries $A + \frac{x}{1}d' + \frac{x}{1}\cdot\frac{x-1}{2}d''$ &c, by which a term was aſſigned, A repreſented the 1ſt term; d' the 1ſt of the 1ſt differences, and x the interval between the firſt term and the laſt; we are to write 0 inſtead of A, A inſtead of d', d' inſtead of d'', d'' inſtead of d''', &c, alſo $x + 1$ inſtead of x; which being done, the ſeries expreſſing the ſums will be

$$0 + \frac{x+1}{1}A + \frac{x+1}{1}\cdot\frac{x}{2}d' + \frac{x+1}{1}\cdot\frac{x}{2}\cdot\frac{x-1}{3}d'', \text{ \&c.}$$

Or, if the real number of terms of the lines be called z, that is, if $z = x + 1$, or $x = z - 1$, the ſum of the ſeries will be $Az + \frac{z}{1}\cdot\frac{z-1}{2}d' + \frac{z}{1}\cdot\frac{z-1}{2}\cdot\frac{z-2}{3}d''$ &c. See De Moivre's Doct. of Chances, pa. 59, 60; or his Miſcel. Analyt. pa. 153; or Simpſon's Eſſays, pa. 95.

For ex. to find the ſum of ſix terms of the ſeries of ſquares $1 + 4 + 9 + 16 + 25 + 36$, of the natural numbers.

Terms	d′	d″	d‴
1			
	3		
4		2	
	5		0
9		2	
	7		0
16		2	
	9		
25			

Here $A = 1$, $d' = 3$, $d'' = 2$, d''' &c $= 0$, and $z = 6$; therefore the ſum is $6 + \frac{6}{1}\cdot\frac{5}{2}\cdot 3 + \frac{6}{1}\cdot\frac{5}{2}\cdot\frac{4}{3}\cdot 2 = 6 + 45 + 40 = 91$ the ſum required, viz. of $1 + 4 + 9 + 16 + 25 + 36$.

A variety of examples may be ſeen in the places above cited, or in Stirling's Methodus Differentialis, &c.

As to the Differential method, it may be obſerved, that though Newton and ſome others have treated it as a method of deſcribing an algebraic curve, at leaſt of the parabolic kind, through any number of given points; yet the conſideration of curves is not at all eſſential to it, though it may help the imagination. The deſcription of a parabolic curve through given points, is the ſame problem as the finding of quantities from their given differences, which may always be done by Algebra, by the reſolution of ſimple equations. See Stirling's Method. Differ. pa. 97. This ingenious author has treated very fully of the differential method, and ſhewn its uſe in the ſolution of ſome very difficult problems. See alſo SERIES.

DIFFERENTIAL *Scale*, in Algebra, is uſed for the ſcale of relation ſubtracted from unity. See *Recurring* SERIES.

DIFFRACTION, a term firſt uſed by Grimaldi, to denote that property of the rays of light, which others have called Inflection; the diſcovery of which is attributed by ſome to Grimaldi, and by others to Dr. Hook.

DIGBY (*Sir* KENELM), a famous Engliſh philoſopher, was born at Gothurſt in Buckinghamſhire, 1603. He was deſcended of an ancient family: his great grandfather, with ſix of his brothers, fought valiantly at Boſworth-field on the ſide of Henry the 7th, againſt Richard the 3d. His father, Everard, engaged in the gunpowder plot againſt James the 1ſt, for which he was beheaded. His ſon, however, was reſtored to his eſtate; and had afterwards ſeveral appointments under king Charles the 1ſt. He granted him letters of repriſal againſt the Venetians, from whom he took ſeveral prizes with a ſmall fleet which he commanded. He fought the Venetians near the port of Scanderoon, and bravely made his way through them with his booty.

In the beginning of the civil wars, he exerted himſelf greatly in the king's cauſe. He was afterwards impriſoned, by order of the parliament; but was ſet at liberty in 1643. He afterward compounded for his eſtate; but being baniſhed from England, he retired to France, and was ſent on two embaſſies to Pope Innocent the 10th, from the queen, widow to Charles the 1ſt, whoſe chancellor he then was. On the reſtoration of Charles the 2d, he returned to London; where he died in 1665, at 62 years of age.

Digby was a great lover of learning, and tranſlated ſeveral authors into Engliſh, as well as publiſhed ſeveral works of his own; as, 1. *Obſervations upon Dr. Brown's Religio Medici*, 1643.—2. *Obſervations on part of Spenſer's Fairy Queen*, 1644.—3. *A Treatiſe of the Nature of Bodies*, 1644.—4. *A Treatiſe declaring the Operations and Nature of Man's Soul, out of which the Immortality of reaſonable ſouls is evinced:* works that diſcover great penetration and extenſive knowledge.

He applied much to chemiſtry; and found out ſeveral uſeful medicines, which he diſtributed with a liberal hand. He particularly diſtinguiſhed himſelf by his ſympathetic powder for the cure of wounds at a diſtance; his diſcourſe concerning which made great noiſe for a while. He held ſeveral conferences with Des Cartes, about the nature of the ſoul, and the principles of things. At the beginning of the Royal Society, he became a diſtinguiſhed member, being one of the firſt council. And he had at his own houſe regular levees or meetings of learned men, to improve themſelves in knowledge, by converſing with one another.

This eminent perſon was, for the early pregnancy of his talents, and his great proficiency in learning, compared to the celebrated Picus de Mirandola, who was one of the wonders of human nature. Yet his knowledge, though various and extenſive, probably appeared greater than it really was; as he had all the powers of elocution and addreſs to recommend it. He knew how to ſhine in a circle, either of ladies or philoſophers; and was as much attended to when he ſpoke on the moſt trivial ſubjects, as when he ſpoke on the moſt important. It has been ſaid that one of the princes of Italy, who had no child, was deſirous that his princeſs ſhould bring him a ſon by Sir Kenelm, whom he eſteemed a juſt model of perfection.

DIGGES (Leonard), a conſiderable mathematician in the 16th century, was deſcended from an ancient family, and born at Digges-court in the pariſh of Barham in Kent; but in what year is not known; and died about the year 1574. He was educated for ſome time at Oxford, where he laid a good foundation of learning. Retiring from thence, he proſecuted his ſtudies, and became an excellent mathematician, a ſkilful architect, and an expert ſurveyor of land, &c. He compoſed ſeveral books: as, 1. *Tectonicum: briefly ſhewing the exact Meaſuring, and ſpeedy Reckoning of all manner of Lands, Squares, Timber, Stones, Steeples, &c*; 1556, 4to. Augmented and publiſhed again by his ſon Thomas Digges, in 1592; and alſo reprinted in 1647. —2. *A Geometrical Practical Treatiſe, named Pantometria, in three books.* This he left in manuſcript; but after his death, his ſon ſupplied ſuch parts of it as were obſcure and imperfect, and publiſhed it in 1591, folio; ſubjoining, "A Diſcourſe Geometrical of the five regular and Platonic bodies, containing ſundry theoretical and practical propoſitions, ariſing by mutual conference of theſe ſolids, Inſcription, Circumſcription, and Tranſformation."—3. *Prognoſtication Everlaſting of right good effect: or Choice Rules to judge the Weather by the Sun, Moon, and Stars, &c*; in 4to. 1555, 1556, and 1564: corrected and augmented by his ſon, with divers general tables, and many compendious rules, in 4to, 1592.

DIGGES (Thomas), only ſon of Leonard Digges, after a liberal education from his tendereſt years, went and ſtudied for ſome time at Oxford; and by the improvements he made there, and the ſubſequent inſtructions of his learned father, became one of the beſt mathematicians of his age. When queen Elizabeth ſent ſome forces to aſſiſt the oppreſſed inhabitants of the Netherlands, Mr. Digges was appointed muſter-maſter general of them; by which he became well ſkilled in military affairs; as his writings afterward ſhewed. He died in 1595.

Mr. Digges, beſide reviſing, correcting, and enlarging ſome pieces of his father's, already mentioned, wrote and publiſhed the following learned works himſelf: viz, 1. *Alæ ſive Scalæ Mathematicæ: or Mathematical Wings or Ladders*, 1573, 4to. A book which contains ſeveral demonſtrations for finding the parallaxes of any comet, or other celeſtial body, with a correction of the errors in the uſe of the radius aſtronomicus.—2. *An Arithmetical Military Treatiſe, containing ſo much of Arithmetic as is neceſſary towards military diſcipline*, 1579, 4to.—3. *A Geometrical Treatiſe, named Stratioticos, requiſite for the perfection of Soldiers*, 1579, 4to. This was begun by his father, but finiſhed by himſelf. They were both reprinted together in 1590, with ſeveral additions and amendments, under this title: "An Arithmetical Warlike Treatiſe, named Stratioticos, compendiouſly teaching the ſcience of Numbers, as well in Fractions as Integers, and ſo much of the Rules and Equations Algebraical, and art of Numbers Coſſical, as are requiſite for the profeſſion of a ſouldier. Together with the Moderne militaire diſcipline, offices, lawes, and orders in every well-governed campe and armie, inviolably to be obſerved." At the end of this work there are two pieces; the firſt, "A briefe and true report of the proceedings of the Earle of Leyceſter, for the reliefe of the towne of Sluce, from his arrival at Vliſhing, about the end of June 1587, untill the ſurrendrie thereof 26 Julii next enſuing. Whereby it ſhall plainelie appear, his excellencie was not in anie fault for the loſſe of that towne:" the ſecond, "A briefe diſcourſe what orders were beſt for repulſing of foraine forces, if at any time they ſhould invade us by ſea in Kent, or elſewhere."—4. *A perfect Deſcription of the Celeſtial Orbs, according to the moſt ancient doctrine of the Pythagoreans, &c.* This was placed at the end of his father's "Prognoſtication Everlaſting, &c." printed in 1592, 4to.—5. *A humble Motive for Aſſociation to maintain the religion eſtabliſhed*, 1601, 8vo. To which is added, *his Letter to the ſame purpoſe to the archbiſhops and biſhops of England.*—6. *England's Defence: or, A Treatiſe concerning Invaſion.* This is a tract of the ſame nature with that printed at the end of his Stratioticos, and called, A briefe Diſcourſe, &c. It was written in 1599, but not publiſhed till 1686.—7. *A Letter printed before Dr. John Dee's Parallaticæ Commentationis praxeoſque nucleus quidam*, 1573, 4to.—Beſide theſe, and his *Nova Corpora*, he left ſeveral mathematical treatiſes ready for the preſs; which, by reaſon of lawſuits and other avocations, he was hindered from publiſhing.

If our author was great in himſelf, he was not leſs ſo in his ſon, Sir Dudley Digges, ſo celebrated as a politician and elegant writer.

DIGIT, in Arithmetic, one of the ten characters, 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, by means of which all numbers are expreſſed.

Digit, in Aſtronomy, is the meaſure by which the part of the luminaries in eclipſes is eſtimated, being the 12th part of the diameter of the luminary. Thus, an eclipſe is ſaid to be of 10 digits, when 10 parts out of 12 of the diameter are in the eclipſed part; when the whole of the luminary is juſt all covered, the digits eclipſed are juſt 12; and when the luminary is more than covered, as often happens in lunar eclipſes, then more than 12 digits are ſaid to be eclipſed: Thus, if the diameter or breadth of the earth's ſhadow, where the moon paſſes through, be equal to one diameter and a half of the moon, then 18° or digits are ſaid to be eclipſed.

Theſe digits are by Wolfius, and ſome others, called *digiti ecliptici*.

Digit is alſo a meaſure taken from the breadth of the finger; being eſtimated at 3-4ths of an inch, and equal to 4 grains of barley, laid breadth-ways, ſo as to touch each other.

DILATATION, a motion of the parts of a body by which it expands, or opens itſelf, ſo as to occupy a greater ſpace.

Many authors confound dilatation with rarefaction; but the more accurate writers diſtinguiſh between them; defining dilatation as the expanſion of a body into a greater bulk, by its own elaſtic power; and rarefaction, the like expanſion produced by means of heat.

The moderns have obſerved, that bodies which, after being compreſſed, and again left at liberty, reſtore themſelves perfectly, do endeavour to dilate themſelves with the ſame force by which they are compreſſed; and accordingly they ſuſtain a force, and raiſe a weight equal to that with which they are compreſſed.

Again, bodies, in dilating by their elaſtic power, exert a greater force at the beginning of their dilatation, than towards the end; as being at firſt more compreſſed; and the greater the compreſſion, the greater the elaſtic power and endeavour to dilate. So that theſe three, the compreſſing power, the compreſſion, and the elaſtic power, are always equal.

Finally, the motion by which compreſſed bodies reſtore themſelves, is uſually accelerated: thus, when compreſſed air begins to reſtore itſelf, and dilate into a greater ſpace, it is ſtill compreſſed; and conſequently a new impetus is ſtill impreſſed upon it, from the dilatative cauſe; and the former remaining, with the increaſe of the cauſe, the effect, that is the motion and velocity, muſt be increaſed likewiſe. Indeed it may happen, that where the compreſſion is only partial, the motion of dilatation ſhall not be accelerated, but retarded; as is evident in the compreſſion of a ſpunge, ſoft bread, gauze, &c.

DILUTE. To dilute a body, is to render it liquid; or, if it were liquid before, to render it more ſo, by the addition of a thinner to it.

DIMENSION, the extenſion of a body, conſidered as meaſurable. Hence, as we conceive a body extended, and meaſurable in length, breadth, and depth, dimenſion is conſidered as threefold, viz, length, breadth, and thickneſs. So a line has one dimenſion only, viz length; a ſuperficies two, length and breadth; and a body or ſolid has three, viz, length, breadth, and thickneſs.

Dimension is alſo particularly uſed with regard to the powers of quantities in equations. Thus, in a ſimple equation, $x = a + b$, the unknown quantity is only of one dimenſion; in a quadratic equation, $x^2 = a^2 + b^2$, it is of two dimenſions; in a cubic, $x^3 = a^3 + b^3$, it is of three dimenſions; and ſo on.

DIMETIENT, has ſometimes been uſed for diameter.

DIMINISHED *Angle*, a term in Fortification. See Angle.

DIMINUTION, in Muſic, is the abating ſomething of the full value or quantity of any note.

Diminution, in Architecture, is a contraction of the upper part of a column, by which its diameter is made leſs than that of the lower part.

DINOCRATES, a celebrated ancient architect of Macedonia, of whom ſeveral extraordinary things are related. He was taken, by Alexander the Great, into Egypt, where he employed him in marking out and building the city of Alexandria. He formed a deſign, in which Mount Athos was to be laid out into the form of a man, in whoſe left hand were deſigned the walls of a great city, and all the rivers of the mount flowing into his right, and from thence into the ſea. Another memorable inſtance of Dinocrates's architectonic ſkill, is his reſtoring and building, in a more auguſt and magnificent manner than before, the celebrated temple of Diana at Epheſus, after Heroſtratus, for the ſake of immortalizing his name, had deſtroyed it by fire. A third inſtance, more extraordinary and wonderful than either of the former, is related by Pliny in his Natural Hiſtory; who ſays he had formed a ſcheme, by building the dome of the temple of Arſinoe at Alexandria of loadſtone, to make her image all of iron to hang in the middle of it, as if it were in the air; but the king's death, and his own, prevented the execution or attempt of this project.

DIONYSIUS, the *Periegetic*, an ancient geographer and poet. Pliny ſays, he was a native of the Perſian Alexandria, afterwards called Antioch, and at laſt Charrax; that he was ſent by Auguſtus, to ſurvey the eaſtern part of the world. Dionyſius wrote a great number of pieces, enumerated by Suidas and his commentator Euſtathius: but his *Periegeſis*, or *Survey of the World*, is the only one now extant; which may be well eſteemed one of the moſt exact ſyſtems of ancient geography, ſince Pliny himſelf propoſed it as his pattern.

DIONYSIAN, or *Victorian Period*. See Period.

DIOPHANTUS, a celebrated mathematician of Alexandria; who has been reputed to be the inventor of Algebra; at leaſt his is the earlieſt work extant on that ſcience. It is not certain when Diophantus lived. Some have placed him before Chriſt, and ſome after, in the reigns of Nero and the Antonines; but all with equal uncertainty. It ſeems he is the ſame Diophantus who wrote the *Canon Aſtronomicus*, which Suidas ſays was commented on by the celebrated Hypatia, daughter of Theon of Alexandria. His reputation muſt have been very high among the ancients, ſince they ranked him with Pythagoras and Euclid in mathematical learning. Bachet, in his notes upon the 5th book *De Arithmeticis*, has collected, from Diophantus's epitaph in the Anthologia, the following circumſtances of his life; namely, that he was married when he was 33 years old, and had a ſon born 5 years after; that this ſon died when he was 42 years of age, and that his father did not ſurvive him above 4 years; from which it appears, that Diophantus was 84 years old when he died.

DIOPHANTUS wrote 13 books of Arithmetic, or Algebra, which Regiomontanus in his preface to Alfraganus, tells us, are ſtill preſerved in manuſcript in the Vatican library. Indeed Diophantus himſelf tells us that his work conſiſted of 13 books, viz, at the end of his addreſs to Dionyſius, placed at the beginning of the work; and from hence Regiomontanus might be led to ſay the 13 books were in that library. No more than 6 whole books, with part of a ſeventh, have ever been publiſhed; and I am of opinion there are no more in being; indeed Bombelli, in the preface to his Algebra, written 1572, ſays there were but 6 of the books then in the library, and that he and another were about a tranſlation of them.

Thoſe 6 books, with the imperfect 7th, were firſt publiſhed at Baſil by Xylander in 1575, but in a Latin verſion only, with the Greek ſcholia of Maximus Planudes upon the two firſt books, and obſervations of his own. The ſame books were afterwards publiſhed in Greek and Latin at Paris in 1621, by Bachet, an ingenious and learned Frenchman, who made a new Latin verſion of the work, and enriched it with very learned commentaries. Bachet did not entirely neglect the notes of Xylander in his edition, but he treated the ſcholiaſt Planudes with the utmoſt contempt. He ſeems to intimate, in what he ſays upon the 28th queſtion of the 2d book, that the 6 books which we have of Diophantus, may be nothing more than a collection made by ſome novice, of ſuch propoſitions as he judged proper, out of the whole 13: but Fabricius thinks there is no juſt ground for ſuch a ſuppoſition.

DIOPHANTINE *Problems*, are certain queſtions relating to ſquare and cubic numbers, and to right-angled triangles, &c; the nature of which were firſt and chiefly treated of by Diophantes, in his Arithmetic, or rather Algebra.

In theſe queſtions, it is chiefly intended to find commenſurable numbers to anſwer indeterminate problems; which often bring out an infinite number of incommenſurable quantities. For example, let it be propoſed to find a right-angled triangle, whoſe three ſides x, y, z are expreſſed by rational numbers; from the nature of the figure it is known that $x^2 + y^2 = z^2$, where z denotes the hypothenuſe. Now it is plain that x and y may alſo be ſo taken, that z ſhall be irrational; for if $x = 1$, and $y = 2$, then is $z = \surd 5$.

Now the art of reſolving ſuch problems, conſiſts in ordering the unknown quantity or quantities, in ſuch a manner, that the ſquare or higher power may vaniſh out of the equation, and then by means of the unknown quantity in its firſt dimenſion, the equation may be reſolved without having recourſe to incommenſurables. For ex. in the equation above, $x^2 + y^2 = z^2$, ſuppoſe $z = x + u$, then is $x^2 + y^2 = x^2 + 2xu + u^2$, out of which equation x^2 vaniſhes, and then it is $y^2 = 2xu + u^2$, which gives $x = \frac{y^2 - u^2}{2u}$. Hence, aſſuming y and u equal to any numbers at pleaſure, the three ſides of the triangle will be y, $\frac{y^2 - u^2}{2u}$, and $\frac{y^2 + u^2}{2u}$, which are all rational whenever y and u are rational. For ex. if $y = 3$, and $u = 1$, then $\frac{y^2 - u^2}{2u} = 4$, and $x + u$ or $\frac{y^2 + u^2}{2u} = 5$. It is evident that this problem admits of infinite numbers of ſolutions, as y or u may be aſſumed infinitely various. See ALGEBRA, and DIOPHANTUS.

Abundant information on this ſort of problems may be found in the writings of a great many authors, particularly Fermat, Bachet, Ozanam, Kerſey, Saunderſon, Euler, &c.

DIOPTER, or DIOPTRA, the ſame with the index or alhidade of an aſtrolabe, or other ſuch inſtrument.

DIOPTRA was an inſtrument invented by Hipparchus, which ſerved for ſeveral uſes; as, to level water-courſes; to take the height of towers, or places at a diſtance; to determine the places, magnitudes, and diſtances of the planets, &c.

DIOPTRICS, called alſo *anaclaſtics*, is the doctrine of refracted viſion; or that part of Optics which explains the effects of light as refracted by paſſing through different mediums, as air, water, glaſs, &c, and eſpecially lenſes.

Dioptrics is one of the moſt uſeful and pleaſant of all the human ſciences; bringing the remoteſt objects near hand, enlarging the ſmalleſt objects ſo as to ſhew their minute parts, and even giving ſight to the blind; and all this by the ſimple means of the attractive power in glaſs and water, cauſing the rays of light in their paſſage through them to alter their courſe according to the different ſubſtances of the medium; whence it happens, that the object ſeen through them, do, in appearance, alter their magnitude, diſtance, and ſituation.

The ancients have treated of direct and reflected viſion; but what we have of refracted viſion, is very imperfect. J. Baptiſta Porta wrote a treatiſe on refraction, in 9 books, but without any great improvement. Kepler was the firſt who ſucceeded in any great degree, on this ſubject; having demonſtrated the properties of ſpherical lenſes very accurately, in a treatiſe firſt publiſhed anno 1611. After Kepler, Galileo gave ſomewhat of this doctrine in his Letters; as alſo an Examination of the Preface of Johannes Pena upon Euclid's Optics, concerning the uſe of optics in aſtronomy. Des Cartes alſo wrote a treatiſe on Dioptrics, commonly annexed to his Principles of Philoſophy, which is one of his beſt works; in which the true manner of viſion is more diſtinctly explained than by any former writer, and in which is contained the true law of refraction, which was found out by Snell, though the name of the inventer is ſuppreſſed: here are alſo laid down the properties of elliptical and hyperbolical lenſes, with the practice of grinding glaſſes. Dr. Barrow has treated on Dioptrics in a very elegant manner, though rather too briefly, in his Optical Lectures, read at Cambridge. There are alſo Huygens's Dioptrics, an excellent work of its kind. Molyneux's Dioptrics, a work rather heavy and dull. Hartſoeker's Eſſai de Dioptrique. Cherubin's Dioptrique Oculaire, et La Viſion Parfaite. David Gregory's Elements of Dioptrics. Traber's Nervus Opticus. Zahn's Oculus Artificialis Teledioptricus. Dr. Smith's Optics, a complete work of its kind. Wolfius's Dioptrics, contained in his Elementa Matheſeos Univerſalis. But over all, the Treatiſe on Optics, and the Optical Lectures of Newton, in whoſe experiments are contained far more diſcoveries than in all the former writers. Laſtly, this ſcience was perfected by Dollond's diſcovery of the acromatic glaſſes, by which the colours are obviated in refracting teleſcopes.

The laws of Dioptrics ſee delivered under the article REFRACTION, LENS, &c; and the application of it in the conſtruction of teleſcopes, miſcroſcopes, and other dioptrical inſtruments, under the articles Telescope, and Microscope.

DIP *of the Horizon.* See Depression.

Dipping-*Needle*, or *Inclinatory Needle*, a magnetical needle, ſo hung, as that, inſtead of playing horizontally, and pointing out north and ſouth, one end dips, or inclines to the horizon, and the other points to a certain degree of elevation above it.

The inventor of the Dipping-needle was one Robert Norman, a compaſs-maker at Ratcliffe, about the year 1580; this is not only teſtified by his own account, in his New Attractive, but alſo by Dr. Gilbert, Mr. William Burrowes, Mr. Henry Bond, and other writers of that time, or ſoon after it. The occaſion of the diſcovery he himſelf relates, viz, that it being his cuſtom to finiſh, and hang the needles of his compaſſes, before he touched them, he always found that, immediately after the touch, the north point would dip or decline downward, pointing in a direction under the horizon; ſo that, to balance the needle again, he was always forced to put a piece of wax on the ſouth end, as a counterpoiſe. The conſtancy of this effect led him, at length, to obſerve the preciſe quantity of the dip, or to meaſure the greateſt angle which the needle would make with the horizon. This, in the year 1576, he found at London was 71° 50′. It is not quite certain whether the dip varies, as well as the horizontal direction, in the ſame place. Mr. Graham made a great many experiments with the dipping-needle in 1723, and found the dip between 74 and 75 degrees. Mr. Nairne, in 1772, found it ſomewhat above 72°. And by many obſervations made ſince that time at the Royal Society, the medium quantity is 72°½. The trifling difference between the firſt obſervations of Norman, and the laſt of Mr. Nairne and the Royal Society, lead to the opinion that the dip is unalterable; and yet it may be difficult to account for the great difference between theſe and Mr. Graham's numbers, conſidering the well-known accuracy of that ingenious gentleman. Philoſ. Tranſ. vol. 45, pa. 279, vol. 62, pa. 476. vol. 69, 70, 71.

It is certain however, from many experiments and obſervations, that the dip is different in different latitudes, and that it increaſes in going northward. It appears from a table of obſervations made with a marine dipping needle of Mr. Nairne's, in a voyage towards the north pole, in 1773, that

in latitude 60° 18′ the Dip was 75° 0′,
in latitude 70 45 the Dip was 77 52,
in latitude 80 12 the Dip was 81 52, and
in latitude 80 27 the Dip was 82 2½.

See Phipps's Voyage, pa. 122. See alſo the Obſervations of Mr. Hutchins, made in Hudſon's Bay and Straits, Philoſ. Tranſ. vol. 65, pa. 129.

Burrowes, Gilbert, Ridley, Bond, &c, endeavoured to apply this diſcovery of the dip to the finding of the latitude; and Bond, going ſtill farther, firſt of any propoſed finding the longitude by it; but for want of obſervations and experiments, he could not go any length. Mr. Whiſton, being furniſhed with the farther obſervations of colonel Windham, Dr. Halley, Mr Pound, Mr. Cunningham, M. Noel, M. Feuille, and his own, made great improvements in the doctrine and uſe of the dipping-needle, brought it to more certain rules, and endeavoured in good earneſt to find the longitude by it. For this purpoſe, he obſerves, 1ſt, That the true tendency of the north or ſouth end of every magnetic needle, is not to that point of the horizon, to which the horizontal needle points, but towards another, directly under it, in the ſame vertical, and in different degrees under it, in different ages, and at different places. 2dly, That the power by which the horizontal needle is governed, and all our navigation uſually directed, it is proved is only one quarter of the power by which the dipping-needle is moved; which ſhould render the latter far the more effectual and accurate inſtrument. 3dly, That a dipping-needle of a foot long will plainly ſhew an alteration of the angle of inclination, in theſe parts of the world, in half a quarter of a degree, or 7½ geographical miles; and a needle of 4 feet, in 2 or 3 miles; i. e. ſuppoſing theſe diſtances taken along, or near a meridian. 4thly, A dipping-needle, 4 feet long, in theſe parts of the world, will ſhew an equal alteration along a parallel, as another of a foot long will ſhew along a meridian; i. e. that will, with equal exactneſs, ſhew the longitude, as this the latitude.

This depends on the poſition of the lines of equal dip, in theſe parts of the world, which it is found do lie about 14 or 15 degrees from the parallels. Hence he argues, that as we can have needles of 5, 6, 7, 8, or more feet long, which will move with ſtrength ſufficient for exact obſervation; and ſince microſcopes may be applied for viewing the ſmalleſt diviſions of degrees on the limb of the inſtrument, it is evident that the longitude at land may thus be found to leſs than 4 miles.

And as there have been many obſervations made at ſea with the ſame inſtrument by Noel, Feuille, &c, which have determined the dip uſually within a degree, ſometimes within ½ or ⅓ of a degree, and this with ſmall needles, of 5 or 6, or at the moſt 9 inches long; it is inferred, that the longitude may be found, even at ſea, to leſs than half a quarter of a degree. This premiſed, the obſervation itſelf follows.

To find the Longitude or Latitude by the Dipping-Needle.—If the lines of equal dip, below the horizon, be drawn on maps, or ſea-charts, from good obſervations, it will be eaſy, from the longitude known, to find the latitude; and from the latitude known, to find the longitude either at ſea or land.

Suppoſe, for example, a perſon travelling or ſailing along the meridian of London, ſhould find that the angle of dip, with a needle of one foot, was 75°; the chart will ſhew that this meridian, and the line of dip, meet in the latitude of 53° 11′; which is therefore the latitude ſought.

Or if he be travelling or ſailing along the parallel of London, i. e. in 51° 31′ north latitude, and find the angle of dip 74°; then this parallel, and the line of this dip, will meet on the map in 1° 46′ of eaſt longitude from London; which therefore is the longitude ſought.

DIRECT, in Arithmetic, is when the proportion of any terms, or quantities, is in the natural or direct order in which they ſtand; being the oppoſite to *inverſe*, which conſiders the proportion in the inverted order of the terms. So, 3 : 4 : : 6 : 8 *directly*; or 3 : 4 : : 8 : 6 inverſely.

Rule of Three Direct, is when both pairs of terms are in *direct* proportion.

Direct, in Aſtronomy. A planet is ſaid to be direct, or its motion direct, when it goes forward by its proper motion in the zodiac, according to the ſucceſſion or order of the ſigns; or when it appears ſo to do, to an obſerver ſtanding upon the earth. Whereas it is ſaid to be, or to move *retrograde*, when it appears to go the contrary way, or backward; and to be *ſtationary*, when it ſeems not to move either way.

Direct *Dials*. See Dial and Dialling.

Direct, in Optics. *Direct* viſion is that performed by direct rays; in contradiſtinction to viſion by refracted, or reflected rays. Direct viſion is the ſubject of Optics, which preſcribes the laws and rules of it.

Direct *Rays*, are thoſe which paſs on in right lines from the object to the eye, without being turned out of their rectilinear direction by any intervening body, either opaque or pellucid; or without being either reflected or refracted.

Direct *Sphere*. See Right *Sphere*.

DIRECTION, in Aſtronomy, the motion and other phenomena of a planet, when direct.

Direction, in Aſtrology, is a kind of calculus, by which they pretend to find the time in which any notable accident ſhall befall the perſon whoſe horoſcope is drawn.

For inſtance, having eſtabliſhed the ſun, moon, or aſcendant, as maſters or ſignificators of life; and Mars or Saturn as promiſers or portenders of death; the direction is a calculation of the time in which the ſignificator ſhall meet the portender.

The ſignificator they likewiſe call *apheta*, or giver of life; and the promiſer, *anereta*, *promiſſor*, or giver of death.

They work the directions of all the principal points of the heavens and ſtars, as the aſcendant, mid-heaven, ſun, moon, and part of fortune. The like is done for the planets and fixed ſtars; but all differently, according to the different authors.

Line of Direction, in Gunnery, is the direct line in which a piece is pointed. Sometimes a line of direction is marked on the upper ſide of the gun, by a ſmall notch or ſlit, or knob, in the baſe and muzzle rings: but, unleſs the two wheels of the carriage ſtand equally high, this line will be fallacious; for which reaſon the gunners commonly find a new line of direction every time, by means of a plummet.

Line of Direction, in Mechanics, denotes the line in which a body moves, or endeavours to proceed.

Angle of Direction, is that comprehended between the lines of direction of two conſpiring powers.

Quantity of Direction, is uſed for the product ariſing from multiplying the velocity of the common centre of gravity, in a ſyſtem of bodies, by the ſum of their maſſes. In the colliſion of bodies the quantity of direction is the ſame both before and after the impulſe.

Direction *of the Load Stone*, that property by which the magnet, or a needle touched by it, always preſents one of its ends toward one of the poles of the world, and the oppoſite end to the other pole. This is alſo called the *polarity* of the magnet or needle.

The attractive property of the magnet was known long before its directive; and the directive long before the inclinatory.

Number of Direction, is the number of days that Septuageſima Sunday falls after the 17th of January. See Number.

DIRECTLY, in Geometry: we ſay two lines lie directly againſt each other, when they are parts of the ſame right line. Alſo quantities are ſaid to be directly proportional, when the proportion is according to the order of the terms; in contradiſtinction to *inverſely*, or *reciprocally* proportional, which is taking the proportion contrary to the order of the terms.

In Mechanics, a body is ſaid to ſtrike or impinge *directly* againſt another body, when the ſtroke is in a direction perpendicular to the ſurface at the point of impact.

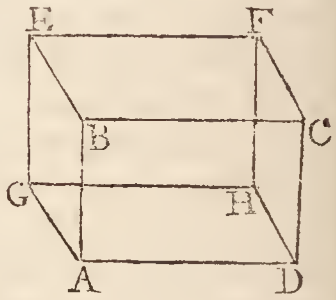
And a ſphere in particular ſtrikes directly againſt another, when the line of direction paſſes through both their centres.

DIRECTRIX. See Dirigent.

Directrix, in a Parabola, a line perpendicular to the axis produced, at the diſtance of the focus without the vertex.

DIRIGENT, a term expreſſing the line of motion, along which a deſcribent line, or ſurface, is carried in the geneſis of any plane or ſolid figure.

Thus, if the line AB move parallel to itſelf, and along the line AD, ſo that the point A always keeps in the line AD, and the point B in the line BC; a parallelogram ABCD will be formed; of which the line AB is the deſcribent, and the line AD the dirigent. So alſo, if the ſurface ABEG be ſuppoſed carried along the line AD, in a poſition always parallel to itſelf at its firſt ſituation, the ſolid AF will be formed; where the ſurface AE is the deſcribent, and the line AD is the dirigent.



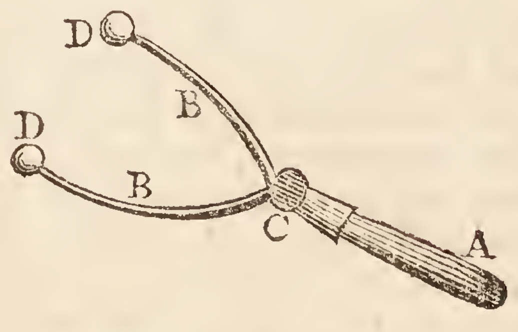
DISC, or Disk, the body or face of the ſun or moon; ſuch as it appears to us; for though they be really ſpherical bodies, they are apparently circular planes.

The diameter of the diſc is conſidered as divided into 12 equal parts, called digits; by means of which it is, that the magnitude of an eclipſe is meaſured, or eſtimated.—In a total eclipſe of either of thoſe luminaries, the whole diſc is obſcured, or darkened; in a partial eclipſe, only part of them.

Illuminated Disc of the Earth. See Circle *of Illumination*.

Disc, in Optics, the magnitude of a teleſcope glaſs, or the width of its aperture, whatever its figure be, whether a plane, convex, meniſcus, or the like.

DISCHARGER, or Discharging *Rod*, in Electricity, conſiſts of a handle of glaſs or baked wood, A, and two bent metal rods BB, terminating in points, and capable of being ſcrewed into the knobs DD, which move



move by a joint C, fixed to the handle A. Thus it may be uſed either with the points or balls, as occaſion requires; and by being made moveable on a joint, it may be applied to larger or ſmaller jars at pleaſure. By bringing one of theſe knobs or points to one coated ſide of a charged electric, and the other to the other ſide, or to any conductor connected with it, the communication is completed between the two ſides, and the electric is diſcharged.

For the deſcription and uſe of an Univerſal Diſcharger, by Mr. Henly, with which many curious experiments may be performed, ſee Cavallo's Electricity, pa. 164.

DISCORD, the relation of two ſounds which are always, and of themſelves, diſagreeable, whether applied in ſucceſſion or conſonance.

DISCOUNT, or *Rebate*, is uſed for an allowance made on a bill, or any other debt not yet become due, in conſideration of making preſent payment of the bill or debt.

Among merchants and traders, it is uſual to allow a ſum for diſcount that is equal to the intereſt of the debt, calculated for the time till it becomes due: but this is not juſt; for as the true value of the diſcount is equal to the difference between the debt and its preſent worth, it is equal only to the intereſt of that preſent worth, inſtead of the intereſt on the whole debt. And therefore the rule for finding the true diſcount is this:

As the amount of 100l, for the given rate and time:
Is to the given ſum or debt : :
So is the intereſt of 100l, for the given rate and time:
To the diſcount of the debt.

So that, if p be the principal or debt, r the rate of intereſt per cent. and t the time;

then as $100+rt : p :: rt : \frac{prt}{100+rt}$, which is the true diſcount. Hence alſo $p - \frac{prt}{100+rt} = \frac{100p}{100+rt}$ is the preſent worth, or ſum to be received.

For ex. Suppoſe it be required to find the diſcount of 250l, for five months, at the rate of 5 per cent. per annum intereſt. Here $p=250$, $r=5$, and $t=\frac{5}{12}$ or 5 months; then

$$\frac{prt}{100+rt} = \frac{250\times5\times\frac{5}{12}}{100+5\times\frac{5}{12}} = \frac{250\times25}{1200+25} = \frac{250}{49} =$$

5*l.* 2*s.* $\frac{2}{49}$ the diſcount ſought.

A Table of Diſcounts may be ſeen in Smart's Tables of Intereſt, the uſe of which makes calculations of diſcount very eaſy.

Discount is alſo the name of a rule in books of Arithmetic, by which calculations of Diſcount are made.

DISCRETE, or Disjunct, *Proportion*, is that in which the ratio between two or more pairs of numbers is the ſame, and yet the proportion is not continued, ſo as that the ratio may be the ſame between the conſequent of one pair and the antecedent of the next pair.

Thus, if the numbers or proportion 6 : 8 : : 3 : 4 be conſidered: the ratio of 6 to 8 is the ſame as that of 3 to 4, and therefore theſe four numbers are proportional: but it is only *diſcretely* or *disjunctly*, and not continued; for 8 to 3 is not the ſame ratio as the former; that is, the proportion is broken off between 8 and 3, and not continued all along, as it is in theſe following four numbers, which are called *continual proportionals*, viz, 3 : 6 : : 12 : 24.

Discrete *Quantity*, is ſuch as is not continued and joined together. Such for inſtance is any number; for its parts, being diſtinct units, cannot be united into one *continuum*; for in a *continuum* there are no actual determinate parts before diviſion, but they are potentially infinite: ſo that it is uſually and truly ſaid that continued quantity is diviſible *in infinitum*.

DISDIAPASON, in Muſic, a compound concord, in the quadruple ratio of 4 to 1, being a fifteenth or double eighth, and is produced when the voice goes from the firſt tone to the fifteenth.

DISJUNCT *Proportion*. See Discrete *Proportion*.

DISK. See Disc.

DISPART, a term in Gunnery, uſed for a mark ſet upon the muzzle-ring of a piece of ordnance, of ſuch height, that a ſight-line taken from the top of the baſe ring near the vent or touch-hole to the top of the diſpart near the muzzle, may be parallel to the axis of the concave cylinder; for which reaſon it is evident that the height of the diſpart is equal to the difference between the radii of the piece at the baſe and muzzle-rings, or to half the difference of the diameters there. Hence comes the common method of diſparting the gun, which is this: Take, with the calipers, the two diameters, viz, of the baſe ring and the place where the diſpart is to ſtand, ſubtract the leſs from the greater, and take half the difference, which will be the length of the diſpart, which is commonly cut to that length from a ſmall bit of wood, and ſo fixed upright in its place with a bit of wax or pitch.

DISPERSION, in Dioptrics, is the divergency of refracted rays of light.

Point of Dispersion, is a point from which refracted rays begin to diverge, when their refraction renders them divergent.—It is called *point of diſperſion* in oppoſition to the *point of concourſe*, or point in which converging rays concur after refraction. But it is more uſual to call the latter the *focus*, and the former the *virtual focus*.

Dispersion *of Light*, occaſioned by the refrangibility of the rays, or the nature of the refracting medium. See Aberration, and Inflection.

DISSIPATION, in Phyſics, a gradual, ſlow, inſenſible loſs or conſumption of the minute parts of a body; or, more properly, the flux by which they fly off and are loſt. See Effluvia.

Circle of Dissipation, or Aberration, in Optics, denotes that circular ſpace upon the retina of the eye, which is occupied by the rays of each pencil in indiſtinct viſion: thus, if the diſtance of the object, or the conſtitution of the eye, be ſuch, that the image falls beyond the retina, as when objects are too near; or before the retina, when the rays have not a ſufficient divergency, the rays of a pencil, inſtead of being collected into a central point, will be diſſipated over this circular ſpace: and, all other circumſtances being alike, this circle will be greater or leſs, according to the diſtances from the retina of the foci of refracted rays. But this circle cauſes no perceptible difference in the diſtinctneſs of viſion, unleſs it exceed a certain magnitude; as ſoon as that is the caſe, we begin to perceive an indiſtinctneſs, which increaſes as that circle increaſes, till at length the object is loſt in confuſion. This circle is alſo greater or leſs, according to the greater or leſs magnitude of the viſible object: and though it be not eaſy to aſſign the diameter of the ſaid circle, it ſeems very probable that viſion continues diſtinct for all ſuch diſtances, or ſo long as theſe circles, or the pencils of light from them, do not touch one another upon the retina; and the indiſtinctneſs begins when the ſaid circles begin to interfere. It has been often obſerved, that a preciſe union of the reſpective rays upon the retina, is not neceſſary to diſtinct viſion; but the firſt author who aſcertained the fact beyond all doubt, was Dr. Jurin. See a variety of obſervations and experiments on this ſubject, in his Eſſay on Diſtinct and Indiſtinct Viſion, in Smith's Optics, Appendix. In the Philoſ. Tranſ. for 1789, pa. 256, is an excellent paper on this ſubject by Dr. Maſkelyne; in which he computes the diameter of the circle of diſſipation at ·002667 of an inch, making it anſwer to an external angle of 15°, which he ſhews is very compatible with diſtinct viſion. See alſo Moon, and Vision.

Radius of Dissipation, is the radius of the circle of diſſipation.

DISSOLVENT, ſomething that diſſolves; i. e. divides, and reduces a body into its ſmalleſt parts.

DISSOLUTION, is a ſeparation of the ſtructure of a body, into ſmall or minute parts.

According to Newton, and others, this is effected by certain powerful attractions.

DISSONANCE, or Discord, is a falſe conſonance or concord; being produced by the mixture or meeting of two ſounds which are diſagreeable to the ear.

DISTANCE, properly ſpeaking, denotes the ſhorteſt line between two points, objects, &c.

Distance, in Aſtronomy, as of the ſun, planets, comets, &c.

The Real Diſtances are found from the parallaxes of the planets, &c. See Parallax, and Planet, and Transit. The diſtance of the earth from the ſun has been determined at 95 millions of miles, by the late tranſits of Venus; and from this one real diſtance, and the ſeveral relative diſtances, by analogy are found all the other real diſtances, as in the table below.

The Proportional or Relative Diſtances of the planets are very well deduced from the theory of gravity: for Kepler has long ſince diſcovered, and Newton has demonſtrated, that the ſquares of their periodical times are proportional to the cubes of their diſtances. Kepler's Epit. Aſtron. lib. 4; Newton's Principia, lib. 3, phæn. 4; and Gregory's Aſtron. book 1, prop. 40. If therefore the mean diſtance of the earth from the ſun be aſſumed, or ſuppoſed 10000, we ſhall then, from the foregoing analogy, and the known periodical times, obtain the relative diſtances of the other planets: thus,

The Periodical Revolutions in Days and Parts.

Mercury.	Venus.	Earth.	Mars.	Jupiter.	Saturn.	Herſchel.
$87\frac{23}{24}$	$224\frac{17}{24}$	$365\frac{1}{4}$	$686\frac{23}{24}$	$4332\frac{1}{2}$	$10759\frac{7}{24}$	30445
Relative Mean Diſtances from the Sun.						
3871	7233	10000	15237	52010	95401	190818
Real Diſtance in Millions of Miles.						
37	66	95	145	$493\frac{1}{2}$	$903\frac{1}{2}$	1813

For the diſtances of the ſecondary planets from the centres of their reſpective primaries, ſee Satellites.

As to that of the fixed ſtars, as having no ſenſible parallax, we can do little more than gueſs at.

Distance *of the Sun from the Moon's node,* or *apogee,* is an arch of the ecliptic, intercepted between the ſun's true place and the moon's node, or apogee. See Node.

Curtate Distance. See Curtate.

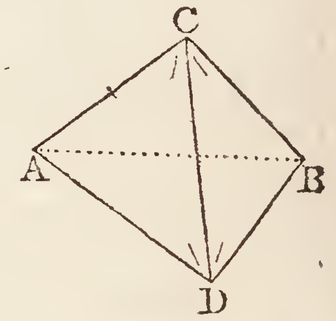
Distance *of the Baſtions,* in Fortification, is the ſide of the exterior polygon.

Acceſſible Distances, in Geometry, are meaſured with the chain, decempeda or ten-foot rod, or the like.

Inacceſſible Distances, are found by taking bearings to them, from the two extremities of a line whoſe length is given. Various ways of performing this may be ſeen in my Treatiſe on Menſuration, ſect. 3, on Heights and Diſtances.

Distance, in Geography, is the arch of a great circle intercepted between two places.

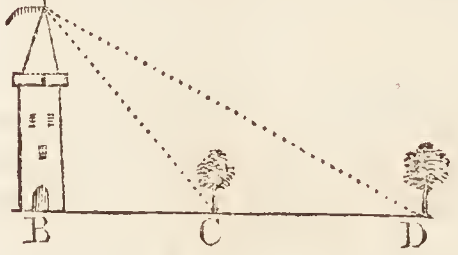
To find the diſtance of two places, A and B, far remote from each other. Aſſume two ſtations, C and D, from which both the places A and B may be ſeen; and there, with a theodolite, obſerve the quantity of the angles ACD, BCD, ADC, BDC, and meaſure any diſtance as AC.

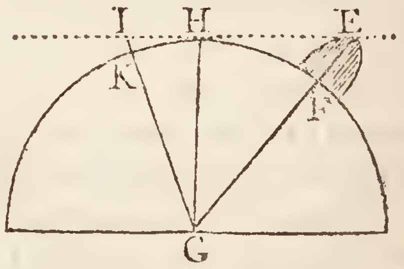


Then, in the triangle ACD, there are given the angles ACD, ADC, and the ſide AC; to find the ſide CD.—Next, in the triangle BCD, there are given the angles BCD, BDC, and the ſide CD; to find the ſide BC.—Laſtly, in the triangle ABC, there are given the angle ACB, and the ſides AC, CB; to find the ſide AB, which is the diſtance ſought.

And in theſe operations, the triangles may be computed either as plane triangles, or as ſpherical ones, as the caſe may require, or according to the magnitude of the diſtances.

The Distance *of a remote object may alſo be found from its height.* This admits of ſeveral caſes, according as the diſtances are large or ſmall, &c. 1ſt, Suppoſe that from the top of a tower at A, whoſe height AB is 120 feet, there be taken the angle BAC = 33°, and the angle BAD = 64½°, to two trees, or other objects, C, D; to find the diſtance between them CD, and the diſtance of each from the bottom of the tower at B.





Firſt, rad. : tang. ∠ BAD : : AB : BD = 251·585,
next, rad. : tang. ∠ BAC : : AB : BC = 77·929,

their difference is the diſt. CD = 173·656.

2d, Suppoſe it be required to find the diſtance to which an object can be ſeen, by knowing its altitude; *ex. gr.* the Pike of Teneriffe, whoſe height is accounted 3 miles above the level of the ſea, ſuppoſing the circumference of the earth 25,000 miles, or the diameter 7958 miles. Let FG be the radius = 3979, EF = 3 the height of the mountain, and EI a tangent to the earth at the point H, which is the fartheſt diſtant point to which the top of the mountain E canbe ſeen. Here in the right angled triangle EGH, are given the hypothenuſe EG = 3982, and the leg FG = 3979; to find the other leg HE = 154$\frac{1}{2}$ miles = the diſtance ſought nearly. Or, rather, as EG : GH : : rad. : coſin. ∠ G = 2° 13′$\frac{1}{2}$; then as 360° : 2° 13′$\frac{1}{2}$: : 25,000 : 154$\frac{1}{2}$ miles = the arch of diſt. HF ſought, the ſame as before.

3d, If the eye, inſtead of being in the horizon at H, were elevated above it at I, any known height, as ſuppoſe 264 feet, or $\frac{1}{20}$th of a mile, as on the top of a ſhip's maſt, &c; then the mountain can be ſeen much farther off along the line IE, and the diſtance will be the two tangents IH and HE, or rather the two arcs KH and HF. Hence, as above, as IG : GH : : rad. coſin. ∠ IGH = 17′$\frac{2}{7}$; then as 360° : 17′$\frac{2}{7}$: : 25,000 : 20 miles = the arc KH: this added to the former arc HF = 154$\frac{1}{2}$, makes the whole arc KF = 174$\frac{1}{2}$ miles, for the whole diſtance to which the top of the mountain can be ſeen in this caſe.

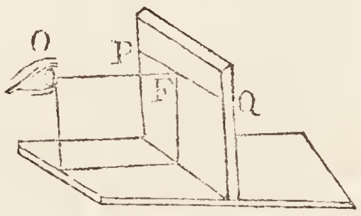
Apparent DISTANCE, in Optics, that diſtance which we judge an object is placed at when ſeen afar off, being uſually very different from the true diſtance; becauſe we are apt to think that all very remote objects, whoſe parts cannot well be diſtinguiſhed, and which have no other object in view near them, are at the ſame diſtance from us, though perhaps the one is thouſands of miles nearer than the other, as is the caſe with regard to the ſun and moon.

M. De la Hire enumerates five circumſtances, which aſſiſt us in judging of the diſtance of objects; viz, their apparent magnitude, the ſtrength of the colouring, the direction of the two eyes, the parallax of the objects, and the diſtinctneſs of their ſmall parts. On the contrary, Dr. Smith maintains, that we judge of diſtance principally, or ſolely, by the apparent magnitude of objects; and concludes univerſally, that the apparent diſtance of an object ſeen in a glaſs, is to its apparent diſtance ſeen by the naked eye, as the apparent magnitude to the naked eye is to its apparent magnitude in the glaſs: But it was long ſince obſerved by Alhazen, that we do not judge of diſtance merely by the angle under which objects are ſeen; and Mr. Robins clearly ſhews that Dr. Smith's hypotheſis is contrary to fact, in the moſt common and ſimple caſes. Thus, if a double convex glaſs be held upright before ſome luminous object, as a candle, there will be ſeen two images, one erect, and the other inverted; the firſt is made ſimply by reflexion from the neareſt ſurface; the ſecond by reflexion from the farther ſurface, the rays undergoing a refraction from the firſt ſurface both before and after the reflexion. If this glaſs has not too ſhort a focal diſtance, when it is held near the object, the inverted image will appear larger than the other, and alſo nearer; but if the glaſs be carried off from the object, though the eye remain as near to it as before, the inverted image will be diminiſhed ſo much faſter than the other, that at length it will appear much leſs than it, but ſtill nearer. Here, ſays Mr. Robins, two images of the ſame object are ſeen under one view, and their apparent diſtances immediately compared; and it is evident that thoſe diſtances have no neceſſary connexion with the apparent magnitude. This experiment may be made ſtill more convincing, by ſticking a piece of paper on the middle of the lens, and viewing it through a ſhort tube. He obſerves farther, that the apparent magnitude of very diſtant objects is neither determined by the magnitude of the angle only under which they are ſeen, nor is the exact proportion of that angle compared with their true diſtance, but is compounded alſo with a deception concerning that diſtance; ſo that if we had no idea of difference in the diſtance of objects, each would appear in magnitude proportional to the angle under which it was ſeen; and if our apprehenſion of the diſtance were always juſt, our idea of their magnitude would be unvaried, in all diſtances; but in proportion as we err in our conception of their diſtance, the greater angle ſuggeſts a greater magnitude. By not attending to this compound effect, Mr. Robins apprehends that Dr. Smith was led into his miſtake.

Dr. Porterfield has made ſeveral remarks on the five methods of judging concerning the diſtance of objects above recited from M. De la Hire; and he has alſo added to them one more, viz. the conformation of each eye. See *Circle of* DISSIPATION. This, he ſays, can be of no uſe to us, with reſpect to objects that are placed without the limits of diſtinct viſion. But the greater or leſs confuſion with which the object appears, as it is more or leſs removed from thoſe limits, will aſſiſt the mind in judging of its diſtance: the more confuſed it appears, the farther will it be thought diſtant. However, this confuſion has its limits; for when an object is placed at a certain diſtance from the eye, to which the breadth of the pupil bears no ſenſible proportion, the rays proceeding from a point in the object may be conſidered as parallel; in which caſe, the picture on the retina will not be ſenſibly more confuſed, though the object be removed to a much greater diſtance. The moſt univerſal, and often the moſt ſure means of judging of the diſtance of objects, he ſays, is the angle made by the optic axes: our two eyes are like two different ſtations, by the aſſiſtance of which, diſtances are taken; and this is the reaſon why thoſe perſons who have loſt the ſight of one eye, ſo frequently miſs their mark in pouring liquor into a glaſs, ſnuffing a candle, and ſuch other actions as require that the diſtance be exactly diſtinguiſhed. With reſpect to the method of judging by the apparent magnitude of objects, he obſerves that this can only ſerve when we are otherwiſe acquainted with their real magnitude. Thus he accounts for the deception to which we are liable in eſtimating diſtances, by any extraordinary magnitudes that terminate them; as, in travelling towards a large city, caſtle, or cathedral, we fancy they are nearer than they really are. Hence alſo, animals and ſmall objects ſeen in a valley contiguous to large mountains, or on the top of a mountain or high building, appear exceedingly ſmall. Dr. Jurin accounts for the laſt recited phenomenon, by obſerving that we have no diſtinct idea of diſtance in that oblique direction, and therefore judge of them merely by their pictures on the eye.

Dr. Porterfield obſerves, with reſpect to the ſtrength of colouring, that if we are aſſured they are of a ſimilar colour, and one appears more bright and lively than the other, we judge that the brighter object is the nearer. When the ſmall parts of objects appear confuſed, or do not appear at all, we judge that they are at a great diſtance, and *vice verſa;* becauſe the image of any object, or part of an object, diminiſhes as the diſtance of it increaſes. Finally, we judge of the diſtance of objects by the number of intervening bodies, by which it is divided into ſeparate and diſtinct parts; and the more this is the caſe, the greater will the diſtance appear. Thus diſtances upon uneven ſurfaces appear leſs than upon a plane, becauſe the inequalities do not appear, and the whole apparent diſtance is diminiſhed by the parts that do not appear in it: and thus the banks of a river appear contiguous to a diſtant eye, when the river is low and not ſeen. Accidens de la Vue, pa. 358. Smith's Optics, vol. 1, pa. 52, and Rem. pa. 51. Robins's Tracts, vol. 2, pa. 230, 247, 251. Porterfield on the Eye, vol. 1, pa. 105, vol. 2, pa. 387. See Prieſtley's Hiſt. of Viſion, pa. 205, and pa. 689.

DISTANCE, in Navigation, is the number of miles or leagues that a ſhip has ſailed from any point or place. See SAILING.



Line of DISTANCE, in Perſpective, is a right line drawn from the eye to the principal point: as the line OF, drawn between the eye at O, and the principal point F. As this is perpendicular to the plane, or table, it is therefore the diſtance of the eye from the table.

Point of DISTANCE, in Perſpective, is a point in the horizontal line at the ſame diſtance from the principal point, as the eye is from the ſame. Such are the points P and Q, in the horizontal line PQ, whoſe diſtance from the principal point F, is equal to that of the eye from the ſame F.

DISTINCT *Baſe*, in Optics, is that diſtance from the pole of a convex glaſs, at which objects, beheld through it, appear diſtinct and well defined: ſo that the Diſtinct baſe is the ſame with what is otherwiſe called the focus.

The Diſtinct baſe is cauſed by the collection of the rays proceeding from a ſingle point in the object, into a ſingle point in the repreſentation: and therefore concave glaſſes, which do not unite, but ſcatter and diſſipate the rays, can have no real Diſtinct baſe.

DISTINCT *Viſion*. See VISION.

DITCH, in Fortification, called alſo *Foſs*, and *Moat*, is a trench dug round the rampart, or wall of a fortified place, between the ſcarp and counterſcarp.

Ditches are either *dry*, or *wet*, that is having water in them; both of which have their particular advantages. The earth dug out of the ditch ſerves to raiſe the rampart.

The ditch in front ſhould be of ſuch breadth as that tall trees may not reach over it, being from 12 to 24 fathoms wide, and 7 or 8 feet deep. The ditches on the ſides are made ſmaller. But the moſt general rule is perhaps, that the dimenſions of the ditch be ſuch as that the earth dug out may be ſufficient to build the rampart of a proper magnitude. The ſpace ſometimes left between the rampart and ditch, being about 6 or 8 feet, is called the *berm*, or *liſt*, ſerving to paſs and repaſs, and to prevent the earth from rolling into the ditch.

DITONE, in Muſic, an interval comprehending two tones, a greater and a leſs. The ratio of the ſounds that form the Ditone, is of 4 to 5; and that of the ſemi-ditone, of 5 to 6.

DITTON (HUMPHREY), an eminent mathematician, was born at Saliſbury, May 29, 1675. Being an only ſon, and his father obſerving in him an extraordinary good capacity, determined to cultivate it with a good education. For this purpoſe he placed him in a reputable private academy; upon quitting of which, he, at the deſire of his father, though againſt his own inclination, engaged in the profeſſion of divinity, and began to exerciſe his function at Tunbridge in the county of Kent, where he continued to preach ſome years; during which time he married a lady of that place.

But a weak conſtitution, and the death of his father, induced Mr. Ditton to quit that profeſſion. And at the perſuaſion of Dr. Harris and Mr. Whiſton, both eminent mathematicians, he engaged in the ſtudy of mathematics, a ſcience to which he had always a ſtrong inclination. In the proſecution of this ſcience, he was much encouraged by the ſucceſs and applauſe he received: being greatly eſteemed by the chief profeſſors of it, and particularly by Sir Iſaac Newton, by whoſe intereſt and recommendation he was elected maſter of the new Mathematical School in Chriſt's Hoſpital; where he continued till his death, which happened in 1715, in the 40th year of his age, much regretted by the philoſophical world, who expected many uſeful and ingenious diſcoveries from his aſſiduity, learning, and penetrating genius.

Mr. Ditton publiſhed ſeveral mathematical and other tracts, as below.—1. *Of the Tangents of Curves*, &c. Philoſ. Tranſ. vol. 23.

2. A Treatiſe on *Spherical Catoptrics*, publiſhed in the Philoſ. Tranſ. for 1705; from whence it was copied and reprinted in the Acta Eruditorum 1707, and alſo in the Memoirs of the Academy of Sciences at Paris.

3. *General Laws of Nature and Motion;* 8vo, 1705. Wolfius mentions this work, and ſays, that it illuſtrates and renders eaſy the writings of Galileo, Huygens, and the Principia of Newton. It is alſo noticed by La Roche, in the Memoires de Literature, vol. 8, pa. 46.

4. *An Inſtitution of Fluxions, containing the firſt Principles, Operations, and Applications, of that admirable Method, as invented by Sir Iſaac Newton;* 8vo, 1706. This work, with additions and alterations, was again publiſhed by Mr. John Clarke, in the year 1726.

5. In 1709 he publiſhed the *Synopſis Algebraica* of John Alexander, with many additions and corrections.

6. His *Treatiſe on Perſpective* was publiſhed in 1712. In this work he explained the principles of that art mathematically; and beſides teaching the methods then generally practiſed, gave the firſt hints of the new

method afterward enlarged upon and improved by Dr. Brook Taylor; and which was publiſhed in the year 1715.

7. In 1714, Mr. Ditton publiſhed ſeveral pieces, both theological and mathematical; particularly his *Diſcourſe on the Reſurrection of Jeſus Chriſt; and The New Law of Fluids, or a Diſcourſe concerning the Aſcent of Liquids, in exact Geometrical Figures, between two nearly contiguous Surfaces.* To this was annexed a tract, to demonſtrate the impoſſibility of thinking or perception being the reſult of any combination of the parts of matter and motion: a ſubject much agitated about that time. To this work alſo was added an advertiſement from him and Mr. Whiſton, concerning a method for diſcovering the longitude, which it ſeems they had publiſhed about half a year before. This attempt probably coſt our author his life; for although it was approved and countenanced by Sir Iſaac Newton, before it was preſented to the Board of Longitude, and the method has been ſucceſsfully put in practice, in finding the longitude between Paris and Vienna, yet that Board then determined againſt it: ſo that the diſappointment, together with ſome public ridicule (particularly in a poem written by Dean Swift), affected his health, ſo that he died the enſuing year, 1715.

In an account of Mr. Ditton, prefixed to the German tranſlation of his Diſcourſe on the Reſurrection, it is ſaid that he had publiſhed, in his own name only, another method for finding the longitude; but which Mr. Whiſton denied. However, Raphael Levi, a learned Jew, who had ſtudied under Leibnitz, informed the German editor, that he well knew that Ditton and Leibnitz had correſponded upon the ſubject; and that Ditton had ſent to Leibnitz a delineation of a machine he had invented for that purpoſe; which was a piece of mechaniſm conſtructed with many wheels like a clock, and which Leibnitz highly approved of for land uſe; but doubted whether it would anſwer on ſhipboard, on account of the motion of the ſhip.

DIVERGENT *Point.* See *Virtual* FOCUS.

DIVERGENT, or DIVERGING *Lines*, in Geometry, are thoſe whoſe diſtance is continually increaſing.—Lines which diverge one way, converge the other way.

DIVERGENT, or DIVERGING, in Optics, is particularly applied to rays which, iſſuing from a radiant point, or having, in their paſſage, undergone a refraction, or reflexion, do continually recede farther from each other.

In this ſenſe the word is oppoſed to convergent, which implies that the rays approach each other, or that they tend to a centre, where they interſect, and, being continued, go on diverging. Indeed all interſecting rays, or lines, diverge both ways from the centre, or point of interſection.

Concave glaſſes render the rays diverging; and convex ones, converging.—Concave mirrors make the rays converge; and convex ones, diverge.—It is demonſtrated in Optics, that as the diameter of a pretty large pupil does not exceed $\frac{1}{5}$ of a digit; diverging rays, flowing from a radiant point, will enter the pupil as parallel, to all intents and purpoſes, if the diſtance of the radiant from the eye amount to 40,000 feet. See FOCUS, LIGHT, and VISION.

DIVERGING HYPERBOLA, is one whoſe legs turn their convexities toward each other, and run out quite contrary ways. See HYPERBOLA.

DIVERGING *Parabola.* See *Diverging* PARABOLA.

DIVIDEND, in Arithmetic, is the number given to be divided by ſome other number, called the *diviſor*. Or it is the number given to be divided, or ſeparated, into a certain number of equal parts, viz. as many as the diviſor contains units; and the number of ſuch equal parts is called the *quotient*. Or, more generally, the dividend contains the diviſor, as many times as the quotient contains unity.

The Dividend is the numerator of a fraction, whoſe denominator is the diviſor, and the quotient is the value of the fraction. Thus, $\frac{8}{2} = 4$, and $\frac{3}{4} = \cdot75$.

DIVIDUAL. By this name ſome authors diſtinguiſh the ſeveral parts of a dividend, from which each ſeparate figure of the quotient is found.

DIVING, the art, or act of deſcending under water, to conſiderable depths, and remaining there a competent time.

The uſes of Diving are very conſiderable, particularly in the fiſhing for pearls, corals, ſponges, &c.

Various methods have been propoſed, and engines contrived, to render the buſineſs of diving more ſafe and eaſy. The great point in all theſe, is to furniſh the diver with freſh air, without which he muſt either make but a ſhort ſtay, or periſh.

Thoſe who dive for ſponges in the Mediterranean, help themſelves by carrying down ſponges dipt in oil in their mouths. But conſidering the ſmall quantity of air that can be contained in the pores of a ſponge, and how much that little will be contracted by the preſſure of the incumbent water, ſuch a ſupply cannot long ſubſiſt the diver. For it is found by experiment, that a gallon of air included in a bladder, and by a pipe reciprocally inſpired and expired by the lungs, becomes unfit for reſpiration in little more than one minute of time. For though its elaſticity be but little altered in paſſing the lungs, yet it loſes its vivifying ſpirit, and is rendered effete. In effect, a naked diver, Dr. Halley aſſures us, without a ſponge, cannot remain above two minutes incloſed in water; nor much longer with one, without ſuffocating; nor without long practice, near ſo long; ordinary perſons beginning to be ſuffocated in about half a minute. Beſides, if the depth be conſiderable, the preſſure of the water on the veſſels makes the eyes blood ſhotten, and frequently occaſions a ſpitting of blood. Hence, where there has been occaſion to continue long at the bottom, ſome have contrived double flexible pipes, to circulate air down into a cavity incloſing the diver, as with armour, both to furniſh air, and to bear off the preſſure of the water, and give leave to his breaſt to dilate upon inſpiration; the freſh air being forced down one of the pipes with bellows, and returning by the other, not unlike to an artery, and vein.

But this method is impracticable when the depth exceeds three fathoms; the water embracing the bare limbs ſo cloſely, as to obſtruct the circulation of the blood in them; and withal preſſing ſo ſtrongly on all the junctures where the armour is made tight with leather; that if there be the leaſt defect in any of them, the water ruſhes in, and inſtantly fills the whole engine, to the great danger of the diver's life.

Diving-*Bell*, is a machine contrived to remedy all theſe inconveniencies. In this the diver is ſafely conveyed to any reaſonable depth, and may ſtay more or leſs time under the water, as the bell is greater or leſs. It is moſt conveniently made in form of a truncated cone, the ſmalleſt baſe being cloſed, and the larger open. It is to be poiſed with lead, and ſo ſuſpended, that it may ſink full of air, with its open baſis downward, and as near as may be in a ſituation parallel to the horizon, ſo as to cloſe with the ſurface of the water all at once.

Under this covercle the diver ſitting, ſinks down with the included air to the depth deſired; and if the cavity of the veſſel can contain a ton of water, a ſingle man may remain a full hour, without much inconvenience, at five or ſix fathoms deep. But the lower he goes, ſtill the more the included air contracts itſelf, according to the weight of the water that compreſſes it; ſo that at thirty-three feet deep, the bell becomes half full of water; the preſſure of the incumbent water being then equal to that of the atmoſphere; and at all other depths, the ſpace occupied by the compreſſed air in the upper part of its capacity, is to the ſpace filled with water, as thirty-three feet to the depth of the ſurface of the water in the bell below the common ſurface of it. And this condenſed air, being taken in with the breath, ſoon inſinuates itſelf into all the cavities of the body, and has no ill effect, provided the bell be permitted to deſcend ſo ſlowly as to allow time for that purpoſe.

One inconvenience that attends it, is found in the ears, within which there are cavities which open only outwards, and that by pores ſo ſmall, as not to give admiſſion even to the air itſelf, unleſs they be dilated and diſtended by a conſiderable force. Hence, on the firſt deſcent of the bell, a preſſure begins to be felt on each ear, which, by degrees, grows painful, till the force overcoming the obſtacle, what conſtringes theſe pores, yields to the preſſure, and letting ſome condenſed air ſlip in, preſently eaſe enſues. The bell deſcending lower, the pain is renewed, and afterwards it is again eaſed in the ſame manner. But the greateſt inconvenience of this engine is, that the water entering it, contracts the bulk of air into ſo ſmall a compaſs, that it ſoon heats, and becomes unfit for reſpiration: ſo that there is a neceſſity for its being drawn up to recruit it; beſides the uncomfortable abiding of the diver, who is almoſt covered with water.

To obviate the difficulties of the diving-bell, Dr. Halley, to whom we owe the preceding account, contrived ſome further apparatus, by which not only to recruit and refreſh the air from time to time, but alſo to keep the water wholly out of it at any depth; which he effected after the following manner:

His diving-bell (plate vii, fig. 6.) was of wood, three feet wide at top, five feet at bottom, and eight feet high, containing about ſixty-three cubic feet in its concavity, coated externally with lead ſo heavy, that it would ſink empty; a particular weight being diſtributed about its bottom R, to make it deſcend perpendicularly, and no otherwiſe. In the top was fixed a meniſcus glaſs D, concave downwards, like a window, to let in light from above; with a cock, as at B, to let out the hot air; and a circular ſeat, as at LM, for the divers to ſit on: and, below, about a yard under the bell, was a ſtage ſuſpended from it by three ropes, each charged with a hundred weight, to keep it ſteady, and for the divers to ſtand upon to do their buſineſs. The machine was ſuſpended from the maſt of a ſhip by a ſprit, which was ſecured by ſtays to the maſt-head, and was directed by braces to carry it overboard clear of the ſide of the ſhip, and to bring it in again.

To ſupply air to this bell when under water, he had a couple of barrels, as C, holding thirty-ſix gallons each, caſed with lead, ſo as to ſink empty, each having a bung-hole at bottom, to let in the water as they deſcended, and let it out again as they were drawn up. In the top of the barrels was another hole, to which was fixed a leathern pipe, or hoſe, well prepared with bees wax and oil, long enough to hang below the bung-hole; being kept down by a weight appended. So that the air driven to the upper part of the barrel by the encroachment of the water, in the deſcent, could not eſcape up this pipe, unleſs the lower end were lifted up.

Theſe air-barrels were fitted with tackle, to make them riſe and fall alternately, like two buckets; being directed in their deſcent by lines faſtened to the under edge of the bell: ſo that they came readily to the hand of a man placed on the ſtage, to receive them; and who taking up the ends of the pipes, as ſoon as they came above the ſurface of the water in the barrels, all the air included in the upper part of it was blown forcibly into the bell; the water taking its place.

One barrel thus received, and emptied; upon a ſignal given, it was drawn up, and at the ſame time the other let down; by which alternate ſucceſſion, freſh air was furniſhed ſo plentifully, that the learned Doctor himſelf was one of five, who were all together in nine or ten fathoms deep of water for above an hour and a half, without the leaſt inconvenience; the whole cavity of the bell being perfectly dry.

All the precaution he obſerved, was, to be let down gradually about twelve feet at a time, and then to ſtop, and drive out the water that had entered, by taking in three or four barrels of freſh air, before he deſcended farther. And, being arrived at the depth intended, he let out as much of the hot air that had been breathed, as each barrel would replace with cold, by means of the cock B, at the top of the bell, through whoſe aperture, though very ſmall, the air would ruſh with ſo much violence, as to make the ſurface of the ſea boil.

Thus, he found, any thing could be done that was required to be done underneath. And by taking off the ſtage, he could, for a ſpace as wide as the circuit of the bell, lay the bottom of the ſea ſo far dry as not to be over ſhoes in water. Beſides, by the glaſs window ſo much light was tranſmitted, that, when the ſea was clear, and eſpecially when the ſun ſhone, he could ſee perfectly well to write or read, much more to faſten, or lay hold of any thing under him that was to be taken up. And by the return of the air barrel he often ſent up orders written with an iron pen on a plate of lead, directing how he would be moved from place to place.

At other times, when the water was troubled and thick, it would be as dark as night below; but in ſuch caſes he was able to keep a candle burning in the bell.

Dr. Halley obſerves, that they were ſubject to one inconvenience in this bell; they felt at firſt a ſmall pain in their ears, as if the end of a tobacco pipe were thruſt into them; but after a little while there was a ſmall puff of air, with a little noiſe, and they were eaſy.

This he ſuppoſes to be occaſioned by the condenſed air ſhutting up a valve leading from ſome cavity in the ear, full of common air; but when the condenſed air preſſed harder, it forced the valve to yield, and filled every cavity. One of the divers, in order to prevent this preſſure, ſtopped his ear with a pledget of paper; which was puſhed in ſo far, that a ſurgeon could not extract it without great difficulty.

The ſame author intimates, that by an additional contrivance he has found it practicable for a diver to go out of the bell to a good diſtance from it; the air being conveyed to him in a continued ſtream by ſmall flexible pipes, which ſerve him as a clue to direct him back again to the bell. For this purpoſe, one end of theſe pipes, kept open againſt the preſſure of the ſea, by a ſmall ſpiral wire, and made tight without by painted leather, and ſheep's guts drawn over it, being open, was faſtened in the bell, as at P, to receive air, and the other end was fixed to a leaden cap on the man's head, reaching down below his ſhoulders, open at bottom, to ſerve him as a little bell, full of air, for him to breathe at his work, which would keep out the water from him, when at the level of the great bell, becauſe of the ſame denſity as the air in the great bell. But when he ſtooped down lower than the level of the great bell, he ſhut the cock F, to cut off the communication between the two bells. Phil. Tranſ. abr. vol. iv. part ii. p. 188, &c. vol. vi. p. 550, &c.

The air in this bell would ſerve him for a minute or two; and he might inſtantly change it, by raiſing himſelf above the great bell, and opening the cock F. The diver was furniſhed with a girdle of large leaden weights, and clogs of lead for the feet, which, with the weight of the leaden cap, kept him firm on the ground; he was alſo well clothed with thick flannels, which being firſt made wet, and then warmed in the bell by the heat of his body, kept off the chill of the cold water for a conſiderable time, when he was out of the bell.

Mr. Martin Triewald, F. R. S. and military architect to the king of Sweden, contrived to conſtruct a diving-bell on a ſmaller ſcale, and leſs expence, than that of Dr. Halley, and yet capable of anſwering the ſame intents and purpoſes. This bell, AB (fig. 7.) ſinks with leaden weights DD, ſuſpended from the bottom of it. It is made of copper, and tinned all over on the inſide; three ſtrong convex lenſes GGG, defended by the copper lids HHH, illuminate this bell. The iron plate E ſerves the diver to ſtand upon, when he is at work; this is ſuſpended by chains FFF, at ſuch a diſtance from the bottom of the bell, that when he ſtands upright, his head is juſt above the water in the bell, where he has the advantage of air fitter for reſpiration, than when he is much higher up; but as there is occaſion for the diver to be wholly in the bell, and conſequently his head in the upper part of it, Mr. Triewald has contrived, that, even there, after he has breathed the hot air as long as he well can, by means of a ſpiral copper tube placed cloſe to the inſide of the bell, he may draw the cooler and freſher air from the lowermoſt parts; for which purpoſe a flexible leather pipe, about two feet long, is fixed to the upper end of the tube at *b*; and to the other end of the pipe is faſtened an ivory mouth-piece, for the diver to hold in his mouth, by which to reſpire the air from below. We ſhall only remark, that as air rendered effete by reſpiration is ſomewhat heavier than common air, it muſt naturally ſubſide in the bell; but it may probably be reſtored by the agitation of the ſea-water, and thus become fitter for reſpiration. See Fixed Air. Phil. Tranſ. abr. vol. viii. p. 634. Or Deſaguliers's Exper. Phil. vol. ii. p. 220, &c.

The famous Corn. Drebell had an expedient in ſome reſpects ſuperior even to the diving bell, if what is related of it be true. He contrived not only a veſſel to be rowed under water, but alſo a liquor to be carried in the veſſel, which ſupplied the place of freſh air.

The veſſel was made for king James I. carrying twelve rowers, beſides the paſſengers. It was tried in the river Thames; and one of the perſons in that ſubmarine navigation, then living, told it one, from whom Mr. Boyle had the relation.

As to the liquor, Mr. Boyle aſſures us, he diſcovered by a phyſician, who married Drebell's daughter, that it was uſed from time to time, when the air in that ſubmarine boat was clogged by the breath of the company, and rendered unfit for reſpiration: at which time, by unſtopping the veſſel full of this liquor, he could ſpeedily reſtore to the troubled air ſuch a proportion of vital parts, as would make it ſerve again a good while. The ſecret of this liquor Drebell would never diſcloſe to above one perſon, who himſelf aſſured Mr. Boyle what it was. Boyle's Exp. Phyſ. Mech. of the Spring of the Air.

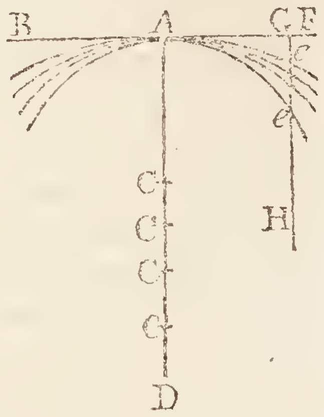
We have had many projects of diving machines, and diving ſhips of various kinds, which have proved abortive.

Diving-*Bladder*, a term uſed by Borelli for a machine which he contrived for diving under the water to great depths, with great facility, which he prefers to the common diving-bell. The veſica, or bladder, as it is uſually called, is to be of braſs or copper, and about two feet in diameter. This is to contain the diver's head, and is to be fixed to a goat's ſkin habit, exactly fitted to the ſhape of the body of the perſon. Within this veſica there are pipes, by means of which a circulation of air is contrived; and the perſon carries an air pump by his ſide, by means of which he may make himſelf heavier or lighter, as the fiſhes do, by contracting or dilating their air bladder: by this means, the objections all other diving machines are liable to are obviated, and particularly that of the air; the moiſture by which it is clogged in reſpiration, and by which it is rendered unfit for the ſame uſe again, being here taken from it by its circulation through the pipes, to the ſides of which it adheres, and leaves the air as free as before. Borelli Opera Poſthuma.

DIVISIBILITY, a property in quantity, body, or extenſion, by which it becomes ſeparable into parts; either actually, or at leaſt mentally.

Such diviſibility is infinite, if not actually, at leaſt potentially; for no part can be conceived ſo ſmall, but another may be conceived ſtill ſmaller; for every part of matter muſt have ſome finite extenſion, and that extenſion may be biſected, or otherwiſe divided; for the ſame reaſon, theſe parts may be divided again, and ſo on without end.

We are not here contending for the poſſibility of an actual diviſion *in infinitum*: it is only aſſerted that however ſmall a body is, it may be ſtill farther divided; which it is preſumed may be called a diviſion *in infinitum*, becauſe what has no limits, is called *infinite*.



The infinite, or indefinite diviſibility of mathematical quantity is thus proved, and illuſtrated by mathematicians: Suppoſe a line AD perpendicular to BF; and another as GH alſo perpendicular to the ſame BF; with the centres C, C, C, &c, and diſtances CA, CA, &c, deſcribe circles cutting the line GH in the points *e*, *e*, &c. Now, the greater the radius AC is, the leſs is the part *e*G; but the radius may be augmented in infinitum, and therefore the part *e*G may be diminiſhed in the ſame manner; and yet it can never be reduced to nothing, becauſe the circle can never coincide with the right line BF. Conſequently the parts of any magnitude may be diminiſhed in infinitum.

All that is ſuppoſed, in ſtrict geometry, concerning the diviſibility of magnitude, amounts to no more, than that a given magnitude may be conceived as divided into a number of parts, equal to any given or propoſed number.

It is true that there are no ſuch things as parts infinitely ſmall; yet the ſubtilty of the particles of ſeveral bodies is ſuch, that they far ſurpaſs our conception; and there are innumerable inſtances in nature of ſuch parts actually ſeparated from one another.

Several inſtances of this are given by Mr. Boyle. He ſpeaks of a ſilken thread 300 yards long, that weighed but two grains and a half. He meaſured leaf-gold, and found by weighing it, that 50 ſquare inches weighed but one grain: if the length of an inch be divided into 200 parts, the eye may diſtinguiſh them all; therefore in one ſquare inch there are 40,000 viſible parts; and in one grain of it there are two millions of ſuch parts; which viſible parts no one will deny are ſtill farther diviſible.

Again, an ounce weight of ſilver may be gilt over with 8 grains of gold, which may be afterwards drawn into a wire 13,000 feet long, and ſtill be all covered with the ſame gilding.

In odoriferous bodies a ſtill greater ſubtilty of parts is perceived, and even ſuch as are actually ſeparated from one another: ſeveral bodies ſcarce loſe any ſenſible part of their weight in a long time, and yet continually fill a very large ſpace with odoriferous particles. Dr. Keil, in his Vera Phyſica, Lect. 5, has calculated the magnitude of a particle of Aſſafœtida, which will be the $\frac{57}{10000000000000000}$th part of a cubic inch. And in the ſame Lecture he ſhews that the particles of the blood in animalculæ, obſerved in fluids by means of microſcopes, muſt be leſs than that part of a cubic inch which is expreſſed by a fraction whoſe numerator is 8, and denominator unity with 30 ciphers after it.

The particles of light, if light conſiſt of real particles, furniſh another ſurpriſing inſtance of the minuteneſs of ſome parts of matter. A ſmall lighted candle placed on a plain, will be viſible two miles, and conſequently its light fills a ſphere of 4 miles diameter, before it has loſt any ſenſible part of its weight. Now, as the force of any body is directly in proportion to its quantity of matter multiplied by its velocity; and ſince it is demonſtrated that the velocity of the particles of light is at leaſt a million of times greater than the velocity of a cannon-ball, it is plain, that if a million of theſe particles were round, and of the ſize of a ſmall grain of ſand, we durſt no more open our eyes to the light, than expoſe them to ſand ſhot point-blank from a cannon.

By help of microſcopes, ſuch objects as would otherwiſe eſcape our ſight, appear very large: there are ſome ſmall animals ſcarce viſible with the beſt microſcopes; and yet theſe have all the parts neceſſary for life, as blood, and other fluids. How wonderful then muſt the ſubtilty of the parts be, which make up ſuch fluids!

Whence is deducible the following theorem:

Any particle of matter, how ſmall ſoever, and any finite ſpace, how large ſoever, being given; it is poſſible for that ſmall ſand, or particle of matter, to be diffuſed through all that great ſpace, and to fill it in ſuch manner, as that there ſhall be no pore in it, whoſe diameter ſhall exceed any given line; as is demonſtrated by Dr. Keil. Introduct. ad Ver. Phyſ.

Divisible, the faculty or quality of being capable of being divided.

DIVISION, is one of the four principal Rules of Arithmetic, being that by which we find out how often one quantity is contained in another, ſo that Diviſion is in reality only a compendious method of Subtraction; its effect being to take one number from another as often as poſſible; that is, as often as it is contained in it. There are therefore three numbers concerned in Diviſion: 1ſt, That which is given to be divided, called the *dividend*; 2d, That by which the dividend is to be divided, called the *diviſor*; 3d, That which expreſſes how often the diviſor is contained in the dividend; or the number reſulting from the diviſion of the dividend by the diviſor, called the *quotient*.

There are various ways of performing Diviſion, one called the Engliſh, another the Flemiſh, another the Italian, another the Spaniſh, another the German, and another the Indian way, all equally juſt, as finding the quotient with the ſame certainty, and only differing in the manner of arranging and diſpoſing the numbers.

There is alſo diviſion in integers, diviſion in fractions, and diviſion in ſpecies, or algebra.

Diviſion is performed by ſeeking how often the diviſor is contained in the dividend; and when the latter conſiſts of a greater number of figures than the former, the dividend muſt be taken into parts, beginning on the left, and proceeding to the right, and ſeeking how often the diviſor is found in each of thoſe parts.

For ex. If it be required to divide 6758 by 3. Firſt ſeek how often 3 is contained in 6, which is 2 times; then how often in 7, which is likewiſe 2 times, with 1 remaining; which joined to the next figure 5 makes 15, then the 3's in 15 are 5 times; and laſtly the 3's in 8 are 2 times, and 2 remaining. All the numbers expreſſing how often 3 is contained in each of thoſe parts, are to be written down according to the order of the parts of the dividend, or from left to right, and ſeparated from the dividend itſelf by a crooked line, thus:

Diviſor Dividend Quotient

$$3\ )\ 6758\ (\ 2252\tfrac{2}{3}$$

It appears therefore, that 3 is contained 2252 times in 6758, with 2 remaining over; or that 6758 being divided into 3 parts, each part will be $2252\frac{2}{3}$, viz, the figures of the quotient before found, together with the fraction $\frac{2}{3}$ formed of the remainder and the diviſor.

When the diviſor is a ſingle digit, or even as large as the number 12, the diviſion is eaſily performed by ſetting down only the quotient as above. But when the diviſor is a larger number, it is neceſſary to ſet down the ſeveral remainders and products &c. This proceſs may be ſeen at large in moſt books of arithmetic, as well as various contractions adapted to particular caſes: ſuch as, 1ſt, when the diviſor has any number of ciphers at the end of it, they are cut off, as well as the ſame number of figures from the end of the dividend, and then the work is performed without them both, annexing only the figures laſt cut off, to the laſt remainder; 2d, when the diviſor is equal to the product of ſeveral ſingle digits, it is eaſier to divide ſucceſſively by thoſe digits, inſtead of the diviſor at once; 3d, when it is required to continue a quotient to a great many places of figures, as in decimals, a very expeditious method of performing it, is as follows: Suppoſe it were required to divide 1 by 29, to a great many places of decimals. Adding ciphers to the 1, firſt divide 10000 by 29 in the common way, till the remainder become a ſingle figure, and annex the fractional ſupplement to complete the quotient, which gives $\frac{1}{29} = 0{\cdot}03448\frac{8}{29}$: next multiply each of theſe by the numerator 8, ſo ſhall $\frac{8}{29} = 0{\cdot}27584\frac{64}{29}$ or rather $0{\cdot}27586\frac{6}{29}$; which figures ſubſtituted inſtead of the fraction $\frac{8}{29}$ in the firſt value of $\frac{1}{29}$, it becomes $\frac{1}{29} = 0{\cdot}0344827586\frac{6}{29}$: again, multiply both of theſe by the laſt numerator 6, and it will be $\frac{6}{29} = 0{\cdot}2068965517\frac{7}{29}$; which figures ſubſtituted for $\frac{6}{29}$ in the laſt-found value of $\frac{1}{29}$, it becomes $\frac{1}{29} = 0{\cdot}03448275862068965517\frac{7}{29}$: and again, multiplying theſe by the numerator 7, gives $\frac{7}{29} = 0{\cdot}24137931034482758620\frac{10}{29}$; which figures ſubſtituted inſtead of $\frac{7}{29}$ in the laſt-found value of $\frac{1}{29}$, it becomes $\frac{1}{29} =$ $0{\cdot}0344827586206896551724137931034482758620\frac{10}{29}$ and ſo on; where every operation will at leaſt double the number of figures before found by the laſt one.

Proof of Division. In every example of diviſion, unity is always in the ſame proportion to the diviſor, as the quotient is to the dividend; and therefore the product of the diviſor and quotient is equal to the product of 1 and the dividend, that is, the dividend itſelf. Hence, to prove diviſion, multiply the diviſor by the quotient, to the product add the remainder, and the ſum will be equal to the dividend when the work is right; if not, there is a miſtake.

Division, *in Decimal Fractions*, is performed the ſame way as in integers, regard being had to the number of decimals, viz, making as many in the quotient as thoſe of the dividend exceed thoſe in the diviſor.

Division, *in Vulgar Fractions*, is performed by dividing the numerators by each other, and the denominators by each other, if they will exactly divide; but if not, then the dividend is multiplied by the reciprocal of the diviſor, that is, having its terms inverted; for, taking the reciprocal of any quantity, converts it from a diviſor to a multiplier, and from a multiplier to a diviſor. For ex. $\frac{15}{16} \div$ by $\frac{5}{8}$ gives $\frac{3}{2}$, by dividing the numerators and denominators; but $\frac{15}{16} \div$ by $\frac{4}{3}$ is the ſame as $\frac{15}{16} \times \frac{3}{4}$, which is $= \frac{45}{64}$. Where × is the ſign of multiplication, and the character ÷ is the mark of diviſion. Or diviſion is alſo denoted like a vulgar fraction; ſo 3 divided by 2, is $\frac{3}{2}$.

Division, in *Algebra*, or *Species*, is performed like that of common numbers, either making a fraction of the dividend and diviſor, and cancelling or dividing by the terms or parts that are common to both; or elſe dividing after the manner of long diviſion, when the quantities are compound ones. Thus,

ab divided by a, gives b for the quotient:

and $12ab$ divided by $4b$, gives $3a$ for the quotient:

and $16abc^2$ divided by $8ac$, gives $2bc$:

and a divided by $3b$, gives $\dfrac{a}{3b}$:

and $15abc^3$ divided by $12bc^2$, gives $\dfrac{15abc^3}{12bc^2} = \dfrac{5ac}{4}$

and $a^2 - b^2$ by $a + b$ gives $a - b$; thus

$$\begin{array}{l} a+b\ )\ a^2 - b^2\ (\ a-b \\ \qquad\ \ a^2 + ab \\ \hline \qquad\ \ -ab - b^2 \\ \qquad\ \ -ab - b^2 \\ \hline \end{array}$$

again, to divide $x^6 - 8x^4 - 124x^2 - 64$ by $x^2 - 16$;

$$\begin{array}{l} x^2 - 16\ )\ x^6 - 8x^4 - 124x^2 - 64\ (\ x^4 + 8x^2 + 4 \\ \qquad\qquad x^6 - 16x^4 \\ \hline \qquad\qquad\qquad 8x^4 - 124x^2 \\ \qquad\qquad\qquad 8x^4 - 128x^2 \\ \hline \qquad\qquad\qquad\qquad 4x^2 - 64 \\ \qquad\qquad\qquad\qquad 4x^2 - 64 \\ \hline \end{array}$$

In ſome caſes, the quotient will run out to an infinite ſeries; and then, after continuing it to any certain number of terms, it is uſual to annex, by way of a fraction, the remainder with the diviſor ſet under it.

It is to be noted that, in dividing any terms by one another, if the ſigns be both alike, either both plus, or both minus, the ſign of the quotient will be plus; but when the ſigns are different, the one plus and the other minus, the ſign of the quotient will be minus.

Division *by Logarithms*. See Logarithms.

Division *of Mathematical Inſtruments*. See Graduation, and Mural *Arc* or *Quadrant*.

Division in Muſic, is the dividing the interval of an octave into a number of leſſer intervals.

Division *by Napier's Bones*. See Napier's *Bones*.

Division *of Powers*, is performed by ſubtracting their exponents. Thus, $a^6 \div a^4$ is $= a^2$; and $4a^{\frac{3}{4}}b^2 \div 2a^{\frac{1}{2}}b^{\frac{4}{3}}$ is $= 2a^{\frac{1}{4}}b^{\frac{2}{3}}$.

Division *of Proportion*, is comparing the difference between the antecedent and conſequent, with either of them. Thus,

$$\begin{array}{ll} \text{if} & \left\{\begin{array}{ccccccc} a & : & b & :: & c & : & d, \\ 12 & : & 4 & :: & 6 & : & 2, \end{array}\right. \\ \text{then by diviſion} & \left\{\begin{array}{ccccccc} a-b & : & a & :: & c-d & : & c, \\ 8 & : & 12 & :: & 4 & : & 6, \end{array}\right. \\ \text{or} & \left\{\begin{array}{ccccccc} a-b & : & b & :: & c-d & : & d, \\ 8 & : & 4 & :: & 4 & : & 2. \end{array}\right. \end{array}$$

Divisor, is the dividing number; or that which ſhews how many parts the dividend is to be divided into.

Common Divisor. See Common *Diviſor.*

DIURNAL, ſomething relating to the day; in oppoſition to nocturnal, relating to the night.

Diurnal *Arch*, is the arch deſcribed by the ſun, moon, or ſtars, between their riſing and ſetting.

Diurnal *Circle*, is the apparent circle deſcribed by the ſun, moon, or ſtars, in conſequence of the rotation of the earth.

Diurnal *Motion of a Planet*, is ſo many degrees and minutes &c as any planet moves in 24 hours.

The Diurnal *Motion of the Earth*, is its rotation round its axis, the duration of which conſtitutes the natural day.—The reality of the diurnal rotation of the earth is now paſt all diſpute.

Diurnal *Parallax*. See Parallax.

Diurnal is alſo uſed in ſpeaking of what belongs to the nycthemeron, or natural day of 24 hours: in which ſenſe it is oppoſed to annual, menſtrual, &c.

The diurnal phenomena of the heavenly bodies are ſolved from the diurnal revolution of the earth; that is, from the rotation of the earth round its own axis in 24 hours. This rotation is equable, and from weſt to eaſt, about an axis whoſe inclination to the ecliptic is now 66° 32′. Since the earth is an opaque body, that ſmall part of its ſurface which comes at the ſame time under the confined view of the ſpectator, though really ſpherical, will ſeem to be extended like a plane: and the eye, taking a view of the heavens all around, defines a concave ſpherical ſuperficies, concentric with the earth, or rather with the eye, which the ſaid plane of the earth's ſuperficies will divide into two equal parts, the one of which is viſible, but the other, becauſe of the earth's opacity, hid from the view.

And as the earth revolves about its axis, the ſpectator, ſtanding upon it, together with the ſaid plane he ſtands upon, called his horizon, dividing the viſible from the inviſible hemiſphere of the heavens, is carried round the ſame way, viz, towards the eaſt. From hence it is, that the ſun and ſtars, placed towards the eaſt, being before hid, now become viſible, the horizon as it were ſinking below them; and the ſtars &c towards the weſt are covered or hid, and become inviſible, the horizon being elevated above them. So that the former ſtars, to the ſpectator, who reckons the place he ſtands on as immoveable, will appear to aſcend above the horizon, or riſe; and the latter to deſcend below the horizon, or ſet.

Since the earth, with the horizon of the ſpectator fixed to it, continues to move always towards the ſame parts, and about the ſame axis equally; all bodies, and all phenomena, that do not partake of the ſaid motion, (that is, all ſuch things as are entirely ſeparate from the earth) will ſeem to move in the ſame time uniformly, but towards the oppoſite parts, or from eaſt to weſt: and every one of theſe objects, according to ſenſe, will deſcribe the circumference of a circle, whoſe plane is perpendicular to the axis of the earth. And becauſe all theſe circles, together with the viſible objects deſcribing them, appear to be in the concave ſpherical ſuperficies of the heavens, every viſible object will ſeem to deſcribe a greater or leſs circle, according to its greater or leſs diſtance from the poles, or extremities of the earth's axis produced; the middle circle between theſe poles, called the equator, is conſequently the greateſt.

It may farther be obſerved, that whereas, by the diurnal revolution of the earth, all the ſeveral luminaries ſeem to move in the heavens from eaſt to weſt, hence this ſeeming diurnal motion of the celeſtial lights is called their common motion, as being common to all of them. Beſides which, all the luminaries, except the ſun, have a proper motion; from which ariſe their proper phenomena: as for the proper phenomena of the ſun, they likewiſe ſeem to ariſe from the proper motion of the ſun; though they are really produced by another motion, which the earth has, and by which it moves round the ſun once every year, and thence called the annual motion of the earth.

DODECAGON, a regular polygon of 12 equal ſides and angles.

If the ſide of a Dodecagon be 1, its area will be equal to 3 times the tang. of 75° $= 3 \times \overline{2 + \sqrt{3}} =$ 11·1961524 nearly; and, the areas of plane figures being as the ſquares of their ſides, therefore 11·1961524 multiplied by the ſquare of the ſide of any Dodecagon, will give its area. See my Menſuration, pa. 114, 2d ed.

To inſcribe a Dodecagon in a given Circle. Carry the radius 6 times round the circumference, which will divide it into 6 equal parts, or will make a hexagon; then biſect each of thoſe parts, which will divide the whole into 12 parts, for the Dodecagon. See alſo other methods of deſcribing the ſame figure in my Menſur. pa. 26, &c. See Polygon.

DODECAHEDRON, one of the Platonic bodies, or five regular ſolids, being contained under a ſurface compoſed of twelve equal and regular pentagons.

To form a Dodecahedron. See *Regular* Body.

If the ſide, or linear edge, of a Dodecahedron be s, its ſurface will be

$$15s^2\sqrt{1+\tfrac{2}{5}\sqrt{5}} = 20{\cdot}6457788\,s^2$$

and its ſolidity $5s^3\sqrt{\dfrac{47+21\sqrt{5}}{40}} = 7{\cdot}66311896\,s^3$

If the radius of the ſphere that circumſcribes a Dodecahedron be r, then is

its ſide or linear edge $= \dfrac{\sqrt{15}-\sqrt{3}}{3}r,$

its ſuperficies $= 10r^2\sqrt{2-\tfrac{2}{5}\sqrt{5}},$

and its ſolidity $= \dfrac{20r^3}{3}\sqrt{\dfrac{3+\sqrt{5}}{30}}.$

The ſide of a Dodecahedron inſcribed in a ſphere, is equal to the greater part of the ſide of a cube inſcribed in the ſame ſphere, and cut according to extreme and mean proportion.

If a line be cut according to extreme and mean proportion, and the leſſer ſegment be taken for the ſide of a Dodecahedron, the greater ſegment will be the ſide of a cube inſcribed in the ſame ſphere.

The ſide of the cube is equal to the right line which ſubtends the angle of a pentagon of the Dodecahedron, inſcribed in the ſame ſphere. See POLYHEDRON; alſo my Menſur. pa. 253, &c.

DODECATEMORY, the 12 houſes or parts of the zodiac of the primum mobile. Alſo the 12 ſigns of the zodiac are ſometimes ſo called, becauſe they contain each the 12th part of the zodiac.

DOG, a name common to two conſtellations, called the Great and Little Dog; but more uſually Canis Major, and Canis Minor.

DOME, is a round, vaulted, or arched roof, of a church, hall, pavilion, veſtibule, ſtair-caſe, &c, by way of crowning, or acroter.

DOMINICAL *Letter*, otherwiſe called the SUNDAY *Letter*, is one of theſe firſt ſeven letters of the alphabet ABCDEFG, uſed in almanacs &c, to mark or denote the Sundays throughout the year.

The reaſon for uſing ſeven letters, is becauſe that is the number of days in a week; and the method of uſing them is this: the firſt letter A is ſet oppoſite the 1ſt day of the year, the 2d letter B oppoſite the 2d day of the year, the 3d letter C oppoſite the 3d day of the year, and ſo on through the ſeven letters; after which they are repeated over and over again, all the way to the end of the year, the letter A denoting the 8th day, the letter B the 9th, &c. Then whichever of the letters ſo placed, falls oppoſite the firſt Sunday in the year, the ſame letter, it is evident, will fall oppoſite every future Sunday throughout the year, becauſe the number of the letters is the ſame as the number of days in the week, being both 7 in number; that is, in common years; for as to leap years, an interruption of the order takes place in them. For, on account of the intercalary day, either the letters muſt be thruſt out of their places for the whole year afterwards, ſo as that the letter, for ex. which anſwers to the 1ſt of March, ſhall likewiſe anſwer to the 2d &c; or elſe the intercalary day muſt be denoted by the ſame letter as the preceding one. This latter expedient was judged the better, and accordingly all the Sundays in the year after the intercalary day have another Dominical letter.

The Dominical letters were introduced into the calendar by the primitive chriſtians, inſtead of the nundinal letters in the Roman calendar; and in this manner were thoſe ſeven letters ſet oppoſite the days of the year, to denote the days in the week, in moſt of our common almanacs, till the year 1771, when the initial letters of the days of the week were generally introduced inſtead of them, excepting the Sunday letter itſelf, which is ſtill retained.

From the foregoing account it follows that,

1ſt, As the common year conſiſts of 365 days, or 52 weeks, and one day over; the letters go one day backwards every common year: ſo that in ſuch a year, if the beginning or firſt day fall on a Sunday, the next year it will fall on Saturday, the next on Friday, and ſo on. Conſequently, if G be the Dominical letter for the preſent year, F will be that for the next year; and ſo on, in a retrograde order.

2d, As the biſſextile or leap year conſiſts of 366 days, or 52 weeks, and 2 days over, the beginning of the year next after biſſextile goes back 2 days. Whence, if in the beginning of the biſſextile year, the Dominical letter were G, that of the following year will be E.

3d, Since in leap-years the intercalary day falls on the 24th of February, in which caſe the 24th and 25th days are conſidered as one day, and denoted by the ſame letter; after the 24th day of February the Dominical letter goes back one place: thus, if in the beginning of the year the Dominical letter be G, it will afterwards change to the letter F for the remaining Sundays of the year. With us however, this day is now added at the end of February, and from thence it is that the change takes place.

4th, As every fourth year is biſſextile, or leap-year, and as the number of letters is 7; the ſame order of Dominical letters only returns in 4 times 7, or 28 years; which without the interruption of biſſextiles, would return in 7 years.

5th, Hence the invention of the ſolar cycle of 28 years; upon the expiration of which the Dominical letters are reſtored ſucceſſively to the ſame days of the month, or the ſame order of the letters returns.

To find the DOMINICAL *Letter of any given year.* Find the cycle of the ſun for that year, as directed under cycle; and the Dominical letter is found correſponding to it. When there are two letters, the propoſed year is biſſextile; the former of them ſerving till the end of February, and the latter for the reſt of the year.

The Dominical letter for any year of the preſent century may be found by this canon:

Divide the odd years, their fourth and 4, by 7,
What is left take from 7, the letter is given.

Thus, for the year 1794, the odd years 94
their 4th 23
and - 4
divided by 7) 121 (17
remains 2
from 7
leaves 5

which anſwers to E the 5th letter in the alphabet.

The Dominical letter may be found univerſally, for any year of any century, thus:

Divide the centuries by 4; and twice what does remain
Take from 6; and then add to the number you gain
Their odd years and their 4th; which dividing by 7,
What is left take from 7, the letter is given.

Thus, for the year 1878 the letter is F.

For the centuries 18 divided by 4, leave 2; the double of which taken from 6 leaves 2 again; to which add the odd years 78, and their 4th part 19, the ſum 99 divided by 7 leaves 1; which taken from 7, leaves 6 anſwering to F the 6th letter in the alphabet.

By the reformation of the calendar under pope Gregory the 13th, the order of the Dominical letters was again diſturbed in the Gregorian year; for the year

1582, which had G for its Dominical letter at the beginning; by retrenching 10 days after the 4th of October, came to have C for its Dominical letter: by which means the Dominical letter of the ancient Julian calendar is 4 places before that of the Gregorian, the letter A in the former anſwering to D in the latter.

DONJON, in Fortification, uſually denotes a large ſtrong tower, or redoubt, of a fortreſs, where the garriſon may retreat in caſe of neceſſity, and capitulate with greater advantage. See Dungeon.

DORADO, a ſouthern conſtellation, not viſible in our latitude; called alſo *Xiphias*, or the *Sword-fiſh*. The ſtars of this conſtellation, in Sharp's catalogue, are ſix.

DORIC *Order*, of Architecture, is the ſecond of the five orders, being placed between the Tuſcan and the Ionic. The Doric ſeems the moſt natural, and beſt proportioned, of all the orders; all its parts being founded on the natural poſition of ſolid bodies; for which reaſon it is moſt proper to be uſed in great and maſſy buildings, as the outſide of churches and public places.

The Doric order has no ornaments on its baſe, nor its capital. Its column is 8 diameters high, and its freeze is divided between triglyphs and metopes.

DORMER, or Dormant, in Architecture, denotes a window made in the roof of a building, or above the entablature; being raiſed upon the rafters.

DOUBLE *Aſpect*, *Baſtion*, *Concave*, *Convex*, *Cone*, *Deſcant*, *Eccentricity*, *Poſition*, *Ratio*, *Tenaille*, *&c*. See the reſpective words.

Double *Horizontal Dial*, one with a double gnomon, the one pointing out the hour on the outer circle, the other the hour on the ſtereographic projection drawn upon it. This dial finds the meridian, the hour, the ſun's place, riſing, ſetting, &c, and many other problems of the globe.

Double *Point*, in the Higher Geometry, is a point which is common to two parts or legs or branches of ſome curve of the 2d or higher order: ſuch as, an infinitely ſmall oval, or a cuſp, or the cruciform interſection, &c, of ſuch curves. See Newton's Enumeratio Linearum &c, de Curvarum Punctis Duplicibus.

Doubling *a Cape*, or *Point of Land*, in Navigation, ſignifies the coming up with it, paſſing by it, and leaving it behind the ſhip.

The Portugueſe pretend that they firſt doubled the Cape of Good Hope, under their admiral Vaſquez de Gama; but there are accounts in hiſtory, particularly in Herodotus, that the Egyptians, Carthaginians, &c, had done the ſame long before them.

DOUCINE, in Architecture, is an ornament on the higheſt part of the cornice, or a moulding cut in the figure of a wave, half convex, and half concave.

DOVE-*tail*, in Carpentry, is a method of faſtening boards or timbers together, by letting or indenting one piece into another, with a Dove-tail joint, or a joint in form of a Dove's tail.

DRACHM, or Dram, is the name of a ſmall weight uſed with us, and is of two kinds, viz, the 8th part of an ounce in Apothecaries weight, and the 16th part of an ounce in Avoirdupois weight.

DRACO, the Dragon, a conſtellation of the northern hemiſphere, whoſe origin is variouſly fabled by the Greeks; ſome of them repreſenting it as the Dragon which guarded the Heſperian fruit, or golden apples, but being killed by Hercules, Juno, as a reward for its faithful ſervices, took it up to heaven, and ſo formed this conſtellation; while others ſay, that in the war of the giants, this Dragon was brought into the combat, and oppoſed to Minerva, when the goddeſs taking the Dragon in her hand, threw him, twiſted as he was, up to the ſkies, and fixed him to the axis of the heavens, before he had time to unwind his contortions.

The ſtars in this conſtellation are, according to Ptolomy, 31; according to Tycho, 32; according to Hevelius, 40; according to Bayer, 33; and according to Flamſteed, 80.

DRAGON, in Aſtronomy. See Draco.

Dragon's *Head*, and *Tail*, are the nodes of the planets, but more particularly of the moon, being the points in which the ecliptic is interſected by her orbit, in an angle of about $5^\circ 18'$.

One of theſe points looks northward, the moon beginning then to have north latitude; and the other ſouthward, where ſhe commences ſouth latitude; the former point being repreſented by the knot ☊ for the head, and the other by the ſame reverſed, or ☋ for the tail. And near theſe points it is that all eclipſes of the ſun and moon happen.

This deviation of the path of the moon from the ecliptic ſeems, according to the fancy of ſome, to make a figure like that of a dragon, whoſe belly is at the part where ſhe has the greateſt latitude; the interſections repreſenting the head and tail, from which reſemblance the denomination ariſes.

Theſe interſections are not always in the ſame two points of the ecliptic, but ſhift by a retrograde motion, at the rate of $3' \, 11''$ per day, and completing their circle in 18 years 225 days.

Dragon-*Beams*, in Architecture, are two ſtrong braces or ſtruts, ſtanding under a breaſt-ſummer, and meeting in an angle on the ſhoulder of the king-piece.

DRAM. See Drachm.

DRAUGHT-*Compaſſes*, thoſe provided with ſeveral moveable points, to draw fine draughts in architecture &c. See Compass.

DRAUGHT-*Hooks*, are large hooks of iron, fixed on the cheeks of a gun-carriage, two on each ſide, one near the trunnion hole, and the other at the train.

DRAW-*Bridge*, a bridge made after the manner of a floor, to be drawn up, or let down, as occaſion requires, before the gate of a town or caſtle.

DRIFT, in Navigation, denotes the angle which the line of a ſhip's motion makes with the neareſt meridian, when ſhe drives with her ſide to the wind and waves, and is not governed by the power of the helm; and alſo the diſtance which the ſhip drives on that line, ſo called only in a ſtorm.

DRIP, in Architecture. See Larmier.

DRY-*Moat*. See Moat.

DUCTILITY, a property of certain bodies, by which they are capable of being beaten, preſſed, drawn, or ſtretched forth, without breaking; or by which they are capable of great alterations in their figure and dimenſions, and of gaining in one way as they loſe in another.

Such are metals, which, being urged by the hammer, gain in length and breadth what they loſe in thickneſs; or, being drawn into wire through holes in iron, grow longer as they become more ſlender. Such alſo are gums, glues, reſins, and ſome other bodies; which, though not malleable, may yet be denominated ductile, in as much as, when ſoftened by water, fire, or ſome other menſtruum, they may be drawn into threads.

Some bodies are ductile both when they are hot and cold, and in all circumſtances: ſuch are metals, and eſpecially gold and ſilver; other bodies are ductile only when they have a certain degree of heat; ſuch as glaſs, and wax, and ſuch like ſubſtances: others again are ductile only when cold, and brittle when hot; as ſome kinds of iron, viz, thoſe called by workmen redſhort, as alſo braſs, and ſome metallic alloys.

The cauſe of ductility is very obſcure; as depending much on hardneſs, a quality whoſe nature we know very little about. It is true, it is uſual to account for hardneſs from the force of attraction between the particles of the hard body; and for ductility, from the particles of the ductile body being, as it were jointed, and entangled with each other. But without dwelling on any fanciful hypotheſes about ductility, we may amuſe ourſelves with ſome truly amazing circumſtances and phenomena of it, in the inſtances of gold, glaſs, and ſpider's-webs. Obſerving however that the ductility of metals decreaſes in the following order: gold, ſilver, copper, iron, tin, lead.

Ductility *of Gold.* One of the properties of gold is, to be the moſt ductile of all bodies; of which the gold beaters and gold wire-drawers, furniſh us with abundant proof.

Fa. Merſenne, M. Rohault, Dr. Halley, &c, have made computations of it: but they truſted to the reports of the workmen. M. Reaumur, in the Memoires de l'Academie Royale des Sciences, an. 1713, took a ſurer way; he made the experiment himſelf. A ſingle grain of gold, he found, even in the common gold leaf, uſed in moſt of our gildings, is extended into 36 and a half ſquare inches; and an ounce of gold, which, in form of a cube, is not half an inch either high, broad, or long, is beat under the hammer into a ſurface of 146 and a half ſquare feet; an extent almoſt double to what could be done in former times. In Fa. Merſenne's time, it was deemed prodigious, that an ounce of gold ſhould form 1600 leaves; which, together, only made a ſurface of 105 ſquare feet.

But the diſtenſion of gold under the hammer (how conſiderable ſoever) is nothing to that which it undergoes in the drawing-iron. There are gold leaves, in ſome parts ſcarce the $\frac{1}{360000}$ part of an inch thick; but $\frac{1}{360000}$ part of an inch is a conſiderable thickneſs, in compariſon of that of the gold ſpun on ſilk in our gold thread.

To conceive this prodigious ductility, it is neceſſary to have ſome idea of the manner in which the wire drawers proceed. The wire, and thread we commonly call gold thread, &c, (which is only ſilver wire gilt, or covered over with gold), is drawn from a large ingot of ſilver, uſually about thirty pounds weight. This they round into a cylinder, or roll, about an inch and a half in diameter, and twenty-two inches long, and cover it with the leaves prepared by the gold beater, laying one over another, till the cover is a good deal thicker than that in our ordinary gilding; and yet, even then, it is very thin; as will be eaſily conceived from the quantity of gold that goes to gild the thirty pounds of ſilver: two ounces ordinarily do the buſineſs; and, frequently, little more than one.

In effect, the full thickneſs of the gold on the ingot rarely exceeds $\frac{1}{600}$ or $\frac{1}{500}$ part; and, ſometimes not $\frac{1}{1000}$ part of an inch.

But this thin coat of gold muſt be yet vaſtly thinner: the ingot is ſucceſſively drawn through the holes of ſeveral irons, each ſmaller than the other, till it be as fine as, or finer than a hair. Every new hole leſſens its diameter; but it gains in length what it loſes in thickneſs; and, of conſequence, increaſes in ſurface: yet the gold ſtill covers it; it follows the ſilver in all its extenſion, and never leaves the minuteſt part bare, not even to the microſcope. Yet, how inconceivably muſt it be attenuated while the ingot is drawn into a thread, whoſe diameter is 9000 times leſs than that of the ingot.

M. Reaumur, by exact weighing, and rigorous calculation, found, that one ounce of the thread was 3232 feet long; and the whole ingot 1163520 feet, Paris meaſure, or 96 French leagues; equal to 1264400 Engliſh feet, or 240 miles Engliſh; an extent which far ſurpaſſes what Fa. Merſenne, Furetiere, Dr. Halley, &c, ever dreamt of.

Merſenne ſays, that half an ounce of the thread is 100 toiſes, or fathoms long; on which footing, an ounce would only be 1200 feet: whereas, M. Reaumur finds it 3232. Dr. Halley makes 6 feet of the wire one grain in weight, and one grain of the gold 98 yards; and, conſequently, the ten thouſandth part of a grain, above one third of an inch. The diameter of the wire he found one-186th part of an inch; and the thickneſs of the gold one-154500th part of an inch. But this, too, comes ſhort of M. Reaumur; for, on this principle, the ounce of wire would only be 2680 feet.

But the ingot is not yet extended to its full length. The greateſt part of our gold thread is ſpun, or wound on ſilk; and, before it is ſpun, they flat it, by paſſing it between two rolls, or wheels of exceedingly well poliſhed ſteel; which wheels, in flatting it, lengthen it by above one ſeventh. So that our 240 miles are now got to 274.

The breadth, now, of theſe laminæ, or plates, M. Reaumur finds, is only one-8th of a line, or one-96th of an inch; and their thickneſs one-3072d. The ounce of gold, then, is here extended to a ſurface of 1190 ſquare feet; whereas, the utmoſt the gold beaters can do, we have obſerved, is to extend it to 146 ſquare feet. But the gold, thus exceedingly extended, how thin muſt it be! From M. Reaumur's calculus, it is found to be one-175000th of a line, or one-2100000th of an inch; which is ſcarce one-13th of the thickneſs of Dr. Halley's gold.

But he adds, that this ſuppoſes the thickneſs of the gold every where equal, which is no ways probable; for in beating the gold leaves, whatever care they can beſtow, it is impoſſible to extend them equally. This we eaſily find, by the greater opacity of ſome parts than others; for, where the leaf is thickeſt, it will gild the wire the thickeſt.

M. Reaumur, computing what the thickneſs of the gold muſt be where thinneſt, finds it only one-3150000th part of an inch. But what is the one-3150000th part of an inch? Yet this is not the utmoſt ductility of gold; for, inſtead of two ounces of gold to the ingot, which we have here computed upon, a ſingle one might have been uſed; and, then, the thickneſs of the gold, in the thinneſt places, would only be the 6300000th part of an inch.

And yet, as thin as the plates are, they might be made twice as thin, yet ſtill be gilt; by only preſſing them more between the flatter's wheels, they are extended to double the breadth and proportionably in length. So that their thickneſs, at laſt, will be reduced to one thirteen or fourteen millionth part of an inch.

Yet, with this amazing thinneſs of the gold, it is ſtill a perfect cover for the ſilver: the beſt eye, or even the beſt microſcope, cannot diſcover the leaſt chaſm, or diſcontinuity. There is not an aperture to admit alcohol of wine, the ſubtileſt fluid in nature, or even light itſelf, unleſs it be owing to cracks occaſioned by repeated ſtrokes of the hammer. Add, that if a piece of this gold thread, or gold plate be laid to diſſolve in aquafortis, the ſilver will be all excavated, or eat out, and the gold left entire, in little tubules.

It ſhould be obſerved, that gold, when it has been ſtruck for ſome time by a hammer, or violently compreſſed, as by gold wire drawers, becomes more hard, elaſtic and ſtiff, and leſs ductile, ſo that it is apt to be cracked or torn: the ſame thing happens to the other metals by percuſſion and compreſſion. But ductility and tractability may be reſtored to metals in that ſtate, by annealing them, or making them red hot. Gold ſeems to be more affected by percuſſion and annealing, than any other metals.

As to the DUCTILITY *of ſoft bodies*, it is not yet carried to that pitch. The reader, however, muſt not be ſurpriſed that, among the ductile bodies of this claſs, we give the firſt place to the moſt brittle of all other, glaſs.

DUCTILITY *of Glaſs*. We all know, that, when well penetrated with the heat of the fire, the workmen can figure and manage glaſs like ſoft wax; but what is moſt remarkable, it may be drawn, or ſpun out into threads exceedingly fine and long.

Our ordinary ſpinners do not form their threads of ſilk, flax, or the like, with half the eaſe, and expedition, as the glaſs ſpinners do threads of this brittle matter. We have ſome of them uſed in plumes for children's heads, and divers other works, much finer than any hair, and which bend and wave like it with every wind.

Nothing is more ſimple and eaſy than the method of making them: there are two workmen employed; the firſt holds one end of a piece of glaſs over the flame of a lamp; and, when the heat has ſoftened it, a ſecond operator applies a glaſs hook to the metal thus in fuſion; and, withdrawing the hook again, it brings with it a thread of glaſs, which ſtill adheres to the maſs: then, fitting his hook on the circumference of a wheel about two feet and a half in diameter, he turns the wheel as faſt as he pleaſes; which, drawing out the thread winds it on its rim; till, after a certain number of revolutions, it is covered with a ſkain of glaſs thread.

The maſs in fuſion over the lamp diminiſhes inſenſibly: being wound out, as it were, like a pelotoon, or clue of ſilk, upon the wheel; and the parts, as they recede from the flame, cooling, become more coherent to thoſe next to them; and this by degrees: the parts neareſt the fire are always the leaſt coherent, and, of conſequence, muſt give way to the effort the reſt make to draw them towards the wheel.

The circumference of theſe threads is uſually a flat oval, being three or four times as broad as thick: ſome of them ſeem ſcarce bigger than the thread of a ſilk worm, and are ſurpriſingly flexible. If the two ends of ſuch threads be knotted together, they may be drawn and bent, till the aperture, or ſpace in the middle of the knot, doth not exceed one-4th of a line, or one-48th of an inch diameter.

Hence M. Reaumur advances, that the flexibility of glaſs increaſes in proportion to the fineneſs of the threads; and that, probably, had we but the art of drawing threads as fine as a ſpider's web, we might weave ſtuffs and cloths of them for wear. Accordingly, he made ſome experiments this way: and found he could make threads fine enough, viz, as fine, in his judgment, as ſpider's thread, but he could never make them long enough to do any thing with them.

DUCTILITY *of Spider's-webs*. See WEB.

DUNGEON, DONJON, in Fortification, the higheſt part of a caſtle built after the ancient mode; ſerving as a watch-tower, or place of obſervation; and alſo for the retreat of a garriſon, in caſe of neceſſity, ſo that they may capitulate with greater advantage.

DUPLE, or DOUBLE *Ratio*, is that in which the antecedent term is double the conſequent; or, where the exponent of the ratio is 2. Thus, 6 to 3 is in a duple ratio.

Sub-DUPLE *Ratio*, is that in which the conſequent is double the antecedent; or, in which the exponent of the ratio is $\frac{1}{2}$. As in 3 to 6, which is in ſubduple ratio.

DUPLICATE *Ratio*, is the ſquare of a ratio, or the ratio of the ſquares of two quantities. Thus, the duplicate ratio of a to b, is the ratio of a^2 to b^2, or of the ſquare of a to the ſquare of b.—In a ſeries of geometrical proportionals, the 1ſt term is to the 3d, in a duplicate ratio of the 1ſt to the 2d, or as the ſquare of the firſt to the ſquare of the 2d: Thus, in the geometricals 2, 4, 8, 16, &c, the ratio of 2 to 8, is the duplicate of that of 2 to 4, or as the ſquare of 2 to the ſquare of 4, that is, as 4 to 16. So that duplicate ratio is the ratio of the ſquares, as triplicate ratio is the ratio of the cubes, &c.

DUPLICATION, is the doubling of a quantity, or multiplying it by 2, or adding it to itſelf.

DUPLICATION *of a Cube*, is finding out the ſide of a cube that ſhall be double in ſolidity to a given cube: which is a celebrated problem, much cultivated by the ancient geometricians, about 2000 years ago.

It was firſt propoſed by the oracle of Apollo at Delphos; which, being conſulted about the manner of ſtopping a plague then raging at Athens, returned for anſwer, that the plague ſhould ceaſe when Apollo's altar, which was cubical, ſhould be doubled. Upon this, they applied themſelves in good earneſt, to ſeek the duplicature of the cube, which from thence was called the *Delian problem*.

This problem cannot be effected geometrically, as it requires the ſolution of a cubic equation, or requires the finding of two mean proportionals, viz, between the ſide of the given cube and the double of the ſame, the firſt of which two mean proportionals is the ſide of the double cube, as was firſt obſerved by Hippocrates of Chios. For, let a be the ſide of the given cube, and z the ſide of the double cube ſought; then it is $z^3 = 2a^3$, or $a^2 : z^2 :: z : 2a$; ſo that, if a and z be the firſt and 2d terms of a ſet of continued proportionals, then $a^2 : z^2$ is the ratio of the ſquare of the 1ſt to the ſquare of the 2d, which, it is known, is the ſame as the ratio of the 1ſt term to the 3d, or of the 2d to the 4th, that is of z to $2a$; therefore z being the 2d term, $2a$ will be the 4th. So that z, the ſide of the cube ſought, is the 2d of four terms in continued proportion, the 1ſt and 4th being a and $2a$, that is, the ſide of the double cube is the firſt of two mean proportionals between a and $2a$.

Eutocius, in his Commentaries on Archimedes, gives ſeveral ways of performing this by the meſolabe. In Pappus too are found three different ways; the firſt according to Archimedes, the ſecond according to Hero, and the 3d by an inſtrument invented by Pappus, which gives all the proportions required. The ſieur de Comiers has likewiſe publiſhed a demonſtration of the ſame problem, by means of a compaſs with three legs. But all theſe methods are only mechanical. See Valerius Maximus, lib. 8; alſo Eutocius's Com. on lib. 2. Archimedes de Sphæra & Cylindro; and Pappus, lib. 3, prop. 5, & lib. 4, prop. 22.

DURER (ALBERT), deſcended of an Hungarian family, but born at Nuremberg, in 1471, was one of the beſt engravers, painters, and practical geometricians of his age. He was at the ſame time a man of letters and a philoſopher; and he was an intimate friend of Eraſmus, who reviſed ſome of the pieces which he publiſhed. He was alſo a man of buſineſs, and for many years the leading magiſtrate of Nuremberg.

Though not the inventor, he was one of the firſt and greateſt improvers of the art of engraving. He was however the inventor of cutting in wood, which he deviſed and practiſed in great perfection, uſing this way for expedition, as he had a multitude of deſigns to execute; and as his work was uſually done in the moſt exquiſite manner, his pieces took him up much time. For in many of thoſe prints which he executed on copper, the engraving is elegant to a great degree. His Hell-Scene in particular, engraven in the year 1513, is as highly finiſhed a print as ever was engraved, and as happily executed. In his wooden prints too it is ſurpriſing to ſee ſo much meaning in ſo early a maſter. In fact, Durer was a man of a very extenſive genius. His pictures were excellent; as well as his prints, which were very numerous. They were much admired, from the firſt, and eagerly bought up; which put his wife, who was another Xantippe, upon urging him to ſpend more time upon engraving than he was inclined to do: for he was rich; and choſe rather to practiſe his art as an amuſement, than as a buſineſs. He died at Nuremberg, in 1528, at 57 years of age.

Albert Durer wrote ſeveral books, in the German language, which were tranſlated into Latin by other perſons, and publiſhed after his death. viz,

1. His book upon the rules of painting, intitled, *De Symmetria Partium in rectis formis Humanorum Corporum*, is one of them: printed in folio, at Nuremberg, in 1532, and at Paris in 1557. An Italian verſion alſo was publiſhed at Venice, in 1591.
2. *Inſtitutiones Geometricæ*; Paris 1532.
3. *De Urbibus, Arcibus, Caſtelliſque condendis & muniendis*; Paris 1531.
4. *De Varietate Figurarum, et Flexuris Partium, et Geſtibus Imaginum*; Nuremberg 1534.

The figures in theſe books, which are from wooden plates, are very numerous, and moſt admirably well executed, indeed far beyond any thing of the kind done in our own days. Some of them alſo are of a very large ſize, as much as 16 inches in length, and of a proportional breadth, which being exquiſitely worked, muſt have coſt great labour. His geometry is chiefly of the practical kind, conſiſting of the moſt curious deſcriptions, inſcriptions, and circumſcriptions of geometrical lines, planes, and ſolids. We here meet, for the firſt time, with the plane figures, which folded up make the five regular or platonic bodies, as well as that curious conſtruction of a pentagon, being the laſt method in prob. 23 of my Menſuration.

DYE, in Architecture, the trunk of the pedeſtal, or that part between the baſe and the cornice, being ſo called, becauſe it is often made in the form of a dye or cube.

DYMANICS, is the ſcience of moving powers; more particularly of the motion of bodies that mutually act on one another. See MECHANICS, MOTION, COMMUNICATION *of Motion*, OSCILLATION, PERCUSSION, &c.

DYPTERE, or DIPTERE, was a kind of temple, encompaſſed round with a double row of columns; and the pſeudo-diptere, or falſe diptere, was the ſame, only this was encompaſſed with a ſingle row of columns, inſtead of a double row.

E.

EAGLE, *Aquila*, is a conſtellation of the northern hemiſphere, having its right wing contiguous to the equinoctial. For the ſtars in this conſtellation, ſee Aquila.

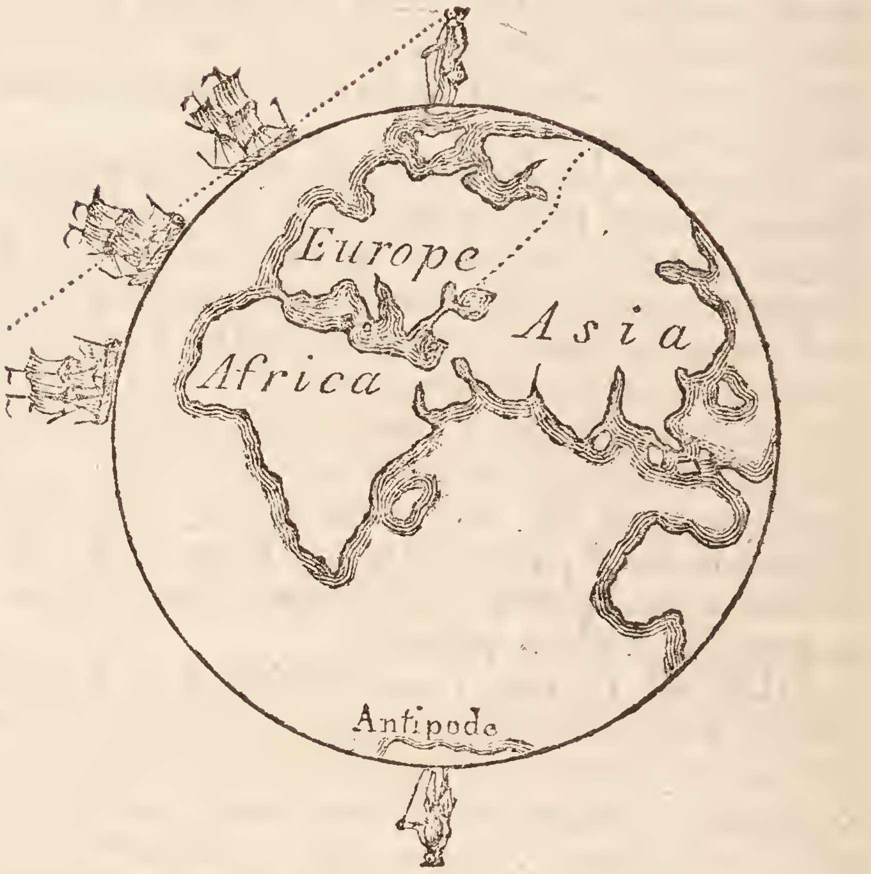
EARTH, *Terra*, in Natural Philoſophy, one of the four vulgar, or Peripatetical elements; defined a ſimple, dry, and cold ſubſtance; and, as ſuch, an ingredient in the compoſition of all natural bodies.

The Earth, in Geography, this terraqueous globe or ball, which we inhabit, conſiſting of land and ſea.

Figure of the Earth. The ancients had various opinions as to the figure of the earth: ſome, as Anaximander and Leucippus, held it cylindrical, or in form of a drum: but the principal opinion was, that it was flat; that the viſible horizon was the bounds of the earth, and the ocean the bounds of the horizon; that the heavens and earth above this ocean were the whole viſible univerſe; and that all beneath the ocean was Hades: and of this ſame opinion were alſo ſome of the Chriſtian fathers, as Lactantius, St. Auguſtine, &c. See Lactan. lib. 3, cap. 24; St. Aug. lib. 16, de Civitate Dei; Ariſtotle de Cœlo, lib. 2, cap. 13.

Such of the ancients however as underſtood any thing of aſtronomy, and eſpecially the doctrine of eclipſes, muſt have been acquainted with the round figure of the earth; as the ancient Babylonian aſtronomers, who had calculated eclipſes long before the time of Alexander, and Thales the Grecian, who predicted an eclipſe of the ſun. It is now indeed agreed on all hands, unleſs perhaps by the moſt vulgar and ignorant, that the form of the terraqueous globe is globular, or very nearly ſo.

That the exterior of the earth is round, or rotund, is manifeſt to the moſt common perception, in the caſe of a ſhip ſailing either from the land, or towards it; for when a perſon ſtands upon the ſhore, and ſees a ſhip ſail from the land, out to ſea; at firſt he loſes ſight of the hull and lower parts of the ſhip, next the rigging and middle parts, and laſtly of the tops of the maſts themſelves, in every caſe the rotundity of the ſea between the ſhip and the eye being very viſible: the contrary happens when a ſhip ſails towards us; we firſt ſee the tops of the maſts appear juſt over the rotundity of the ſea; next we perceive the rigging, and laſtly the hull of the ſhip itſelf: all which is well illuſtrated by the following figure.



The round figure of the earth is alſo evident from the eclipſes of the ſun and moon; for in all eclipſes of the moon, which are cauſed by the moon paſſing through the earth's ſhadow, that ſhadow always appears circular upon the face of the moon, what way ſoever it be projected, whether eaſt, weſt, north, or ſouth, and howſoever its diameter vary, according to the greater or leſs diſtance from the earth. Hence it follows, that the ſhadow of the earth, in all ſituations, is really conical; and conſequently the body that projects it, i. e. the earth, is at leaſt nearly ſpherical.

The ſpherical figure of the earth is alſo evinced from the riſing and ſetting of the ſun, moon, and ſtars; all which happen ſooner to thoſe who live to the eaſt, and later to thoſe living weſtwardly; and that more or leſs ſo, according to the diſtance and roundneſs of the earth.

So alſo, going or ſailing to the northward, the north pole and northern ſtars become more elevated, and the ſouth pole and ſouthern ſtars more depreſſed; the elevation northerly increaſing equally with the depreſſion ſoutherly; and either of them proportionably to the diſtance gone. The ſame thing happens in going to the ſouthward. Beſides, the oblique aſcenſions, deſcenſions,

ſcenſions, emerſions, and amplitudes of the riſing and ſetting of the ſun and ſtars in every latitude, are agreeable to the ſuppoſition of the earth's being of a ſpherical form: all which could not happen if it was of any other figure.

Moreover, the roundneſs of the earth is farther confirmed by its having been often ſailed round: the firſt time was in the year 1519, when Ferd. Magellan made the tour of the whole globe in 1124 days. In the year 1557 Francis Drake performed the ſame in 1056 days: in the year 1586, Sir Tho. Cavendiſh made the ſame voyage in 777 days; Simon Cordes, of Rotterdam, in the year 1590; in the year 1598, Oliver Noort, a Hollander, in 1077 days; Van Schouten, in the year 1615, in 749 days; Jac. Heremites and Joh. Huygens, in the year 1623, in 802 days: and many others have ſince performed the ſame navigation, particularly Anſon, Bougainville, and Cook. Sometimes ſailing round by the eaſtward, ſometimes to the weſtward; till at length they arrived again in Europe, from whence they ſet out; and in the courſe of their voyage, obſerved that all the phenomena, both of the heavens and earth, correſpond to, and evince this ſpherical figure.

The ſame globular figure is likewiſe inferred from the operation of Levelling, in which it is found neceſſary to make an allowance for the difference between the apparent and the true level.

The natural cauſe of this ſphericity of the globe is, according to Sir Iſaac Newton, the great principle of attraction, which the Creator has ſtamped on all the matter in the univerſe; and by which all bodies, and all the parts of bodies, mutually attract one another.—And the ſame is the cauſe of the ſphericity of the drops of rain, quickſilver, &c.

What the earth loſes of its ſphericity by mountains and valleys, is nothing conſiderable; the higheſt eminence being ſcarce equivalent to the minuteſt protuberance on the ſurface of an orange. Its difference from a perfect ſphere however is more conſiderable in another reſpect, by which it approaches nearly to the ſhape of an orange, or to an oblate ſpheroid, being a little flatted at the poles, and raiſed about the equatorial parts, ſo that the axis from pole to pole is leſs than the equatorial diameter. What gave the firſt occaſion to the diſcovery of this figure of the earth, was the obſervations of ſome French and Engliſh philoſophers in the Eaſt-Indies, and other parts, who found that pendulums, the nearer they came to the equator, performed their vibrations ſlower: from whence it follows, that the velocity of the deſcent of bodies by gravity, is leſs in countries nearer to the equator; and conſequently that thoſe parts are farther removed from the centre of the earth, or from the common centre of gravity. See the Hiſtory of the Royal Academy of Sciences, by Du Hamel, p. 110, 156, 206; and l'Hiſt. de l'Acad. Roy. 1700 and 1701.—This circumſtance put Huygens and Newton upon finding out the cauſe, which they attributed to the revolution of the earth about its axis. If the earth were in a fluid ſtate, its rotation round its axis would neceſſarily make it put on ſuch a figure, becauſe the centrifugal force being greateſt towards the equator, the fluid would there riſe and ſwell moſt; and that its figure really ſhould be ſo now, ſeems neceſſary, to keep the ſea in the equinoctial regions from overflowing the earth about thoſe parts. See this curious ſubject well handled by Huygens, in his diſcourſe De Cauſa Gravitatis, pa. 154, where he ſtates the ratio of the polar diameter to that of the equator, as 577 to 578. And Newton, in his Principia, firſt publiſhed in 1686, demonſtrates, from the theory of gravity, that the figure of the earth muſt be that of an oblate ſpheroid generated by the rotation of an ellipſe about its ſhorteſt diameter, provided all the parts of the earth were of an uniform denſity throughout, and that the proportion of the polar to the equatorial diameter of the earth, would be that of 689 to 692, or nearly that of 229 to 230, or as ·9956522 to 1.

This proportion of the two diameters was calculated by Newton in the following manner. Having found that the centrifugal force at the equator is $\frac{1}{289}$th of gravity, he aſſumes, as an hypotheſis, that the axis of the earth is to the diameter of the equator as 100 to 101, and thence determines what muſt be the centrifugal force at the equator to give the earth ſuch a form, and finds it to be $\frac{4}{505}$ths of gravity: then, by the rule of proportion, if a centrifugal force equal to $\frac{4}{505}$ths of gravity would make the earth higher at the equator than at the poles by $\frac{1}{100}$th of the whole height at the poles, a centrifugal force that is the $\frac{1}{289}$th of gravity will make it higher by a proportional exceſs, which by calculation is $\frac{1}{229}$th of the height at the poles; and thus he diſcovered that the diameter at the equator is to the diameter at the poles, or the axis, as 230 to 229. But this computation ſuppoſes the earth to be every where of an uniform denſity; whereas if the earth is more denſe near the centre, then bodies at the poles will be more attracted by this additional matter being nearer; and therefore the exceſs of the ſemi-diameter of the equator above the ſemi-axis, will be different. According to this proportion between the two diameters, Newton farther computes, from the different meaſures of a degree, that the equatorial diameter will exceed the polar, by 34 miles and $\frac{1}{5}$.

Nevertheleſs, Meſſrs Caſſini, both father and ſon, the one in 1701, and the other in 1713, attempted to prove, in the Memoirs of the Royal Academy of Sciences, that the earth was an oblong ſpheroid; and in 1718, M. Caſſini again undertook, from obſervations, to ſhew that, on the contrary, the longeſt diameter paſſes through the poles; which gave occaſion for Mr. John Bernoulli, in his Eſſai d'une Nouvelle Phyſique Celeſte, printed at Paris in 1735, to triumph over the Britiſh philoſopher, apprehending that theſe obſervations would invalidate what Newton had demonſtrated. And in 1720, M. De Mairan advanced arguments, ſuppoſed to be ſtrengthened by geometrical demonſtrations, farther to confirm the aſſertions of Caſſini. But in 1735 two companies of mathematicians were employed, one for a northern, and another for a ſouthern expedition, the reſult of whoſe obſervations and meaſurement plainly proved that the earth was flatted at the poles. See Degree.

The proportion of the equatorial diameter to the polar, as ſtated by the gentlemen employed on the northern expedition for meaſuring a degree of the meridian, is as 1 to 0·9891; by the Spaniſh mathematicians as 266 to 265, or as 1 to 0·99624; by M. Bouguer as 179 to 178, or as 1 to 0·99441.

As to all concluſions however deduced from the length of pendulums in different places, it is to be obſerved that they proceed upon the ſuppoſition of the uniform denſity of the earth, which is a very improbable circumſtance; as juſtly obſerved by Dr. Horſley in his letter to Capt. Phipps. "You finiſh your article, he concludes, relating to the pendulum with ſaying, 'that theſe obſervations give a figure of the earth nearer to Sir Iſaac Newton's computation, than any others that have hitherto been made;' and then you ſtate the ſeveral figures given, as you imagine, by former obſervations, and by your own. Now it is very true, that *if* the meridians be ellipſes, or *if* the figure of the earth be that of a ſpheroid generated by the revolution of an ellipſis, turning on its ſhorter axis, the particular figure, or the ellipticity of the generating ellipſis, which your obſervations give, is nearer to what Sir Iſaac Newton ſaith it ſhould be, if the globe were homogeneous, than any that can be derived from former obſervations. But yet it is not what you imagine. Taking the gain of the pendulum in latitude 79° 50′ exactly as you ſtate it, the difference between the equatorial and the polar diameter, is about as much leſs than the Newtonian computation makes it, and the hypotheſis of homogeneity would require, as you reckon it to be greater. The proportion of 212 to 211 ſhould indeed, according to your obſervations, be the proportion of the force that acts upon the pendulum at the poles, to the force acting upon it at the equator. But this is by no means the ſame with the proportion of the equatorial diameter to the polar. If the globe were homogeneous, the equatorial diameter would exceed the polar by $\frac{1}{230}$ of the length of the latter: and the polar force would alſo exceed the equatorial by the like part. But if the difference between the polar and equatorial force be greater than $\frac{1}{230}$, (which may be the caſe in an heterogeneous globe, and ſeems to be the caſe in ours,) then the difference of the diameters ſhould, according to theory, be leſs than $\frac{1}{230}$, and vice verſa.

"I confeſs this is by no means obvious at firſt ſight; ſo far otherwiſe, that the miſtake, which you have fallen into, was once very general. Many of the beſt mathematicians were miſled by too implicit a reliance upon the authority of Newton, who had certainly confined his inveſtigations to the homogeneous ſpheroid, and had thought about the heterogeneous only in a looſe and general way. The late Mr. Clairault was the firſt who ſet the matter right, in his elegant and ſubtle treatiſe on the figure of the earth. That work hath now been many years in the hands of mathematicians, among whom I imagine there are none, who have conſidered the ſubject attentively, that do not acquieſce in the author's concluſions.

"In the 2d part of that treatiſe, it is proved, that putting P for the polar force, Π for the equatorial, δ for the true ellipticity of the earth's figure, and ε for the ellipticity of the homogeneous ſpheriod,

$$\frac{P - \Pi}{\Pi} = 2\varepsilon - \delta : \text{therefore } \delta = 2\varepsilon - \frac{P - \Pi}{\Pi};$$

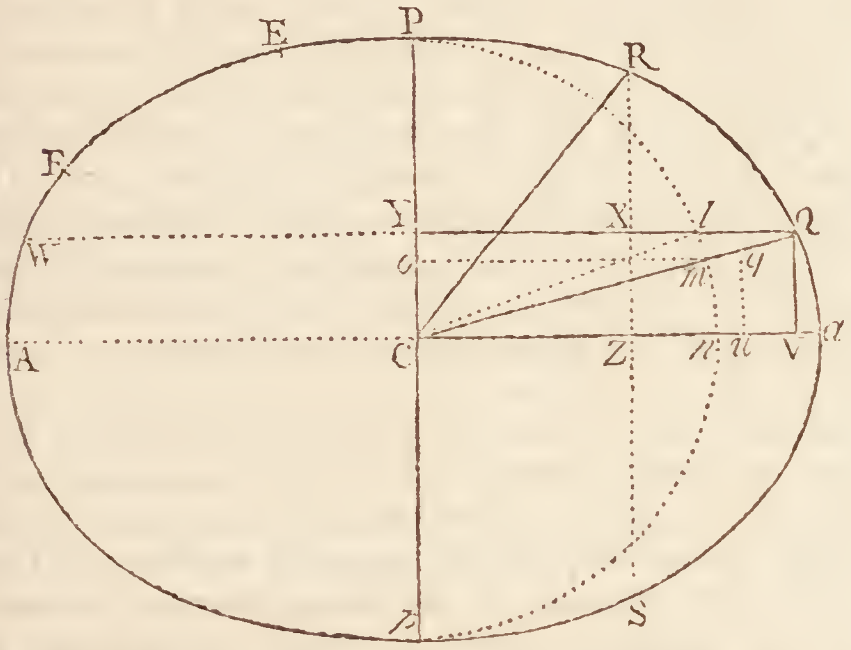
and therefore, according to your obſervation, $\delta = \frac{1}{251}$. This is the juſt concluſion from your obſervations of the pendulum, taking it for granted, that the meridians are ellipſes: which is an hypotheſis, upon which all the reaſonings of theory have hitherto proceeded. But plauſible as it may ſeem, I muſt ſay, that there is much reaſon from experiment to call it in queſtion. If it were true, the increment of the force which actuates the pendulum, as we approach the poles, ſhould be as the ſquare of the ſine of the latitude: or, which is the ſame thing, the decrement, as we approach the equator, ſhould be as the ſquare of the coſine of the latitude. But whoever takes the pains to compare together ſuch of the obſervations of the pendulum in different latitudes, as ſeem to have been made with the greateſt care, will find that the increments and decrements do by no means follow theſe proportions; and in thoſe which I have examined, I find a regularity in the deviation which little reſembles the mere error of obſervation. The unavoidable concluſion is, that the true figure of the meridians is not elliptical. If the meridians are not ellipſes, the difference of the diameters may indeed, or it may not, be proportional to the difference between the polar and the equatorial force; but it is quite an uncertainty, what relation ſubſiſts between the one quantity and the other; our whole theory, except ſo far as it relates to the homogeneous ſpheroid, is built upon falſe aſſumptions, and there is no ſaying what figure of the earth any obſervations of the pendulum give."

He then lays down the following table, which ſhews the different reſults of obſervations made in different latitudes; in which the firſt three columns contain the names of the ſeveral obſervers, the places of obſervation, and the latitude of each; the 4th column ſhews the quantity of $P - \Pi$ in ſuch parts as Π is 100000, as deduced from comparing the length of the pendulum at each place of obſervation, with the length of the equatorial pendulum as determined by M. Bouguer, upon the ſuppoſition that the increments and decrements of force, as the latitude is increaſed or lowered, obſerve the proportion which theory aſſigns. Only the 2d and the laſt value of $P - \Pi$ are concluded from comparifons with the pendulum at Greenwich and at London, not at the equator. The 5th column ſhews the value of δ correſponding to every value of $P - \Pi$, according to Clairault's theorem:

Obſervers.	*Places.*	*Latitudes.*	$P - \Pi$	δ
Bouguer	Equator	0° 0′		
Bouguer	Porto Bello	9 34	741·8	$\frac{1}{784}$
Green	Otaheitee	17 29	563·2	$\frac{1}{326}$
Bouguer	San Domingo	18 27	591·0	$\frac{1}{358}$
Abbé de la Caille	Cape of Good Hope	33 55	731·5	
- - - -	Paris	48 50	585·1	$\frac{1}{351}$
The Academicians	Pello	66 48	565·9	$\frac{1}{329}$
Capt. Phipps	- - - -	79 50	471·2	$\frac{1}{251}$

"By this table it appears, that the obſervations in the middle parts of the globe, ſetting aſide the ſingle one at the Cape, are as conſiſtent as could reaſonably be expected; and they repreſent the ellipticity of the earth as about $\frac{1}{340}$. But when we come within 10 degrees of the equator, it ſhould ſeem that the force of gravity ſuddenly becomes much leſs, and within the like diſtance of the poles much greater than it could be in ſuch a ſpheroid."

The following problem, communicated by Dr Leatherland to Dr. Pemberton, and publiſhed by Mr. Robertſon, ſerves for finding the proportion between the axis and the equatorial diameter, from meaſures taken of a degree of the meridian in two different latitudes, ſuppoſing the earth an oblate ſpheroid.



Let AP*ap* be an ellipſe repreſenting a ſection of the earth through the axis P*p*; the equatorial diameter, or the greater axis of the ellipſe, being A*a*; let E and F be two places where the meaſure of a degree has been taken; theſe meaſures are proportional to the radii of curvature in the ellipſe at thoſe places; and if CQ, CR be conjugates to the diameters whoſe vertices are E and F, CQ will be to CR in the ſubtriplicate ratio of the radius of curvature at E to that at F, by Cor. 1, prop. 4, part 6 of Milnes's Conic Sections, and therefore in a given ratio to one another; alſo the angles QCP, RCP are the latitudes of E and F; ſo that, drawing QV parallel to P*p*, and QXYW to A*a*, theſe angles being given, as well as the ratio of CQ to CR, the rectilinear figure CVQXRY is given in ſpecies; and the ratio of $VC^2 - ZC^2$ ($= QX \times XW$) to $RZ^2 - QV^2$ ($= RX \times XS$) is given, which is the ratio of CA^2 to CP^2; therefore the ratio of CA to CP is given.

Hence, if the ſine and coſine of the greater latitude be each augmented in the ſubtriplicate ratio of the meaſure of the degree in the greater latitude to that in the leſſer, then the difference of the ſquares of the augmented ſine, and the ſine of the leſſer latitude, will be to the difference of the ſquares of the coſine of the leſſer latitude and the augmented coſine, in the duplicate ratio of the equatorial to the polar diameter. For, C*q* being taken in CQ equal to CR, and *qv* drawn parallel to QV, C*v* and *vq*, CZ and ZR will be the ſigns and coſines of the reſpective latitudes to the ſame radius; and CV, VQ will be the augmentations of C*v* and C*q* in the ratio named.

Hence, to find the ratio between the two axes of the earth, let E denote the greater, and F the leſſer of the two latitudes, M and N the reſpective meaſures taken in each; and let P denote $\sqrt[3]{\frac{M}{N}}$: then

$$\sqrt{\frac{\text{coſ.}^2\,F - P^3 \times \text{coſ.}^2\,E}{P^2 \times \text{ſin.}^2\,E - \text{ſin.}^2\,F}} \text{ is } = \frac{\text{leſſer axis}}{\text{greater axis}}.$$

It alſo appears by the above problem, that when one of the degrees meaſured, is at the equator, the coſine of the latitude of the other being augmented in the ſubtriplicate ratio of the degrees, the tangent of the latitude will be to the tangent anſwering to the augmented coſine, in the ratio of the greater axis to the leſs. For ſuppoſing E the place out of the equator; then if the ſemi-circle P*lmnp* be deſcribed, and *l*C joined, and *mo* drawn parallel to *a*C: C*o* is the coſine of the latitude to the radius CP, and CY that coſine augmented in the ratio before-named; YQ being to Y*l*, that is C*a* to C*n* or CP, as the tangent of the angle YCQ, the latitude of the point E, to the tangent of the angle YC*l*, belonging to the augmented coſine. Thus, if M repreſent the meaſure in a latitude denoted by E, and N the meaſure at the equator, let A denote an angle whoſe meaſure is

$$\text{coſ. } E \times \sqrt[3]{\frac{M}{N}}. \text{ Then } \frac{\text{tan. } A}{\text{tan. } E} \text{ is } = \frac{\text{leſſer axis}}{\text{greater axis}}.$$

But M, or the length of a degree, obtained by actual menſuration in different latitudes, is known from the following table:

Names.	*Latit.* ° ′	*Value of M.* toiſes	
Maupertuis and Aſſoc.	66 20	M = 57438	
Caſſini and La Caille	49 22	M = 57074	
	45 00	M = 57050	
Boſcovich	43 00	M = 56972	
De la Caille	33 18	M = 57037	
Juan and Ulloa	00 00	M = 56768	at the equator.
Bouguer	00 00	M = 56753	
Condamine	00 00	M = 56749	

Now, by comparing the 1ſt with each of the following ones; the 2d with each of the following; and in like manner the 3d, 4th, and 5th, with each of the following; there will be obtained 25 reſults, each ſhewing the relation of the axes or diameters; the arithmetical means of all of which will give that ratio as 1 to 0·9951989.

If the meaſures of the latitudes of 49° 22′, and of 45°, which fall within the meridian line drawn through France, and which have been re-examined and corrected ſince the northern and ſouthern expedition, be compared with thoſe of Maupertuis and his aſſociates in the north, and that of Bouguer at the equator, there will reſult 6 different values of the ratio of the two axes, the arithmetical mean of all which is that of 1 to 0·9953467, which may be conſidered as the ratio of the greater axis to the leſs; which is as 230 to 228·92974, or 215 to 214, or very near the ratio as aſſigned by Newton.

Now, the magnitude as well as the figure of the earth, that is the polar and equatorial diameters, may be deduced from the foregoing problem. For, as half the latus rectum of the greater axis A*a* is the radius of curvature at A, it is given in magnitude from the degree meaſured there, and thence the axes themſelves are given. Thus, the circular arc whoſe length is equal to the radius being 57·29578 degrees, if this number be multiplied by 56750 toiſes, the meaſure of a degree at the equator, as Bouguer has ſtated it, the product will be the radius of curvature there, or half the latus rectum of the greater axis; and this is to half the leſſer axis in the ratio of the leſs axis to the greater, that is, as 0·9953467 to 1; whence the two axes are 6533820 and 6564366 toiſes, or 7913 and 7950 Engliſh miles; and the difference between the two axes about 37 miles. See Robertſon's Navigation, vol. 2, pa. 206 &c. See alſo Suite des Mem. de l'Acad. 1718, pa. 247, and Maclaurin's Fluxions, vol. 2, book 1, ch. 14.

And very nearly the ſame ratio is deduced from the lengths of pendulums vibrating in the ſame time, in different latitudes; provided it be again allowed that the meridians are real ellipſes, or the earth a true ſpheroid, which however can only take place in the caſe of an uniform gravity in all parts of the earth.

Thus, in the new Peterſburgh Acts, for the years 1788 and 1789, are accounts and calculations of experiments relative to this ſubject, by M. Krafft. Theſe experiments were made at different times and in various parts of the Ruſſian empire. This gentleman has collected and compared them, and drawn the proper concluſions from them: thus he infers that the length x of a pendulum that ſwings ſeconds in any given latitude λ, and in a temperature of 10 degrees of Reaumur's thermometer, may be determined by this equation:

$x = 439{\cdot}178 + 2{\cdot}321$ ſine $^2\lambda$, lines of a French foot,
or $x = 39{\cdot}0045 + 0{\cdot}206$ ſine $^2\lambda$, in Engliſh inches,
in the temperature of 53 of Fahrenheit's thermometer.

This expreſſion nearly agrees, not only with all the experiments made on the pendulum in Ruſſia, but alſo with thoſe of Mr. Graham in England, and thoſe of Mr. Lyons in 79° 50′ north latitude, where he found its length to be 431·38 lines. It alſo ſhews the augmentation of gravity from the equator to the parallel of a given latitude λ: for, putting g for the gravity under the equator, G for that under the pole, and y for that under the latitude λ, M. Krafft finds

$y = (1 + 0{\cdot}0052848$ ſine $^2\lambda)\, g$; and theref. $G = 1{\cdot}0052848\, g$.

From this proportion of gravity under different latitudes, the ſame author infers that, in caſe the earth is a homogeneous ellipſoid, its oblateneſs muſt be $\frac{1}{191}$; inſtead of $\frac{1}{230}$, which ought to be the reſult of this hypotheſis: but on the ſuppoſition that the earth is a heterogeneous ellipſoid, he finds its oblateneſs, as deduced from theſe experiments, to be $\frac{1}{297}$; which agrees with that reſulting from the meaſurement of ſome of the degrees of the meridian. This confirms an obſervation of M. De la Place, that, if the hypotheſis of the earth's homogeneity be given up, then theory, the meaſurement of degrees of latitude, and experiments with the pendulum, all agree in their reſult with reſpect to the oblateneſs of the earth. See Memoires de l' Acad. 1783, pa. 17.

In the Philoſ. Tranſ. for 1791, pa. 236, Mr. Dalby has given ſome calculations on meaſured degrees of the meridian, from whence he infers, that thoſe degrees meaſured in middle latitudes, will anſwer nearly to an ellipſoid whoſe axes are in the ratio aſſigned by Newton, viz, that of 230 to 229. And as to the deviations of ſome of the others, viz, towards the poles and equator, he thinks they are cauſed by the errors in the obſerved celeſtial arcs.

Tacquet draws ſome pretty little inferences, in the form of paradoxes, from the round figure of the earth; as, 1ſt, That if any part of the ſurface of the earth were quite plane, a man could no more walk upright upon it, than on the ſide of a mountain. 2d, That the traveller's head goes a greater ſpace than his feet; and a horſeman than a footman; as moving in a greater circle. 3d, That a veſſel, full of water, being raiſed perpendicularly, ſome of the water will be continually flowing out, yet the veſſel ſtill remain full; and on the contrary, if a veſſel of water be let perpendicularly down, though nothing flow out, yet it will ceaſe to be full: conſequently there is more water contained in the ſame veſſel at the foot of a mountain, than on the top; becauſe the ſurface of the water is compreſſed into a ſegment of a ſmaller ſphere below than above. Tacq. Aſtron. lib. 1, cap. 2.

Changes of the EARTH. Mr. Boyle ſuſpects that there are great, though ſlow internal changes, in the maſs of the earth. He argues from the varieties obſerved in the change of the magnetic needle, and from the obſerved changes in the temperature of climates. But as to the latter, there is reaſon to doubt that he could not have diaries of the weather ſufficient to direct his judgment. Boyle's Works abr. vol. 1, pa. 292, &c.

Magnetiſm of the EARTH. The notion of the magnetiſm of the earth was ſtarted by Gilbert; and Boyle ſuppoſes magnetic effluvia moving from one pole to the other. See his Works abr. vol. 1, pa. 285, 290.

Dr. Knight alſo thinks that the earth may be conſidered as a great loadſtone, whoſe magnetical parts are diſpoſed in a very irregular manner; and that the ſouth pole of the earth is analogous to the north pole in magnets, that is, the pole by which the magnetical ſtream enters. See MAGNET.

He obſerves, that all the phenomena attending the direction of the needle, in different parts of the earth, in great meaſure correſpond with what happens to a needle, when placed upon a large terrella; if we make allowances for the different diſpoſitions of the magnetical parts, with reſpect to each other, and conſider the ſouth pole of the earth as a north pole with regard to magnetiſm. The earth might become magnetical by the iron ores it contains, for all iron ores are capable of magnetiſm. It is true, the globe might notwithſtanding have remained unmagnetical, unleſs ſome cauſe had exiſted capable of making that repellent matter producing magnetiſm move in a ſtream through the earth.

Now the doctor thinks that ſuch a cauſe does exiſt. For if the earth revolves round the ſun in an ellipſis, and the ſouth pole of the earth is directed towards the ſun, at the time of its deſcent towards it, a ſtream of repellent matter will thence be made to enter at the ſouth pole, and iſſue out at the north. And he ſuggeſts, that the earth's being in its perihelion in winter may be one reaſon why magnetiſm is ſtronger in this ſeaſon than in ſummer.

This cauſe here aſſigned for the earth's magnetiſm muſt continue, and perhaps improve it, from year to year. Hence the doctor thinks it probable, that the earth's magnetiſm has been improving ever ſince the creation, and that this may be one reaſon why the uſe of the compaſs was not diſcovered ſooner. See Dr. Knight's Attempt to demonſtrate, that all the phenomena in nature may be explained by Attraction and Repulſion, prop. 87.

Magnitude and Conſtitution of the EARTH. This has been variouſly determined by different authors, both ancient and modern. The uſual way has been, to meaſure the length of one degree of the meridian, and multiply it by 360, for the whole circumference. See DEGREE. Diogenes Laertius informs us, that Anaximander, a ſcholar of Thales, who lived about 550 years before the birth of Chriſt, was the firſt who gave an account of the circumference of the ſea and land; and it ſeems his meaſure was uſed by the ſucceeding mathematicians, till the time of Eratoſthenes. Ariſtotle, at the end of lib. 2 De Cœlo, ſays, the mathematicians who have attempted to meaſure the circuit of the earth, make it 40000 ſtadiums: which it is thought is the number determined by Anaximander.

Eratoſthenes, who lived about 200 years before Chriſt, was the next who undertook this buſineſs; which, as Cleomedes relates, he performed by taking the ſun's zenith diſtances, and meaſuring the diſtance between two places under the ſame meridian; by which he deduced for the whole circuit about 250000 ſtadiums, which Pliny ſtates at 31500 Roman miles, reckoning each at 1000 paces. But this meaſure was accounted falſe by many of the ancient mathematicians, and particularly by Hipparchus, who lived 100 years afterwards, and who added 25000 ſtadiums to the circuit of Eratoſthenes.

Poſſidonius, in the time of Cicero and Pompey the Great, next meaſured the earth, viz, by means of the altitudes of a ſtar, and meaſuring a part of a meridian; and he concluded the circumference at 240000 ſtadiums, according to Cleomedes, but only at 180000 according to Strabo.

Ptolomy, in his Geography, ſays that Marinus, a celebrated geographer, attempted ſomething of the ſame kind; and, in lib. 1, cap. 3, he mentions that he himſelf had tried to perform the buſineſs in a way different from any other before him, which was by means of places under different meridians: but he does not ſay how much he made the number; for he ſtill made uſe of the 180000, which had been found out before him.

Snellius relates, from the Arabian Geographer Abelfedea, who lived about the 1300th year of Chriſt, that about the 800th year of Chriſt, Almaimon, an Arabian king, having collected together ſome ſkilful mathematicians, commanded them to find out the circumference of the earth. Accordingly theſe made choice of the fields of Meſopotamia, where they meaſured under the ſame meridian from north to ſouth, till the pole was depreſſed one degree lower: which meaſure they found equal to 56 miles, or 56½: ſo that according to them the circuit of the earth is 20160 or 20340 miles.

It was a long time after this before any more attempts were made in this buſineſs. At length however, the ſame Snell, above mentioned, profeſſor of mathematics at Leyden, about the year 1620, with great ſkill and labour, by meaſuring large diſtances between two parallels, found one degree equal to 28500 perches, each of which is 12 Rhinland feet, amounting to 19 Dutch miles, and ſo the whole periphery 6840 miles; a mile being, according to him, 1500 perches, or 18000 Rhinland feet. See his treatiſe called *Eratoſthenes Batavus*.

The next that undertook this meaſurement, was Richard Norwood, who in the year 1635, by meaſuring the diſtance from London to York with a chain, and taking the ſun's meridian altitude, June 11th old ſtyle, with a ſextant of about 5 feet radius, found a degree contained 367200 feet, or 69 miles and a half and 14 poles; and thence the circumference of a great circle of the earth is a little more than 25036 miles, and the diameter a little more than 7966 miles. See the particulars of this meaſurement in his *Seaman's Practice*.

The meaſurement of the earth by Snell, though very ingenious and troubleſome, and much more accurate than any of the ancients, being ſtill thought by ſome French mathematicians, as liable to certain ſmall errors, the buſineſs was renewed, after Snell's manner, by Picard and other mathematicians, by the king's command; uſing a quadrant of 3⅙ French feet radius; by which they found a degree contained 342360 French feet. See Picard's treatiſe, *La Meſure de la Terre*.

M. Caſſini the younger, in the year 1700, by the king's command alſo, renewed the buſineſs with a quadrant of 10 feet radius, for taking the latitude, and another of 3⅛ feet for taking the angles of the triangles; and found a degree, from his calculation, contained 57292 toiſes, or almoſt 69½ Engliſh miles.

See the reſults of many other meaſurements under the article DEGREE. From the mean of all which, the following dimenſions may be taken as near the truth:

the circumference	25000 miles,
the diameter	7957¾ miles,
the ſuperficies	198944206 ſquare miles,
the ſolidity	263930000000 cubic miles.

Alſo the ſeas and unknown parts of the earth, by a meaſurement of the beſt maps, contain 160522026 ſquare miles, the inhabited parts 38922180; of which Europe contains 4456065; Aſia, 10768823; Africa, 9654807; and America, 14110874.

It is now generally granted that the terraqueous globe has two motions, beſides that on which the preceſſion of the equinoxes depends; the one diurnal around its own axis in the ſpace of 24 hours, which conſtitutes the natural day or nycthemeron; the other annual, about the ſun, in an elliptical orbit or track, in 365 days 6 hours, conſtituting the year. From the former ariſe the diverſities of night and day; and from the latter, the viciſſitudes of ſeaſons, ſpring, ſummer, autumn, winter.

The terraqueous globe is diſtinguiſhed into three parts or regions, viz, 1ſt, The external part or cruſt, being that from which vegetables ſpring and animals are nurſed. 2d, The middle, or intermediate part, which is poſſeſſed by foſſils, extending farther than human labour ever yet penetrated. 3d, The internal or central part, which is utterly unknown to us, though by many authors ſuppoſed of a magnetic nature; by others, a maſs or ſphere of fire; by others, an abyſs or collection of waters, ſurrounded by the ſtrata of earth; and by others, a hollow, empty ſpace, inhabited by animals, who have their ſun, moon, planets, and other conveniences within the ſame. But others divide the body of the globe into two parts, viz, the external part, called the cortex, including the internal, which they call the nucleus, being of a different nature from the former, and poſſeſſed by fire, water, or more probably by a conſiderable portion of metals, as it has been found, by calculation, that the mean denſity of the whole earth is near double the denſity of common ſtone. See my determination of it, Philoſ. Tranſ. 1778, pa. 781.

The external part of the globe either exhibits inequalities, as mountains and valleys; or it is plane and level; or dug in channels, fiſſures, beds, &c, for rivers, lakes, ſeas, &c. Theſe inequalities in the face of the earth moſt naturaliſts ſuppoſe have ariſen from a rupture or ſubverſion of the earth, by the force either of the ſubterraneous fires or waters. The earth, in its natural and original ſtate, it has been ſuppoſed by Des Cartes, and after him, Burnet, Steno, Woodward, Whiſton, and others, was perfectly round, ſmooth, and equable; and they account for its preſent rude and irregular form, principally from the great deluge. See Deluge.

In the external, or cortical part of the earth, there appear various ſtrata, ſuppoſed the ſediments of ſeveral floods; the waters of which, being replete with matters of divers kinds, as they dried up, or oozed through, depoſited theſe different matters, which in time hardened into ſtrata of ſtone, ſand, coal, clay, &c.

Dr. Woodward has conſidered the circumſtances of theſe ſtrata with great attention, viz, their order, number, ſituation with reſpect to the horizon, depth, interſections, fiſſures, colour, conſiſtence, &c. He aſcribes the origin and formation of them all, to the great flood or cataclyſmus. At that terrible revolution he ſuppoſed that all ſorts of terreſtrial bodies had been diſſolved and mixed with the waters, forming all together a chaos or confuſed maſs. This maſs of terreſtrial particles, intermixed with water, he ſuppoſes was at length precipitated to the bottom; and that generally according to the order of gravity, the heavieſt ſinking firſt, and the lighteſt afterwards. By ſuch means were the ſtrata formed of which the earth conſiſts; which, attaining their ſolidity and hardneſs by degrees, have continued ſo ever ſince. Theſe ſediments, he farther concludes, were at firſt all parallel and concentrical; and the ſurface of the earth formed of them, perfectly ſmooth and regular; but that in courſe of time, divers changes happening, from earthquakes, volcanos, &c, the order and regularity of the ſtrata was diſturbed and broken, and the ſurface of the earth by ſuch means brought to the irregular form in which it now appears.

M. De Buffon ſurmiſes that the earth, as well as the other planets, are parts ſtruck off from the body of the ſun by the colliſion of comets; and that when the earth aſſumed its form, it was in a ſtate of liquefaction by fire. But that could not be the method of producing the planets; for if they were ſtruck off from the body of the ſun, they would move in orbits that would always paſs through the ſun, inſtead of having the ſun for their focus, or centre, as they are now found; ſo that having been ſtruck off they would fall down into the ſun again, terminating their career as it were after one revolution only.

Earth, *in Aſtronomy*, is one of the primary planets, according to the ſyſtem of Copernicus, or Pythagoras; its aſtronomical character or mark being ⊖: but according to the Ptolomaic hypotheſis, the earth is the centre of the ſyſtem. For, whether the earth move or remain at reſt, that is, whether it be fixed in the centre, having the ſun, the heavens and ſtars moving round it from eaſt to weſt; or, theſe being at reſt, whether the earth only moves from weſt to eaſt, is the great article that diſtinguiſhes the Ptolomaic ſyſtem from the Copernican.

Motion of the Earth. It is now univerſally agreed that, beſides the ſmall motion of the earth which cauſes the preceſſion of the equinoxes, the earth has two great and independent motions; viz, the one by which it turns round its own axis, in the ſpace of 24 hours nearly, and cauſing the continual ſucceſſion of day and night; and the other an abſolute motion of its whole maſs in a large orbit about the ſun, having that luminary for its centre, in ſuch manner that its axis keeps always parallel to itſelf, inclined in the ſame angle to its path, and by that means cauſing the viciſſitudes of ſeaſons, ſpring, ſummer, autumn, winter.

It is indeed true that, as to ſenſe, the earth appears to be fixed in the centre, with the ſun, ſtars and heavens moving round it every day; and ſuch doubtleſs would be conſidered as the true nature of the motions in the rude ages of mankind, as they are ſtill by the rude and unlearned. But to a thinking and learned mind, the contrary will ſoon appear.

Indeed there are traces of the knowledge of theſe motions in the earlieſt age of the ſciences. Cicero, in his Tuſc. Quæſt. ſays that Nicetas of Syracuſe firſt diſcovered that the earth had a diurnal motion, by which it revolved round its axis every 24 hours; and Plutarch, de Placit. Philoſoph. informs, that Philolaus diſcovered its annual motion round the ſun; and Ariſtarchus, about 100 years after Philolaus, propoſed the motion of the earth in ſtronger and clearer terms, as we are aſſured by Archimedes, in his Arenarius. And the ſame, we are farther aſſured, was the opinion and doctrine of Pythagoras.

But the religious opinions of the heathen world prevented this doctrine from being more cultivated. For, Ariſtarchus being accuſed of ſacrilege by Cleanthes for moving Veſta and the tutelar deities of the univerſe out of their places, the philoſophers were obliged to diſſemble, and ſeem to relinquiſh ſo perilous a poſition.

Many ages afterwards, Nic. Cuſanus revived the ancient ſyſtem, in his Doct. de Pignorant. and aſſerted the motion of the earth: but the doctrine gained very little ground till the time of Copernicus, who ſhewed its great uſe and advantages in aſtronomy; and who had immediately all the philoſophers and aſtronomers on his ſide, who dared to differ from the crowd, and were not afraid of eccleſiaſtical cenſure, which was not leſs dangerous under the chriſtian diſpenſation, than it had been under that of the heathen. For, becauſe certain parts of ſcripture make mention of the ſtability of the earth, and of the motion of the ſun, as the riſing and ſetting, &c, the fathers of the church thought their religion required that they ſhould defend, with all its power, what they conceived to be its doctrines, and to cenſure and puniſh every attempt at innovation on ſuch points. They have now however been pretty generally convinced that in ſuch inſtances the expreſſions are only to be conſidered as accommodated to appearances, and the vulgar notions of things.

By the diurnal rotation of the earth on its axis, the ſame phenomena will take place as if it had no ſuch motion, and as if the ſun and ſtars moved round it. For, turning round from weſt to eaſt, cauſes the ſun and all the viſible heavens to ſeem to move the contrary way, or from eaſt to weſt, as we daily ſee them do. So, when in its rotation it has brought the ſun or a ſtar to appear juſt in the horizon in the eaſt, they are then ſaid to be riſing; and as the earth continues to revolve more and more towards the eaſt, other ſtars ſeem to riſe and advance weſtwards, paſſing the meridian of the obſerver, when they are due ſouth from him, and at their greateſt altitude above his horizon; after which, by a continuance of the ſame motions, viz, of the earth's rotation eaſtwards, and the luminaries apparent counter motion weſtwards, theſe decline from the meridian, or ſouth point, towards the weſt, where being arrived, they are ſaid to ſet and deſcend below it; and ſo on continually from day to day; thus making it day while the ſun is above the horizon, and night while he is below it.

While the earth is thus turning on its axis, it is at the ſame time carried by its proper motion in its orbit round the ſun, as one of the planets, namely, between the orbits of Venus and Mars, having the orbits of Venus and Mercury within its own, or between it and the ſun, in the centre, and thoſe of Mars, Jupiter, Saturn, &c, without or above it; which are therefore called ſuperior planets, and the others the inferior ones. This is called the annual motion of the earth, becauſe it is performed in a year, or 365 days 6 hours nearly; or rather 365 days 5^h 49^m, from any equinox or ſolſtice to the ſame again, making the tropical year; but from any fixed ſtar to the ſame again, as ſeen from the ſun, in 365 days 6 hours 9 minutes, which is called the ſidereal year. The figure of this orbit is elliptical, having the ſun in one focus, the mean diſtance being about 95 millions of miles, which is upon the ſuppoſition that the ſun's parallax is about $8''_3$, or the angle under which the earth's ſemi-diameter would appear to an obſerver placed in the ſun: and the eccentricity of the orbit, or diſtance of the ſun, in the focus, from the centre of this elliptic orbit, is about $\frac{1}{60}$th of the mean diſtance.

Now this annual motion is performed in ſuch a manner, that the earth's axis is every where parallel, or in the ſame direction in every part of the orbit; by which means it happens, that at one time of the year the ſun enlightens more of the north polar parts, and at the oppoſite ſeaſon of the year more of the ſouthern parts, thus ſhewing all the varieties of ſeaſons, ſpring, ſummer, autumn and winter; which may be illuſtrated in the following manner: Let the candle I (fig. 1, plate viii) repreſent the ſun, about which the earth E, or F, &c, is moved in its elliptical orbit ABCD, or ecliptic, and cutting the equator *abcd* in the nodes E and G: then, ſuſpending the terrella by its north pole, and moving it, ſo ſuſpended, round the ecliptic, its axis will always be parallel to its firſt poſition, and the various ſeaſons will be repreſented at the different parts of the path. Thus, when the earth is at ♋ or F, the enlightened half of it includes the ſouth pole, and leaves the north pole in darkneſs, making our winter; at G it is ſpring, and the two poles are equally illuminated, and the days are every where of the ſame length; at H or ♑ it is our ſummer, the north polar parts being in the illuminated hemiſphere, and the ſouthern in the dark one; laſtly at E it is autumn, the poles being equally illuminated again, and the days of equal length every where.

Earth, *its Quantity of Matter, Denſity, and Attractive Power.* Although the relative denſities of the earth and moſt of the other planets have been known a conſiderable time, it is but very lately that we have come to the knowledge of the abſolute gravity or denſity of the whole maſs of the earth. This I have calculated and deduced from the obſervations made by Dr. Maſkelyne, Aſtronomer Royal, at the mountain Schehallien, in the years 1774, 5, and 6. The attraction of that mountain on a plummet, being obſerved on both ſides of it, and its maſs being computed from a number of ſections in all directions, and conſiſting of ſtone; theſe data being then compared with the known attraction and magnitude of the earth, gave by proportion its mean denſity, which is to that of water as 9 to 2, and to common ſtone as 9 to 5: from which very conſiderable mean denſity, it may be preſumed that the internal parts contain ſome great quantities of metals.

From the denſity, now found, its quantity of matter becomes known, being equal to the product of its denſity by its magnitude. From various experiments too, we know that its attractive force, at the ſurface, is ſuch, that bodies fall there through a ſpace of $16\frac{1}{12}$ feet in the firſt ſecond of time: from whence the force at any other place, either within or without it, becomes known; for the force at any part within it, is directly as its diſtance from the centre; but the force of any part without it, reciprocally as the ſquare of its diſtance from the centre.

EAST, one of the cardinal points of the horizon, or of the compaſs, being the middle point of it between north and ſouth, on that ſide where the ſun riſes, or the point in which it is interſected on that ſide by the prime vertical.

EASTER, a feaſt of the church, held in memory of our Saviour's reſurrection. This feaſt has been annually celebrated ever ſince the time of the apoſtles, and is one of the moſt conſiderable feſtivals in the chriſtian calendar; being that which regulates and determines the times of all the other moveable feaſts.

The rule for the celebration of Eaſter, fixed by the council of Nice, in the year 325, is, that it be held on the Sunday which falls upon, or next after, the full moon which happens next after the 21ſt of March; that is, the Sunday which falls upon, or next after the firſt full moon after the vernal equinox. The reaſon of which decree was, that the chriſtians might avoid celebrating their Eaſter at the ſame time with the Jewiſh Paſſover, which, according to the inſtitution of Moſes, was held the very day of the full moon.

To find EASTER *according to the Old, or Julian Style.*

Gold. Num.	Paſchal full moons.		Dom. letter.
16	March	21	C
5		22	D
		23	E
13		24	F
2		25	G
		26	A
10		27	B
		28	C
18		29	D
7		30	E
		31	F
15	April	1	G
4		2	A
		3	B
12		4	C
1		5	D
		6	E
9		7	F
		8	G
17		9	A
6		10	B
		11	C
14		12	D
3		13	E
		14	F
11		15	G
		16	A
19		17	B
8		18	C

In the annexed table, find the golden number, with the day of the paſchal full moon, and the Sunday letter annexed; compare this letter with the dominical letter of the given year, that it may appear how many days are to be added to the day of the paſchal full moon, to give Eaſter-day.

For ex. In the year 1715, the dominical letter is B, and the golden number is 6, oppoſite to which ſtands April 10 for the day of the paſchal full moon; oppoſite to which is the Sunday letter B, which happening to be the ſame with that of the year given, that day is a Sunday; and therefore Eaſter will fall 7 days after, viz, on the 17th of April.

But in this computation, the vernal equinox is ſuppoſed fixed to the 21ſt of March; and the cycle of 19 years, or golden numbers, is ſuppoſed to point out the places of the new and full moons exactly; both which ſuppoſitions are erroneous: ſo that the Julian Eaſter never happens at its due time, unleſs by accident. For inſtance, in the above example the vernal equinox falls on the 10th of March, eleven days before the rule ſuppoſes it; and the paſchal full moon on the 7th of April, or 3 days earlier than was ſuppoſed: and therefore Eaſter-day ſhould be held on the 10th of April, inſtead of the 17th.

This error had grown to ſuch a height, that pope Gregory the 13th thought it neceſſary to correct it; and accordingly, in the year 1582, by the advice of Aloyſius Lilius and others, he ordered 10 days to be thrown out of October, to bring the vernal equinox back again to the 21ſt of March: and hence ariſe the terms Gregorian calendar, Gregorian year, &c.

This correction however did not entirely remove the error; for the equinoxes and ſolſtices ſtill anticipate 28′ 20″ in every 100 Gregorian years; but the difference is ſo inconſiderable as not to amount to a whole day, or 24 hours, in leſs than 5082 Gregorian years.

The Gregorian, or New Style, was not introduced into England till the year 1752, when eleven days were thrown out, viz, between the 3d and 14th of September, the error amounting then to that quantity.

Gold. Num.	Paſchal full moon.		Sund. letter.
14	March	21	C
3		22	D
		23	E
11		24	F
		25	G
19		26	A
8		27	B
		28	C
16		29	D
5		30	E
		31	F
13	April	1	G
2		2	A
		3	B
10		4	C
		5	D
18		6	E
7		7	F
		8	G
15		9	A
4		10	B
		11	C
12		12	D
1		13	E
		14	F
9		15	G
		16	A
17		17	B
6		18	C

To find EASTER *according to the New or Gregorian Style, till the year* 1900 *excluſive.* Look for the golden number of the year in the firſt column of the table, againſt which ſtands the day of the paſchal full moon; then look in the 3d column for the Sunday letter, next after the day of the full moon, and the day of the month ſtanding againſt that Sunday letter is Eaſter-day. When the full moon happens on a Sunday, then the next Sunday after is Eaſter-day.

For Ex. For the year 1790, the golden number is 5; againſt which ſtands March the 30th, and the next Sunday letter, which is C, below that, ſtands oppoſite April 4, which is therefore the Eaſter-day for the year 1790.

Though the Gregorian calendar be much preferable to the Julian, it is yet not without its defects. It cannot, for inſtance, keep the equinox fixed on the 21ſt of March, but it will ſometimes fall on the 19th, and ſometimes on the 23d. Add, that the full moon happening on the 20th of March, might ſometimes be paſchal; yet it is not allowed as ſuch in the Gregorian computation; as on the contrary, the full moon of the 22d of March may be allowed for paſchal, which it is not. Scaliger and Calviſius have alſo pointed out other inaccuracies in this calendar. An excellent paper on this ſubject by the earl of Macclesfield, may be ſeen in the Philoſ. Tranſ. vol. 40, pa. 417.

EASTER *Term.* See TERM.

EAVES, the margin or lower edge of the roof of a houſe, being the loweſt courſe of tiles, ſlates, or the like, which hang over the walls to throw off the water to a diſtance from the wall.

EAVES-*Board,* or EAVES-*Lath,* a thick featheredged board, uſually nailed round the eaves of a houſe for the lowermoſt tiles, ſlate, or ſhingles, to reſt upon.

EBBING *and* FLOWING *of the Sea.* See TIDES.

ECCENTRIC. See EXCENTRIC.

ECCENTRICITY. See EXCENTRICITY.

ECHO, or ECCHO, a ſound reflected, or reverberated from ſome body, and thence returned or repeated to the ear.

For an echo to be heard, the ear muſt be in the line of reflection; that the perſon who made the ſound, may hear the echo, it is neceſſary he ſhould be in a perpendicular to the place which reflects it; and for a multiple or tautological echo, it is neceſſary there be a number of walls and vaults, rocks, and cavities, either placed behind each other, or fronting each other. Thoſe murmurs in the air, that are occaſioned by the diſcharge of great guns, &c, are a kind of indefinite echoes, and are produced from the vaporous particles ſuſpended in the atmoſphere, which reſiſt the undulations of ſound, and reverberate them to the ear.

There can be no echo, unleſs the direct and reflex ſounds follow one another at a ſufficient diſtance of time; for if the reflex ſound arrive at the ear before the impreſſion of the direct ſound ceaſes, the ſound will not be doubled, but only rendered more intenſe. Now if we allow that 9 or 10 ſyllables can be pronounced in a ſecond, in order to preſerve the ſounds articulate and diſtinct, there ſhould be about the 9th part of a ſecond between the times of their appulſe to the ear; or, as ſound flies about 1142 feet in a ſecond, the ſaid difference ſhould be $\frac{1}{9}$ of 1142, or 127 feet; and therefore every ſyllable will be reflected to the ear at the diſtance of about 70 feet from the reflecting body; but as, in the ordinary way of ſpeaking, 3 or 4 ſyllables only are uttered in a ſecond, the ſpeaker, that he may have the echo returned as ſoon as they are expreſſed, ſhould ſtand about 500 feet from the reflecting body; and ſo in proportion for any other number of ſyllables. Merſenne allows for a monoſyllable the diſtance of 69 feet; Morton, 90 feet; for a diſſyllable 105 feet, a triſyllable 160 feet, a tetraſyllable 182 feet, and a pentaſyllable 204 feet. Nat. Hiſt. Northampton, cap. 5, pa. 358.

From what has been ſaid, it follows that echoes may be applied for meaſuring inacceſſible diſtances. Thus, Mr. Derham, ſtanding upon the banks of the Thames, oppoſite to Woolwich, obſerved that the echo of a ſingle ſound was reflected back from the houſes in 3 ſeconds; conſequently the ſum of the direct and reflex rays muſt have been $1142 \times 3 = 3426$ feet, and the half of it, 1713 feet, the breadth of the river in that place.

It alſo follows that the echoing body being removed farther off, it reflects more of the ſound than when nearer; which is the reaſon why ſome echoes repeat but one ſyllable, or one word, and ſome many. Of theſe, ſome are tonical, which only return a voice when modulated into ſome particular muſical tone; and others polyſyllabical. That fine echo in Woodſtock park, Dr. Plot aſſures us, in the day-time will return very diſtinctly 17 ſyllables, and in the night 20. Nat. Hiſt. Oxf. cap. 1, pa. 7.

Echoing bodies may be ſo contrived, and placed, as that reflecting the ſound from one to the other, a multiple echo, or many echoes, ſhall ariſe.—At Roſneath, near Glaſgow, in Scotland, there is an echo that repeats a tune played with a trumpet three times completely and diſtinctly.—At the ſepulchre of Metella, wife of Craſſus, there was an echo, which repeated what a man ſaid five times.—Authors mention a tower at Cyzicus, where the echo repeated ſeven times.—There is an echo at Bruſſels, that anſwers 15 times.

One of the fineſt echoes we read of, is that mentioned by Barthius, in his notes on Statius's Thebais, lib. 6, ver. 30, which repeated the words a man uttered 17 times. This was on the banks of the Naha, between Coblentz and Bingen. And whereas, in common echoes, the repetition is not heard till ſome time after hearing the words ſpoken, or the notes ſung; in this, the perſon who ſpeaks, or ſings, is ſcarce heard at all; but the repetition very clearly, and always in ſurpriſing varieties; the echo ſeeming ſometimes to approach nearer, and ſometimes farther off; ſometimes the voice is heard very diſtinctly, and ſometimes ſcarce at all: one perſon hears only one voice, and another ſeveral; one hears the echo on the right, and the other on the left, &c.

Addiſon, and other travellers in Italy, mention an echo at Simonetta palace, near Milan, ſtill more extraordinary, returning the ſound of a piſtol 56 times. The echo is heard behind the houſe, which has two wings; the piſtol is diſcharged from a window in one of theſe wings, the ſound is returned from a dead wall in the other wing, and heard from a window in the back-front. See Addiſ. Travels, pa. 32; Miſſon, Voyag. d'Ital. tom. 2, pa. 196; Philoſ. Tranſ. N° 480, pa. 220.

Farther, a multiple echo may be made, by ſo placing the echoing bodies, at unequal diſtances, as that they may reflect all one way, and not one on the other; by which means, a manifold ſucceſſive ſound will be heard; one clap of the hands like many; one *ha* like a laughter; one ſingle word like many of the ſame tone and accent; and ſo one muſical inſtrument like many of the ſame kind, imitating each other.

Laſtly, echoing bodies may be ſo ordered, that from any one ſound given, they ſhall produce many echoes, different both as to tone and intenſion. By which means a muſical room may be ſo contrived, that not only one inſtrument playing in it ſhall ſeem many of the ſame ſort and ſize, but even a concert of different ones; this may be contrived by placing certain echoing bodies ſo, as that any note played, ſhall be returned by them in 3ds, 5ths, and 8ths.

Echo is alſo uſed for the place where the repetition of the ſound is produced, or heard. This is either natural or artificial.

In echoes, the place where the ſpeaker ſtands, is called the *centrum phonicum*; and the object or place that returns the voice, the *centrum phonocampticum*.

Echo, in Architecture, is applied to certain vaults and arches, moſtly of elliptical or parabolical figures; uſed to redouble ſounds, and produce artificial echoes.—The method of making them is taught by F. Blancani, in his Echometria, at the end of his book on the Sphere.

Vitruvius tells us, that in divers parts of Greece and Italy there were brazen veſſels, artfully ranged under the ſeats of the theatres, to render the ſound of the actors' voices more clear, and make a kind of echo; by which means, every one of the prodigious multitude of perſons, preſent at thoſe ſpectacles, might hear with eaſe and pleaſure.

ECLIPSAREON, an inſtrument invented by Mr. Ferguſon for ſhewing the phenomena of eclipſes; as their time, quantity, duration, progreſs, &c. Ferguſon's Aſtron., or Philoſ. Tranſ. vol. 48, pa. 520.

ECLIPSE, a privation of the light of one of the luminaries, by the interpoſition of ſome opaque body, either between it and the eye, or between it and the ſun.

The ancients had terrible ideas of eclipſes; ſuppoſing them preſages of ſome dreadful events. Plutarch aſſures us, that at Rome it was not allowed to talk publicly of any natural cauſes of eclipſes; the popular opinion running ſo ſtrongly in favour of their ſupernatural production, at leaſt thoſe of the moon; for as to thoſe of the ſun, they had ſome idea that they were cauſed by the interpoſition of the moon between us and the ſun; but were at a loſs for a body to interpoſe between us and the moon, which they thought muſt be the way, if the eclipſes of the moon were produced by natural cauſes. They therefore made a great noiſe with brazen inſtruments, and ſet up loud ſhouts, during eclipſes of the moon; thinking by that means to eaſe her in labour: whence Juvenal, ſpeaking of a talkative woman, ſays, *Una laboranti poterit ſuccurrere lunæ.* Others attributed the eclipſe of the moon to the arts of magicians, who, by their inchantments, plucked her out of heaven, and made her ſkim over the graſs.

The natives of Mexico keep faſt during eclipſes; and particularly their women, who beat and abuſe themſelves, drawing blood from their arms, &c; imagining the moon has been wounded by the ſun, in ſome quarrel between them.

The Chineſe fancy that eclipſes are occaſioned by great dragons, who are ready to devour the ſun and moon; and therefore when they perceive an eclipſe, they rattle drums and braſs kettles, till they think the monſter, terrified by the noiſe, lets go his prey.

The ſuperſtitious notions entertained of eclipſes, were once of conſiderable advantage to Chriſtopher Columbus, the diſcoverer of America, who being driven on the iſland of Jamaica in the year 1493, and diſtreſſed for want of proviſions, was refuſed relief by the natives; but having threatened them with a plague, and foretelling an eclipſe as a token of it, which happened according to his prediction, the barbarians were ſo terrified, that they ſtrove who ſhould be the firſt in bringing him ſupplies, throwing them at his feet, and imploring forgiveneſs.

Duration of an ECLIPSE, is the time of its continuance, or between the immerſion and emerſion.

Immerſion, or Incidence, of an ECLIPSE, is the moment when the eclipſe begins, or when part of the ſun, moon, or planet firſt begins to be obſcured.

Emerſion, or Expurgation, of an ECLIPSE, is the time when the eclipſed luminary begins to re-appear, or emerge out of the ſhadow.

Quantity of an ECLIPSE, is the part of the luminary eclipſed. To determine the quantity eclipſed, it is uſual to divide the diameter of the luminary into 12 equal parts, called digits; whence the eclipſe is ſaid to be of ſo many digits according to the number of them contained in that part of the diameter which is eclipſed or obſcured.

Eclipſes are divided, with reſpect to the luminary eclipſed, into *Eclipſes of the ſun, of the moon,* and *of the ſatellites;* and with reſpect to the circumſtances, into *total, partial, annular, central,* &c.

Annular ECLIPSE, is when the whole is eclipſed, except a ring, or annulus, which appears round the border or edge.

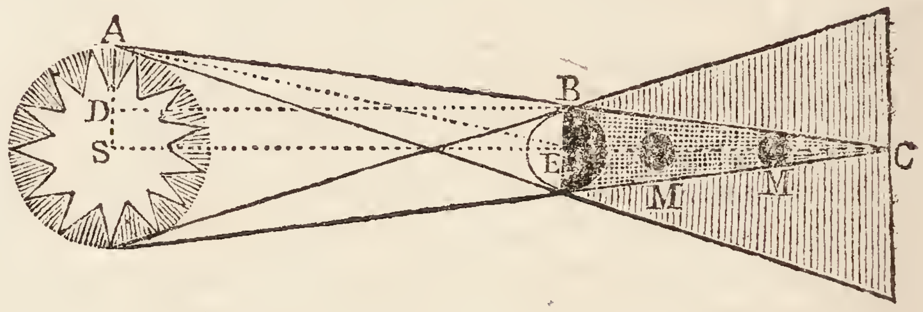
Central ECLIPSE, is one in which the centres of the two luminaries and the earth come into the ſame ſtraight line.

Partial ECLIPSE, is when only a part of the luminary is eclipſed. And a

Total ECLIPSE, is that in which the whole luminary is darkened.

ECLIPSE *of the Moon,* is a privation of the light of the moon, occaſioned by an interpoſition of the body of the earth directly between the ſun and moon, and ſo intercepting the ſun's rays that they cannot arrive at the moon, to illuminate her. Or, the obſcuration of the moon may be conſidered as a ſection of the earth's conical ſhadow, by the moon paſſing through ſome part of it.

The manner of this eclipſe is repreſented in this figure, where S is the ſun, E the earth, and M or M the moon.



Lunar Eclipſes only happen at the time of full moon; becauſe it is only then the earth is between the ſun and moon: nor do they happen every full moon, becauſe of the obliquity of the moon's path with reſpect to the ſun's; but only in ſuch full moons as happen either at the interſection of thoſe two paths, called the moon's nodes, or very near them; viz, when the moon's latitude, or diſtance between the centres of the earth and moon, is leſs than the ſum of the apparent ſemidiameters of the moon and the earth's ſhadow.

The chief Circumſtances in Lunar Eclipſes, are the following:—1. All lunar Eclipſes are univerſal, or viſible in all parts of the earth which have the moon above their horizon; and are every where of the ſame magnitude, with the ſame beginning and end.—2. In all lunar eclipſes, the eaſtern ſide is what firſt immerges and emerges again, i. e. the left-hand ſide of the moon as we look towards her, from the north; for the proper motion of the moon being ſwifter than that of the earth's ſhadow, the moon approaches it from the weſt, overtakes and paſſes through it with the moon's eaſt ſide foremoſt, leaving the ſhadow behind, or to the weſtward.—3. Total eclipſes, and thoſe of the longeſt duration, happen in the very nodes of the ecliptic; becauſe the ſection of the earth's ſhadow, then falling on the moon, is conſiderably larger than her diſc. There may however be total eclipſes within a ſmall diſtance of the nodes; but their duration is the leſs as they are farther from it; till they become only partial ones, and at laſt, none at all.—4. The moon, even in the middle of an eclipſe, has uſually a faint appearance of light, reſembling tarniſhed copper; which Gaſſendus, Ricciolus, Kepler, &c, attribute to the light of the ſun, refracted by the earth's atmoſphere, and ſo tranſmitted thither.—Laſtly, ſhe grows ſenſibly paler and dimmer, before entering into the real ſhadow; owing to a penumbra which ſurrounds that ſhadow to ſome diſtance.

Aſtronomy of Lunar ECLIPSES, *or the method of calculating their times, places, magnitudes, and other phenomena.*

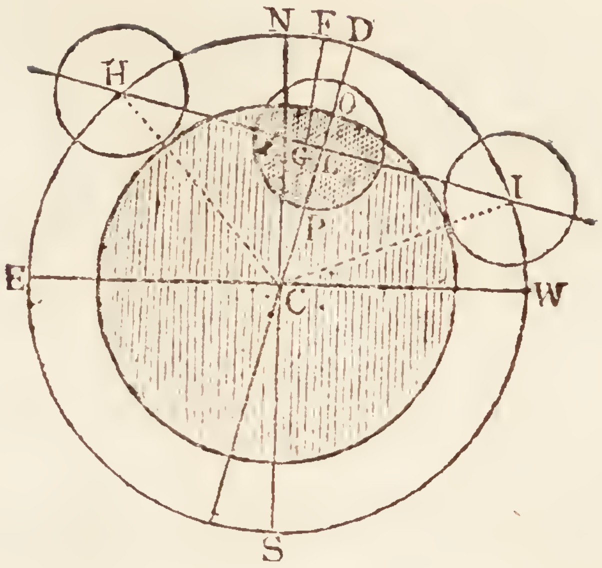
The 1ſt preliminary is to find the length of the earth's conical ſhadow. This may be found either from the diſtance between the earth and ſun, and the proportion of their diameters, or from the angle of the ſun's apparent magnitude at the time. Thus, ſuppoſe the ſemiaxis of the earth's orbit 95,000000 miles, and the eccentricity of the orbit 1,377000 miles, making the greateſt diſtance 96,377000 miles, or 24194 ſemidiameters of the earth; and the ſun's ſemidiameter being to the earth's, as 112 to 1; then as AD : BE : : DB : EC, that is, 111 : 1 : : 24194 : 218 ſemidiameters of the earth = EC the length of the earth's ſhadow. Otherwiſe, ſuppoſe the angle AES, or the ſun's apparent ſemidiameter be 15′ 56″, and the angle BAE, or the ſun's parallax 8·6″, then is their difference, or the angle ACE = 15′ 47·4″; hence, as tang. 15′ 47·4″ : radius : : BE or 1 : 218 nearly = CE, the ſame diſtance as before. Hence, as the moon's leaſt diſtance from the earth is ſcarce 56 ſemidiameters, and the greateſt not more than 64, the moon, when in oppoſition to the ſun, in or near the nodes, will fall into the earth's ſhadow, and will be eclipſed, as the length of the ſhadow is almoſt 4 times the moon's diſtance.

2. *To find the apparent ſemidiameter of the earth's ſhadow*, in the place where the moon paſſes through it, at any given time. Add together the ſun and moon's parallaxes, and from the ſum ſubtract the apparent ſemidiameter of the ſun; ſo ſhall the remainder be the apparent ſemidiameter of the ſhadow at the place of the moon's paſſage. For ex. the 28th of April 1790, at midnight, the moon's parallax is 61′ 9″, to which add 8·6″, or 9″, for the ſun's parallax, from the ſum 61′ 18″ take 15′ 56″, the ſun's apparent ſemidiameter, and the remainder 45′ 22″ is the ſemidiameter of the ſhadow at the place where the moon paſſes through at that time. N. B. Some omit the ſun's parallax, as of no conſequence; but increaſe the apparent ſemidiameter of the ſhadow by one whole minute, for the ſhadow of the atmoſphere; which would give the ſemidiameter of the ſhadow, in the caſe above, 46′ 13″.

3. There muſt alſo be had, the true diſtance of the moon from the node, at the mean oppoſition; alſo the true time of the oppoſition, with the true place of the ſun and moon, reduced to the ecliptic; likewiſe the moon's true latitude at the time of the true oppoſition; the angle of the moon's way with the ecliptic, and the true horary motions of the ſun and moon: from which all the circumſtances of her eclipſe may be computed by common arithmetic and trigonometry.

To Conſtruct an ECLIPSE *of the Moon.*

Let EW be a part of the ecliptic, and C the centre of the earth's ſhadow, through which draw perpendicular to EW, the line CN towards the north, if the moon have north latitude at the time of the eclipſe, or CS ſouthward, if ſhe have ſouth latitude. Make the angle NCD equal to the angle of the moon's way with the ecliptic, which may be always taken at 5° 35′, on an average, without any ſenſible error; and biſect this angle by the right line CF; in which line it is that the true equal time of oppoſition of the ſun and moon falls, as given by the tables.



From a convenient ſcale of equal parts, repreſenting minutes of a degree, take the moon's latitude at the true time of full moon, and ſet it from C to G, on the line CF; and through the point G, at right-angles to CD, draw the right line HKGLI for the path of the moon's centre. Then is L the point in the earth's ſhadow, where the moon's centre is at the middle of the eclipſe; G the point where her centre is at the tabular time of her being full; and K the point where her centre is at the inſtant of her ecliptic oppoſition: alſo I the moon's centre at the moment of immerſion, and H her centre at the end of the eclipſe.

With the moon's ſemidiameter as a radius, and the points I, L, H, as centres, deſcribe circles for the moon at the beginning, middle, and end of the eclipſe.

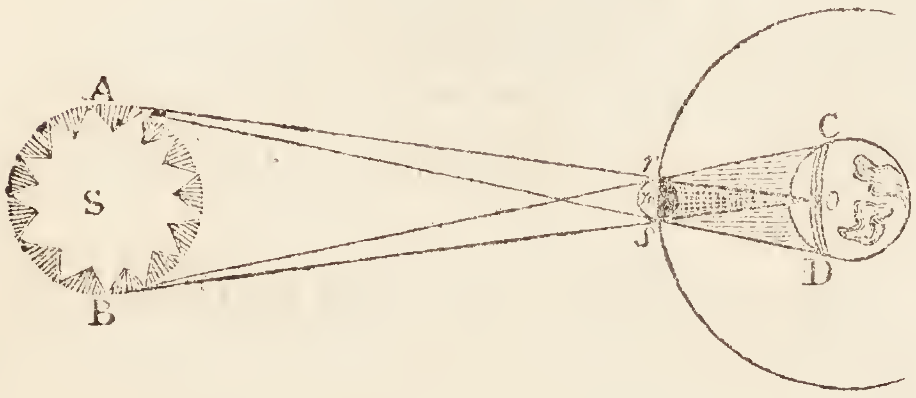
Finally, the length of the line of path IH, meaſured on the ſame ſcale, will ſerve to determine the duration of the eclipſe, viz. by ſaying, As the moon's horary motion from the ſun is to IH : : 1 hour or 60 min. to the whole duration of the eclipſe.

To Compute a Lunar ECLIPSE. This will be very eaſy from the foregoing conſtruction. For, 1ſt, in the triangle CGL, right-angled at L, there are given the hypothenuſe CG = the moon's latitude at the time of full moon, and the angle GCL = the half of 5° 35′; to find the legs CL and LG.—2d, In the right-angled triangle CHL or CIL, are given the leg CL, and CH or CI, the ſum of the ſemidiameters of the moon and the earth's ſhadow; to find LH or LI, half the difference of the ſun's and moon's motions during the time of the eclipſe.—3d, As the difference of the horary motions of the luminaries is to one hour, or 60 min. : : HL to the ſemiduration of the eclipſe, and : : GL to the difference between the oppoſition and middle of the eclipſe; this laſt therefore taken from the time of full moon, gives the time of the middle of the eclipſe; from which ſubtracting the time in LI, or ſemiduration before found, gives the beginning of the eclipſe; or add the ſame, and it gives the end of it.—Laſtly, from CO the ſemidiameter of the ſhadow, take CL, leaves LO; to which add LP, the moon's ſemidiameter, when neceſſary, gives OP the quantity eclipſed.

Note, When the moon's diſtance from the node exceeds 12°, there can be no eclipſe of the moon; or, more accurately, the limit is from $10\frac{1}{2}$ and $12\frac{1}{30}$ degrees, according to the diſtances of the ſun, earth, and moon.

ECLIPSE *of the Sun*, is an occultation of the ſun's body, occaſioned by an interpoſition of the moon between the earth and ſun. On which account it is by ſome conſidered as an eclipſe of the earth, ſince the light of the ſun is hid from the earth by the moon, whoſe ſhadow involves a part of the earth.

The manner of a ſolar eclipſe is repreſented in this figure; where S is the ſun, *m* the moon, and CD the earth, *rmso* the moon's conical ſhadow, travelling over a part of the earth C*o*D, and making a complete eclipſe to all the inhabitants reſiding in that track, but no where elſe; excepting that for a large ſpace around it there is a fainter ſhade, included within all the ſpace *r*CD*s*, which is called the *Penumbra*.



Hence, Solar Eclipſes happen when the moon is in conjunction with the ſun, or at the new moon, and alſo in the nodes or near them, the limit being about 17 degrees on each ſide of it; and ſuch eclipſes only happen when the latitude of the moon, viewed from the earth, is leſs than the ſum of the apparent ſemidiameters of the ſun and moon; becauſe the moon's way is oblique to the ecliptic, or ſun's path, making an angle of nearly 5° 35′ with it.

In the nodes, when the moon has no viſible latitude, the occultation is total; and with ſome continuance, when the diſc of the moon in perigee appears greater than that of the ſun in apogee, and its ſhadow is extended beyond the ſurface of the earth; and without continuance at moderate diſtances when the cuſp, or point of the moon's ſhadow, barely touches the earth. Laſtly, out of the nodes, but near them, the eclipſes are partial.

The other circumſtances of ſolar eclipſes are, 1. That none of them are univerſal; that is, none of them are ſeen throughout the whole hemiſphere which the ſun is then above; the moon's diſc being much too little, and much too near the earth, to hide the ſun from the whole diſc of the earth. Commonly the moon's dark ſhadow covers only a ſpot on the earth's ſurface, about 180 miles broad when the ſun's diſtance is greateſt, and the moon's leaſt. But her partial ſhadow, or penumbra, may then cover a circular ſpace of 4900 miles in diameter, within which the ſun is more or leſs eclipſed, as the places are nearer to or farther from the centre of the penumbra. In this caſe the axis of the ſhade paſſes through the centre of the earth, or the new moon happens exactly in the node, and then it is evident that the ſection of the ſhadow is circular; but in every other caſe the conical ſhadow is cut obliquely by the ſurface of the earth, and the ſection will be an oval, and very nearly a true ellipſis.

2. Nor does the Eclipſe appear the ſame in all parts of the earth, where it is ſeen; but when in one place it is total, in another it is only partial. Farther, when the moon appears much leſs than the ſun, as is chiefly the caſe when ſhe is in apogee and he in perigee, the vertex of the lunar ſhadow is then too ſhort to reach the earth, and though ſhe be in a central conjunction with the ſun, is yet not large enough to cover his whole diſc, but lets his limb appear like a lucid ring or bracelet, and ſo cauſes an *Annular Eclipſe*.

3. A Solar Eclipſe does not happen at the ſame time, in all places where it is ſeen; but appears more early to the weſtern parts, and later to the eaſtern; as the motion of the moon, and conſequently of her ſhadow, is from weſt to eaſt.

4. In moſt Solar Eclipſes the moon's diſc is covered with a faint light; which is attributed to the reflexion of the light from the illuminated part of the earth.

Laſtly, in total Eclipſes of the ſun, the moon's limb is ſeen ſurrounded by a pale circle of light; which ſome aſtronomers conſider as an indication of a lunar atmoſphere; but others as the atmoſphere of the ſun, becauſe it has been obſerved to move equally with the ſun, and not with the moon; and beſides, it is generally believed that the moon is without any atmoſphere, unleſs it be one that is very ſmall, and very rare.

To determine the Bounds of a Solar ECLIPSE.

If the moon's parallax were inſenſible, the bounds of a ſolar eclipſe would be determined after the ſame manner as thoſe of a lunar; but becauſe here is a ſenſible parallax, the method is a little altered, viz.

1. Add together the apparent ſemidiameters of the luminaries, both in apogee and perigee; which gives 33′ 6″ for the greateſt ſum of them, and 30′ 31″ for the leaſt ſum.

2. Since the parallax diminiſhes the northern latitude and augments the ſouthern, therefore let the greateſt parallax in latitude be added to the former ſums, and alſo ſubtracted from them: Thus in each caſe there will be had the true latitude, beyond which there can be no eclipſe. This latitude being given, the moon's diſtance from the nodes, beyond which eclipſes cannot happen, is found as for a lunar eclipſe. This limit is nearly between 16½ and 18⅓ degrees diſtance from the nodes.

To find the Digits eclipſed. Add the apparent ſemidiameters of the luminaries into one ſum; from which ſubtract the moon's apparent latitude; the remainder is the ſcruples, or parts of the diameter eclipſed. Then ſay, As the ſemidiameter of the ſun is to the ſcruples eclipſed; ſo are 6 digits reduced into ſcruples, viz 360 ſcruples or minutes, to the digits &c eclipſed.

To determine the Duration of a Solar ECLIPSE. Find the horary motion of the moon from the ſun, for one hour before the conjunction, and another hour after: then ſay, As the former horary motion is to the ſeconds in an hour, ſo are the ſcruples of half duration (found as in a lunar eclipſe) to the time of immerſion; and as the latter horary motion is to the ſame ſeconds, ſo are the ſame ſcruples of half duration to the time of emerſion. Laſtly, adding the times of immerſion and emerſion together, the aggregate is the total duration.

The moon's apparent diameter when largeſt, exceeds the ſun's when leaſt, only 2′ of a degree; and at the greateſt ſolar eclipſe that can happen at any time and place, the total darkneſs cannot continue any longer than whilſt the moon is moving through this 2′ from the ſun in her orbit, which is about 4 minutes of time: for the motion of the ſhadow on the earth's diſc is equal to the moon's motion from the ſun, which on account of the earth's rotation on its axis towards the ſame way, or eaſtward, is about 30½ minutes of a degree every hour, at a mean rate; but ſo much of the moon's orbit is equal to 30½ degrees of a great circle on the earth, becauſe the circumference of the moon's orbit is about 60 times that of the earth; and therefore the moon's ſhadow goes 30½ degrees, or 1830 geographical miles in an hour, or 30½ miles in a minute.

To determine the Beginning, Middle, and End, of a Solar Eclipſe. From the moon's latitude, for the time of conjunction, find the arch GL (laſt fig. but one), or the diſtance of the greateſt obſcurity. Then ſay, as the horary motion of the moon from the ſun, before the conjunction, is to 1 hour; ſo is the diſtance of the greateſt darkneſs, to the interval of time between the greateſt darkneſs and the conjunction. Subtract this interval, in the 1ſt and 3d quarter of the anomaly, from the time of the conjunction; and in the other quarters, add it to the ſame; the reſult is the time of the greateſt darkneſs. Laſtly, from the time of the greateſt darkneſs ſubtract the time of incidence, and add it to the time of emerſion; the difference in the firſt caſe will be the beginning; and the ſum, in the latter caſe, the end of the eclipſe.

To Calculate Eclipses *of the Sun.* Firſt, find the mean new moon, and thence the true one; with the place of the luminaries for the apparent time of the true one.—2. For the apparent time of the true new moon, compute the apparent time of the new moon obſerved.—3. For the apparent time of the new moon ſeen, compute the latitude ſeen.—4. Thence determine the digits eclipſed.—5. Find the times of the greateſt darkneſs, immerſion, and emerſion.—6. Thence determine the beginning, and ending of the eclipſe.

From the foregoing problems, it is evident that all the trouble and fatigue of the calculus ariſes from the parallaxes of longitude and latitude, without which, the calculation of ſolar eclipſes would be the ſame with that of lunar ones.

See the Conſtruction and Calculation of Eclipſes by Flamſteed in Sir Jonas Moor's Syſtem of Mathematics, and in Ferguſon's Aſtronomy, &c.

In the Philoſ. Tranſ. N° 461 is a contrivance to repreſent ſolar eclipſes, by means of the terreſtrial globe, by M. Seguer, profeſſor of Mathematics at Gottingen. And Mr. Ferguſon has fitted a terreſtrial globe, ſo as to ſhew the time, quantity, duration, and progreſs of ſolar eclipſes, at any place of the earth where they are viſible; which he calls the Eclipſareon. He has alſo given a large catalogue of ancient and modern eclipſes, including thoſe recorded in hiſtory, from 721 years before Chriſt, to A. D. 1485; alſo computed eclipſes from 1485 to 1700, and all the eclipſes viſible in Europe from 1700 to 1800. See his Aſtron.

The Number of Eclipses, of both luminaries, in any year, cannot be leſs than two, nor more than ſeven; the moſt uſual number is 4, and it is rare to have more than 6. The reaſon is obvious; becauſe the ſun paſſes by both the nodes but once in a year, unleſs he paſs by one of them in the beginning of the year; in which caſe he will paſs by the ſame again a little before the end of the year; becauſe the nodes move backwards 19⅓ degrees every year, and therefore the ſun will come to either of them 173 days after the other. And if either node be within 17° of the ſun at the time of new moon, the ſun will be eclipſed; and at the ſubſequent oppoſition, the moon will be eclipſed in the other node, and come round to the next conjunction before the former node be 17° beyond the ſun, and eclipſe him again. When three eclipſes happen about either node, the like number commonly happens about the oppoſite one; as the ſun comes to it in 173 days afterward, and 6 lunations contain only 4 days more. Thus there may be two eclipſes of the ſun, and one of the moon, about each of the nodes. But when the moon changes in either of the nodes, ſhe cannot be near enough the other node at the next full, to be eclipſed; and in 6 lunar months afterward ſhe will change near the other node; in which caſe there cannot be more than two eclipſes in a year, both of the ſun.

Period of Eclipses, is the period of time in which the ſame eclipſes return again; and as the nodes move backwards 19⅓ degrees every year, they would ſhift through every point of the ecliptic in 18 years and 225 days; and this would be the regular period of their return, if any complete number of lunations were finiſhed without a fraction; but this is not the caſe. However, in 223 mean lunations, after the ſun, moon, and nodes have been once in a line of conjunction, they return ſo nearly to the ſame ſtate again, as that the ſame node which was in conjunction with the ſun and moon at the beginning of the firſt of theſe lunations, will be within 28′ 12″ of the line of conjunction with the ſun and moon again, when the laſt of theſe lunations is completed; and in this period there will be a regular return of eclipſes for many ages. To the mean time of any ſolar or lunar eclipſe, by adding this period, or 18 Julian years 11 days 7 hours 43 minutes 20 ſeconds, when the laſt day of February in leap years is 4 times included, or a day leſs when it occurs 5 times, we ſhall have the mean time of the return of the ſame eclipſe. In an interval of 6890 mean lunations, containing 557 years 21 days 18 hours 30 minutes 11 ſeconds, the ſun and node meet ſo nearly, as to be diſtant only 11 ſeconds.

The Uſe of Eclipses. In Aſtronomy, eclipſes of the moon determine the ſpherical figure of the earth; they alſo ſhew that the ſun is larger than the earth, and the earth than the moon. Eclipſes alſo, that are ſimilar in all circumſtances, and that happen at conſiderable intervals of time, ſerve to aſcertain the period of the moon's motion. In Geography, eclipſes diſcover the longitude of different places; for which purpoſe thoſe of the moon are the more uſeful, becauſe they are more often viſible, and the ſame lunar eclipſe is of equal magnitude and duration at all places where it is ſeen. In Chronology, both ſolar and lunar eclipſes ſerve to determine exactly the time of any paſt event.

Eclipses *of the Satellites.* See Satellites *of Jupiter.*

The chief circumſtances here obſerved, are, 1. That the ſatellites of Jupiter undergo two or three kinds of eclipſes; the firſt of which are proper, being ſuch as happen when Jupiter's body is directly interpoſed between them and the ſun: and theſe happen almoſt every day. Various authors have given tables for computing eclipſes of the ſatellites of Jupiter; as Flamſteed, Caſſini, &c, but the lateſt and beſt of all, are thoſe of profeſſor Wargentin of Upſal.

The ſecond ſort are occultations, rather than obſervations; when the ſatellites, coming too near the body of Jupiter, are loſt in his light; which Riccioli calls *occidere zeuſiace, ſetting jovially*. In which caſe, the neareſt or firſt ſatellite exhibits a third kind of eclipſe; being obſerved like a round macula, or dark ſpot, tranſiting the diſc of Jupiter, with a motion contrary to that of the ſatellite; like as the moon's ſhadow projected on the earth, will appear to do, to the lunar inhabitants.

The eclipſes of Jupiter's ſatellites furniſh very good means of finding the longitude at ſea. Thoſe eſpecially of the firſt ſatellite are much ſurer than the eclipſes of the moon, and they alſo happen much oftener: the manner of applying them is alſo very eaſy. See LONGITUDE.

ECLIPTIC, in Aſtronomy, a great circle of the ſphere conceived to paſs through the middle of the zodiac. It is ſometimes called the *via ſolis*, or *ſun's path*, being the track which he appears to deſcribe among the fixed ſtars; though more properly it is the apparent path of the earth, as viewed from the ſun, and thence called the heliocentric circle of the earth. It is called the ecliptic, becauſe all the eclipſes of the ſun or moon happen when the moon croſſes it, or is nearly in one of thoſe two parts of her orbit where it croſſes the ecliptic, which points are called the moon's nodes.

Upon the ecliptic are marked and counted the 12 celeſtial ſigns, Aries, Taurus, Gemini, &c; and upon it is counted the longitude of the planets and ſtars. It is placed obliquely with reſpect to the equator, which it cuts in two oppoſite points, viz, the beginning of Aries and Libra, which are directly oppoſite to each other, and called the equinoxes, making the one half of the ecliptic to the north, and the other half on the ſouth ſide of the equator; the two extreme points of it, to the north and ſouth, which are oppoſite to each other, and at a quadrant diſtance from the equinoctial points both ways, are called the ſolſtices, or ſolſtitial points, or alſo the two tropics, which are at the beginning of Cancer and Capricorn, and which are at the fartheſt diſtance of any points of it from the equator, which diſtance is the meaſure of the ſun's greateſt declination, which is the ſame with the obliquity of the ecliptic, or the angle it makes with the equator.

This obliquity of the ecliptic is not permanent, but is continually diminiſhing, by the ecliptic approaching nearer and nearer to a paralleliſm with the equator, at the rate of half a ſecond in a year nearly, or from 50″ to 55″ in 100 years, as is deduced from ancient and modern obſervations compared together; and as the mean obliquity of the ecliptic was 23° 28′ about the end of the year 1788, or beginning of 1789, by adding half a ſecond for each preceding year, or ſubtracting the ſame for each following year, the mean obliquity will be found nearly for any year either before or ſince that period. The quantity however of this change is variouſly ſtated by different authors, from 50″ to 60″ or 70″ for each century or 100 years. Hipparchus, almoſt two thouſand years ſince, obſerved the obliquity of the ecliptic, and found it about 23° 51′; and all ſucceeding aſtronomers, to the preſent time, having obſerved the ſame, have found it always leſs and leſs; being now rather under 23° 28′; a difference of about 23′ in 1950 years; which gives a medium of 70″ in 100 years. There is great reaſon however to think that the diminution is variable.

This diminution of the obliquity of the ecliptic to the equator, according to Mr. Long and ſome others, is chiefly owing to the unequal attraction of the ſun and moon on the protuberant matter about the earth's equator. For if it be conſidered, ſay they, that the earth is not a perfect ſphere, but an oblate ſpheroid, having its axis ſhorter than its equatorial diameter; and that the ſun and moon are conſtantly acting obliquely upon the greater quantity of matter about the equator, drawing it, as it were, towards a nearer and nearer coincidence with the ecliptic; it will not appear ſtrange that theſe actions ſhould gradually diminiſh the angle between the planes of thoſe two circles. Nor is it leſs probable that the mutual attractions of all the planets ſhould have a tendency to bring their orbits to a coincidence: though this change is too ſmall to become ſenſible in many ages.

It is now however well known that this change in the obliquity of the ecliptic, is wholly owing to the actions of the planets upon the earth, and eſpecially the planets Venus and Jupiter, but chiefly the former. See La Grange's excellent paper upon this ſubject in the Memoirs of the French Academy for 1774; Caſſini's in 1778; and La Lande's Aſtron. vol. 3, art. 2737. According to La Grange, who proceeds upon theory, the annual change of obliquity is variable, and has its limits: about 2000 years ago, he thinks it was after the rate of about 38″ in 100 years; that it is now, and will be for 400 years to come, 56″ per century; but 2000 years hence, 49″ per century. According to Caſſini, who computes from obſervations of the obliquity between the years 1739 and 1778, the annual change at preſent is 60″ or 1′ in 100 years. But according to La Lande, the diminution is at the rate of 88″ per century; while Dr. Maſkelyne makes it only 50″ in the ſame time.

Beſide the regular diminution of the obliquity of the ecliptic, at the rate of near 50 ſeconds in a century, or half a ſecond a year, which ariſes from a change of the ecliptic itſelf, it is ſubject to two periodical inequalities, the one produced by the unequal force of the ſun in cauſing the preceſſion of the equinoxes, and the other depending on the nutation of the earth's axis. See the Explanation and Uſe of Dr. Maſkelyne's Tables and Obſervations, pa. vi, where we are ſhewn how to calculate thoſe inequalities, and where he ſhews that, from his own obſervations, the mean obliquity of the ecliptic to the beginning of the year 1769, was 23° 28′ 9″·7.

To find the Obliquity of the Ecliptic, or the greateſt declination of the ſun: about the time of the ſummer ſolſtice obſerve very carefully the ſun's zenith diſtance for ſeveral days together; then the difference between this diſtance and the latitude of the place, will be the obliquity ſought, when the ſun and equator are both on one ſide of the place of obſervation; but their ſum will be the obliquity when they are on different ſides of it. Or, it may be found by obſerving the meridian altitude, or zenith diſtance, of the ſun's centre, on the days of the ſummer and winter ſolſtice; then the difference of the two will be the diſtance between the tropics, the half of which will be the obliquity ſought.

By the ſame method too, the declination of the ſun from the equator for any other day may be found; and thus a table of his declination for every day in the year might be conſtructed. Thus alſo the declination of the ſtars might be found.

Authors' Names	Years before Chriſt	Obliquity		
		°	′	″
Pytheas	324	23	49	23
Eratoſthenes and Hipparchus	230 & 140	23	51	20
	after Chriſt			
Ptolomy	140	23	48	45
Almahmon	832	23	35	
Albategnius	880	23	35	
Thebat	911	23	33	30
Abul Waſi and Hamed	999	23	35	
Perſian Tables in Chryſococca	1004	23	35	
Albatrunius	1007	23	35	
Arzachel	1104	23	33	30
Almæon	1140	23	33	30
Choja Naſſir Oddin	1290	23	30	
Prophatius the Jew	1300	23	32	
Ebn Shattir	1363	23	31	
Purbach and Regiomontanus	1460	23	30	
Ulugh Beigh	1463	23	30	17
Walther	1476	23	30	
Do. corrected by refraction &c	——	23	29	8
Werner	1510	23	28	30
Copernicus	1525	23	28	24
Egnatio Danti	1570	23	29	
Prince of Heſſe	1570	23	31	
Rothmann and Byrge	1570	23	30	20
Tycho Brahe	1584	23	31	30
Ditto corrected	——	23	29	
Wright	1594	23	30	
Kepler	1627	23	30	30
Gaſſendus	1630	23	31	
Ricciolus	1646	23	30	20
Ditto corrected	1655	23	29	
Hevelius	1653	23	30	20
Ditto corrected	1661	23	28	52
Caſſini	1655	23	29	15
Montons, corrected, &c	1660	23	29	3
Richer corrected	1672	23	28	52
De la Hire	1686	23	29	
Ditto corrected	——	23	29	28
Flamſteed	1690	23	29	
Bianchini	1703	23	28	25
Roemer	1706	23	28	41
Louville	1715	23	28	24
Godin	1730	23	28	20
Bradley	1750	23	28	18
Mayer	1756	23	28	16
Maſkelyne	1769	23	28	10
Hornſby	1772	23	28	8

The obſervations of aſtronomers of all ages, on the obliquity of the ecliptic, have been collected together; and although ſome of them may not be quite accurate, yet they ſufficiently ſhew the gradual and continual decreaſe of the obliquity from the times of the earlieſt obſervations down to the preſent time. The chief of thoſe obſervations may be ſeen in the foregoing table; where the firſt column contains the name of the obſerver, or author, the 2d the year before or after Chriſt, and the 3d the obliquity of the ecliptic for that time.

See Ptolom. Alm. lib. 1, cap. 10; Riccioli Alm. vol. 1, lib. 3, cap. 27; Flamſteed Proleg. Hiſt. Cœl. vol. 3; Philoſ. Tranſ. number 163; ib. vol. 63, pt. 1; Long's Aſtron. vol. 1, cap. 16; Memoirs of the Acad. an. 1716, 1734, 1762, 1767, 1774, 1778; Acta Erud. Lipſiæ 1719; Naut. Alm. 1779; Maſkelyne's Obſerv. Explan. pa. vi; &c.

According to an ancient tradition of the Egyptians, mentioned by Herodotus, the ecliptic had formerly been perpendicular to the equator: they were led into this notion by obſerving, for a long ſeries of years, that the obliquity was continually diminiſhing; or, which amounts to the ſame thing, that the ecliptic was continually approaching to the equator. From thence they took occaſion to ſuſpect that thoſe two circles, in the beginning, had been as far off each other as poſſible, that is, perpendicular to each other. Diodorus Siculus relates, that the Chaldeans reckoned 403,000 years from their firſt obſervations to the time of Alexander's entering Babylon. This enormous account may have ſome foundation, on the ſuppoſition that the Chaldeans built on the diminution of the obliquity of the ecliptic at the rate of a minute in 100 years. M. de Louville, taking the obliquity ſuch as it muſt have been at the time of Alexander's entrance into Babylon, and going back to the time when the ecliptic, at that rate, muſt have been perpendicular to the equator, actually finds 402,942 Egyptian, or Chaldean years; which is only 58 years ſhort of that epocha. Indeed there is no way of accounting for the fabulous antiquity of the Egyptians, Chaldeans, &c, ſo probable, as from the ſuppoſition of long periods of very ſlow celeſtial motions, a ſmall part of which they had obſerved, and from which they calculated the beginning of the period, making the world and their own nation to commence together. Or perhaps they ſometimes counted months or days for years.

Should the diminution always continue at the rate it has lately done, viz. at 50″ or 56″ a century, it would take 96,960 years, from the year 1788, to bring the ecliptic exactly to coincide with the equator.

Ecliptic, *in Geography*, a great circle on the terreſtrial globe, in the plane of, or directly under, the celeſtial ecliptic.

Ecliptic, *Eclipticus*, ſomething belonging to the ecliptic, or to eclipſes; as ecliptic conjunction, oppoſition, &c.

Ecliptic *Bounds*, or *Limits*, are the greateſt diſtances from the nodes at which the ſun or moon can be eclipſed; namely, near 18 degrees for the ſun, and 12 degrees for the moon.

Ecliptic *Digits*, *digiti ecliptici*. See Digits.

Poles of the Ecliptic, are the two oppoſite points of the ſphere which are each everywhere equally diſtant from the ecliptic quite around, or 90° diſtant from it. The diſtance of the poles of the ecliptic from the poles of the equator, or of the world, is always equal to the varying diſtance of the obliquity of the ecliptic, and at the beginning of the year 1789 it was juſt 23° 28′.

Reduction to the Ecliptic. See Reduction.

EFFECT, the reſult or conſequence of the application of a cauſe, or agent, on ſome ſubject. It is one of the great axioms in philoſophy, that full effects are always proportional to the powers of their adequate cauſes.

EFFECTION, denotes the geometrical conſtruction of a propoſition. The term is alſo uſed in reference to problems and practices; which, when they are deducible from, or founded upon ſome general propoſitions, are called the *geometrical effection* of them.

EFFERVESCENCE, is popularly uſed for a light ebullition, or a briſk inteſtine motion, produced in a liquor by the firſt action of heat, with any remarkable ſeparation of its parts.

EFFICIENT *Cauſe*, is that which produces an effect. See Cause and Effect.

Efficients, in Arithmetic, are the numbers given for an operation of multiplication, and are otherwiſe called the factors. Hence the term coefficients in Algebra, which are the numbers prefixed to, or that multiply the letters or algebraic quantities.

EFFLUVIUM, a flux or exhalation of minute particles from any body ; or an emanation of ſubtile corpuſcles from a mixed, ſenſible body, by a kind of motion of tranſpiration.

ELASTIC, an appellation given to all bodies endowed with the property of elaſticity or ſpringineſs.

Elastic *Body*, is that which changes its figure, and yields to any impulſe or preſſure, but endeavours by its own nature and force to reſtore the ſame again ; or, it is a ſpringy body, which, when compreſſed or condenſed, or the like, makes an effort to ſet itſelf at liberty, and to repel the body that conſtrained it. Such, for inſtance, as a bow, or a ſword blade, &c, which are eaſily bent, but preſently return to their former figure and extenſion. All bodies partake of this property in ſome degree, though perhaps none are perfectly elaſtic, as none are found to reſtore themſelves with a force equal to that with which they are compreſſed.

The principal phenomena obſervable in Elaſtic bodies, are 1, That an elaſtic body (i. e. a body perfectly elaſtic, if any ſuch there be) endeavours to reſtore itſelf with the ſame force with which it is preſſed or bent. 2, An elaſtic body exerts its force equally towards all ſides ; though the effect is chiefly found on that ſide where the reſiſtance is weakeſt ; as is evident in the caſe of a gun exploding a ball, a bow ſhooting out an arrow, &c.—3, Elaſtic bodies, in what manner ſoever ſtruck, or impelled, are inflected and rebound after the ſame manner : thus a bell yields the ſame muſical ſound, in what manner, or on what ſide ſoever it be ſtruck ; the ſame of a tenſe or muſical chord ; and a body rebounds from a plane in the ſame angle in which it meets or ſtrikes it, making the angle of incidence equal to the angle of reflection, whether the intenſity of the ſtroke be greater or leſs.—4, A body perfectly fluid, if any ſuch there be, cannot be elaſtic, if it be allowed that its parts cannot be compreſſed.—5, A body perfectly ſolid, if any ſuch there be, cannot be elaſtic ; becauſe, having no pores, it is incapable of being compreſſed.—6, The elaſtic properties of bodies ſeem to differ, according to their greater or leſs denſity or compactneſs, though not in an equal degree : thus, metals are rendered more compact and elaſtic by being hammered : tempered ſteel is much more elaſtic than ſoft ſteel ; and the denſity of the former is to that of the latter as 7809 to 7738 : cold condenſes ſolid bodies, and renders them more elaſtic ; whilſt heat, that relaxes them, has the oppoſite effect : but, on the contrary, air, and other elaſtic fluids, are expanded by heat, and rendered more elaſtic.—For the laws of Motion and Percuſſion in Elaſtic bodies, ſee Motion, and Percussion.

Elastic *Curve*. See Catenaria.

Elastic *Fluids*. See Air, Electricity, Gas, Elastic Vapours, &c.

Elastic *Gum*. The ſame as Caoutchouc, or *Indian Rubber*.

Elastic *Vapours*, or *Fluids*, are ſuch as may be compreſſed mechanically into a leſs ſpace, and which reſume their former ſtate when the compreſſing force is withdrawn. Such as atmoſpherical air, and all the aerial fluids, with all kinds of fumes raiſed by means of heat, whether from ſolid or fluid bodies.

Of theſe, ſome remain elaſtic only while a conſiderable degree of heat is applied to them, or to the ſubſtance which produces them ; while others continue elaſtic in every degree of cold that has yet been obſerved. Of the former kind, are the vapours of water, ſpirit of wine, mercury, ſal-ammoniac, and all kinds of ſublimable ſalts : of the latter, thoſe of ſpirit of ſalt, mixtures of vitriolic acid and iron, nitrous acid, and various other metals, and in ſhort the ſeveral ſpecies of aerial fluids indiſcriminately.

The elaſtic force with which any one of theſe fluids is endowed, has not yet been calculated, as being ultimately greater than any obſtacle we can put in its way. Thus, on compreſſing the atmoſpherical air, we find that for ſome little time at firſt it eaſily yields to any force applied ; but at every ſucceeding moment the reſiſtance becomes always the ſtronger, and a greater and greater force muſt be applied, to compreſs it farther. As the compreſſion goes on, the veſſel containing the air becomes hot ; but no power whatever has yet been able in any degree to deſtroy the elaſticity of the contained fluid ; for, upon removing the preſſure, it is always found to occupy the very ſame ſpace that it did before. The caſe is the ſame with the ſteam of water, to which a ſufficient heat is applied to keep it from condenſing into water.

Elasticity, or Elastic *Force*, that property of bodies by which they reſtore themſelves to their former figure, after any external preſſure.

The cauſe or principle of this important property, elaſticity, is variouſly accounted for. The Carteſians aſcribe it to their ſubtile matter making an effort to paſs through pores that are too narrow for it. Thus, ſay they, in bending or compreſſing a hard elaſtic body, as a bow, for inſtance, its parts recede from each other on the convex ſide, and approach on the concave one ;

conſequently the pores are contracted or ſtraitened on the concave ſide; and, if they were before round, are now perhaps oval: ſo that the materia ſubtilis, or matter of the ſecond element, endeavouring to paſs out of the pores thus ſtraitened, muſt make an effort, at the ſame time, to reſtore the body to the ſtate it was in when the pores were rounder, i. e. before the bow was bent: and in this conſiſts its Elaſticity.

Other later philoſophers account for Elaſticity much after the ſame manner as the Carteſians; with this only difference, that inſtead of the ſubtile matter of the Carteſians, theſe ſubſtitute *Ether*, or a fine ethereal medium that pervades all bodies.

Others, ſetting aſide the precarious notion of a materia ſubtilis, account for Elaſticity from the great law of nature, Attraction, or the cauſe of the cohesion of the parts of ſolid and firm bodies. Thus, ſay they, when a hard body is ſtruck or bent, ſo that the component parts are moved a little from each other, but not quite disjointed or broken off, or ſeparated ſo far as to be out of the power of that attracting force by which they cohere; they muſt, on removing the external violence, ſpring back to their former natural ſtate.

Others again reſolve Elaſticity into the preſſure of the atmoſphere: for a violent tenſion, or compreſſion, though not ſo great as to ſeparate the conſtituent particles of bodies far enough to let in any foreign matter, muſt yet occaſion many little vacuola between the ſeparated ſurfaces; ſo that on the removal of the force they will cloſe again by the preſſure of the aerial fluid upon the external parts.

Laſtly, others attribute the Elaſticity of all hard bodies to the power of reſilition in the air included within them; and ſo make the elaſtic force of the air the principle of Elaſticity in all other bodies. See Deſaguliers's Exper. Philoſ. vol. 2, pa. 38, &c.

The ELASTICITY *of Fluids* is accounted for from their particles being all endowed with a centrifugal force; whence Sir Iſaac Newton demonſtrates, prop. 23, lib. 2, that particles, which naturally avoid or fly off from one another by ſuch forces as are reciprocally proportional to the diſtances of their centres, will compoſe an elaſtic fluid, whoſe denſity ſhall be proportional to its compreſſion; and vice verſa, if any fluid be compoſed of particles that fly off or avoid one another, and have its denſity proportional to its compreſſion, then the centrifugal forces of thoſe particles will be reciprocally proportional to the diſtances of their centres.

ELASTICITY *of the Air* is the force with which that element endeavours to expand, and with which it does actually dilate itſelf, on removing the force that compreſſed it. See AIR, and ATMOSPHERE.

The Elaſticity or ſpring of the air was firſt diſcovered by lord Bacon, and farther eſtabliſhed by Galileo. Its exiſtence is proved by this experiment of that philoſopher: An extraordinary quantity of air being intruded, by means of a ſyringe, into a hollow ball or ſhell of glaſs or metal, till ſuch time as the ball, with this acceſſion of air, weigh conſiderably more in the balance than it did before; then, opening the mouth of the ball, the air ruſhes out, till the ball ſink to its former weight. From hence we infer, that there is juſt as much air gone out, as compreſſed air had been crowded in. Air therefore returns to its former degree of expanſion, upon removing the force that compreſſed or reſiſted its expanſion; and conſequently it is endowed with an elaſtic force. It may be added, that as the air is found to ruſh out in every ſituation or direction of the orifice, the elaſtic force acts every way, or in every direction alike.

The cauſe of Elaſticity in air hath been uſually aſcribed to a repulſion between its particles; but what is the cauſe of that repulſion? The term repulſion, like that of attraction, requires to be defined; and probably it will be found in moſt caſes to be the effect of the action of ſome other fluid. Thus, it is found that the Elaſticity of the atmoſphere is very conſiderably affected by heat. Suppoſing a quantity of air heated to ſuch a degree as to raiſe Fahrenheit's thermometer to 212, it will then occupy a conſiderable ſpace; but if it be cooled again to ſuch a degree, as to ſink the thermometer to 0, it will ſhrink up to leſs than half the former bulk. The quantity of repulſive power therefore acquired by the air, while paſſing from one of theſe ſtates to the other, is evidently owing to the heat added to it, or taken away from it. Nor does there ſeem to be any reaſon to ſuppoſe, that the quantity of Elaſticity or repulſive power it ſtill poſſeſſes, is owing to any other cauſe than the fire contained in it. The ſuppoſition that repulſion is a primary cauſe, independent of all others, has given riſe to many erroneous theories, and very much embarraſſed philoſophers in accounting for the phenomena of Elaſticity.

The Elaſticity of the air is not only proportional to its denſity, but is always equal to the force which compreſſes it, becauſe theſe two exactly balance each other. This Elaſticity, in the atmoſpheric air, is meaſured by the height of the barometer at any time, allowing for its heat or temperature, after this rate, viz. the 434th part for each degree of Fahrenheit's thermometer, above or below ſome mean temperature, as 55°; for by that part of the whole it is that air expands or contracts, or elſe increaſes or decreaſes in its Elaſticity, for each degree of the thermometer. Sir Geo. Shuckburgh, in the Philoſ. Tranſ. for 1777, pa. 561.

ELECTIONS, or *Choice*, ſignify the ſeveral different ways of taking any number of things propoſed, either ſeparately, or as combined in pairs, in threes, in fours, &c; not as to the order, but only as to the number and variety of them. Thus, of the things a, b, c, d, e, &c, the elections of

one thing are $(a,)$ $1 = 2^1 - 1$,
two things are (a, b, ab) $3 = 2^2 - 1$,
three things are $(a, b, c, ab, ac, bc, abc)$ $7 = 2^3 - 1$,

&c; and of any number, n, all the elections are $2^n - 1$; that is, one leſs than the power of 2 whoſe exponent is n, the number of ſingle things to be choſen, either ſeparately or in combination.

ELECTRIC, in Phyſics, is a term applied to thoſe ſubſtances, in which the electric fluid can be excited, and accumulated, without tranſmitting it; and which are therefore called *non-conductors*. They are alſo called *original Electrics*, and *Electrics per ſe*.

The word is derived from ηλεκτρον, *amber*, one of the moſt obſervable non-conductors. To this claſs alſo belong glaſs, and all vitrifications, even of metals; all precious ſtones, of which the moſt tranſparent are the beſt; all reſins, and reſinous compoſitions; alſo ſulphur, baked wood, all bituminous ſubſtances, wax, ſilk, cotton, all dry animal ſubſtances, as feathers, wool, hair, &c; alſo paper, white ſugar, and ſugar-candy; likewiſe air, oils, chocolate, calces of metals and ſemi-metals, the aſhes of animal and vegetable ſubſtances, the ruſt of metals, all dry vegetable ſubſtances, and ſtones, of which the hardeſt are the beſt.

Subſtances of this kind may be excited, ſo as to exhibit the Electric appearances of attracting and repelling light bodies, emitting a ſpark of light, attended with a ſnapping noiſe, and yielding a current of air, the ſenſation of which reſembles that of a ſpider's web drawn over the face, &c, and a ſmell like that of phoſphorus; and this exciting may be either by friction, or by heating and cooling, or by melting, and pouring one melted ſubſtance into another.

The term is peculiarly applied to the electric, viz. the globe, or cylinder, &c, uſed in electrical machines, to collect the electrical matter by rubbing it.

Electrical *Air Thermometer*, an inſtrument contrived by Mr. Kinnerſley of Philadelphia, and uſed in determining the effects of the electrical exploſion upon air. The deſcription may be ſeen in Franklin's Letters, &c, pa. 389, 4to, 1769.

Electrical *Apparatus*, conſiſts of glaſs tubes, about 3 feet long, and an inch and a half in diameter, one of which ſhould be cloſed at one end, and furniſhed at the other end with a braſs cap and ſtop-cock, to rarefy or condenſe the incloſed air; ſticks of ſealing wax, or tubes of rough glaſs, or glaſs tubes covered with ſealing-wax, or cylinders of baked wood for producing the negative electricity; with proper rubbers, as black oiled ſilk, with amalgam upon it for the former, and ſoft new flannel, or hare ſkins, or cat ſkins, tanned with the hair on, for the latter; coated jars, or plates of glaſs, either ſingle, or combined in a battery, for accumulating electricity; metal rods, as diſchargers; an electrical machine; electrometers, and inſulated ſtools, ſupported by pillars of glaſs, covered with ſealing-wax, or baked wood, varniſhed or boiled in linſeed oil.

Electrical *Atmoſphere*, is a ſtream or maſs of the Electrical fluid which ſurrounds an excited or electrified body, to ſome diſtance.

Electrical *Balls*. See Balls and Electrometer.

Electrical *Battery*, conſiſts of a large quantity of coated jars, placed near each other in a convenient manner. Theſe being charged, or electrified, and connected with each other, are then ſuddenly exploded or diſcharged, with a prodigious effect.

Electrical *Fluid*, is a fine rare fluid which iſſues from, and ſurrounds electrified bodies.

Electrical *Kite*, was contrived by Dr. Franklin, to verify his hypotheſis of the identity of electricity and lightning. It conſiſted of a large thin ſilk handkerchief, extended and faſtened at the four corners to two ſlender ſtrips of cedar, and accommodated with a tail, loop, and ſtring, ſo as to riſe in the air like a paper kite. To the top of the upright ſtick of the croſs was fixed a very ſharp-pointed wire, riſing a foot or more above the wood; and to the end of the twine, next the hand, a ſilk ribband was tied. From a key ſuſpended at the junction of the twine and ſilk, when the kite is raiſed during a thunder-ſtorm, a phial may be charged, and electric fire collected, as is uſually done by means of a rubbed glaſs tube or globe. Philoſ. Tranſ. vol. 47, pa. 565, or Franklin's Letters, pa. 111 and 112.

Kites made of paper, covered with varniſh, or with well boiled linſeed oil, to preſerve them from the rain, with a ſtick and cane bow, like the common ones uſed by boys, will anſwer the purpoſe extremely well, and are very uſeful in determining the electricity of the atmoſphere. See Conductor.

Electrical *Machine*, is a part of the Electrical apparatus, contrived for collecting a great quantity of electricity, and exhibiting its effects in a very ſenſible manner. It conſiſts of the electric, the moving engine, the rubber, and the prime conductor. In the early ſtate of this ſcience, for the electric, was uſed ſealing-wax, ſulphur, or rough glaſs; but, ſince the method of inſulating the rubber, and ſo producing negative electricity, was introduced, ſmooth glaſs has been uſed. The form is commonly either that of a globe, or of a cylinder. Each figure has its advantages, and its inconveniences. Dr. Van Marum, a late German writer, has conſtructed a machine, in which gumlac, in the form of a diſc, is uſed as an electric inſtead of glaſs; which has the effect of depending very little on the temperature of the air; deſcribed in his Verhandeling over het Electrizeeren, &c, or a Treatiſe concerning the method of electrifying. Groningen, 1776. But he has ſince procured ſome others to be made by Mr. Cuthbertſon, a very ingenious artiſt, of large diſcs, or round plates of glaſs: one of theſe is now placed in Teyler's Muſeum at Harlem, having two of theſe glaſs plates, of 65 inches diameter, excited, on both ſides of them, by rubbers of waxed taffaty; with which, effects are produced that are truly aſtoniſhing and tremendous. See his Deſcription of this machine, and its effects, publiſhed in 4to, at Harlem, 1785, &c.

There have been various contrivances for giving motion to the electric of a machine. The common method is by a wheel turned by a winch or handle; a cord going round a groove in the periphery of the wheel, and over a pulley in the neck of the globe or cylinder. Others have uſed multiplying wheels, which are eaſily turned by a winch; and others again make uſe of a wheel and pinion, or a wheel and endleſs ſcrew. But Van Marum's machine it ſeems has the completeſt movement, its operation being very uniform, and eaſily worked; it is kept in motion by a weight, which, after being wound up to the height of 12 feet, will continue the motion uniformly for 6 hours; yielding alſo a negative power, as well as the poſitive; and the conductors annexed to it ſerving eaſily to convey the electrical power wherever it is required, without the addition of any chain, or wires, &c.

The Rubber is the next material part of a machine. Theſe were formerly made of red baſil ſkins, ſtuffed with hair, wool, flax, or bran: Dr. Nooth introduced ſilk cuſhions ſtuffed with hair, over which is laid a piece of leather, rubbed with amalgam, which are better than the others. The rubber may be inſulated in any way that beſt ſuits the conſtruction of the machine:

chine : and a chain or wire may eaſily be ſuſpended from it, to communicate with the floor, whenever the inſulation is not neceſſary ; and thus poſitive and negative electricity may be produced at pleaſure. Van Marum uſes mercury to his rubbers.

The Prime Conductor is another neceſſary appendage to the Electrical Machine : its uſe is to receive the electricity from the electric, as it is produced, and accumulate it as in a magazine, ready to be drawn off and employed on all occaſions. See *Prime* Conductor.

Deſcription of the moſt uſeful Electrical Machines.

Fig. 1, plate ix, repreſents Dr. Prieſtley's Machine, a very extenſively uſeful one, deſcribed in his Hiſtory of Electricity; in which *g* is the globe, or electric ; *f* the rubber ; in the two pillars *d*, *d*, of baked wood, are ſeveral holes to receive the ſpindles of different globes or cylinders, ſeveral of which may be put on together, to increaſe the electricity : *klm* is the prime conductor, being a copper tube, ſupported on a ſtand of glaſs or baked wood.

Fig. 2 is Dr. Watſon's Machine, for uſing ſeveral globes at once, to accumulate a great quantity of electricity.

Fig. 3 repreſents a very portable Electrical Machine invented by Mr. Read, and improved by Mr. Lane. A is the glaſs cylinder, moved vertically by means of the pulley at the lower end of the axis, the pulley being turned by the large wheel B parallel to the table; there are ſeveral pulleys, of different ſizes, either of which may be uſed, according as the motion is required to be quicker or ſlower. The conductor C is furniſhed with points to collect the fluid, and is ſcrewed to the wire of a coated jar D. The figure ſhews alſo the manner of applying Mr. Lane's electrometer to this machine.

Electrical Machines have of late years undergone ſome very eſſential alterations and improvements ; both from the ſuggeſtions of private electricians, and the inventions of Meſſrs. Adams, Nairne, and Jones, inſtrument makers in London ; ſome of which are as follow:

Fig. 4 repreſents a very convenient machine for practice. The frame of this machine conſiſts of the bottom board ABCD ; which, when the machine muſt be uſed, is faſtened to the table by two metal cramps. EF are two round pillars, of baked wood, which ſupport the cylinder G by the axles of the braſs or wooden caps H, turned ſometimes by a ſimple winch I, and ſometimes by a pulley and wheel, as in the next fig. The rubber is fixed to a glaſs pillar K, which is faſtened to a wooden baſis L at the bottom. The conductor N is uſually made of braſs or tin japanned, and is inſulated by a glaſs pillar, ſcrewed into a wooden baſis or foot, which is moſt conveniently placed parallel to the cylinder.

Fig. 5 repreſents an Electrical Machine, with a conductor in the ſhape of a T ; and an improved medical apparatus, where it is neceſſary to give the ſhock in the arms.

Fig. 6 ſhews Mr. Nairne's patent machine for medical purpoſes. Its glaſs cylinder is about 7 inches in diameter, and 12 long, with two conductors parallel to it. The rubber is faſtened to the conductor R ; and conſiſts of a cuſhion of leather ſtuffed, having a piece of ſilk glewed to its under part. The conductors are of tin covered with black lacker, each of them containing a large coated glaſs jar, and likewiſe a ſmaller one, or a coated tube, which are viſible when the caps NN are removed. To each conductor is fixed a knob O, for the occaſional ſuſpenſion of a chain to produce poſitive or negative electricity. That part of the winch C which acts as a lever in turning the cylinder, is of glaſs. Thus every part of the machine is inſulated, the cylinder itſelf and its braſs caps not excepted ; by which means very little of the electricity is diſſipated, and hence of courſe the effects are likely to be the more powerful. And to this the inventor has adapted ſome flexible conducting joints, a diſcharging electrometer, &c, for the practice of medical electricity.

The large Electrical Machine placed in Teyler' Muſeum at Harlem, has been partly deſcribed above. It was conſtructed by Mr. John Cuthbertſon, an Engliſh inſtrument maker; and it has, for the electric, two glaſs plates of 65 inches diameter, made of French glaſs, as this is found to produce the moſt electricity next to Engliſh flint glaſs, which could not be made of a ſufficient ſize : theſe plates are ſet on the ſame horizontal axis, at the diſtance of $7\frac{1}{2}$ inches, and are excited by 8 rubbers, each $15\frac{1}{2}$ inches long ; and both ſides of the plates are covered with a reſinous ſubſtance to the diſtance of $16\frac{1}{2}$ inches from the centre, both to ſtrengthen the plates, and to prevent any electricity from being carried off by the axis. Its battery of jars contains 225 ſquare feet of coated ſurface, and its effects are aſtoniſhingly great.

Electrical *Phial*. See Leyden *Phial*.

Electrical *Rubber*. See Electrical *Apparatus*, and Electrical *Machine*.

Electrical *Shock*, is the ſudden exploſion between the oppoſite ſides of a charged electric ; ſo called becauſe if the diſcharge be made through the body of an animal, it occaſions a ſudden motion by the contraction of the muſcles through which it paſſes, accompanied with a diſagreeable ſenſation. The force of this ſhock is proportioned to the quantity of coated ſurface, the thinneſs of the glaſs, and the power of the machine by which it is charged. Its velocity is almoſt inſtantaneous, and it has not been found to take up the leaſt ſenſible time in paſſing to the greateſt diſtances.

It has been obſerved that the Electrical Shock is weakened by being communicated through ſeveral perſons in contact with one another. Indeed it is obſtructed in its paſſage, even through the beſt conductors, as it will prefer a ſhort paſſage through the air to a long one through the moſt perfect conductors ; and if the circuit be interrupted, either by electrics, or very imperfect conductors of a moderate thickneſs, the ſhock will rend them in its paſſage, diſperſe them in every direction, and exhibit the appearance of a ſudden expanſion of the air about the centre of the ſhock. A ſtrong ſhock made to paſs through or over the belly of a muſcle, forces it to contract ; and ſent through a ſmall animal body, deprives it inſtantly of life, and haſtens putrefaction. It gives polarity to magnetic needles, reverſes their poles, and produces effects preciſely ſimilar, though inferior in degree, to thoſe of lightning.

Electrical *Star*. See Star.

ELECTRICITY, or Electrical *Force*, is that power or property, which was firſt obſerved in amber, the lyncurium, or tourmalin, and which ſealing-wax, glaſs, and a variety of other ſubſtances, called electrics, are now known to poſſeſs, of attracting light bodies, when excited by heat or friction; and which is alſo capable of being communicated in particular circumſtances to other bodies.

Electricity alſo denotes the ſcience, or that part of natural philoſophy, which propoſes to inveſtigate the nature and effects of this power. From ηλεκτρον, the Greek name for amber, is derived the term Electricity, which is now very extenſively applied, not only to the power of attracting light bodies inherent in amber, but to other ſimilar powers, and their various effects, in whatever bodies they reſide, or to whatever bodies they may be communicated.

Muſchenbroek and Æpinus have obſerved a conſiderable analogy, in a variety of particulars, between the powers of Electricity and Magnetiſm; and they have alſo pointed out many inſtances in which they differ.

Hiſtory of Electricity.——The property which amber poſſeſſes of attracting light bodies, was very anciently obſerved. Thales of Miletus, 600 years before Chriſt, concluded from hence that it was animated. But the firſt perſon who expreſsly mentioned this ſubſtance, was Theophraſtus, about 300 years before Chriſt. The attractive property of amber is alſo occaſionally noticed by Pliny, and other later naturaliſts, particularly Gaſſendus, Kenelm Digby, and Sir Thomas Brown. But it was generally apprehended that this quality was peculiar to amber and jet, and perhaps agate, till W. Gilbert, a native of Colcheſter, and a phyſician in London, publiſhed his treatiſe *De Magnete*, in the year 1600. Dr. Gilbert made many conſiderable experiments and diſcoveries, conſidering the then infant ſtate of the ſcience. He enlarged the liſt both of electrics, and of the bodies on which they act: he remarked, that a dry air was moſt favourable to electrical appearances, whilſt a moiſt air almoſt annihilates the electric virtue: he alſo obſerved the conical figure aſſumed by electrified drops of water: he conſidered electrical attraction ſeparately from repulſion, which he thought had no place in Electricity, as a phenomenon ſimilar to the attraction of coheſion, and he imagined, that electrics were brought into contact with the bodies on which they act by their effluvia, excited by friction.

The ingenious Mr. Boyle added to the catalogue of electric ſubſtances; but he thought that glaſs poſſeſſed this power in a very low degree: he found, that the Electricity of all bodies, in which it might be excited, was increaſed by wiping and warming them before they were rubbed; that an excited electric was acted upon by other bodies as ſtrongly as it acted upon them; that diamonds rubbed againſt any kind of ſtuff, emitted light in the dark; and that feathers would cling to the fingers, and to other ſubſtances, after they had been attracted by electrics. He accounted for electrical attraction, by ſuppoſing a glutinous effluvia emitted from electrics, which laid hold of ſmall bodies, in its way, and carried them back to the body from which it proceeded.

Otto Guericke, the celebrated inventor of the air-pump, lived about the ſame time. This ingenious philoſopher diſcovered, by means of a globe of ſulphur, that a body once attracted by an electric, was next repelled, and continued in this ſtate of repulſion till it ſhould be touched by ſome other body: he alſo obſerved the ſound and light produced by the excitation of his globe; and that bodies immerged in electrical atmoſpheres are themſelves electrified with an electricity oppoſite to that of the atmoſphere.

The light emitted by electrical bodies was, not long after, obſerved to much greater advantage by Dr. Wall, who aſcribes to light the electrical property which they poſſeſs; and he ſuggeſts a ſimilarity between the effects of electricity and lightning.

Sir Iſaac Newton was not inattentive to this ſubject: he obſerved that excited glaſs attracts light bodies on the ſide oppoſite to that on which it is rubbed; and he aſcribes the action of electric bodies to an elaſtic fluid, which freely penetrates glaſs, and the emiſſion of it to the vibratory motions of the parts of excited bodies.

Mr. Hawkſbee wrote on this ſubject in the year 1709, when a new æra commenced in the hiſtory of this ſcience. He firſt took notice of the great electrical power of glaſs, and the light proceeding from it; though others had before obſerved the light proceeding from other electrified ſubſtances: he alſo noted the noiſe occaſioned by it, with a variety of phenomena relating to electrical attraction and repulſion. He firſt introduced a glaſs globe into the electrical apparatus, to which circumſtance it was that many of his important diſcoveries were owing.

After his time there was an interval of near 20 years in the progreſs of this ſcience, till Mr. Stephen Grey eſtabliſhed a new æra in the hiſtory of Electricity. To him we owe the capital diſcovery of communicating the power of native electrics to other bodies, in which it cannot be excited, by ſupporting them on ſilken lines, hair lines, cakes of reſin or glaſs; and a more accurate diſtinction than had hitherto obtained between electrics and non-electrics: he alſo ſhewed the effect of electricity on water much more obviouſly than Gilbert had done in the infancy of this ſcience.

The experiments of Mr. Grey were repeated by M. du Fay, member of the Academy of Sciences at Paris, to which he added many new experiments and diſcoveries of his own. He obſerved, that electrical operations are obſtructed by great heat, as well as by a moiſt air; that all bodies, both ſolid and fluid, would receive electricity, when placed on warm or dry glaſs, or ſealing-wax; that thoſe bodies which are naturally the leaſt electric, have the greateſt degree of electricity communicated to them by the approach of the excited tube. He tranſmitted the electric virtue through a diſtance of 1256 feet; and firſt obſerved the electric ſpark from a living body, ſuſpended on ſilken lines, and noted ſeveral circumſtances attending it. M. du Fay alſo eſtabliſhed a principle, firſt ſuggeſted by Otto Guericke, that electric bodies attract all thoſe that are not ſo, and repel them as ſoon as they are become electric, by the vicinity or contact of the electric body. He likewiſe inferred from other experiments, that there were two kinds of electricity; one of which he called the *vitreous*, belonging to glaſs, rock cryſtal, &c ; and the other *reſinous*, as that of amber, gumlac, &c, diſtinguiſhed by their repelling thoſe of the ſame kind, and attracting each other. He farther obſerved, that communicated electricity had the ſame property as the excited ; and that electric ſubſtances attract the dew more than conductors.

Mr. Grey, reſuming his experiments in 1734, ſuſpended ſeveral pieces of metal on ſilken lines, and found that by electrifying them they gave ſparks ; which was the origin of metallic conductors : and on this occaſion he diſcovered a cone or pencil of electric light, ſuch as is now known to iſſue from an electrified point. From other experiments he concludes, that the electric power ſeems to be of the ſame nature with that of thunder and lightning.

Dr. Deſaguliers ſucceeded Mr. Grey in the proſecution of this ſcience. The account of his firſt experiments is dated in 1739. To him we owe thoſe technical terms of *conductors* or *non-electrics*, and *electrics per ſe ;* and he firſt ranked pure air among the electrics per ſe, and ſuppoſed its Electricity to be of the vitreous kind.

After the year 1742, in which Dr. Deſaguliers concluded his experiments, the ſubject was taken up and purſued in Germany : the globe was ſubſtituted for the tube, which had been uſed ever ſince the time of Hawkſbee, and a cuſhion was ſoon after uſed as a rubber, inſtead of the hand. About this time too, ſome uſed cylinders inſtead of the globes ; and ſome of the German electricians made uſe of more globes than one at the ſame time. By thus increaſing the electrical power, they were the firſt who ſucceeded in ſetting fire to inflammable ſubſtances : this was firſt done by Dr. Ludolf, in the beginning of the year 1744, who, with ſparks excited by the friction of a glaſs tube, kindled the ethereal ſpirit of Frobenius. Winkler did the ſame by a ſpark from his own finger, by which he kindled French brandy, and other ſpirits, after previouſly heating them. Mr. Gralath fired the ſmoke of a candle juſt blown out, and ſo lighted it again ; and Mr. Boze fired gun-powder, by means of its inflammable vapour. About this time Ludolf the younger demonſtrated, that the luminous barometer was made perfectly electrical by the motion of the quickſilver. The electrical ſtar and electrical bells were alſo of German invention.

In England Dr. Watſon made a diſtinguiſhed figure from this period in the hiſtory of Electricity : he fired a variety of ſubſtances by the electrical ſpark, and firſt diſcovered that they are capable of being fired by the repulſive power of Electricity. In the year 1745, the accumulation of the electrical power in glaſs, by means of the Leyden phial, was firſt diſcovered. See LEYDEN *phial :* and for the method practiſed about this time, of meaſuring the diſtance to which the electrical ſhock may be conveyed, ſee *Electrical* CIRCUIT. Dr. Watſon diſcovered that the glaſs tubes and globes do not contain the electric matter in themſelves, but only ſerve as *firſt-movers* or *determiners*, as he expreſſes it, of that power ; which was alſo confirmed towards the end of 1746, by Mr. Benjamin Wilſon, who made the ſame diſcovery, that the electric fluid does not come from the globe, but from the earth, and other non-electric bodies about the apparatus. Dr. Watſon alſo diſcovered what Dr. Franklin obſerved about the ſame time in America, and called the *plus* and *minus* in Electricity. He likewiſe ſhewed that the electric matter paſſed through the ſubſtance of the metal of communication, and not merely over the ſurface. The hiſtory of medical Electricity commenced in the year 1747. We muſt omit other experiments, and concluſions drawn from them, by Mr Wilſon, Mr. Smeaton, and Dr. Miles in England, and by the Abbé Nollet, with regard to the effect of Electricity on the evaporation of fluids, on ſolids, and on animal and other organized bodies, in France.

Whilſt the philoſophers of Europe were buſily employed in electrical experiments and purſuits, thoſe of America, and Dr. Franklin in particular, were equally induſtrious, and no leſs ſucceſsful. His diſcoveries and obſervations in Electricity were communicated in ſeveral letters to a friend ; the firſt of which is dated in 1747, and the laſt in 1754 ; and the particulars of his ſyſtem may be ſeen under the articles, *Theory of* ELECTRICITY, LEYDEN *Phial*, POINTS, CHARGING, CONDUCTORS, ELECTRICS, &c.

The ſimilarity between Electricity and Lightning had been ſuggeſted by ſeveral writers : Dr. Franklin firſt propoſed a method of bringing the matter to the teſt of experiment, by raiſing an electrical kite ; and he ſucceeded in collecting electrical fire by this means from the clouds, in 1752, one month after the ſame theory had been verified in France, and without knowing what had been done there : and to him we owe the practical application of this diſcovery, in ſecuring buildings from the damage of lightning, by erecting metallic conductors. See CONDUCTORS, and LIGHTNING.

In the ſubſequent period of the hiſtory of this ſcience, Mr. Canton in England, and Signior Beccaria in Italy, acquired diſtinguiſhed reputation. They both diſcovered, independently of each other, that air is capable of receiving Electricity by communication, and of retaining it when received. Mr. Canton alſo, towards the latter end of the year 1753, purſued a ſeries of experiments, which prove that the appearances of poſitive and negative Electricity, which had hitherto been deemed eſſential and unchangeable properties of different ſubſtances, as of glaſs and ſealing-wax for inſtance, depend upon the ſurface of the electrics, and that of the rubber.

This hypotheſis, verified by numerous experiments, occaſioned a controverſy between Mr. Canton, and Mr. Delaval, who ſtill maintained that theſe different powers depended entirely on the ſubſtances themſelves. About this time too, ſome curious experiments were performed by four of the principal electricians of that period, viz. Dr. Franklin, and Meſſrs Canton, Wilcke, and Æpinus, to aſcertain the nature of electric atmoſpheres ; the reſult of which ſee under that article.

The theory of two electric fluids, always co-exiſtent and counteracting each other, though not abſolutely independent, was maintained by a courſe of experiments on ſilk ſtockings of different colours, communicated to the Royal Society by Mr. Symmer, in the year 1759, which were farther purſued by Mr. Cigna of Turin, who publiſhed an account of them in the Memoirs of the Academy at Turin for the year 1765.

Many inſtances occur in the hiſtory of the ſcience about this period, of the aſtoniſhing force of the electric ſhock, in melting wires, and producing other ſimilar effects: but the moſt remarkable is an experiment of S. Beccaria, in which he thus revivified metals. Several experiments were alſo made by Dr. Watſon, Mr. Smeaton, Mr. Canton, and others, on the paſſage of the electric fluid through a vacuum, and its luminous appearance, and on the power poſſeſſed by certain ſubſtances of retaining the light communicated to them by an electric exploſion. Mr. Canton, S. Beccaria, and others, made many experiments to identify Electricity and lightning, to aſcertain the ſtate of the atmoſphere at different times, and to explain the various phenomena of the Aurora Borealis, Water-Spouts, Hurricanes, &c, on the principles of this ſcience.

Thoſe who are deſirous of farther information with reſpect to the hiſtory of electrical experiments and diſcoveries, may conſult Dr. Prieſtley's Hiſtory and Preſent State of Electricity. This author however is not merely an hiſtorian: his work contains many original experiments and diſcoveries made by himſelf. He aſcertained the conducting power of charcoal, and of hot glaſs; the Electricity of fixed and inflammable air, and of oil; the difference between new and old glaſs, with reſpect to the diffuſion of Electricity over its ſurface; the lateral exploſion in electrical diſcharges; a new method of fixing circular-coloured ſpots on the ſurfaces of metals, and the moſt probable difference between electrics and conductors, &c. The ſcience is alſo greatly indebted to many other perſons, either for their experiments and improvements of it, or for treatiſes and other writings upon it; as Mr. Henley, to whom we owe ſeveral curious experiments and obſervations on the electrical and conducting quality of different ſubſtances, as chocolate, vapour, &c, with the reaſon of the difference between them; the fuſion of platina; the nature of the electric fluid, and its courſe in a diſcharge; the method of eſtimating the quantity of it in electrical bodies by an electrometer; the influence of points; &c, &c. Alſo Meſſrs Van Marum, Van Swinden, Ferguſon, Cavallo, Lord Mahon, Nairne, &c, &c, for their ſeveral treatiſes on the ſubject of Electricity, any of which may be conſulted with advantage for the experiments and principles of the ſcience.

Medical ELECTRICITY. It is natural to imagine that a power of ſuch efficacy as that of electricity would be applied to medical purpoſes; eſpecially, ſince it has been found invariably to increaſe the ſenſible perſpiration, to quicken the circulation of the blood, and to promote the glandular ſecretion: accordingly, many inſtances occur in the latter period of the hiſtory of this ſcience, in which it has been applied with conſiderable advantage and ſucceſs. And among the variety of caſes in which it has been tried, there are none in which it has been found prejudicial except thoſe of pregnancy and the venereal diſeaſe. In moſt diſorders, in which it has been uſed with perſeverance, it has given at leaſt a temporary and partial relief, and in many it has effected a total cure. Of which numerous inſtances may be ſeen in the Philoſ. Tranſ. and the writings on this ſcience by Meſſrs Lovet, Weſtley, Ferguſon, Cavallo, &c. &c.

Theory of ELECTRICITY. It is hardly neceſſary to recite the ancient hypotheſes on this ſubject; ſuch as that of the ſympathetic powder of the Peripatetics; that of unctuous effluvia emitted by excited bodies, and returning to them again, adopted by Gilbert, Gaſſendus, Sir Kenelm Digby, &c; or that of the Carteſians, who aſcribed electricity to the globules of the firſt elements, diſcharged through the pores of the rubbed ſubſtance, and in their return carrying with them thoſe light bodies, in whoſe pores they were entangled: theſe hypotheſes were framed in the infancy of the ſcience, and of philoſophy in general, and have long ſince been exploded. In the more advanced ſtate of electricity there have been two principal theories, each of which has had its advocates. The one, is that of two diſtinct electric fluids, repulſive with reſpect to themſelves, and attractive of one another, adopted by M. du Fay, on diſcovering the two oppoſite ſpecies of electricity, viz, the vitreous and reſinous, and ſince new-modelled by Mr. Symmer. It is ſuppoſed that theſe two fluids are equally attracted by all bodies, and exiſt in intimate union in their pores; and that in this ſtate they exhibit no mark of their exiſtence. But that the friction of an electric by a rubber ſeparates theſe fluids, and cauſes the vitreous electricity of the rubber to paſs to the electric, and then to the prime conductor of a machine, while the reſinous electricity of the conductor and electric is conveyed to the rubber: and thus the quality of the electric fluid, poſſeſſed by the conductor and the rubber, is changed, while the quantity remains the ſame in each. In this ſtate of ſeparation, the two electric fluids will exert their reſpective powers; and any number of bodies charged with either of them will repel each other, attract thoſe bodies that have leſs of each particular fluid than themſelves, and be ſtill more attracted by bodies that are wholly deſtitute of it, or that are loaded with the contrary. According to this theory, the electric ſpark makes a double current; one fluid paſſing to an electrified conductor from any ſubſtance preſented to it, whilſt the ſame quantity of the other fluid paſſes from it; and when each body receives its natural quantity of both fluids, the balance of the two powers is reſtored, and both bodies are unelectrified. For a further account of the explication of ſome of the principal phenomena of electricity by this theory, ſee Dr. Prieſtley's Hiſtory, vol. 2, § 3.

The other theory is commonly diſtinguiſhed under the denomination of *poſitive and negative electricity*, being firſt ſuggeſted by Dr. Watſon, but digeſted, illuſtrated, and confirmed by Dr. Franklin; and ſince that it has been known by the appellation of the Franklinian hypotheſis. It is here ſuppoſed that all the phenomena of electricity depend on one fluid, *ſui generis*, extremely ſubtile and elaſtic, diſperſed through the pores of all bodies, by which the particles of it are as ſtrongly attracted as they are repelled by one another. When bodies poſſeſs their natural ſhare of this fluid, or ſuch a quantity as they can retain by their non-attraction, it is then ſaid they are in an unelectrified ſtate; but when the equilibrium is diſturbed, and they either acquire an additional quantity *from* other bodies, or loſe part of their own natural ſhare by communication *to* other bodies, they exhibit electrical appearances. In the former caſe it is ſaid they are electrified poſitively, or *plus ;* and in the other negatively, or *minus*. This electric fluid, it is ſuppoſed, moves with great eaſe in thoſe bodies that are called conductors, but with extreme difficulty and ſlowneſs in the pores of electrics ; whence it comes to paſs, that all electrics are impermeable to it. It is farther ſuppoſed that electrics contain always an equal quantity of this fluid, ſo that there can be no ſurcharge or increaſe on one ſide without a proportionable decreaſe or loſs on the other, and vice verſa ; and as the electric does not admit the paſſage of the fluid through its pores, there will be an accumulation on one ſide, and a correſponding deficiency on the other. Then when both ſides are connected together by proper conductors, the equilibrium will be reſtored by the ruſhing of the redundant fluid from the overcharged ſurface to the exhauſted one. Thus alſo, if an electric be rubbed by a conducting ſubſtance, the electricity is only conveyed from one to the other, the one giving what the other receives ; and if one be electrified poſitively, the other will be electrified negatively, unleſs the loſs be ſupplied by other bodies connected with it, as in the caſe of the electric and inſulated rubber of a machine. This theory ſerves likewiſe to illuſtrate the other phenomena and operations in the ſcience of electricity. Thus, bodies differently electrified will naturally attract each other, till they mutually give and receive an equal quantity of the electric fluid, and the equilibrium is reſtored between them. Beccaria ſuppoſes, that this effect is produced by the electric matter making a vacuum in its paſſage, and the contiguous air afterwards collapſing, and ſo puſhing the bodies together.

The influence of points, in drawing or throwing off the electric fluid, depends on the leſs reſiſtance it finds to enter or paſs off through fewer particles than through a greater number, whoſe reſiſtance is united in flat or round ſurfaces. The electric light is ſuppoſed to be part of the electric fluid, which appears when it is properly agitated ; and the ſound of an exploſion is produced by vibrations, occaſioned by the air's being diſplaced by the electric fluid, and again ſuddenly collapſing.

As to the nature of the electric fluid, philoſophers have entertained very different ſentiments : ſome, and among them Mr. Wilſon, have ſuppoſed that it is the ſame with the ether of Sir Iſaac Newton, to which the phenomena of attraction and repulſion are aſcribed ; whilſt the light, ſmell, and other ſenſible qualities of the electric fluid, are referred to the groſſer particles of bodies, driven from them by the forcible action of this ether ; and other appearances are explained by means of a ſubtile medium diffuſed over the ſurfaces of all bodies, and reſiſting the entrance and exit of the ether ; which medium, it is ſuppoſed, is the ſame with the electric fluid, and is more rare on the ſurfaces of conductors, and more denſe and reſiſting on thoſe of electrics : but Dr. Prieſtley remarks that, though they may poſſeſs ſome common properties, they have others eſſentially diſtinct ; the ether is repelled by all other matter, whereas the electric fluid is ſtrongly attracted by it. Others have had recourſe to the element of fire ; and from the ſuppoſed identity of fire and the electric fluid, as well as from the ſimilarity of ſome of their effects, the latter has been uſually called the electric fire : but moſt electricians have ſuppoſed that it is a fluid *ſui generis*. Mr. Cavendiſh has publiſhed an attempt to deduce and explain ſome of the principal phenomena of electricity in a mathematical and ſyſtematic manner, from the nature of this fluid, conſidered as compoſed of particles that repel each other, and attract the particles of all other matter, with a force inverſely as ſome leſs power of the diſtance than the cube, whilſt the particles of all other matter repel each other, and attract thoſe of the electric fluid, according to the ſame law. Philoſ. Tranſ. vol. 61, pa. 584—677. And a ſimilar hypotheſis and method of reaſoning was alſo propoſed by M. Æpinus, in his Tentamen Theoriæ Electricitatis & Magnetiſmi.

Dr. Prieſtley concludes, from experiments, that the electric matter either is phlogiſton, or contains it, ſince he found that both produced ſimilar effects. Mr. Henley alſo apprehends, that the electric fluid is a modification of that element, which, in its quieſcent ſtate, is called phlogiſton ; in its firſt active ſtate, electricity ; and when violently agitated, fire. Perhaps we may be allowed to enlarge our views, and conſider the ſun as the fountain of the electric fluid, and the zodiacal light, the tails of comets, the aurora borealis, lightning, and artificial electricity, as its various and not very diſſimilar modifications. On this ſubject, ſee Prieſtley's Hiſt. of Electr. vol. 2, part 3, § 1, 2, 3 ; Wilſon's Eſſay towards an Explication of the Phenomena of Electricity, &c ; Wilſon and Hoadley's Obſ. &c, pa. 55, 1759 ; Freke's Eſſay on the Cauſe of Electricity, 1746 ; Prieſtley on Air, vol. 1, pa. 186, 274, &c ; Philoſ. Tranſ. vol. 67, pa. 129 ; and Mr. Eeles's Letters, on the ſame ſubject.

ELECTROMETER, is an inſtrument that meaſures the quantity, and determines the quality of electricity, in any electrified body. Previous to the invention of inſtruments of this kind, Mr. Canton eſtimated the quantity of electricity in a charged phial, by preſenting the phial with one hand to an inſulated conductor, and giving it a ſpark, which he took off with the other ; proceeding in this manner till the phial was diſcharged, when he determined the height of the charge by the number of ſparks. Electrometers are of 4 kinds : 1, the ſingle thread ; 2, the cork or pith balls ; 3, the quadrant ; and 4, the diſcharging Electrometer.

The 1ſt, or moſt ſimple Electrometer, is a linen thread, called by Dr. Deſaguliers, the *thread of trial* ; which, being brought near an electrified body, is attracted by it : but this does little more than determine whether the body is in any degree electrified or not ; without determining with any preciſion its quantity, much leſs the quality of it. The Abbé Nollet uſed two threads, ſhewing the degree of electricity by the angle of their divergency exhibited in their ſhadow on a board placed behind them.

Mr. Canton's Electrometer conſiſted of two balls of cork, or pith of elder, about the ſize of a ſmall pea, ſuſpended by fine linen threads, about 6 inches long, which may be wetted in a weak ſolution of ſalt. See fig. 7. If the box containing theſe balls be inſulated, by placing it on a drinking glaſs, &c, and an excited ſmooth glaſs tube be brought near them, they will firſt be attracted by it, and then be repelled both from the glaſs, and from each other; but on the approach of excited wax, they will gradually approach and come together; and vice verſa. This apparatus will alſo ſerve to determine the electricity of the clouds and air, by holding them at a ſufficient diſtance from buildings, trees, &c; for if the electricity of the clouds or air be poſitive, their mutual repulſion will increaſe by the approach of excited glaſs, or decreaſe by the approach of amber or ſealing-wax; on the contrary, if it be negative, their repulſion will be diminiſhed by the former, and increaſed by the latter. See Philoſ. Tranſ. vol. 48, part 1 and 2, for an account of Mr. Canton's curious experiments with this apparatus.

If two balls of this kind be annexed to a prime conductor, they will ſerve to determine both the degree and quality of its electriſication, by their mutual repulſion and divergency.

The Diſcharging Electrometer, fig. 3, plate ix, was invented by Mr. Lane. It conſiſts of braſs work G, the lower part of which is incloſed in the pillar F, made of baked wood, and boiled in linſeed oil, and bored cylindrically about two-thirds of its length; the braſs work is fixed to the pillar by the ſcrew H, moveable in the groove I; and through the ſame is made to paſs a ſteel ſcrew L, to the end of which, and oppoſite to K, a poliſhed hemiſpherical piece of braſs, attached to the prime conductor, is fixed a ball of braſs M well poliſhed. To this ſcrew is annexed a circular plate O, divided into 12 equal parts. The uſe of this Electrometer is to diſcharge a jar D, or any battery connected with the conductor, without a diſcharging rod, and to give ſhocks ſucceſſively of the ſame degree of ſtrength; on which account it is very fit for medical purpoſes. Then, if a perſon holds a wire faſtened to the ſcrew H in one hand, and another wire fixed to E, a loop of braſs wire paſſing from the frame of the machine to a tin plate, on which the phial D ſtands, he will perceive no ſhock, when K and M are in contact; and the degree of the exploſion, as well as the quantity of electricity accumulated in the phial, will be regulated by the diſtance between K and M. Philoſ. Tranſ. vol. 57, pa. 451.—Mr. Henley much improved Mr. Lane's Electrometer, by taking away the ſcrew, the double milled nut, and the ſharp-edged graduated plate, and adding other contrivances in their ſtead. Mr. Henley's diſcharger of this kind has two tubes, one ſliding within the other, to lengthen and accommodate it to larger apparatus.

The Quadrant Electrometer of Mr. Henley, conſiſts of a ſtem, terminating at its lower end with a braſs ferrule and ſcrew, for faſtening it upon any occaſion; and its upper part ends in a ball. Near the top is fixed a graduated ſemicircle of ivory, on the centre of which the index, being a very light rod with a cork ball at its extremity, reaching to the braſs ferrule of the ſtem, is made to turn on a pin in the braſs piece, ſo as to keep near the graduated limb of the ſemicircle. When the Electrometer is not electrified, the index hangs parallel to the ſtem; but as ſoon as it begins to be electrified, the index, repelled by the ſtem, will begin to move along the graduated edge of the ſemicircle, and ſo mark the degree to which the conductor is electrified, or the height to which the charge of any jar or battery is advanced.

Mr. Cavallo has alſo contrived ſeveral ingenious Electrometers, for different uſes; as may be ſeen in his Treatiſe on Electricity, pa. 370, &c, and in the Philoſ. Tranſ. vol. 67, pa. 48 and 399.

ELECTROPHOR, or ELECTROPHORUS, an inſtrument for ſhewing perpetual electricity; which was invented by Mr. Volta, of Como, near Milan, in Italy. The machine conſiſts of two plates, fig. 8, one of which B is a circular plate of glaſs, covered on one ſide with ſome reſinous electric, and the other A is a plate of braſs, or a circular board, coated with tinfoil, and furniſhed with a glaſs handle I, which may be ſcrewed into its centre by means of a ſocket. If the plate B be excited by rubbing it with new white flannel, and the plate A be applied to its coated ſide, a finger, or any other conductor, will receive a ſpark on touching this plate; and if the plate A be then ſeparated, by means of the handle I, it will be found ſtrongly electrified, with an electricity contrary to that of the plate B. By replacing the plate A, touching it with the finger, and ſeparating it again, it will be found electrified as before, and give a ſpark to any conductor, attended with a ſnapping noiſe; and by this means a coated phial may be charged. The ſame phenomena may repeatedly be exhibited, without any renewed excitation of the electric plate B; the electric power of B having continued for ſeveral days, and even weeks, after excitation; though there is no reaſon to imagine that it is perpetual.

Mr. Cavallo prepares this machine by coating the glaſs plate with ſealing-wax; and Mr. Adams, philoſophical inſtrument maker, prepares them with plates formed from a compoſition of two parts of ſhell-lac, and one of Venice turpentine, without any glaſs plate.

The action of this plate depends on a principle diſcovered and illuſtrated by the experiments of Franklin, Canton, Wilcke, and Æpinus, viz, that an excited electric repels the electricity of another body, brought within its ſphere of action, and gives it a contrary electricity. Thus the plate A, touched by a conductor, whilſt in contact with the plate B, electrified negatively, will acquire an additional quantity of the electric fluid from the conductor; but if it were in contact with a plate electrified poſitively, it would part with its electricity to the conductor connected with it. See an account of ſeveral curious experiments with this machine, by Mr. Henley, Mr. Cavallo, and Dr. Ingenhouſz, in the Philoſ. Tranſ. vol. 66, pa. 513; vol. 67, pa. 116 and 389; and vol. 68, pa. 1027 and 1049.

ELEMENTARY, ſomething that relates to the principles or elements of bodies, or ſciences; as Elementary Air, Fire, Geometry, Muſic, &c.

ELEMENTS, the firſt principles, of which all bodies and things are compoſed. Theſe are ſuppoſed few in number, unchangeable, and by their combinations producing that extenſive variety of objects to be met with in the works of nature.

Democritus ſtands at the head of the Elementary Philoſophers, in which he is followed by Epicurus, and many others after them, of the Epicurean and corpuſcular philoſophers.

Among thoſe who hold the Elements corruptible, ſome will have only one, and ſome ſeveral. Of the former, the principal are Heraclitus, who held fire; Anaximenes, air; Thales Mileſius, water; and Heſiod, earth; as the only Element. Heſiod is followed by Bernardin, Teleſius; and Thales by many of the chemiſts.

Among thoſe who admit ſeveral corruptible Elements, the principal are the Peripatetics; who, after their leader Ariſtotle, contend for four Elements, viz, fire, air, water, and earth. Ariſtotle took the notion from Hippocrates; Hippocrates from Pythagoras; and Pythagoras from Ocellus Lucanus, who it ſeems was the firſt author of it.

The Carteſians admit only three Elements, fire, air, and earth. See Cartesian *Philoſophy*.

Newton obſerves, that it ſeems probable that God, in the beginning, formed matter in ſolid, maſſive, hard, impenetrable, moveable particles, of ſuch ſizes and figures, &c, as moſt conduced to the end for which he formed them; and that theſe primitive particles, being ſolids, are incomparably harder than any porous body compounded of them; even ſo hard as never to wear out; no ordinary power being able to divide what God made one in the firſt creation. While the particles remain entire, they may compoſe bodies of one and the ſame nature and texture in all ages; but ſhould they wear away, or break in pieces, the nature of things, depending on them, would be changed; water and earth, compoſed of old worn particles, and fragments of particles, would not be of the ſame nature and texture now, with water and earth compoſed of entire particles in the beginning. And therefore, that things may be laſting, the changes of corporeal things are to be placed only in the various ſeparations, and new aſſociations and motions of thoſe permanent particles; compound bodies being apt to break, not in the midſt of ſolid particles, but where thoſe particles are laid together, and only touch in a few points. It ſeems to him likewiſe, that theſe particles have not only a vis inertiæ, with the paſſive laws of motion thence reſulting, but are alſo moved by certain active principles; ſuch as gravity, and the cauſe of fermentation, and the coheſion of bodies.

Elements, a term alſo uſed for the firſt grounds and principles of arts and ſciences; as the Elements of geometry, Elements of mathematics, &c. So Euclid's Elements, or ſimply the Elements, as they were anciently and peculiarly named, denotes the treatiſe on the chief properties of geometrical figures by that author.

The Elements of Mathematics have been delivered by ſeveral authors in their courſes, ſyſtems, &c. The firſt work of this kind is that of Herigon, in Latin and French, and publiſhed in 1664, in 10 tomes; which contains Euclid's Elements and Data, Apollonius, Theodoſius, &c; with the modern Elements of arithmetic, algebra, trigonometry, architecture, geography, navigation, optics, ſpherics, aſtronomy, muſic, perſpective, &c. The work is remarkable for this, that a kind of real and univerſal characters are uſed throughout; ſo that the demonſtrations may be underſtood by ſuch as only remember the characters, without any dependence on language or words at all.

Since Herigon, the Elements of the ſeveral parts of mathematics have been alſo delivered by others; particularly the Jeſuit Schottus, in his Curſus Mathematicus, in 1674; De Chales, in his Curſus, 1674; Sir Jonas Moore, in his New Syſtem of Mathematics, in 1681; Ozanam, in his Cours de Mathematique, in 1699; Jones, in his Synopſis Palmariorum Matheſeos, in 1706; and many others, but above all, Chriſt. Wolfius, or Wolf, in his Elementa Matheſeos Univerſæ, in 2 vols 4to, the 1ſt publiſhed in 1713, and the 2d in 1715; a very excellent work of the kind. Another edition of the work was publiſhed at Geneva, in 5 vols 4to, of the ſeveral dates 1732, 1733, 1735, 1738, and 1741.

The Elements of Euclid, as they were the firſt, ſo they continue ſtill the beſt ſyſtem of geometry, are in 15 books. There have been numerous editions and commentaries of this work. Proclus wrote a commentary on it. Orontius Fineus firſt gave a printed edition of the firſt 6 books, in 1530, with notes, to explain Euclid's ſenſe. Peletarius did the ſame in 1557. Nic. Tartaglia, about the ſame time, made a comment on all the 15 books, with the addition of many things of his own. And the ſame was alſo done by Billingſley in 1570; and by Fluſſates Candalla, a noble Frenchman, in the year 1578, with conſiderable additions as to the compariſon and inſcriptions of ſolid bodies; which work was afterwards republiſhed with a prolix commentary, by Clavius. Commandine gave alſo a good edition of it. In 1703, Dr. Gregory publiſhed an edition of the whole works of Euclid, in Greek and Latin, including his Elements. But it would be endleſs to relate all the other editions of theſe Elements, either the whole, or in part, that have been given; ſome of the beſt of which are thoſe of De Chales, Tacquet, Ozanam, Whiſton, Stone, and moſt eſpecially that of Dr. Rob. Simſon, of Glaſgow.

Other writers on the Elements of Geometry are almoſt out of number, in all nations.

Elements, in the Higher or Sublime Geometry, are the infinitely ſmall parts, or differentials, of a right line, curve, ſurface, or ſolid.

Elements, in Aſtronomy, are thoſe principles deduced from aſtronomical obſervations and calculations, and thoſe fundamental numbers, which are employed in the conſtruction of tables of the planetary motions. Thus, the Elements of the theory of the ſun, or rather of the earth, are his mean motion and eccentricity, with the motion of the aphelia. And the Elements of the theory of the moon, are her mean motion, that of the node and apogee, the eccentricity, the inclination of her orbit to the plane of the ecliptic; &c.

ELEVATION, the height or altitude of any thing.

Elevation, in Architecture, denotes a draught or deſcription of the principal face or ſide of a building; called alſo its upright or orthography.

Elevation, in Aſtronomy and Geography, is various; as Elevation of the equator, of the pole, of a ſtar, &c.

Elevation *of the Equator*, is the height of the equator above the horizon; or an arc of the meridian intercepted between the equator and the horizon of the place.—The Elevation of the equator and of the pole together always make up a quadrant; the one being the complement of the other. Therefore, the Elevation of the pole being found, and ſubtracted from 90°, leaves the elevation of the equator.

Elevation *of the Pole*, is its height above the horizon ; or an arc of the meridian comprehended between the equator and the horizon of the place.

The elevation of the pole is always equal to the latitude of the place ; that is, the arc of the meridian intercepted between the pole and the horizon, is every where equal to the arc of the ſame meridian intercepted between the equator and the zenith. Thus the north pole is elevated 51° 31′ above the horizon of London ; and the diſtance, or number of degrees, is the ſame between London and the equator ; ſo that London is alſo in 51° 31′ of north latitude.

ELEVATION *of a Star*, or of any other point in the ſphere, is the angular height above the horizon ; or an arc of the vertical circle intercepted between the ſtar and the horizon. The meridian altitude of any ſuch point, or its altitude when in the meridian, is the greateſt of all.

ELEVATION *of a Cannon*, or *Mortar*, is the angle which the bore or the axis of the piece makes with the horizontal plane.

Angle of ELEVATION, is the angle which any line of direction makes above a horizontal line.

ELEVATION is alſo uſed by ſome writers on Perſpective, for the ſcenography, or perſpective repreſentation of the whole body or building.

ELLIPSE, or ELLIPSIS, is one of the conic ſections, popularly called an oval ; being called an Ellipſe or Ellipſis by Apollonius, the firſt and principal author on the conic ſections, becauſe in this figure the ſquares of the ordinates are *leſs* than, or *defective* of, the rectangles under the parameters and abſciſſes.

This figure is differently defined by different authors ; either from ſome of its properties, or from mechanical conſtruction, or from the ſection of a cone, which is the beſt and moſt natural way. Thus ;

Fig. 1.

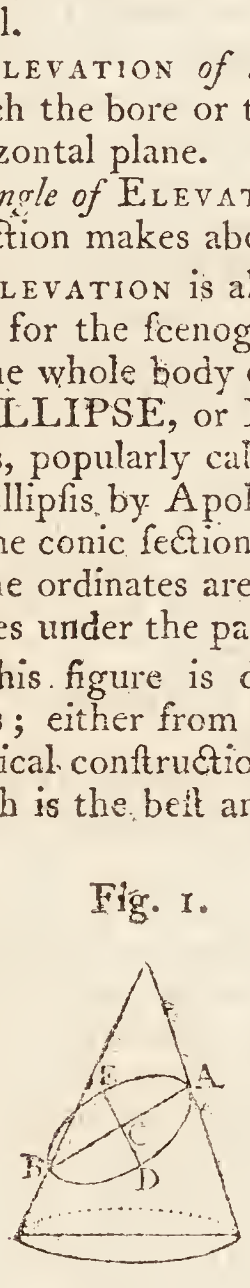
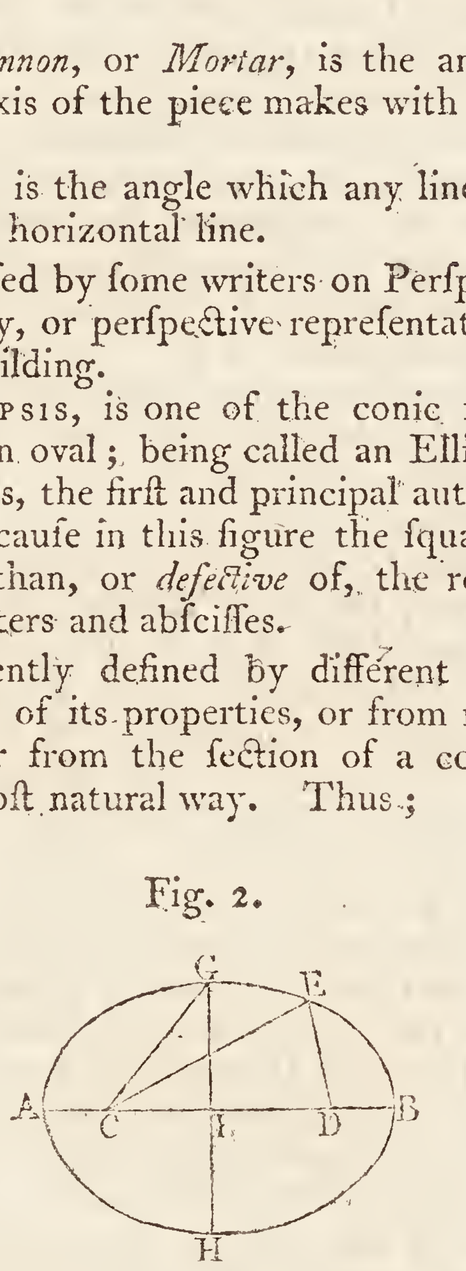


Fig. 2.



1. An Ellipſe is a plane figure made by cutting a cone by a plane obliquely through the oppoſite ſides of it ; or ſo as that the plane makes a leſs angle with the baſe than the ſide of the cone makes with it ; as ABD, fig. 1.

The line AB connecting the uppermoſt and loweſt points of the ſection, is the tranſverſe axis ; the middle of it C, is the centre ; and the perpendicular to it DCE, through the centre, is the conjugate axis. The parameter, or latus rectum, is a 3d proportional to the tranſverſe and conjugate axes ; and the foci are two points in the tranſverſe axis, at ſuch equal diſtances from the centre, that the double ordinates paſſing through thoſe points, and perpendicular to the tranſverſe, are equal to the parameter.

2. The Ellipſe is alſo variouſly deſcribed from ſome of its properties. As firſt, That it is a figure of ſuch a nature that if two lines be drawn from two certain points C and D in the axis, fig. 2, to any point E in the circumference, the ſum of thoſe two lines CE and DE will be every where equal to the ſame conſtant quantity, viz, the axis AB. Or, ſecondly, that it is a figure of ſuch a nature, that the rectangle AG × GB (fig. 3) of the abſciſſes, tending contrary ways, is to GH^2 the ſquare of the ordinate, as AB^2 to IK^2, the ſquare of the tranſverſe axis to the ſquare of the conjugate, or, which is the ſame thing, as the tranſverſe axis is to the parameter. And ſo of other properties.

Fig. 3.

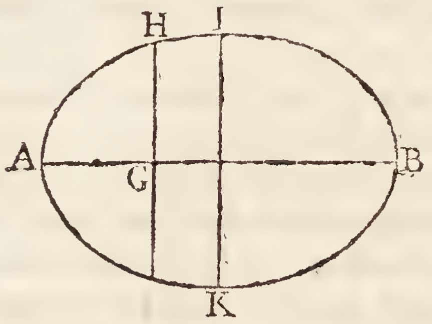
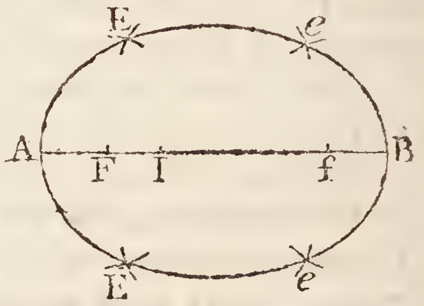


Fig. 4.



3. Or the Ellipſe is alſo variouſly deſcribed from its mechanical conſtructions, which alſo depend on ſome of its chief properties. Thus ; 1ſt, If in the axis AB, there be taken any point I (fig. 4) ; and if with the radii AI, BI, and centres F and *f*, the two foci, arcs be deſcribed, theſe arcs will interſect in certain points E, E, *e*, *e*, which will be in the curve or circumference of the figure : and thus ſeveral points I being taken in the axis AB, as many more points E, *e*, &c, will be found ; then the curve line drawn through all theſe points E, *e*, will be an Ellipſe. Or, thus ; if there be taken a thread of the exact length of the tranſverſe axis AB, and the ends of the thread be fixed by pins in the two foci F and *f* ; (fig. 5) then moving a pen or pencil within the thread, ſo as to keep it always ſtretched out, it will deſcribe the curve called an Ellipſe.

Fig. 5.

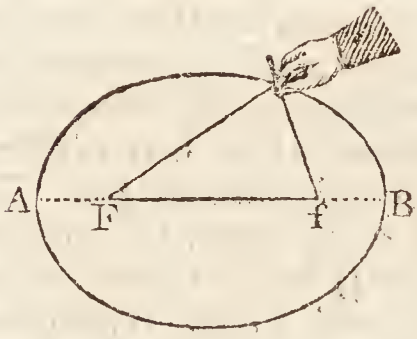
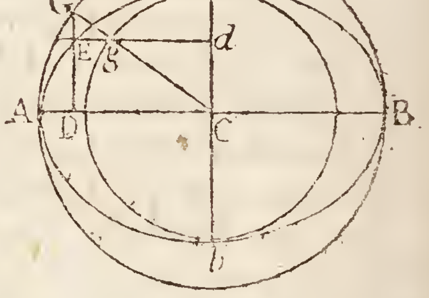


Fig. 6.



To Conſtruct an ELLIPSE. There are many other ways of deſcribing or conſtructing an Ellipſe, beſides thoſe juſt now given : as

1ſt. If upon the given tranſverſe axis there be deſcribed a circle AGB (fig. 6), to which draw any ordinate DG, and DE a 4th proportional to the tranſverſe, the conjugate, and the ordinate DG ; then E is a point in the curve. Or if the circle *agb* be deſcribed on the conjugate axis *ab*, to which any ordinate *dg* is drawn, in which taking *d*E in like manner a 4th proportional to the conjugate, the tranſverſe, and ordinate *dg*, then ſhall E be in the curve. Or, having deſcribed the two circles, and drawn the common radius C*g*G cutting them in G and *g*; then *dg*E drawn parallel to the tranſverſe, and DGE parallel to the conjugate, the interſection E of theſe two lines will be in the curve of the ellipſe. And thus ſeveral points E being found, the curve may be drawn through them all with a ſteady hand.

Fig. 7.

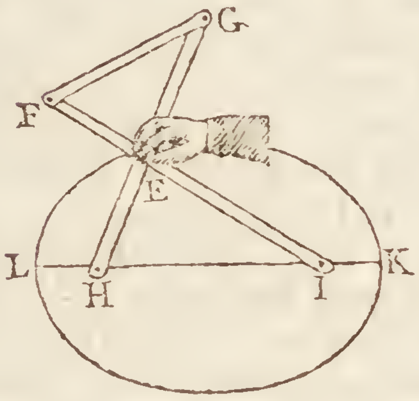
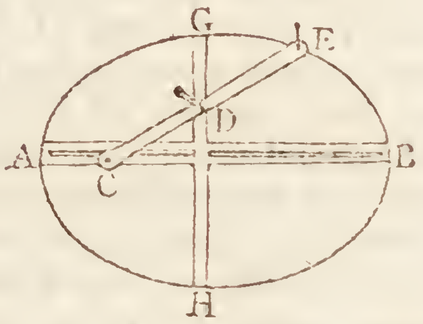


Fig. 8.



2. If there be provided three rulers, of which the two GH and FI (fig. 7) are of the length of the tranſverſe axis LK, and the third FG equal to HI the diſtance between the foci; then connecting theſe rulers ſo as to be moveable about the foci H and I, and about the points F and G, their interſection E will always be in the curve of the ellipſe; ſo that by moving the rulers about the joints, with a pencil paſſed through the ſlits made in them, it will trace out the Ellipſe.

Fig. 9.

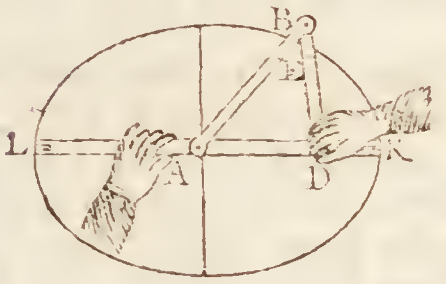
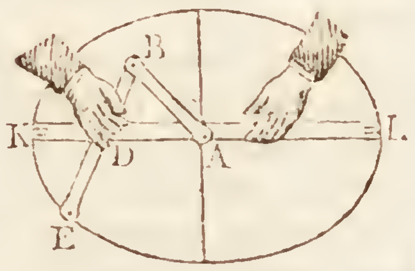


Fig. 10.



3. If one end A of any two equal rulers AB, DB, (fig. 9 and 10) which are moveable about the point B, like a carpenter's joint-rule, be faſtened to the ruler LK, ſo as to be moveable about the point A; and if the end D of the ruler DB be drawn along the ſide of the ruler LK; then any point E, taken in the ſide of the ruler DB, will deſcribe an ellipſe, whoſe centre is A, conjugate axis = 2DE, and tranſverſe = 2AB + 2BE.

Another method of deſcription is by the Elliptical Compaſs. See that article, below.

Some of the more Remarkable Properties of the Ellipſe.—1. The rectangles under the abſciſſes are proportional to the ſquares of their ordinates; or as the ſquare of any axis, or any diameter, is to the ſquare of its conjugate, ſo is the rectangle under two abſciſſes of the former, to the ſquare of their ordinate parallel to the latter; or again, as any diameter is to its parameter, ſo is the ſaid rectangle under two abſciſſes of that diameter, to the ſquare of their ordinate. So that if d be any diameter, c its conjugate, p its parameter $= \frac{c^2}{d}$, x the one abſciſs, $d-x$ the other, and y the ordinate; then,

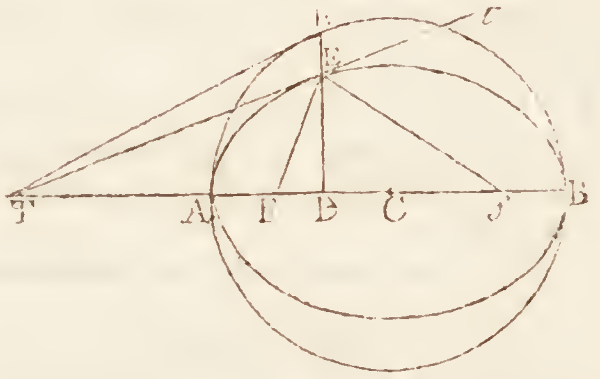
as $d^2 : c^2 :: x . \overline{d-x} : y^2$, or $d^2y^2 = c^2x . \overline{d-x}$;

or as $d : p :: x . \overline{d-x} : y^2$, or $dy^2 = px . \overline{d-x}$.

From either of which equations, called the equation of the curve, any one of the quantities may be found, when the other three are given.

2. The ſum of two lines drawn from the foci to meet in any point of the curve, is always equal to the tranſverſe axis; that is, CE + DE = AB, in the 2d fig. Conſequently the line CG drawn from the focus to the end of the conjugate axis, is equal to AI the ſemi-tranſverſe.

Fig. 11.



3. If from any point of the curve, there be an ordinate to either axis, and alſo a tangent meeting the axis produced; then half that axis will be a mean proportional between the diſtances from the centre to the two points of interſection; viz, CA a mean proportional between CD and CT. And conſequently all the tangents TE, TE, meet in the ſame point of the axis produced, which are drawn from the extremities E, E, of the common ordinates DE, DE, of all Ellipſes deſcribed on the ſame axis AB.

4. Two lines drawn from the foci to any point of the curve, make equal angles with the tangent at that point: that is, the $\angle$ FET = $\angle f$E*t*.

5. All the parallelograms are equal to each other, that are circumſcribed about an Ellipſis; and every ſuch parallelogram is equal to the rectangle of the two axes.

6. The ſum of the ſquares of every pair of conjugate diameters, is equal to the ſame conſtant quantity, viz, the ſum of the ſquares of the two axes.

7. If a circle be deſcribed upon either axis, and from any point in that axis an ordinate be drawn both to the circle and ellipſis; then ſhall the ordinate of the circle be to the ordinate of the Ellipſe, as that axis is to the other axis: viz,

AB : *ab* : : DG : DE,

and *ab* : AB : : *dg* : *d*E. (in the 6th fig.)

And in the ſame proportion is the area of the circle to the area of the Ellipſe, or any correſponding ſegments ADG, ADE. Alſo the area of the Ellipſe is a mean proportional between the areas of the inſcribed and circumſcribed circles. Hence therefore,

8. *To find the Area of an Ellipſe.* Multiply the two axes together, and that product by ·7854, for the area. Or

9. *To find the Area of any Segment* ADE. Find the

area of the correſponding ſegment ADG of a circle on the ſame diameter AB; then ſay, as the axis AB : its conj. *ab* : : circ. ſeg. ADG : elliptic ſeg. ADE.

10. *To find the length of the whole circumference of the Ellipſe.* Multiply the circumference of the circumſcribing circle by the ſum of the ſeries

$$1 - \frac{d}{2^2} - \frac{3d^2}{2^2.4^2} - \frac{3^2.5d^3}{2^2.4^2.6^2} - \frac{3^2.5^2.7d^4}{2^2.4^2.6^2.8^2} \ \&c,$$

for the area: where d is the difference between an unit and the ſquare of the leſs axis divided by the ſquare of the greater.

Or, for a near approximation, take the circumference of the circle whoſe diameter is an arithmetical mean between the two axes, or half their ſum: that is, $\frac{t+c}{2} \times 3{\cdot}1416 =$ the perimeter nearly; being about the 200th part too little; where t denotes the tranſverſe, and c the conjugate axis.

Or, again, take the circumference of the circle, the ſquare of whoſe diameter is half the ſum of, or an arithmetical mean between the ſquares of the two axes: that is, $\sqrt{\frac{t^2+c^2}{2}} \times 3{\cdot}1416 =$ the perimeter nearly; being about the 200th part too great.

Hence combining theſe two approximate rules together, the periphery of the Ellipſe will be very nearly equal to half their ſum, or equal to $\frac{3{\cdot}1416}{2} \times \left(\frac{t+c}{2} + \sqrt{\frac{t^2+c^2}{2}}\right)$, within about the 30000th part of the truth.

For the length of any particular arc, and many other parts about the Ellipſe, ſee my Menſuration, pa. 283, &c, 2d edit. See alſo my Conic Sections, for many other properties of the Ellipſe, eſpecially ſuch as are common to the hyperbola alſo, or to the conic ſections in general.

Infinite ELLIPSES. See ELLIPTOIDE.

ELLIPSOID, is an elliptical ſpheroid, being the ſolid generated by the revolution of an ellipſe about either axis. See SPHEROID.

ELLIPTIC or ELLIPTICAL, ſomething relating to an ellipſe.

ELLIPTIC *Compaſſes*, or ELLIPTICAL *Compaſs*, is an inſtrument for deſcribing ellipſes at one revolution of the index. It conſiſts of a croſs ABGH (fig. 8.) with grooves in it, and an index CE, ſliding in dovetail grooves; by which motion the end E deſcribes the curve of an ellipſe.

ELLIPTICAL *Conoid*, is ſometimes uſed for the ſpheroid.

ELLIPTICAL *Dial*, an inſtrument uſually made of braſs, with a joint to fold together, and the gnomon to fall flat, for the convenience of the pocket. By this inſtrument are found the meridian, the hour of the day, the riſing and ſetting of the ſun, &c.

ELLIPTOIDE, an infinite or indefinite Ellipſis, defined by the indefinite equation $ay^{m+n} = bx^m . \overline{a-x}^n$ when m or n are greater than 1: for when they are each $= 1$, it denotes the common ellipſe.

There are ſeveral kinds or degrees of Elliptoides, denominated from the exponent $m+n$ of the ordinate y. As the cubical Elliptoide, expreſſed by $ay^3 = bx^2 . \overline{a-x}$; the biquadratic, or ſurſolid $ay^4 = bx^2 . \overline{a-x}^2$; &c.

ELONGATION, in Aſtronomy, the diſtance of a planet from the ſun, with reſpect to the earth; or the angle formed by two lines drawn from the earth, the one to the ſun, and the other to the planet; or the arc meaſuring that angle: Or it is the difference between the ſun's place and the geocentric place of the planet.

The Greateſt Elongation, is the greateſt diſtance to which the planets recede from the ſun, on either ſide. This is chiefly conſidered in the inferior planets, Venus and Mercury; the Greateſt Elongation of Venus being about 48 degrees, and of Mercury only about 28 degrees; which is the reaſon that this planet is ſo rarely ſeen, being uſually loſt in the light of the ſun.

EMBER-*Days*, are certain days obſerved by the church at four different ſeaſons of the year; viz, the Wedneſday, Friday, and Saturday next after Quadrageſima Sunday, or the 1ſt Sunday in Lent; after Whitſunday; after Holyrood, or Holycroſs, the 14th day of September; and after St. Lucy, the 13th day of December. The name, it ſeems, is derived from Embers, or aſhes, which it is ſuppoſed were ſtrewed on the head, on theſe ſolemn faſts.

EMBER-*Weeks*, are thoſe weeks in which the Ember-days fall. Theſe Ember-weeks are now chiefly noticed on account of the ordination of prieſts and deacons; becauſe the canon appoints the Sundays next after the Ember-weeks for the ſolemn times of ordination; though the biſhops, if they pleaſe, may ordain on any Sunday or holiday.

EMBOLIMÆAN, and EMBOLISMIC, *Intercalary*, is chiefly uſed in ſpeaking of the additional months inſerted by chronologiſts to form the lunar cycle of 19 years.

The 19 ſolar years conſiſting of 6939 days and 18 hours, and the 19 lunar years only making 6726 days, it was found neceſſary to intercalate or inſert 7 lunar months, containing 209 days; which, with the 4 biſſextile days happening in the lunar cycle, make 213 days, and the whole 6939 days, the ſame as the 19 ſolar years, which make the lunar cycle.

In the courſe of 19 years there are 228 common moons, and 7 Emboliſmic moons, which are diſtributed in this manner, viz, the 3d, 6th, 9th, 11th, 14th, 17th, and 19th years, are Emboliſmic, and ſo contain 384 days each. And this was the method of computing time among the Greeks: though they did not keep regularly to it, as it ſeems the Jews did. And the method of the Greeks was followed by the Romans till the time of Julius Cæſar.

The Emboliſmic months, like other lunar months, are ſometimes of 30 days, and ſometimes only 29 days.

The *Emboliſmic Epacts* are thoſe between 19 and 29; which are ſo called, becauſe, with the addition of the epact 11, they exceed the number 30: or rather, becauſe the years which have theſe epacts, are Emboliſmic; having 13 moons each, the 13th being the Emboliſmic.

EMBOLISMUS, in Chronology, ſignifies intercalation. As the Greeks uſed the lunar year, which contains only 354 days, that they might bring it to the ſolar year, of 365 days, they had an Emboliſm every two or three years, when they added a 13th lunar month.

EMBOLUS, the moveable part of a pump or ſyringe; called alſo the piſton, and popularly the ſucker. The pipe or barrel of a ſyringe, &c, being cloſe ſhut, the embolus cannot be drawn up without a very conſiderable force; which force being withdrawn, the embolus returns again with violence; owing to the greater preſſure of air above than below it.

EMBRASURE, in Architecture, an enlargement of the aperture or opening of a door, or window, within ſide the wall, ſloping back inwards, to give the greater play for the opening of the door, caſement, &c, or to take in the more light.

EMBRASURES, in Fortification, are the apertures or holes through which the cannon are pointed, whether in caſemates, batteries, or in the parapets of walls. In the navy, theſe are called port-holes. The Embraſures are placed 12 or 15 feet apart from each other; being made ſloping or opening outwards, from 6 to 9 feet wide on the outſide of the wall, and from 2 to 3 within, to allow the gun to traverſe from ſide to ſide. Their baſe is about $2\frac{1}{2}$ or 3 feet above the platform on the inſide of the wall, but ſloping down outwards, ſo as to be only about $1\frac{1}{2}$ above it on the outſide; in order that the muzzle on occaſion may be depreſſed, and ſo the gun ſhoot low, or downwards.

EMERGENT *Year*, in Chronology, is the epoch, or date, from whence any people begin to compute their time or dates. So, our Emergent year is ſometimes the year of the creation, but more uſually the year of the birth of Chriſt. The Jews uſed that of the Deluge, or the Exodus, &c. The Emergent year of the Greeks, was the beginning of the Olympic games; while that of the Romans was the date of the building of their city.

EMERSION, in Aſtronomy, is the re-appearance of the ſun, moon, or other planet, after having been eclipſed, or hid by the interpoſition of the moon, earth, or other body.

The Emerſions and immerſions of Jupiter's firſt ſatellite, are particularly uſeful for finding the longitudes of places; the immerſions being obſerved from Jupiter's conjunction with the ſun, till his oppoſition; and the Emerſions from the oppoſition till the conjunction. But within 15 days of the conjunction, both before and after it, they cannot be obſerved, becauſe the planet and his ſatellites are then loſt in the ſun's light.

EMERSION is alſo uſed when a ſtar, after being hid by the ſun, begins to re-appear, and to get out of his rays.

Minutes or Scruples of EMERSION, an arc of the moon's orbit, which her centre paſſes over, from the time ſhe begins to emerge out of the earth's ſhadow, to the end of the eclipſe.

EMERSION, in Phyſics, the riſing of any ſolid above the ſurface of a fluid that is ſpecifically heavier than the ſolid, into which it had been violently immerged, or puſhed.

It is one of the known laws of hydroſtatics, that a lighter ſolid, being forced down into a heavier fluid, immediately endeavours to emerge; and that with a force equal to the exceſs of the weight of a quantity of the fluid above that of an equal bulk of the ſolid. Thus, if the body be immerged in a fluid of double its ſpecific gravity, it will emerge again till half its bulk be above the ſurface of the fluid.

EMERSON (WILLIAM), a late eminent mathematician, was born in June 1701, at Hurworth, a village about three miles ſouth of Darlington, on the borders of the county of Durham; at leaſt it is certain that he reſided here from his childhood. His father Dudley Emerſon taught a ſchool, and was a tolerable proficient in the mathematics; and without his books and inſtructions, perhaps his ſon's genius, though eminently fitted for mathematical ſtudies, might never have been unfolded. Beſide his father's inſtructions, our author was aſſiſted in the learned languages by a young clergyman, then curate of Hurworth, who was boarded at his father's houſe. In the early part of his life he attempted to teach a few ſcholars: but whether from his conciſe method (for he was not happy in explaining his ideas), or the warmth of his natural temper, he made no progreſs in his ſchool; he therefore ſoon left it off; and ſatisfied with a moderate competence left him by his parents, he devoted himſelf to a ſtudious retirement, which he thus cloſely purſued, in the ſame place, through the courſe of a long life, being moſtly very healthy, till towards the latter part of his days, when he was much afflicted with the ſtone. Toward the cloſe of the year 1781, being ſenſible of his approaching diſſolution, he diſpoſed of the whole of his mathematical library to a bookſeller at York; and on May the 20th, 1782, his lingering and painful diſorder put an end to his life at his native village, being near 81 years of age.

Mr. Emerſon, in his perſon, was rather ſhort, but ſtrong and well made, with an open countenance and ruddy complexion, being of a healthy and hardy diſpoſition. He was very ſingular in his behaviour, dreſs, and converſation. His manner and appearance were that of a rude and rather booriſh country man; he was of very plain converſation, and indeed ſeemingly rude, commonly mixing oaths in his ſentences, though without any ill intention. He had ſtrong good natural mental parts, and could diſcourſe ſenſibly on any ſubject, but was always poſitive and impatient of any contradiction. He ſpent his whole life in cloſe ſtudy, and writing books, from the profits of which, he redeemed his little patrimony from ſome original incumbrance. In his dreſs he was as ſingular as in every thing elſe. He poſſeſſed commonly but one ſuit of clothes at a time, and thoſe very old in their appearance. He ſeldom uſed a waiſtcoat; and his coat he wore open before, except the lower button; and his ſhirt quite the reverſe of one in common uſe, the hind-ſide turned foremoſt, to cover his breaſt, and buttoned cloſe at the collar behind. He wore a kind of ruſty coloured wig, without a crooked hair in it, which probably had never been tortured with a comb from the time of its being made. A hat he would make to laſt him the beſt part of a life time; gradually leſſening the flaps, bit by bit, as it loſt its elaſticity and hung down, till little or nothing but the crown remained.

He often walked up to London when he had any book to be publiſhed, reviſing ſheet by ſheet himſelf:—truſting no eye but his own was always a favourite maxim with him. In mechanical ſubjects, he always tried the propoſitions practically, making all the different parts himſelf on a ſmall ſcale; ſo that his houſe was filled with all kinds of mechanical inſtruments, together or disjointed. He would frequently ſtand up to his middle in water while fiſhing; a diverſion he was remarkably fond of. He uſed to ſtudy inceſſantly for ſome time, and then for relaxation take a ramble to any pot-alehouſe where he could get any body to drink with and talk to. The late Mr. Montagu was very kind to Mr. Emerſon, and often viſited him, being pleaſed with his converſation, and uſed often to come to him in the fields where he was working, and accompany him home, but could never perſuade him to get into a carriage: on theſe occaſions he would ſometimes exclaim, "Damn your whim-wham! I had rather walk." He was a married man, and his wife uſed to ſpin on an old-faſhioned wheel, of his own making, a drawing of which is given in his Mechanics.

Mr. Emerſon, from his ſtrong vigorous mind and cloſe application, had acquired a deep knowledge of all the branches of mathematics and phyſics, upon all parts of which he wrote good treatiſes, though in a rough and unpoliſhed ſtyle and manner. He was not remarkable however for genius or diſcoveries of his own, as his works ſhew hardly any traces of original invention. He was well ſkilled in the ſcience of muſic, the theory of ſounds, and the various ſcales both ancient and modern; but he was a very poor performer, though he could make and repair ſome inſtruments, and ſometimes went about the country tuning harpſichords.

The following is a liſt of Mr. Emerſon's works; all of them printed in 8vo, excepting his Mechanics and his Increments in 4to, and his Navigation in 12mo. 1. The Doctrine of Fluxions.—2. The Projection of the Sphere, orthographic, ſtereographic, and gnomonical. —3. The Elements of Trigonometry.—4. The Principles of Mechanics.—5. A Treatiſe of Navigation on the Sea.—6. A Treatiſe on Arithmetic.—7. A Treatiſe on Geometry.—8. A Treatiſe of Algebra, in 2 books.—9. The Method of Increments.—10. Arithmetic of Infinites, and the Conic Sections, with other Curve Lines.—11. Elements of Optics and Perſpective.—12. Aſtronomy.—13. Mechanics, with Centripetal and Centrifugal Forces.—14. Mathematical Principles of Geography, Navigation, and Dialling.—15. Commentary on the Principia, with the Defence of Newton.—16. Tracts.—17. Miſcellanies.

EMINENTIAL *Equation*, a term uſed by ſome algebraiſts, in the inveſtigation of the areas of curvilineal figures, for a kind of aſſumed equation that contains another equation Eminently, the latter being a particular caſe of the former. Hayes's Flux. pa. 97.

ENCEINTE, a French term, in Fortification, ſignifying the whole incloſure, circumference, or compaſs of a fortified place, whether built with ſtone or brick, or only made of earth, and whether with or without baſtions, &c.

ENCYCLOPÆDIA, the circle or chain of arts and ſciences; ſometimes alſo written Cyclopædia.

ENDECAGON, a plane geometrical figure of eleven ſides and angles, otherwiſe called Undecagon. If each ſide of this figure be 1, its area will be 9·3656399 $= \frac{11}{4}$ of the tang. of $73\frac{7}{11}$ degrees, to the radius 1. See my Menſuration, pa. 114, &c, 2d edit. See alſo REGULAR *Figure*.

ENFILADE, a French term, applied to thoſe trenches, and other lines, that are ranged in a right line, and ſo may be ſcoured or ſwept by the cannon lengthways, or in the direction of the line.

To ENFILADE, is to ſweep lengthways by the firing of cannon, &c.

*A Battery d'*ENFILADE, is that where the cannon ſweep a right line.

A *Poſt*, or *Command d'*ENFILADE, is a height from whence a whole line may be ſwept at once.

ENGINE, in Mechanics, a compound machine, conſiſting of ſeveral ſimple ones, as wheels, ſcrews, levers, or the like, combined together, in order to lift, caſt, or ſuſtain a weight, or produce ſome other conſiderable effect, ſo as to ſave either force or time.

There are numberleſs kinds of engines; of which ſome are for war, as the Baliſta, Catapulta, Scorpio, Aries or Ram, &c.; others for the arts of peace, as Mills, Cranes, Preſſes, Clocks, Watches, &c, &c.

ENGINEER, or INGINEER, is applied to a contriver or maker of any kind of uſeful engines or machines; or who is particularly ſkilled or employed in them. And he is denominated either a civil or military Engineer, according as the objects of his profeſſion reſpect civil or military purpoſes.

A military Engineer ſhould be an expert mathematician and draughtſman, and particularly verſed in fortification and gunnery, being the perſon officially employed to direct the operations both for attacking and defending works. When at a ſiege the Engineers have narrowly ſurveyed the place, they are to make their report to the general, or commander, by acquainting him which part they judge the weakeſt, and where approaches may be made with moſt ſucceſs. It is their buſineſs alſo to draw the lines of circumvallation and contravallation; alſo to mark out the trenches, places of arms, batteries, and lodgments, and in general to direct the workmen in all ſuch operations.

ENGONASIS, in Aſtronomy, the ſame as Hercules, one of the northern conſtellations; which ſee.

ENGYSCOPE, the ſame as MICROSCOPE.

ENHARMONIC, the laſt of the three kinds of muſic. It abounds in dieſes, or the leaſt ſenſible diviſions of a tone. See Philoſ. Tranſ. number 481; alſo Wallis Appendix ad Ptolom. pa. 165, 166.

ENNEADECATERIS, in Chronology, a cycle or period of 19 ſolar years, being the ſame as the Golden Number and Lunar Cycle, or Cycle of the Moon; which ſee; as alſo EMBOLISMIC.

ENNEAGON, a plane geometrical figure of 9 ſides and angles; and is otherwiſe called a Nonagon. If each ſide of this figure be 1, its area will be 6·1818242 $= \frac{9}{4}$ of the tang. of 70 degrees, to the radius 1. See my Menſuration, pa. 114, 2d edit. See alſo the article REGULAR *Figure*.

ENTABLATURE, in Architecture, is that part of an order of column which is over the capital, comprehending the Architrave, Frize, and Corniche.

ENTABLATURE, or ENTABLEMENT, is ſometimes alſo uſed for the laſt row of ſtones on the top of the wall of a building; on which the timber and covering

ing reſt; ſometimes alſo called the Drip, becauſe it projects a little, to throw the water off.

ENVELOPE, in Fortification, is a mound of earth, ſometimes raiſed in the ditch of a place, and ſometimes beyond it, being either in form of a ſingle parapet, or of a ſmall parapet bordered with a parapet. Theſe Envelopes are made only to cover weak parts with ſingle lines, without advancing towards the field, which cannot be done without works that require a great deal of room, ſuch as horn-works, half-moons, &c. Envelopes are ſometimes called Sillons, Contregards, Conſerves, Lunettes, &c.

ENUMERATION, a numbering or counting. Sir Iſaac Newton wrote an ingenious treatiſe, being an Enumeration of the lines of the 3d order.

EOLIPILE. See ÆOLIPILE.

EPACT, in Chronology, the exceſs of the ſolar month above the lunar ſynodical month; or of the ſolar year above the lunar year of 12 ſynodical months; or of ſeveral ſolar months above as many ſynodical months; or of ſeveral ſolar years above as many dozen of ſynodical months.

The Epacts then are either Annual or Menſtrual.

Menſtrual EPACTS, are the exceſſes of the civil calendar month above the lunar month. Suppoſe, for example, it were new moon on the firſt day of January: then ſince the month of January contains 31 days,

and the lunar month - 29^{ds} 12^{h} 44^{m} 3^{s};
the menſtrual Epact is - 1 11 15 57

Annual EPACTS, are the exceſſes of the ſolar year above the lunar. Hence,

as the Julian ſolar year is 365^{ds} 6^{h} 0^{m} 0^{s},
and the Julian lunar year 354 8 48 38,
the annual Epact will be 10 21 11 22,

that is, almoſt 11 days. Conſequently the Epact of 2 years, is 22 days; of 3 years, 33 days; or rather 3, ſince 30 days make an emboliſmic, or intercalary month. Then, adding ſtill 11, the Epact of 4 years is 14 days; and ſo of the reſt as in the following table, where they do not become 30, or 0 again, till the 19th year; ſo that at the 20th year the Epact is 11 again; and hence the cycle of Epacts expires with the Golden Number, or Lunar Cycle of 19 years, and begins with the ſame again.

TABLE of Julian Epacts.

Golden Numb.	Epacts	Golden Numb.	Epacts	Golden Numb.	Epacts
I	11	VIII	28	XV	15
II	22	IX	9	XVI	26
III	3	X	20	XVII	8
IV	14	XI	1	XVIII	19
V	25	XII	12	XIX	30
VI	6	XIII	23		or 0
VII	17	XIV	4		

Again, as the new moons are the ſame, or fall on the ſame day, every 19 years, ſo the difference between the ſolar and lunar years is the ſame every 19 years. And becauſe the ſaid difference is always to be added to the lunar year, to adjuſt or make it equal to the ſolar year; hence the ſaid difference reſpectively belonging to each year of the moon's cycle, is called the *Epact of the ſaid year*, that is, the number to be added to the ſaid year, to make it equal to the ſolar year. Upon this mutual reſpect between the cycle of the moon and the cycle of the Epacts, is founded this

Rule for finding the Julian Epact, belonging to any year of the Moon's Cycle.

Multiply the Golden Number, or the given year of the Moon's Cycle, by 11, and the product will be the Epact if it be leſs than 30; but if it exceed 30, then throw out as many 30's as the product contains, and the remainder will be the Epact.

Rule to find the Gregorian Epact.

1ſt, The difference between the Julian and Gregorian years being equal to the difference between the ſolar and lunar year, or 11 days, therefore the Gregorian Epact for any year is the ſame with the Julian Epact for the preceding year; and hence the Gregorian Epact will be found, by ſubtracting 1 from the golden number, multiplying the remainder by 11, and rejecting the 30's. This rule will ſerve till the year 1900; but after that year, the Gregorian Epact will be found by this rule: Divide the centuries of the given year by 4; multiply the remainder by 17; then to this product add 43 times the quotient, and alſo the number 86, and divide the whole ſum by 25, reſerving the quotient: next multiply the golden number by 11, and from the product ſubtract the reſerved quotient, ſo ſhall the remainder, after rejecting all the 30's contained in it, be the Epact ſought.

The following table contains the Golden Numbers, with their correſponding Epacts, till the year 1900.

TABLE of Gregorian Epacts.

Golden Numb.	Epacts	Golden Numb.	Epacts	Golden Numb.	Epacts
I	0	VIII	17	XV	4
II	11	IX	28	XVI	15
III	22	X	9	XVII	26
IV	3	XI	20	XVIII	7
V	14	XII	1	XIX	18
VI	25	XIII	12	I	0
VII	6	XIV	23		

On the ſubject of Epacts, ſee Wolfius's Elementa Chronologiæ, apud Opera, tom. 4, pa. 133; alſo Philoſ. Tranſ. vol. 46. pa. 417, or numb. 495, art. 5.

EPAULE, or ESPAULE, in Fortification, the ſhoulder of the baſtion, or the angle made by the face and flank, otherwiſe called the Angle of the Epaule.

EPAULEMENT, in Fortification, a ſide-work haſtily thrown up, to cover the cannon or the men; and is made either of earth thrown up, or bags filled with earth or ſand, or of gabions, or faſcines, &c, with earth: of which latter ſort are commonly the Epaulements of the places of arms for the cavalry behind the trenches.

EPAULEMENT, is alſo uſed for a demi-baſtion, conſiſting of a face and flank, placed at the point of a horn-work or crown-work. Alſo for a little flank added to the ſides of horn-works, to defend them when they are too long. Alſo for the redoubts made on a right line, to fortify it. And laſtly, for an orillon, or maſs of earth almoſt ſquare, faced and lined with a wall, and deſigned to cover the cannon of a caſemate.

EPHEMERIS, Ephemerides, tables calculated by aſtronomers, ſhewing the preſent ſtate of the heavens for every day at noon; that is, the places in which all the planets are found at that time; differing but little from an Aſtronomical Almanac. It is from ſuch tables as theſe that the eclipſes, conjunctions, and aſpects of the planets are made out; as alſo horoſcopes, or celeſtial ſchemes, conſtructed, &c.

There have been Ephemerides of Origan, Kepler, Argoli, Heckerus, Mezzarachis, Wing, Gadbury, Parker, De la Hire, &c.

In France the Academy of Sciences have publiſhed annually, from the beginning of the preſent century, a kind of Ephemeris, under the title of Connoiſſance des Temps, which is ſtill continued, and is in great eſteem; as are alſo the Ephemerides, publiſhed there every ten years, by M. Deſplaces, and De la Lande.

There are now publiſhed ſuch Ephemerides by the Academies of ſeveral other nations; but that which is in moſt eſteem for its accuracy and uſe in finding the longitude, is the Nautical Almanac, or Aſtronomical Ephemeris, publiſhed in England by the Board of Longitude, under the direction of the Rev. Dr. Maſkelyne, Aſtronomer Royal, which commenced with the year 1767.

EPICHARMUS, an ancient poet and philoſopher, born in Sicily, was a ſcholar of Pythagoras, and flouriſhed in the time of Hiero, in whoſe reign it is ſaid he introduced comedy at Syracuſe. He wrote alſo treatiſes concerning philoſophy and medicine; but none of his works have been preſerved. He died at 90 years of age, according to Laertius, who has preſerved four verſes inſcribed on his ſtatue.

EPICURUS, a celebrated ancient philoſopher, was born at Gargettium in Attica, in the 109th Olympiad, or about 340 years before Chriſt. He ſettled at Athens in a fine garden he had bought; where he lived with his friends in much tranquillity, and educated a great number of diſciples; who lived all in common with their maſter. His ſchool was never divided, but his doctrine was followed as an oracle; and the reſpect which his followers paid to his memory is admirable; his birth-day being ſtill kept in Pliny's time, and even the very month he was born in obſerved as a continued feſtival, and his picture placed every where. He wrote a great many books, and valued himſelf upon making no quotations. He raiſed the atomical ſyſtem to great reputation, though he was not the inventor of it, but only made ſome change in that of Democritus. As to his doctrine concerning the ſupreme good or happineſs, it was very liable to be miſrepreſented, and ſome ill effects proceeded from thence, which diſcredited his ſect, though undeſervedly. He was charged with perverting the worſhip of the gods, and inciting men to debauchery. But he did not forget himſelf on this occaſion: he publiſhed his opinions to the whole world; wrote ſome books of devotion; recommended the veneration of the gods, ſobriety, and chaſtity, living in an exemplary manner, and conformably to the rules of philoſophical wiſdom and frugality. He died of a ſuppreſſion of urine, at 72 years of age.—Gaſſendus has given us all he could collect from the ancients concerning the perſon and doctrine of this philoſopher.

Epicurean *Philoſophy*, the doctrine, or ſyſtem of philoſophy maintained by Epicurus and his followers. This conſiſted of three parts; canonical, phyſical, and ethical. The firſt reſpected the canons or rules of judging; in which ſoundneſs and ſimplicity of ſenſe, aſſiſted by ſome natural reflections, chiefly formed his art. His ſearch after truth proceeded only by the ſenſes; to the evidence of which he gave ſo great a certainty, that he conſidered them as an infallible rule of truth, and termed them the Firſt natural light of mankind.

In the 2d part of his philoſophy he laid down atoms, ſpace, and gravity, as the firſt principles of all things. He aſſerted the exiſtence of God, whom he accounted a bleſſed immortal being, but who did not concern himſelf with human affairs.

As to his ethics, he made the ſupreme good of man to conſiſt in pleaſure, and conſequently ſupreme evil in pain. Nature itſelf, ſays he, teaches us this truth; and prompts us from our birth to procure whatever gives us pleaſure, and avoid what gives us pain. To this end he propoſes a remedy againſt the ſharpneſs of pain, which was to divert the mind from it, by turning our whole attention upon the pleaſures we have formerly enjoyed. He held that the wiſe man muſt be happy, as long as he is wiſe: the pain, not depriving him of his wiſdom, cannot deprive him of his happineſs: from which it would ſeem that his pleaſure conſiſted rather in intellectual than in ſenſual enjoyments: though this is a point ſtrongly conteſted.

EPICUREANS, the ſect of philoſophers holding or following the principles and doctrine of Epicurus. As the nature of the pleaſure, in which the chief happineſs is ſuppoſed to be ſeated, is a great problem in the morals of Epicurus, there hence ariſe two kinds of Epicureans, the rigid and the remiſs: the firſt were thoſe who underſtood Epicurus's notion of pleaſure in the beſt ſenſe, and placed all their happineſs in the pure pleaſures of the mind, ariſing from the practice of virtue: while the looſe or remiſs Epicureans, taking the words of that philoſopher in a groſs ſenſe, placed all their happineſs in bodily pleaſures or debauchery.

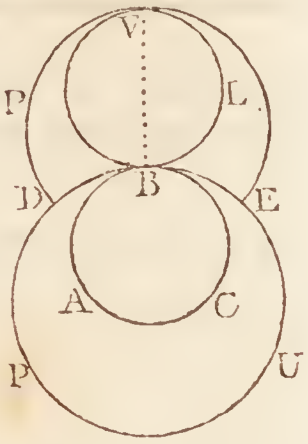
EPICYCLE, in the ancient aſtronomy, a little circle having its centre in the circumference of a greater one: or a ſmall orb or ſphere, which being fixed in the deferent of a planet, is carried along with it, and yet, by its own peculiar motion, carries the planet faſtened to it round its proper centre.

It was by means of Epicycles that Ptolomy and his followers ſolved the various phenomena of the planets, but more eſpecially their ſtations and retrogradations.

EPICYCLOID, is a curve generated by the revolution of a point of the periphery of a circle, which rolls along or upon the circumference of another circle, either on the convex or concave ſide of it.

When a circle rolls along a ſtraight line, a point in its circumference deſcribes the curve called a cycloid.

But if, inſtead of the right line, the circle roll along the circumference of another circle, either equal to the former or not, then the curve deſcribed by any point in its circumference is what is called the Epicycloid.



If the generating circle roll along the convexity of the circumference, the curve is called an *Upper*, or *Exterior* Epicycloid; but if along the concavity, it is called a *Lower*, or *Interior* Epicycloid. Alſo the circle that revolves is called the *Generant;* and the arc of the other circle along which it revolves, is called the *Baſe* of the Epicycloid. Thus, ABC or BLV is the Generant; DPVE the Exterior Epicycloid, its axis BV; DPUE the Interior Epicycloid; and DBE their common Baſe.

For the Length of the Curve.

The length of any part of the curve of an Epicycloid, which any given point in the revolving circle has deſcribed, from the poſition where it touched the circle upon which it revolved, is to double the verſed ſide of half the arc which all the time of revolving touched the quieſcent circle, as the ſum of the diameters of the circles, is to the ſemidiameter of the quieſcent circle in the Exterior Cycloid; or as the difference of the diameters is to that ſemidiameter, for the Interior one.

For the Area of the Epicycloid.

Dr. Halley has given a general propoſition for the meaſuring of all cycloids and Epicycloids: thus, the area of a cycloid, or Epicycloid, either primary, or contracted, or prolate, is to the area of the generating circle; and alſo the areas of the parts generated in thoſe curves, to the areas of analogous ſegments of the circle; as the ſum of double the velocity of the centre and the velocity of the circular motion, is to this velocity of the circular motion. See the Demonſtr. in the Philoſ. Tranſ. number 218.

Spherical EPICYCLOIDS are formed by a point of the revolving circle, when its plane makes a conſtant angle with the plane of the circle on which it revolves. Meſſrs. Bernoulli, Maupertuis, Nicole, and Clairaut, have demonſtrated ſeveral properties of theſe Epicycloids, in Hiſt. Acad. Sci. for 1732.

Parabolic, *Elliptic*, &c. EPICYCLOIDS.

If a parabola roll upon another equal to it; its focus will deſcribe a right line perpendicular to the axis of the quieſcent parabola: alſo the vertex of the rolling parabola will deſcribe the ciſſoid of Diocles; and any other point of it will deſcribe ſome one of Newton's defective hyperbolas, having a double point in the like point of the quieſcent parabola.

In like manner, if an ellipſe revolve upon another ellipſe, equal and ſimilar to it, its focus will deſcribe a circle, whoſe centre is in the other focus, and conſequently the radius is equal to the axis of the ellipſis; and any other point in the plane of the ellipſe will deſcribe a line of the 4th order.

The ſame may be ſaid alſo of an hyperbola, revolving upon another, equal and ſimilar to it; for one of the foci will deſcribe a circle, having its centre in the other focus, and the radius will be the principal axis of the hyperbola; and any other point of the hyperbola will deſcribe a line of the 4th order.

Concerning theſe lines, ſee Newton's Principia, lib. 1; alſo De la Hire's Memoires de Mathematique &c, where he ſhews the nature of this line, and its uſe in Mechanics; ſee alſo Maclaurin's Geometria Organica.

EPIPHANY, a chriſtian feſtival, otherwiſe called the Manifeſtation of Chriſt to the Gentiles, obſerved on the 6th of January, in honour of the appearance of our Saviour to the three magi or wiſe men, who came to adore him and bring him preſents.

EPISTYLE, in the ancient Architecture, a term uſed by the Greeks for what we call Architrave, viz a maſſive ſtone, or a piece of wood, laid immediately over the capital of a column.

EPOCHA, or EPOCH, a term or fixed point of time, from whence the ſucceeding years are numbered or reckoned.

Different nations make uſe of different Epochs. The chriſtians chiefly uſe the Epoch of the nativity or incarnation of Jeſus Chriſt; the Mahometans, that of the Hegira; the Jews, that of the creation of the world, or that of the Deluge; the ancient Greeks, that of the Olympiads; the Romans, that of the building of their city; the ancient Perſians and Aſſyrians, that of Nabonaſſar; &c.

The doctrine and uſe of Epochs is of very great extent in chronology. To reduce the years of one Epoch to thoſe of another, i. e. to find what year of one correſponds to a given year of another; a period of years has been invented, which, commencing before all the known Epochs, is, as it were, a common receptacle of them all, called the Julian Period. To this period all the Epochs are reduced; i. e. the year of this period when each Epoch commences, is determined. So that, adding the given year of one Epoch to the year of the period correſponding with its riſe, and from the ſum ſubtracting the year of the ſame period correſponding to the other Epoch, the remainder is the year of that other Epoch.

EPOCH *of Chriſt*, is the common Epoch throughout Europe, commencing at the ſuppoſed time of our Saviour's nativity, December 25; or rather, according to the uſual account, from his circumciſion, or the 1ſt of January. The author of this Epoch was an Abbot of Rome, one Dionyſius Exiguus, a Scythian, about the year 507 or 527. Dionyſius began his account from the conception or incarnation, uſually called the Annunciation, or Lady Day; which method obtained in the dominions of Great Britain till the year 1752, before which time the Dionyſian was the ſame as the Engliſh Epoch: but in that year the Gregorian calendar having been admitted by act of parliament, they now reckon from the firſt of January, as in the other parts of Europe, except in the court of Rome, where the Epoch of the Incarnation ſtill obtains for the date of their bulls.

A TABLE of the Years of the moſt remarkable Epochs or Eras and Events.

N. B. The years before Chriſt, are thoſe before the reputed year of his birth, and not reckoned back from the firſt year of his age, as is generally done in ſuch tables.	Julian Period.	Year of the World.	Years before Chriſt.
The Creation of the World	706	0	4007
The Deluge, or Noah's flood	2362	1656	2351
Aſſyrian monarchy founded by Nimrod	2537	1831	2176
The birth of Abraham	2714	2008	1999
Kingdom of Athens founded by Cecrops	3157	2451	1556
Entrance of the Iſraelites into Canaan	3262	2556	1451
The deſtruction of Troy	3529	2823	1184
Solomon's temple founded	3701	2995	1012
The Argonautic expedition	3776	3070	937
Lycurgus formed his laws	3829	3103	884
Arbaces, 1ſt king of the Medes	3838	3132	875
Olympiads of the Greeks began	3938	3232	775
Rome built, or Roman Era	3961	3255	752
Era of Nabonaſſar	3967	3261	746
Firſt Babyloniſh captivity, by Nebuchadnezzar	4107	3401	606
The 2d ditto, and birth of Cyrus	4114	3408	599
Solomon's temple deſtroyed	4125	3419	588
Cyrus began to reign in Babylon	4177	3471	536
Peloponneſian war began	4282	3576	431
Alexander the great died	4390	3684	323
Captivity of 100,000 Jews by Ptolomy	4393	3687	320
Archimedes killed at Syracuſe	4506	3800	207
Julius Cæſar invaded Britain	4659	3953	54
He corrected the calendar	4667	3961	46
The true year of Chriſt's birth	4709	4003	4
The Chriſtian Era begins here.			Years ſince Chriſt.
Dionyſian, or vulgar era of Chriſt's birth	4717	4007	0
Chriſt crucified, Friday April 3d	4746	4040	33
Jeruſalem deſtroyed	4783	4077	70
Adrian's wall built in Britain	4833	4127	120
Diocleſian Epoch, or that of Martyrs	4997	4291	284
The council of Nice	5038	4332	325
Conſtantine the great died	5050	4344	337
The Saxons invited into Britain	5158	4452	445
Hegira, or flight of Mohammed	5335	4629	622
Death of Mohammed	5343	4637	630
The Perſian Yeſdegird	5344	4638	631
Sun, Moon, and Planets ♄, ♃, ♂, ♀, ☿ in ♎, ſeen from the earth	5899	5193	1186
Art of printing diſcovered	6153	5447	1440
The reformation begun by Martin Luther	6230	5524	1517
The Calendar corrected by pope Gregory	6295	5589	1582
Oliver Cromwell died	6371	5665	1658
Sir Iſaac Newton born, Dec. 25	6355	5649	1642
Made Preſident of the Royal Society	6416	5710	1703
Died, March 20th	6440	5734	1727
New Planet diſcovered by Herſchel	6494	5788	1781

EQUABLE *Motion*, Celerity, Velocity, &c, is that which is uniform, or without alteration, or by which equal ſpaces are paſſed over in equal times. Hence, the ſpaces, paſſed over in Equable motions, are proportional to the times. So that if a body paſs over 20 feet in 1 ſecond of time, it will paſs over 40 feet in 2 ſeconds, and ſo on.

Equably *Accelerated* or *Retarded*, &c, is when the motion or change is increaſed or decreaſed by equal quantities or degrees in equal times.

EQUAL, a term of relation between different things, but of the ſame kind, magnitude, quantity, or quality.——Wolfius defines Equals to be thoſe things that may be ſubſtituted for each other, without any alteration of their quantity.—It is an axiom in mathematics &c, that two things which are equal to the ſame third, are alſo equal to each other. And if Equals be equally altered, by Equal addition, ſubtraction, multiplication, diviſion, &c, the reſults will be alſo Equal.

Equal *Circles*, are thoſe whoſe diameters are equal.

Equal *Angles*, are thoſe whoſe ſides are equally inclined, or which are meaſured by ſimilar arcs of circles.

Equal *Lines*, are lines of the ſame length.

Equal *Plane Figures*, are thoſe whoſe areas are equal; whether the figures be of the ſame form or not.

Equal *Solids*, are ſuch as are of the ſame ſpace, capacity, or ſolid content; whether they be of the ſame kind or not.

Equal *Curvatures*, are ſuch as have the ſame or equal radii of curvature.

Equal *Ratios*, are thoſe whoſe terms are in the ſame proportion.

Equal, in Optics, is ſaid of things that are ſeen under equal angles.

EQUALITY, the exact agreement of two things in reſpect of quantity. Thoſe figures are equal which may occupy the ſame ſpace, or may be conceived to poſſeſs the ſame ſpace, by the flexion or tranſpoſition of their parts. See a learned diſcourſe upon this, by Dr. Barrow, in the 11th and 12th of his Mathematical Lectures.

Equality, in Algebra, the relation or compariſon between two quantities that are really or effectually equal. See Equation.

Equality, in Algebra, is uſually denoted by two equal parallel lines, as = : thus $2 + 3 = 5$, i. e. 2 plus 3, are equal to 5. This character =, was firſt introduced by Robert Recorde. Des Cartes, and ſome others after him, uſe the mark ∝ inſtead of it: as $2 + 3 \propto 5$.

Equality, in Aſtronomy. *Circle of* Equality, or the Equant. See Circle and Equant.

Ratio or *Proportion* of Equality, is that between two equal numbers or quantities.

Proportion of Equality *evenly ranged*, or *ex æquo ordinata*, is that in which two terms, in a rank or ſeries, are proportional to as many terms in another ſeries, compared to each other in the ſame order, i. e. the firſt of one rank to the firſt of another, the 2d to the 2d, &c.

Proportion of Equality *evenly diſturbed*, called alſo *ex æquo perturbata*, is that in which more than two terms of one rank, are proportional to as many terms of another, compared to each other in a different and interrupted order; viz, the 1ſt of one rank to the 2d of another, the 2d to the 3d, &c.

EQUANT, or Æquant, in Aſtronomy, a circle formerly conceived by aſtronomers, in the plane of the deferent, or eccentric; for regulating and adjuſting certain motions of the planets, and reducing them more eaſily to a calculus.

EQUATED *Anomaly*. See Anomaly.

Equated *Bodies*. On Gunter's Sector there are ſometimes placed two lines, anſwering to one another, and called the Lines of Equated Bodies. They are ſituated between the lines of ſuperficies and ſolids, and are marked with the letters D, I, C, S, O, T, to ſignify dodecahedron, icoſahedron, cube, ſphere, octahedron, and tetrahedron.

The uſes of theſe lines are, 1ſt, When the diameter of the ſphere is given, to find the ſides of the five regular bodies, each equal to that ſphere; 2d, From the ſide of any one of thoſe bodies being given, to find the diameter of the ſphere, and the ſides of the other bodies, which ſhall be each equal to the firſt given body. So, when the ſphere is given, take its diameter, and apply it over on the ſector in the points S, S; but when one of the other five bodies is given, apply its ſide over in its proper points; then the parallels taken from between the points of the other bodies, or ſphere, ſhall be the ſides or diameter, equal ſeverally to the ſphere or body firſt given.

EQUATION, in Algebra, an expreſſion of equality between two different quantities; or two quantities, whether ſimple or compound, with the mark of equality between them: as $2 + 3 = 7 - 2$; or $2 \times 3 = 6$; or $5 \times 3 = \frac{30}{2}$; or $a + b = c$; or $x^2 + ax = b$; &c. When the things, or two ſides of the equation, are the ſame, the expreſſion becomes an Identity, as $5 = 5$, or $a = a$, &c. Sometimes the quantities are placed all on one ſide, and made equal to 0 or nothing on the other ſide; as $5 - 5 = 0$, or $a - b = 0$: which is no more than ſetting down the difference of two equal quantities equal to nothing.

The character or ſign uſually employed to denote an Equation, is =, which is placed between the two equal quantities, called the two ſides of the Equation.

The *Terms* of an Equation, are the ſeveral quantities or parts of which it is compoſed. Thus, of the Equation $a + b = c$, the terms are a, b, and c: and the tenor or import of the expreſſion is, that ſome quantity repreſented by c, is equal to two others repreſented by a and b.

Equations are either Simple or Affected.

A *Simple* Equation is that which has only one power of the unknown quantity: as $a + x = 3b$, or $ax^2 = bc$, $a^3 + 2x^3 = 5b$, &c; where x denotes the unknown quantity, and the other letters known ones. But

An *Affected*, or *Adfected* Equation contains two or more different powers: as $x^2 + ax = b$, or $3x - 4x^2 + x^3 = 25$, &c.

Again, Equations are denominated from the higheſt power contained in them; as quadratic, cubic, biquadratic, &c. Thus,

A *Quadratic* EQUATION, is that in which the unknown quantity riſes to two dimenſions, or to the ſquare or 2d power: as $x^2 + 20x = 200$, or $ax - x^2 = b$.

A *Cubic* EQUATION, is that in which the unknown quantity is of three dimenſions, or riſes to the cube or 3d power: as $x^3 = 25$, or $x^3 - 2x^2 = 27$, or $x^3 - ax^2 + bx = c$.

A *Biquadratic* EQUATION, is that in which the unknown quantity is of 4 dimenſions, or riſes to the 4th or biquadratic power: as $x^4 = 25$, or $x^4 - 20x = 10$, or $x^4 - ax^3 + bx^2 - cx = d$.

And ſo for other higher orders of Equations.

The *Root* of an EQUATION, is the value of the unknown letter or quantity contained in it. And this value being ſubſtituted in the terms of the Equation inſtead of that letter or quantity, will cauſe both ſides to vaniſh, or will make the one ſide exactly equal to the other. So the root of the Equation $\frac{3x}{5} + 18 = 24$, is 10; becauſe that uſing 10 for x, it becomes $\frac{3 \times 10}{5} + 18 = 6 + 18 = 24$.

Every Equation has as many roots as it has dimenſions, or as it contains units in the index of the higheſt power, when the powers are all reduced to integral exponents. So the ſimple Equation of the 1ſt power, has only one root; but the quadratic has 2, the cubic 3, the biquadratic 4, &c. Thus the two roots of this equation $x^2 - 4x = -3$ are 1 and 3; for either of theſe ſubſtituted for x makes $x^2 - 4x$ come out equal to -3. Alſo the three roots of $x^3 - 4x^2 - 11x = -30$, or $x^3 - 4x^2 - 11x + 30 = 0$, are 2, 5, and -3; as will appear by ſubſtituting each of theſe inſtead of x in the equation, which will make all the terms on one ſide equal to the other ſide. And ſo of others.

The Relation between the Roots of Equations, and the Coefficients of their Terms.——In every Equation, when the terms are ranged in order according to the order of the powers, the greater before the leſs; the firſt term or higheſt power freed from its coefficient, by dividing all the terms by it, and all brought to one ſide, and made equal to nothing on the other ſide, when it will appear in this form,

$$x^n + ax^{n-1} + bx^{n-2} + cx^{n-3} \; . \; . \; . \; . = 0;$$

then the relations between the roots and coefficients, are as follow:

1ſt, The coefficient a of the 2d term, is equal to the ſum of all the roots.

2d, The coefficient b of the 3d term, is equal to the ſum of all the products of the roots that can be made by multiplying every two of them together. In like manner,

3d, The coefficients c, d, e, &c, of the following terms, are reſpectively equal to the ſum of the products of the roots made by multiplying every three together, or every four together, or every five together, &c, the ſigns of all the roots being changed. All which will appear below, in the Generation of Equations.

The Roots of Equations are Poſitive or Negative, and Real or Imaginary. Thus, the two roots of the Equation $x^2 - 4x = -3$, are 1 and 3, real and both poſitive; but the roots of the Equation $x^3 - 4x^2 - 11x = -30$, are 2, 5, & -3, are real, two poſitive and one negative; and the roots of the equation $x^3 + 9x = 10$, are 1 and $-\frac{1}{2} \pm \frac{1}{2}\sqrt{-39}$, one real and two imaginary.

The *Generation* of EQUATIONS, is the multiplying of certain aſſumed ſimple equations together, to produce compound ones, with intent to ſhew the nature of theſe; a method invented by Harriot, which is this: Suppoſe x to denote the unknown quantity of any equation, and let the roots of that equation, or the values of x, be, a, b, c, d, &c; that is $x = a$, and $x = b$, and $x = c$, &c; or $x - a = 0$, and $x - b = 0$, and $x - c = 0$, &c; then multiply theſe laſt equations together, thus,

$$\begin{array}{l}
x - a = 0 \\
x - b = 0 \\
\hline
x^2 - ax \\
\quad - bx + ab \\
\hline
\left.\begin{array}{l} x^2 - a \\ \quad - b \end{array}\right\} . \, x + ab = 0, \text{ the quadratic Equation,} \\
x - c \; - \; - \; - \; - \; - \; = 0 \\
\hline
x^3 - \overline{a + b} \, . \, x^2 + abx \\
\qquad - c \, . \, x^2 + \overline{ac + bc} \, . \, x - abc \\
\hline
\end{array}$$

$$\begin{array}{llllll}
x^3 & - & a \, . \, x^2 + ab \, . & & x - abc = 0, \text{ the cubic Equ.} \\
 & - & b & + ac & \\
 & - & c & + bc & \\
x & - & d & & = 0 \\
\hline
\end{array}$$

$$\begin{array}{llllll}
x^4 & - & a \, . \, x^3 & + ab \, . \, x^2 & - abc \, . \, x & + abcd = 0, \text{ the} \\
 & - & b & + ac & - abd & \text{biquadratic Equ.} \\
 & - & c & + bc & - acd & \\
 & - & d & + ad & - bcd & \\
 & & & + bd & & \\
 & & & + cd & &
\end{array}$$

Now the roots of theſe equations are a, b, c, d, &c; and it is obvious that the ſum of all the roots is the coefficient of the 2d term, the ſum of all the products of every two is the coefficient of the 3d term, the ſum of all the products of every three that of the 4th term, and ſo on, to the laſt term, which is the continual product of all the roots.

Reduction of EQUATIONS, is the transforming or changing them to their ſimpleſt and moſt commodious form, to prepare them for finding or extracting their roots. The moſt convenient form is, that the terms be ranged according to the powers of the unknown letter, the higheſt power foremoſt next the left hand, and that term to have only + 1 for its coefficient; alſo all the terms containing the unknown letter to be on one ſide of the equation, and the abſolute known term only on the other ſide.

Now this reduction chiefly reſpects the firſt term, or that which contains the higheſt power of the unknown quantity; and the general rule for reducing it is, to conſider in what manner it is involved or connected with other quantities, and then perform the counter or oppoſite relation or operation; for every operation is undone or counteracted by the reverſe of it; as addition by ſubtraction, multiplication by diviſion, involution by evolution, &c: then bring all the unknown terms to one ſide, and the known term to the other ſide, changing the ſigns, from + to −, or from − to +, of thoſe terms which are changed from one ſide to the other; and laſtly divide by the coefficient of the firſt term, with its ſign.

Thus, for ex. if $5x-12=3x+4$:
then is $5x-3x=12+4$, or $2x=16$;
and ſo $x=\frac{16}{2}=8$.

Again, if $ax-b=cx+d$:
then $ax-cx=b+d$,
and $x=\dfrac{b+d}{a-c}$

And, if $6x^2-20=12x+2x^2$:
then $6x^2-2x^2=12x+20$,
or $4x^2-12x=20$,
and $x^2-3x=5$.

Alſo, if $\dfrac{a}{a+x}+\dfrac{b}{x}=3$:
then $ax+ab+bx=3ax+3x^2$,
and $-3x^2+ax+bx-3ax=-ab$,
or $-3x^2+bx-2ax=-ab$,
and $x^2+\dfrac{2a-b}{3}x=\dfrac{ab}{3}$.

Alſo, if $\sqrt{a^2+x^2}-3b=\dfrac{2ab}{x}$:
then $\sqrt{a^2+x^2}=3b+\dfrac{2ab}{x}=\dfrac{3bx+2ab}{x}$,
and $a^2+x^2=\dfrac{9b^2x^2+12ab^2x+4a^2b^2}{x^2}$,
and $a^2x^2+x^4=9b^2x^2+12ab^2x+4a^2b^2$,
and $x^4+\overline{a^2-9b^2}.x^2-12ab^2x=4a^2b^2$.

Extracting or finding the Roots of EQUATIONS.

This is finding the value or values of the unknown letter in an Equation, the rules for which are various, according to the degree of the Equation.

1. *For the Root of a Simple* EQUATION.

Having reduced the equation as above, by bringing the unknown terms to one ſide, and the known ones to the other, freeing the former from radicals and fractions, by their counter operations, and laſtly dividing by the coefficients of the unknown quantity, the value of it is then found: as in the firſt and 2d examples of reduction above given.

2. *For the Roots of Quadratic* EQUATIONS.

Theſe are uſually found by what is called completing the ſquare; which conſiſts in ſquaring half the coefficient of the 2d term, and adding it to both ſides of the equation; for then the unknown ſide is a complete ſquare of a binomial, and the other ſide conſiſts only of known quantities. Therefore, extract the root on both ſides, ſo ſhall the root of the firſt ſide be a binomial, one part of which is the unknown letter, and the other a known or given quantity, and the root of the other ſide is taken either + or −, ſince the ſquare of either of theſe is the ſame given quantity: laſtly, bringing over the known part of the binomial root to the other ſide, with a contrary ſign, gives the two roots or values of the unknown letter ſought.

Thus, if $x^2+2ax=b^2$ be a general quadratic Equation, $2a$ being the coefficient of the 2d term, and b^2 the abſolute known term, both with their ſigns. Then, a is half that coefficient, and a^2 its ſquare; which being added, gives $x^2+2ax+a^2=a^2+b^2$; and the root extracted gives $x+a=\pm\sqrt{a^2+b^2}$; then, tranſpoſing a, it is $x=-a\pm\sqrt{a^2+b^2}$, the two roots, or values of x.

And if $x^2-4x=5$: then, adding 2^2 or 4,
it is $x^2-4x+4=9$;
the root is $x-2=\pm 3$,
then $x=2\pm 3=5$ or -1,
the two roots, or values of x.

And if $x^2+4x=5$:
then $x^2+4x+4=9$;
and $x+2=\pm 3$;
or $x=-2\pm 3=1$, or -5, the two roots, which are the ſame numbers as before, but with the ſigns changed.

Alſo, if $x^2-4x=-5$:
then $x^2-4x+4=4-5=-1$;
and $x^2-2=\pm\sqrt{-1}$;
or $x=2\pm\sqrt{-1}$, two imaginary roots.

3. *For the Roots of Cubic* EQUATIONS.

A Cubic EQUATION is that in which the unknown letter aſcends to the 3d power; as $x^3+a^3=b^3$, or $x^3+ax^2+bx=c$.

The 2d term of every Cubic Equation being taken away, thoſe equations may all be reduced to this form, $x^3+ax=b$; and the general value of one root is
$x=\sqrt[3]{\tfrac{1}{2}b+\sqrt{\tfrac{1}{4}b^2+\tfrac{1}{27}a^3}}+\sqrt[3]{\tfrac{1}{2}b-\sqrt{\tfrac{1}{4}b^2+\tfrac{1}{27}a^3}}$.
This rule is uſually called Cardan's, becauſe firſt publiſhed by him, but it was invented both by Scipio Ferreus, and Nich. Tartalea, by whom it was communicated to Cardan. See the article ALGEBRA.

When the 2d term is negative, or the equation of this form, $x^3-ax=\pm b$, the radical $\sqrt{\tfrac{1}{4}b^2+\tfrac{1}{27}a^3}$ becomes $\sqrt{\tfrac{1}{4}b^2-\tfrac{1}{27}a^3}$, which will be imaginary or impoſſible when $\tfrac{1}{27}a^3$ is greater than $\tfrac{1}{4}b^2$, for $\sqrt{\tfrac{1}{4}b^2-\tfrac{1}{27}a^3}$ will then be the ſquare root of a negative quantity, which is impoſſible: and yet, in this caſe, the root x is a real quantity; though algebraiſts have never been able to find a real finite general expreſſion for it. And this is called the Irreducible or Impracticable Caſe.

This caſe may indeed be reſolved by the triſection of an arc or angle; or by any of the uſual methods of converging; or by general expreſſions in infinite ſeries. See Saunderſon's Algebra, pa. 713; Philoſ. Tranſ. vol. 18, pa. 136, or Abr. vol. 1, pa. 87; alſo vol. 70, pa. 415. See alſo the article CUBIC *Equations*.

Mr. Cotes obſerves, in his Logometria, pa. 29, that the ſolution of all cubic Equations depends either upon the triſection of a ratio, or of an angle. See this method explained in Saunderſon's Alg. p. 718.

Biquadratic Equations, or thoſe that are of 4 dimenſions, are reſolved after various methods. The firſt rule was given by Lewis Ferrari, the companion of Cardan, which is one of the beſt. A 2d method was given by Des Cartes, and another by Mr. Simpſon and Dr. Waring. For the explanation of which, ſee Biquadratic *Equations*.

Equations *of the Higher Degrees or Orders.*

There is no general rule to expreſs algebraically the roots of Equations above thoſe of the 4th degree; and therefore methods of approximation are here made uſe of, which, though not accurately, are yet practically true. Some of theſe excel in eaſe and ſimplicity, and others in quickneſs of converging. Among theſe may be reckoned firſt, Double Poſition, or Trial-and-Error, both in reſpect of eaſe and univerſality, as it applies in the ſimpleſt manner to all ſorts of Equations whatever, not excepting even exponential ones, radical expreſſions of ever ſo complex a form, expreſſions of logarithms, of arches by the ſines or tangents, of arcs of curves by the abſciſſes, or any other fluents, or roots of fluxional Equations. For an explanation of this and other methods of converging to the roots of equations, by Halley, Newton, Raphſon, &c, &c, ſee Approximation, and Converging.

Beſides the methods above adverted to, there have been ſome others, given in the Memoirs of ſeveral Academies, and elſewhere. As, by M. Daniel Bernoulli, in the Acta Petropolitana, tom. 3, p. 92; and by M. L. Euler, in the ſame, vol. 6, New Series, and tom. 5, p. 63 & 82; by Mr. Thos. Simpſon, in his Eſſays, p. 82; in his Diſſertations, p. 102; in his Algebra, p. 158; and in his Select Exerciſes, p. 215.

Abſolute Equation. See Absolute.

Adfected, or *Affected* Equation. See Affected.

Differential Equation, is the Equation of Differences or Fluxions.

Eminential Equation. See Eminential.

Exponential Equation, one in which the exponents of the powers are variable or unknown quantities. See Exponential.

Fluential Equation, is the Equation of the fluents.

Fluxional Equation, is the Equation of the fluxions.

Literal Equation, is a general Equation expreſſed in letters, as contradiſtinguiſhed from a

Numeral Equation, one expreſſed in numbers.

Tranſcendental Equation. See Transcendental.

Equation, in Aſtronomy, as *Annual* Equation, is either of the mean motion of the ſun and moon, or of the moon's apogee and nodes.

The Annual Equation of the ſun's centre being given, the other three correſponding annual Equations, will be alſo given, and therefore a table of the firſt will ſerve for all of them. Thus, if the annual Equation of the ſun's centre, taken from ſuch a table for any time, be called s; and if $\frac{1}{10}s = A$, and $\frac{1}{6}s = B$; then ſhall the other annual Equations for that time be thus,

$A + \frac{1}{30}A = m$, that of the moon's mean motion;
and $B + \frac{1}{30}B = a$, that of the moon's apogee;
and $\frac{1}{2}B - \frac{1}{120}B = n$, that of her nodes.

And here note, that when s, or the Equation of the ſun's centre, is additive; then m is negative, a is poſitive, and n is negative. But on the contrary, when s is negative or ſubductive; then m is poſitive, a negative, and n poſitive.

There is alſo an *Equation of the moon's mean motion*, depending on the ſituation of her apogee in reſpect of the ſun; which is greateſt when the moon's apogee is in an octant with the ſun, and is nothing at all when it is in the quadratures or ſyzygies. This Equation when greateſt, and the ſun in perigee, is 3′ 56″. But it is never above 3′ 34″ when the ſun is in apogee. At other diſtances of the ſun from the earth, this Equation when greateſt, is reciprocally as the cube of that diſtance. But when the moon's apogee is any where out of the octants, this Equation grows leſs, and is moſtly, at the ſame diſtance between the earth and ſun, as the ſine of double the diſtance of the moon's apogee from the next quadrature or ſyzygy, is to radius. This is to be added to the moon's motion while her apogee paſſes from a quadrature with the ſun to a ſyzygy; but is to be ſubtracted from it, while the apogee moves from the ſyzygy to the quadrature.

There is moreover another *Equation of the moon's motion*, which depends on the aſpect of the nodes of the moon's orbit with reſpect to the ſun: and this is greateſt when her nodes are in octants to the ſun, and quite vaniſhes when they come to their quadratures or ſyzygies. This Equation is proportional to the ſine of double the diſtance of the node from the next ſyzygy or quadrature; and at the greateſt is only 47″. This muſt be added to the moon's mean motion while the nodes are paſſing from the ſyzygies with the ſun to their quadratures; but ſubtracted while they paſs from the quadratures to the ſyzygies.

From the ſun's true place ſubtract the equated mean motion of the lunar apogee, as was ſhewn above, the remainder will be the annual argument of the ſaid apogee; from whence the eccentricity of the moon and the 2d Equation of her apogee may be compared. See *Theory of the* Moon's *motions*, &c.

Equation *of the Centre*, called alſo *Proſthaphereſis*, and *Total Proſthaphereſis*, is the difference between the true and mean place of a planet, or the angle made by the lines of the true and mean place; or, which amounts to the ſame, between the mean and equated anomaly.

The greateſt Equation of the Centre may be obtained by finding the ſun's longitude at the times when he is near his mean diſtances, for then the difference will give the true motion for that interval of time: next find the ſun's mean motion for the ſame interval of time; then half the difference between the true and mean motions will ſhew the greateſt Equation of the Centre.

For Example, by obſervations made at the Royal Obſervatory at Greenwich, it appears that at the following mean times the ſun's longitudes were as annexed; viz,

Mean times.	Sun's longitudes.
1769 Oct. 1 at $23^h\ 49^m\ 12^s$ - -	$6^s\ 9°\ 32'\ 0{\cdot}6''$
1770 Mar. 29 at 0 4 50 . - -	0 8 50 27·5
dif. of time 178^d 0 15 38;	true dif. lon. 5 29 18 27

tropical year $= 365^d\ 5^h\ 48^m\ 42^s = 365{\cdot}2421527$;
obſerved interval $= 178\ \ 0\ \ 15\ \ 38 = 178{\cdot}0108565$:

Then
365·2421527 : 178·01085648 : : 360° : 175·455948
or 175° 27′ 21″ the mean motion.
Therefore - 175° 27′ 21″ of mean motion,
anſwers to - 179 18 27 of true motion;
their difference is 3 51 6
and its half - 1 55 33
is the greateſt Equation of the Centre according to theſe obſervations.

To find the Equation of the Centre, or to reſolve Kepler's problem, is a very troubleſome operation, eſpecially in the more eccentric orbits. How this is to be done, has been ſhewn by Newton, Gregory, Keil, Machin, La Caille, and others, by methods little differing from one another; which conſiſt chiefly in finding a certain intermediate angle, called the eccentric anomaly; having known the mean anamoly, and the dimenſions of the ſun's orbit. The mean anomaly is eaſily found, by determining the exact time when the ſun is in the aphelion, and uſing the following proportion, viz,

As the time of a tropical revolution, or ſolar year,
Is to the interval between the aphelion and given time,
So is 360 degrees, to the degrees of the mean anomaly.
Or it may be found by taking the ſun's mean motion, at the given time out of tables.

To find the Eccentric Anomaly, ſay,
As the aphelion diſtance,
Is to the perihelion diſtance;
So is the tangent of half the mean anomaly,
To the tangent of an arc.

Which arc added to half the mean anomaly, gives the eccentric anomaly. Then,

To find the True Anomaly, ſay,
As the ſquare root of the aphelion diſtance,
Is to the ſquare root of the perihelion diſtance;
So is the tangent of half the eccentric anomaly,
To the tangent of half the true anomaly.

Then, the difference between the true and mean anomaly, gives the Equation of the Centre, ſought. Which is ſubtractive, from the aphelion to the perihelion, or in the firſt 6 ſigns of anomaly; and additive, from the perihelion to the aphelion, or in the laſt 6 ſigns of anomaly; and hence called *Proſthaphereſis*.

By this problem a table may eaſily be formed. When the Equations of the Centre for every degree of the firſt 6 ſigns of mean anomaly are found, they will ſerve alſo for the degrees of the laſt 6 ſigns, becauſe equal anomalies are at equal diſtances on both ſides of their apſes. Then ſet theſe equations orderly to their ſigns and degrees of anomaly; the firſt 6 being reckoned from the top of the table downwards, and ſigned *ſubtract*; the laſt 6, for which the ſame Equations ſerve, in a contrary order, being reckoned from the bottom upwards, and marked *add*. Let alſo the difference between every adjacent two Equations, called Tabular Differences, be ſet in another column. Hence, from theſe Equations of the Centre, augmented or diminiſhed by the proportional parts of their reſpective tabular differences, for any given minutes and ſeconds, may eaſily be deduced Equations of the Centre to any mean anomaly propoſed. Robertſon's Elem. of Navig. book 5, p. 286, 290, 295, and 308, where ſuch a table of Equations is given.

The late excellent Mr. Euler has particularly conſidered this ſubject, in the Mem. de l'Acad. de Berlin, tom. 2, p. 225 & ſeq. where he reſolves the following problems:

1. To find the true and mean anomaly correſponding to the planet's mean diſtance from the ſun; that is, where the planet is in the extremity of the conjugate axis of its orbit.

2. The eccentricity of a planet being given, to find the eccentric anomaly correſponding to the greateſt Equation.

3. The eccentricity being given, to find the mean anomaly correſponding to the greateſt Equation.

4. From the ſame data, to find the true anomaly correſponding to this Equation.

5. From the ſame data, to find the greateſt Equation.

6. The greateſt Equation being given, to find the eccentricity.

Mr. Euler obſerves, that this problem is very difficult, and that it can only be reſolved by approximation and tentatively, in the manner he mentions: but if the eccentricity be not great, it may then be found directly from the greateſt Equation. Thus, if the greateſt Equation be $= m$, and the eccentricity $= n$;

then is $$m = 2n + \frac{11}{2^4 \cdot 3} n^3 + \frac{599}{2^7 \cdot 3 \cdot 5 \cdot 7} n^5 + \&c.$$

Whence by reverſion
$$n = \tfrac{1}{2} m - \frac{11}{2^8 \cdot 3} m^3 - \frac{587}{2^{10} \cdot 3 \cdot 5} m^5 - \&c.$$

Where the greateſt equation m muſt be expreſſed in parts of the radius, which may be done by reducing the angle m into ſeconds, and adding 4·6855749 to the log. of the reſulting number, which will be the log. of the number m.

The mean anomaly to which this greateſt equation correſponds, will be

$$x = 90° + \tfrac{5}{8} m - \frac{5}{2^9 \cdot 3} m^3 - \frac{1}{2^9 \cdot 5} m^5 - \&c.$$ Whence, if 90° be added to $\frac{5}{8}$ of the greateſt Equation, the ſum will be the mean anomaly ſufficiently exact.

Mr. Euler ſubjoins a table, by which may be found the greateſt Equations, with the mean and eccentric anomalies correſponding to theſe greateſt Equations for every 100th part of unity, which he ſuppoſes equal to the greateſt eccentricity, or when the tranſverſe and diſtance of the foci become infinite. The laſt column of the table gives alſo the logarithm of that diſtance of the planet from the ſun where its Equation is greateſt. By means of this table, any eccentricity being given, by interpolation will be found the correſponding greateſt Equation. But the chief uſe of the table is to determine the eccentricity when the greateſt Equation is known; and without this help Mr. Euler thinks the problem cannot be reſolved.

EQUATION *of Time*, denotes the difference between mean and apparent time, or the reduction of the apparent unequal time, or motion of the ſun or a planet, to equal and mean time, or motion; or the Equation of time is the difference between the ſun's mean motion, and his right aſcenſion. Apparent time is that which takes its beginning from the paſſage of the ſun's centre over the meridian of any place; and had the ſun no motion in the ecliptic, or was his motion reduced to the equator or in right aſcenſion uniform, he would always return to the meridian after equal intervals of time. But his apparent motion in the ecliptic being continually varying, and his motion in right aſcenſion being rendered farther unequal on account of the obliquity of the ecliptic to the equator, from theſe cauſes it ariſes that the intervals of his return to the meridian become unequal, and the ſun will gradually come too ſlow or too ſoon to the meridian for an equable motion, ſuch as that of clocks and watches ought to be; and this retardation or acceleration of the ſun's coming to the meridian, is called the equation of time.

Now, computing the celeſtial motions according to equal time, it is neceſſary to turn that time back again into apparent time, that they may correſpond to obſervation: on the contrary, any phenomenon being obſerved, the apparent time of it muſt be converted into equal time, to have it correſpond with the times marked in the aſtronomical tables.

The Equation of time is nothing at four different times in the year, when the whole mean and unequal motions exactly agree; viz, about the 15th of April, the 15th of June, the 31ſt of Auguſt, and the 24th of December: at all other times the ſun is either too faſt or too ſlow for mean, equal, or clock time, by a certain number of minutes and ſeconds, which at the greateſt is 16′ 14″, and happens about the 1ſt of November; every other day throughout the year having a certain quantity of this difference belonging to it; which however is not exactly the ſame every year, but only every 4th year; for which reaſon it is neceſſary to have 4 tables of this Equation, viz, one for each of the four years in the period of leap years. Inſtead of theſe, may be here inſerted, as follows, one general equation of time, according to the place of the ſun, in every point of the ecliptic: where it is to be obſerved, that the ſign of the ecliptic is placed at the tops of the columns, and the particular degree of the ſun's place, in each ſign, in the firſt and laſt columns; and in the angle of meeting in all the other columns, is the equation of time, in minutes and ſeconds, when the ſun has any particular longitude: ſuppoſing the obliquity of the ecliptic 23° 28′, and the ſun's apogee in 9° of ♋.

A Table of the Equation of Time, for every Degree of the Sun's Longitude.

Deg.	♈ 0		♉ 1		♊ 2		♋ 3		♌ 4		♍ 5		♎ 6		♏ 7		♐ 8		♑ 9		♒ 10		♓ 11		Deg.
	m	s	m	s	m	s	m	s	m	s	m	s	m	s	m	s	m	s	m	s	m	s	m	s	
0	7	+36	1	−9	3	−51	1	+13	5	+57	2	+20	7	−38	15	−31	13	−33	1	−11	11	+28	14	+19	0
1	7	17	1	23	3	47	1	26	5	59	2	4	7	58	15	39	13	17	0	42	11	45	14	13	1
2	6	58	1	36	3	41	1	40	6	0	1	48	8	19	15	46	13	0	0	12	12	1	14	6	2
3	6	39	1	48	3	37	1	53	6	1	1	31	8	40	15	52	12	42	0	+17	12	17	13	59	3
4	6	20	2	0	3	32	2	7	6	1	1	14	9	1	15	57	12	23	0	46	12	32	13	51	4
5	6	1	2	11	3	26	2	20	6	0	0	56	9	21	16	2	12	4	1	16	12	46	13	43	5
6	5	42	2	22	3	19	2	33	5	59	0	38	9	41	16	6	11	44	1	45	12	59	13	34	6
7	5	24	2	32	3	12	2	45	5	57	0	20	10	1	16	9	11	23	2	14	13	12	13	24	7
8	5	5	2	42	3	4	2	58	5	54	0	1	10	20	16	11	11	1	2	43	13	24	13	14	8
9	4	47	2	51	2	56	3	11	5	51	0	−18	10	39	16	13	10	39	3	11	13	35	13	3	9
10	4	28	3	0	2	47	3	23	5	47	0	37	10	57	16	13	10	16	3	39	13	45	12	51	10
11	4	9	3	8	2	38	3	35	5	42	0	57	11	15	16	13	9	53	4	7	13	54	12	39	11
12	3	50	3	16	2	29	3	46	5	37	1	17	11	33	16	12	9	29	4	35	14	2	12	27	12
13	3	32	3	23	2	19	3	58	5	31	1	38	11	51	16	10	9	5	5	2	14	9	12	14	13
14	3	13	3	30	2	8	4	9	5	24	1	58	12	8	16	7	8	40	5	29	14	16	12	0	14
15	2	55	3	36	1	57	4	19	5	17	2	19	12	25	16	4	8	14	5	56	14	22	11	46	15
16	2	37	3	41	1	46	4	29	5	9	2	40	12	41	16	0	7	48	6	22	14	27	11	31	16
17	2	19	3	46	1	35	4	39	5	1	3	1	12	57	15	55	7	22	6	48	14	31	11	16	17
18	2	1	3	50	1	23	4	48	4	52	3	22	13	12	15	49	6	55	7	13	14	35	11	1	18
19	1	43	3	53	1	11	4	57	4	43	3	44	13	27	15	42	6	28	7	37	14	38	10	46	19
20	1	26	3	56	0	59	5	5	4	33	4	5	13	42	15	35	6	0	8	1	14	40	10	30	20
21	1	9	3	58	0	46	5	13	4	22	4	26	13	56	15	26	5	32	8	24	14	41	10	14	21
22	0	52	4	0	0	34	5	20	4	11	4	47	14	9	15	17	5	4	8	47	14	42	9	58	22
23	0	36	4	1	0	21	5	27	3	59	5	9	14	21	15	7	4	36	9	9	14	41	9	41	23
24	0	20	4	1	0	8	5	33	3	46	5	30	14	33	14	56	4	8	9	31	14	40	9	24	24
25	0	4	4	1	0	+5	5	39	3	33	5	52	14	44	14	44	3	39	9	53	14	39	9	6	25
26	0	−11	4	0	0	19	5	44	3	19	6	13	14	53	14	31	3	10	10	14	14	37	8	48	26
27	0	26	3	59	0	31	5	48	3	4	6	35	15	5	14	17	2	41	10	34	14	34	8	30	27
28	0	40	3	57	0	46	5	52	2	50	6	56	15	14	14	3	2	11	10	53	14	30	8	12	28
29	0	53	3	54	0	59	5	55	2	35	7	17	15	23	13	48	1	41	11	11	14	25	7	54	29
30	1	9	3	51	1	13	5	57	2	20	7	28	15	31	13	33	1	11	11	28	14	19	7	36	30

The Equations with +, are to be added to the apparent time, to have the mean time; thoſe with −, are to be ſubtracted from apparent time, to give the mean time.

The preceding mark, whether + or −, at the top of any column, belongs to all the numbers or equations in that column till the ſign changes; after which, the remainder of the column belongs to the contrary ſign.

The Equation anſwering to any point of longitude between one degree and another, or any number of minutes or parts of a degree, is to be found by proportion in the uſual way, viz, as 1° or 60′, to that number of minutes, ſo is the whole difference in the Equation from the given whole degree of longitude to the next degree, to the proportional part of it anſwering to the given number of minutes.

See Tables of the Equation of Time computed for every year, in the Nautical Almanac, by a method propoſed and illuſtrated by Dr. Maſkelyne, the aſtronomer royal, viz, by taking the difference between the ſun's true right aſcenſion and his mean longitude, corrected by the Equation of the equinoxes in right aſcenſion, and turning it into time at the rate of 1 minute of time to 15′ of right aſcenſion. Philoſ. Tranſ. vol. 54, p. 336.

Equation *of a Curve,* is an Equation ſhewing the nature of a curve by expreſſing the relation between any abſciſs and its correſponding ordinate, or elſe the relation of their fluxions, &c. Thus, the Equation to the circle, is $ax - x^2 = y^2$, where a is its diameter, x any abſciſs, or part of that diameter, and y the ordinate at that point of the diameter; the meaning being, that whatever abſciſs is denoted by x, then the ſquare of its correſponding ordinate will be $ax - x^2$. In like manner the Equation

of the ellipſe is $\frac{p}{a} \cdot \overline{ax - x^2} = y^2$,

of the hyperbola is $\frac{p}{a} \cdot \overline{ax + x^2} = y^2$,

of the parabola is - - - $px = y^2$.

Where a is an axis, and p the parameter.

And in like manner for any other curves.

This method of expreſſing the nature of curves by algebraical equations, was firſt introduced by Des Cartes, who, by thus connecting together the two ſciences of algebra and geometry, made them mutually aſſiſting to each other, and ſo laid the foundation of the greateſt improvements that have been made in every branch of them ſince that time. See Des Cartes's Geometry; alſo Newton's Lines of the 3d Order, and many other ſimilar works on curve lines, by ſeveral authors.

EQUATOR, in Geography, a great circle of the earth, equally diſtant from its two poles, and dividing it into two equal parts, or hemiſpheres, the northern and ſouthern. The Equator is ſometimes ſimply called the Line.

The circle in the heavens conceived directly over the Equator, is the Equinoctial. See Equinoctial.

The greateſt height of the Equator above the horizon, is equal to the latitude of the place.

Table for turning Degrees and Minutes into Time, and the Contrary.

D	H	M	D	H	M	D	H	M
M	M	S	M	M	S	M	M	S
1	0	4	61	4	4	121	8	4
2	0	8	62	4	8	122	8	8
3	0	12	63	4	12	123	8	12
4	0	16	64	4	16	124	8	16
5	0	20	65	4	20	125	8	20
6	0	24	66	4	24	126	8	24
7	0	28	67	4	28	127	8	28
8	0	32	68	4	32	128	8	32
9	0	36	69	4	36	129	8	36
10	0	40	70	4	40	130	8	40
11	0	44	71	4	44	131	8	44
12	0	48	72	4	48	132	8	48
13	0	52	73	4	52	133	8	52
14	0	56	74	4	56	134	8	56
15	1	0	75	5	0	135	9	0
16	1	4	76	5	4	136	9	4
17	1	8	77	5	8	137	9	8
18	1	12	78	5	12	138	9	12
19	1	16	79	5	16	139	9	16
20	1	20	80	5	20	140	9	20
21	1	24	81	5	24	141	9	24
22	1	28	82	5	28	142	9	28
23	1	32	83	5	32	143	9	32
24	1	36	84	5	36	144	9	36
25	1	40	85	5	40	145	9	40
26	1	44	86	5	44	146	9	44
27	1	48	87	5	48	147	9	48
28	1	52	88	5	52	148	9	52
29	1	56	89	5	56	149	9	56
30	2	0	90	6	0	150	10	0
31	2	4	91	6	4	151	10	4
32	2	8	92	6	8	152	10	8
33	2	12	93	6	12	153	10	12
34	2	16	94	6	16	154	10	16
35	2	20	95	6	20	155	10	20
36	2	24	96	6	24	156	10	24
37	2	28	97	6	28	157	10	28
38	2	32	98	6	32	158	10	32
39	2	36	99	6	36	159	10	36
40	2	40	100	6	40	160	10	40
41	2	44	101	6	44	161	10	44
42	2	48	102	6	48	162	10	48
43	2	52	103	6	52	163	10	52
44	2	56	104	6	56	164	10	56
45	3	0	105	7	0	165	11	0
46	3	4	106	7	4	166	11	4
47	3	8	107	7	8	167	11	8
48	3	12	108	7	12	168	11	12
49	3	16	109	7	16	169	11	16
50	3	20	110	7	20	170	11	20
51	3	24	111	7	24	171	11	24
52	3	28	112	7	28	172	11	28
53	3	32	113	7	32	173	11	32
54	3	36	114	7	36	174	11	36
55	3	40	115	7	40	175	11	40
56	3	44	116	7	44	176	11	44
57	3	48	117	7	48	177	11	48
58	3	52	118	7	52	178	11	52
59	3	56	119	7	56	179	11	56
60	4	0	120	8	0	180	12	0

As one whole revolution of the earth, or of the 360° of the Equator, is performed in 24 hours, which is at the rate of 15° to the hour; hence the number of degrees of the Equator, anſwering to any other given time, or the time anſwering to any given degrees of the Equator, will be eaſily found by proportion, viz,

as 1hr. : 15° : : any time : its degrees,
or as 15° : 1hr. : : any degrees : their time.

And thus is computed the foregoing Table for turning time into degrees of the Equator, and the contrary.

EQUATORIAL, *Univerſal*, or Portable Observatory, is an inſtrument intended to anſwer a number of uſeful purpoſes in practical aſtronomy, independent of any particular obſervatory. It may be employed in any ſteady room or place, and it performs moſt of the uſeful problems in the ſcience of aſtronomy. The following is the deſcription of one lately invented, and named the *Univerſal Equatorial*.

The principal parts of this inſtrument (fig. 2, plate viii.) are, 1ſt, The azimuth or horizontal circle A, which repreſents the horizon of the place, and moves on a long axis B, called the vertical axis. 2d, The Equatorial or hour-circle C, repreſenting the Equator, placed at right angles to the polar axis D, or the axis of the earth, upon which it moves. 3d, The ſemicircle of declination E, on which the teleſcope is placed, and moving on the axis of declination, or the axis of motion of the line of collimation F. Which circles are meaſured and divided as in the following Table:

Meaſures of the ſeveral circles and diviſions on them:	Radius Inches	Limb divided to	Non. of 30 gives ſeconds	Divid. on limb into pts. of Inc.	Divid. by Non. into pts. of Inc.
Azimuth or horizontal circle	5·1	15′	30″	45th	1350th
Equatorial or hour circle	5·1	15′, or 1 m. in time	30″ 2″	45th	1350th
Vertical ſemicircle for declination or latitude	5·	15′	30″	42d	1260th

4th, The teleſcope, which is an achromatic refractor with a triple object-glaſs, whoſe focal diſtance is 17 inches, and its aperture 2·45 inc., and it is furniſhed with 6 different eye-tubes; ſo that its magnifying powers extend from 44 to 168. The teleſcope in this Equatorial may be brought parallel to the polar axis, as in the figure, ſo as to point to the pole-ſtar in any part of its diurnal revolution; and thus it has been obſerved near noon, when the ſun has ſhone very bright. 5th, The apparatus for correcting the error in altitude occaſioned by refraction, which is applied to the eye-end of the teleſcope, and conſiſts of a ſlide G moving in a groove or dove-tail, and carrying the ſeveral eye-tubes of the teleſcope, on which ſlide there is an index correſponding to five ſmall diviſions engraved on the dove-tail; a very ſmall circle, called the refraction circle H, moveable by a finger-ſcrew at the extremity of the eye-end of the teleſcope; which circle is divided into half minutes, one whole revolution of it being equal to 3′ 18″, and by its motion it raiſes the centre of the croſs-hairs on a circle of altitude; and alſo a quadrant I of 1½ inc. radius, with diviſions on each ſide, one expreſſing the degree of altitude of the object viewed, and the other expreſſing the minutes and ſeconds of error occaſioned by refraction, correſponding to that degree of altitude. To this quadrant is joined a ſmall round level K, which is adjuſted partly by the pinion that turns the whole of this apparatus, and partly by the index of the quadrant; for which purpoſe the refraction circle is ſet to the ſame minute &c, which the index points to on the limb of the quadrant; and if the minute &c, given by the quadrant, exceed the 3′ 18″ contained in one entire revolution of the refraction circle, this muſt be ſet to the exceſs above one or more of its entire revolutions; then the centre of the croſs-hairs will appear to be raiſed on a circle of altitude to the additional height which the error of refraction will occaſion at that altitude.

The principal adjuſtment in this inſtrument, is that of making the line of collimation to deſcribe a portion of an hour circle in the heavens: in order to which, the azimuth circle muſt be truly level; the line of collimation, or ſome correſponding line repreſented by the ſmall braſs rod M parallel to it, muſt be perpendicular to the axis of its own proper motion; and this laſt axis muſt be perpendicular to the polar axis. On the braſs rod M there is occaſionally placed a hanging level N, the uſe of which will appear in the following adjuſtments:

The azimuth circle may be made level by turning the inſtrument till one of the levels be parallel to an imaginary line joining two of the feet ſcrews; then adjuſt that level with theſe two feet ſcrews; turn the circle 180°, or half round; and if the bubble be not then right, correct half the error by the ſcrew belonging to the level, and the other half error by the two foot ſcrews, repeating this operation till the bubble come right; then turn the circle 90° from the two former poſitions, and ſet the bubble right, if it be wrong, by the foot ſcrew at the end of the level; when this is done, adjuſt the other level by its own ſcrew, and the azimuth circle will be truly level. The hanging level muſt then be fixed to the braſs rod by two hooks of equal length, and made truly parallel to it: for this purpoſe, make the polar axis perpendicular or nearly perpendicular to the horizon; then adjuſt the level by the pinion of the declination ſemicircle: reverſe the level, and if it be wrong, correct half the error by a ſmall ſteel ſcrew that lies under one end of the level, and the other half error by the pinion of the declination-ſemicircle, repeating the operation till the bubble be right in both poſitions. To make the braſs rod, on which the level is ſuſpended, at right angles to the axis of motion of the teleſcope, or line of collimation, make the polar axis horizontal, or nearly ſo; ſet the declination ſemicircle to 0°, and turn the hour-circle till the bubble come right; then turn the declination-circle to 90°; adjuſt the bubble by raiſing or depreſſing the polar axis (firſt by hand till it be nearly right, afterwards tighten with an ivory key the ſocket which runs on the arch with the polar axis, and then apply the ſame ivory key to the adjuſting ſcrew at the end of the ſaid arch till the bubble come quite right); then turn the declination-circle to the oppoſite 90°; if the level be not then right, correct half the error by the aforeſaid adjuſting ſcrew at the end of the arch, and the other half error by the two ſcrews that raiſe or depreſs the end of the braſs rod. The polar axis remaining nearly horizontal as before, and the declination-ſemicircle at 0°, adjuſt the bubble by the hour-circle; then turn the declination-ſemicircle to 90°, and adjuſt the bubble by raiſing or depreſſing the polar axis; then turn the hour-circle 12 hours; and if the bubble be wrong, correct half the error by the polar axis, and the other half error by the two pair of capſtan ſcrews at the feet of the two ſupports on one ſide of the axis of motion of the teleſcope; and thus this axis will be at right angles to the polar axis. The next adjuſtment, is to make the centre of the croſs hairs remain on the ſame object, while the eye-tube is turned quite round by the pinion of the refraction apparatus: for this adjuſtment, ſet the index on the ſlide to the firſt diviſion on the dove-tail; and ſet the diviſion marked 18″ on the refraction-circle to its index; then look through the teleſcope, and with the pinion turn the eye-tube quite round; then if the centre of the hairs does not remain on the ſame ſpot during that revolution, it muſt be corrected by the four ſmall ſcrews, 2 and 2 at a time, which will be found upon unſcrewing the neareſt end of the eye-tube that contains the firſt eye-glaſs; repeating this correction till the centre of the hairs remain on the ſpot looked at during a whole revolution. To make the line of collimation parallel to the braſs rod on which the level hangs, ſet the polar axis horizontal, and the declination-circle to 90°, adjuſt the level by the polar axis; look through the teleſcope on ſome diſtant horizontal object, covered by the centre of the croſs hairs: then invert the teleſcope, which is done by turning the hour-circle half round; and if the centre of the croſs hairs does not cover the ſame object as before, correct half the error by the uppermoſt and lowermoſt of the 4 ſmall ſcrews at the eye-end of the large tube of the teleſcope; this correction will give a ſecond object now covered by the centre of the hairs, which muſt be adopted inſtead of the firſt object; then invert the teleſcope as before; and if the ſecond object be not covered by the centre of the hairs, correct half the error by the ſame two ſcrews as were uſed before: this correction will give a third object, now covered by the centre of the hairs, which muſt be adopted inſtead of the ſecond object; repeat this operation till no error remain; then ſet the hour-circle exactly to 12 hours, the declination-circle remaining a 90° as before; and if the centre of the croſs hairs do not cover the laſt object fixed on, ſet it to that object by the two remaining ſmall ſcrews at the eye-end of the large tube, and then the line of collimation will be parallel to the braſs rod. For rectifying the nonius of the declination and Equatorial circles, lower the teleſcope as many degrees &c below 0° or Æ on the declination-ſemicircle as are equal to the complement of the latitude; then elevate the polar axis till the bubble be horizontal; and thus the Equatorial circle will be elevated to the co-latitude of the place: ſet this circle to 6 hours; adjuſt the level by the pinion of the declination-circle; then turn the Equatorial circle exactly 12 hours from the laſt poſition; and if the level be not right, correct one half of the error by the Equatorial circle, and the other half by the declination-circle: then turn the Equatorial circle back again exactly 12 hours from the laſt poſition; and if the level be ſtill wrong, repeat the correction as before, till it be right, when turned to either poſition: that being done, ſet the nonius of the Equatorial circle exactly to 6 hours, and the nonius of the declination-circle exactly to 0°.

The chief uſes of this Equatorial are,

1ſt, To find the meridian by one obſervation only: for this purpoſe, elevate the Equatorial circle to the co-latitude of the place, and ſet the declination-ſemicircle to the ſun's declination for the day and hour of the day required; then move the azimuth and hour-circles both at the ſame time, either in the ſame or contrary direction, till you bring the centre of the croſs hairs in the teleſcope exactly to cover the centre of the ſun; when that is done, the index of the hour-circle will give the apparent or ſolar time at the inſtant of obſervation; and thus the time is gained, though the ſun be at a diſtance from the meridian; then turn the hour-circle till the index points preciſely at 12 o'clock, and lower the teleſcope to the horizon, in order to obſerve ſome point there in the centre of the glaſs; and that point is the meridian mark, found by one obſervation only. The beſt time for this operation is 3 hours before, or 3 hours after 12 at noon.

2d, To point the teleſcope on a ſtar, though not on the meridian, in full day-light. Having elevated the equatorial circle to the co-latitude of the place, and ſet the declination-ſemicircle to the ſtar's declination, move the index of the hour-circle till it ſhall point to the preciſe time at which the ſtar is then diſtant from the meridian, found in the tables of the right aſcenſion of the ſtars, and the ſtar will then appear in the glaſs.

Beſides theſe uſes, peculiar to this inſtrument, it may alſo be applied to all the purpoſes to which the principal aſtronomical inſtruments are applied; ſuch as a tranſit inſtrument, a quadrant, and an equal-altitude inſtrument.

See the deſcription and drawing of an Equatorial teleſcope, or portable obſervatory, invented by Mr. Short, in the Philoſ. Tranſ. number 493, or vol. 46, p. 242; and another by Mr. Nairne, vol. 61, p. 107.

EQUIANGULAR *Figure*, is one that has all its angles equal among themſelves; as the ſquare, and all the regular figures.

An equilateral figure inſcribed in a circle, is always Equiangular. But an Equiangular figure inſcribed in a circle, is not always equilateral, except when it has an odd number of ſides: If the ſides be of an even number, then they may either be all equal, or elſe half of them will always be equal to each other, and the other half to each other, the equals being placed alternately. See the demonſtration in my Mathematical Miſcellany, pa. 272.

Equiangular, is alſo ſaid of any two figures of the ſame kind, when each angle of the one is equal to a correſponding angle in the other, whether each figure, ſeparately conſidered in itſelf, be an equiangular figure or not, that is, having all its angles equal to each other. Thus, two triangles are Equiangular to each other, if, ex. gr. one angle in each be of 30°, a ſecond angle in each of 50°, and the third angle of each equal to 100 degrees.

Equiangular triangles have not their like ſides neceſſarily equal, but only proportional to each other; and ſuch triangles are always ſimilar to each other.

Equicrural *Triangle*, is one that has two of its ſides equal to each other; and is more uſually called an Iſoſceles triangle.

Equiculus, Equuleus, or Equus *Minor*, a conſtellation of the northern hemiſphere. See Equuleus.

EQUIDIFFERENT, are ſuch things as have equal differences, or arithmetically proportional. If the terms have all the ſame difference, viz, the 1ſt and 2d, the 2d and 3d, the 3d and 4th, &c, they are ſaid to be Continually Equidifferent; as the numbers 3, 6, 9, 12, &c, where the common difference is 3. But if the ſeveral different couplets only have the ſame difference, as the 1ſt and 2d, the 3d and 4th, the 5th and 6th, &c, they are ſaid to be Diſcretely Equidifferent; as the terms 3 and 6, 7 and 10, 9 and 12, &c. See Arithmetical *Progreſſion* and *Proportion*.

EQUILATERAL *Figure*, is one that has all its ſides equal to each other. Such as the ſquare, and all the regular figures or polygons, or a triangle that has all its ſides equal. See Equiangular.

Equilateral *Hyperbola*, is that which has the two axes equal to each other, and every pair of conjugate diameters alſo equal to each other. The aſymptotes alſo are at right angles to each other, and make each half a right angle with either axis. Alſo, ſuch an hyperbola is equal to its oppoſite hyperbola, and likewiſe to its conjugate hyperbola; ſo that all the four conjugate hyperbolas are mutually equal to each other.

Moreover, as the 3d proportional to the two axes is the parameter, therefore in ſuch a figure, the parameter and two axes are all three equal to one another. Hence, as the general equation to hyperbolas is

$y^2 = \frac{p}{t} . \overline{tx + x^2}$ or $= \frac{c^2}{t^2} . \overline{tx + x^2}$, where t is the tranſverſe axis, c the conjugate, p the parameter, x the abſciſs, and y the ordinate; then making t, c, and p all equal, the equation, for the Equilateral Hyperbola, becomes $y^2 = tx + x^2$; differing from the equation of the circle only in the ſign of the term x^2, which in the circle is $-$.

EQUILIBRIUM, is an equality between two equal forces acting in oppoſite directions; ſo that they mutually balance each other; like the two equal arms, or ſcales, of a balance, &c.

Equilibrium, in ſolid bodies, forms a conſiderable part of the ſcience of Statics. And Equilibrium of fluids, a conſiderable part of the doctrine of Hydroſtatics.

EQUIMULTIPLES, the products of quantities equally multiplied.

Thus, $3a$ and $3b$ are Equimultiples of a and b;
and ma and mb are Equimultiples of a and b.

Equimultiples of any quantities, have the ſame ratio as the quantities themſelves.

Thus $a : b :: 3a : 3b :: ma : mb$.

EQUINOCTIAL, a great circle in the heavens under which the equator moves in its diurnal motion. The poles of this circle are the poles of the world. It divides the ſphere into two equal parts, the northern and ſouthern. It cuts the horizon, of any place, in the eaſt and weſt points; and at the meridian its elevation above the horizon is equal to the co-latitude of the place. The Equinoctial has alſo various other properties; as,

1. Whenever the ſun comes to this circle, he makes equal days and nights all round the globe; becauſe he then riſes due eaſt, and ſets due weſt. Hence it has the name Equinoctial. All ſtars which are under this circle, or have no declination, do alſo riſe due eaſt, and ſet due weſt.

2. All people living under this circle, or upon the equator, or line, have their days and nights at all times equal to each other.

3. From this circle, on the globe, is counted, upon the meridian, the declination in the heavens, and the latitude on the earth.

4. Upon the Equinoctial, or equator, is counted the longitude, making in all 360° quite round, or elſe 180° eaſt, and 180° weſt.

5. And as the time of one whole revolution is divided into 24 hours; therefore 1 hour anſwers to 15°, or the 24th part of 360°. Hence,

1° of longitude anſwers to 4 min. of time,
15′ - - - - - - - - to 1 min. of time,
1′ - - - - - - - - to 4 ſec. of time, &c.

6. The ſhadows of thoſe people who live under this circle are caſt to the ſouthward of them for one half of the year, and to the northward of them during the other half; and twice in a year, viz, at the time of the equinoxes, the ſun at noon caſts no ſhadow, being exactly in their zenith.

Equinoctial *Colure*, is the great circle paſſing through the poles of the world and the Equinoctial points, or firſt points of Aries and Libra.

Equinoctial *Dial*, is one whoſe plane is parallel to the Equinoctial. The properties or principles of this dial are,

1. The hour lines are all equally diſtant from one another, quite round the circumference of a circle; and the ſtyle is a ſtraight pin, or wire, ſet up in the centre of the circle, perpendicular to the plane of the dial.

2. The ſun ſhines upon the upper part of this dial-plane from the 21ſt of March to the 22d of September, and upon the under part of the plane the other half of the year.

Some of theſe dials are made of braſs, &c; and ſet up in a frame, to be elevated to any given latitude.

Equinoctial *Points*, are the two oppoſite points where the Ecliptic and Equinoctial croſs each other;

the one point being in the beginning of Aries, and called the Vernal point, or Vernal Equinox; and the other in the beginning of Libra, and called the Autumnal point, or Autumnal Equinox.

It is found by obſervation, that the Equinoctial points, and all the other points of the ecliptic, are continually moving backwards, or in antecedentia, i. e. weſtwards. This retrograde motion of the Equinoctial points, is that phenomenon called the Preceſſion of the Equinoxes, and is made at the rate of 50″ every year nearly.

EQUINOXES, the times when the ſun enters the Equinoctial points, or about the 21ſt of March and 22d of September: the former being the Vernal or Spring Equinox, and the latter time the Autumnal Equinox.

As the ſun's motion is unequal, being ſometimes quicker and ſometimes ſlower, it hence happens that there are about 8 days more from the vernal to the autumnal Equinox, or while the ſun is on the northern ſide of the equator, than while he is in moving through the ſouthern ſigns from the autumnal to the vernal Equinox, or on the ſouthern ſide of the equator. According to the obſervations of M. Caſſini, the

ſun is $186^d\ 14^h\ 53^m$ in the northern ſigns,
and only 178 14 56 in the ſouthern ſigns,
ſo that 7 23 57 is the difference of them, or nearly 8 days.

EQUINUS *Barbatus*, a kind of comet. See Hippeus.

EQUULEUS, Equiculus, and Equus *Minor*, *Equi ſectio*, the *Horſe's Head*, one of the 48 old conſtellations, in the northern hemiſphere. Its ſtars, in Ptolomy's catalogue, are 4, in Tycho's 4, in Hevelius's 6, and in Flamſteed's 10.

ERECT *Viſion*. See Vision.

Erect *Dials*, ſuch as ſtand perpendicular to the horizon, and are of various kinds; as Erect Direct, when they face exactly one of the four cardinal points, eaſt, weſt, north, ſouth; and Erect Declining, when declined from the cardinal points. See Dial.

To Erect *a Perpendicular*, is a popular problem in practical geometry, and denotes to raiſe a perpendicular from a given line &c, as diſtinguiſhed from Demitting or letting a perpendicular fall upon a line &c, from ſome point out of it. See Perpendicular.

ERIDANUS, the *River*, a conſtellation of the ſouthern hemiſphere, and one of the 48 old aſteriſms. The ſtars in this conſtellation, in Ptolomy's catalogue, are 34, in Tycho's 19, and in the Britiſh catalogue 84.

ERRATIC, an epithet applied to the planets, which are called Erratic or wandering ſtars, in contradiſtinction to the fixed ſtars.

ESCALADE, or Scalade, a furious attack of a wall or a rampart; carried on with ladders, to paſs the ditch, or mount the rampart; without proceeding in form, breaking ground, or carrying on regular works to ſecure the men.

ESPAULE, or Epaule. See Epaule.

ESPLANADE, in Fortification, called alſo Glacis, a part which ſerves as a parapet to the counterſcarp, or covert way; being a declivity or ſlope of earth, commencing from the top of the counterſcarp, and loſing itſelf inſenſibly in the level of the champaign.

Eſplanade alſo means the ground which has been levelled from the glacis of the counterſcarp, to the firſt houſes; or the vacant ſpace between the works and the houſes of the town.

The term is alſo applied, in the general, to any piece of ground that is made flat or level, and which before had ſome eminence that incommoded the place.

ESTIVAL *Occident*, *Orient*, or *Solſtice*. See Occident, Orient, Solstice.

EVAPORATION, the act of diſſipating the humidity of a body in fumes or vapour; differing from exhalation, which is properly a diſperſion of dry particles iſſuing from a body.

Evaporation is uſually produced by heat, and by the change of air: thus, common ſalt is formed by evaporating all the humidity in the brine or ſalt water; which evaporation is either performed by the heat of the ſun, as in the ſalt-works on the ſea-coaſt, &c; or by means of fire, as at the ſalt-ſprings, &c: and it is well known how uſeful a briſk wind is in drying wet clothes, or the ſurface of the ground; while in a calm, ſtill atmoſphere, they dry extreme ſlowly.

But, though Evaporation be generally conſidered as an effect of the heat and motion of the air, yet M. Gauteron, in the Memoires de l'Acad. des Scienc. an. 1705, ſhews, that a quite oppoſite cauſe may have the ſame effect, and that fluids loſe more of their parts in the ſevereſt froſt than when the air is moderately warm: thus, in the great froſt of the year 1708, he found that the greater the cold, the more conſiderable the evaporation; and that ice itſelf loſt full as much as the warmer liquors that did not freeze.

There are indeed few ſubjects of philoſophical inveſtigation that have occaſioned a greater variety of opinion than the theory of Evaporation, or of the aſcent of water in ſuch a fluid as air, between 8 and 9 hundred times lighter than itſelf, to different heights according to the different denſities of the air; in which caſe it muſt be ſpecifically lighter than the air through which it aſcends. The Carteſians account for it by ſuppoſing, that by the action of the ſun upon the water, ſmall particles of the water are formed into hollow ſpheres and filled with the *materia ſubtilis*, which renders them ſpecifically lighter than the ambient air, ſo that they are buoyed up by it.

Dr. Nieuwentyt, in his Religious Philoſopher, cont. 19, and ſeveral others, have alleged, that the ſun emits particles of fire which adhere to thoſe of water, and form moleculæ, or ſmall bodies, lighter than an equal bulk of air, which conſequently aſcend till they come to a height where the air is of the ſame ſpecific gravity with themſelves; and that theſe particles being ſeparated from the fire with which they are incorporated, coaleſce and deſcend in dew or rain.

Dr. Halley has advanced another hypotheſis, which has been more generally received: he imagined, that by the action of the ſun on the ſurface of the water, the aqueous particles are formed into hollow ſpherules, that are filled with a finer air highly rarefied, ſo as to become ſpecifically lighter than the external air. Philoſ. Tranſ. number 192, or Abr. vol. 2, p. 126.

Dr. Deſaguliers, diſſatisfied with theſe two hypotheſes, propoſes another in the Philoſ. Tranſ. number 407, or Abr. vol. 7, pa. 61. See alſo his Courſe of

Experimental Philoſophy, vol. 2, p. 336. He ſuppoſes that heat acts more powerfully on water than on common air; that the ſame degree of heat which rareſies air two-thirds, will rarefy water near 14,000 times; and that a very ſmall degree of heat will raiſe a ſteam or vapour from water, even in winter, whilſt it condenſes the air; and thus the particles of water are converted into vapour by being made to repel each other ſtrongly, and, deriving electricity from the particles of air to which they are contiguous, are repelled by them and by each other, ſo as to form a fluid which, being lighter than the air, riſes in it, according to their relative gravities. The particles of this vapour retain their repellent force for a conſiderable time, till, by ſome diminution of the denſity of the air in which they float, they are precipitated downwards, and brought within the ſphere of each other's attraction of coheſion, and ſo join again into drops of water.

Many objections have been urged againſt this opinion, by Mr. Clare in his Treatiſe of the Motion of Fluids, pa. 294, and by Mr. Rowning in his Syſtem of Philoſophy, part 2, diſſ. 6; to which Dr. Hamilton has added the two following, viz, that if heat were the only cauſe of evaporation, water would evaporate faſter in a warm cloſe room, than when expoſed in a colder place, where there is a conſtant current of air; which is contrary to experience; and that the evaporation of water is ſo far from depending on its being rarefied by heat, that it is carried on even whilſt water is condenſed by the coldneſs of the air, till it freezes; and ſince it evaporates even when frozen into hard ice, it muſt alſo evaporate in all the leſſer degrees of cold. And therefore heat does not ſeem to be the principal, much leſs the only cauſe of Evaporation.

Others have more ſucceſsfully accounted for the phenomena of Evaporation on another principle, viz that of ſolution; and ſhewn, from a variety of experiments, that what we call Evaporation, is nothing more than a gradual ſolution of water in air, produced and ſupported by the ſame means, viz, attraction, heat, and motion, by which other ſolutions are effected.

It ſeems the Abbé Nollet firſt ſtarted this opinion, though without much purſuing it, in his Leçons de Phyſique Experimentale, firſt publiſhed in 1743: he offers it as a conjecture, that the air of the atmoſphere ſerves as a ſolvent or ſponge, with regard to the bodies that encompaſs it, and receives into its pores the vapours and exhalations that are detached from the maſſes to which they belong in a fluid ſtate; and he accounts for their aſcent on the ſame principles with the aſcent of liquors in capillary tubes. On his hypotheſis, the condenſation of the air contributes, like the ſqueezing of a ſponge, to their deſcent.

Dr. Franklin, in a paper of Philoſophical and Meteorological Obſervations, Conjectures and Suppoſitions, delivered to the Royal Society about the year 1747, and read in 1756, ſuggeſted a ſimilar hypotheſis: he obſerves, that air and water mutually attract each other; and hence he concludes, that water will diſſolve in air, as ſalt in water; every particle of air aſſuming one or more particles of water; and when too much is added, it precipitates in rain. But as there is not the ſame contiguity between the particles of air as of water, the ſolution of water in air is not carried on without a motion of the air, ſo as to cauſe a freſh acceſſion of dry particles. A ſmall degree of heat ſo weakens the coheſion of the particles of water, that thoſe on the ſurface eaſily quit it, and adhere to the particles of air: a greater degree of heat is neceſſary to break the coheſion between water and air; for its particles being by heat repelled to a greater diſtance from each other, thereby more eaſily keep the particles of water that are annexed to them from running into coheſions that would obſtruct, refract, or reflect the heat: and hence it happens that when we breathe in warm air, though the ſame quantity of moiſture may be taken up from the lungs as when we breathe in cold air, yet that moiſture is not ſo viſible. On theſe principles he accounts for the production and different appearances of fogs, miſts, and clouds. He adds, that if the particles of water bring electrical fire when they attach themſelves to air, the repulſion between the particles of water electrified, joins with the natural repulſion of the air to force its particles to a greater diſtance, ſo that the air being more dilated, riſes and carries up with it the water; which mutual repulſion of the particles of air is increaſed by a mixture of common fire in the particles of water. When air, loaded with ſurrounding particles of water, is compreſſed by adverſe winds, or by being driven againſt mountains, &c, or condenſed by taking away the fire that aſſiſted it in expanding, the particles will approach one another, and the air with its water will deſcend as a dew; or if the water ſurrounding one particle of air come in contact with the water ſurrounding another, they coaleſce and form a drop, producing rain; and ſince it is a well-known fact, that vapour is a good conductor of electricity, as well as of common fire, it is reaſonable to conclude with Mr. Henley, that Evaporation is one great cauſe of the clouds becoming at times ſurcharged with this fluid. Philoſ. Tranſ. vol. 67, pa. 134. See alſo vol. 55, p. 182, or Franklin's Letters and Papers on Philoſophical Subjects, p. 42 &c, and pa. 182, ed. 1769.

M. le Roi, of the Acad. of Sciences at Paris, has alſo advanced the ſame opinion, and ſupported it by a variety of facts and obſervations in the Memoirs for the year 1751. He ſhews, that water does undergo in the air a real diſſolution, forming with it a tranſparent mixture, and poſſeſſing the ſame properties with the ſolutions of moſt ſalts in water; and that the two principal cauſes which promote the ſolution of water in the air, are heat and wind; that the hotter the air is, within a certain limit, the more water it will diſſolve; and that at a certain degree of heat the air will be ſaturated with water; and by determining at different times the degree of the air's ſaturation, he eſtimates the influence of thoſe cauſes on which the quantity depends that is ſuſpended in the air in a ſtate of ſolution. Accordingly, the air, heated by evaporating ſubſtances to which it is contiguous, becomes more rare and light, riſes and gives way to a denſer air; and, by being thus removed, contributes to accelerate the Evaporation. The fixed air contained in the internal parts of evaporating bodies, put into action by heat, ſeems alſo to increaſe their Evaporation. The wind is another cauſe of the increaſe of Evaporation, chiefly by changing and renewing the air which immediately encompaſſes the evaporating ſubſtances; and from the conſideration of theſe two cauſes combined, it appears why the quantity of vapour raiſed in the night is leſs than that of the day, ſince the air is then both leſs heated and leſs agitated. To the objection urged againſt this hypotheſis, on account of the Evaporation of water in a vacuum, this ingenious writer replies, that the water itſelf contains a great quantity of air, which gradually diſengages itſelf, and cauſes the Evaporation; and that it is impoſſible that a ſpace containing water which evaporates ſhould remain perfectly free from air. To this objection a late writer, Dr. Dobſon of Liverpool, replies, that though air appears, by unqueſtionable experiments, to be a chemical ſolvent of water, and as ſuch, is to be conſidered as one cauſe of its Evaporation, heat is another cauſe, acting without the intervention of air, and producing a copious Evaporation in an exhauſted receiver; agreeably to an experiment of Dr. Irving, who ſays, that in an exhauſted receiver water riſes in vapour more copiouſly at 180° of Fahrenheit's thermometer, than in the open air at 212°, its boiling point. Dr. Dobſon farther adds, that water may exiſt in air in three different ſtates; in a ſtate of perfect ſolution, when the air will be clear, dry, and heavy, and its powers of ſolution ſtill active; in a ſtate of beginning precipitation, when it becomes moiſt and foggy, its powers of ſolution are diminiſhed, and it becomes lighter in proportion as its water is depoſited; and alſo, when it is completely precipitated, which may happen either by a ſlower proceſs, when the diſſolved water falls in a drizzling rain, or by a more ſudden proceſs, when it deſcends in briſk ſhowers. Philoſ. Tranſ. vol. 67, p. 257, and Phipps's Voyage towards the North Pole, p. 211.

Dr. Hamilton, profeſſor of philoſophy in the univerſity of Dublin, tranſmitted to the Royal Society in 1765, a long Diſſertation on the nature of Evaporation, in which he propoſes and eſtabliſhes this theory of ſolution; and though other writers had been prior in their conjectures, and even in their reaſoning on this ſubject, Dr. Hamilton aſſures us, that he has not repreſented any thing as new which he was conſcious had ever been propoſed by any one before him, even as a conjecture. Dr. Hamilton having evinced the agreement between Solution and Evaporation, concludes, that Evaporation is nothing more than a gradual ſolution of water in air, produced and promoted by attraction, heat, and motion, juſt as other ſolutions are effected.

To account for the aſcent of aqueous vapours into the atmoſphere, this ingenious writer obſerves, that the loweſt part of the air being preſſed by the weight of the upper againſt the ſurface of the water, and continually rubbing upon it by its motion, attracts and diſſolves thoſe particles with which it is in contact, and ſeparates them from the reſt of the water. And ſince the cauſe of ſolution in this caſe is the ſtronger attraction of the particles of water towards the air, than towards each other, thoſe that are already diſſolved and taken up, will be ſtill farther raiſed by the attraction of the dry air that lies over them, and thus will diffuſe themſelves, riſing gradually higher and higher, and ſo leave the loweſt air not ſo much ſaturated but that it will ſtill be able to diſſolve and take up freſh particles of water; which proceſs is greatly promoted by the motion of the wind. When the vapours are thus raiſed and carried by the winds into the higher and colder parts of the atmoſphere, ſome of them will coaleſce into ſmall particles, which ſlightly attracting each other, and being intermixed with air, will form clouds; and theſe clouds will float at different heights, according to the quantity of vapour borne up, and the degree of heat in the upper parts of the atmoſphere: and thus clouds are generally higher in ſummer than in winter. When the clouds are much increaſed by a continual addition of vapours, and their particles are driven cloſe together by the force of the winds, they will run into drops heavy enough to fall down in rain. If the clouds be frozen before their particles are gathered into drops, ſmall pieces of them being condenſed and made heavier by the cold, fall down in thin flakes of ſnow. When the particles are formed into drops before they are frozen, they become hailſtones. When the air is replete with vapours, and a cold breeze ſprings up, which checks the ſolution of them, clouds are formed in the lower parts of the atmoſphere, and compoſe a miſt or fog, which uſually happens in a cold morning, and is diſperſed when the ſun has warmed the air, and made it capable of diſſolving theſe watry particles. Southerly winds commonly bring rain, becauſe, being warm and replete with aqueous vapours, they are cooled by coming into a colder climate; and therefore they part with ſome of them, and ſuffer them to precipitate in rain: whereas northerly winds, being cold, and acquiring additional heat by coming into a warmer climate, are ready to diſſolve and receive more vapour than they before contained; and therefore, by long continuance, they are dry and parching, and commonly attended with fair weather.

Changes of the air, with reſpect to its denſity and rarity, as well as its heat and cold, will produce contrary effects in the ſolution of water, and the conſequent aſcent or fall of vapours. Several experiments prove that air, when rarefied, cannot keep ſo much water diſſolved as it does in a more condenſed ſtate; and therefore when the atmoſphere is ſaturated with water, and changes from a denſer to a rarer ſtate, the high and colder parts of it will let go ſome of the water before diſſolved, forming new clouds, and diſpoſing them to fall down in rain: but a change from a rarer to a denſer ſtate will ſtop the precipitation of the water, and enable the air to diſſolve, either in whole or in part, ſome of thoſe clouds that were formed before, and render their particles leſs apt to run into drops and fall down in rain: on this account, we generally find that the rarefied and condenſed ſtates of the atmoſphere are reſpectively attended with rain or fair weather. See more on this ſubject in the Philoſ. Tranſ. vol. 55, pa. 146, or Hamilton's Philoſophical Eſſays, p. 33.

Dr. Halley, before mentioned, has furniſhed ſome experiments on the Evaporation of water; the reſult of which is contained in the following articles: 1. That water ſalted to about the ſame degree as ſea-water, and expoſed to a heat equal to that of a ſummer's day, did, from a circular ſurface of about 8 inches diameter, evaporate at the rate of 6 ounces in 24 hours: whence by a calculus he finds that, in ſuch circumſtances, the water evaporates 1-10th of an inch deep in 12 hours:

which quantity, he obſerves, will be found abundantly ſufficient to furniſh all the rains, ſprings, dews, &c. By this experiment, every 10 ſquare inches of ſurface of the water yield in vapour *per diem* a cubic inch of water: and each ſquare foot half a wine pint; every ſpace of 4 feet ſquare, a gallon; a mile ſquare, 6914 tuns; and a ſquare degree, of 69 Engliſh miles, will evaporate 33 millions of tuns a day; and the whole Mediterranean, computed to contain 160 ſquare degrees, at leaſt 5280 millions of tuns each day. Philoſ. Tranſ. number 189, or Abridg. vol. 2, pa. 108.—2. A ſurface of 8 ſquare inches, evaporated purely by the natural warmth of the weather, without either wind or ſun, in the courſe of a whole year, 16292 grains of water, or 64 cubic inches; conſequently, the depth of water thus evaporated in one year, amounts to 8 inches. But this being too little to anſwer the experiments of the French, who found that it rained 19 inches of water in one year at Paris; or thoſe of Mr. Townley, who found the annual quantity of rain in Lancaſhire above 40 inches; he concludes, that the ſun and wind contribute more to Evaporation than any internal heat or agitation of the water. In effect, Dr. Halley fixes the annual Evaporation of London at 48 inches; and Dr. Dobſon ſtates the ſame for Liverpool at 36¼ inches. Philoſ. Tranſ. vol. 67, p. 252.

3. The effect of the wind is very conſiderable, on a double account; for the ſame obſervations ſhew a very odd quality in the vapours of water, viz, that of adhering and hanging to the ſurface that exhaled them, which they clothe as it were with a fleece of vapourous air; which once inveſting the vapour, it afterwards riſes in much leſs quantity. Whence, the quantity of water loſt in 24 hours, when the air is very ſtill, was very ſmall, in proportion to what went off when there was a ſtrong gale of wind abroad to diſſipate the fleece, and make room for the emiſſion of vapour; and this, even though the experiment was made in a place as cloſe from the wind as could be contrived. Add, that this fleece of water, hanging to the ſurface of waters in ſtill weather, is the occaſion of very ſtrange appearances, by the refraction of the vapours differing from and exceeding that of common air: whence every thing appears raiſed, as houſes like ſteeples, ſhips as on land above the water, the land raiſed, and as it were lifted from the ſea, &c.

4. The ſame experiments ſhew that the Evaporation in May, June, July, and Auguſt, which are nearly equal, are about three times as great as thoſe in the months of November, December, January, and February. Philoſ. Tranſ. numb. 212, or Abr. vol. 2, pa. 110.

Dr. Brownrigg, in his Art of making common ſalt, pa. 189, fixes the Evaporation of ſome parts of England at 73·8 inches during the months of May, June, July, and Auguſt; and the Evaporation of the whole year at more than 140 inches. But the Evaporation of the four ſummer months at Liverpool, on a medium of 4 years, was found to be only 18·88 inches. Alſo Dr. Hales calculates the greateſt annual Evaporation from the ſurface of the earth in England at 6·66 inches; and therefore the annual Evaporation from a ſurface of water, is to the annual Evaporation from the ſurface of the earth at Liverpool, nearly as 6 to 1. Philoſ. Tranſ. vol. 67, ubi ſupra.

In the Tranſactions of the American Philoſophical Society, vol. 3, pa. 125, there is an ingenious paper on Evaporation, by Dr. Wiſtar. It is there ſhewn, that evaporation ariſes when the moiſt body is warmer than the medium it is incloſed in. And, on the contrary, it acquires moiſture from the air, when the body is the colder. This carrying off, and acquiring of moiſture, it is ſhewn, is by the paſſage of heat out of the body, or into it.

EUCLID, of Megara, a celebrated philoſopher and logician; he was a diſciple of Socrates, and flouriſhed about 400 years before Chriſt. The Athenians having prohibited the Megarians from entering their city on pain of death, this philoſopher diſguiſed himſelf in women's clothes to attend the lectures of Socrates. After the death of Socrates, Plato and other philoſophers went to Euclid at Megara, to ſhelter themſelves from the tyrants who governed Athens.—This philoſopher admitted but one chief good; which he at different times called *God*, or the *Spirit*, or *Providence*.

EUCLID, the celebrated mathematician, according to the account of Pappus and Proclus, was born at Alexandria, in Egypt, where he flouriſhed and taught mathematics, with great applauſe, under the reign of Ptolomy Lagos, about 280 years before Chriſt. And here, from his time, till the conqueſt of Alexandria by the Saracens, all the eminent mathematicians were either born, or ſtudied; and it is to Euclid, and his ſcholars, we are beholden for Eratoſthenes, Archimedes, Apollonius, Ptolomy, Theon, &c, &c. He reduced into regularity and order all the fundamental principles of pure mathematics, which had been delivered down by Thales, Pythagoras, Eudoxus, and other mathematicians before him, and added many others of his own diſcovering: on which account it is ſaid he was the firſt who reduced arithmetic and geometry into the form of a ſcience. He likewiſe applied himſelf to the ſtudy of mixed mathematics, particularly to aſtronomy and optics.

His works, as we learn from Pappus and Proclus, are the *Elements*, *Data*, *Introduction to Harmony*, *Phenomena*, *Optics*, *Catoprics*, *a Treatiſe of the Diviſion of Superficies*, *Poriſms*, *Loci ad Superficiem*, *Fallacies*, *and Four books of Conics*. The moſt celebrated of theſe, is the firſt work, the Elements of Geometry; of which there have been numberleſs editions, in all languages; and a fine edition of all his works, now extant, was printed in 1703, by David Gregory, Savilian profeſſor of aſtronomy at Oxford.

The Elements, as commonly publiſhed, conſiſt of 15 books, of which the two laſt it is ſuſpected are not Euclid's, but a comment of Hypſicles of Alexandria, who lived 200 years after Euclid. They are divided into three parts, viz, the Contemplation of Superficies, Numbers, and Solids: The firſt 4 books treat of planes only; the 5th of the proportions of magnitudes in general; the 6th of the proportion of plane figures; the 7th, 8th, and 9th give us the fundamental properties of numbers; the 10th contains the theory of commenſurable and incommenſurable lines and ſpaces; the 11th, 12th, 13th, 14th, and 15th, treat of the doctrine of ſolids.

There is no doubt but, before Euclid, Elements of Geometry were compiled by Hippocrates of Chius, Eudoxus, Leon, and many others, mentioned by Proclus, in the beginning of his ſecond book; for he affirms that Euclid new ordered many things in the Elements of Eudoxus, completed many things in thoſe of Theatetus, and beſides ſtrengthened ſuch propoſitions as before were too ſlightly, or but ſuperficially eſtabliſhed, with the moſt firm and convincing demonſtrations.

Hiſtory is ſilent as to the time of Euclid's death, or his age. He is repreſented as a perſon of a courteous and agreeable behaviour, and in great eſteem and familiarity with king Ptolomy; who once aſking him, whether there was any ſhorter way of coming at geometry than by his Elements, Euclid, as Proclus teſtifies, made anſwer, that there was no royal way or path to geometry.

EUDIOMETER, an inſtrument for determining the purity of the air, or the quantity of pure and dephlogiſticated or vital air contained in it, chiefly by means of its diminution on a mixture with nitrous air.

Inſtruments of this kind have been but lately made, and that in conſequence of the experiments and diſcoveries of Dr. Prieſtley, for determining the ſalubrity of different kinds of air. That writer having diſcovered, that when nitrous air is mixed with any other air, their original bulk is diminiſhed; and that the diminution is nearly, if not exactly, in proportion to its ſalubrity; he was hence led to adopt nitrous air as a true teſt of the purity of reſpirable air; and nothing more ſeemed to be neceſſary but an eaſy, expeditious, and accurate method of eſtimating the degree of diminution in different caſes; and for this purpoſe, the Eudiometer was contrived; of which ſeveral kinds have been invented, the principal of which are the following.

I. The Eudiometer originally uſed by Dr. Prieſtley is a divided glaſs tube, into which, after having filled it with common water, and inverted it into the ſame, one meaſure or more of common air, and an equal quantity of nitrous air, are introduced by means of a ſmall phial, which is called the meaſure; and thus the diminution of the volume of the mixture, which is ſeen at once by means of the graduations of the tube, inſtantly diſcovers the purity of the air required.

II. The diſcovery of Dr. Prieſtley was announced to the public in the year 1772; and ſeveral perſons, both at home and abroad, preſently availed themſelves of it, by framing other more accurate inſtruments. The firſt of theſe was contrived by M. Landriani; an account of which is publiſhed in the 6th volume of Roſier's Journal, for the year 1775. It conſiſts of a glaſs tube, fitted by grinding to a cylindrical veſſel, to which are joined two glaſs cocks and a ſmall baſon; the whole being fitted to a wooden frame. In this inſtrument quickſilver is uſed inſtead of water; though that is attended with an inconvenience, becauſe the nitrous air acts upon the metal, and renders the experiment ambiguous.

III. In 1777, Mr. Magellan publiſhed an account of three Eudiometers invented by himſelf, conſiſting of glaſs veſſels of rather difficult conſtruction, and troubleſome uſe. Mr. Cavallo obſerves, that the conſtruction of all the three is founded on a ſuppoſition, that the mixture of nitrous and atmoſpherical air, having continued for ſome time to diminiſh, afterwards increaſes again; which it ſeems is a miſtake: neither do they give accurate or uniform reſults in any two experiments made with nitrous and common air of preciſely the ſame quality.

IV. A preferable method of diſcovering the purity of the air by means of an Eudiometer, is recommended by M. Fontana, of very great accuracy. The inſtrument is originally nothing more than a divided glaſs tube, though the inventor afterwards added to it a complicated apparatus, perhaps of little or no uſe. The firſt ſimple Eudiometer conſiſted only of a glaſs tube, uniformly cylindrical, about 18 inches long, and 3-4ths of an inch diameter within ſide, the outſide being marked with a diamond at ſuch diſtances as are exactly filled by equal meaſures of elaſtic fluids: and when any parts of theſe diviſions are required, the edge of a ruler, divided into inches and ſmaller parts, is held againſt the tube, ſo as that the firſt diviſion of the ruler may coincide with one of the marks on the tube. The nitrous and atmoſpherical air are introduced into this tube, in order to be diminiſhed, and thence the purity of the atmoſpheric air aſcertained.

V. M. Sauſſure of Geneva has alſo invented an Eudiometer, which he thinks is more exact than any of thoſe before deſcribed; the apparatus of which is as follows: 1. A cylindrical glaſs bottle, with a ground ſtopple, containing about $5\frac{1}{2}$ ounces, which ſerves as a receiver for mixing the two airs.—2. A ſmall glaſs phial, to ſerve as a meaſure, and is about one-third the ſize of the receiver.—3. A ſmall pair of ſcales that may weigh very exactly.—4. Several glaſs bottles, for containing the nitrous or other air to be uſed, and which may ſupply the place of the recipient when broken. The method of uſing it is as follows: The receiver is to be filled with water, cloſed exactly with its glaſs ſtopper, wiped dry on the outſide, and then weighed very nicely. Being then immerged in a veſſel of water, and held with the mouth downwards, the ſtopple is removed, and, by means of a funnel, two meaſures of common and one of nitrous air are introduced into it, one after another: theſe diminiſh as ſoon as they come into contact; in conſequence of which the water enters the recipient in proportionable quantity. After being ſtopped and well ſhaken, to promote the diminution, the receiver is to be opened again under water; then ſtopped and ſhaken again, and ſo on for three times ſucceſſively, after which the bottle is ſtopped for the laſt time under water, then taken out, wiped very clean and dry, and exactly weighed as before. It is plain that now, the bottle being filled partly with elaſtic fluid and partly with water, it muſt be lighter than when quite full of water; and the difference between thoſe two weights, ſhows nearly what quantity of water would fill the ſpace occupied by the diminiſhed elaſtic fluid. Now, in making experiments with airs of different degrees of purity, the ſaid difference will be greater when the diminution is leſs, or when the air is leſs pure, and vice verſa; by which means the comparative purity between two different kinds of air is determined.

VI. But as this method, notwithſtanding the encomiums beſtowed on it by the inventor, is ſubject to ſeveral errors and inconveniences; to remedy all theſe, another inſtrument was invented by Mr. Cavallo; the deſcription of which, being long, may be ſeen in his Treatiſe on the Nature and Properties of Air, pa. 344.

Other conſtructions of the Eudiometer have alſo been given by Mr. Cavendiſh and Mr. Scheele. For farther information, ſee Magellan's Letter to Dr. Prieſtley, containing the Deſcription of a Glaſs Apparatus, &c, and of New Eudiometers &c, 1777, pa. 15 &c; Prieſtley's Exp. and Obſ. on Air, vol. 3, preface and appendix; the methods of Dr. Ingenhouſz in Philoſ. Tranſ. vol. 66, art. 15; ſee alſo the Philoſ. Tranſ. vol. 73; and Cavallo's Treatiſe on Air, pa. 274, 315, 316, 317, 328, 340, 344, and 834.

EUDOXUS, of Cnidus, a city of Caria in Aſia Minor, flouriſhed about 370 years before Chriſt. He learned geometry from Archytas, and afterwards travelled into Egypt to learn aſtronomy and other ſciences. There he and Plato ſtudied together, as Laertius informs us, for the ſpace of 13 years; and afterwards came to Athens, fraught with all ſorts of knowledge, which they had imbibed from the mouths of the prieſts. Here Eudoxus opened a ſchool; which he ſupported with ſo much glory and renown, that even Plato, though his friend, is ſaid to have envied him. Eudoxus compoſed Elements of Geometry, from whence Euclid liberally borrowed, as mentioned by Proclus. Cicero calls Eudoxus the greateſt aſtronomer that had ever lived: and Petronius ſays, he ſpent the latter part of his life upon the top of a very high mountain, that he might contemplate the ſtars and the heavens with more convenience and leſs interruption: and we learn from Strabo, that there were ſome remains of his obſervatory at Cnidus, to be ſeen even in his time. He died in the 53d year of his age.

EVECTION, is uſed by ſome aſtronomers for the Libration of the moon; being an inequality in her motion, by which, at or near the quadratures, ſhe is not in a line drawn through the centre of the earth to the ſun, as ſhe is at the ſyzygies, or conjunction and oppoſition, but makes an angle with that line of about 2° 51′. The motion of the moon about her axis only is equable, which rotation is performed exactly in the ſame time as ſhe revolves about the earth; for which reaſon it is that ſhe turns always the ſame face towards the earth nearly, and would do ſo exactly were it not that her menſtrual motion about the earth, in an elliptic orbit, is not equable; on which account the moon, ſeen from the earth, appears to librate a little upon her axis, ſometimes from eaſt to weſt, and ſometimes from weſt to eaſt; or ſome parts in the eaſtern limb of the moon go backwards and forwards a ſmall ſpace, and ſome that were conſpicuous, are hid, and then appear again.

The term Evection is uſed by ſome aſtronomers to denote that equation of the moon's motion, which is proportional to the ſine of double the diſtance of the moon from the ſun, diminiſhed by the moon's anomaly: this equation is not yet accurately determined; ſome ſtate it at 1° 30′, others at 1° 16′, &c. It is the greateſt of all the moon's equations, except the Equation of the Centre.

EVEN *Number*, is that which can be divided into two equal whole numbers; ſuch as the ſeries of alternate numbers 2, 4, 6, 8, 10, &c.

EVENLY *Even Number*, is that which an even number meaſures by an even number; as 16, which the even number 8 meaſures by the even number 2.

Evenly *Odd Number*, is that which an even number meaſures by an odd one; as 30, which the even number 6 meaſures by the odd number 5.

EVERARD's *Sliding Rule*, a particular ſort of one invented by Mr. Thomas Everard, for the purpoſe of gauging. See Sliding Rule.

EULER (Leonard), one of the moſt extraordinary, and even prodigious, mathematical geniuſes, that the world ever produced. He was a native of Baſil, and was born April 15, 1707. The years of his infancy were paſſed at Richen, where his father was miniſter. He was afterwards ſent to the univerſity of Baſil; and as his memory was prodigious, and his application regular, he performed his academical taſks with great rapidity; and all the time that he ſaved by this, was conſecrated to the ſtudy of mathematics, which ſoon became his favourite ſcience. The early progreſs he made in this ſtudy, added freſh ardour to his application; by which too he obtained a diſtinguiſhed place in the attention and eſteem of profeſſor John Bernoulli, who was then one of the chief mathematicians in Europe.

In 1723, M. Euler took his degree as maſter of arts; and delivered on that occaſion a Latin diſcourſe, in which he drew a compariſon between the philoſophy of Newton and the Carteſian ſyſtem, which was received with the greateſt applauſe. At his father's deſire, he next applied himſelf to the ſtudy of theology and the oriental languages: and though theſe ſtudies were foreign to his predominant propenſity, his ſucceſs was conſiderable even in this line: however, with his father's conſent, he afterward returned to mathematics as his principal object. In continuing to avail himſelf of the counſels and inſtructions of M. Bernoulli, he contracted an intimate friendſhip with his two ſons Nicholas and Daniel; and it was chiefly in conſequence of theſe connections that he afterwards became the principal ornament of the philoſophical world.

The project of erecting an academy at Peterſburg, which had been formed by Peter the Great, was executed by Catharine the 1ſt; and the two young Bernoullis being invited to Peterſburg in 1725, promiſed Euler, who was deſirous of following them, that they would uſe their endeavours to procure for him an advantageous ſettlement in that city. In the mean time, by their advice, he made cloſe application to the ſtudy of philoſophy, to which he made happy applications of his mathematical knowledge, in a diſſertation on the nature and propagation of ſound, and an anſwer to a prize queſtion concerning the maſting of ſhips; to which the Academy of Sciences adjudged the *acceſſit*, or ſecond rank, in the year 1727. From this latter diſcourſe, and other circumſtances, it appears that Euler had very early embarked in the curious and uſeful ſtudy of naval architecture, which he afterward enriched with ſo many valuable diſcoveries. The ſtudy of mathematics and philoſophy however did not ſolely engage his attention, as he in the mean time attended the medical and botanical lectures of the profeſſors at Baſil.

Euler's merit would have given him an eaſy admiſſion to honourable preferment either in the magiſtracy or univerſity of his native city, if both civil and academical honours had not been there diſtributed by lot. The lot being againſt him in a certain promotion, he left his country, ſet out for Peterſburg, and was made joint profeſſor with his countrymen Hermann and Daniel Bernoulli in the univerſity of that city.

At his firſt ſetting out in his new career, he enriched the academical collection with many memoirs, which excited a noble emulation between him and the Bernoullis; an emulation that always continued, without either degenerating into a ſelfiſh jealouſy, or producing the leaſt alteration in their friendſhip. It was at this time that he carried to new degrees of perfection the integral calculus, invented the calculation by ſines, reduced analytical operations to a greater ſimplicity, and thus was enabled to throw new light on all the parts of mathematical ſcience.

In 1730, M. Euler was promoted to the profeſſorſhip of natural philoſophy; and in 1733 he ſucceeded his friend D. Bernoulli in the mathematical chair. In 1735, a problem was propoſed by the academy, which required expedition, and for the calculation of which ſome eminent mathematicians had demanded the ſpace of ſome months. The problem was undertaken by Euler, who completed the calculation in three days, to the great aſtoniſhment of the academy: but the violent and laborious efforts it coſt him threw him into a fever, which endangered his life, and deprived him of the uſe of his right eye, which afterward brought on a total blindneſs.

The Academy of Sciences at Paris, which in 1738 had adjudged the prize to his memoir Concerning the Nature and Properties of Fire, propoſed for the year 1740 the important ſubject of the Tides of the Sea; a problem whoſe ſolution comprehended the theory of the ſolar ſyſtem, and required the moſt arduous calculations. Euler's ſolution of this queſtion was adjudged a maſter-piece of analyſis and geometry; and it was more honourable for him to ſhare the academical prize with ſuch illuſtrious competitors as Colin Maclaurin and Daniel Bernoulli, than to have carried it away from rivals of leſs magnitude. Seldom, if ever, did ſuch a brilliant competition adorn the annals of the academy; and perhaps no ſubject, propoſed by that learned body, was ever treated with ſuch force of genius and accuracy of inveſtigation, as that which here diſplayed the philoſophical powers of this extraordinary triumvirate.

In the year 1741, M. Euler was invited to Berlin to direct and aſſiſt the academy that was there riſing into fame. On this occaſion he enriched the laſt volume of the Miſcellanies *(Melanges)* of Berlin with five memoirs, which form an eminent, perhaps the principal, figure in that collection. Theſe were followed, with amazing rapidity, by a great number of important reſearches, which are diſperſed through the memoirs of the Pruſſian academy; a volume of which has been regularly publiſhed every year ſince its eſtabliſhment in 1744. The labours of Euler will appear more eſpecially aſtoniſhing, when it is conſidered, that while he was enriching the academy of Berlin with a profuſion of memoirs, on the deepeſt parts of mathematical ſcience, containing always ſome new points of view, often ſublime truths, and ſometimes diſcoveries of great importance; he ſtill continued his philoſophical contributions to the Peterſburg academy, whoſe memoirs diſplay the marvellous fecundity of his genius, and which granted him a penſion in 1742.

It was with great difficulty that this extraordinary man, in 1766, obtained permiſſion from the king of Pruſſia to return to Peterſburg, where he wiſhed to paſs the remainder of his days. Soon after his return, which was graciouſly rewarded by the munificence of Catharine the 2d, he was ſeized with a violent diſorder, which ended in the total loſs of his ſight. A cataract, formed in his left eye, which had been eſſentially damaged by the loſs of the other eye, and a too cloſe application to ſtudy, deprived him entirely of the uſe of that organ. It was in this diſtreſſing ſituation that he dictated to his ſervant, a taylor's apprentice, who was abſolutely devoid of mathematical knowledge, his Elements of Algebra; which by their intrinſic merit in point of perſpicuity and method, and the unhappy circumſtances in which they were compoſed, have equally excited wonder and applauſe. This work, though purely elementary, plainly diſcovers the proofs of an inventive genius; and it is perhaps here alone that we meet with a complete theory of the analyſis of Diophantus.

About this time M. Euler was honoured by the Academy of Sciences at Paris with the place of one of the foreign members of that learned body; after which, the academical prize was adjudged to three of his memoirs, *Concerning the Inequalities in the Motions of the Planets.* The two prize queſtions propoſed by the ſame Academy for 1770 and 1772 were deſigned to obtain from the labours of aſtronomers *a more perfect Theory of the Moon.* M. Euler, aſſiſted by his eldeſt ſon, was a competitor for theſe prizes, and obtained them both. In this laſt memoir, he reſerved for farther conſideration ſeveral inequalities of the moon's motion, which he could not determine in his firſt theory, on account of the complicated calculations in which the method he then employed had engaged him. He afterward reviſed his whole theory, with the aſſiſtance of his ſon and Meſſrs Krafft and Lexell, and purſued his reſearches till he had conſtructed the new tables, which appeared, together with the great work, in 1772. Inſtead of confining himſelf, as before, to the fruitleſs integration of three differential equations of the ſecond degree, which are furniſhed by mathematical principles, he reduced them to the three ordinates, which determine the place of the moon: he divided into claſſes all the inequalities of that planet, as far as they depend either on the elongation of the ſun and moon, or upon the eccentricity, or the parallax, or the inclination of the lunar orbit. All theſe means of inveſtigation, employed with ſuch art and dexterity as could only be expected from a genius of the firſt order, were attended with the greateſt ſucceſs; and it is impoſſible to obſerve without admiration, ſuch immenſe calculations on the one hand, and on the other the ingenious methods employed by this great man to abridge them, and to facilitate their application to the real motion of the moon. But this admiration will become aſtoniſhment, when we conſider at what period and in what circumſtances all this was effectuated. It was when our author was totally blind, and conſequently obliged to arrange all his computations by the ſole powers of his memory and his genius: it was when he was embarraſſed in his domeſtic affairs by a dreadful fire, that had conſumed great part of his ſubſtance, and forced him to quit a ruined houſe, every corner of which was known to him by habit, which in ſome meaſure ſupplied the want of ſight. It was in theſe circumſtances that Euler compoſed a work which alone was ſufficient to render his name immortal.

Some time after this, the famous oculiſt Wentzell, by couching the cataract, reſtored ſight to our author; but the joy produced by this operation was of ſhort duration. Some inſtances of negligence on the part of his ſurgeons, and his own impatience to uſe an organ, whoſe cure was not completely finiſhed, deprived him a ſecond time and for ever of his ſight: a relapſe which was alſo accompanied with tormenting pain. With the aſſiſtance of his ſons, however, and of Meſſrs Krafft and Lexell, he continued his labours; neither the infirmities of old age, nor the loſs of his ſight, could quell the ardour of his genius. He had engaged to furniſh the academy of Peterſburg with as many memoirs as would be ſufficient to complete its acts for 20 years after his death. In the ſpace of 7 years he tranſmitted to the Academy above 70 memoirs, and above 200 more, left behind him, were reviſed and completed by a friend. Such of theſe memoirs as were of ancient date were ſeparated from the reſt, and form a collection that was publiſhed in the year 1783, under the title of *Analytical Works*.

The general knowledge of our author was more extenſive than could well be expected in one who had purſued, with ſuch unremitting ardour, mathematics and aſtronomy as his favourite ſtudies. He had made a very conſiderable progreſs in medical, botanical, and chemical ſcience. What was ſtill more extraordinary, he was an excellent ſcholar, and poſſeſſed in a high degree what is generally called *erudition*. He had attentively read the moſt eminent writers of ancient Rome; the civil and literary hiſtory of all ages and all nations was familiar to him; and foreigners, who were only acquainted with his works, were aſtoniſhed to find in the converſation of a man, whoſe long life ſeemed ſolely occupied in mathematical and phyſical reſearches and diſcoveries, ſuch an extenſive acquaintance with the moſt intereſting branches of literature. In this reſpect, no doubt, he was much indebted to a very uncommon memory, which ſeemed to retain every idea that was conveyed to it, either from reading or from meditation. He could repeat the Æneid of Virgil, from the beginning to the end, without heſitation, and indicate the firſt and laſt line of every page of the edition he uſed.

Several attacks of a vertigo, in the beginning of September 1783, which did not prevent his computing the motions of the aeroſtatic globes, were however the forerunners of his mild paſſage out of this life. While he was amuſing himſelf at tea with one of his grandchildren, he was ſtruck with an apoplexy, which terminated his illuſtrious career at 76 years of age.

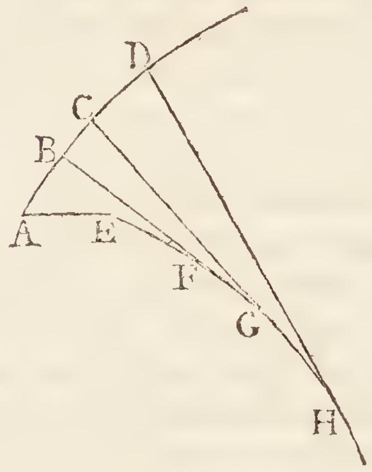
M. Euler's conſtitution was uncommonly ſtrong and vigorous. His health was good; and the evening of his long life was calm and ſerene, ſweetened by the fame that follows genius, the public eſteem and reſpect that are never withheld from exemplary virtue, and ſeveral domeſtic comforts which he was capable of feeling, and therefore deſerved to enjoy.

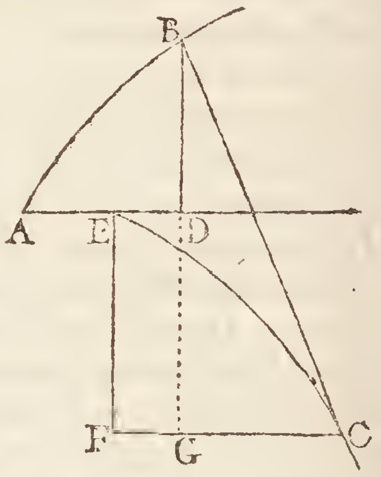
The catalogue of his works has been printed in 50 pages, 14 of which contain the manuſcript works.—The printed ones conſiſt of works publiſhed ſeparately, and works to be found in the memoirs of ſeveral Academies, viz, in 38 volumes of the Peterſburg Acts, (from 6 to 10 papers in each volume);—in ſeveral volumes of the Paris Acts;—in 26 volumes of the Berlin Acts, (about 5 papers to each volume);—in the Acta Eruditorum, in 2 volumes;—in the Miſcellanea Taurinenſia;—in vol. 9 of the Society of Ulyſſingue;—in the Ephemerides of Berlin;—and in the Memoires de la Société Oeconomique for 1766.

EVOLVENT, in the Higher Geometry, a term uſed by ſome writers for the Involute, or curve reſulting from the evolution of a curve, in contradiſtinction to that evolute, or curve ſuppoſed to be opened or evolved. See EVOLUTE, and INVOLUTE.

EVOLUTE, in the Higher Geometry, a curve firſt propoſed by M. Huygens, and ſince much ſtudied by the later mathematicians. It is any curve ſuppoſed to be evolved or opened, by having a thread wrapped cloſe upon it, faſtened at one end, and beginning to evolve or unwind the thread from the other end, keeping the part evolved, or wound off, tight ſtretched; then this end of the thread will deſcribe another curve called the Involute. Or the ſame involute is deſcribed the contrary way, by wrapping the thread upon the Evolute, keeping it always ſtretched.

Thus, if EFGH be any curve, and AE either a part of the curve, or a right line; then if a thread be wound cloſe upon the curve from A to H, where it is fixed, and then be unwound from A; the curve AEFGH, from which it is evolved, is called the Evolute; and the other curve ABCD deſcribed by the end of the thread, as it evolves or unwinds, is the Involute. Or, if the thread HD, fixed at H, be wound or wrapped upon the Evolute HGFEA, keeping it always tight, as at the ſeveral poſitions of it HD, GC, FB, EA, the extremity will deſcribe the Involute curve DCBA.





From this deſcription it appears, 1. That the parts of the thread at any poſitions, as EA, FB, GC, HD, &c, are radii of curvature, or oſculatory radii, of the involute curve, at the points A, B, C, D.

2. The ſame parts of the thread are alſo equal to the correſponding lengths AE, AEF, AEFG, &c, of the Evolute; that is,

AE = AE is the rad. of curvature to the point A,
BF = AF - - - - - B,
CG = AG - - - - - C,
DH = AH - - - - - D.

3. Any radius of curvature BF, is perpendicular to the involute at the point B, and is a tangent to the Evolute curve at the point F.

4. The Evolute is the locus of the centre of curvature of the involute curve.

The finding the radii of Evolutes, is a matter of great importance in the higher ſpeculations of geometry; and is even ſometimes uſeful in practice; as is ſhewn by Huygens, the inventor of this theory, in applying it to the pendulum. Horol. Oſcil. part 3. The doctrine of the Oſcula of Evolutes is owing to M. Leibnitz, who firſt ſhewed the uſe of Evolutes in the meaſuring of curvatures.

To find the EVOLUTE *and Involute Curves*, the one from the other.

For this purpoſe, put

x = AD the abſcis of the involute,

y = DB its ordinate,

z = AB the involute curve,

r = BC its radius of curvature,

v = EF the abſcis of the Evolute,

u = FC its ordinate, and

a = AE a given line, (fig. 2 above).

Then, by the nature of the radius of curvature, it is

$$r = \frac{\dot{z}^3}{\dot{y}\ddot{x} - \dot{x}\ddot{y}} = \text{BC} = \text{AE} + \text{EC}\text{; alſo by ſim. triangles,}$$

$$\dot{z} : \dot{x} :: r : \text{GB} = \frac{r\dot{x}}{\dot{z}} = \frac{\dot{x}\dot{z}^2}{\dot{y}\ddot{x} - \dot{x}\ddot{y}},$$

$$\dot{z} : \dot{y} :: r : \text{GC} = \frac{r\dot{y}}{\dot{z}} = \frac{\dot{y}\dot{z}^2}{\dot{y}\ddot{x} - \dot{x}\ddot{y}}.$$

Hence $\text{EF} = \text{GB} - \text{DB} = \dfrac{\dot{x}\dot{z}^2}{\dot{y}\ddot{x} - \dot{x}\ddot{y}} - y = v$; and

$$\text{FC} = \text{AD} - \text{AE} + \text{GC} = x - a + \frac{\dot{y}\dot{z}^2}{\dot{y}\ddot{x} - \dot{x}\ddot{y}} = u;$$

which are the values of the abſciſs and ordinate of the Evolute curve EC; and therefore theſe may be found when the involute is given.

On the other hand, if v and u, or the Evolute be given: then, putting the given curve EC = s; ſince CB = AE + EC, or $r = a + s$, this gives r the radius of curvature. Alſo, by ſimilar triangles, there reſult theſe proportions, viz,

$$\dot{s} : \dot{v} :: r : \frac{r\dot{v}}{\dot{s}} = \frac{a+s}{\dot{s}}\dot{v} = \text{GB},$$

$$\dot{s} : \dot{u} :: r : \frac{r\dot{u}}{\dot{s}} = \frac{a+s}{\dot{s}}\dot{u} = \text{GC}\text{; theref.}$$

$$\text{AD} = \text{AE} + \text{FC} - \text{GC} = a + u - \frac{a+s}{\dot{s}}\dot{u} = x,$$

$$\text{and DB} = \text{GB} - \text{EF} = \frac{a+s}{\dot{s}}\dot{v} - v = y;$$

which are the abſciſs and ordinate of the involute curve, and which may therefore be found when the Evolute is given. Where it may be noted that $\dot{s}^2 = \dot{v}^2 + \dot{u}^2$, and $\dot{z}^2 = \dot{x}^2 + \dot{y}^2$. Alſo either of the quantities x, y, may be ſuppoſed to flow equably, in which caſe the reſpective ſecond fluxion $\ddot{x}$ or $\ddot{y}$ will be nothing, and the correſponding term in the denominator $\dot{y}\ddot{x} - \dot{x}\ddot{y}$ will vaniſh, leaving only the other term in it; which will have the effect of rendering the whole operation ſimpler.

For Ex. Suppoſe it were required to find the Evolution EC when the given involute AB is the common parabola, whoſe equation is $px = y^2$, the parameter being p.

Here $y = \sqrt{px}$, $\dot{y} = \frac{1}{2}p^{\frac{1}{2}}x^{-\frac{1}{2}}\dot{x}$, and $\ddot{y} = -\frac{1}{4}p^{\frac{1}{2}}x^{-\frac{3}{2}}\dot{x}$, making $\ddot{x} = 0$. Then, to find firſt AE the radius of curvature of the parabola AB at the vertex, when $\ddot{x} = 0$, the general value of the radius of curvature above given becomes $r = \dfrac{\dot{z}^3}{-\dot{x}\ddot{y}} = \dfrac{\overline{\dot{x}^2 + \dot{y}^2}|^{\frac{3}{2}}}{-\dot{x}\ddot{y}}$ = (by ſubſtituting the value of $\dot{y}$ and $\ddot{y}$ &c) $\dfrac{\overline{p + 4x}|^{\frac{3}{2}}}{2\sqrt{p}}$ which is the general value of r or BC, the radius of curvature, for any value of x or AD; and when x or AD is = 0 or nothing, the value of r, or AE, becomes then

$a = \dfrac{p^{\frac{3}{2}}}{2\sqrt{p}} = \frac{1}{2}p$ only; that is half the parameter of the axis is the radius of curvature at the vertex of the parabola.

Again, in the general values of v and u above given, by ſubſtituting the values of $\dot{y}$, $\ddot{y}$, and $\dot{z}$, alſo 0 for $\ddot{x}$, and $\frac{1}{2}p$ for a; thoſe quantities become

$$v = \frac{\dot{z}^2}{-\ddot{y}} - y = \frac{\dot{x}^2 + \dot{y}^2}{-\ddot{y}} - y = \frac{4x^{\frac{3}{2}}}{p^{\frac{1}{2}}} = 4x\sqrt{\frac{x}{p}}\text{; and}$$

$$u = x - a + \frac{\dot{y}\dot{z}^2}{-\dot{x}\ddot{y}} = 3x + \tfrac{1}{2}p - a = 3x.$$

Hence then, comparing the values of v and u, there is found $3p^{\frac{1}{2}}v = 4x^{\frac{1}{2}}u$, and $27pv^2 = 16u^3$; which is the equation between the abſciſs and ordinate of the Evolute curve EC, ſhewing it to be the ſemicubical parabola.

In like manner the Evolute to any other curve is found.—The Evolute to the common cycloid, is an equal cycloid; a property firſt demonſtrated by Huygens, and which he uſed as a contrivance to make a pendulum vibrate in the curve of a cycloid. See his Horolog. Oſcil. See alſo, on the ſubject of Evolute and Involute Curves, the Fluxions of Newton, Maclaurin, Simpſon, De l'Hôpital, &c, Wolf. Elem. Math. tom. 1, &c, &c.

M. Varignon has applied the doctrine of the radius of the Evolute, to that of central forces; ſo that having the radius of the Evolute of any curve, there may be found the value of the central force of a body; which, moving in that curve, is found in the ſame point where that radius terminates; or reciprocally, having the central force given, the radius of the Evolute may be determined. Hiſt. de l'Acad. an. 1706.

The variation of curvature of the line deſcribed by the Evolution of a curve, is meaſured by the ratio of the radius of curvature of the Evolute, to the radius of curvature of the line deſcribed by the Evolution. See Maclaurin's Flux. art. 402, prop. 36.

Imperfect EVOLUTE, a name given by M. Reaumur to a new kind of Evolute. The mathematicians had hitherto only conſidered the perpendiculars let fall from the Involute on the convex ſide of the Evolute: but if other lines not perpendicular be drawn upon the ſame points, provided they be all drawn under the ſame angle, the effect will ſtill be the ſame; that is, the oblique lines will all interſect in the curve, and by their interſections form the infinitely ſmall ſides of a new curve, to which they would be ſo many tangents.—Such a curve is a kind of Evolute, and has its radii; but it is an Imperfect one, ſince the radii are not perpendicular to the firſt curve, or Involute. Hiſt. de l'Acad. &c, an. 1709.

EVOLUTION, in Arithmetic and Algebra, denotes the Extraction of the roots out of powers. In which ſenſe it ſtands oppoſed to Involution, which is the raiſing of powers. The note or character that has been uſed by ſome Algebraiſts, to denote Evolution, is *ɯ*; as the ſign of involution is : characters I think firſt uſed by Dr. Pell.

Evolution, in Geometry, the opening, or unfolding of a curve, and making it deſcribe an Evolvent.

The equable Evolution of the periphery of a circle, or other curve, is ſuch a gradual approach of the circumference to rectitude, as that its parts do all concur, and equally evolve or unbend; ſo that the ſame line becomes ſucceſſively a leſs arc of a reciprocally greater circle; till at laſt they change into a ſtraight line.—In the Philoſ. Tranſ. N° 260, a new quadratix to the circle is found by this means, being the curve deſcribed by the equable Evolution of its periphery.

EURYTHMY, in Architecture, Painting, and Sculpture, is a kind of majeſty, elegance, and eaſineſs appearing in the compoſition of certain members or parts of a body, building, or painting, and reſulting from the fine and exact proportions of them.

EUSTYLE, is the beſt manner of placing columns, with regard to their diſtance; which, according to Vitruvius, ſhould be four modules, or two diameters and a quarter.

EXAGON. See Hexagon.

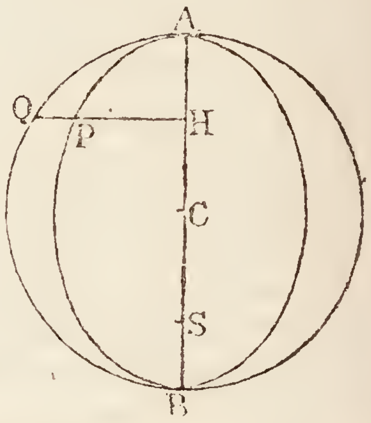
EXALTATION, in Aſtrology, is a dignity which a planet acquires in certain ſigns of the zodiac; which dignity, it is ſuppoſed, gives the planet an extraordinary virtue, efficacy, and influence. The oppoſite ſide of the zodiac is called the Dejection of the planet.——Thus, the 15th degree of Cancer is the Exaltation of Jupiter, according to Albumazar, becauſe it was the aſcendant of that planet at the time of the creation; that of the ſun is in the 19th degree of Aries; and its dejection in Libra; that of the moon is in Taurus, &c. Ptolomy gives the reaſon of this in his firſt book De Quadripartita.

EXCENTRIC, is applied to ſuch figures, circles, ſpheres, &c, as have not the ſame centre; as oppoſed to Concentric, which have the ſame centre.

Excentric, or *Excentric Circle*, in the ancient Ptolomaic aſtronomy, was the very orbit of the planet itſelf, which it was ſuppoſed to deſcribe about the earth, and which was conceived Excentric with it; called alſo the Deferent.

Inſtead of theſe Excentric Circles round the earth, the moderns make the planets deſcribe elliptic orbits about the ſun; which accounts for all the irregularities of their motions, and their various diſtances from the earth, &c, more juſtly and naturally.

Excentric, or *Excentric Circle*, in the New Aſtronomy, is the circle deſcribed from the centre of the orbit of a planet, with half the greateſt axis as a radius; or it is the circle that circumſcribes the elliptic orbit of the planet; as the circle AQB.



Excentric *Anomaly*, or *Anomaly of the Centre*, is an arc AQ of the Excentric circle, intercepted between the aphelion A, and the right line QH, drawn through the centre P of the planet perpendicular to the line of the apſes AB.

Excentric *Equation*, in the Old Aſtronomy, is an angle made by a line drawn from the centre of the earth, with another line drawn from the centre of the Excentric, to the body or place of any planet. This is the ſame with the proſthapheréſis; and is equal to the difference, accounted in an arch of the ecliptic, between the real and apparent place of the ſun or planet. See Equation *of the Centre*.

Excentric *Place of a planet*, in its orbit, is the Heliocentric place, or that in which it appears as ſeen from the ſun.

Excentric *Place in the ecliptic*, is the point of the ecliptic to which the planet is referred as viewed from the ſun; and which coincides with the heliocentric longitude.

EXCENTRICITY, is the diſtance between the centres of two circles, or ſpheres, which have not the ſame centre.

Excentricity, *in the Old Aſtronomy*, is the diſtance between the centre of a planet and the centre of the earth.—That the planets have ſuch an Excentricity, is allowed on all ſides, and may be evinced from various circumſtances; and eſpecially this, that the planets at ſome times appear larger, and at others leſs; which can only proceed from hence, that their orbits being Excentric to the earth, in ſome parts of thoſe orbits the planets are nearer to us, and in others more remote. And as to the Excentricities of the ſun and moon, it is thought they are ſufficiently proved, both from eclipſes, from the moon's greater and leſs parallax at the ſame diſtance from the zenith, and from the ſun's continuing longer by 8 days in the northern hemiſphere than in the ſouthern one.

Excentricity, *in the New Aſtronomy*, is the diſtance CS between the ſun S and the centre C of a planet's orbit; or the diſtance of the centre from the focus of the elliptic orbit; called alſo the *Simple* or *Single Excentricity*.

When the greateſt Equation of the centre is given, the Excentricity of the earth's orbit may be found by the following proportion; viz,

As the diameter of a circle in degrees,
Is to the diameter in equal parts;
So the greateſt equat. of the centre in degrees,
To the Excentricity in equal parts. Thus,

Greateſt equat. of the cent. $1^\circ\,55'\,33'' = 1^\circ{\cdot}9258333$ &c.
The diam. of a circ. being 1, its circumf. is 3·1415926.
Then 3·1415926 : 1 : : 360° : 114°·5915609 diam. in deg.
And 114·5915609 : 1 : : 1·9258333 : 0·016806, the Ex

Hence, by adding this to 1, and ſubtracting it from 1, gives $1\cdot016806 = AS$ the aphelion diſtance,
and $0\cdot983194 = BS$ the perihelion diſtance.
See Robertſon's Elem. of Navig. book 5, pa. 286.

Otherwiſe, thus: Since it is found that the ſun's greateſt apparent ſemi-diameter is to his leaſt, as $32' \, 43''$ to $31' \, 38''$, or as $1963''$ to $1898''$; the ſun's greateſt diſtance from the earth will be to his leaſt, or AS to SB, as 1963 to 1898; of which,
the half dif. is $32\frac{1}{2} = CS$,
and half ſum $1930\frac{1}{2} = CB$; wherefore,
as $1930\frac{1}{2} : 32\frac{1}{2} :: 1 : \cdot016835 = CS$ the Excentricity, to the mean diſtance or ſemi-axis 1; which is nearly the ſame as before.

The Excentricities of the orbits of the ſeveral planets, in parts of their own mean diſtances 1000, and alſo in Engliſh miles, are as below, viz, the Excentricity of the orbit of

	Parts.	Miles.
Mercury - -	210 - -	7,730,000
Venus - -	7 - -	482,000
Earth - -	17 - -	1,618,000
Mars - -	93 - -	13,486,000
Jupiter - -	48 - -	23,760,000
Saturn - -	55 - -	49,940,000
Georgian - -	$47\frac{1}{2}$ - -	86,000,000

Double EXCENTRICITY, is the diſtance between the two foci of the elliptic orbit, and is equal to double the Single Excentricity above given.

EXCHANGE, in Arithmetic, is the bartering or exchanging the money of one place for that of another; or the finding what quantity of the money of one place is equal to a given ſum of another, according to a given courſe of exchange.

The ſeveral operations in this caſe are only different applications of the Rule of Three. See moſt books of Arithmetic.

Arbitration of EXCHANGE, is the method of remitting to, and drawing upon, foreign places, in ſuch a manner as ſhall turn out the moſt profitable.

Arbitration is either Simple or Compound.

Simple Arbitration reſpects three places only. Here, by comparing the par of arbitration between a firſt and ſecond place, and between the 1ſt and a 3d, the rate between the 2d and 3d is diſcovered; from whence a perſon can judge how to remit or draw to the moſt advantage, and to determine what that advantage is.

Compound Arbitration reſpects the caſes in which the exchanges among three, four, or more places are concerned. A perſon who knows at what rate he can draw or remit directly, and alſo has advice of the courſe of exchange in foreign parts, may trace out a path for circulating his money, through more or fewer of ſuch places, and alſo in ſuch order, as to make a benefit of his ſkill and credit: and in this lies the great art of ſuch negociations. See my Arithmetic, pa. 105, &c.

EXCURSION, in Aſtronomy. See ELONGATION.

Circles of EXCURSION. See CIRCLES.

EXEGESIS, or EXEGETICA, in Algebra, is the finding, either in numbers or lines, the roots of the equation of a problem, according as the problem is either numeral or geometrical.

EXHALATION, a fume or ſteam Exhaling, or iſſuing, from a body, and diffuſing itſelf in the atmoſphere.

The terms Exhalation and Vapour are often uſed indifferently; but the more accurate writers diſtinguiſh them, appropriating the term Vapour to the moiſt fumes raiſed from water and other liquid bodies; and the term Exhalation to the dry ones emitted from ſolid bodies; as earth, fire, minerals, &c. In this ſenſe, Exhalations are dry and ſubtle corpuſcles, or effluvia, looſened from hard terreſtrial bodies, either by the heat of the ſun, or the action of the air, or ſome other cauſe: being emitted upwards to a certain height in the atmoſphere, where, mixing with the vapours, they help to conſtitute clouds, and return back in dews, miſts, rains, &c.

Sir Iſaac Newton thinks, that true and permanent air is formed from the Exhalations raiſed from the hardeſt and moſt compact bodies.

EXHAUSTED *Receiver*, is a glaſs, or other veſſel, applied on the plate of an air-pump, to have the air extracted out of it by the working of the pump.—Things placed in ſuch an Exhauſted Receiver, are ſaid to be *in vacuo*.

EXHAUSTIONS, or the *Method of* EXHAUSTIONS, a method of demonſtration founded upon a kind of Exhauſting a quantity by continually taking away certain parts of it.

The method of Exhauſtions was of frequent uſe among the ancient mathematicians; as Euclid, Archimedes, &c. It is founded on what Euclid ſays in the 10th book of his Elements; viz, that thoſe quantities are equal, whoſe difference is leſs than any aſſignable quantity. Or thus, two quantities A and B are equal, when, if to or from one of them as A, any other quantity as d be ſubtracted, however ſmall it be, then the ſum or difference is reſpectively greater or leſs than the other quantity B: viz, d being an indefinitely ſmall quantity,
if $A + d$ be greater than B,
and $A - d$ leſs than B,
then is A equal to B.

This principle is uſed in the 1ſt prop. of the 10th book, which imports, that if from the greater of two quantities be taken more than its half, and from the remainder more than its half, and ſo on; there will at length remain a quantity leſs than either of thoſe propoſed. On this foundation it is demonſtrated, that if a regular polygon of infinite ſides be inſcribed in a circle, or circumſcribed about it; then the ſpace, which is the difference between the circle and the polygon, will by degrees be quite exhauſted, and the circle become ultimately equal to the polygon. And in this way it is that Archimedes demonſtrates, that a circle is equal to a right-angled triangle, whoſe two ſides about the right angle, are equal, the one to the ſemidiameter, and the other to the perimeter of the circle. Prop. 1 De Dimenſione Circuli.

Upon the Method of Exhauſtions depends the Method of Indiviſibles introduced by Cavalerius, which is but a ſhorter way of expreſſing the method of Exhauſtions; as alſo Wallis's Arithmetic of Infinites, which is a farther improvement of the Method of Indiviſibles; and hence alſo the Methods of Increments, Differentials, Fluxions, and Infinite Series. See ſome account of the Method of Exhauſtions in Wallis's Algebra, chap. 73, and in Ronayne's Algebra, part 3, pa. 395.

EXPANSION, is the dilating, ſtretching, or ſpreading out of a body; whether from any external cauſe, as the cauſe of rarefaction, or from an internal cauſe, as elaſticity. Bodies naturally expand by heat beyond their dimenſions when cold; and hence it happens that their dimenſions and ſpecific gravities are different in different temperatures and ſeaſons of the year. Air compreſſed or condenſed, as ſoon as the compreſſing or condenſing force is removed, expands itſelf by its elaſtic power to its former dimenſions.

In ſome few caſes indeed bodies ſeem to expand as they grow cold, as water in the act of freezing: but it ſeems this is owing to the extrication of a number of air bubbles from the fluid at a certain time; and is not at all a regular and gradual expanſion like that of metals, &c, by means of heat. Mr. Boyle, in his Hiſtory of Cold, ſays that ice takes up one 12th part more ſpace than water; but by Major Williams's experiments on the force of freezing water, I have found it occupies but about the 17th or 18th part more ſpace. Tranſac. of the R. Soc. of Edinb. vol. 2, pa. 28. In certain metals alſo, an Expanſion takes place when they paſs from a fluid to a ſolid ſtate: but this too is not to be accounted any proper effect of cold, but of the arrangement of the parts of the metal in a certain manner; and is therefore to be accounted a kind of cryſtallization, rather than any thing elſe.

The Expanſion of different bodies by heat is very various; and many experiments upon it are to be met with in the volumes of the Philoſ. Tranſ. and elſewhere. In the 48th vol. in particular, Mr. Smeaton has given a table of the Expanſion of many different ſubſtances, as determined by experiment, from which the following particulars are extracted. Where it is to be noted, that the quantities of Expanſion which anſwer to 180 degrees of Fahrenheit's thermometer, are expreſſed in ten-thouſandth parts of an Engliſh inch, each ſubſtance being 1 foot or 12 inches in length.

White glaſs barometer tube	100
Martial regulus of antimony	130
Bliſtered ſteel	138
Hard ſteel	147
Iron	151
Biſmuth	167
Copper hammered	204
Copper 8 parts, mixed with 1 of tin	218
Caſt braſs	225
Braſs 16 parts, with tin 1	229
Braſs wire	232
Speculum metal	232
Spelter ſolder, viz. braſs 2 parts, zink 1	247
Fine pewter	274
Grain tin	298
Soft ſolder, viz. lead 2, tin 1	301
Zink 8 parts, tin 1, a little hammered	323
Lead	344
Zink or Spelter	353
Zink hammer'd $\frac{1}{2}$ an inch per foot	373

By other experiments too it has been found that, for each degree of heat of the thermometer, mercury, water, and air, expand by the following parts of their own bulk, viz,

Mercury the 9600th
Water the 6666th } part of its bulk.
Air the 435th

From the foregoing table it appears, that there is no general rule for the degree of Expanſion to which bodies are ſubject by the ſame degree of heat, either from their ſpecific gravity or otherwiſe. Zink, which is much lighter than lead, expands more with heat; while glaſs, which is lighter than either, expands much leſs; and copper, which is heavier than a mixture of braſs and tin, expands leſs.

It ſeems too that metals obſerve a proportion of Expanſion in a fluid ſtate, quite different from what they do in a ſolid one: For regulus of antimony ſeemed to ſhrink in fixing, after being melted, conſiderably more than zink.

But of all known ſubſtances, thoſe of the aërial kind expand moſt by an equal degree of heat; and in general the greater quantity of latent heat that any ſubſtance contains, the more eaſily is it expanded; though even here no general rule can be formed. It is indeed certain that the denſeſt fluids, ſuch as mercury, oil of vitriol, &c, are leſs expanſible than water, ſpirit of wine, or ether. Which laſt is ſo eaſily expanded, that were it not for the preſſure of the atmoſphere it would be in a continual ſtate of vapour. And indeed this is the caſe, in ſome meaſure, with perhaps all fluids; as it has been found, by experiments with the beſt air-pumps, that water, and other fluids, aſcend in vapours the more as the exhauſtion is the more perfect; from which it would ſeem that water would wholly riſe in vapour, in any temperature, if the preſſure of the atmoſphere was entirely taken off.

After bodies are reduced to a vaporous ſtate, their Expanſion ſeems to go on without any limitation, in proportion to the degree of heat applied; though it may be impoſſible to ſay what would be the ultimate effects of that principle upon them in this way. The force with which theſe vapours expand on the application of high degrees, is very great; nor does it appear that any obſtacle whatever is inſuperable by them.

On this principle depend the ſteam engines, ſo much uſed in various mechanical operations; likewiſe ſome hydraulic machines; and the inſtruments called manometers, which ſhew the variation of gravity in the external atmoſphere, by the expanſion or condenſation of a ſmall quantity of air confined in a proper veſſel. On this principle alſo, perpetual movements might be conſtructed ſimilar to thoſe invented by Mr. Coxe, on the principle of the barometer. And a variety of other curious machines may be conſtructed on the principle of aërial expanſion; an account of ſome of which is given under HYDROSTATICS and PNEUMATICS.

On the principle of the Expanſion of fluids are conſtructed Thermometers. And for the effects of the different Expanſions of metals in correcting the errors of machines for meaſuring time, ſee the article PENDULUM.

The Expanſion of ſolid bodies is meaſured by an inſtrument called the Pyrometer; and the force with which they expand is ſtill greater than that of aërial vapours; the flame of a farthing candle produces an

Expanſion in a bar of iron capable of counteracting a weight of 500 pounds. The quantity of expanſion however is ſo ſmall, that it has never been applied to the movement of any mechanical engine.

EXPECTATION, in the Doctrine of Chances, is applied to any contingent event, upon the happening of which ſome benefit &c is expected. This is capable of being reduced to the rules of computation: for a ſum of money in Expectation when a particular event happens, has a determinate value before that event happens. Thus, if a perſon is to receive any ſum, as 10l, when an event takes place which has an equal chance or probability of happening and failing, the value of the Expectation is half that ſum or 5l.: but if there are 3 chances for failing, and only 1 for its happening, or one chance only in its favour out of all the 4 chances; then the probability of its happening is only 1 out of 4, or $\frac{1}{4}$, and the value of the Expectation is but $\frac{1}{4}$ of 10l. which is only 2l. 10s. or half the former ſum. And in all caſes, the value of the Expectation of any ſum is found by multiplying that ſum by the fraction expreſſing the probability of obtaining it. So the value of the Expectation on 100l. when there are 3 chances out of 5 for obtaining it, or when the probability of obtaining it is $\frac{3}{5}$, is $\frac{3}{5}$ of 100l. which is 60l. And if s be any ſum expected on the happening of an event, h the chances for that event happening, and f the chances for its failing; then, there being h chances out of $f + h$ for its happening, the probability will be $\frac{h}{f + h}$, and the value of the expectation is $\frac{h}{f + h} \times s$. See Simpſon's or De Moivre's Doctrine of Chances.

EXPECTATION *of Life*, in the Doctrine of Life Annuities, is the ſhare, or number of years of life, which a perſon of a given age may, upon an equality of chance, expect to enjoy.

By the Expectation or ſhare of life, ſays Mr. Simpſon (Select Exerciſes pa. 273), is not here to be underſtood that particular period which a perſon hath an equal chance of ſurviving; this laſt being a different, and more ſimple conſideration. The Expectation of a life, to put it in the moſt familiar light, may be taken as the number of years at which the purchaſe of an annuity, granted upon it, without diſcount of money, ought to be valued. Which number of years will differ more or leſs from the period above-mentioned, according to the different degrees of mortality to which the ſeveral ſtages of life are incident. Thus it is much more than an equal chance, according to the table of the probability of the duration of life (p. 254 ut ſupra), that an infant, juſt come into the world, arrives not to the age of 10 years; yet the Expectation or ſhare of life due to it, upon an average, is near 20 years. The reaſon of which wide difference, is the great exceſs of the probability of mortality in the firſt tender years of life, above that reſpecting the more mature and ſtronger ages. Indeed if the numbers that die at every age were to be the ſame, the two quantities above ſpecified would alſo be equal; but when the ſaid numbers become continually leſs and leſs, the Expectation muſt of conſequence be the greater of the two.

Mr. Simpſon has given a table and rule for finding this Expectation, pa. 255 and 273 as above. Thus,

A Table of the EXPECTATIONS *of Life in London.*

Age	Expectation	Age	Expectation	Age	Expectation
1	27·0	28	24·6	55	14·2
2	32·0	29	24·1	56	13·8
3	34.0	30	23·6	57	13·4
4	35·6	31	23·1	58	13·1
5	36·0	32	22·7	59	12·7
6	36·0	33	22·3	60	12·4
7	35.8	34	21·9	61	12·0
8	35·6	35	21·5	62	11·6
9	35·2	36	21·1	63	11·2
10	34·8	37	20·7	64	10·8
11	34·3	38	20·3	65	10·5
12	33·7	39	19·9	66	10·1
13	33·1	40	19·6	67	9·8
14	32·5	41	19·2	68	9·4
15	31·9	42	18·8	69	9·1
16	31·3	43	18·5	70	8·8
17	30·7	44	18·1	71	8·4
18	30·1	45	17·8	72	8·1
19	29·5	46	17·4	73	7·8
20	28·9	47	17·0	74	7·5
21	28·3	48	16·7	75	7·2
22	27·7	49	16·3	76	6·8
23	27·2	50	16·0	77	6·4
24	26·6	51	15·6	78	6·0
25	26·1	52	15·2	79	5·5
26	25·6	53	14·9	80	5·0
27	25·1	54	14·5		

For Example, if it be required to find the Expectation or ſhare of life, due to a perſon of 30 years old. Oppoſite the given age in the firſt column of the table, ſtands 23·6 in the ſecond col. for the years in the Expectation ſought.

See De Moivre's Doctrine of Chances applied to the Valuation of Annuities, p. 288; or Dr. Price's Obſervations on Reverſionary Payments, p. 168, 364, 374, &c; or Philoſ. Tranſ. vol. 59, p. 89.

EXPERIMENT, in Philoſophy, a trial of the effect or reſult of certain applications and motions of natural bodies, in order to diſcover ſomething of their laws and relations, &c.

The making of experiments is grown into a kind of art; and there are now many collections of them, moſtly under the denomination of Courſes of Experimental Philoſophy. Sturmius made a curious collection of the principal Diſcoveries and Experiments of the laſt age, under the title of Collegium Experimentale. Other Courſes of Experiments have been publiſhed by Graveſande, Deſaguliers, Helſham, Cotes, and others.

EXPERIMENTAL *Philoſophy*, is that which proceeds on Experiments, or which deduces the laws of nature and the properties and powers of bodies, and their actions upon each other, from ſenſible Experiments and obſervations.

Experiments are of the utmoſt importance in philoſophy; and the great advantages the modern phyſics have over the ancient, is chiefly owing to this, viz, that we abound much more in Experiments, and that we make more uſe of the Experiments we have. The method of the ancients, was chiefly to begin with the cauſes of things, and thence argue to the phenomena and effects; on the contrary, that of the moderns proceeds from Experiments and Obſervations, from whence the properties and laws of natural things are deduced, and general theories are formed.

Several of the ancients indeed thought as highly of Experiments as the moderns, and practiſed them alſo. Plato omits no occaſion of ſpeaking of the advantages of them; and Ariſtotle's hiſtory of animals bears ample teſtimony for *him*. Democritus's great employment was to make experiments; and even Epicurus himſelf owes part of his glory to the ſame cauſe.

Among the moderns, the making of Experiments was chiefly begun by Friar Bacon, in the 13th century, who it ſeems ſpent a great deal of money and labour in this way. After him, the lord chancellor Bacon is looked upon as the founder of the preſent mode of philoſophiſing by Experiments. And his method has been proſecuted with laudable emulation by the Academy del Cimento, the Royal Society, the Royal Academy at Paris; by Mr. Boyle, and, over all, by Sir Iſaac Newton, with many other illuſtrious names.

Indeed, Experiments, within the laſt century, are come ſo much into vogue, that nothing will paſs in philoſophy, but what is either founded on Experiments, or confirmed by them; ſo that the new philoſophy is almoſt wholly Experimental.

Yet there are ſome, even among the learned, who ſpeak of Experiments in a different manner, or perhaps rather of the abuſe of them, and in deriſion of the pretenders to this practice. Thus, though Dr. Keil allows that philoſophy has received very conſiderable advantages from the makers of Experiments; yet he complains of their diſingenuity, in too often wreſting and diſtorting their Experiments and Obſervations to favour ſome darling theories they had eſpouſed. Nay more, M. Hartſoeker, in his Recueil de pluſieurs Pieces de Phyſique, undertakes to ſhew, that ſuch as employ themſelves in the making of Experiments, are not properly philoſophers, but as it were the labourers or operators of philoſophers, that work under them, and for them, furniſhing them with materials to build their ſyſtems and hypotheſes upon. And the learned M. Dacier, in the beginning of his diſcourſe on Plato, at the head of his tranſlation of the works of that philoſopher, deals ſtill more ſeverely with the makers of Experiments. He breaks out with a kind of indignation at a tribe of idly curious people, whoſe ſole employment conſiſts in making Experiments on the gravity of the air, the equilibrium of fluids, the loadſtone, &c, and yet arrogate to themſelves the noble title of philoſophers. But his honeſt indignation would have exceeded all bounds, had he lived to ſee the contemptible fall of one of the principal ſocieties above-mentioned; while its members firſt amuſe themſelves with magnetical conundrums, ſpinning electrical wheels, torturing the unſeen and unknown phlogiſtic particles; and finally polluting the ſource of ſcience, and the ſtreams of wiſdom, with the folly of hunting after cockle-ſhells, caterpillars, and butterflies!

EXPLOSION, a ſudden and violent expanſion of an elaſtic fluid, by which it inſtantly throws off any obſtacle that happens to be in the way, ſometimes with aſtoniſhing force and rapidity, as the Exploſion of fired gun-powder, &c.

Exploſion differs from expanſion, in that the latter is a gradual and continued power, acting uniformly for ſome certain time; whereas the former is always ſudden, and only of momentary or immenſurably ſhort duration. The expanſions of ſolid ſubſtances do not terminate in violent exploſions, on account of their ſlowneſs, and the ſmall ſpace through which the expanding ſubſtance moves; though their ſtrength may be equally great with that of the moſt active aerial fluids. Thus we find that though wedges of wood, when wetted, will cleave ſolid blocks of ſtone, they never throw them to any diſtance, as is the caſe with gun-powder. On the other hand, it is ſeldom that the expanſion of any elaſtic fluid burſts a ſolid ſubſtance without throwing the fragments of it to a conſiderable diſtance, with effects that are often very terrible.

The moſt part of exploſive ſubſtances are either aerial, or convertible into ſuch, and raiſed into an elaſtic fluid. Thus gun-powder, whoſe eſſence ſeems to conſiſt in common air fixed in the nitre, or at leaſt an air of ſimilar elaſticity, where it is condenſed into a bulk many hundred times leſs than the natural ſtate of the atmoſphere; which air being ſuddenly diſengaged by the firing of the gun-powder, and the decompoſition of its parts, it rapidly expands itſelf again with a force proportioned to the degree of its condenſation when fixed in the gun-powder, and ſo explodes, and produces all thoſe terrible effects that attend the exploſion. The elaſtic fluid generated by the fired gun-powder expands itſelf with a velocity of about 10,000 feet per ſecond, and with a force more than 1000 times greater than the preſſure of the atmoſphere on the ſame baſe.

The Electric Exploſions ſeem to be ſtill much more ſtrong and aſtoniſhing; as in the caſes of lightning, earthquakes, and volcanoes; and even in the artificial electricity produced by the ordinary machines. The aſtoniſhing ſtrength of electric exploſions, which is beyond all poſſible means of meaſuring it, manifeſts itſelf by the many tremendous effects we hear of fire-balls and lightning.

In caſes where the electric matter acts like common fire, the force of the exploſions, though very great, is capable of meaſurement, by comparing the diſtances to which bodies are thrown, with their weight. This is moſt evident in volcanoes, where the projections of the burning rocks and lava manifeſt the greatneſs of the power, at the ſame time that they afford a method of meaſuring it: and theſe exploſions are owing to the extrication of aerial vapours, and their rarefaction by intenſe heat.

Next in ſtrength to the aerial vapours, are thoſe of aqueous and other liquids. Very remarkable effects of theſe are obſerved in ſteam-engines; and there is one caſe from which it has been inferred that aqueous ſteam is even vaſtly ſtronger than fired gun-powder. This is when water is thrown upon melted copper: for here the exploſion is ſo ſtrong as almoſt to exceed imagination; and the moſt terrible accidents have happened, even from ſo ſlight a cauſe as one of the workmen ſpitting ſing is the furnace where copper was melting; ariſing probably from a ſudden decompoſition of the water.' Exploſions happen alſo from the application of water to other melted metals, though in a lower degree, when the fluid is applied in ſmall quantities, and even to common fire itſelf, as every perſon's own experience muſt have informed him; and this ſeems to be occaſioned by the ſudden rarefaction of the water into ſteam. Examples of this kind often occur when workmen are faſtening cramps of iron into ſtones; where, if there happen to be a little water in the hole into which the lead is poured, this will fly out in ſuch a manner as ſometimes to burn them ſeverely. Terrible accidents of this kind have ſometimes happened in founderies, when large quantities of melted metal have been poured into wet or damp moulds. In theſe caſes, the ſudden expanſion of the aqueous ſteam has thrown out the metal with great violence; and if any decompoſition has taken place at the ſame time, ſo as to convert the aqueous vapour into an aerial one, the exploſion muſt be ſtill greater.

To this laſt kind of exploſion muſt be referred that which takes place on pouring cold water into boiling or burning oil or tallow, or in pouring the latter upon the former; the water however being always uſed in a ſmall quantity.

Another remarkable kind of Exploſion is that produced by inflammable and dephlogiſticated air, when mixed together, and ſet on fire; a kind of exploſion that often happens in coal mines, &c. This differs from any of the caſes before mentioned; for here is an abſolute condenſation rather than an expanſion throughout the whole of the operation; and could the airs be made to take fire throughout their whole ſubſtance abſolutely at the ſame inſtant, there would be no Exploſion, but only a ſudden production of heat.

Though Exploſions be ſometimes very deſtructive, they are likewiſe of conſiderable uſe in life, as in removing obſtacles that could ſcarcely be overcome by any mechanical power whatever. The principal of theſe are the blowing up of rocks, the ſeparating of ſtones in quarries, and other purpoſes of that kind. The deſtruction occaſioned by them in times of war, and the machines formed upon the principle of Exploſion for the deſtruction of the human race, are well known; and if we cannot call theſe uſeful, they muſt be allowed at leaſt to be neceſſary evils.

The effects of Exploſions, when violent, are felt at a conſiderable diſtance, by reaſon of the concuſſions they give to the atmoſphere. Sir Wm. Hamilton relates, that at the exploſions of Veſuvius, in 1767, the doors and windows of the houſes at Naples flew open if unbolted, and one door was burſt open that had been locked, though at the diſtance of 6 miles: and the exploſion of a powder-magazine, or a powder-mill, it is well known, ſpreads deſtruction for many miles round; and even kills people by the mere concuſſion of the air. A curious effect of them too is, that they electrify the air, and even glaſs windows, at a conſiderable diſtance. This is always obſervable in firing the guns at the Tower of London: and ſome years ago, after an Exploſion of ſome powder-mills near that city, many people were alarmed by a rattling and breaking of their china-ware. In this reſpect however, the effects of electrical Exploſions are the moſt remarkable, though not in the uncommon way juſt mentioned; but it is certain that the influence of a flaſh of lightning is diffuſed for a great way round the place where the Exploſion happens, producing very perceptible changes both on the animal and vegetable creation.

EXPONENT *of a Power*, in Arithmetic and Algebra, denotes the number or quantity expreſſing the degree or elevation of the power, or which ſhews how often a given power is to be divided by its root before it be brought down to unity or 1. Thus, the Exponent or index of a ſquare number, or the 2d power, is 2; of a cube 3; and ſo on; the ſquare being a power of the 2d degree; the cube, of a 3d, &c. It is otherwiſe called the Index.

Exponents, as now uſed, are rather of modern invention. Diophantus, with the Arabian and the firſt European authors, denoted the powers of quantities by ſubjoining an abbreviation of the name of the power; though with ſome variation, and difference from one another. The names of the powers, and the marks for denoting them, according to Diophantus, are as follow: viz.

Names, μοναϛ, αριθμος, δυναμις, κυβος, δυναμοδυναμις,
Marks, μ^{o} , ϛ , δ^{υ} , κ^{υ} , $\delta^{\upsilon}\,\delta^{\upsilon}$,
δυναμοκυβος , κυβοκυβος, &c. ;
$\delta^{\upsilon}\,\kappa^{\upsilon}$ $\kappa^{\upsilon}\,\kappa^{\upsilon}$

which we now denote by

1, a, a^2, a^3, a^4, a^5, a^6, &c.

F. Lucas Paciolus, or De Burgo, for the root, ſquare, cube, &c, uſes the terms *coſa*, *cenſo*, *cubo*, *relato* (*primo*, *ſecundo*, *tertio*, &c), or the abbreviations *co. ce. cu.*; and ℞ for root or radicality.

Cardan uſed the Latin contractions of the names of the powers; and other contemporary, as well as ſucceeding, authors, eſpecially the Germans, as Stifelius, Scheubelius, Pelitarius, &c, uſed the like contractions, but ſomewhat varied, as thus:

ȣ, ꝝ, ʒ, ȼ, ʒʒ, ſʒ, ʒȼ, &c.
or 1, ꝝ, ʒ, ȼ, ʒʒ, *ſs*, ʒȼ, &c.
or 1, ℞, *q*, ȼ, *qq*, *ſs*, *q*ȼ, &c.
Exp. 0, 1, 2, 3, 4, 5, 6, &c.

But beſides that way, the ſame authors alſo made uſe of the numbers as in the laſt line here above, and it was Stifelius who firſt called them by the name *Exponent*.

Bombelli, whoſe Algebra was publiſhed in 1579, denotes the *res*, or unknown quantity, by this mark $\smile$, and the powers by numeral Exponents ſet over it, thus: $\overset{1}{\smile}$, $\overset{2}{\smile}$, $\overset{3}{\smile}$, &c. And

Stevinus, who publiſhed his Arithmetic in 1585, and his Algebra ſoon afterwards, has ſuch another method, but inſtead of $\smile$ he uſes a ſmall circle ○, within which he places the numeral Exponent of the power; thus ⓪, ①, ②, ③, &c: and in this way he extends his notation to fractional Exponents, and even to radical ones; thus $\textcircled{\tfrac{1}{2}}$, $\textcircled{\tfrac{1}{3}}$, $\textcircled{\tfrac{1}{4}}$, $\textcircled{\tfrac{2}{3}}$, &c.

Vieta after this uſed words again to denote the powers. Afterwards Harriot denoted the powers by a repetition of the root; as *a*, *aa*, *aaa*, for the 1ſt, 2d, and 3d powers. Inſtead of which, Des Cartes again reſtored the numeral Exponents, placing them after the root, when the power is high, to avoid a too frequent repetition of the letter of the root; as a^3 a^4, &c, as at preſent. Alſo Albert Girard, in 1629, uſed the Exponents to roots, thus; $\sqrt{}$, $\sqrt[2]{}$, $\sqrt[3]{}$, &c.

The notation of powers and roots by the preſent way of Exponents, has introduced a new and general arithmetic of Exponents or powers; for hence powers are multiplied by only adding their Exponents, divided by ſubtracting the Exponents, raiſed to other powers, or roots of them extracted, by multiplying or dividing the Exponent by the index of the power or root.—

So $a^2 \times a^3 = a^5$, and $a^{\frac{1}{2}} \times a^{\frac{1}{4}} = a^{\frac{3}{4}}$;
$a^5 \div a^3 = a^2$, and $a^{\frac{3}{4}} \div a^{\frac{1}{2}} = a^{\frac{1}{4}}$;
the 2d power of a^3 is a^6,
and the 3d root of a^6 is a^2.

This algorithm of powers led the way to the invention of logarithms, which are only the indices or Exponents of powers: and hence the addition and ſubtraction of logarithms, anſwer to the multiplication and diviſion of numbers; while the raiſing of powers, and extracting of roots, is effected by multiplying the logarithm by the index of the power, or dividing the logarithm by the index of the root.

EXPONENT *of a Ratio*, is, by ſome, underſtood as the quotient ariſing from the diviſion of the antecedent of the ratio by the conſequent: in which ſenſe, the Exponent of the ratio of 3 to 2 is $\frac{3}{2}$; and that of the ratio of 2 to 3 is $\frac{2}{3}$.

But others, and thoſe among the beſt mathematicians, underſtand logarithms as the Exponents of ratios; in which ſenſe they coincide with the idea of meaſures of ratios, as delivered by Kepler, Mercator, Halley, Cotes, &c.

EXPONENTIAL *Calculus*, the method of differencing, or finding the fluxions of, Exponential quantities, and of ſumming up thoſe differences, or finding their fluents. See CALCULUS, FLUXIONS, and FLUENTS.

EXPONENTIAL *Curve*, is that whoſe nature is defined or expreſſed by an Exponential equation; as the curve denoted by $a^x = y$, or by $x^x = y$.

EXPONENTIAL *Equation*, is one in which is contained an exponential quantity: as the equation $a^x = b$, or $x^x = ab$, &c.

Exponential Equations are commonly beſt reſolved by means of logarithms, viz, firſt taking the log. of the given equation: thus, taking the log. of the equation $a^x = b$, it is $x \times$ log. of $a =$ log. of b; and hence $x = \frac{\text{log. } b}{\text{log. } a}$

Alſo, the log. of the equation $x^x = ab$, is $x \times$ log. $x =$ log. ab; and then x is eaſily found by trial-and-error, or the double rule of poſition.

EXPONENTIAL *Quantity*, is that whoſe power is a variable quantity; as the expreſſion a^x, or x^x.

Exponential quantities are of ſeveral degrees, and orders, according to the number of exponents or powers, one over another. Thus,
a^x is an Exponential of the 1ſt order,
a^{x^y}, is one of the 2d order,
$a^{x^{y^z}}$ is one of the 3d order, and ſo on.
See Bernoulli Oper. tom. 1, pa. 182, &c.

EXPRESSION, in Algebra, is any algebraical quantity, ſimple or compound: as the expreſſion, $3a$, or $2ab$, or $\sqrt{a^2 + c^2}$.

EXTENSION, one of the common and eſſential properties of body; or that by which it poſſeſſes or takes up ſome part of univerſal ſpace, called the place of that body.

The extenſion of a body, is properly in every direction whatever; but it is uſual to conſider it as extended only in length, breadth, and thickneſs.

EXTERIOR *Polygon*, or *Talus*, is the outer or circumſcribing one. See POLYGON and TALUS.

EXTERMINATION, or EXTERMINATING, in Algebra, is the taking away, or expelling of ſomething from an expreſſion, or from an equation: as to Exterminate ſurds, fractions, or any particular letter or quantity out of equations.

Thus, to take away the fractional form from this equation $\frac{a^2 + x^2}{2c} = \frac{ab}{d}$; multiply each numerator by the other's denominator, and the equation becomes $a^2d + dx^2 = 2abc$, out of fractions.

Alſo, to take away the radicality from the equation $3\sqrt{a^2 - x^2} = 2c$, raiſe each to the 2d power, and it becomes $9a^2 - 9x^2 = 4c^2$.

For Exterminating any quantity out of equations, there are various rules and methods, according to the form of the equations; of which many excellent ſpecimens may be ſeen in Newton's Algebra, pa. 60, ed. 1738; or in Maclaurin's Algebra, part 1, chap. 12. For example, to Exterminate y out of theſe two equations, $a + x = b + y$,
and $3b = 2x + y$;
ſubtract the upper equation from the under, ſo ſhall there ariſe $3b - a - x = 2x - b$; then, by the known methods of tranſpoſition &c, there is obtained $4b - a = 3x$, and hence $x = \frac{4b - a}{3}$.

EXTERNAL *Angles*, are the angles formed without-ſide of a figure, by producing its ſides out.

In a triangle, any External angle is equal to the ſum of both the two internal oppoſite angles taken together: and, in any right-lined figure, the ſum of all the external angles, is always equal to 4 right angles.

EXTRA-*Conſtellary Stars*, ſuch as are not properly included in any conſtellation.

EXTRA-*Mundane Space*, is the infinite, empty, void ſpace, which is by ſome ſuppoſed to be extended beyond the bounds of the univerſe, and conſequently in which there is really nothing at all.

EXTRACTION *of Roots*, is the finding the roots of given numbers, or quantities, or equations.

The roots of quantities are denominated from their powers; as the ſquare or 2d root, the cubic or 3d root, the biquadratic or 4th root, the 5th root, &c; which are the roots of the 2d, 3d, 4th, 5th, &c powers. The Extraction of roots has always made a part of arithmetical calculation, at leaſt as far back as the compoſition of powers has been known: for the compoſition of powers always led to their reſolution, or Extraction of roots, which is performed by the rules exactly reverſe of the former. Thus, if any root be conſidered as conſiſting of two parts $a + x$, of which the former a is known, and the latter x unknown, then the ſquare of this root being $a^2 + 2ax + x^2$, which is its compoſition, this indicated the method of reſolution, ſo as to find out the unknown part x; for having ſubtracted the neareſt ſquare a^2 from the given quantity, there remains $2ax + x^2$ or $\overline{2a + x} \times x$; therefore divide this remainder by $2a$, the double of the firſt member of the root, the quotient will be nearly x the other member; then to $2a$ add this quotient x, and multiply the ſum $2a + x$ by x, and the product will make up the remaining part $2ax + x^2$ of the given power.

The compoſition of the cubic or 3d power next preſented itſelf, which conſiſts of theſe four terms $a^3 + 3a^2x + 3ax^2 + x^3$; by means of which the cubic roots of numbers have been extracted: viz, by ſubtracting the neareſt cube a^3 from the given power, dividing the remainder by $3a^2$, which gives x nearly for the quotient; then completing the diviſor up to $3a^2 + 3ax + x^2$, multiply it by x for the other part of the power to be ſubtracted. And this was the extent of the Extraction of roots in the time of Lucas de Burgo, who, from 1470 to 1500, wrote ſeveral pieces on arithmetic and algebra, which were the firſt works of this kind printed in Europe.

It was not long however before the nature and compoſition of all the higher powers became known, and general tables of coefficients formed for raiſing them; the firſt of which is contained in Stifelius's arithmetic, printed at Norimberg in 1543, where he fully explains their uſe in Extracting the roots of all powers whatever, by methods ſimilar to thoſe for the ſquare and cubic roots, as above deſcribed; and thus completed the Extraction of all ſorts of roots of numbers, at leaſt ſo far as reſpects that method of reſolution. Since that time, however, many new methods of Extraction have been deviſed, as well as improvements made in the old way.

The Extraction of roots of equations followed cloſely that of known numbers. In De Burgo's time they extracted the roots of quadratic equations, the ſame way as at preſent. Ferreus, Tartalea, and Cardan extracted the roots of cubic equations, by general rules. Soon afterwards the roots of higher equations were extracted, at leaſt in numbers, by approximation. And the late improvements in analytics have furniſhed general rules for Extracting the roots, in infinite ſeries, of all equations whatever. All which methods may be ſeen in moſt books of arithmetic and algebra. Of which it may ſuffice to give here a ſhort ſpecimen of ſome of the eaſieſt rules for Extracting the roots of quantities and equations, as they here follow.

I. *To Extract the Square Root of any Number.*—Point off, or divide the number, from the place of units, into portions of two figures each, as here of the number 99856, ſetting a point or mark over the ſpace between each portion of two figures. Then, beginning at the left hand, take the greateſt root 3, of the firſt part 9, placing it on the right hand for the firſt figure of the root, and ſubtracting its ſquare 9 from the ſaid firſt part; to the remainder, which here is 0, bring down the 2d part 98, and on the left hand of it place 6 the double of the firſt figure 3, for a diviſor; conceive a cipher added to this, making it 60, and then divide the 98 by the 60, the quotient is 1 for the ſecond figure of the root, which is accordingly placed there, after the 3, alſo in the diviſor after the 6 and below the ſame; then multiply theſe as they ſtand, the 61 by the 1, and the product 61 ſet below the 98, and ſubtract it from the ſame, which leaves 37 for the next remainder; to this bring down the 3d period 56, making 3756 for the next reſolvend: then form its diviſor as before, viz, doubling the root 31, or adding, as they ſtand in the diviſor, the 1 to the 61, either way making 62, which with a cipher makes 620, by which divide the reſolvend 3756; the quotient of this diviſion is 6, to be placed, as before, both as the next figure of the root, and at the end of the diviſor 62, and below itſelf there; then multiply as they ſtand the whole diviſor 626 by the 6, the product 3756 is exactly the ſame as the reſolvend, and therefore the number 316 is accurately the ſquare root of the given number 99856, as required.

```
        .  .   root
      99856 ( 316
      9
     ------
 61 | 98
  1 | 61
 ------------
626 | 3756
  6 | 3756
 ------------
```

When the root is to be carried into decimals, couplets of ciphers are to be added, inſtead of figures, as far as may be wanted. In which caſe too, a good abbreviation is made, after the work has been carried on to half the number of figures, by continuing it to the other half only by the contracted way of diviſion; as here in the annexed example for the ſquare root of 2 to eight decimals, or nine places of figures in all.

```
      2 ( 1·41421356
      1
     -------
   24 | 100
    4 |  96
 ------------
  281 |  400
    1 |  281
 ------------
 2824 | 11900
    4 | 11296
 --------------
28282 |  60400
    2 |  56564
 --------------
28284 ) 3836(·1356
 ....   2828
        ----
        1008
         848
        ----
         160
         141
        ----
          19
          17
        ----
           2
```

II. *To extract the cubic root, or any other root whatever.* This is eaſieſt done by one general rule, which I have invented, and publiſhed in my Tracts, vol. 1, pa. 49, which is to this effect: Let N be any number or power, whoſe nth root is to be extracted; and let R be the neareſt rational root of N, of the ſame kind, or R^n the neareſt rational power to N, either greater or leſs than it; then ſhall the true root be very nearly equal to $\dfrac{\overline{n+1}\,.\,N + \overline{n-1}\,.\,R^n}{\overline{n-1}\,.\,N + \overline{n+1}\,.\,R^n} \times R$; which rule is general for any root whoſe index is denoted by n. And by expounding n ſucceſſively by all the numbers 2, 3, 4, 5, &c, this theorem will give the following particular rules for the ſeveral roots, viz, the

2d or ſqu. root, $\dfrac{3N + R^2}{N + 3R^2} \times R$;

3d or cube root, $\dfrac{4N + 2R^3}{2N + 4R^3} \times R$, or $\dfrac{2N + R^3}{N + 2R^3} \times R$;

4th root $\dfrac{5N + 3R^4}{3N + 5R^4} \times R$;

$$\begin{array}{ll}
\text{5th root} & \frac{6N+4R^5}{4N+6R^5}\times R, \text{ or } \frac{3N+2R^5}{2N+3R^5}\times R; \\
\text{6th root} & \frac{7N+5R^6}{5N+7R^6}\times R; \\
\text{7th root} & \frac{8N+6R^7}{6N+8R^7}\times R, \text{ or } \frac{4N+3R^7}{3N+4R^7}\times R; \\
\quad \&c. & \quad \&c.
\end{array}$$

Or the theorem may be ſtated in the form of a proportion, thus:

as $\overline{n-1}\,.\,N+\overline{n+1}\,.\,R^n : \overline{n+1}\,.\,N+\overline{n-1}\,.\,R^n :: R$: the root ſought very nearly.

For ex. ſuppoſe the problem propoſed, of doubling the cube, or to find the cube root of the number 2. Here $N = 2$, $n = 3$, and the neareſt power, and root too, is 1:

$$\begin{array}{rl}
\text{Hence } 2N+R^3 & = 4+1 = 5, \\
\text{and } N+2R^3 & = 2+2 = 4;
\end{array}$$

then $4 : 5 :: 1 : \frac{5}{4} = 1{\cdot}25$ the firſt approximation.

Again, taking $R = \frac{5}{4}$, and conſeq. $R^3 = \frac{125}{64}$:

$$\begin{array}{rl}
\text{Hence } 2N+R^3 & = 4 + \frac{125}{64} = \frac{381}{64}, \\
\text{and } N+2R^3 & = 2 + \frac{250}{64} = \frac{378}{64};
\end{array}$$

then $378 : 381 :: \frac{5}{4} : \frac{635}{504} = 1{\cdot}259921$, for the cube root of 2, which is exact in the very laſt figure.

And again by taking $\frac{635}{504}$ for the value of R, a great many more figures may be found.

III. *To Extract the Roots of Algebraic Quantities.*—This is done by the ſame rules, and in the ſame manner as for the roots of numbers in arithmetic, as above taught. Thus, to Extract the ſquare root of $4a^2 + 12ax + 9x^2$.

$$\begin{array}{r|l}
 & 4a^2+12ax+9x^2\ (2a+3x \text{ the root} \\
 & 4a^2 \\ \hline
4a+3x & 12ax+9x^2 \\
3x & 12ax+9x^2 \\ \hline
\end{array}$$

So alſo the root is carried out in an infinite ſeries, in imitation of the like Extraction of numbers in infinite decimals: thus, for the ſquare root of a^2+x^2,

$$\begin{array}{r|l}
 & a^2+x^2\ (a+\frac{x^2}{2a}-\frac{x^4}{8a^3}+\frac{x^6}{16a^5}\ \&c. \\
 & a^2 \\ \hline
2a+\frac{x^2}{2a} & x^2 \\
\frac{x^2}{2a} & x^2+\frac{x^4}{4a^2} \\ \hline
2a+\frac{x^2}{a}-\frac{x^4}{8a^3} & -\frac{x^4}{4a^2} \\
-\frac{x^4}{8a^3} & -\frac{x^4}{4a^2}-\frac{x^6}{8a^4}+\frac{x^8}{4a^6} \\ \hline
2a+\frac{x^2}{a}-\frac{x^4}{4a^3}+\frac{x^6}{16a^5} & \frac{x^6}{8a^4}-\frac{x^8}{4a^6} \\
\frac{x^6}{16a^5} & \frac{x^6}{8a^4}+\frac{x^8}{16a^6}\ \&c. \\ \hline
 & \&c.
\end{array}$$

To extract the cube root of a^3-x^3 by the general rule in the 2d article.—Here $N = a^3-x^3$, $R = a$, or $R^3 = a^3$; hence $N + 2R^3 = 3a^3 - x^3$, and $2N + R^3 = 3a^3-2x^3$; therefore, by the rule,

$$3a^3-x^3 : 3a^3-2x^3 :: a : a - \frac{x^3}{3a^2} - \frac{x^6}{9a^5} - \frac{x^9}{27a^7}\ \&c,$$

which is the cube root of a^3-x^3 very nearly.

But theſe ſorts of roots are beſt extracted by the Binomial Theorem; which ſee.

IV. *To Extract the Roots of Equations.*—This is the ſame thing as to find the value of the unknown quantity in an equation; which is effected by various means, depending on the form of the equation, and the height of the higheſt power of the unknown quantity in it: for which, ſee the reſpective terms, EQUATION, ROOT, QUADRATIC, CUBIC, &c.

The moſt general, as well as the moſt eaſy, method of Extracting the roots of all equations, is by Double Poſition, or Trial-and-Error; as it eaſily applies to all ſorts of equations whatever, be they ever ſo complex, even logarithmic and exponential ones. There are alſo ſeveral other good methods of approximating to the roots of equations, given by Newton, Halley, Raphſon, De Moivre, &c; of which the moſt general is a rule for Extracting the root of the following indefinite equation,

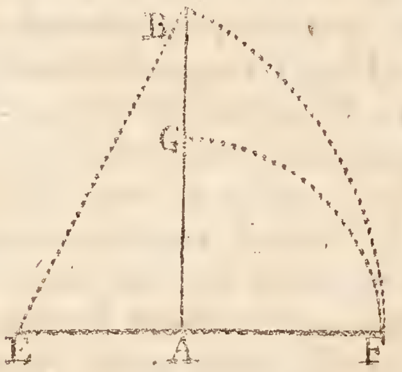
$$\begin{array}{l}
\text{viz, } az + bz^2 + cz^3 + dz^4 + ez^5\ \&c \\
\quad = gy + hy^2 + iy^3 + ky^4 + ly^5\ \&c,
\end{array}$$

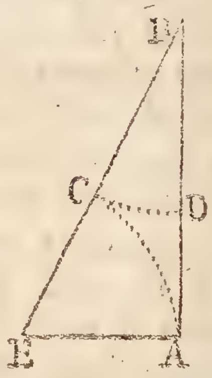
given by M. De Moivre in the Philoſ. Tranſ. vol. 20, p. 190, or Abr. vol. 1, pa. 101.

EXTRADOS, the outſide of an arch of a bridge, vault, &c.

EXTREME-*and-Mean Proportion*, is when a line, or any quantity is ſo divided, as that the whole line is to the greater part, as that greater part is to the leſs part. Hence, in any line ſo divided, the rectangle of the whole line and the leſs ſegment, is equal to the ſquare of the greater ſegment.

Euclid ſhews how to divide a line in Extreme-and-mean ratio, in his Elements, book 2, prop. 11, to this effect: Let AB be the given line; to which draw AE perpendicular and equal to half AB; in EA produced take EF = EB, ſo ſhall AF be equal to the greater part; conſequently if AG be taken equal to AF, the line AB will be divided in G as required.





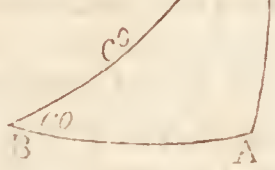
The ſame may be done otherwiſe thus:

As before, make AE (fig. 2.) perpendicular and = ½AB; join EB, on which take EC = EA, and then take BD = BC, ſo ſhall the line be divided in D as required.

No number can be divided into extreme and mean proportion, ſo that its two parts ſhall be rational; as is well demonſtrated by Clavius, in his Commentary upon the 9th book of Euclid's Elements; and the ſame thing will alſo appear from the following algebraical ſolution of the ſame problem: Let a denote the whole line, and x the greater part; then ſhall $a-x$ be the leſs part, and the rectangle of the whole and leſs part being put equal to the ſquare of the greater part, gives this equation, $x^2 = a \times \overline{a-x} = a^2 - ax$; hence $x^2 + ax = a^2$ and by completing the ſquare, and extracting the root, &c, there is at laſt $x = \frac{\sqrt{5}-1}{2}a$ the greater part; conſequently $a - x = \frac{3-\sqrt{5}}{2}a$ is the leſs part. And as the ſquare root of 5, which cannot be exactly extracted, makes a portion of both theſe parts, it is manifeſt that neither of them can be obtained in rational numbers.

Euclid makes great uſe of this problem, viz, in ſeveral parts of the 13th book of the Elements; and by means of it he conſtructs that notable propoſition, viz the 10th of the 4th book, which is to conſtruct an iſoſceles triangle having each angle at the baſe double the angle at the vertex.

EXTREMES *Conjunct*, and Extremes Diſjunct, in Spherical Trigonometry, are the former the two circular parts that lie next the aſſumed middle part, and the latter are the two that lie remote from the middle part. Theſe were terms applied by lord Napier, in his univerſal theorem for reſolving all right-angled and quadrantal ſpherical triangles, and publiſhed in his Logarithmorum Canonis Deſcriptio, an. 1614. In this theorem, Napier condenſes into one rule, in two parts, the rules for all the caſes of right-angled ſpherical triangles, which had been ſeparately demonſtrated by Pitiſcus, Lanſbergius, Copernicus, Regiomontanus, and others. In this theorem, neglecting the right angle, Napier calls the other five parts, circular parts, which are, the two legs about the right angle, and the complements of the other three, viz of the hypothenuſe, and the two oblique angles. Then, taking any three of theſe five parts, one of them will be in the middle between the other two, and theſe two are the Extremes Conjunct when they are immediately adjacent to that middle part, or they are the Extremes Diſjunct when they are each ſeparated from the middle one by another part. Thus, the five parts being AB, AC, and the complements of BC and of the two angles B and C: then if the three parts be AB, and the complements of the angle B and hypothenuſe BC be taken, theſe three are contiguous to each other, the angle B lying in the middle between the other two: therefore the comp. of B is middle part, and AB with the comp. of BC the Extremes Conjunct. But if the three ſides be taken; BC is equally ſeparated from the two legs AB and AC, by two angles B and C; and therefore theſe two legs AB and AC are Extremes Diſjunct, and the comp. of BC the middle part.



Napier's rule for reſolving each caſe is in two parts, as below:

The rectangle contained by radius and the ſine of the middle part, is equal to the rectangle of the tangents of the Extremes conjunct, or equal to the rectangle of the ſines of the Extremes disjunct. Which rule comprehends all the caſes that can happen in right-angled ſpherical triangles; in the application of which rule, the equal rectangles are divided into a proportion or analogy, in ſuch manner that the term ſought may be the laſt of the four terms that are concerned, and conſequently its correſponding term in the ſame rectangle muſt be the firſt of thoſe terms.

EYE, the organ of ſight, conſiſting of ſeveral parts, and of ſuch forms as beſt to anſwer the purpoſe for which it was formed.

As viſion or ſight is effected by a refraction of light through the humours of the eye to the bottom or farther internal part of it, where the images of external objects are formed on a fine expanſion of the optic nerve, called the *Retina*, and therefore the forepart of the eye muſt be of a convex figure, and of ſuch a preciſe degree of convexity as the particular refractive power of the ſeveral humours require for forming the image of an object at a given focal diſtance, viz, the diameter of the eye. Hence we find,

1ſt; The external part of the eye-ball CD (Plate 2, fig. 8.) is a ſtrong pellucid ſubſtance, properly convex, and which, when dried, has ſome reſemblance to a piece of tranſparent horn, for which reaſon it is called the *Cornea*, or horny coat of the eye.

2dly; Immediately behind this coat there is a fine clear humour which, from its likeneſs to water, is called the *Aqueous* or watery humour, and is contained in the ſpace between CD and GFE.

3dly; In this ſpace there is a membrane or diaphragm, called the *Uvea*, with a hole in the middle as at F, called the *Pupil*, of a muſcular contexture for altering the dimenſions of that hole, for the adjuſting or admitting a due quantity of light.

4thly; Juſt behind this diaphragm is placed a lenticular-formed ſubſtance GE, of a conſiderable conſiſtence, called from its tranſparency the *Cryſtalline* humour. This is contained in a fine tunic called the *Choroides*, and is ſuſpended in the middle of the eye by a ring of muſcular fibres called the *Ligamentum Ciliare*, as at G and E; by which means it is moved a little nearer to, or farther from, the bottom of the eye, to alter the focal diſtance.

5thly; All the remaining interior part of the eye, conſtituting the great body of it, from GHE to IMK, is made up of a large quantity of a jelly-like ſubſtance, called the *Vitreous* or glaſſy humour; though it reſembles glaſs in nothing except its tranſparency; it being moſt like the white of an egg of any thing.

6thly; On one ſide of the hinder part of the Eye, as at K, the optic nerve enters it from the brain, and is expanded over all the interior part of the eye to G and E quite around, the expanſion being named the *Retina*. On this delicate membrane, the image IM of every external object OB, is formed according to the optic laws of nature, in the following manner.

Let OB be any very diſtant object. Then a pencil of rays proceeding from any point L, will fall on the cornea CD, and be refracted by the aqueous humour under it, to a point in the axis of that pencil continued out. Then the radius of convexity of the cornea being nearly $\frac{1}{3}$ of an inch; and the ſine of incidence in air to that of refraction in the aqueous humour, being nearly as 4 to 3, ſuppoſing the rays parallel, or the object very far diſtant, the focal diſtance after the firſt refraction, by the proper theorem $\frac{mr}{m-n}$, will be found $1\frac{1}{3}$ inch from the cornea: r being the radius $\frac{1}{3}$, and m to n as 4 to 3.

The rays thus refracted by the cornea, fall converging on the cryſtalline humour, and tend to a point 1·228 inch behind it; alſo the radii of convexity in the ſaid humour are $\frac{1}{3}$ and $\frac{1}{4}$ reſpectively; and the ſine of incidence to that of refraction of the aqueous into the cryſtalline humour, being as 13 to 12; therefore, by this theorem $\frac{mdr}{md - nd + nr}$, the focal diſtance after refraction in the cryſtalline, will be 1·02 inch from the fore part of it: where $m = 13$, $n = 12$, $r = \frac{1}{3}$, and $d = 1{\cdot}228$.

The rays now paſs from the cryſtalline to the vitreous humour ſtill in a converging ſtate, and the ſines of incidence and refraction being here as 12 to 13, as found by experiment; and ſince the ſurface of the vitreous humour is concave which receives the rays, and is the ſame with the convexity of the hinder ſurface of the cryſtalline, the radius will be the ſame, viz $\frac{1}{4}$ of an inch. Therefore the focal diſtance after this third refraction will be found, by this theorem, $\frac{mdr}{nd - md + nr}$, to be 6 tenths of an inch nearly from the hinder part of the cornea: where $m = 12$, $n = 13$, $r = \frac{1}{4}$, and $d = {\cdot}82$; the thickneſs of the lens of the cornea being nearly $\frac{1}{5}$ of an inch.

Now experience ſhews that the diſtance of the retina in the back part of the eye, behind the cornea, is nearly equal to that focal diſtance; and therefore it follows that all objects at a great diſtance have their images formed on the retina in the bottom or hinder part of the eye, and thus diſtinct viſion is produced by this wonderful organ of optic ſenſation.

When the diſtance of objects is not very great, the focal diſtance, after the laſt refraction in the vitreous humour, will be a little increaſed; and to do this we can move the cryſtalline a little nearer the cornea by means of the ligamentum ciliare, and thus on all occaſions it may be adjuſted for a due focal diſtance for every diſtance of objects, excepting that which is leſs than 6 or 7 inches, in good eyes. Many are of opinion, however, that this is effected by a power in the eye to alter the convexity of the cryſtalline humour as occaſion requires; though this is rather doubtful.

By what has been ſaid it appears, that rays of light flowing from every part of an object OB, placed at a proper diſtance from the eye, will have an image IM thereby formed on the retina in the bottom of the eye; and ſince the rays OM, BI, which come from the extreme parts of the object, croſs each other in the middle of the pupil, the poſition of the image IM will be contrary to that of the object, or inverted, as in the caſe of a lens.

The apparent place of any part of an object is in the axis, and conjugate focus of that pencil of rays by which that part or point is formed in the image. Thus, OM is the axis, and O the focus proper to the rays by which the point M in the image is made; therefore the ſenſation of the place of that part will be conceived in the mind to be at O; in like manner the idea of place belonging to the point I, will be referred in the axis IB, to the proper focus B; therefore the apparent place of the whole image IM, will be conceived in the mind to occupy all the ſpace between O and B, and at the diſtance AL from the eye.

Hence likewiſe appears the reaſon, why we ſee an object upright by means of an inverted image; for ſince the apparent place of every point M will be in the axis MO at O; and this axis croſſing the axis of the eye HL in the pupil, it follows, that the ſenſible place O of that point will lie, without the eye, on the contrary ſide of the axis of the eye, to that of the point in the eye; and ſince this is true of all other parts or points in the image, it is evident that the poſition of every part of the object will be on the contrary ſide of the axis to every correſponding part in the image, and therefore the whole object OB will have a contrary poſition to that of the image IM, or will appear upright.

If the convexity of the cornea CD happens not exactly to correſpond to the diameter of the eye, conſidered as the natural focal diſtance, then the image will not be formed on the retina, and conſequently no diſtinct viſion can be effected in ſuch an eye.

If the cornea be too convex, the focal diſtance in the eye will be leſs than its diameter, and the image will be formed ſhort of the retina. Hence the reaſon why people having ſuch eyes are obliged to hold things very near them, to lengthen the focal diſtances; and alſo why they uſe concave glaſſes to counteract or remedy the exceſs of convexity, in order to view diſtant objects diſtinctly.

When the eye has leſs than a juſt degree of convexity, or is too flat, as is generally the caſe with old eyes, by a natural deficiency of the aqueous humour, then the rays tend to a point or focus beyond the retina or bottom of the eye; and to ſupply this want of convexity in the cornea, we uſe convex lenſes in thoſe frames called ſpectacles, or viſual glaſſes.

Since the rays of light OA, BA, which conſtitute the viſual angle OAB, will, when they are intercepted by a lens, be refracted ſooner to the axis; the ſaid angle will thereby be enlarged, and the object of courſe become magnified; which is the reaſon why thoſe lenſes are called magnifiers, or reading-glaſſes.

The dimenſions, or magnitude, of an object OB, are judged of by the quantity of the angle OAB which it ſubtends at the eye. For if the ſame object be placed at two different diſtances L and N, the angles OAB, *oAb*, which in theſe two places it ſubtends at the eye, will be of different magnitude; and the lineal dimenſions, viz length and breadth, will be at N and at L, as the angle *oAb* is to the angle OAB. But the ſurfaces of the objects will be as the ſquares of thoſe angles, and the ſolidities as the cubes of them.

It is found by experience, that two points O, L, in any object, will not be diſtinctly ſeen by the Eye, till they are near enough to ſubtend an angle OAL of one minute. And hence when objects, however large they may be, are ſo remote as not to be ſeen under an angle of one minute, they cannot properly be ſaid to have any apparent dimenſions or magnitude at all; ſuch as is the caſe of the large bodies of the planets, comets, and fixed ſtars. But the optic ſcience has ſupplied means of enlarging this natural ſmall angle under which moſt diſtant objects appear, and thereby increaſing their apparent magnitudes to a very ſurpriſing degree, in the inſtance of that noble inſtrument the teleſcope.

On the other hand, there are in creation an infinity of objects, of ſuch ſmall dimenſions, that they will not ſubtend the requiſite angle, if brought to the neareſt limits of diſtinct viſion, viz 6, 7, or 8 inches from the Eye, as found by experience; and therefore to render them viſible at a very near diſtance, we have a variety of glaſſes, and inſtruments of different conſtructions, uſually called microſcopes, by which thoſe minute objects appear many thouſand times larger than to the naked Eye; and thereby enrich the mind with diſcoveries of the ſublimeſt nature, in regard to creative power, wiſdom, and œconomy.

EYE-*glaſs*, in Optical Inſtruments, is that which is next the Eye in uſing the machine. This is uſually a lens convex on both ſides; but Euſtachia Divini long ſince invented a microſcope of this kind, the power of which he places very greatly above that of the common ſort; and this chiefly depending on the Eye-glaſs, which was double, conſiſting of two plano-convex glaſſes, ſo placed as to touch one another in the middle of their convex ſurface. This inſtrument is well ſpoken of by Fabri in his Optics, and as poſſeſſing this peculiar excellence, that it ſhews all the objects flat, and not crooked, and takes in a large area, though it magnifies very much.

Bull's EYE, a ſtar of the firſt magnitude, in the Eye of the conſtellation Taurus, the bull, and by the Arabs called *Aldebaran*.

F.

FACE, or Façade, in Architecture, is ſometimes uſed for the front or outward part of a building, which immediately preſents itſelf to the eye; or the ſide where the chief entrance is, or next the ſtreet, &c.

Face, Facia, or Fascia, alſo denotes a flat member, having a conſiderable breadth, and but a ſmall projecture. Such are the bands of an architrave, larmier, &c.

Face, in Aſtrology, is uſed for the 3d part of a ſign.—Each ſign is ſuppoſed to be divided into three faces, of 10 degrees each: the firſt 10 degrees compoſe the firſt face; the next 10 degrees, the 2d face; and the laſt 10, the 3d face.—Venus is in the 3d face of Taurus; that is, in the laſt 10 degrees of it.

Faces *of a Baſtion*, in Fortification, are the two foremoſt ſides, reaching from the flanks to the outermoſt point of the baſtion, where they meet, and form the ſaliant angle of the baſtion. Theſe are uſually the firſt parts that are undermined, or beaten down; becauſe they reach the fartheſt out, are the leaſt flanked, and are therefore the weakeſt.

Face *of a Place*, is the extent between the outermoſt points of two adjacent baſtions; containing the curtain, the two flanks, and the two faces of thoſe baſtions that look towards each other. This is otherwiſe called the Tenaille of the place.

Face *Prolonged*, is that part of a line of defence raſant, which is between the angle of the epaule or ſhoulder of a baſtion and the curtain; or the line of a defence raſant diminiſhed by the face of the baſtion.

FACIA, in Architecture. See Face, and Fascia.

FACTORS, in Multiplication, in Arithmetic, a name given to the two numbers that are multiplied together, viz, the multiplicand and multiplier; ſo called becauſe they are to facere productum, make or conſtitute the factum or product.

FACTUM, the product of two quantities multiplied together. As, the factum of 3 and 4 is 12; and the factum of $2a$ and $5b$ is $10ab$.

FACULÆ, in Aſtronomy, a name given by Scheiner, and others after him, to certain bright ſpots on the ſun's diſc, that appear more bright and lucid than the reſt of his body.

Hevelius aſſures us that, on July 20, 1634, he obſerved a facula whoſe breadth was equal to a 3d part of the ſun's diameter. He ſays too that the maculæ often change into Faculæ; but theſe ſeldom or never into maculæ. And ſome authors even contend that all the maculæ degenerate into Faculæ before they quite diſappear. Many authors, after Kircher and Scheiner, have repreſented the ſun's body full of bright, fiery ſpots, which they conceive to be a ſort of volcanos in the body of the ſun: but Huygens, and others of the lateſt and beſt obſervers, finding that the beſt teleſcopes diſcover nothing of the matter, agree entirely to explode the phenomena of Faculæ. All the foundation he could ſee for the notion of Faculæ, he ſays, was, that in the darkiſh clouds which frequently ſurround the maculæ, there are ſometimes ſeen little points or ſparks brighter than the reſt. Their cauſe is attributed by theſe authors to the tremulous agitation of the vapours near our earth; the ſame as ſometimes ſhews a little unevenneſs in the circumference of the ſun's diſc when viewed through a teleſcope. Strictly then, the Faculæ are not eructations of fire and flame, but refractions of the ſun's rays in the rarer exhalations, which, being condenſed near that ſhade, ſeem to exhibit a light greater than that of the ſun.

FACULTY, denotes the ſeveral parts of an univerſity, divided according to the arts or ſciences taught or profeſſed there. In moſt univerſities there are four Faculties; that of arts, which includes philoſophy and the humanities or languages, and is the moſt ancient and extenſive; the 2d is that of theology; the 3d, that of medicine; and the 4th, juriſprudence, or laws.

FALCATED, one of the phaſes of the planets, vulgarly called horned. The aſtronomers ſay, the moon, or any planet, is Falcated, when the enlightened part appears in form of a creſcent, like a ſickle, or reaping-hook, which by the Latins is called falx. The moon is Falcated while ſhe moves from the 3d quarter to the conjunction, and ſo on from hence to the firſt quarter; the bright part appearing then like a creſcent, viz during the firſt and laſt quarters. But during the 2d and 3d quarters, the light part appears gibbous, and the dark part Falcated.

FALCON or Faucon, and Falconet or Fauconet, certain old ſpecies of cannon, now long diſuſed.

FALL, the deſcent or natural motion of bodies towards the centre of the earth, &c. Galileo firſt diſcovered the ratio of the acceleration of falling bodies; viz, that the ſpaces deſcended from reſt are as the ſquares of the times of deſcent; or, which comes to the ſame thing, that if the whole time of Falling be divided into any number of equal parts, whatever ſpace it falls through in the firſt part of the time, it will Fall 3 times as far in the 2d part of time, and 5 times as far in the 3d portion of time, and ſo on, according to the uneven numbers 1, 3, 5, 7, &c. See Acceleration, Descent, Gravity, &c.

FALSE-braye, in Fortification. See Fausse-braye.

False *Poſition*, in Arithmetic. See Position.

False *Root*, a name given by Cardan, to the negative roots of equations, and numbers. So the root of 9 may be either 3 or − 3, the former he calls the true, and the latter the falſe or fictitious root; alſo of this equation $x^2 - x = 6$, the two roots are 3 and −2, the former true, and the latter Falſe.

FASCIA, in Architecture. See Facia and Face.

FASCIÆ, in Aſtronomy, are certain ſtripes or rows of bright parts, obſerved on the bodies of ſome of the planets, like ſwathes, bands, or belts; eſpecially on the planet Jupiter.

The Faſciæ, or belts of Jupiter, are more lucid than the reſt of the diſc, and are terminated by parallel lines. They are ſometimes broader and ſometimes narrower; nor do they always poſſeſs the ſame part of the diſc.

M. Huygens alſo obſerved a very large kind of Faſcia in Mars, in the year 1656; but it was darker than the reſt of the diſc, and occupied the middle part of it.

FASCINES, in Fortification, are faggots made of the twigs and ſmall branches of trees and bruſh wood, bound up in bundles; theſe, being mixed with earth, ſerve to fill up ditches, to make the parapets of trenches, batteries, &c.

FATHOM, an Engliſh meaſure of the length of 6 feet or 2 yards; and is taken from the utmoſt extent of both arms when ſtretched into a right line.

FATUUS *Ignis*. See Ignis *Fatuus*.

FAUCON, and Fauconet, the ſame as Falcon and Falconet; the old names of certain ſpecies of ordnance; which, as well as many other names, are now no longer in uſe, as it has been for ſome time the practice to denominate the ſeveral ſizes of cannon from the weight of their ball, inſtead of calling them by thoſe fanciful and unmeaning names.

FAUSSE-Braye, in Fortification, an elevation of earth, about three feet above the level ground; round the foot of the rampart on the outſide, defended by a parapet about four or five fathoms diſtant from the upper parapet, which parts it from the berme, and the edge of the ditch. The Fauſſe-braye is the ſame with what is otherwiſe called Chemin des rondes, and Baſſe enceinte; and its uſe is for the defence of the ditch.

FEATHER-edged, is a term uſed by workmen, for ſuch boards as are thicker on one edge, or ſide, than on the other.

FEBRUARY, the 2d month of the year, containing 28 days for three years, and every fourth year 29 days.—In the firſt ages of Rome, February was the laſt month of the year, and preceded January, till the Decemviri made an order that February ſhould be the 2d month of the year, and come after January.

FELLOWSHIP, Company, or Partnership, is a rule in arithmetic, of great uſe in balancing accounts among merchants, and partners in trade, teaching how to aſſign to every one of them his due ſhare of the gain or loſs, in proportion to the ſtock he has contributed, and the time it has been employed, or according to any other conditions. Or, more generally, it is a method of dividing a given number, or quantity, into any number of parts, that ſhall have any aſſigned ratios to one another. And hence comes this general rule: Having added into one ſum the ſeveral numbers that expreſs the proportions of the parts, it will be,

As that ſum of the proportional numbers :
Is to the given quantity that is to be divided : :
So is each proportional number :
To the correſponding ſhare of the given quantity.

For Ex. Suppoſe it be required to divide the number 120 into three parts that ſhall be in proportion to each other as the numbers 1, 2, 3.—Here 120 is the quantity to be divided, and 6 is the ſum of the numbers 1, 2, and 3, which expreſs the proportions of the parts; therefore as

$$6 : 120 :: \begin{cases} 1 : 20 \text{ the 1ſt part,} \\ 2 : 40 \text{ the 2d part,} \\ 3 : 60 \text{ the 3d part.} \end{cases}$$

This rule is uſually diſtinguiſhed into two caſes, one in which time is concerned, or in which the ſtocks of partners are continued for different times; and the other in which time is not conſidered; this latter being called Single Fellowſhip, and the former Double Fellowſhip.

Single FELLOWSHIP, or FELLOWSHIP *without Time*, is the caſe in which the times of continuance of the ſhares of partners are not conſidered, becauſe they are all the ſame; and in this caſe, the rule will be as above, viz,

As the whole ſtock of the partners :
Is to the whole gain or loſs : :
So is each one's particular ſtock :
To his ſhare of the gain or loſs.

Ex. Two partners, A and B, form a joint ſtock, of which A contributed 75l, and B 45l; with which they gain 30l: how much of it muſt each one have?

$$\text{As } 120 : 30 :: \begin{cases} 75 : 18\text{l. } 15\text{s.} = \text{A's ſhare,} \\ 45 : 11\text{l. } 5\text{s.} = \text{B's ſhare.} \end{cases}$$

Double FELLOWSHIP, or FELLOWSHIP *with Time*, is the caſe in which the times of the ſtocks continuing are conſidered, becauſe they are not all the ſame.

In this caſe, the ſhares of the gain or loſs muſt be proportional, both to the ſeveral ſhares of the ſtock, and to the times of their continuance, and therefore proportional to the products of the two. Hence this Rule: Multiply each particular ſhare of the ſtock by the time of its continuance, and add all the products together into one ſum; then ſay,

As that ſum of the products :
Is to the whole gain or loſs : :
So is each ſeveral product :
To the correſponding ſhare of the gain or loſs.

For Ex. A had in company 50l. for 4 months, and B 60l. for 5 months; and their gain was 24l: how muſt it be divided between them?

$$\begin{array}{cc} 50 & 60 \\ 4 & 5 \\ \hline 200 & 300 \\ 300 & \\ \hline \end{array}$$

$$\text{As } 500 : 24 :: \begin{cases} 200 : 9\text{l. } 12\text{s.} = \text{A's ſhare,} \\ 300 : 14\text{l. } 8\text{s.} = \text{B's ſhare.} \end{cases}$$

FERGUSON (JAMES), an eminent experimental philoſopher, mechaniſt, and aſtronomer, was born in Bamffshire, in Scotland, 1710, of very poor parents. At the very earlieſt age his extraordinary genius began to unfold itſelf. He firſt learned to read, by overhearing his father teach his elder brother: and he had made this acquiſition before any one ſuſpected it. He ſoon diſcovered a peculiar taſte for mechanics, which firſt aroſe on ſeeing his father uſe a lever. He purſued this ſtudy a conſiderable length, while he was yet very young; and made a watch in wood-work, from having once ſeen one. As he had at firſt no inſtructor, nor any help from books, every thing he learned had all the merit of an original diſcovery; and ſuch, with inexpreſſible joy, he believed it to be.

As ſoon as his age would permit, he went to ſervice; in which he met with hardſhips, which rendered his conſtitution feeble through life. While he was ſervant to a farmer (whoſe goodneſs he acknowledges in the modeſt and humble account of himſelf which he prefixed to one of his publications), he contemplated and learned to know the ſtars, while he tended the ſheep; and began the ſtudy of aſtronomy, by laying down, from his own obſervations only, a celeſtial globe. His kind maſter, obſerving theſe marks of his ingenuity, procured him the countenance and aſſiſtance of ſome neighbouring gentlemen. By their help and inſtructions he went on gaining farther knowledge, having by their means been taught arithmetic, with ſome algebra, and practical geometry. He had got ſome notion of drawing, and being ſent to Edinburgh, he there began to take portraits in miniature, at a ſmall price; an employment by which he ſupported himſelf and family for ſeveral years, both in Scotland and England, while he was purſuing more ſerious ſtudies. In London he firſt publiſhed ſome curious aſtronomical tables and calculations; and afterwards gave public lectures in experimental philoſophy, both in London and moſt of the country towns in England, with the higheſt marks of general approbation. He was elected a fellow of the Royal Society, and was excuſed the payment of the admiſſion fee and the uſual annual contributions. He enjoyed from the king a penſion of 50 pounds a year, beſides other occaſional preſents, which he privately accepted and received from different quarters, till the time of his death; by which, and the fruits of his own labours, he left behind him a ſum to the amount of about ſix thouſand pounds, inſtead of which all his friends had always entertained an idea of his great poverty. He died in 1776, at 66 years of age, though he had the appearance of many more years.

Mr. Ferguſon muſt be allowed to have been a very uncommon genius, eſpecially in mechanical contrivances and executions, for he executed many machines himſelf in a very neat manner. He had alſo a good taſte in aſtronomy, with natural and experimental philoſophy, and was poſſeſſed of a happy manner of explaining himſelf in an eaſy, clear, and familiar way. His general mathematical knowledge, however, was little or nothing. Of algebra he underſtood but little more than the notation; and he has often told me he could never demonſtrate one propoſition in Euclid's Elements; his conſtant method being to ſatisfy himſelf, as to the truth of any problem, with a meaſurement by ſcale and compaſſes. He was a man of a very clear judgment in any thing that he profeſſed, and of unwearied application to ſtudy: benevolent, meek, and innocent in his manners as a child: humble, courteous, and communicative: inſtead of pedantry, philoſophy ſeemed to produce in him only diffidence and urbanity.

The liſt of Mr. Ferguſon's public works, is as follows:

1. Aſtronomical Tables and Precepts, for calculating the true times of New and Full Moons, &c; 1763. —2. Tables and Tracts, relative to ſeveral arts and ſciences; 1767.—3. An Eaſy Introduction to Aſtronomy, for Young Gentlemen and Ladies: 2d edit. 1769.—4. Aſtronomy explained upon Sir Iſaac Newton's Principles; 5th edit. 1772.—5. Lectures on Select Subjects in Mechanics, Hydroſtatics, Pneumatics, and Optics; 4th edit. 1772.—6. Select Mechanical Exerciſes; with a ſhort Account of the life of the author, by himſelf; 1773.—7. The Art of Drawing in Perſpective made eaſy; 1775.—8. An Introduction to Electricity; 1775.—9. Two Letters to the Rev. Mr. John Kennedy; 1775.—10. A Third Letter to the Rev. Mr. John Kennedy; 1775.

FERMAT (Peter), counſellor of the parliament of Toulouſe, in France, who flouriſhed in the 17th century, and died in 1663. He was a general ſcholar, and a univerſal genius, cultivating juriſprudence, poetry, and mathematics, but eſpecially the latter, for his amuſement. He was contemporary and intimately connected with Merſenne, Torricelli, Des Cartes, Paſcal, Roberval, Huygens, Frenicle, and Carcavi, and ſeveral others the moſt celebrated philoſophers of their time. He was a firſt-rate mathematician, and poſſeſſed the fineſt taſte for pure and genuine geometry, which he contributed greatly to improve, as well as algebra.

Fermat was author of 1. A Method for the Quadrature of all ſorts of Parabolas.—2. Another on Maximums and Minimums: which ſerves not only for the determination of plane and ſolid problems; but alſo for drawing tangents to curve lines, finding the centres of gravity in ſolids, and the reſolution of queſtions concerning numbers: in ſhort a method very ſimilar to the Fluxions of Newton.—3. An Introduction to Geometric Loci, plane and ſolid.—4. A Treatiſe on Spherical Tangencies: where he demonſtrates in the ſolids, the ſame things as Vieta demonſtrated in planes.—5. A Reſtoration of Apollonius's two books on Plane Loci.—6. A General Method for the dimenſion of Curve Lines. Beſides a number of other ſmaller pieces, and many letters to learned men; ſeveral of which are to be found in his *Opera Varia Mathematica*, printed at Toulouſe, in folio, 1679.

FERMENTATION, an inteſtine motion, ariſing ſpontaneouſly among the ſmall and inſenſible particles of a mixed body, thereby producing a new diſpoſition, and a different combination of thoſe parts. Fermentation differs from diſſolution, as the cauſe from its effect, the latter being only a reſult or effect of the former.

FESTOON, in Architecture &c, a decoration in form of a garland or cluſter of flowers.

FICHANT Flank. See Flank.

Fichant *Line of Defence*. See Fixed *Line of Defence*.

FIELD-Fort. See Fortine.

Field-*Pieces*, are ſmall cannon, uſually carried along with an army in the field: ſuch as, one pounders, one and a half, two, three, four, ſix, nine, and 12 pounders; which, being light and ſmall, are eaſily carried.

Field-*Staff*, is a ſtaff carried by the gunners, in which they ſcrew lighted matches, when they are on ſervice; which is called arming the Field-ſtaffs. See Linstock.

Field *of View*, or *of Viſion*, is the whole ſpace or extent within which objects can be ſeen through an optical machine, or at one view of the eye without turning it.

The preciſe limits of this ſpace are not eaſily aſcertained, for the natural view of the eye. In looking at a ſmall diſtance, we have an imperfect glimpſe of objects through almoſt the extent of a hemiſphere, or at leaſt for above 60 degrees each way from the optic axis; but towards the extremity of this ſpace, objects are very imperfectly ſeen; and the diameter of the field of diſtinct viſion does not ſubtend an angle of more than 5 degrees at moſt, ſo that the diameter of a diſtinct image on the retina is leſs than $\frac{1}{100}$ of an inch; but it is probably much leſs.

Field-*Book*, in Surveying, a book uſed for ſetting down angles, diſtances, and other things, remarkable in taking ſurveys.

The pages of the Field-book may be conveniently divided into three columns. In the middle column are to be entered the angles taken at the ſeveral ſtations by the theodolite, with the diſtances meaſured from ſtation to ſtation. And the offsets, taken with the offset-ſtaff, on either ſide of the ſtation line, are to be entered in the columns on either ſide of the middle column, according to their poſition, on the right or left, with reſpect to that line: alſo on the right or left of theſe are to be ſet down the names and characters of the objects, with proper remarks, &c. See a ſpecimen in my Treatiſe on Menſuration, pa. 517, ed. 2d.

FIFTH, in Muſic, one of the harmonical intervals or concords; called by the ancients Diapente.

The Fifth is the 3d in order of the concords, and the ratio of the chords that produce it, is that of 3 to 2. It is called Fifth, becauſe it contains five terms, or ſounds, between its extremes, and four degrees; ſo that in the natural ſcale of muſic it comes in the 5th place, or order, from the fundamental.—The imperfect, or defective Fifth, by the ancients called Semidiapente, is leſs than the Fifth by a mean ſemitone.

FIGURAL, the ſame as Figurate numbers; which ſee.

FIGURATE *Numbers*, ſuch as do or may repreſent ſome geometrical figure, in relation to which they are always conſidered; as triangular, pentagonal, pyramidal, &c, numbers.

Figurate numbers are diſtinguiſhed into orders, according to their place in the ſcale of their generation, being all produced one from another, viz, by adding continually the terms of any one, the ſucceſſive ſums are the terms of the next order, beginning from the firſt order which is that of equal units 1, 1, 1, 1, &c; then the 2d order conſiſts of the ſucceſſive ſums of thoſe of the 1ſt order, forming the arithmetical progreſſion 1, 2, 3, 4, &c; thoſe of the 3d order are the ſucceſſive ſums of thoſe of the 2d, and are the triangular numbers 1, 3, 6, 10, 15, &c; thoſe of the 4th order are the ſucceſſive ſums of thoſe of the 3d, and are the pyramidal numbers 1, 4, 10, 20, 35, &c; and ſo on, as below:

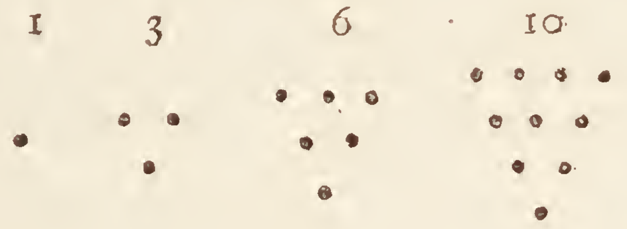
Order.	*Name.*	*Numbers.*					
1.	Equals.	1,	1,	1,	1,	1,	&c.
2.	Arithmeticals,	1,	2,	3,	4,	5,	&c.
3.	Triangulars,	1,	3,	6,	10,	15,	&c.
4.	Pyramidals,	1,	4,	10,	20,	35,	&c.
5.	2d Pyramidals,	1,	5,	15,	35,	70,	&c.
6.	3d Pyramidals,	1,	6,	21,	56,	126,	&c.
7.	4th Pyramidals,	1,	7,	28,	84,	210,	&c.

The above are all conſidered as different ſorts of triangular numbers, being formed from an arithmetical progreſſion whoſe common difference is 1. But if that common difference be 2, the ſucceſſive ſums will be the ſeries of ſquare numbers: if it be 3, the ſeries will be pentagonal numbers, or pentagons; if it be 4, the ſeries will be hexagonal numbers, or hexagons; and ſo on. Thus:

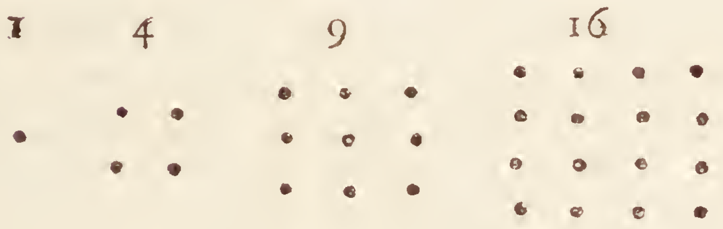
Arithmeticals.	1ſt *Sums*, or *Polygons.*	2d *Sums*, or 2d *Polygons.*
1, 2, 3, 4,	Tri. 1, 3, 6, 10	1, 4, 10, 20
1, 3, 5, 7,	Sqrs. 1, 4, 9, 16	1, 5, 14, 30
1, 4, 7, 10,	Pent. 1, 5, 12, 22	1, 6, 18, 40
1, 5, 9, 13,	Hex. 1, 6, 15, 28	1, 7, 22, 50
&c.		

And the reaſon of the names triangles, ſquares, pentagons, hexagons, &c, is, that thoſe numbers may be placed in the form of theſe regular figures or polygons, as here below :

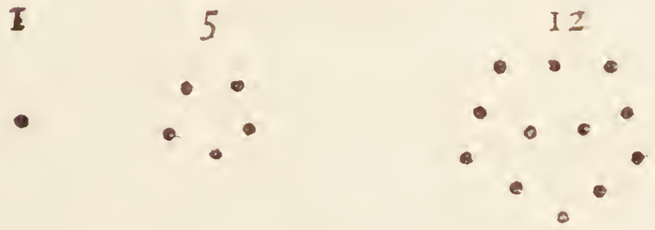
Triangles.



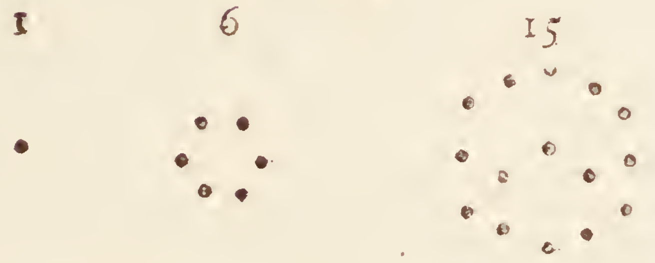
Squares.



Pentagons.



Hexagons.



But the Figurate numbers of any order may alſo be found without computing thoſe of the preceding orders ; which is done by taking the ſucceſſive products of as many of the terms of the arithmeticals 1, 2, 3, 4, 5, &c, in their natural order, as there are units in the number which denominates the order of Figurates required, and dividing thoſe products always by the firſt product : thus, the triangular numbers are found by dividing the products 1 × 2, 2 × 3, 3 × 4, 4 × 5, &c, each by the 1ſt pr. 1 × 2 ; the firſt pyramids by dividing the products 1 × 2 × 3, 2 × 3 × 4, 3 × 4 × 5, &c, by the firſt 1 × 2 × 3. And, in general, the figurate numbers of any order n, are found by ſubſtituting ſucceſſively 1, 2, 3, 4, 5, &c, inſtead of x in this general expreſſion $\frac{x \cdot x+1 \cdot x+2 \cdot x+3 \cdot \&c}{1 \cdot 2 \cdot 3 \cdot 4 \cdot \&c}$; where the factors in the numerator and denominator are ſuppoſed to be multiplied together, and to be continued till the number in each be leſs by 1 than that which expreſſes the order of the Figurates required. See Maclaurin's Fluxions, art. 351, in the notes ; alſo Simpſon's Algebra, pa. 213 ; or Malcolm's Arithmetic, pa. 396, where the ſubject of Figurates is treated in a very extenſive and perſpicuous manner.

FIGURE, in general, denotes the ſurface or terminating extremes of a body.——All finite bodies have ſome figure, form, or ſhape ; whence, figurability is reckoned among the eſſential properties of body, or matter : a body without Figure, would be an infinite body.

FIGURES, in Architecture and Sculpture, denote repreſentations of things made in ſolid matter ; ſuch as ſtatues, &c.

FIGURES, in Arithmetic, are the numeral characters, by which numbers are expreſſed or written, as the ten digits, 1, 2, 3, 4, 5, 6, 7, 8, 9, 0. Theſe are uſually called the Arabic, and Indian figures, from which people it is ſuppoſed they have been derived. They were brought into Europe by the Moors of Spain, and into England about 1130, as Dr. Wallis apprehends : ſee his Algebra, pa. 9. However, from ſome ancient dates, ſuppoſed to conſiſt wholly or in part of Arabian figures, ſome have concluded that theſe Figures, originally Indian, were known and uſed in this country at leaſt as early as the 10th century. The oldeſt date diſcovered by Dr. Wallis, was on a chimney piece, at Helmdon, in Northamptonſhire, thus M133, that is 1133. Other dates diſcovered ſince, are 1090, at Colcheſter, in Eſſex ; M16 or 1016, at Widgel-hall, near Buntingford, in Hertfordſhire ; 1011 on the north front of the pariſh church of Rumſey in Hampſhire ; and 975 over a gate-way at Worceſter.

Dr. Ward, however, has urged ſeveral objections againſt the antiquity of theſe dates. As no example occurs of the uſe of theſe figures in any ancient manuſcript, earlier than ſome copies of Johannes de Sacro Boſco, who died in 1256, he thinks it ſtrange that theſe Figures ſhould have been uſed by artificers ſo long before they appear in the writings of the learned ; and he alſo diſputes the fact. The Helmdon date, according to him, ſhould be 1233 ; the Colcheſter date 1490 ; that at Widgel-hall has in it no Arabic Figures, the 1 and 6 being I and G, the initial letters of a name ; and the date at Worceſter conſiſts, he ſuppoſes, of Roman numerals, being really MXV. Martyn's Abridg. Philoſ. Tranſ. vol. 9, pa. 420.

Mr. Gibbon obſerves (in his Hiſtory of the Decline and Fall of the Roman Empire, vol. v. pa. 321). that " under the reign of the caliph Waled, the Greek language and characters were excluded from the accounts of the public revenue. If this change was productive of the invention or familiar uſe of our preſent numerals, the Arabic characters or cyphers, as they are commonly ſtyled, a regulation of office has promoted the moſt important diſcoveries of arithmetic, algebra, and the mathematical ſciences."'

On the other hand it may be obſerved that, " according to a new, though probable notion, maintained by M. de Villaiſon) Anecdota Græca, tom. ii. p. 152, 157), our cyphers are not of Indian or Arabic invention. They were uſed by the Greek and Latin arithmeticians long before the age of Boethius. After the extinction of ſcience in the weſt, they were adopted in the Arabic verſions from the original manuſcripts, and reſtored to the Latins about the eleventh century.

Figure, in Aſtrology, a deſcription, draught, or conſtruction of the ſtate and diſpoſition of the heavens, at a certain point of time; containing the places of the planets and ſtars, marked down in a Figure of 12 triangles, called houſes. This is alſo called a Horoſcope, and Theme.

Figure, *of an Eclipſe*, in Aſtronomy, denotes a repreſentation upon paper &c, of the path or orbit of the ſun or moon, during the time of the eclipſe; with the different phaſes, the digits eclipſed, and the beginning, middle, and end of darkneſs, &c.

Figure, or *Delineation*, of the full moon, ſuch as, viewed through a teleſcope with two convex glaſſes, is of conſiderable uſe in obſervations of eclipſes, and conjunctions of the moon with other luminaries. In this Figure are uſually repreſented the maculæ or ſpots of the moon, marked by numbers; beginning with the ſpots that uſually enter firſt within the ſhade at the time of the eclipſes, and alſo emerge the firſt.

Figure, in Conic Sections, according to Apollonius, is the rectangle contained under the latus-rectum and the tranſverſe axis, in the ellipſe and hyperbola.

Figure, in Fortification, is the plan of any fortified place; or the interior polygon, &c. When the ſides, and the angles, are all equal, it is called a regular Figure; but when unequal, an irregular one.

Figure, in Geomancy, is applied to the extremes of points, lines, or numbers, thrown or caſt at random: on the combinations or variations of which, the ſages of this art found their divinations.

Figure, in Geometry, denotes a ſurface or ſpace incloſed on all ſides; and is either ſuperficial or ſolid; ſuperficial when it is incloſed by lines, and ſolid when it is incloſed or bounded by ſurfaces.

Figures are either ſtraight, curved, or mixed, according as their bounds are ſtraight, or curved, or both.—The exterior bounds of a Figure, are called its ſides; the loweſt ſide, its baſe; and the angular point oppoſite the baſe, the vertex of the Figure; alſo its height, is the diſtance of the vertex from the baſe, or the perpendicular let fall upon it from the vertex.

For *Figures*, equal, equiangular, equilateral, circumſcribed, inſcribed, plane, regular, irregular, ſimilar, &c; ſee the reſpective adjectives.

Apparent Figure, in Optics, that Figure, or ſhape, under which an object appears, when viewed at a diſtance. This is often very different from the true figure; for a ſtraight line viewed at a diſtance may appear but as a point; a ſurface as a line; a ſolid as a ſurface; and a crooked figure as a ſtraight one. Alſo, each of theſe may appear of different magnitudes, and ſome of them of different ſhapes, according to their ſituation with regard to the eye. Thus an arch of a circle may appear a ſtraight line; a ſquare or parallelogram, a trapezium, or even a triangle; a circle, an ellipſis; angular magnitudes, round; a ſphere, a circle; &c.

Alſo any ſmall light, as a candle, ſeen at a diſtance in the dark, will appear magnified, and farther off than it really is. Add to this, that when ſeveral objects are ſeen at a diſtance, under angles that are ſo ſmall as to be inſenſible, as well as each of the angles ſubtended by any one of them, and that next to it; then all theſe objects appear not only as contiguous, but as conſtituting and ſeeming but one continued magnitude.

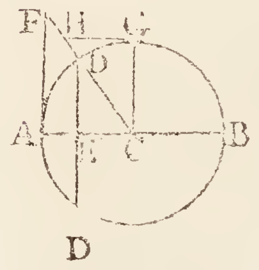
Figure *of the Sines*, *Coſines*, *Verſed-ſines*, *Tangents*, or *Secants*, *&c*, are Figures made by conceiving the circumference of a circle extended out in a right line, upon every point of which are erected perpendicular ordinates equal to the Sines, Coſines, &c, of the correſponding arcs; and then drawing the curve line through the extremity of all theſe ordinates; which is then the Figure of the Sines, Coſines, &c.

It would ſeem that theſe Figures took their riſe from the circumſtance of the extenſion of the meridian line by Edward Wright, who computed that line by collecting the ſucceſſive ſums of the ſecants, which is the ſame thing as the area of the Figure of the ſecants, this being made up of all the ordinates, or ſecants, by the conſtruction of the Figure. And in imitation of this, the Figures of the other lines have been invented. By means of the Figure of the ſecants, James Gregory ſhewed how the logarithmic tangents may be conſtructed, in his Exercitationes Geometricæ, 4to, 1668.

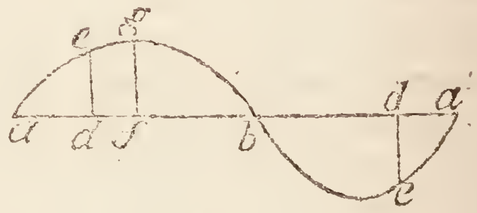
Conſtruction of the Figures of Sines, Coſines, &c.

Let ADB &c (fig. 1) be the circle, AD an arc, DE its ſine, CE its coſine, AE the verſed ſine, AF the tangent, GH the cotangent, CF the ſecant, and CH the coſecant. Draw a right line *aa* equal to the whole circumference ADGBA of the circle, upon which lay off alſo the lengths of ſeveral arcs, as the arcs at every 10°, from 0 at *a*, to 360° at the other end at *a*; upon theſe points raiſe perpendicular ordinates, upwards or downwards, according as the ſine, coſine, &c, is affirmative or negative in that part of the circle; laſtly, upon theſe ordinates ſet off the length of the ſines, coſines, &c, correſponding to the arcs at thoſe points of the line or circumference *aa*, drawing a curve line through the extremities of all theſe ordinates; which will be the Figure of the ſines, coſines, verſedſines, tangents, cotangents, ſecants, and coſecants, as in the annexed Figures. Where it may be obſerved, that the following curves are the ſame, viz, thoſe of the ſines and coſines, thoſe of the tangents and cotangents, and thoſe of the ſecants and coſecants; only ſome of their parts a little differently placed.

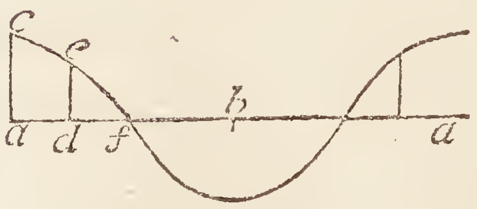
Fig. 1.



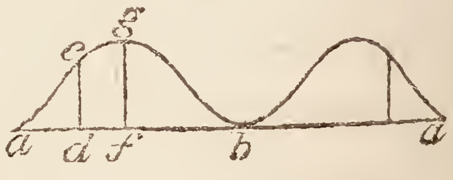
Sines.



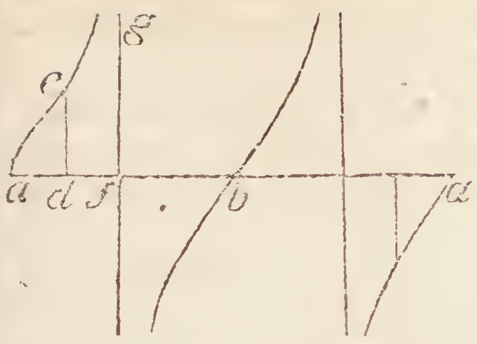
Coſines.



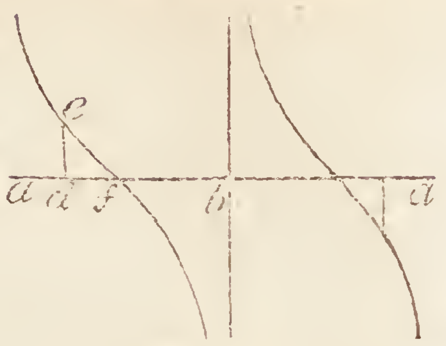
Verſedſines.



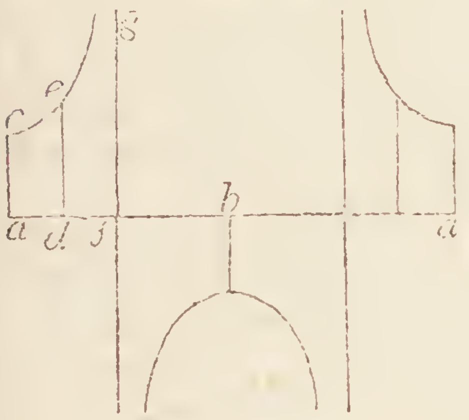
Tangents.



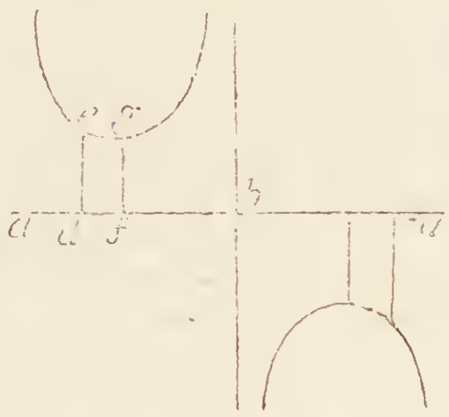
Cotangents.



Secants.



Coſecants.



It may be known when any of theſe lines, viz, the ſines, coſines, &c, are affirmative or negative, i. e. to be ſet upwards or downwards, by obſerving the following general rules for thoſe lines in the 1ſt, 2d, 3d, and 4th quadrants of the circle.

The ſines	in the 1ſt and 2d are	affirmative,
	in the 3d and 4th	negative:
The coſines	in the 1ſt and 4th are	affirmative,
	in the 2d and 3d	negative:
The tangents	in the 1ſt and 3d are	affirmative,
	in the 2d and 4th	negative:
The cotangents	in the 1ſt and 3d are	affirmative,
	in the 2d and 4th	negative:
The ſecants	in the 1ſt and 4th are	affirmative,
	in the 2d and 3d	negative:
The coſecants	in the 1ſt and 2d are	affirmative,
	in the 3d and 4th	negative:

And all the verſedſines are affirmative.

To find the Equation and Area, &c, to each of theſe Curves.

Draw any ordinate *de;* putting $r =$ the radius AC of the given circle, $x = ad$ or AD any abſciſs or arc, and $y = de$ its ordinate, which will be either the ſine DE $= s$, coſine CE $= c$, verſedſine AE $= v$, tangent AF $= t$, cotangent GH $= \tau$, ſecant CF $= ſ$, or coſecant CH $= \sigma$, according to the nature of the particular conſtruction. Now, from the nature of the circle, are obtained theſe following general equations, expreſſing the relations between the fluxions of a circular arc and its ſine, or coſine, &c.

$$\dot{x} = \frac{r\dot{s}}{\sqrt{r^2 - s^2}} = \frac{-r\dot{c}}{\sqrt{r^2 - c^2}} = \frac{r\dot{v}}{\sqrt{2rv - vv}} = \frac{r^2\dot{t}}{r^2 + t^2}$$

$$= \frac{-r^2\dot{\tau}}{r^2 + \tau^2} = \frac{r^2\dot{ſ}}{ſ\sqrt{ſ^2 - r^2}} = \frac{-r^2\dot{\sigma}}{\sigma\sqrt{\sigma^2 - r^2}}.$$

And theſe alſo expreſs the relation between the abſciſs and ordinate of the curves in queſtion, each in the order in which it ſtands; where x is the common abſciſs to all of them, and the reſpective ordinates are s, c, v, t, τ, $ſ$, and σ. And hence the area &c, of any of theſe curves may be found, as follows:

1. *In the* Figure *of Sines.*—Here $x = ad$, and $s =$ the ordinate *de*; and the equation of the curve, as above, is $\dot{x} = \dfrac{r\dot{s}}{\sqrt{r^2 - s^2}}$. Hence the fluxion of the area, or $s\dot{x}$ is $\dfrac{rs\dot{s}}{\sqrt{r^2 - s^2}}$; the correct fluent of which is $r^2 \mp r\sqrt{r^2 - s^2} = r^2 - rc = rv$ the rectangle of radius and verſ. i. e. − or + as s is increaſing or decreaſing; which is a general expreſſion for the area *ade* in the Figure of ſines. When $s = 0$, as at *a* or *b*, this expreſſion becomes 0 or $2r^2$; that is 0 at *a*, and $2r^2 =$ the area *acb*; or $r^2 =$ the area of *afg* when *ad* becomes a quadrant *af*.

2. *In the* Figure *of Coſines.*—Here $x = ad$, and $c = de$; and the equation of the curve is

$$\dot{x} = \frac{-r\dot{c}}{\sqrt{r^2 - c^2}}.$$ Hence the fluxion of the area is

$$c\dot{x} = \frac{-rc\dot{c}}{\sqrt{r^2 - c^2}};$$ and the fluent of this is $r\sqrt{r^2 - c^2} = rs$, the rectangle of radius and ſine, for the general area *adec*. When $s = r$, or $c = 0$, this becomes $r^2 =$ the area *afc*, whoſe abſciſs *af* is equal to a quadrant of the circumference; the ſame as in the Figure of the ſines, upon an equal abſciſs.

3. *In the* Figure *of Verſedſines.*—Here $x = ad$, and $v = de$; and the equation of the curve is

$$\dot{x} = \frac{r\dot{v}}{\sqrt{2rv - vv}}.$$ Hence the fluxion of the area is

$$v\dot{x} = \frac{rv\dot{v}}{\sqrt{2rv - vv}} = \frac{r\dot{v}\sqrt{v}}{\sqrt{2r - v}};$$ and the fluent of this is $rx - rs = r \times \overline{\text{AD} - \text{DE}}$ for the area *ade* in the Figure of verſed ſines. When AD or *ad* is a quadrant AG or *af*, this becomes $\frac{3{\cdot}1416}{2}r^2 - r^2 =$ the area *afg*. And when AD or *ad* is a ſemicircle *ab*, it becomes $3{\cdot}1416r^2 =$ the area *abg* in the Figure of verſedſines.

4. *In the* Figure *of Tangents.*—Here $x = ad$, and $t = de$; and the equation of the curve is $\dot{x} = \dfrac{r^2\dot{t}}{r^2 + t^2}$. Hence the fluxion of the area is $t\dot{x} = \dfrac{r^2t\dot{t}}{r^2 + t^2}$; and the correct fluent of this is $\frac{1}{2}r^2 \times$ hyp. log. of $\dfrac{r^2 + t^2}{r^2} = r^2 \times$ hyp. log. of $\dfrac{\sqrt{r^2 + t^2}}{r} = r^2 \times$ hyp. log. of $\dfrac{ſ}{r}$. And hence the Figure of the tangents may be uſed for conſtructing the logarithmic ſecants; a property that was remarked by Gregory at the end of his Exercit. Geomet.

When *ad* becomes a quadrant *af*, t being then infinite, this becomes infinite for the area *afg*. And the ſame for the Figure of cotangents, beginning at *f* inſtead of *a*.

5. *For the* Figure *of the Secants.*—Here $x = ad$, and $ſ = de$; and the equation of the curve is $\dot{x} = \frac{r^2\dot{ſ}}{ſ\sqrt{ſ^2 - r^2}}$.

Hence the fluxion of the area is $ſ\dot{x} = \frac{r^2\dot{ſ}}{\sqrt{ſ^2 - r^2}}$; the fluent of which is $r^2 \times$ hyp. log. of $\frac{ſ + \sqrt{ſ^2 - r^2}}{r}$ for the general area *ade*. And when *ad* becomes the quadrant *af*, this expreſſion becomes infinite for the area *afg*.

The ſame proceſs will ſerve for the Figure of coſecants, beginning at *f* inſtead of *a*.

From hence the meridional parts in Mercator's chart may be calculated for any latitude AD or *ad*: For the merid. parts : are to the arc of latitude AD : : as the ſum of the ſecants : to the ſum of as many radii or : : as the area *ade* : to *ad* × radius *ac* or AD × AC in the firſt figure.

FILLET, in Architecture, any little ſquare member or ornament uſed in crowning a larger moulding.

FINÆUS (Orontius), in French *Finé*, profeſſor of mathematics in the Royal-college of Paris, was the ſon of a phyſician, and was born at Briançon in Dauphiné in 1494. He went young to Paris, where his friends procured him a place in the college of Navarre. He applied himſelf there to philoſophy and polite literature; but more eſpecially to mathematics, in which, having a natural propenſity, he made a conſiderable proficiency. Particularly he made a good progreſs in mechanics; in which, having both a genius to invent inſtruments, and a ſkilful hand to make them, he gained much reputation by the ſpecimens he gave of his ingenuity.

Finæus firſt made himſelf publicly known by correcting and publiſhing Siliceus's *Arithmetic*, and the *Margarita Philoſophica*. He afterwards read private lectures in mathematics, and then taught that ſcience publicly in the college of Gervais: from the reputation of which, he was recommended to Francis the 1ſt, as the propereſt perſon to teach mathematics in the new college which that prince had founded at Paris. And here, though he ſpared no pains to improve his pupils, he yet found time to publiſh a great many books upon moſt parts of the mathematics. But neither his genius, his labours, his inventions, and the eſteem which numberleſs perſons ſhewed him, could ſecure him from that fate which ſo often befalls men of letters. He was obliged to ſtruggle all his life-time with poverty; and when he died, left a numerous family deeply in debt. However, as merit muſt always be eſteemed in ſecret, though it ſeldom has the luck to be rewarded openly; ſo Finæus's children found Mecænaſes, who for their father's ſake aſſiſted his family.—He died in 1555, at 61 years of age.

Like all other mathematicians and aſtronomers of thoſe times, he was greatly addicted to aſtrology; and had the misfortune to be a long time impriſoned for having predicted ſome things, that were not acceptable to the court of France. He was alſo one of thoſe, who vainly boaſted of having found out the quadrature of the circle. An edition of his works, tranſlated into the Italian language, was publiſhed in 4to, at Venice, 1587; conſiſting of Arithmetic, Practical Geometry, Coſmography, Aſtronomy, and Dialling.

FINITE, the property of any thing that is bounded or limited, either in its power, or extent, or duration, &c; as diſtinguiſhed from the property of infinite, or without bounds.

FINITOR, the horizon; being ſo called, becauſe it finiſhes or bounds the ſight or proſpect.

FIRE, is that ſubtile inviſible cauſe by which bodies are made hot to the touch, and expanded or enlarged in bulk; by which fluids are rarefied into vapour; or ſolid bodies become fluid, and at laſt either diſſipated and carried off in vapour, or elſe melted into glaſs. It ſeems alſo to be the chief agent in nature on which animal and vegetable life have an immediate dependence.

The diſputes concerning fire, which long exiſted among philoſophers, have now in a great meaſure ſubſided. Thoſe celebrated philoſophers of the laſt century, Bacon, Boyle, and Newton, were of opinion, that Fire was not a ſubſtance diſtinct from other bodies, but that it entirely conſiſted in the violent motion of the parts of any body, which was produced by the mechanical force of impulſion, or of attrition. So Boyle ſays, when a piece of iron becomes hot by hammering, "there is nothing to make it ſo, except the forcible motion of the hammer impreſſing a vehement and variouſly determined agitation on the ſmall parts of the iron." And Bacon defines heat, which he makes ſynonymous with Fire, an "expanſive undulatory motion in the minute particles of a body, whereby they tend with ſome rapidity from a centre towards a circumference, and at the ſame time a little upwards." And according to Newton, Fire is a body heated ſo hot as to emit light copiouſly; for what elſe, ſays he, is red-hot iron, but Fire? and what elſe is a fiery coal than red-hod wood? by which he ſuggeſts, that bodies which are not Fire, may be changed and converted into it.

On the other hand, the chemiſts ſtrenuouſly contended that Fire was a fluid of a certain kind, diſtinct from all others, and univerſally preſent throughout the whole globe. Boerhaave particularly maintained this doctrine; and in ſupport of it brought this argument, that flint and ſteel would ſtrike fire, and produce the ſame degree of heat in Nova Zembla as they would do under the equator. Other arguments were drawn from the increaſed weight of metallic calces, which they thought proceeded from the fixing of the element of Fire in the ſubſtance whoſe weight was thus increaſed. For a long time however, the matter was moſt violently diſputed; but the mechanical philoſophers at laſt prevailed through the deference paid to the principles of Newton, though he himſelf had ſcarcely taken any active part in the conteſt.

The experiments of Dr. Black however ſeemed to bring the diſpute to a deciſion, and that in favour of the chemiſts, concerning what he called latent-heat. From theſe diſcoveries it appears, that Fire may exiſt in bodies in ſuch a manner, as not to diſcover itſelf in any other way than by its action on the minute parts of the body; but that ſuddenly this action may be changed in ſuch a manner, as no longer to be directed upon the particles of the body itſelf, but upon external objects; in which caſe we then perceive its action by our ſenſe of feeling, or diſcover it by the thermometer, and call it heat, or ſenſible Fire.

From this diſcovery, and others in electricity, it is now pretty generally allowed, that Fire is a diſtinct fluid, capable of being transferred from one body to another. But when this was diſcovered, another queſtion no leſs perplexing aroſe, viz, what kind of a fluid it was; or whether it bears any analogy to thoſe with which we are better acquainted. Now there are found two fluids, viz, the ſolar light, and the electric matter, both of which occaſionally act as fire, and which therefore ſeem likely to be all the ſame at the bottom; and popularly the matter has been long ſince determined; the ſolar rays and the electric fluid having been indifferently accounted elementary Fire. Some indeed have imagined both theſe fluids to be mere phlogiſton itſelf, or at leaſt containing a large portion of it; and Mr. Scheele went ſo far in this way as to form an hypotheſis, which he endeavoured to ſupport by experiments, that Fire is compoſed of phlogiſton and dephlogiſticated air. But it is now aſcertained beyond doubt, that the reſult of ſuch a combination is not Fire, but fixed air.

It was long ſince obſerved by Newton, that heat was certainly conveyed by a medium more ſubtile than the common air; for two thermometers, one included in the vacuum of an air-pump, the other placed in the open air, at an equal diſtance from the fire, would grow equally hot in nearly the ſame time. This and other experiments ſhew, that Fire exiſts and acts where there is no other matter, and of conſequence it is a fluid per ſe, independent of every terreſtrial ſubſtance, without being generated or compounded of any thing we are yet acquainted with. To determine the nature of the fluid, we have only to conſider whether any other can be diſcovered which will paſs through the perfect vacuum juſt mentioned, and act there as Fire. Such a fluid is found in the ſolar light, which is well known to act even in vacuo as the moſt violent Fire. The ſolar light will likewiſe act in the very ſame manner in the moſt intenſe cold; for M. de Sauſſure has found, that on the cold mountain top the ſun-beams are equally powerful as on the plain below, if not more ſo. It appears therefore, that the ſolar light will produce heat independent of any other ſubſtance whatever; that is, where no other body is preſent, at leaſt as far as we can judge, except the light itſelf, and the body to be acted upon. We cannot therefore avoid concluding, that a certain modification of the ſolar light is the cauſe of heat, expanſion, vapour, &c, and anſwers to the reſt of the characters given in the foregoing definition of Fire, and that independent of any other ſubſtance whatever.

It is very probable too, that the electric matter is no other than the ſolar light abſorbed by the earth, and thus becoming ſubject to new laws, and aſſuming many properties apparently different from what it has when it acts as light. Even in this caſe it manifeſts its identity with Fire or light, viz, by producing a moſt intenſe heat where a large quantity of it paſſes through a ſmall ſpace. So that at any rate, the experiments which have already been made, and the proofs drawn from the phenomena of nature, ſhew ſuch a ſtrong affinity between the elements of Fire, light and electricity, that we may not only aſſert their identity upon the moſt probable grounds, but lay it down as a poſition againſt which at preſent no argument of any weight has an exiſtence.

Fire-*Arrow*, is a ſmall iron dart, furniſhed with ſprings and bars, and alſo a match impregnated with ſulphur and powder, which is wound about its ſhaft. It is chiefly uſed by privateers and pirates to fire the ſails of the enemy's ſhip, and for this purpoſe it is diſcharged from a muſketoon, or a ſwivel gun. The match being kindled by the exploſion, it communicates the flame to the ſail againſt which it is directed, where the arrow faſtens itſelf by means of its bars and ſprings. This weapon is peculiar to hot climates, particularly the Weſt Indies; the ſails being very dry, are quickly ſet on Fire, and the Fire is ſoon conveyed to the maſts, rigging, and finally to the veſſel itſelf.

Fire-*Balls*, in Artillery, are certain balls compoſed of combuſtible matters, ſuch as fine or mealed powder, ſulphur, ſaltpetre, roſin, pitch, &c. Theſe are thrown into the enemy's works in the night time, to diſcover where they are; or to ſet Fire to houſes, galleries, or blinds of the beſiegers, &c. They are ſometimes armed with ſpikes or hooks of iron, that they may not roll off, but ſtick or hang where they are to take effect.

Fire-*Balls*, or *Fiery-Meteors*, in Meteorology, a kind of luminous bodies uſually appearing at a great height above the earth, with a ſplendor ſurpaſſing that of the moon, and ſometimes apparently as large. They have not been found to obſerve any regular courſe or motion, but, on the contrary, moving in all directions, and with very different degrees of celerity; frequently breaking into ſeveral ſmaller ones; ſometimes making a ſtrong hiſſing ſound, ſometimes burſting or vaniſhing with a loud report, and ſometimes not.

Theſe luminous appearances doubtleſs conſtitute one part of the ancient prodigies, blazing ſtars, or comets, which laſt they ſometimes reſemble in being attended with a train; but more often they appear round. The firſt of theſe of which we have any accurate account, was obſerved by Dr. Halley, and ſome other philoſophers at different places, in the year 1719, the height of which above the ſurface of the earth was computed at more than 70 miles. Many others have been pretty accurately obſerved ſince that time, and deſcribed by philoſophers, as in the French Memoirs, and in the Philoſ. Tranſ. vols. 30, 41, 42, 43, 46, 47, 48, 51, 53, 54, 63, 74, &c. The velocities, directions, appearances, and heights of all theſe were found to be very various; though the height of all of them was ſuppoſed above the limits aſſigned to our atmoſphere, or where it loſes its refractive power. The moſt remarkable of thoſe on record, appeared on the 18th of Auguſt 1783, about 9 o'clock in the evening. It was ſeen to the northward of Shetland, and took a ſouth-eaſterly direction for an immenſe ſpace, being obſerved as far as the ſouthern provinces of France, and by ſome it was ſaid to have been ſeen at Rome, paſſing over a ſpace of 1000 miles in about half a minute of time, and at a very great height. During its courſe it appeared ſeveral times to change its ſhape; ſometimes appearing in the form of one ball, ſometimes of two or more; ſometimes with a train, and ſometimes without one.

There are divers opinions concerning the nature and origin of theſe Fire-balls. The firſt thing that occurred to philoſophers on this ſubject was, that the meteors in queſtion were burning bodies riſing from the ſurface of the earth, and flying through the atmoſphere with great rapidity. But this hypotheſis was ſoon rejected, on conſidering that there was no power known by which ſuch bodies could either be raiſed to a ſufficient height, or projected with the velocity of the meteors. The next hypotheſis was, that, inſtead of one ſingle body, they conſiſt of a train of ſulphureous vapours, extending a vaſt way through the atmoſphere, and being kindled at one end, diſplay the luminous appearances in queſtion by the fire running from one end of the train to the other. But it is not eaſy to conceive how ſuch matters can exiſt and be diſpoſed in ſuch lines in ſo rare a part of the atmoſphere, and even to burn there, in an almoſt perfect vacuum. For which reaſon this hypotheſis was abandoned, for another, which was, that thoſe meteors are permanent ſolid bodies, not riſing from the earth, but revolving round it in very excentric orbits, and thus in their perigeon moving with vaſt rapidity. But as the various appearances of one and the ſame meteor, to obſervers at different places, are not compatible with the idea of a ſingle body ſo revolving, this hypotheſis has alſo been given up in its turn. Another hypotheſis that has ſometimes been advanced is this; viz, that theſe meteors are a kind of bodies that take fire as ſoon as they come within the atmoſphere of the earth. But this cannot be ſuppoſed without implying a previous knowledge of theſe bodies, which it is impoſſible we can have. The only opportunity we can have of ſeeing them, is when they are on fire. Before that time they are in an inviſible and unknown ſtate; and it is ſurely improper to argue concerning them in this ſtate, or pretend to determine any one of their properties, when it is not in our power at all to ſee or inveſtigate them. As theſe meteors therefore never manifeſt themſelves to our ſenſes but when they are on fire, the only rational concluſion we can draw from thence is, that they have no exiſtence in any other ſtate; and conſequently that their ſubſtance muſt be compoſed of that fluid which, when acting after a certain manner, becomes luminous and ſhews itſelf as fire; remaining inviſible and eluding our reſearches in every other caſe. On this hypotheſis, it is now pretty generally concluded, that the Fire-balls are great bodies of electric matter, moving from one part of the heavens where, to our conception, it is ſuperabundant, to another where it is deficient: a concluſion attended with much probability, from the analogy obſerved between electricity and the phenomena of theſe meteors: and hence theſe Fire-balls appear to be of the ſame family with ſhooting-ſtars, lightning, the aurora-borealis, &c, being all referred to the ſame origin, viz, the electricity of the atmoſphere.

FIRE-*Engine*, is a machine for extinguiſhing accidental Fires by means of a ſtream or jet of water. The common ſquirting Fire-engine conſiſts of a lifting pump placed in a veſſel of water, and wrought by two levers that act always together. During the ſtroke, the water raiſed by the piſton of the pump ſpouts forcibly through a pipe joined to the pump-barrel, and made capable of any degree of elevation by means of a yielding leather pipe, or by a ball and ſocket turning every way, ſcrewed on the top of the pump. The veſſel containing the water is covered with a ſtrainer, to prevent the mud, &c, which is poured into it with the water, from choking the pump-work. Between the ſtrokes of this engine the ſtream is diſcontinued, for want of an air veſſel. However, in ſome caſes, Engines of this conſtruction have their uſe, becauſe the ſtream, though interrupted, is much ſmarter than when the engine is made to throw water in a continued ſtream. See theſe Engines particularly deſcribed in Deſaguliers's Exper. Philoſ. vol. 2, pa. 505; or Martin's Philoſ. Britan. vol. 2, pa. 69. See alſo the figure of them, plate viii. fig. 3.

FIRE-*Engine*, is alſo ſometimes uſed for the machine employed in raiſing water by ſteam, and more properly called STEAM-*Engine*; which ſee.

FIRE-*Lock*, or *Fuſil*, a ſmall gun or muſket, which fires with a flint and ſteel; as diſtinguiſhed from the old muſket, or match-lock, which was fired with a match. The Fire-lock is now in common uſe with the European armies, and carried by the foot-ſoldiers. It is uſually about 3 feet 8 inches in the barrel; and, including the ſtock, 4 feet 8 inches, carrying a leaden bullet, of which 29 make 2lb. The diameter of the ball is ·55, and that of the barrel ·56 parts of an inch. The time of the invention of Fire-locks is uncertain; but they were uſed in 1690.

FIRE-*Places*, are contrivances for communicating heat to rooms, and alſo for anſwering various purpoſes of art and manufacture.

The general properties of air and fire, on which their conſtruction chiefly depends, are the following, viz, that air is rarefied by heat, and condenſed by cold; i. e. the ſame quantity of air takes up more ſpace when warm than when cold. Air rarefied and expanded by heat, is ſpecifically lighter than it was before, and will riſe in other air of greater denſity: ſo that a fire being made in any chimney, the air about and over the fire is rarefied by the heat, thence becomes lighter, and ſo riſes in the funnel, and goes out at the top of the chimney: the other air in the room, flowing towards the chimney, ſupplies its place, is then rarefied in its turn, and riſes likewiſe; and the place of the air thus carried out of the room, is ſupplied by freſh air coming in through doors and windows, or, if they be ſhut, through every crevice with violence; or if the avenues to the room be ſo cloſed up, that little or no freſh ſupply of air can be obtained, the current up the funnel muſt flag; and the ſmoke, no longer driven up, float about in the room.

Upon theſe principles, various contrivances and kinds of Fire-grates and ſtoves have been deviſed, from the old very open and wide chimney places, down to the preſent modiſh ones, which are much narrowed in the front, opening, by ſide and back jambs, and a low breaſt or mantle, beſides the convenience of a flap, called a regiſter, that covers the top of the Fire-ſtove, but opening to any degree with a ſmall winch, which lifts the back part ſloping upwards, and ſo throws the ſmoke freely up the funnel, and yet admitting as little air to paſs as you pleaſe; by which ſimple means the warm air is kept very much in the room, while the very narrow and ſloping orifice promotes the briſk aſcent of the ſmoke, and yet prevents its return down again, for the ſame reaſon.

Another very ingenious, but more complex, apparatus, called the Penſylvania Fire-place, was invented by Dr. Franklin, by which a room is kept very warm by a conſtant ſupply of freſh hot air, that paſſes into it through the ſtove itſelf. See the deſcription in his Letters and Papers on Philoſophical Subjects.

FIRE-*Pot*, in the Military Art, is a ſmall earthen pot, into which is put a grenade, filled with fine powder till the grenade be covered; the pot is then covered with a piece of parchment, and two pieces of match laid acroſs and lighted. This pot being thrown where it is deſigned to do execution, breaks and fires the powder, and this again fires the powder in the grenade, which ought to have no fuze, that its operation may be the quicker.

Raſant, or *Razant* FIRE, is a fire from the artillery and ſmall arms, directed parallel to the horizon, or to thoſe parts of the works of a place that are defended.

Running FIRE, is when ranks of men fire one after another; or when the lines of an army are drawn out to fire on account of a victory; in which caſe each ſquadron or battalion takes the fire from that on its right, from the right of the firſt line to the left, and from the left to the right of the ſecond line, &c.

FIRE-*Ships*, in the Navy, are veſſels charged with combuſtible materials or artificial Fire-works; which having the wind of an enemy's ſhip, grapple her, and ſet her on fire.

Anderſon, in his Hiſtory of Commerce, vol. 1, pa. 432, aſcribes the invention to the Engliſh, in this inſtance, viz, ſome veſſels being filled with combuſtible matter, and ſent among the Spaniſh ſhips compoſing the Invincible Armada in 1588; and hence aroſe it is ſaid the terrible invention of Fire-ſhips.

But Livy informs us, that the Rhodians had invented a kind of Fire-ſhips, which were uſed in junction with the Roman fleet in their engagement with the Syrians, in the year 190 before Chriſt: cauldrons of combuſtible and burning materials were hung out at their prows, ſo that none of the enemies' ſhips durſt approach them: for theſe fell on the enemies' gallies, ſtruck their beaks into them, and at the ſame time ſet them on fire. Livy, lib. 37, cap. 30.

Wild-FIRE, is a kind of artificial or factitious fire, that burns even under water, and that with greater violence than out of it. It is compoſed of ſulphur, naphtha, pitch, gum, and bitumen, and it is only extinguiſhable by vinegar, mixed with ſand and urine, or by covering it with raw hides. It is ſaid its motion is contrary to that of natural fire, always following the direction in which it is thrown, whether it be downwards, ſideways, or otherwiſe.

The French call it Greek Fire, or Feu Gregeois, becauſe firſt uſed by the Greeks about the year 660, as is obſerved by the Jeſuit Petavius, on the authority of Nicetas, Theophanes, Cedrenus, &c. The inventor, according to the ſame author, was an engineer of Heliopolis, in Syria, named Callinicus, who firſt applied it in the ſea-fight commanded by Conſtantine Pogonates, againſt the Saracens, near Cyzicus, in the Helleſpont; and with ſuch effect, that he burnt the whole fleet, which contained 30,000 men. But others refer it to a much older date, and aſcribe the invention to Marcus Gracchus; an opinion which is ſupported by ſeveral paſſages, both in the Greek and Roman writers; which ſhew that it was anciently uſed by both theſe nations in their wars. See Scaliger againſt Cardan.

The ſucceſſors of Conſtantine uſed it on ſeveral occaſions, with great advantage: and it is remarkable that they were able to keep the ſecret of the compoſition to themſelves; ſo that no other nation knew it in the year 960.

It is recorded by Chorier, in his Hiſt. de Dauph. that Hugh, king of Burgundy, demanding ſhips of the emperor Leo for the ſiege of Freſne, deſired alſo the Greek Fire.

And F. Daniel gives a good deſcription of the Greek Fire, in his account of the ſiege of Damietta, under St. Louis. Every body, ſays he, was aſtoniſhed with the Greek Fire, which the Turks then prepared; and the ſecret of which is now loſt. They threw it out of a kind of mortar, and ſometimes ſhot it with an odd ſort of croſs-bow, which was ſtrongly bent by means of a handle, or winch, of much greater force than the bare arm. That which was thrown from the mortar ſometimes appeared in the air of the ſize of a tun, with a long tail, and a noiſe like that of thunder. The French, by degrees, got the ſecret of extinguiſhing it; in which they ſucceeded ſeveral times.

After all, perhaps the invention of the Wild-Fire is to be aſcribed to other nations, and to a ſtill older date, and that it was the ſame as that uſed among the Indians in Alexander's invaſion, when it was ſaid they fought with thunder and lightning, or ſhot Fire with a terrible noiſe.

FIRE-*Works*, otherwiſe called Pyrotechnia, are artificial Fires, or preparations made of gunpowder, ſulphur, and other inflammable and combuſtible ingredients, uſed on occaſion of public rejoicings, and other ſolemnities. The principal of theſe are rockets, ſerpents, ſtars, hail, mines, bombs, garlands, letters, and other devices.

The invention of Fire-works is attributed, by M. Mahudel, to the Florentines and people of Sienna; who found out likewiſe the method of adding to them decorations of ſtatues, with fire iſſuing from their eyes and mouths.

FIRKIN, an Engliſh meaſure of capacity; being the 4th part of a barrel; and containing 8 gallons of ale, ſoap, butter, or herrings; or 9 gallons of beer.

FIRLOT, a dry meaſure uſed in Scotland. The oat-firlot contains 21¼ pints of that country, or about 85 Engliſh pints; and the barley-firlot, 31 ſtandard pints. The wheat firlot contains about 2211 cubic inches; and therefore exceeds the Engliſh buſhel by 60 cubic inches, or almoſt an Engliſh quart.

FIRMAMENT, by ſome old aſtronomers, is the orb of the fixed ſtars, or the higheſt of all the heavens. But in ſcripture and common language it is uſed for the middle regions, or the ſpace or expanſe appearing like an arch quite around or above us in the heavens. Many ancients and moderns alſo accounted the Firmament a fluid matter; but thoſe who gave it the name of Firmament muſt have taken it for a ſolid one.

FIRMNESS, is the conſiſtence of a body; or that ſtate when its ſenſible parts cohere, or are united together, ſo that the motion of one part induces a motion

tion of the reſt. In which ſenſe firmneſs ſtands oppoſed to fluidity.

The firmneſs of bodies then depends on the connexion or coheſion of their particles ; and the cauſe of coheſion the Newtonians hold to be an attractive force, inherent in bodies, which binds their ſmall particles together ; exerting itſelf only at the points of contact, or extremely near them, and vaniſhing at greater diſtances.

FIRST *Mover*, in the old Aſtronomy, is the Primum Mobile, or that which gives motion to the other parts of the univerſe.

FISSURES, in the Hiſtory of the Earth, are certain interruptions, moſtly parallel to each other, that divide or ſeparate the ſtrata of it from one another, in nearly horizontal directions ; and the parts of the ſame ſtratum in nearly vertical directions.

FIXED *Line of Defence*, a line drawn along the face of the baſtion, and terminating in the curtain.

Fixed *Signs of the Zodiac*, according to ſome, are the four ſigns Taurus, Leo, Scorpio, Aquarius. They are ſo called becauſe the ſun paſſes them reſpectively in the middle of each quarter, when that ſeaſon is more ſettled and Fixed than under the ſigns which begin and end it.

Fixed *Stars*, are ſuch as conſtantly retain the ſame poſition and diſtance with reſpect to each other ; by which they are contradiſtinguiſhed from erratic or wandering ſtars, which are continually varying their ſituation and diſtance.—The Fixed Stars only are properly and abſolutely called ſtars ; the reſt having their peculiar denomination of planet or comet.

FIXITY, or Fixedness, the quality of a body which determines it fixed ; or a property which enables it to endure the fire and other violent agents.

A body may be ſaid to be fixed in two reſpects : 1ſt, When on being expoſed to the fire, or a corroſive menſtruum, its particles are indeed ſeparated, and the body rendered fluid, but without being reſolved into its firſt elements. The 2d, when the body ſuſtains the active force of the fire or menſtruums whilſt its integral parts are not carried off in fumes. Each kind of Fixity is the reſult of a ſtrong or intimate coheſion between the particles.

FLAME, the ſubtleſt and brighteſt part of the fuel, aſcending above it in a pyramidal or conical figure, and heated red-hot. Sir Iſaac Newton defines Flame as only red-hot ſmoke, or the vapour of any ſubſtance raiſed from it by fire, and heated to ſuch a degree as to emit light copiouſly. Is not Flame, ſays he, a vapour, fume, or exhalation, heated red-hot ; that is, ſo hot as to ſhine ? For bodies do not Flame without emitting a copious fume ; and this fume burns in the Flame. The ignis fatuus is a vapour ſhining without heat ; and is there not the ſame difference between this vapour and Flame, as between rotten wood ſhining without heat, and burning coals of fire ? In diſtilling hot ſpirits, if the head of the ſtill be taken off, the vapour which aſcends will take fire at the Flame of a candle, and turn into Flame. Some bodies, heated by motion or fermentation, if the heat grow intenſe, fume copiouſly ; and if the heat be great enough, the fumes will ſhine, and become Flame. Metals in fuſion do not Flame, for want of a copious fume. All flaming bodies, as oil, tallow, wax, wood, foſſil coal, pitch, ſulphur, &c, by burning, waſte in ſmoke, which at firſt is lucid ; but at a little diſtance from the body ceaſes to be ſo, and only continues hot. When the Flame is put out, the ſmoke is thick, and frequently ſmells ſtrongly : but in the Flame it loſes its ſmell ; and, according to the nature of the fuel, the Flame is of divers colours. That of ſulphur and ſpirit of wine is blue ; that of copper opened with ſublimate, green ; that of tallow, yellow ; of camphire, white ; &c. Newton's Optics, p. 318.

FLAMSTEED (John), an eminent Engliſh aſtronomer, being indeed the firſt aſtronomer royal, for whoſe uſe the royal obſervatory was built at Greenwich, thence called Flamſteed Houſe. He was born at Denby in Derbyſhire the 19th of Auguſt 1646. He was educated at the free ſchool of Derby, where his father lived ; and at 14 years of age was afflicted with a ſevere illneſs, which rendered his conſtitution tender ever after, and prevented him then from going to the univerſity, for which he was intended. He nevertheleſs proſecuted his ſchool education with the beſt effect ; and then, in 1662, on quitting the grammar ſchool, he purſued the natural bent of his genius, which led him to the ſtudy of aſtronomy, and cloſely peruſed Sacroboſco's book *De Sphæra*, which fell in his way, and which laid the groundwork of all that mathematical and aſtronomical knowledge, for which he became afterward ſo juſtly famous. He next procured other more modern books of the ſame kind, and among them Streete's *Aſtronomia Carolina*, then lately publiſhed, from which he learned to calculate eclipſes and the planets' places. Some of theſe being ſhewn to a Mr. Halton, a conſiderable mathematician, he lent him Riccioli's *Almageſtum Novum*, and Kepler's *Tabulæ Rudolphinæ*, which he profited much by. In 1669, having calculated ſome remarkable eclipſes of the moon, he ſent them to lord Brouncker, preſident of the Royal Society, which were greatly approved by that learned body, and procured him a letter of thanks from Mr. Oldenburg their ſecretary, and another from Mr. John Collins, with whom, and other learned men, Mr. Flamſteed for a long time afterwards kept up a correſpondence by letters on literary ſubjects. In 1670, his father, obſerving he held correſpondence with theſe ingenious gentlemen, adviſed him to take a journey to London, to make himſelf perſonally acquainted with them ; an offer which he gladly embraced, and viſited Mr. Oldenburg and Mr. Collins, who introduced him to Sir Jonas Moore, which proved the means of his greateſt honour and preferment. He here got the knowledge and practice of aſtronomical inſtruments, as teleſcopes, micrometers, &c. On his return, he called at Cambridge, and viſited Dr. Barrow, Mr. Iſaac Newton, and other learned men there, and entered himſelf a ſtudent of Jeſus College. In 1672 he extracted ſeveral obſervations from Mr. Gaſcoigne's and Mr. Crabtree's letters, which improved him greatly in dioptrics. In this year he made many celeſtial obſervations, which, with calculations of appulſes of the moon and planets to fixed ſtars for the year following, he ſent to Mr. Oldenburg, who publiſhed them in the Philoſ. Tranſ.

In 1673, Mr. Flamſteed wrote a ſmall tract concerning the true diameters of all the planets, when at their greateſt and leaſt diſtances from the earth ; which he lent to Mr. Newton in 1685, who made ſome uſe of it in the 4th book of his Principia.—In 1674 he wrote an ephemeris, to ſhew the falſity of Aſtrology, and the ignorance of thoſe who pretended to it: with calculations of the moon's riſing and ſetting; alſo occultations and appulſes of the moon and planets to the fixed ſtars. To which, at Sir Jonas Moore's requeſt, he added a table of the moon's ſouthings for that year; from which, and from Philips's theory of the tides, the high-waters being computed, he found the times come very near. In 1674 too, he drew up an account of the tides, for the uſe of the king. Sir Jonas alſo ſhewed the king, and the duke of York, ſome barometers and thermometers that Mr. Flamſteed had given him, with the neceſſary rules for judging of the weather; and otherwiſe took every opportunity of ſpeaking favourably of Flamſteed to them, till at length he brought him a warrant to be the king's aſtronomer, with a ſalary of 100l. per annum, to be paid out of the office of Ordnance, becauſe Sir Jonas was then Surveyor General of the Ordnance. This however did not abate our author's propenſity for holy orders, and he was accordingly ordained at Ely by biſhop Gunning.

On the 10th of Auguſt, 1675, the foundation of the Royal Obſervatory at Greenwich was laid; and during the building of it, Mr. Flamſteed's temporary obſervatory was in the queen's houſe, where he made his obſervations of the appulſes of the moon and planets to the fixed ſtars, and wrote his Doctrine of the Sphere, which was afterward publiſhed by Sir Jonas, in his Syſtem of the Mathematics.

About the year 1684 he was preſented to the living of Burſlow in Surry, which he held as long as he lived. Mr. Flamſteed was equally reſpected by the great men his contemporaries, and by thoſe who have ſucceeded ſince his death. Dr. Wotton, in his Reflections upon Ancient and Modern Learning, ſtyles our author one of the moſt accurate Obſervers of the Planets and Stars, and ſays he calculated tables of the eclipſes of the ſeveral ſatellites, which proved very uſeful to the aſtronomers. And Mr. Molyneux, in his *Dioptrica Nova*, gives him a high character; and, in the admonition to the reader prefixed to the work, obſerves, that the geometrical method of calculating a ray's progreſs is quite new, and never before publiſhed; and for the firſt hint of it, ſays he, I muſt acknowledge myſelf obliged to my worthy friend Mr. Flamſteed. He wrote ſeveral ſmall tracts, and had many papers inſerted in the Philoſophical Tranſactions, viz, ſeveral in almoſt every volume, from the 4th to the 29th, too numerous to be mentioned in this place particularly.

But his great work, and that which contained the main operations of his life, was the *Hiſtoria Cœleſtis Britannica*, publiſhed in 1725, in 3 large folio volumes. The firſt of which contains the obſervations of Mr. William Gaſcoigne, the firſt inventor of the method of meaſuring angles in a teleſcope by means of ſcrews, and the firſt who applied teleſcopical ſights to aſtronomical inſtruments, taken at Middleton, near Leeds in Yorkſhire, between the years 1638 and 1643; extracted from his letters by Mr. Crabtree; with ſome of Mr. Crabtree's obſervations about the ſame time; and alſo thoſe of Mr. Flamſteed himſelf, made at Derby between the years 1670 and 1675; beſides a multitude of curious obſervations, and neceſſary tables to be uſed with them, made at the Royal Obſervatory, between the years 1675 and 1689.—The 2d volume contains his obſervations, made with a mural arch of near 7 feet radius, and 140 degrees on the limb, of the meridional zenith diſtances of the fixed ſtars, ſun, moon, and planets, with their tranſits over the meridian; alſo obſervations of the diameters of the ſun and moon, with their eclipſes, and thoſe of Jupiter's ſatellites, and variations of the compaſs, from 1689 to 1719: with tables ſhewing how to render the calculation of the places of the ſtars and planets eaſy and expeditious. To which are added, the moon's place at her oppoſitions, quadratures, &c; alſo the planets' places, derived from the obſervations.—The 3d volume contains a catalogue of the right-aſcenſions, polar-diſtances, longitudes, and magnitudes of near 3000 fixed ſtars, with the correſponding variations of the ſame. To this volume is prefixed a large preface, containing an account of all the aſtronomical obſervations made before his time, with a deſcription of the inſtruments employed; as alſo of his own obſervations and inſtruments; with a new Latin verſion of Ptolomy's catalogue of 1026 fixed ſtars; and Ulegh-beig's places annexed on the Latin page, with the corrections: a ſmall catalogue of the Arabs: Tycho Brahe's of about 780 fixed ſtars: the Landgrave of Heſſe's of 386: Hevelius's of 1534: and a catalogue of ſome of the ſouthern fixed ſtars not viſible in our hemiſphere, calculated from the obſervations made by Dr. Halley at St. Helena, adapted to the year 1726.

This work he prepared in a great meaſure for the preſs, with much care and accuracy: but through a natural weakneſs of conſtitution, and the declines of age, he died of a ſtrangury before he had finiſhed it, December the 19th, 1719, at 73 years of age; leaving the care of finiſhing and publiſhing his work to his friend Mr. Hodgſon.—A leſs perfect edition of the *Hiſtoria Cœleſtis* had before been publiſhed, without his conſent, viz, in 1712, in one volume folio, containing his obſervations to the year 1705.

Thus then, as Dr. Keil obſerved, our author, with indefatigable pains, for more than 40 years watched the motions of the ſtars, and has given us innumerable obſervations of the ſun, moon, and planets, which he made with very large inſtruments, accurately divided, and fitted with teleſcopic ſights; whence we may rely much more on the obſervations he has made, than on former aſtronomers, who made their obſervations with the naked eye, and without the like aſſiſtance of teleſcopes.

FLANK, in Fortification, is that part of the baſtion which reaches from the curtain to the face; and it defends the curtain, with the oppoſite face and flank.

Oblique or *Second* FLANK, or FLANK *of the Curtain*, is that part of the curtain from whence the face of the oppoſite baſtion can be ſeen, being contained between the lines raſant and fichant, or the greater and leſs lines of defence; or the part of the curtain between the Flank and the point where the fichant line of defence terminates.

Covered, *Low*, or *Retired* FLANK, is the platform of the caſemate, which lies hid in the baſtion; and is otherwiſe called the Orillon.

Fichant FLANK, is that from whence a cannon playing, fires directly on the face of the oppoſite baſtion.

Raſant or *Razant* Flank, is the point from whence the line of defence begins, from the conjunction of which with the curtain, the ſhot only raſeth the face of the next baſtion, which happens when the face cannot be diſcovered but from the Flank alone.

Simple Flanks, are lines going from the angle of the ſhoulder to the curtain; the chief office of which is for the defence of the moat and place.

Flank, is alſo a term of war, uſed by way of analogy for the ſide of a battalion, army, &c, in contradiſtinction to the front and rear. So, to attack the enemy in Flank, is to diſcover and fire upon them on one ſide.

FLANKED, is uſed of ſomething that has Flanks, or may be approached on the Flank. As

Flanked *Angle*, which is that formed by the two faces of the baſtion, and forming its point or angle. Alſo,

Flanked *Line of Defence*, Flanked *Tenaille*, &c.

FLANKING, in general, is the diſcovering and firing upon the ſide of a place, body, battalion, &c. To Flank a place, or other work, is to diſpoſe it in ſuch a manner as that every part of it may be played upon both in front and rear.

Flanking means alſo defending. Any fortification that has no defence, but juſt right forwards, is faulty; and to render it complete, one part ought to be made to Flank the other. Hence the curtain is always the ſtrongeſt part of any place, becauſe it is flanked at each end.

Battalions alſo are ſaid to be Flanked by the wings of the cavalry. And a houſe is ſometimes ſaid to be Flanked with two pavilions, or two galleries; meaning it has a gallery, &c, on each ſide.——There are alſo Flanking Angle, Flanking Line of Defence, &c.

FLAT *Baſtion*, is that which is built on a right line, as on the middle of the curtain, &c.

FLEXIBLE, is the property or quality of a body that may be bent.

FLEXION, the ſame as Flexure.

FLEXURE, or Flexion, is the bending or curving of a line or figure.

When a line firſt bends one way, and then gradually changes to a bend the contrary way, the point where the two parts join, or where the bending changes to the other ſide, is called the point of inflexion, or of contrary Flexure.

FLIE, or Fly, that part of the mariner's compaſs, on which the 32 points of the wind are drawn, and over which the needle is placed, and faſtened underneath.

FLOAT-*Boards*, the boards fixed to the outer rim of underſhot water wheels, ſerving to receive the impulſe of the ſtream, by which the wheel is carried round.—There may be too many of theſe boards on a wheel. It is thought to be the beſt rule, to have their diſtance aſunder ſuch, that each of them may come out of the water as ſoon as poſſible, after it has received and acted with its full impulſe; or, which comes to the ſame thing, when the ſucceeding one is in a direction perpendicular to the ſurface of the water.

Floating *Bridge*, is a bridge of boats, caſks, &c, covered with planks, firmly bound together for the paſſage of men, horſes, or carriages, &c.

FLOOD, a Deluge or inundation of water.

Flood, is alſo uſed in ſpeaking of the tide, when it is riſing or flowing up; in contradiſtinction to the Ebb, which is when it is decreaſing or running out.

FLOORING, in Carpentry, is commonly underſtood of the boarding of the Floors. The meaſurement of Flooring is eſtimated in ſquares, of 100 ſquare feet each, or of 10 feet on each ſide every way; for 10 times 10 are 100. Hence the length of the floor being multiplied by its breath, in feet, and two figures cut off on the right-hand, gives the ſquares, and feet, or decimals cut off. Thus, a Floor being 22 feet long, and 16 wide;

then 22 length
16 breadth
―――
132
22
―――
3,52 the content is therefore 3 ſquares, and 52 feet, or decimals.

FLUENT, or *Flowing Quantity*, in the Doctrine of Fluxions, is the variable quantity which is conſidered as increaſing or decreaſing; or the Fluent of a given fluxion, is that quantity whoſe fluxion being taken, according to the rules of that doctrine, ſhall be the ſame with the given fluxion. See Fluxions.

Contemporary Fluents, are ſuch as flow together or for the ſame time. And the ſame is to be underſtood of Contemporary Fluxions.——When Contemporary Fluents are always equal, or in any conſtant ratio; then alſo are their fluxions reſpectively either equal, or in that ſame conſtant ratio. That is, if $x = y$, then is $\dot{x} = \dot{y}$; or if $x : y :: n : 1$, then is $\dot{x} : \dot{y} :: n : 1$; or if $x = ny$, then is $\dot{x} = n\dot{y}$.

It is eaſy to find the fluxions to all the given forms of Fluents; but, on the contrary, it is difficult to find the Fluents of many given fluxions; and indeed there are numberleſs caſes in which this cannot at all be done, excepting by the quadrature and rectification of curve lines, or by logarithms, or infinite ſeries.

This doctrine, as it was firſt invented by Sir Iſaac Newton, ſo it was carried by him to a conſiderable degree of perfection, at leaſt as to the moſt frequent, and moſt uſeful forms of fluents; as may be ſeen in his Fluxions, and in his Quadrature of Curves. Maclaurin, in his Treatiſe of Fluxions, has made ſeveral inquiries into Fluents, reducible to the rectification of the ellipſe and hyperbola: and D'Alembert has purſued the ſame ſubject, and carried it farther, in the Memoires de l'Acad. de Berlin, tom. 2, p. 200. To the celebrated Mr. Euler this doctrine is greatly indebted, in many parts of his various writings, as well as in the Inſtitutio Calculi Integralis, in 3 vols 4to, Petr. 1768. The ingenious Mr. Cotes contributed very much to this doctrine, in his Harmonia Menſurarum, concerning the meaſures of ratios and angles, in a large collection of different forms of fluxions, with their correſponding Fluents. And this ſubject was farther proſecuted in the ſame way by Walmeſley, in his Analyſe des Meſures des Rapportes et des Angles, a large vol. in 4to, 1749. Beſides many other Authors who, by their ingenious labours, have greatly contributed to facilitate and extend the doctrine of Fluents; as Emerſon, Simpſon, Landen, Waring, &c, in this country; with l'Hôpital,

pital, and many other learned foreigners. Laſtly, in 1785 was publiſhed at Vienna, by M. Paccaſſi, a German nobleman, Udhandlung uber eine neue Methode zu Integriren, being a method of integrating, or finding the Fluents of given fluxions, by the rules of the direct method, or by taking again the fluxion of the given fluxion, or the 2d fluxion of the fluent ſought; and then making every flowing quantity its fluxion, and 2d fluxion, in geometrical progreſſion; a method however, which, it ſeems, only holds true in the eaſieſt caſes or forms, whoſe fluents are eaſily had by the moſt common methods. See this method farther explained in the rules following.

As it is only in certain particular forms and caſes that the Fluents of given fluxions can be found; there being no method of performing this univerſally a priori, by a direct inveſtigation; like finding the fluxion of a given fluent quantity; we can do little more than lay down a few rules for ſuch forms of fluxions as are known, from the direct method, to belong to ſuch and ſuch kinds of Fluents or flowing quantities: and theſe rules, it is evident, muſt chiefly conſiſt in performing ſuch operations as are the reverſe of thoſe by which the fluxions are found to given flowing quantities. The principal caſes of which are as follow:

I. *To find the Fluent of a ſimple fluxion*; or that in which there is no variable quantity, and only one fluxional quantity. This is done by barely ſubſtituting the variable or flowing quantity inſtead of its fluxion, and is the reſult or reverſe of the notation only. Thus,

The Fluent of $a\dot{x}$ is ax.
The Fluent of $a\dot{y} + 2\dot{y}$ is $ay + 2y$.
The Fluent of $\sqrt{\dot{a}^2 + \dot{x}^2}$ is $\sqrt{a^2 + x^2}$.

II. *When any power of a flowing quantity is multiplied by the fluxion of the root.* Then, having ſubſtituted, as before, the flowing quantity for its fluxion, divide the reſult by the new index of the power. Or, which is the ſame thing, take out, or divide by, the fluxion of the root; add 1 to the index of the power; and divide by the index ſo increaſed.

So if the fluxion propoſed be - $3x^5\dot{x}$;
Strike out $\dot{x}$, then it is - - $3x^5$;
add 1 to the index, and it is - - $3x^6$;
divide by the index 6, and it is - $\frac{3}{6}x^6$ or $\frac{1}{2}x^6$;
which is the Fluent of the propoſed fluxion $3x^5\dot{x}$.

In like manner, the Fluent
of $4ax\dot{x}$ is $2ax^2$;
of $3x^{\frac{1}{2}}\dot{x}$ is $2x^{\frac{3}{2}}$;
of $ax^n\dot{x}$ is $\frac{a}{n+1}x^{n+1}$;
of $\frac{\dot{z}}{z^2}$ or $z^{-2}\dot{z}$ is $-z^{-1}$ or $\frac{-1}{z}$.
of $(a^3 + z^3)^4 z^2\dot{z}$ is $\frac{1}{15}(a^3 + z^3)^5$.

III. *When the root under a vinculum is a compound quantity; and the index of the part or factor without the vinculum increaſed by 1, is ſome multiple of that under the vinculum:* Put a ſingle variable letter for the compound root; and ſubſtitute its powers and fluxion inſtead of thoſe, of the ſame value, in the given quantity; ſo will it be reduced to a ſimpler form, to which the preceding rule can then be applied.

So, if the given fluxion be $\dot{F} = (a^2 + x^2)^{\frac{2}{3}}x^3\dot{x}$; where 3, the index of the quantity without the vinculum, increaſed by 1, makes 4, which is double of 2, the exponent of x^2 within the ſame; therefore putting $z = a^2 + x^2$, thence $x^2 = z - a^2$, and its fluxion is $2x\dot{x} = \dot{z}$; hence then $x^3\dot{x} = \frac{1}{2}x^2\dot{z} = \frac{1}{2}\dot{z}(z - a^2)$, and the given quantity $\dot{F}$ or $(a^2 + x^2)^{\frac{2}{3}}x^3\dot{x}$ is $=$ $\frac{1}{2}z^{\frac{2}{3}}\dot{z}(z - a^2)$ or $= \frac{1}{2}z^{\frac{5}{3}}\dot{z} - \frac{1}{2}a^2z^{\frac{2}{3}}\dot{z}$; and the Fluent of each term gives

$F = \frac{3}{16}z^{\frac{8}{3}} - \frac{3}{10}a^2z^{\frac{5}{3}} = 3z^{\frac{5}{3}}(\frac{1}{16}z - \frac{1}{10}a)$; or, by ſubſtituting the value of z inſtead of it, the ſame Fluent is $3(a^2 + x^2)^{\frac{5}{3}} \times (\frac{1}{13}x^2 - \frac{3}{80}a^2)$, or $\frac{3}{16} \cdot \overline{a^2 + x^2}|^{\frac{5}{3}} \cdot \overline{x^2 - \frac{3}{5}a^2}$.

IV. *When there are ſeveral terms involving two or more variable quantities, having the fluxion of each multiplied by the other quantity or quantities:* Take the Fluent of each term, as if there was only one variable quantity in it, namely that whoſe fluxion is contained in it, ſuppoſing all the others to be conſtant in that term; then if the Fluents of all the terms ſo found, be the very ſame quantity, that quantity will be the Fluent of the whole.

Thus, if the given fluxion be $\dot{x}y + x\dot{y}$. Then, the Fluent of $\dot{x}y$ is xy, ſuppoſing y conſtant; and the Fluent of $x\dot{y}$ is alſo xy, when x is conſtant; therefore the common reſulting quantity xy is the required Fluent of the given fluxion $\dot{x}y + x\dot{y}$.

And, in like manner, the Fluent of $\dot{x}yz + x\dot{y}z + xy\dot{z}$ is xyz.

V. *When the given fluxional expreſſion is in this form* $\frac{\dot{x}y - x\dot{y}}{y^2}$, *viz, a fraction including two quantities, being the fluxion of the former drawn into the latter, minus the fluxion of the latter drawn into the former, and divided by the ſquare of the latter:* then the Fluent is the fraction $\frac{x}{y}$, or of the former quantity divided by the latter. That is,

The Fluent of $\frac{\dot{x}y - x\dot{y}}{y^2}$ is $\frac{x}{y}$;

and the Fluent of $\frac{2x\dot{x}y^2 - 2x^2y\dot{y}}{y^4}$ is $\frac{x^2}{y^2}$.

Though the examples of this caſe may be performed by the foregoing one. Thus the given fluxion $\frac{\dot{x}y - x\dot{y}}{y^2}$ reduces to $\frac{\dot{x}}{y} - \frac{x\dot{y}}{y^2}$ or $\frac{\dot{x}}{y} - x\dot{y}y^{-2}$; of which the Fluent of $\frac{\dot{x}}{y}$ is $\frac{x}{y}$ when y is conſtant; and the Fluent of $x\dot{y}y^{-2}$ is $+xy^{-1}$ or $\frac{x}{y}$ when x is conſtant; and therefore, by that caſe, $\frac{x}{y}$ is the Fluent of the whole $\frac{\dot{x}y - x\dot{y}}{y^2}$.

VI. *When the fluxion of a quantity is divided by the quantity itſelf:* Then the Fluent is equal to the hyperbolic logarithm of that quantity; or, which is the ſame thing, the Fluent is equal to 2·30258509 &c, multiplied by the common log. of the ſame quantity.

So, the Fluent

of $\frac{\dot{x}}{x}$ or $x^{-1}\dot{x}$ is the hyp. log. of x;

of $\frac{2\dot{x}}{x}$ is 2 × hyp. log. of x, or = h. l. of x^2;

of $\frac{a\dot{x}}{x}$ is a × h. l. of x, or h. l. of x^a;

of $\frac{\dot{x}}{a+x}$ is the h. l. of $a + x$;

of $\frac{3x^2\dot{x}}{a+x^3}$ is the h. l. of $a + x^3$.

VII. Many Fluents may be found by the direct method of fluxions, thus: Take the fluxion again of the given fluxional expreſſion, or the 2d fluxion of the Fluent ſought; into which ſubſtitute $\frac{\dot{x}^2}{x}$ for $\ddot{x}$, and $\frac{\dot{y}^2}{y}$ for $\ddot{y}$, &c, that is, make x, $\dot{x}$, $\ddot{x}$, as alſo y, $\dot{y}$, $\ddot{y}$, &c, in continual proportion, or $x : \dot{x} :: \dot{x} : \ddot{x}$, and $y : \dot{y} :: \dot{y} : \ddot{y}$, &c; then divide the ſquare of the given fluxional expreſſion by the 2d fluxion, juſt found, and the quotient will be the Fluent ſought in many caſes.

Or the ſame rule may be otherwiſe delivered thus: In the given fluxion $\dot{F}$, write x for $\dot{x}$, y for $\dot{y}$, &c, and call the reſult G, taking alſo the fluxion of this quantity, $\dot{G}$; then make $\dot{G} : \dot{F} :: G : F$, ſo ſhall the 4th proportional F be the Fluent as before. And this is the rule of M. Paccaſſi.

It may be proved if this be the true Fluent, by taking the fluxion of it again, which, if it agree with the propoſed fluxion, will ſhew that the Fluent is right; otherwiſe, it is wrong.

Thus, if it be propoſed to find the Fluent of $nx^{n-1}\dot{x}$. Here $\dot{F} = nx^{n-1}\dot{x}$; write firſt x for $\dot{x}$, and it is $nx^{n-1}x$ or $nx^n = G$; the fluxion of this is $\dot{G} = n^2x^{n-1}\dot{x}$; therefore $\dot{G} : \dot{F} :: G : F$ becomes $n^2x^{n-1}\dot{x} : nx^{n-1}\dot{x} :: nx^n : x^n = F$, the Fluent ſought.

For a 2d ex. ſuppoſe it be propoſed to find the Fluent of $\dot{x}y + x\dot{y}$. Here $F = \dot{x}y + x\dot{y}$; then, writing x for $\dot{x}$, and y for $\dot{y}$, it is $xy + xy$ or $2xy = G$; the fluxion of which is $2\dot{x}y + 2x\dot{y} = \dot{G}$; then $\dot{G} : \dot{F} :: G : F$ becomes $2\dot{x}y + 2x\dot{y} : \dot{x}y + x\dot{y} :: 2xy : xy = F$, the Fluent ſought.

VIII. *To find Fluents by means of a table of forms of Fluxions and Fluents.*

In the following table are contained the moſt uſual forms of fluxions that occur in the practical ſolution of problems, with their correſponding Fluents ſet oppoſite to them; by means of which, viz, comparing any propoſed fluxion with the correſponding form here, the Fluent of it will be found.

Where it is to be noted, that the logarithms in the ſaid forms, are the hyperbolic ones, which are found by multiplying the common logs. by 2·302585092994 &c. Alſo the arcs whoſe ſine, or tangent, &c, are mentioned, have the radius 1, and are thoſe in the common tables of ſines, tangents, &c.—And the numbers m, n, &c. are to be ſome quantities, as the forms fail when $n = 0$, or $m = 0$, &c.

Forms	*Fluxions.*	*Fluents.*
I	$x^{n-1}\dot{x}$	$\frac{1}{n}x^n$
II	$(a \pm x^n)^{m-1}x^{n-1}\dot{x}$	$\pm\frac{1}{mn}(a \pm x^n)^m$
III	$\frac{x^{mn-1}\dot{x}}{(a \pm x^n)^{m+1}}$	$\frac{1}{mna} \times \frac{x^{mn}}{(a \pm x^n)^m}$
IV	$\frac{(a \pm x^n)^{m-1}\dot{x}}{x^{mn+1}}$	$\frac{-1}{mna} \times \frac{(a \pm x^n)^m}{x^{mn}}$
V	$(m\dot{x}y + nx\dot{y})\,x^{m-1}y^{n-1}$, or $\left(\frac{m\dot{x}}{x} + \frac{n\dot{y}}{y}\right)x^m y^n$	$x^m y^n$
VI	$mx^{m-1}\dot{x}y^nz^r + nx^my^{n-1}\dot{y}z^r + rx^my^nz^{r-1}\dot{z}$, or $(m\dot{x}yz + nx\dot{y}z + rxy\dot{z})x^{m-1}y^{n-1}z^{r-1}$, or $\left(\frac{m\dot{x}}{x} + \frac{n\dot{y}}{y} + \frac{r\dot{z}}{z}\right)x^my^nz^r$	$x^m y^n z^r$
	$\frac{\dot{x}}{x}$ or $x^{-1}\dot{x}$	log. of x

Forms.	*Fluxions.*	*Fluents.*
VIII	$\frac{x^{n-1}\dot{x}}{a \pm x^n}$	$\pm \frac{1}{n}$ log. of $a \pm x^n$
IX	$\frac{x^{-1}\dot{x}}{a \pm x^n}$	$\frac{1}{na}$ log. of $\frac{x^n}{a \pm x^n}$
X	$\frac{x^{\frac{1}{2}n-1}\dot{x}}{a - x^n}$	$\frac{1}{n\sqrt{a}}$ log. of $\frac{\sqrt{a} + \sqrt{x^n}}{\sqrt{a} - \sqrt{x^n}}$
XI	$\frac{x^{\frac{1}{2}n-1}\dot{x}}{a + x^n}$	$\frac{2}{n\sqrt{a}}$ × arc to tang. $\sqrt{\frac{x^n}{a}}$, or $\frac{1}{n\sqrt{a}}$ × arc to coſine $\frac{a - x^n}{a + x^n}$
XII	$\frac{x^{\frac{1}{2}n-1}\dot{x}}{\sqrt{\pm a + x^n}}$	$\frac{2}{n}$ log. of $\sqrt{x^n} + \sqrt{\pm a + x^n}$
XIII	$\frac{x^{\frac{1}{2}n-1}\dot{x}}{\sqrt{a - x^n}}$	$\frac{2}{n}$ × arc to ſin. $\sqrt{\frac{x^n}{a}}$, or $\frac{1}{n}$ × arc to verſ. $\frac{2x^n}{a}$
XIV	$\frac{x^{-1}\dot{x}}{\sqrt{a \pm x^n}}$	$\frac{1}{n\sqrt{a}}$ log. of $\frac{\pm\sqrt{a \mp x^n} \pm \sqrt{a}}{\sqrt{a \pm x^n} + \sqrt{a}}$
XV	$\frac{x^{-1}\dot{x}}{\sqrt{-a + x^n}}$	$\frac{2}{n\sqrt{a}}$ × arc to ſecant $\sqrt{\frac{x^n}{a}}$, or $\frac{1}{n\sqrt{a}}$ × arc to coſin. $\frac{2a - x^n}{x^n}$
XVI	$\dot{x}\sqrt{dx - x^2}$	$\frac{1}{2}$ circ. ſeg. to diam. d and verſ. x
XVII	$c^{nx}\dot{x}$	$\frac{c^{nx}}{n \text{ log. } c}$
XVIII	$\dot{x}y^x \text{ log. } y + xy^{x-1}\dot{y}$	y
XIX	$(a + x^n)^m x^{rn-1}\dot{x}$ Putting $s = m + r$	$\frac{(a + x^n)^{m+1} x^{rn-n}}{ns} \times \left(\frac{1}{1} - \frac{r-1.a}{s-1.x^n} + \frac{r-1.r-2.a^2}{s-1.s-2.x^{2n}} - \frac{r-1.r-2.r-3.a^3}{s-1.s-2.s-3x^{3n}} \&c\right)$ or $\frac{(a + x^n)^{m+1} x^{rn}}{rna} \times \left(\frac{1}{1} - \frac{s+1.x^n}{r+1.a} + \frac{s+1.s+2.x^{2n}}{r+1.r+2.a^2} - \frac{s+1.s+2.s+3.x^{3n}}{r+1.r+2.r+3.a^3} \&c\right)$

The Uſe of the foregoing Table of Forms of Fluxions and Fluents.—In the uſe of this table, it is to be obſerved, that the firſt column ſerves only to ſhew the number of the form, as a mark of reference; in the 2d column are the ſeveral forms of fluxions, which are of different kinds or claſſes; and in the 3d or laſt column are the correſponding Fluents.

The method of uſing the table is this. Having any fluxion given, whoſe Fluent it is propoſed to find: Firſt, compare the given fluxion with the ſeveral forms of fluxions in the 2d column of the table, till one of the forms be found that agrees with it; which is done by comparing the terms of the given fluxion with the like parts of the tabular fluxion, viz, the radical quantity of the one, with that of the other; and the exponents of the variable quantities of each, both within and without the vinculum; all which, being found to agree or correſpond, will give the particular values of the general quantities in the tabular form. Then ſubſtitute theſe particular values, for the ſame quantities in the general or tabular form of the Fluent, and the reſult will be the particular Fluent ſought; after it is multiplied by any coefficient the propoſed fluxion may have.

For Ex. To find the Fluent of the given fluxional expreſſion $3x^{\frac{5}{3}}\dot{x}$. This agrees with the firſt form; and by comparing the fluxions, it appears that $x = x$, and $n - 1 = \frac{5}{3}$, or $n = \frac{8}{3}$; which being ſubſtituted in the tabular Fluent, or $\frac{1}{n}x^n$, gives, after multiplying by 3 the coefficient, $3 \times \frac{3}{8}x^{\frac{8}{3}}$ or $\frac{9}{8}x^{\frac{8}{3}}$ for the Fluent ſought.

Again, To find the Fluent of $5x^2\dot{x}\sqrt{c^3 - x^3}$, or $5x^2\dot{x} \cdot \overline{c^3 - x^3}|^{\frac{1}{2}}$. This belongs to the 2d form; for $a = c^3 - x^n = - x^3$, $n = 3$ under the vinculum, $m - 1 = \frac{1}{2}$, or $m = \frac{3}{2}$, and the exponent^{n-1} of x^{n-1} without the vinculum, by uſing 3 for n, is $n - 1 = 2$, which agrees with x^2 in the fluxion given; and therefore all the parts of the form are found to anſwer. Then, ſubſtituting theſe values into the general Fluent, $-\frac{1}{mn}(a - x^n)^m$, it becomes $-\frac{5 \times 2}{3 \times 3}(c^3 - x^3)^{\frac{3}{2}} = -\frac{10}{9}(c^3 - x^3)^{\frac{3}{2}}$.

Thirdly, To find the Fluent of $\frac{x^2\dot{x}}{1 + x^3}$. This agrees with the 8th form; where $\pm x^n = + x^3$ in the denominator, or $n = 3$; and the numerator x^{n-1} then becomes x^2, which agrees with the numerator in the given fluxion; alſo $a = 1$. Hence then, by ſubſtituting in the general form of the Fluent $\frac{1}{n}$ logarithm of $a + x^n$, it becomes $\frac{1}{3}$ logarithm of $1 + x^3$.

IX. *To find Fluents by means of Infinite Series.*—When a finite form cannot be found to agree with a propoſed fluxion, it is then uſual to throw it into an infinite ſeries, either by diviſion, or extraction of roots, or by the binomial theorem, &c; after which, the Fluents of all the terms are taken ſeparately.

For Ex. To find the Fluent of $\frac{1 - x}{1 + x - x^2}\dot{x}$. Here, by dividing the numerator by the denominator, this becomes $\dot{x} - 2x\dot{x} + 3x^2\dot{x} - 5x^3\dot{x} + 8x^4\dot{x}$ &c; and, the Fluents of all the terms being taken, give $x - x^2 + x^3 - \frac{5}{4}x^4 + \frac{8}{5}x^5$ &c, for the Fluent ſought.

To Correct a FLUENT.—The Fluent of a given fluxion, found as above, ſometimes wants a correction, to make it contemporary with that required by the problem under conſideration, &c: for the Fluent of any given fluxion, as $\dot{x}$, may be either x (which is found by the rule) or it may be $x \pm c$, that is x plus or minus ſome conſtant quantity c; becauſe both x and $x \pm c$ have the ſame fluxion $\dot{x}$: and the finding of this conſtant quantity, is called correcting the Fluent. Now this correction is to be determined from the nature of the problem in hand, by which we come to know the relation which the Fluent quantities have to each other at ſome certain point or time. Reduce therefore the general Fluential equation, found by the rules above, to that point or time; then if the equation be true at that point, it is correct; but if not, it wants a correction, and the quantity of that correction is the difference between the two general ſides of the equation when reduced to that particular ſtate. Hence the general rule for the correction is this:

Connect the conſtant, but indeterminate, quantity c with one ſide of the Fluential equation, as determined by the foregoing rules; then, in this equation, ſubſtitute for the variable quantities ſuch values as they are known to have at any particular ſtate, place, or time; and then from that particular ſtate of the equation find the value of c, the conſtant quantity of the correction.

Ex. To find the Correct Fluent of $\dot{z} = ax^3\dot{x}$. Firſt the general Fluent of this is $z = ax^4$, or $z = ax^4 + c$, taking in the correction c.

Now if it be known that z and x begin together, or that $z = 0$, when $x = 0$; then writing 0 both for x and z, the general equation becomes $0 = 0 + c$, or $c = 0$; ſo that, the value of c being 0, the Correct Fluents are $z = ax^4$.

But if z be $= 0$, when x is $= b$, any known quantity; then ſubſtituting 0 for z, and b for x, in the general equation, it becomes $0 = ab^4 + c$, from which is found $c = - ab^4$; and this being written for it in the general equation, this becomes $z = ax^4 - ab^4$, for the correct, or contemporary Fluents.

Or laſtly, if it be known that z is $=$ ſome quantity d, when x is equal ſome other quantity, as b; then ſubſtituting d for z, and b for x, in the general Fluential equation $z = ax^4 + c$, it becomes $d = ab^4 + c$; and hence is deduced the value of the correction, viz, $c = d - ab^4$; conſequently, writing this value for c in the general equation, it becomes $z = ax^4 - ab^4 + d$, for the Correct equation of the Fluents in this caſe.

And hence ariſes another eaſy and general way of correcting the Fluents, which is this: In the general equation of the Fluents, write the particular values of the quantities which they are known to have at any certain time; then ſubtract the ſides of the reſulting particular equation, from the correſponding ſides of the general one, and the remainders will give the Correct equation of the Fluents ſought. So, as above,

the general equation being - $z = ax^4$;
write d for z, and b for x, then - $d = ab^4$;
hence by ſubtraction - $z - d = ax^4 - ab^4$,
or $z = ax^4 - ab^4 + d$,

the Correct Fluents as before.

FLUID, or FLUID *Body*, according to Newton, is that whoſe parts yield to the ſmalleſt force impreſſed, and by yielding are eaſily moved among each other; in which ſenſe it ſtands oppoſed to a ſolid. This is the definition of a perfect fluid: if the Fluid require ſome ſenſible force to move its parts, it is imperfect in proportion to that force; ſuch as perhaps all the fluids we know of in nature.

That Fluids have vacuities in their ſubſtance is evident, becauſe certain bodies may be diſſolved in them without increaſing their bulk. Thus, water will diſſolve a certain quantity of ſalt; after which it will receive a little ſugar, and after that a little alum; and all this without increaſing its firſt dimenſions. Which ſhews that the particles of theſe ſolids are ſo far ſeparated as to become ſmaller than thoſe of the Fluid, and to be received and contained in the interſtices between them.

Fluids are either elaſtic, ſuch as air; or non-elaſtic, as water, mercury, &c. Theſe latter occupy the ſame ſpace, or are of the ſame bulk, under all preſſures or forces; but the former dilate and expand themſelves continually by taking off the external preſſure from them; for which reaſon it is that the denſity and elaſticity of ſuch fluids, are proportional to the force or weight that compreſſes them. The doctrine and laws of Fluids are of the greateſt extent in philoſophy: the properties of elaſtic Fluids conſtituting the doctrine of Pneumatics; thoſe of the non-elaſtic ones, that of Hydroſtatics; and their motions, Hydraulics. For which ſee theſe reſpective articles. Alſo,

For the laws of the preſſure and gravitation in Fluids ſpecifically heavier or lighter than the bodies immerged, ſee Specific *Gravity*.

For the laws of the reſiſtance of Fluids, or the retardation of ſolid bodies moving in Fluids, ſee Resistance. And

For the aſcent of Fluids in capillary tubes, or between glaſs planes, &c, ſee Ascent, and Capillary *Tubes*.

FLUTES, or Flutings, are certain channels or cavities cut along the ſhaft of a column, or pilaſter.

FLUIDITY, that ſtate or affection of bodies, which denominates or renders them Fluid; or that property by which they yield to the ſmalleſt force impreſſed: in contradiſtinction to Solidity or Firmneſs.

Fluidity is to be carefully diſtinguiſhed from Liquidity or Humidity, which latter implies wetting or adhering. Thus, air, ether, mercury, and other melted metals, and even ſmoke and flame itſelf, are Fluid bodies, but not Liquid ones; whilſt water, beer, milk, urine, &c, are both Fluids and Liquids at the ſame time.

The nature and cauſes of Fluidity have been variouſly aſſigned. The Gaſſendiſts, and ancient corpuſcularians, require only three conditions as neceſſary to it; viz, a ſmallneſs and ſmoothneſs of the particles of the body, vacuities interſperſed between them, and a ſpherical figure. The Carteſians, and after them Dr. Hook, Mr. Boyle, &c, beſide theſe circumſtances, require alſo a certain internal or inteſtine motion of the particles as chiefly contributing to Fluidity. Thus, Mr. Boyle, in his Hiſtory of Fluidity, argues from various experiments: for example, a little dry powder of alabaſter, or plaiſter of Paris, finely ſifted, being put into a veſſel over the fire, ſoon begins to boil like water; exhibiting all the motions and phenomena of a boiling liquor: it will tumble variouſly in great waves like that; will bear ſtirring with a ſtick or ladle like that, without reſiſting; and if ſtrongly ſtirred near the ſide of the veſſel, its waves will apparently daſh againſt it: yet it is all the while a dry parched powder.

The like is obſerved in ſand; a diſh of which being ſet on a drum-head, briſkly beaten by the ſticks, or on the upper ſtone of a mill, it in all reſpects emulates the properties of a Fluid body. A heavy body, ex. gr. will immediately ſink in it to the bottom, and a light one emerge to the top: each grain of ſand has a conſtant vibratory and dancing motion; and if a hole be made in the ſide of the diſh, the ſand will ſpin out like water.

The Carteſians bring divers conſiderations to prove that the parts of Fluids are in continual motion: as 1ſt, The change of ſolids into Fluids, ex. gr. ice into water, and vice verſa; the chief difference between the body in thoſe two ſtates conſiſting in this, that the parts, being fixed and at reſt in the one, reſiſt the touch; whereas in the other, being already in motion, they give way to the ſlighteſt impulſe. 2dly, The effects of Fluids, which commonly proceed from motion: ſuch are the inſinuation of Fluids among the pores of bodies; the ſoftening and diſſolving hard bodies; the actions of corroſive menſtruums; &c: Add, that no ſolid can be brought to a ſtate of Fluidity, without the intervention of ſome moving or moveable body, as fire, air, or water. Air, the ſame gentlemen hold to be the firſt ſpring of theſe cauſes of Fluidity, it being this that gives motion to fire and water, though itſelf receives its motion and action from the ether, or ſubtle medium.

But Boerhaave pleads ſtrenuouſly that fire is the firſt mover, and the cauſe of all Fluidity in other bodies, as air, water, &c: without this, he ſhews that the atmoſphere itſelf would fix into one ſolid maſs. And in like manner, Dr. Black, of Edinburgh, mentions Fluidity as an effect of heat. The different degrees of heat which are required to bring different bodies into a ſtate of Fluidity, he ſuppoſes may depend on ſome particulars in the mixture and compoſition of the bodies themſelves: which is rendered farther probable from conſidering that the natural ſtate of bodies in this reſpect is changed by certain mixtures; thus, when two metals are compounded, the mixture is commonly more fuſible than either of them ſeparately.

Newton's idea of the cauſe of Fluidity is different: he makes it to be the great principle of attraction. The various inteſtine motion and agitation among the particles of Fluid bodies, he thinks is naturally accounted for, by ſuppoſing it a primary law of nature, that as all the particles of matter attract each other when within a certain diſtance; ſo at all greater diſtances, they avoid and fly from one another. For then, though their common gravity, together with the preſſure of other bodies upon them, may keep them together in a maſs, yet their continual endeavour to avoid one another ſingly, and the adventitious impulſes of heat and light, or other external cauſes, may make the particles of Fluids continually move round about one another, and ſo produce this quality.

As therefore the cauſe of coheſion of the parts of ſolid bodies appears to be their mutual attraction; ſo, on this principle, the chief cauſe of Fluidity ſeems to be a contrary motion, impreſſed on the particles of Fluids; by which they avoid and fly from one another, as ſoon as they come at, and as long as they keep at, ſuch a diſtance from each other.

It is obſerved alſo in all Fluids, that the direction of their preſſure againſt the veſſels which contain them, is in lines perpendicular to the ſides of ſuch veſſels; which property, being the neceſſary reſult of the ſpherical figure of the particles of any Fluid, ſhews that the parts of all Fluids are ſo, or of a figure very nearly approaching to it.

FLUX, in Hydrography, a regular and periodical motion of the ſea, happening twice in 24 hours and 48 minutes, nearly; in which time the water is raiſed, and driven violently againſt the ſhores. The Flux, or Flow, is one of the motions of the tide: the other, by which the water ſinks and retires, being called the Reflux, or Ebb. See TIDE.

Between the Flux and Reflux there is a kind of reſt or ceſſation, of about half an hour; during which time the water is at its greateſt height, called High-water.

The Flux of the ſea follows chiefly the courſe of the moon; and is always higheſt and greateſt at new and full moons, particularly near the time of the equinoxes. In ſome parts, as at Mount St. Michael, it riſes 80 or 90 feet, though in the open ſea it never riſes above a foot or two; and in ſome places, as about the Morea, there is no flux at all. It runs up ſome rivers above 120 miles: though up the river Thames it goes only about 80, viz, near to Kingſton in Surry. Above London-bridge, the water flows 4 hours, and ebbs 8; and below the bridge, it flows 5 hours, and ebbs 7.

FLUXION, in the Newtonian analyſis, denotes the rate or proportion at which a flowing or varying quantity increaſes its magnitude or quantity; and it is proportional to the magnitude by which the flowing quantity would be uniformly increaſed, in a given time, by the generating quantity continuing of the invariable magnitude it has at the moment of time for which the Fluxion is taken: by which it ſtands contradiſtinguiſhed from fluent, or the flowing quantity, which is gradually, and indefinitely increaſing, after the manner of a ſpace which a body in motion deſcribes.

Mr. Simpſon obſerves, that there is an advantage in thus conſidering Fluxions, not as mere velocities of increaſe at a certain point, but as the magnitudes which would be uniformly generated in a given finite time: the imagination is not here confined to a ſingle point, and the higher orders of Fluxions are rendered much more eaſy and intelligible. And though Sir Iſaac Newton defines Fluxions to be the velocities of motions, yet he has recourſe to the moments or increments, generated in equal particles of time, to determine thoſe velocities, which he afterwards directs to expound by finite magnitudes of other kinds.

As to the illuſtration of this definition, and the rules for finding the Fluxions of all ſorts of fluent quantities, ſee the following article, or the Method of Fluxions.

Method of FLUXIONS, is the algorithm and analyſis of Fluxions, and fluents or flowing quantities.

Moſt foreigners define this as the method of differences or differentials, being the analyſis of indefinitely ſmall quantities. But Newton, and other Engliſh authors, call theſe infinitely ſmall quantities, moments; conſidering them as the momentary increments of variable quantities; as of a line conſidered as generated by the flux or motion of a point, or of a ſurface generated by the flux of a line. Accordingly, the variable quantities are called Fluents, or flowing quantities; and the method of finding either the Fluxion, or the fluent, the method of Fluxions.

M. Leibnitz conſiders the ſame infinitely ſmall quantities as the differences or differentials, of quantities; and the method of finding thoſe differences, he calls the Differential Calculus.

Beſides this difference in the name, there is another in the notation. Newton expreſſes the Fluxion of a quantity, as of x, by a dot placed over it, thus $\dot{x}$; while Leibnitz expreſſes his differential of the ſame x, by prefixing the initial letter d, as dx. But, ſetting aſide theſe circumſtances, the two methods are juſt alike.

The Method of Fluxions is one of the greateſt, moſt ſubtle, and ſublime diſcoveries of perhaps any age: it opens a new world to our view, and extends our knowledge, as it were, to infinity; carrying us beyond the bounds that ſeemed to have been preſcribed to the human mind, at leaſt infinitely beyond thoſe to which the ancient geometry was confined.

The hiſtory of this important diſcovery, recent as it is, is a little dark, and embroiled. Two of the greateſt men of the laſt age have both of them claimed the invention, Sir I. Newton, and M. Leibnitz; and nothing can be more glorious for the method itſelf, than the zeal with which the partiſans of either ſide have aſſerted their title.

To exhibit a juſt view of this diſpute; and of the pretenſions of each party, we may here advert to the origin of the diſcovery, and mark where each claim commenced, and how it was ſupported.

The principles upon which the Method of Fluxions is founded, or which conducted to it, had been laying, and gradually developing, from the beginning of the laſt century, by Fermat, Napier, Barrow, Wallis, Sluſius, &c, who had methods of drawing tangents, of maxima and minima, of quadratures, &c, in certain particular caſes, as of rational quantities, upon nearly the ſame principles. And it was not wonderful that ſuch a genius as Newton ſhould ſoon after raiſe thoſe faint beginnings into a regular and general ſyſtem of ſcience, which he did about the year 1665, or ſooner.

The firſt time however that the method appeared in print, was in 1684, when M. Leibnitz gave the rules of it in the Leipſic Acts of that year; but without the demonſtrations. The two brothers however, John and James Bernoulli, being greatly ſtruck with this new method, applied themſelves diligently to it, found out the demonſtrations, and applied the calculus with great ſucceſs.

But before this, M. Leibnitz had propoſed his Differential Method, viz, in a letter, dated Jan. 21, 1677, in which he exactly purſues Dr. Barrow's method of tangents, which had been publiſhed in 1670: and Newton communicated his method of drawing tangents to Mr. Collins, in a letter dated Dec. 10, 1672; which letter, together with another dated June 13, 1676, were ſent to Mr. Leibnitz by Mr. Oldenburgh, in 1676. So that there is a ſtrong preſumption that he might avail himſelf of the information contained in theſe letters, and other papers tranſmitted with them, and alſo in 1675, before the publication of his own letter, containing the firſt hint of his differential method. Indeed it ſufficiently appears that Newton had invented his method before the year 1669, and that he actually made uſe of it in his Compendium of Analyſis and Quadrature of Curves before that time. His attention ſeems to have been directed this way, even before the time of the plague which happened in London in 1665 and 1666, when he was about 28 years of age.

This is all that is heard of the method, till the year 1687, when Newton's admirable Principia came out, which is almoſt wholly built on the ſame calculus. The common opinion then was, that Newton and Leibnitz had each invented it about the ſame time: and what ſeemed to confirm it was, that neither of them made any mention of the other; and that, though they agreed in the ſubſtance of the thing, yet they differed in their ways of conceiving it, calling it by different names, and uſing different characters. However, foreigners having firſt learned the method through the medium of Leibnitz's publication, which ſpread the method through Europe, thoſe geometricians were inſenſibly accuſtomed to look upon him as the ſole, or principal inventor, and became ever after ſtrongly prejudiced in favour of his notation and mode of conceiving it.

The two great authors themſelves, without any ſeeming concern, or diſpute, as to the property of the invention, enjoyed the glorious proſpect of the progreſſes continually making under their auſpices, till the year 1699, when the peace began to be diſturbed.

M. Facio, in a treatiſe on the Line of Swifteſt Deſcent, declared, that he was obliged to own Newton as the firſt inventor of the differential calculus, and the firſt by many years; and that he left the world to judge, whether Leibnitz, the ſecond inventor, had taken any thing from him. This preciſe diſtinction between firſt and 2d inventor, with the ſuſpicion it inſinuated, raiſed a controverſy between M. Leibnitz, ſupported by the editors of the Leipſic Acts, and the Engliſh mathematicians, who declared for Newton. Sir Iſaac himſelf never appeared on the ſcene; his glory was become that of the nation; and his adherents, warm in the cauſe of their country, needed not his aſſiſtance to animate them.

Writings ſucceeded each other but ſlowly, on either ſide; probably on account of the diſtance of places; but the controverſy grew ſtill hotter and hotter: till at length M. Leibnitz, in the year 1711, complained to the Royal Society, that Dr. Keil had accuſed him of publiſhing the Method of Fluxions invented by Sir I. Newton, under other names and characters. He inſiſted that nobody knew better than Sir Iſaac himſelf, that he had ſtolen nothing from him; and required that Dr. Keil ſhould diſavow the ill conſtruction which might be put upon his words.

The Society, thus appealed to as a judge, appointed a committee to examine all the old letters, papers, and documents, that had paſſed among the ſeveral mathematicians, relating to the point; who, after a ſtrict examination of all the evidence that could be procured, gave in their report as follows: "That Mr. Leibnitz was in London in 1673, and kept a correſpondence with Mr. Collins by means of Mr. Oldenburgh, till Sept. 1676, when he returned from Paris to Hanover, by way of London and Amſterdam: that it did not appear that Mr. Leibnitz knew any thing of the differential calculus before his letter of the 21ſt of June, 1677, which was a year after a copy of a letter, written by Newton in the year 1672, had been ſent to Paris to be communicated to him, and above 4 years after Mr. Collins began to communicate that letter to his correſpondents; in which the Method of Fluxions was ſufficiently explained, to let a man of his ſagacity into the whole matter: and that Sir I. Newton had even invented his method before the year 1669, and conſequently 15 years before M. Leibnitz had given any thing on the ſubject in the Leipſic Acts." From which they concluded that Dr. Keil had not at all injured M. Leibnitz in what he had ſaid.

The Society printed this their determination, together with all the pieces and materials relating to it, under the title of Commercium Epiſtolicum de Analyſi. Promota, 8vo, Lond. 1712. This book was carefully diſtributed through Europe, to vindicate the title of the Engliſh nation to the diſcovery; for Newton himſelf, as already hinted, never appeared in the affair: whether it was that he truſted his honour with his compatriots, who were zealous enough in the cauſe; or whether he felt himſelf even ſuperior to the glory of it.

M. Leibnitz and his friends however could not ſhew the ſame indifference: he was accuſed of a theft; and the whole Commercium Epiſtolicum either expreſſes it in terms, or inſinuates it. Soon after the publication therefore, a looſe ſheet was printed at Paris, in behalf of M. Leibnitz, then at Vienna. It is written with great zeal and ſpirit; and it boldly maintains that the Method of Fluxions had not preceded the Method of Differences; and even inſinuates that it might have ariſen from it. The detail of the proofs however, on each ſide, would be too long, and could not be underſtood without a large comment, which muſt enter into the deepeſt geometry.

M. Leibnitz had begun to work upon a Commercium Epiſtolicum, in oppoſition to that of the Royal Society; but he died before it was completed.

A ſecond edition of the Commercium Epiſtolicum was printed at London in 1722; when Newton, in the preface, account, and annotations, which were added to that edition, particularly anſwered all the objections which Leibnitz and Bernoulli were able to make ſince the Commercium firſt appeared in 1712; and from the laſt edition of the Commercium, with the various original papers contained in it, evidently appears that Newton had diſcovered his Method of Fluxions many years before the pretenſions of Leibnitz. See alſo Raphſon's Hiſtory of Fluxions.

There are however, according to the opinion of ſome, ſtrong preſumptions in favour of Leibnitz; i. e. that he was no plagiary: for that Newton was at leaſt the firſt inventor, is paſt all diſpute; his glory is ſecure; the reaſonable part, even among the foreigners, allow it: and the queſtion is only, whether Leibnitz took it from him, or fell upon the ſame thing with him; for, in his theory of abſtract notions, which he dedicated to the Royal Academy in 1671, before he had ſeen any thing of Newton's, he already ſuppoſed infinitely ſmall quantities, ſome greater than others; which is one of the great principles of his ſyſtem.

Before proſecuting farther the hiſtory and improvements of this ſcience, it will be proper to premiſe ſomewhat of the principles and practice of it, according to the ideas of the inventor.

Principles of the Method of FLUXIONS.

1. In the doctrine of Fluxions, magnitudes or quantities, of all kinds, are conſidered, not as made up of a

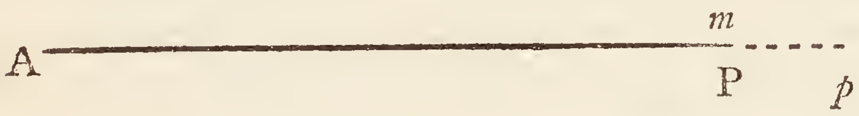
number of ſmall parts, but as generated by continued motion, by means of which they increaſe or decreaſe: as a line by the motion of a point; a ſurface by the motion of a line; and a ſolid by the motion of a ſurface: which is no new principle in geometry; having been uſed by Euclid and Archimedes. So likewiſe, time may be conſidered as repreſented by a line, increaſing uniformly by the motion of a point. And quantities of all kinds whatever, which are capable of increaſe and decreaſe, may in like manner be repreſented by lines, ſurfaces, or ſolids, conſidered as generated by motion.

2. Any quantity, thus generated, and variable, is called a Fluent, or a flowing quantity. And the rate or proportion according to which any flowing quantity increaſes, at any poſition or inſtant, is the Fluxion of the ſaid quantity, at that poſition or inſtant: and it is proportional to the magnitude by which the flowing quantity would be uniformly increaſed, in a given time, with the generating celerity uniformly continued during that time.

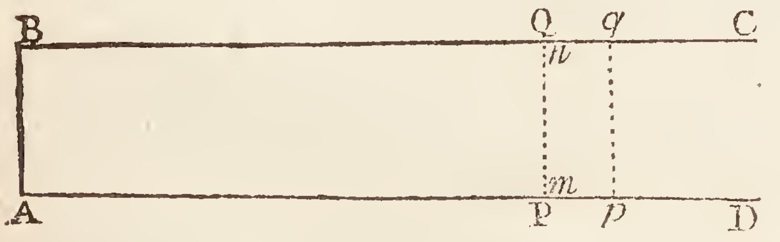
3. The ſmall quantities that are actually generated or deſcribed, in any ſmall given time, and by any continued motion, either uniform or variable, are called Increments.

4. Hence, if the motion of increaſe be uniform, by which increments are generated, the increments will in that caſe be proportional, or equal, to the meaſures of the Fluxions: but if the motion of increaſe be accelerated, the increments ſo generated, in a given finite time, will exceed the Fluxion; and if it be a decreaſing motion, the increment ſo generated, will be leſs than the Fluxion. But if the time be indefinitely ſmall, ſo that the motion be conſidered as uniform for that inſtant; then theſe naſcent increments will always be proportional or equal to the Fluxions, and may be ſubſtituted for them, in any calculation.

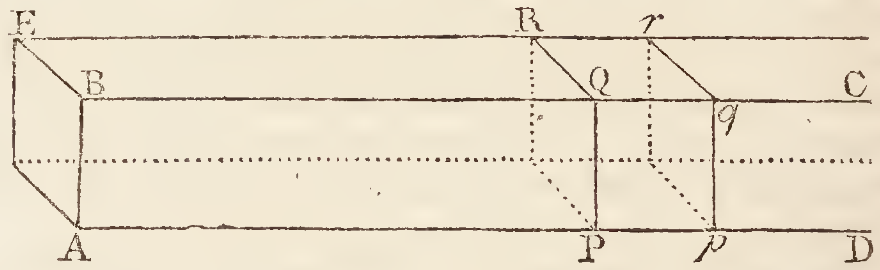
5. To illuſtrate theſe definitions: Suppoſe a point m be conceived to move from the poſition A, and to generate a line AP, with a motion any-how regulated; and ſuppoſe the celerity of the point m, at any poſition P, to be ſuch, as would, if from thence it ſhould become, or continue, uniform, be ſufficient to deſcribe, or paſs uniformly over, the diſtance Pp, in the given time allowed for the Fluxion: then will the ſaid line Pp repreſent the Fluxion of the ſaid fluent or flowing line AP, at that poſition.



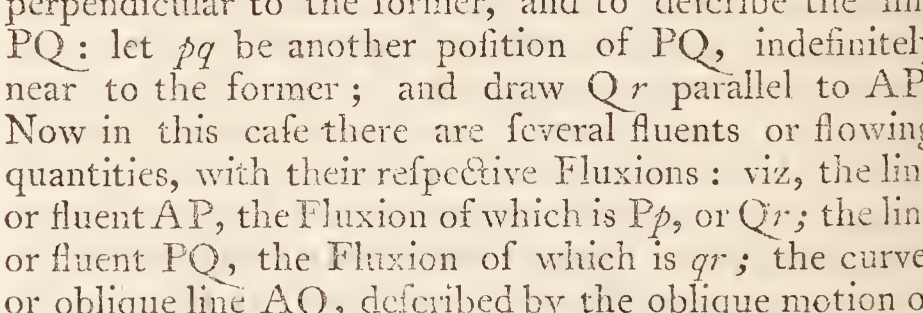
6. Again, ſuppoſe the right line mn to move, from the poſition AB, continually parallel to itſelf, with any continued motion, ſo as to generate the fluent, or flowing rectangle ABQP, whilſt the point m deſcribes the line AP; alſo let the diſtance Pp be taken, as above, to expreſs the Fluxion of the line or baſe AP; and complete the rectangle $PQqp$. Then, like as Pp is the Fluxion of the line AP, ſo is the ſmall parallelogram Pq the Fluxion of the flowing parallelogram, AQ; both theſe Fluxions or increments being uniformly deſcribed in the ſame time.



7. In like manner, if the ſolid AERP be conceived as generated by the plane PQR moving, from the poſition ABE, always parallel to itſelf, along the line AD; and if Pp denote the Fluxion of the line AP. Then, like as the parallelogram Pq, or $Pp \times PQ$, expreſſes the Fluxion of the flowing rectangle AQ, ſo likewiſe ſhall the Fluxion of the variable ſolid or priſm AR be expreſſed by the priſm Pr, or $Pp \times$ the plane PR. And in both theſe laſt two caſes, it appears that the Fluxion of the generated rectangle, or priſm, is equal to the product of the generant, whether line or plane, drawn into the Fluxion of the line along which it moves.



8. Hitherto the generant, or generating line or plane, has been conſidered as of a conſtant or invariable magnitude; in which caſe the fluent, or quantity generated, is a parallelogram, or a priſm, the former being deſcribed by the motion of a line, and the latter by the motion of a plane. In like manner are other figures, whether plane or ſolid, conceived to be deſcribed, by the motion of a variable magnitude, whether it be a line or a plane. Thus, let a variable line PQ be carried with a parallel motion along AP, or whilſt a point P is carried along, and deſcribes, the line AP, ſuppoſe another point Q to be carried by a motion perpendicular to the former, and to deſcribe the line PQ: let pq be another poſition of PQ, indefinitely near to the former; and draw Qr parallel to AP. Now in this caſe there are ſeveral fluents or flowing quantities, with their reſpective Fluxions: viz, the line or fluent AP, the Fluxion of which is Pp, or Qr; the line or fluent PQ, the Fluxion of which is qr; the curve, or oblique line AQ, deſcribed by the oblique motion of the point, the Fluxion of which is Qq; and laſtly the ſurface APQ, deſcribed by the variable line PQ, and the Fluxion of which is the rectangle $PQrp$, or $PQ \times Pp$. And in the ſame manner may any ſolid be conceived to be deſcribed, by the motion of a variable plane parallel to itſelf, ſubſtituting the variable plane for the variable line; in which caſe, the Fluxion of the ſolid, at any poſition, is repreſented by the va-

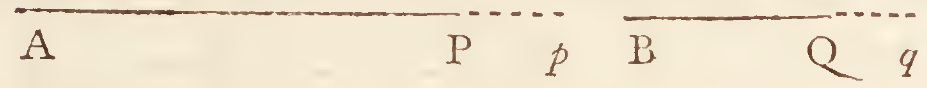


riable plane, at that poſition, drawn into the Fluxion of the line along which it is carried.

9. Hence then it follows generally, that the Fluxion of any figure, whether plane or ſolid, at any poſition, is equal to the ſection of it, at that poſition, drawn into the Fluxion of the axis, or line along which the variable ſection is ſuppoſed to be perpendicularly carried; i. e. the Fluxion of the figure AQP, is equal the plane PQ × Pp when that figure is a ſolid, or to the ordinate PQ × Pp when the figure is a ſurface.

10. It alſo follows, from the ſame premiſes, that, in any curve, or oblique line, AQ, whoſe abſciſs is AP, and ordinate is PQ, the Fluxions of theſe three form a ſmall right-angled plane triangle Q*qr*; for Q*r* = P*p* is the Fluxion of the abſciſs AP, *qr* the Fluxion of the ordinate PQ, and Q*q* the Fluxion of the curve or right line AQ. And conſequently that, in any curve, the ſquare of the Fluxion of the curve, is equal to the ſum of the ſquares of the Fluxions of the abſciſs and ordinate, when theſe two lines are at right angles to each other.

11. From the premiſes it alſo appears, that contemporaneous fluents, or quantities that flow or increaſe together, which are always in a conſtant ratio to each other, have their Fluxions alſo in the ſame conſtant ratio at every poſition. For, let AP and BQ be two contemporaneous fluents, deſcribed in the ſame time by the motion of the points P and Q, the contemporaneous poſitions being P, Q, and *p*, *q*; and let AP be to BQ, or A*p* to B*q*, in the conſtant ratio of *n* to 1.



Then is - - - AP $= n \times$ BQ,
and - - - A*p* $= n \times$ B*q*;
therefore by ſubtraction, - P*p* $= n \times$ Q*q*;
that is, the Fluxion P*p* : Fluxion Q*q* :: *n* : 1,
the ſame as Fluent AP : Fluent BQ :: *n* : 1;
or the Fluxions and Fluents are in the ſame conſtant ratio.

But if the ratio of the fluents be variable, ſo will that of the Fluxions be alſo, though not in the ſame variable ratio with the former, at every poſition.

The Notation, &c, in FLUXIONS.

12. To apply the foregoing principles to the determination of the Fluxions of algebraic quantities, by means of which thoſe of all other kinds are determined, it will be neceſſary firſt to premiſe the notation uſed in this ſcience, with ſome obſervations. As, firſt, that the final letters of the alphabet *z*, *y*, *x*, *w*, &c, are uſed to denote variable or flowing quantities; and the initial letters *a*, *b*, *c*, *d*, &c, conſtant or invariable ones: Thus, the variable baſe AP of the flowing rectangular figure ABQP, at art. 6, may be repreſented by *x*; and the invariable altitude PQ, by *a*: alſo the variable baſe or abſciſs AP, of the figures in art. 8, may be repreſented by *x*; the variable ordinate PQ, by *y*; and the variable curve or line AQ, by *z*.

Secondly, that the Fluxion of a quantity denoted by a ſingle letter, is repreſented by the ſame letter with a point over it: Thus the Fluxion of *x* is expreſſed by $\dot{x}$, that of *y* by $\dot{y}$, and that of *z* by $\dot{z}$. As to the Fluxions of conſtant or invariable quantities, as of *a*, *b*, *c*, &c, they are equal to 0 or nothing, becauſe they do not flow, or change their magnitude.

Thirdly, that the increments of variable or flowing quantities, are alſo denoted by the ſame letters with a ſmall (′) over them: So the increments of *x*, *y*, *z*, are x', y, z'.

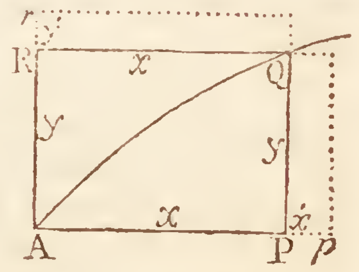
13. From theſe notations, and the foregoing principles, the quantities and their Fluxions, there conſidered, will be denoted as below.

In all the foregoing figures, put
the variable or flowing line - - - AP $= x$,
in art. 6, the conſtant line - - - PQ $= a$,
in art. 8, the variable ordinate - - PQ $= y$,
the variable curve or right line - - AQ $= z$;
Then ſhall the ſeveral Fluxions be thus repreſented, viz,
$\dot{x} =$ P*p* the Fluxion of the line AP,
$a\dot{x} =$ PQ*qp* the Fluxion of ABQP in art. 6,
$y\dot{x} =$ PQ*rp* the Fluxion of APQ in art. 8,
$\dot{z} =$ Q*q* $= \sqrt{\dot{x}^2 + y^2}$ the Fluxion of AQ,
and $a\dot{x} =$ P*r* the Flux. of the ſolid in art. 7, if
a denote the conſtant generating plane PQR.
Alſo $nx =$ BQ in the figure to art. 11,
and $n\dot{x} =$ Q*q* the Fluxion of the ſame.

14. The principles and notation being now laid down, we may proceed to the practice and rules of this doctrine, which conſiſts of two principal parts, called the direct and inverſe method of Fluxions; viz, the direct method, which conſiſts in finding the Fluxion of any propoſed fluent or flowing quantity; and the inverſe method, which conſiſts in finding the fluent of any propoſed Fluxion. As to the former of theſe two problems, it can always be determined, and that in finite algebraic terms; but the latter, or finding of fluents, only in ſome certain caſes, except by means of infinite ſeries.—Firſt then, of

The Direct Method of FLUXIONS.

15. To find the Fluxion of the product or rectangle of two variable quantities; let ARQP $= xy$ be the flowing or variable rectangle, generated by two lines RQ and PQ moving always perpendicular to each other, from the poſitions AP and AR; denoting the one by *x*, and the other by *y*; and ſuppoſe *x* and *y* to be ſo related, that the curve AQ always paſſes through their interſection Q, or the oppoſite angle of the rectangle.



Now this rectangle conſiſts of the two trilineal ſpaces APQ, ARQ, of which the Fluxion of the former is PQ × P*p* or $\dot{x}y$, and that of the latter is RQ × R*r* or $x\dot{y}$, by art. 8; therefore the ſum of the two, $\dot{x}y + x\dot{y}$, is the Fluxion of the whole rectangle xy or ARQP.

The ſame otherwiſe.—Let the ſides of the rectangle, *x* and *y*, by flowing, become $x + x'$ and $y + y'$: then the product of the two, or $xy + x'y + xy' + y'y'$ will be the new or contemporaneous value of the flowing rectangle PR or xy; ſubtract the one value from the other, and the remainder $x\acute{y} + \acute{x}y + \acute{x}\acute{y}$, will be the increment generated in the ſame time as $\acute{x}$ or $\acute{y}$; of which the laſt term $\acute{x}\acute{y}$ is nothing, or indefinitely ſmall in reſpect of the other two terms, becauſe $\acute{x}$ and $\acute{y}$ are indefinitely ſmall in reſpect of x and y; which term being therefore omitted, there remains $x\acute{y} + \acute{x}y$ for the value of that increment: and hence, by ſubſtituting $\dot{x}$ and $\dot{y}$ for $\acute{x}$ and $\acute{y}$, to which they are proportional, there ariſes $x\dot{y} + \dot{x}y$ for the value of the Fluxion of xy; the ſame as before.

17. Hence may be derived the Fluxions of all powers and products, and of all other forms of algebraic quantities whatever. And firſt for the continual products of any number of quantities, as xyz, or $wxyz$, or $vwxyz$, &c. For xyz put q or pz, ſo that $p = xy$, and $xyz = pz = q$. Now, taking the Fluxion of $q = pz$, by the laſt article, it is $\dot{q} = \dot{p}z + p\dot{z}$; but $p = xy$, and ſo $\dot{p} = \dot{x}y + x\dot{y}$ by the ſame article; ſubſtituting therefore theſe values of p and $\dot{p}$ inſtead of them, in the value of $\dot{q}$, this becomes $\dot{q} = \dot{x}yz + x\dot{y}z + xy\dot{z}$, the Fluxion of xyz required; which is therefore equal to the ſum of the products ariſing from the Fluxion of each letter or quantity multiplied by the product of the other two.

Again, to determine the Fluxion of $wxyz$, the continual product of four variable quantities; put this product, viz, $wxyz$ or $qw = r$, where $q = xyz$ as above; then, taking the Fluxion by the laſt article, $\dot{r} = \dot{q}w + q\dot{w}$; and this, by ſubſtituting for q and $\dot{q}$ their values as above, becomes

$\dot{r} = \dot{w}xyz + w\dot{x}yz + wx\dot{y}z + wxy\dot{z}$, the Fluxion of $wxyz$ as required; conſiſting of the Fluxion of each quantity drawn into the products of the other three.

In the very ſame manner it is found that the Fluxion of $vwxyz$ is

$\dot{v}wxyz + v\dot{w}xyz + vw\dot{x}yz + vwx\dot{y}z + vwxy\dot{z}$; and ſo on, for any number of quantities whatever; in which it is always found that there as many terms as there are variable quantities in the propoſed fluent, and that theſe terms conſiſt of the Fluxion of each variable quantity multiplied by the product of all the reſt of the quantities.

18. From hence is eaſily derived the Fluxion of any power of a variable quantity, as of x^2, or x^3, or x^4, &c. For, in the rectangle or product xy, if $x = y$, then is the product $xy = xx$ or x^2, and alſo its Fluxion $\dot{x}y + x\dot{y} = \dot{x}x + x\dot{x}$ or $2x\dot{x}$, the Fluxion of x^2.

Again, if all the three x, y, z be equal; then is the product of the three $xyz = xxx$ or x^3; and its Fluxion $\dot{x}yz + x\dot{y}z + xy\dot{z} = \dot{x}xx + x\dot{x}x + xx\dot{x}$ or $3x^2\dot{x}$, the Fluxion of x^3.

And in the ſame manner it will appear that the Fluxion of x^4 is $= 4x^3\dot{x}$,
that of x^5 is $= 5x^4\dot{x}$,
that of x^n is $= nx^{n-1}\dot{x}$;

where n is any poſitive whole number. That is, the Fluxion of any poſitive integral power, is equal to the exponent of the power (n), multiplied by the next leſs power of the ſame quantity (x^{n-1}), and by the Fluxion of the root ($\dot{x}$).

19. Next, for the Fluxion of any fraction, as $\frac{x}{y}$ of one variable quantity divided by another; put the propoſed fraction $\frac{x}{y} = q$; then multiplying by the denominator, $x = qy$; and, taking the Fluxions, $\dot{x} = \dot{q}y + q\dot{y}$, or $\dot{q}y = \dot{x} - q\dot{y}$; and, by diviſion, $\dot{q} = \frac{\dot{x}}{y} - \frac{q\dot{y}}{y} =$ (by ſubſtituting the value of q, or $\frac{x}{y}$) $\frac{\dot{x}}{y} - \frac{x\dot{y}}{y^2} = \frac{\dot{x}y - x\dot{y}}{y^2}$ the Fluxion of $\frac{x}{y}$, as required. That is, the Fluxion of any fraction, is equal to the Fluxion of the numerator drawn into the denominator, minus the Fluxion of the denominator drawn into the numerator, and the remainder divided by the ſquare of the denominator.

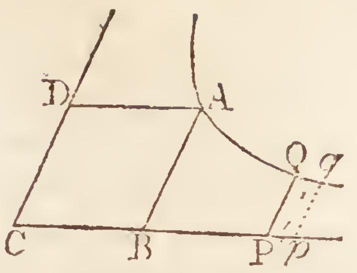
20. Hence too is eaſily derived the Fluxion of any negative integer power of a variable quantity, as of x^{-n} or $\frac{1}{x^n}$, which is the ſame thing. For here the numerator of the fraction is 1, whoſe Fluxion is nothing; and therefore, by the laſt article, the Fluxion of ſuch a fraction, or negative power, is barely equal to minus the Fluxion of the denominator, divided by the ſquare of the ſaid denominator. That is, the Fluxion of x^{-n}, or $\frac{1}{x^n}$, is $-\frac{nx^{n-1}\dot{x}}{x^{2n}}$ or $-\frac{n\dot{x}}{x^{n\,1}}$ or $-n\dot{x}^{-n-1}\dot{x}$; which is the ſame rule as before for integral powers.

Or, the ſame thing is otherwiſe derived immediately from the Fluxion of a rectangle or product, thus: put the propoſed fraction, or quotient, $\frac{1}{x^n} = q$; then is $qx^n = 1$; and, taking the Fluxions, $\dot{q}x^n + qnx^{n-1}\dot{x} = 0$; hence $\dot{q}x^n = -qnx^{n-1}\dot{x}$, and (dividing by x^n),

$\dot{q} = -\frac{qn\dot{x}}{x} =$ (by ſubſtituting $\frac{1}{x^n}$ for q), $\frac{-n\dot{x}}{x^n+1}$ or $-nx^{-n-1}\dot{x}$; the ſame as before.

21. Much in the ſame manner is obtained the Fluxion of any ſurd, or fractional power of a fluent quantity, as of $x^{\frac{m}{n}}$ or $\sqrt[n]{x}^{m}$. For, putting the propoſed quantity - - $x^{\frac{m}{n}} = q$, then,
raiſing each to the n power, $x^m = q^n$;
take the Fluxions, - $mx^{m-1}\dot{x} = nq^{n-1}\dot{q}$;
divide by nq^{n-1}, - $\dot{q} = \frac{mx^{m-1}\dot{x}}{nq^{n-1}} = \frac{m^{m-1}\dot{x}}{nx^{m-\frac{m}{n}}} = \frac{m}{n}x^{\frac{m}{n}-1}\dot{x}$: which is ſtill the ſame rule as before, for finding the Fluxion of any power of a fluent quantity, and which is therefore general, whether the exponent be poſitive or negative, or integral or fractional.

22. For



22. For the Fluxions of Logarithms: Let A be the principal vertex of an hyperbola, having its aſymptotes CD, CP, with the ordinates DA, BA, PQ, &c, parallel to them. Then, from the nature of the hyperbola, and of logarithms, it is known that any ſpace ABPQ is the log. of the ratio of CB to CP, to the modulus ABCD. Now put 1 = CB or BA the ſide of the ſquare or rhombus DB; m = the modulus, or area of DB, or ſine of the angle C to the radius 1; alſo the abſciſs CP = x, and the ordinate PQ = y. Then, by the nature of the hyperbola, CP × PQ is always equal to DB, that is, $xy = m$; hence $y = \frac{m}{x}$, and the fluxion of the ſpace, or $\dot{x}y$ is $\frac{m\dot{x}}{x}$ = PQqp the fluxion of the log. of x, to the modulus m. And in the ordinary hyp. logarithms the modulus m being 1, therefore $\frac{\dot{x}}{x}$ is the fluxion of the hyp. log. of x; which is therefore equal to the Fluxion of the quantity, divided by the quantity itſelf. And the ſame might be brought out in ſeveral other ways, independent of the figure of the hyperbola.

23. By means of the Fluxions of logarithms, are determined thoſe of exponential quantities, i. e. quantities which have their exponent alſo a flowing or variable quantity. Theſe exponentials are of two kinds, viz, when the root is a conſtant quantity, as e^x; and when the root is variable, as y^x.

In the former caſe, put the propoſed exponential $e^x = z$, a ſingle variable quantity; then take the logarithm of each, ſo ſhall log. of $z = x \times$ log. of e; take the fluxions of theſe, ſo ſhall $\frac{\dot{z}}{z} = \dot{x} \times$ log. of e; hence $\dot{z} = z\dot{x} \times$ log. of $e = e^x\dot{x} \times$ log. of e, the fluxion of the propoſed exponential e^x; and which therefore is equal to the ſaid propoſed quantity, drawn into the fluxion of the exponent, and alſo into the log. of the root.

24. Alſo in the 2d caſe, put the exponential $y^x = z$; then the logarithms give log. $z = x \times$ log. y, and the fluxions give $\frac{\dot{z}}{z} = \dot{x} \times$ log. $y + x \times \frac{\dot{y}}{y}$; hence $\dot{z} = z\dot{x} \times$ log. $y + \frac{zx\dot{y}}{y}$ = (by ſubſtituting y^x for z) $y^x\dot{x} \times$ log. $y + xy^{x-1}\dot{y}$, is the fluxion of the propoſed exponential y^x; which therefore conſiſts of two terms, of which the one is the fluxion of the propoſed quantity conſidering the exponent only as conſtant, and the other is the fluxion of the ſame quantity conſidering the root as conſtant.

Of Second, Third, &c Fluxions.—Having explained the manner of conſidering and determining the firſt fluxions of flowing or variable quantities; it remains now to conſider thoſe of the higher orders, as 2d, 3d, 4th, &c, fluxions.

25. If the rate or celerity with which any flowing quantity changes its magnitude, be conſtant, or the ſame, at every poſition; then is the fluxion of it alſo conſtantly the ſame. But if the variation of magnitude be continually changing, either increaſing or decreaſing; then will there be a certain degree of fluxion peculiar to every point or poſition; and the rate of variation or change in the fluxion, is called the *Fluxion of the Fluxion*, or the *ſecond Fluxion* of the given fluent quantity. In like manner, the variation or fluxion of this 2d fluxion is called the *third Fluxion* of the firſt propoſed fluent quantity; and ſo on.

And theſe orders of fluxions are denoted by the fluent letter or quantity, with the correſponding number of points over it; viz, 2 points for the 2d fluxion, 3 for the 3d fluxion, 4 for the 4th fluxion, and ſo on. So the different orders of the fluxions of x, are $\dot{x}$, $\ddot{x}$, $\dddot{x}$, $\ddddot{x}$, &c; where each is the fluxion of the one next before it.

26. This deſcription of the higher orders of fluxions may be illuſtrated by the three figures at the 8th article; where, if x denote the abſciſs AP, and y the ordinate PQ; and if the ordinate PQ or y flow along the abſciſs AP or x, with an uniform motion; then the fluxion of x, viz $\dot{x}$ = Pp or Qr is a conſtant quantity, or $\ddot{x} = 0$, in all the figures. Alſo, in fig. 1, in which AQ is a right line, $\dot{y}$ is = rq, or the fluxion of PQ, is a conſtant quantity, or $\ddot{y} = 0$; for, the angle Q, = the angle A, being conſtant, Qr is to rq, or $\dot{x}$ to $\dot{y}$, in a conſtant ratio. But in the 2d figure, rq, or the fluxion of PQ, continually increaſes more and more; and in fig. 3 it continually decreaſes more and more; and therefore in both theſe caſes y has a 2d fluxion, being poſitive in fig. 2, but negative in fig. 3: and ſo on for the other orders of fluxions.

27. Thus, if for inſtance, the nature of the curve be ſuch, that x^3 is everywhere equal to a^2y; then, taking the fluxions, it is $a^2\dot{y} = 3x^2\dot{x}$; and, conſidering $\dot{x}$ always as a conſtant quantity, and taking always the fluxions, the equations of the ſeveral orders of fluxions will be as below; viz,

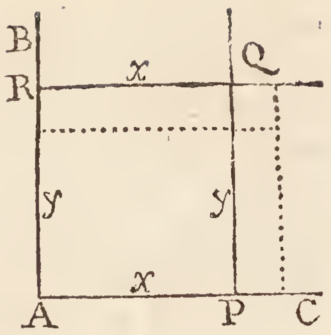
the 1ſt fluxions $a^2\dot{y} = 3x^2\dot{x}$,
the 2d fluxions $a^2\ddot{y} = 6x\dot{x}^2$,
the 3d fluxions $a^2\dddot{y} = 6\dot{x}^3$,
the 4th fluxions $a^2\ddddot{y} = 0$,

and all the higher fluxions = 0 or nothing.

Alſo the higher orders of fluxions are found in the ſame manner as the lower ones. Thus,

The 1ſt flux. of y^3 is - . -	$3y^2\dot{y}$;
its 2d flux. or flux. of $3y^2\dot{y}$, conſidered as the rectangle of $3y^2$ and $\dot{y}$, is - -	$3y^2\ddot{y} + 6y\dot{y}^2$;
and the flux. of this again, or the 3d fluxion of y^3, is -	$3y^2\dddot{y} + 18y\dot{y}\ddot{y} + 6\dot{y}^3$.

28. In the foregoing articles, it has been ſuppoſed that the fluents increaſe; or that their fluxions are poſitive; but it often happens that ſome fluents decreaſe, and that therefore their fluxions are negative: and whenever this is the caſe, the ſign of the fluxion muſt be changed, or made contrary to that of the fluent. So, of the rectangle xy, when both x and y increaſe together, the fluxion is $\dot{x}y + x\dot{y}$: but if one of them, as y, decreaſe, while the other, x, increaſes; then the fluxion of y being $-\dot{y}$, the fluxion of xy will in that caſe be $\dot{x}y - x\dot{y}$. This may be illuſtrated by the annexed rectangle



tangle APQR = xy, ſuppoſed to be generated by the motion of the line PQ from A towards C, and by the motion of the line RQ from B towards A: For, by the motion of PQ, from A towards C, the rectangle is increaſed, and its fluxion is + $\dot{x}y$; but by the motion of RQ, from B towards A, the rectangle is decreaſed, and the fluxion of the decreaſe is $x\dot{y}$; therefore, taking the fluxion of the decreaſe from that of the increaſe, the fluxion of the rectangle xy, when x increaſes and y decreaſes, is $\dot{x}y - x\dot{y}$.

For the Inverſe Method, or the finding of fluents, ſee Fluent. And for the ſeveral applications of this ſcience to Maxima and Minima, the drawing of Tangents, &c, ſee the reſpective articles.

An idea of the principles of Fluxions being now delivered, as above, we may next conſider ſomewhat of the chief writings and improvements that have been made by divers authors, ſince the firſt diſcovery of them: indeed ſome of the chief improvements may be learned by conſulting the preface to Dr. Waring's Meditationes Analyticæ.

The inventor himſelf brought the doctrine of Fluxions to a conſiderable degree of perfection; as may be ſeen by many ſpecimens of this ſcience, given by him; particularly in his Principia, in his Tract on Quadratures, and in his Treatiſe on Fluxions, publiſhed by Mr. Colſon; from all which it will appear, that he not only laid down the whole theory of this method, both direct and inverſe; but alſo applied it in practice, to the ſolution of many of the moſt uſeful and important problems in mathematics and philoſophy.

Various improvements however have been made by many illuſtrious authors on this ſcience; particularly by John Bernoulli, who treated of the fluents belonging to the fluxions of exponential expreſſions; James Bernoulli, Craig, Cheyne, Cotes, Manfredi, Riccati, Taylor, Fagnanus, Clairaut, D'Alembert, Euler, Condorcet, Walmeſley, Le Grange, Emerſon, Simpſon, Landen, Waring, &c. There are ſeveral other treatiſes alſo on the principles of Fluxions, by Hayes, Newyentyt, L'Hôpital, Hodſon, Rowe, &c, &c, delivering the elements of this ſcience in an eaſy and familiar way.

FLY, in Mechanics, a heavy weight applied to certain machines, to regulate their motions, as in a jack, or to increaſe their effect, as in the coining engine, &c; by means of which the force of the power is not only preſerved, but equally diſtributed in all the parts of the revolution.

The Fly is either like a croſs, with heavy weights at the ends of its arms; or like a heavy wheel at right-angles to the axis of motion. It may be applied to various ſorts of engines, whether moved by men, horſes, wind, or water; and is of great uſe in thoſe parts of an engine having a quick circular motion, and where the power or reſiſtance acts unequally in the different parts of a revolution. In this caſe the Fly becomes a moderator, making the motion of revolution almoſt everywhere equal.

FLYERS, in Architecture, ſuch ſtairs as go ſtraight, and do not wind round, nor have the ſteps made tapering, but equally broad at both ends. Hence, if one flight do not riſe to the top of the ſtory &c, there is a broad half pace, and then commonly another ſet of flyers.

FLYING, the progreſſive motion of a bird, or other winged animal, through the air.

The parts of birds chiefly concerned in Flying, are the wings and tail: by the former, the bird ſuſtains and wafts himſelf along; and by the latter he is aſſiſted in aſcending and deſcending, to keep his body poiſed and upright, and ſteady. The wings are extended or ſtretched quite out, and then ſtruck forcibly downwards againſt the air, which by its reſiſtance raiſes the bird upwards; then to make another ſtroke, the wing, by means of its joints, readily cloſes in ſome degree, preſenting the ſharp edge of the pinion foremoſt to cut the air, and drawing the collapſed feathers after it like a flag, to diminiſh the reſiſtance to the aſcent as much as poſſible; the wing and feathers are then ſtretched out horizontally again, and another downward ſtroke made, which raiſes the bird ſtill higher; and ſo on as far as he pleaſes, or as the denſity of the air will ſuſtain him; performing thoſe motions of the wings very rapidly, that the flight may be the quicker.

Artificial Flying, is that attempted by men, &c, by the aſſiſtance of mechanics.

The art of flying has been attempted by ſeveral perſons in all ages. Friar Bacon, about 500 years ago, not only aſſerts the poſſibility of flying, but affirms that he himſelf knew how to make a machine with which a man might be able to convey himſelf through the air like a bird; and further adds, that it had been tried with ſucceſs. Though the fact is to be doubted, if, as it was ſaid, it conſiſted in the following method; viz, in a couple of large thin hollow copper globes, exhauſted of air; which being much lighter than air, would ſuſtain a chair on which a perſon might ſit. Father Franciſco Lana, in his Prodromo, propoſes the ſame thing, as his own thought. He computes, that a round veſſel of plate-braſs, 14 feet in diameter, weighing 3 ounces the ſquare foot, will only weigh 1848 ounces; whereas a quantity of air of the ſame bulk will weigh near 2156 ounces; ſo that the globe will not only be ſuſtained in the air, but will carry with it a weight of 304 ounces; and by increaſing the ſize of the globe, the thickneſs of the metal remaining the ſame, he adds, a veſſel might be made to carry a much greater weight. But the fallacy is obvious: a globe of the dimenſions he deſcribes, as Dr. Hook obſerves, would not ſuſtain the preſſure of the air, but be cruſhed inwards. Indeed it is not probable that ſuch a globe can be made of a thinneſs ſufficient to float in the atmoſphere after it is exhauſted of air, and yet be ſtrong enough to ſuſtain the compreſſing force of the atmoſphere. But for this purpoſe it ſeems that the globe ſhould be filled with an air as elaſtic or ſtrong as the atmoſphere, and yet be very much lighter; ſuch as has lately been uſed in the Mongolfiers and Balloons; the former of which is filled with common air heated, ſo as to be more elaſtic, and leſs heavy; and the latter with inflammable air, which is as elaſtic as the common air, with only about one tenth of its weight. And thus the idea of flying, or rather floating, in the air, has been lately realized by the moderns, uſing however a different ſort of air. See Aerostation.

The *ſame author* deſcribes a machine for Flying, invented by the Sieur Beſnier, a ſmith of Sable, in the county of Main. See Philoſ. Collec. numb. 1.

By the foregoing method however, at beſt, only a method of floating can be obtained, like a log floating in a current; but not of Flying, which conſiſts in moving through the air, independent of any current; and which muſt be effected by ſomething in the nature of wings. Attempts of this latter kind alſo have indeed been made by ſeveral perſons of late years; but it does not appear that any of them have been attended with ſuch ſucceſs as to induce the authors of thoſe attempts to make them public. The philoſophers of king Charles the ſecond's reign were much buſied about this art; and the celebrated biſhop Wilkins was ſo confident of ſucceſs in it, that he ſays, he does not queſtion but in future ages it will be as uſual to hear a man call for his wings, when he is going a journey, as it is now to call for his boots.

The ſtory of the flight of Dædalus is well known.

FLYING *Pinion*, is part of a clock having a fly or fan with which to gather the air, and ſo bridle the rapidity of the clock's motion, when the weight deſcends in the ſtriking part.

FOCAL *Diſtance*, the Diſtance of the Focus, which is ſometimes underſtood as its diſtance from the vertex, as in the parabola; and ſometimes its diſtance from the centre, as in the ellipſe or hyperbola.

FOCUS, in Geometry and the Conic Sections, is applied to certain points in the Ellipſe, Hyperbola, and Parabola, where the rays reflected from all parts of theſe curves do concur or meet; that is, rays iſſuing from a luminous point in the one focus, and falling on all points of the curves, are reflected into the other Focus, or into the line directed to the other Focus, viz, into the other Focus in the ellipſe and parabola, and directly from it in the hyperbola. Which is the reaſon of the name Focus, or Burning-point. Hence, as the one Focus of the parabola is at an infinite diſtance; and conſequently all rays drawn from it, to any finite part of the curve about the vertex, are parallel to one another; therefore if rays from the ſun, or any other object ſo diſtant as that thoſe rays may be accounted parallel, fall upon the curve of a parabola or concave ſurface of a paraboloidal figure, thoſe rays will all be reflected into its Focus.

Thus, the rays P*f*, from the Focus *f*, are reflected in the direction PF, into the other Focus F, in the ellipſe and parabola, and form the Focus F, into FQ, in the hyperbola.

In all the three curves, the double ordinate CD drawn through the Focus F, is the parameter of the axis, or a 3d proportional to AB and *ab*, the tranſverſe and conjugate axes.

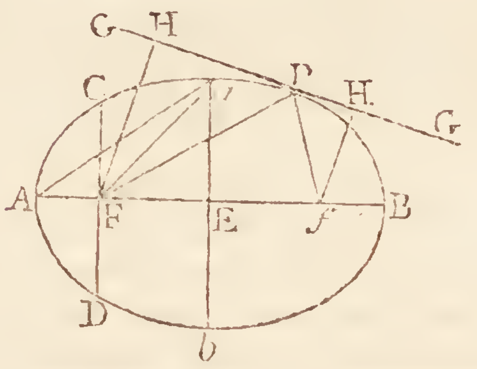
In the ellipſe and parabola, the tranſverſe axis is equal to the ſum of the two lines PF + P*f*, drawn from the Foci to any point P in the curve; but in the hyperbola, the tranſverſe is equal to the difference of thoſe two lines. That is,

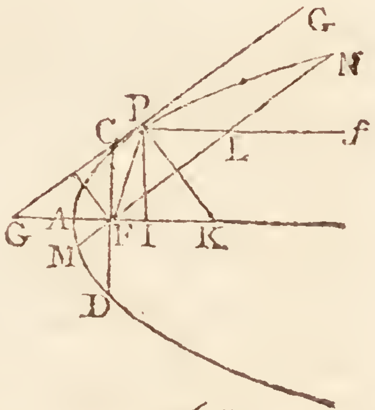
$AB = PF + Pf$ in the ellipſe and parabola,
$AB = PF - Pf$ in the hyperbola.

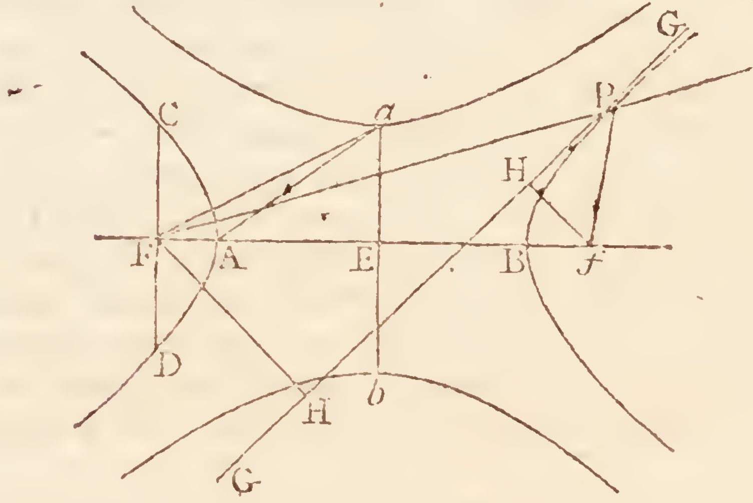
In the ellipſe and parabola, the ſquare of the diſtance between the Foci, is equal to the difference of the ſquares of the two axes; and in the hyperbola, it is equal to the ſum of their ſquares: that is

$Ff^2 = AB^2 - ab^2$ in the ellipſe and parabola.
$Ff^2 = AB^2 + ab^2$ in the hyperbola.

Therefore the two ſemi-axes, with the diſtance of the Focus from the centre, form always a right-angled triangle F*a*E, or A*a*E.







In all the curves, the conjugate ſemi-axis is a mean proportional between the diſtances of either Focus from either end of the tranſverſe axis: that is,

$AF : Ea :: Ea : FB$,
or $Ea^2 = AF . FB$.

If there be any tangent to theſe curves, and two lines drawn from the Foci to the point of contact; theſe two lines will make equal angles with that tangent.

So, if GPG touch the curve at P;
then is the angle $FPG = \angle fPG$.

If a line be drawn from either Focus, perpendicularly upon a tangent; the diſtance of their interſection from the centre will be equal to the ſemi-tranſverſe axis. So, if FH or *f*H be perpendicular to the tangent PH; then is EH = EA or EB. Conſequently, the circle deſcribed on the diameter AB, will paſs through all the points H.

The foregoing are the chief properties that are common to the Foci of all the three conic ſections. To which may be added the following properties which are peculiar to the parabola: viz,

In the parabola, the diſtance from the Focus to the vertex, is equal to $\frac{1}{4}$ of the parameter, or half the ordinate at the Focus: viz, $AF = \frac{1}{2} FC$.

Alſo, a line drawn from the Focus to any point in the curve, is equal to the ſum of the Focal diſtance from the vertex and the abſciſs of the ordinate to that point: i. e. $FP = AF + AI$.

If from any point of a parabola there be drawn a tangent, and a perpendicular to it PK, both to meet the axis produced; then the focus will be equally diſtant from the two interſections and the point of contact: i. e. $FG = FP = FK$.

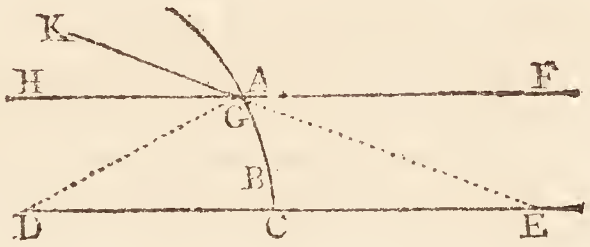
Hence alſo the ſubnormal IK is $= 2AF$ or $= FC$ the ſemi-parameter.

The line drawn from the Focus to any point of the curve, is equal to $\frac{1}{4}$ the parameter of the diameter to that point: i. e. $FP = \frac{1}{4}$ the parameter of the diameter Pf.

If an ordinate to any diameter paſs through the Focus, it will be equal to half its parameter; and its abſciſs equal to $\frac{1}{4}$ of the ſame parameter; or the abſciſs equal to half the ordinate; i. e. $PL = \frac{1}{4}MN = \frac{1}{2}LM$ or $\frac{1}{2}LN$.

Focus, in Optics, is a point in which ſeveral rays meet, and are collected, after being either reflected or refracted. It is ſo called, becauſe the rays being here brought together and united, their force and effect are increaſed, inſomuch as to be able to burn; and therefore it is that bodies are placed in this point to be burnt, or to ſhew the effect of burning glaſſes, or mirrors.—It is to be obſerved however, that in practice, the Focus is not an abſolute point, but a ſpace of ſome ſmall breadth, over which the rays are ſcattered; owing to the different nature and refrangibility of the rays of light, and to the imperfections in the figure of the lens, &c. However, the ſmaller this ſpace is, the better, or the nearer to perfection the machine approaches. Huygens ſhews that the Focus of a lens convex on both ſides, has its breadth equal to $\frac{5}{8}$ of the thickneſs of the lens.

Virtual Focus, or *Point of Divergence*, ſo called by Mr. Molyneux, is the point from whence rays tend, after refraction or reflection; being in this reſpect oppoſed to the ordinary Focus, or Point of Concurrence, where rays are made to meet after refraction or reflection. Thus, the Foci of an hyperbola are mutually Virtual Foci to each other; but, in an ellipſe, they are common Foci to each other: for the rays are reflected *from* the other Focus in the hyperbola, but *towards* it in the ellipſe; as appears by the figures at the beginning of this article.



And, in Dioptrics, let ABC be the concavity of a glaſs, whoſe centre is D, and axis DE: Let FG be a ray of light falling on the glaſs, parallel to the axis DE; this ray FG, after it has paſſed through the glaſs, at its emerſion at G will not proceed directly to H, but be refracted from the perpendicular DG, and will become the ray GK, which being produced to meet the axis in E, this point E is the Virtual Focus, as the ray is refracted directly *from* this point.

Rules for the Foci of Lenſes and Mirrors.

I. *In Catoptrics*, or *Lenſes.*

1. The Focus of a convex glaſs, i. e. the point where parallel rays tranſmitted through a convex glaſs, whoſe ſurface is the ſegment of a ſphere, do unite, is diſtant from the pole or vertex of the glaſs, almoſt a diameter and half of the convexity.—2. In a Plano-Convex glaſs, the Focus of parallel rays is diſtant from the pole of the glaſs a diameter of the convexity, if the ſegment do not exceed 30 degrees. Or the rule in Plano-Convex glaſſes is, As 107 : 193 : : ſo is the radius of convexity : to the refracted ray taken to its concourſe with the axis; which in glaſſes of larger ſpheres is almoſt equal to the diſtance of the Focus taken in the axis.—3. In Double-Convex glaſſes of the ſame ſphere, the Focus is diſtant from the pole of the glaſs about the radius of the convexity, if the ſegment be but 30 degrees. But when the two convexities are unequal, or ſegments of different ſpheres, then the rule is, As the ſum of the radii of both convexities : to the radius of either convexity alone : : ſo is double the radius of the other convexity : to the diſtance of the Focus. —Here obſerve, that the rays which fall nearer the axis of any glaſs, are not united with it ſo near the pole of the glaſs as thoſe farther off: nor will the Focal diſtance be ſo great in a plano-convex glaſs, when the convex ſide is towards the object, as when the plane ſide is towards it. And hence it is truly concluded, that, in viewing any object by a plano-convex glaſs, the convex ſide ſhould always be turned outward; as alſo in burning by ſuch a glaſs.

II. *For the Virtual Focus*, obſerve

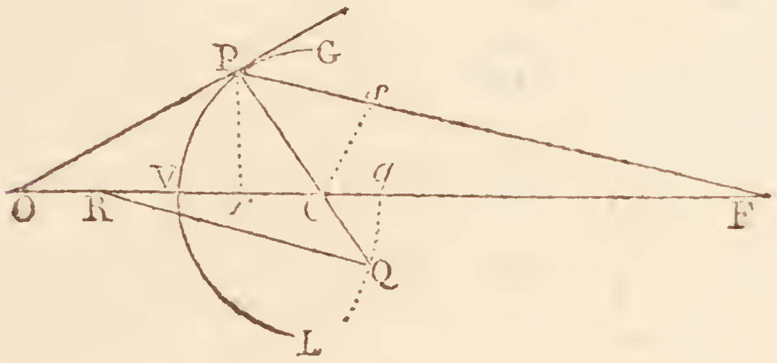
1. That in Concave glaſſes, when a ray falls from air parallel to the axis, the Virtual Focus, by its firſt refraction, becomes at the diſtance of a diameter and a half of the concavity.—2. In Plano-Concave glaſſes, when the rays fall parallel to the axis, the Virtual Focus is diſtant from the glaſs, the diameter of the concavity.—3. In Plano-Concave glaſſes, as 107 : 193 : : ſo is the radius of the concavity : to the diſtance of the Virtual Focus.—4. In Double Concaves of the ſame ſphere, the Virtual Focus of parallel rays is at the diſtance of the radius of the concavity. But, whether the concavities be equal or unequal, the Virtual Focus, or point of divergency of the parallel rays, is determined by this rule; As the ſum of the radii of both concavities : is to the radius of either concavity : : ſo is double the radius of the other concavity : to the diſtance of the Virtual Focus.—5. In Concave glaſſes, expoſed to converging rays, if the point to which the incident ray converges, be farther diſtant from the glaſs than the Virtual Focus of parallel rays, the rule for finding the Virtual Focus of this ray, is this; As the difference between the diſtance of this point from the glaſs, and the diſtance of the Virtual Focus from the glaſs : is to the diſtance of the Virtual Focus : : ſo is the diſtance of this point of convergence from the glaſs : to the diſtance of the Virtual Focus of this converging ray.—6. In Concave glaſſes, if the point to which the incident ray converges, be nearer to the glaſs than the Virtual Focus of parallel rays, the rule to find where it croſſes the axis, is this; As the exceſs of the Virtual Focus, more than this point of convergency : is to the Virtual Focus : : ſo is the diſtance of this point of convergency from the glaſs : to the diſtance of the point where this ray croſſes the axis.

III. *Practical Rules for finding the Foci of Glaſſes.*

1. To find, by experiment, the Focus of a convex ſpherical glaſs, being of a ſmall ſphere; apply it to the end of a ſcale of inches and decimal parts, and expoſe it before the ſun; upon the ſcale may be ſeen the bright interſection of the rays meaſured out: or, expoſe it in the hole of a dark chamber; and where a white paper receives the diſtinct repreſentation of diſtant objects, there is the Focus of the glaſs.—2. For a glaſs of a pretty long Focus, obſerve ſome diſtant object through it, and recede from the glaſs till the eye perceives all in confuſion, or the object begins to appear inverted; then the eye is in the Focus.—3. For a Plano-Convex glaſs: make it reflect the ſun againſt the wall; on the wall will then be ſeen two ſorts of light, a brighter within another more obſcure: withdraw the glaſs from the wall, till the bright image be in its leaſt dimenſions; then is the glaſs diſtant from the wall about a fourth part of its Focal length.—4. For a Double Convex: expoſe each ſide to the ſun in like manner; and obſerve both the diſtances of the glaſs from the wall: then is the firſt diſtance about half the radius of the convexity turned from the ſun; and the ſecond is about half the radius of the other convexity. The radii of the two convexities being thus known, the Focus is then found by this rule; As the ſum of the radii of both convexities : is to the radius of either convexity : : ſo is double the radius of the other convexity : to the diſtance of the Focus.

IV. *To find the Foci of all Glaſſes Geometrically.*

Dr. Halley has given a general method for finding the Foci of ſpherical glaſſes of all kinds, both concave and convex; expoſed to any kind of rays, either parallel, converging, or diverging; as follows: To find the Focus of any parcel of rays diverging from, or converging to, a given point in the axis of a ſpherical lens, and making the ſame angle with it; the ratio of the ſines of refraction being given.



Suppoſe GL a lens; P a point in its ſurface; V its pole; C the centre of the ſpherical ſegment; O the object, or point in the axis, to or from which the rays proceed; and OP a given ray: and ſuppoſe the ratio of refraction to be as r to s. Then making CR to CO, as s to r for the immerſion of a ray, or as r to s for the emerſion (i. e. as the ſines of the angles in the medium which the ray enters, to the correſponding ſines in the medium out of which it comes); and laying CR from C towards O, the point R will be the ſame for all the rays of the point O. Laſtly, drawing the radius PC, continued if neceſſary; with the centre R, and diſtance OP, deſcribe an arc interſecting PC in Q. The line QR, being drawn, ſhall be parallel to the reflected ray; and PF, being made parallel to it, ſhall interſect the axis in the point F, the Focus ſought. —Or, make as CQ : CP : : CR : CF, which will be the diſtance of the Focus from the centre of the ſphere. —And from this general conſtruction, he adverts to a number of particular ſimple caſes.

Dr. Halley gave alſo an univerſal algebraical theorem to find the Focus of all ſorts of optic glaſſes, or lenſes. See the Philoſ. Tranſ. N° 205, or Abr. vol. 1, pa. 191.

V. *In Catoptrics, or Foci by Reflection.*

Theſe are eaſily found for any known curve, from this principle, that the angle of reflection is always equal to the angle of incidence.

The ſame are alſo eaſily found by experiment, being expoſed to any object.

The increaſe of heat from collecting the ſun's rays into a Focus, has been found in many caſes of burning glaſſes, to be aſtoniſhingly great; the effect being increaſed as the ſquare of the diameter of the glaſs exceeds that of the Focus. If, for inſtance, there be a burning glaſs of 12 inches diameter; this will collect or crowd together all the rays of the ſun which fall upon the glaſs into the compaſs of about $\frac{1}{8}$ part of an inch: then, the areas of the two ſpaces being as the ſquare of 12 to the ſquare of $\frac{1}{8}$, or as the ſquare of 96 to the ſquare of 1, that is, as 9216 to 1; it follows, that the heat in the Focus will be 9216 times greater than the ſun's common heat. And this will have an effect as great as the direct rays of the ſun would have on a body placed at the 96th part of the earth's diſtance from the ſun; or the ſame as on a planet that ſhould move round the ſun at but a very little more than a diameter of the ſun's diſtance from him, or that would never appear farther from him than about 36 minutes.

Beſides Dr. Halley in his method for finding the Foci, ſeveral other authors have written upon this ſubject; as Mr. Ditton, in his Fluxions; Dr. Gregory, in his Elements of Dioptrics; M. Carré, and Guiſnée, in the Memoires de l'Acad.: Dr. Barrow and Sir I. Newton have alſo neat and elegant ways of finding geometrically the Foci of ſpherical glaſſes; which may be ſeen in Barrow's Optical Lectures.

FOLIATE, a name given by ſome to a curve of the 2d order, expreſſed by the equation $x^3 + y^3 = axy$, being one ſpecies of defective hyperbolas, with one aſymptote, and conſiſting of two infinite legs croſſing each other, forming a ſort of leaf. It is the 42d ſpecies of Newton's Lines of the 3d Order.

FOLKES (Martin), an Engliſh mathematician, philoſopher, and antiquary, was born at Weſtminſter about 1690; and was greatly diſtinguiſhed as a member of the Royal Society in London, and of the Academy of Sciences at Paris. He was admitted into the former at 24 years of age; made one of their council two years after; named vice-preſident by Sir Iſaac Newton himſelf; and, after Sir Hans Sloane, became preſident. Coins, ancient and modern, were a great object with him; and his laſt production was a book upon the Engliſh Silver Coin, from the Conqueſt to his own times. He died at London in 1754. Dr. Birch had drawn up materials for a life of Mr. Folkes, which are preſerved at large in the Anecdotes of Bowyer, p. 562. There are many memoirs of Mr. Folkes's in the Philoſ. Tranſ. from vol. 30 to vol. 46, both incluſive; viz, 1. Account of an Aurora Borealis, vol. 30.—2. Of Lieuwenhoek's curious Microſcope, vol. 32.—3. On the Standard Meaſures in the Capitol at Rome, vol. 39.—4. Obſervations of three Mock-ſuns, vol. 40.—5. On the freſh-water Polypus, vol. 42.—6. On human bones petrified, vol. 43.—7. On a paſſage in Pliny's Natural Hiſtory, vol. 44.—8. On an Earthquake at London, vol. 46.—9. Ditto at Kenſington, vol. 46.—10. Ditto at Newton, vol. 46.

FOMAHAUT, or FOMALHAUT, a ſtar of the firſt magnitude in the water of the conſtellation Aquarius, or in the mouth of the ſouthern fiſh. Its latitude is $21^\circ\, 6'\, 28''$ ſouth, and mean longitude to the beginning of 1760, $11^s\, 0^\circ\, 28'\, 55''$.

FONTENELLE (BERNARD DE), a celebrated French author, was born at Rouen in 1657, and died in 1756, when he was near 100 years old. He was a univerſal genius: at a very early age he wrote ſeveral comedies and tragedies of conſiderable merit; and he did the ſame at a very advanced age. Voltaire declares him the moſt univerſal genius the age of Lewis the 14th produced; and compares him to lands ſituated in ſo happy a climate, as to produce all ſorts of fruits. His laſt comedies, though they ſhewed the elegance of Fontenelle, were however, little fitted for the ſtage; he then alſo produced an Apology for the Vortices of Des Cartes; upon which Voltaire ſays, "We muſt excuſe his comedies, on account of his great age; and his Carteſian opinions, as they were thoſe of his youth, when they were univerſally received all over Europe."

In his poetical performances and Dialogues of the Dead, the ſpirit of Voiture was diſcerned, though more extended and more philoſophical. His Plurality of Worlds is a work ſingular in its kind: his deſign in this was, to preſent that part of philoſophy to view in a gay and pleaſing dreſs.

Fontenelle applied himſelf alſo to mathematics and natural philoſophy; in which he proved not leſs ſucceſsful than he had been in polite literature. Having been appointed perpetual ſecretary to the Academy of Sciences, he diſcharged that truſt above 40 years with univerſal applauſe: his Hiſtory of the Academy often throws great light upon their memoirs, which are ſometimes obſcure; and it has been ſaid, he was the firſt who introduced elegance into the ſciences. The Eloges, which he pronounced on the deceaſed members of the Academy, have this peculiar merit, that they excite a reſpect for the ſciences as well as for the authors.

Upon the whole, Fontenelle muſt be looked upon as the great maſter of the new art of treating abſtract ſciences, in a manner that make their ſtudy at once eaſy and agreeable: nor are any of his works of other kinds void of merit. All thoſe talents which he poſſeſſed from nature, were aſſiſted by a good knowledge of hiſtory and languages: and he perhaps ſurpaſſes all men of learning who have not had the gift of invention.

Beſide his poetical and theatrical works, with thoſe of Belles lettres, &c, he publiſhed *Elemens de Geometrie de l'Infini*, in 4to, 1727; alſo the *Theorie des Tourbillons Carteſiens*; and *Diſcours moraux & philoſophiques*. All his different works were collected in eleven volumes, 12mo, under the title of *Oeuvres Diverſes*.

FOOT, a meaſure of length, divided into 12 inches, and each inch ſuppoſed to contain 3 barley-corns in length. Geometricians divide the Foot into 10 digits, and the digit into 10 lines, &c. The French divide their Foot, as we do, into 12 inches; but their inch they divide into 12 lines.

It ſeems this meaſure has been taken from the length of the human Foot; but it is of different lengths in different countries. The Paris royal Foot is to the Engliſh Foot, as 4263 to 4000, and exceeds the Engliſh by $9\frac{1}{2}$ lines; the ancient Roman Foot of the Capitol conſiſted of 4 palms; equal to 11 inches and $\frac{7}{10}$ Engliſh; the Rhinland, or Leyden Foot, uſed by the northern nations, is to the Roman Foot, as 19 to 20. For the proportions of the Foot of ſeveral nations, compared with the Engliſh, ſee the article MEASURE.

Square FOOT, is a ſquare whoſe ſide is 1 foot, or 12 inches, and conſequently its area is 144 ſquare inches.

Cubic FOOT, is a cube whoſe ſide is one Foot, or 12 inches, and conſequently it contains 12^3 or 1728 cubic inches.

FOOT-*bank*, or FOOT-*ſtep*, in Fortification. See BANQUETTE.

FORCE, *Vis*, or *Power*, in Mechanics, Philoſophy, &c, denotes the cauſe of the change in the ſtate of a body, with reſpect to motion, reſt, preſſure, &c; as well as its endeavour to oppoſe or reſiſt any change made in ſuch its ſtate. Thus, whenever a body, which was at reſt, begins to move; or when its motion is either not uniform, or not direct; the cauſe of this change in the ſtate of the body, is what is called *Force*, and is an external Force. Or, while a body remains in the ſame ſtate, either of reſt, or of uniform and rectilinear motion, the cauſe of its remaining in ſuch ſtate, is in the nature of the body, being an innate internal Force, and is called its *Inertia*.

Mechanical Forces may be reduced to two ſorts; one of a body at reſt, the other of a body in motion.

The Force of a body at reſt, is that which we conceive to be in a body lying ſtill on a table, or hanging by a rope, or ſupported by a ſpring, &c; and this is called by the names of Preſſure, Tenſion, Force, or Vis Mortua, Solicitatio, Conatus Movendi, Conamen, &c; which kind of Force may be always meaſured by a weight, viz, the weight that ſuſtains it. To this claſs of Forces may alſo be referred Centripetal and Centrifugal Forces, though they reſide in a body in motion; becauſe theſe Forces are homogeneous to weights, preſſures, or tenſions of any kind. The preſſure, or Force of gravity in any body, is proportional to the quantity of matter in it.

The Force of a body in motion, is a power reſiding in that body, ſo long as it continues its motion; by means of which, it is able to remove obſtacles lying in its way; to leſſen, deſtroy, or overcome the Force of any other moving body, which meets it in an oppoſite direction; or to ſurmount any the largeſt dead preſſure or reſiſtance, as tenſion, gravity, friction, &c, for ſome time; but which will be leſſened or deſtroyed by ſuch reſiſtance as leſſens or deſtroys the motion of the body. This is called Vis Motrix, Moving Force, or Motive Force, and by ſome late writers Vis Viva, to diſtinguiſh it from the Vis Mortua ſpoken of before; and by theſe appellations, however different, the ſame thing is underſtood by all mathematicians; namely, that power of diſplacing, of withſtanding oppoſite moving Forces, or of overcoming any dead reſiſtance, which reſides in a moving body, and which, in whole or in part, continues to accompany it, as long as the body moves; and may be otherwiſe called Percuſſive Force, or Momentum.

But concerning the meaſure of this kind of Force, mathematicians have been divided into two parties. It is allowed by both ſides, that the meaſure of this Force depends partly upon the maſs of matter in the body, or its weight, and partly upon the velocity of its motion; ſo that upon any increaſe of either weight or velocity, the moving Force will become greater. It is alſo agreed, that the velocity being given, or being the ſame in two moving bodies, their Forces will be in proportion to their maſſes or weights. But, when two bodies are equal, and the velocities with which they move are different, the two parties no longer agree about the meaſure of the moving Force.

The Carteſians and Newtonians maintain, that, in this caſe, the moving Force is in proportion ſimply as the velocity with which a body moves; ſo that with a double velocity it has a double Force, &c: But the Leibnitians aſſert, that the moving Force is proportional to the ſquare of the velocity; ſo as, with a double velocity to have a quadruple Force, &c. Or, when the bodies are different, the former hold, that the momentum or moving Force of bodies, is in the compound ratio of their weights and velocities: But the Leibnitians hold, that it is in the compound ratio of the weights and ſquares of the velocities.

Though Leibnitz was the firſt who expreſsly aſſerted, that the Force of a body in motion is as the ſquare of its velocity, which was in a paper inſerted in the Leipſic Acts for the year 1686, yet it is thought that Huygens led him into that notion, by ſome demonſtrations in the 4th part of his book De Horologio Oſcillatorio, relating to the centre of oſcillation, and by his diſſertations, in anſwer to the objections of the abbot Catalan, one of which was publiſhed in 1684. This eminent mathematician had demonſtrated, that in the colliſion of two bodies that are perfectly elaſtic, the ſum of the products of each body multiplied by the ſquare of its velocity, was the ſame after the ſhock as before; (though the ſame thing is true of the ſums of the products of the bodies multiplied ſimply by their velocities). Now that propoſition is ſo far general as to obtain in all colliſions of bodies that are perfectly elaſtic: and it is alſo true, when bodies of a perfect elaſticity ſtrike any immoveable obſtacle, as well as when they ſtrike one another; or when they are conſtrained by any power or reſiſtance to move in directions different from thoſe in which they impel one another. Theſe conſiderations might have induced Huygens to lay it down as a general rule, that bodies conſtantly preſerve their Aſcenſional Force, i. e. the product of their maſs by the height to which their centre of gravity can aſcend, which is as the ſquare of the velocity; and therefore, in a given ſyſtem of bodies, the ſum of the ſquares of their velocities will remain the ſame, and not be altered by the action of the bodies among themſelves, nor againſt immoveable obſtacles. Leibnitz's metaphyſical ſyſtem led him to think that the ſame quantity of action or Force ſubſiſted in the univerſe; and finding this impoſſible, if Force were eſtimated by the quantity of motion, he adopted Huygens's principle of the preſervation of the Aſcenſional Force, and made it the meaſure of moving Forces. But it is to be obſerved that Huygens's principle, above-mentioned, is general only when bodies are perfectly elaſtic; and in ſome other caſes which Maclaurin has endeavoured to diſtinguiſh: ſhewing at the ſame time that no uſeful concluſion in mechanics is affected by the diſputes concerning the meaſure of the Force of bodies in motion, which have been objected to mathematicians. Analyſt, Query 9. See Maclaurin's Fluxions, vol. 2, art. 533; Huygens Oper. tom. 1, pa. 248; &c.

Leibnitz's principle was adopted by ſeveral perſons; as Wolfius, the Bernoullis, &c. Mr. Dan. Bernoulli, in his Treatiſe, has aſſumed the preſervation of the Vis Aſcendens of Huygens, or, as others expreſs it, the Conſervatio Virium Vivarum; and, in Bernoulli's own expreſſion, *æqualitas inter deſcenſum actualem aſcenſumque potentialem*, as an hypotheſis of wonderful uſe in mechanics. But a late author contends, that the concluſions drawn from this principle are oftener falſe than true. See De Conſervat. Virium Vivarum Diſſert. Lond. 1744.

Catalan and Papin anſwered Leibnitz's paper publiſhed in 1686; and from that time the controverſy became more general, and was carried on for ſeveral years by Leibnitz, John and Daniel Bernoulli, Poleni, Wolfius, s'Graveſande, Camus, Muſchenbroek, &c, on one ſide; and Pemberton, Eames, Deſaguliers, Dr. S. Clark, M. de Mairan, Jurin, Maclaurin, Robins, &c, on the other. See Act. Erud. 1686, 1690, 1691, 1695; Nouv. de la Rep. des Let. Sept. 1686, 1687, art. 2; Comm. Epiſt. inter Leibn. et Bern. Ep. 24, p. 143; Diſcourſes ſur les Loix de la Comm. du Mouvement, Oper. tom. 3, & Diſſ. de vera Notione Virium Vivarum, ib.; Act. Petropol. tom. 1, p. 131, &c; Hydronamica, ſect. 1; Herman, in Act. Petrop. tom. 1, p. 2, &c; Polen. de Caſtellis; Wolf. in Act. Petrop. tom. 1, p. 217, &c, and in Coſmol. Gener.; Graveſ. in Journ. Lit. and Phyſ. Elem. Math. 1742, lib. 2, cap. 2 and 3; Memoir. de l'Acad. des Sciences 1728; Muſchenbr. Int. ad Phil. Nat. 1762, vol. 1, p. 83 &c; Pemb. &c, in Phil. Tranſ. number 371, 375, 376, 396, 400, 401, or Abridg. vol. 6, p. 216 &c. Mairan in Memo. de l'Acad. des Sc. 1728, Phil. Tranſ. numb. 459, or Abridg. vol. 8, p. 236, Philoſ. Tranſ. vol. 43, p. 423 &c; Maclaurin's Acc. of Newton's Diſcoveries, p. 117 &c, Flux. ubi ſupr. & Recueil des Pieces qui ont emporté le Prix &c. tom. 1; Deſagul. Courſe Exp. Philoſ. vol. 1, p. 393 &c, vol. 2, p. 49 &c; and Robins's Tracts, vol. 2, p. 135.

The nature and limits of this work will not admit of a full account of the arguments and experiments that have been urged on both ſides of this queſtion; but they may be found chiefly in the preceding references. A few of them however may be conſidered, as follows.

The defenders of Leibnitz's principle, beſide the arguments above-mentioned, refer to the ſpaces that bodies aſcend to, when thrown upwards, or the penetrations of bodies let fall into ſoft wax, tallow, clay, ſnow, and other ſoft ſubſtances, which ſpaces are always as the ſquares of the velocities of the bodies. On the other hand, their opponents retort, that ſuch ſpaces are not the meaſures of the force in queſtion, which is rather percuſſive and momentary, as thoſe above are paſſed over in unequal times, and are indeed the joint effect of the Forces and times.

Deſaguliers brings an argument from the familiar experiment of the balance, and the other ſimple mechanic powers, ſhewing that the effect is in proportion to the velocity multiplied by the weight; for example, 4 pounds being placed at the diſtance of 6 inches from the centre of motion of a balance, and 2 pounds at the diſtance of 12 inches; theſe will have a Vis Viva if the balance be put into a ſwinging motion. Now it appears that theſe Forces are equal, becauſe, with contrary directions, they ſoon deſtroy each other; and they are to each other in the ſimple ratio of the velocity multiplied by the maſs, viz $4 \times 6 = 24$, and $2 \times 12 = 24$ alſo.

Mr. Robins, in his remarks on J. Bernoulli's treatiſe, entitled, Diſcours ſur les Loix de la Communication du Mouvement, informs us, that Leibnitz adopted this opinion through miſtake; for though he maintained that the quantity of Force is always the ſame in the univerſe, he endeavours to expoſe the error of Des Cartes, who alſo aſſerted, that the quantity of motion is always the ſame; and in his diſcourſe on this ſubject in the Acta Eruditorum for 1686, he ſays that it is agreed on by the Carteſians, and all other philoſophers and mathematicians, that there is the ſame Force requiſite to raiſe a body of 1 pound to the height of 4 yards, as to raiſe a body of 4 pounds to the height of 1 yard; but being ſhewn how much he was miſtaken in taking that for the common opinion, which would, if allowed, prove the force of the body to be as the ſquare of the velocity it moved with, he afterwards, rather than own himſelf capable of ſuch a miſtake, endeavoured to defend it as true; ſince he found it was the neceſſary conſequence of what he had once aſſerted; and maintained, that the force of a body in motion was proportional to the height from which it muſt fall, to acquire that velocity; and the heights being as the ſquares of the velocities, the Forces would be as the maſſes multiplied by them; whereas, when a body deſcends by its gravity, or is projected perpendicularly upwards, its motion may be conſidered as the ſum of the uniform and continual impulſes of the power of gravity, during its falling in the former caſe, and till they extinguiſh it in the latter. Thus when a body is projected upwards with a double velocity, theſe uniform impulſes muſt be continued for a double time, in order to deſtroy the motion of the body; and hence it follows, that the body, by ſetting out with a double velocity, and aſcending for a double time, muſt ariſe to a quadruple height, before its motion is exhauſted. But this proves that a body with a double velocity moves with a double Force, ſince it is produced or deſtroyed by the ſame uniform power continued for a double time, and not with a quadruple force, though it riſes to a quadruple height; ſo that the error of Leibnitz conſiſted in his not conſidering the time, ſince the velocities alone are not the cauſes of the ſpaces deſcribed, but the times and the velocities together; yet this is the fallacious argument on which he firſt built his new doctrine; and thoſe which have been ſince much inſiſted on, and derived from the indentings or hollows produced in ſoft bodies by others falling into them, are much of the ſame kind. Robins's Tracts, vol. 2, p. 178.

But many of the experiments and reaſonings, that have been urged on both ſides in this controverſy, have been founded in the different ſenſes applied to the term Force. The Engliſh and French philoſophers, by the word Force, mean the ſame thing as they do by momentum, motion, quantity of motion, percuſſion, or inſtantaneous preſſure, which is meaſured by the maſs drawn into the velocity, and may be known by its effect; and when they conſider bodies as moving through a certain ſpace, they allow for the time in which that ſpace is deſcribed: whereas the Dutch, Italian, and German philoſophers, who have eſpouſed the new opinion, mean by the word Force, or Force inherent in a body in motion, that which it is able to produce; or, in other words, the Force is always meaſured by the whole effect produced by the body in motion; until its whole Force be entirely communicated or deſtroyed, without any regard to the time employed in producing this total effect. Thus, ſay they, if a point runs through a determinate ſpace, and preſſes with a certain given Force, or intenſity of preſſure, it will perform the ſame action whether it move faſt or ſlow, and therefore the time of the action in this caſe ought not to be regarded. s'Graveſande, Phyſ. El. Math. § 723—728.

Mr. Euler obſerves, with regard to this diſpute concerning the meaſure of vivid Force, or living Force, as it is ſometimes called, that we cannot abſolutely aſcribe any Force to a body in motion, whether we ſuppoſe this Force proportional to the velocity, or to the ſquare of the velocity: for the Force exerted by a body, ſtriking another at reſt, is different from that which it exerts in ſtriking the ſame body in motion; ſo that this Force cannot be aſcribed to any body conſidered in itſelf, but only relatively to the other bodies it meets with. There is no force in a body abſolutely conſidered, but its inertia, which is always the ſame, whether the body be in motion or at reſt. But if this body be forced by others to change its ſtate, its inertia then exerts itſelf as a Force, properly ſo called, which is not abſolutely determinable; becauſe it depends on the changes that happen in the ſtate of the body.

A ſecond obſervation which has been made by ſome eminent writers, is, that the effect of a ſhock of two or more bodies, is not produced in an inſtant, but requires a certain interval of time. If this be ſo, the heterogeneity between the Vis Viva and Mortua, or Living and Dead Force, will vaniſh; ſince a preſſure may always be aſſigned, which in the ſame time, however little, ſhall produce the ſame effect. If then the Vis Viva be homogeneous to the Vis Mortua, and having a perfect meaſure and knowledge of the latter, we need require no other meaſure of the former than that which is derived from the Vis Mortua equivalent to it.

Now that the change in the ſtate of two bodies, by their ſhock, does not happen in an inſtant, appears evidently from the experiments made on ſoft bodies: in theſe, percuſſion forms a ſmall cavity, viſible after the ſhock, if the bodies have no elaſticity. Such a cavity cannot certainly be made in an inſtant. And if the ſhock of ſoft bodies require a determinate time, we muſt certainly ſay as much of the hardeſt, though this time may be ſo ſmall as to be beyond all our ideas. Neither can an inſtantaneous ſhock agree with that conſtant law of nature, by virtue of which nothing is performed per ſaltum. But it is needleſs to inſiſt farther upon this, ſince the duration of any ſhock may be determined from the moſt certain principles.

There can be no ſhock or colliſion of bodies, without their making mutual impreſſions on each other: theſe impreſſions will be greater or leſs, according as the bodies are more or leſs ſoft, other circumſtances being the ſame. In bodies called hard, the impreſſions are ſmall; but a perfect hardneſs, which admits of no impreſſion, ſeems inconſiſtent with the laws of nature; ſo that while the colliſion laſts, the action of bodies is the reſult of their mutually preſſing each other. This preſſure changes their ſtate; and the Forces exerted in percuſſion are really preſſures, and truly Vires Mortuæ, if we will uſe this expreſſion, which is no longer proper, ſince the pretended infinite difference between the Vires Vivæ and Mortuæ ceaſes.

The Force of percuſſion, reſulting from the preſſures that bodies exert on each other, while the colliſion laſts, may be perfectly known, if theſe preſſures be determined for every inſtant of the ſhock. The mutual action of the bodies begins the firſt moment of their contact; and is then leaſt; after which this action increaſes, and becomes greateſt when the reciprocal impreſſions are ſtrongeſt. If the bodies have no elaſticity, and the impreſſions they have received remain, the Forces will then ceaſe. But if the bodies be elaſtic, aud the parts compreſſed reſtore themſelves to their former ſtate, then will the bodies continue to preſs each other till they ſeparate. To comprehend therefore perfectly the Force of percuſſion, it is requiſite firſt to define the time the ſhock laſts, and then to aſſign the preſſure correſponding to each inſtant of this time; and as the effect of preſſures in changing the ſtate of any body may be known, we may thence come at the true cauſe of the change of motion ariſing from colliſion. The Force of percuſſion therefore is no more than the operation of a variable preſſure during a given time; and to meaſure this Force, we muſt have regard to the time, and to the variations according to which the preſſure increaſes and decreaſes.

Mr. Euler has given ſome calculations relative to theſe particulars; and he illuſtrates their tendency by this inſtance: Suppoſe that the hardneſs of the two bodies, A and B, is equal; and ſuch, that being preſſed together with the force of 100lb, the impreſſion made on each is of the depth of $\frac{1}{1000}$th part of a foot. Suppoſe alſo that B is fixed, and that A ſtrikes it with the velocity of 100 feet in a ſecond; according to Mr. Euler, the greateſt Force of compreſſion will be equivalent to 400lb; and this Force will produce in each of theſe bodies an impreſſion equal to $\frac{1}{25}$ of a foot; and the duration of the colliſion, that is, till the bodies arrive at their greateſt compreſſion, will be about $\frac{1}{800}$ of a ſecond. Mr. Euler, in his calculations, ſuppoſes the hardneſs of a body to be proportional to the Force or preſſure requiſite to make a given impreſſion on it; ſo that the Force by which a given impreſſion is made on a body, is in a compound ratio of the hardneſs of the body and of the quantity of the impreſſion. But he obſerves, that regard muſt be had to the magnitude of the bodies, as the ſame impreſſion cannot be made on the leaſt bodies as on the greateſt, from the defect of ſpace through which their component particles muſt be driven: he conſiders therefore only the leaſt impreſſions, and ſuppoſes the bodies of ſuch magnitudes, that with reſpect to them the impreſſions may be looked upon as nothing. What he ſuppoſes concerning the hardneſs of bodies, neither implies elaſticity nor the want of it, as elaſticity only produces a reſtitution of figure and impreſſion when the preſſing Force ceaſes; but this reſtitution need not be here conſidered. It is alſo ſuppoſed, that the bodies which ſtrike each other, have plane and equal baſes, by which they touch each other in the colliſion; ſo that the impreſſion hereby made diminiſhes the length of each body. It is farther to be obſerved, that in Mr. Euler's calculations, bodies are ſuppoſed ſo conſtituted, that they may not only receive impreſſions from the Forces preſſing them, but that a greater Force is requiſite to make a greater impreſſion. This excludes all bodies, fluid or ſolid, in which the ſame Force may penetrate farther and farther, provided it have time, without ever being in equilibrio with the reſiſtance: thus a body may continually penetrate farther into ſoft wax, although the force impelling it be not increaſed: in theſe, and the like caſes, nothing is required but to ſurmount the firſt obſtacles; which being once done, and the connection of parts broken, the penetrating body always advances, meeting with the ſame obſtacles as before, and deſtroying them by an equal Force. But Mr. Euler only conſiders the firſt obſtacles which exiſt before any ſeparation of parts, and which are doubtleſs ſuch, that a greater impreſſion requires a greater Force. Indeed this chiefly takes place in elaſtic bodies; but it ſeems likewiſe to obtain in all bodies when the impreſſions made on them are ſmall, and the contexture of their parts is not altered.

Theſe things being premiſed, let the maſs or weight of the body A be expreſſed in general by A, and let its velocity before the ſhock be that which it might acquire by falling from the height a. Farther, let the hardneſs of A be expreſſed by M, and that of B by N, and let the area of the baſe, on which the impreſſion is made, be cc; then will the greateſt compreſſion be made with the Force $\sqrt{\frac{\mathrm{MN}cc}{\mathrm{M}+\mathrm{N}} \times \mathrm{A}a}$. Therefore if the hardneſs of the two bodies, and the plane of their contact during the whole time of their colliſion be the ſame, this Force will be as $\sqrt{\mathrm{A}a}$, that is, as the ſquare root of the Vis Viva of the ſtriking body A. And as $\sqrt{a}$ is proportional to the velocity of the body A, the Force of percuſſion will be in a compound ratio of the velocity and of the ſubduplicate ratio of the maſs of the body ſtriking; ſo that in this caſe neither the Leibnitian nor the Carteſian propoſitions take place. But this Force of percuſſion depends chiefly on the hardneſs of the bodies; the greater this is, the greater will the Force of percuſſion be. If M = N, this Force will be as $\sqrt{\mathrm{M}cc \times \mathrm{A}a}$, that is, in a compound ſubduplicate ratio of the Vis Viva of the ſtriking body, of the hardneſs, and of the plane of contact. But if

M, the hardneſs of one of the bodies, be infinite, the Force of percuſſion will be as $\sqrt{Ncc \times Aa}$; at the ſame time, if M = N, this Force will be as $\sqrt{\frac{1}{2}Ncc \times Aa}$. Therefore, all other things being equal, the Force of percuſſion, if the ſtriking body be infinitely hard, will be to the Force of percuſſion when both the bodies are equally hard, as $\sqrt{2}$ to 1.

Mr. Euler farther deduces from his calculation, that the impreſſion received by the bodies A and B will be as follows; viz, as

$$\sqrt{\frac{N \times Aa}{\overline{M+N} \times Mcc}} \text{ and } \sqrt{\frac{M \times Aa}{\overline{M+N} \times Ncc}} \text{ reſpectively.}$$

If therefore the hardneſs of A, that is M, be infinite, it will ſuffer no impreſſion; whereas that on B will extend to the depth of $\sqrt{\frac{Aa}{Ncc}}$. But if the hardneſs of the two bodies be the ſame, or M = N, they will each receive equal impreſſions of the depth $\sqrt{\frac{Aa}{2Ncc}}$. So that the impreſſion received by the body B in this caſe, will be to the impreſſion it receives in the former, as 1 to $\sqrt{2}$.

Mr. Euler has likewiſe conſidered and computed the caſe where the ſtriking body has its anterior ſurface convex, with which it ſtrikes an immoveable body whoſe ſurface is plane. He has alſo examined the caſe when both bodies are ſuppoſed immoveable; and from his formulæ he deduces the known laws of the colliſion of elaſtic and non-elaſtic bodies. He has alſo determined the greateſt preſſures the bodies receive in theſe caſes; and likewiſe the impreſſions made on them. In particular he ſhews, that the impreſſions received by the body ſtruck, or B, if moveable, is to the impreſſion received by the ſame body when immoveable, as $\sqrt{B}$ to $\sqrt{A+B}$.

There are ſeveral curious as well as uſeful obſervations in Deſaguliers's Experimental Philoſophy, concerning the comparative Forces of men and horſes, and the beſt way of applying them. A horſe draws with the greateſt advantage when the line of direction is level with his breaſt; in ſuch a ſituation, he is able to draw 200lb for 8 hours a day, walking about $2\frac{1}{2}$ miles an hour. But if the ſame horſe be made to draw 240lb, he can work only 6 hours a day, and cannot go quite ſo faſt. On a carriage, indeed, where friction alone is to be overcome, a middling horſe will draw 1000lb. But the beſt way to try the Force of a horſe, is to make him draw up out of a well, over a ſingle pulley or roller; and in that caſe, an ordinary horſe will draw about 200lb, as before obſerved.

It is found that 5 men are of equal Force with one horſe, and can with equal eaſe puſh round the horizontal beam of a mill, in a walk 40 feet wide; whereas 3 men will do it in a walk only 19 feet wide.

The worſt way of applying the Force of a horſe is to make him carry or draw up hill: for if the hill be ſteep, 3 men will do more than a horſe, each man climbing up faſter with a burden of 100lb weight, than a horſe that is loaded with 300lb: a difference which is owing to the poſition of the parts of the human body being better adapted to climb, than thoſe of a horſe.

On the other hand, the beſt way of applying the Force of a horſe, is the horizontal direction, in which a man can exert the leaſt Force: thus, a man that weighs 140lb, when drawing a boat along by means of a rope coming over his ſhoulders, cannot draw above 27lb, or exert above 1-7th part of the Force of a horſe employed to the ſame purpoſe; ſo that in this way the Force of a horſe is equal to that of 7 men.

The beſt and moſt effectual poſture in a man, is that of rowing; when he not only acts with more muſcles at once for overcoming the reſiſtance, than in any other poſition; but alſo as he pulls backwards, the weight of his body aſſiſts by way of lever. See Deſaguliers's Exp. Philoſ. vol. 1, p. 241, where ſeveral other obſervations are made relative to Force acquired by certain poſitions of the body; from which that author accounts for moſt feats of ſtrength and activity. See alſo a Memoir on this ſubject by M. De la Hire, in the Mem. Roy. Acad. 1729; or in Deſaguliers's Exp. &c. p. 267 &c, who has publiſhed a tranſlation of part of it with remarks.

FORCE is diſtinguiſhed into Motive and Accelerative or Retardive.

Motive FORCE, otherwiſe called Momentum, or Force of Percuſſion, is the abſolute Force of a body in motion, &c; and is expreſſed by the product of the weight or maſs of matter in the body multiplied by the velocity with which it moves. But

Accelerative FORCE, or *Retardive* FORCE, is that which reſpects the velocity of the motion only, accelerating or retarding it; and it is denoted by the quotient of the motive Force divided by the maſs or weight of the body. So,

if m denote the motive Force,
and b the body, or its weight,
and f the accelerating or retarding Force,

then is f as $\frac{m}{b}$.

Again, Forces are either Conſtant or Variable.

Conſtant FORCES are ſuch as remain and act continually the ſame for ſome determinate time. Such, for example, is the Force of gravity, which acts conſtantly the ſame upon a body while it continues at the ſame diſtance from the centre of the earth, or from the centre of Force, wherever that may be. In the caſe of a conſtant Force F acting upon a body b, for any time t, we have theſe following theorems; putting

f = the conſtant accelerating Force = F ÷ b,
v = the velocity at the end of the time t,
s = the ſpace paſſed over in that time, by the conſtant action of that Force on the body:

and $g = 16\frac{1}{12}$ feet, the ſpace generated by gravity in 1 ſecond, and calling the accelerating Force of gravity 1; then is

$$s = \tfrac{1}{2}tv = gft^2 = \frac{v^2}{4gf};$$

$$v = 2gft = \frac{2s}{t} = \sqrt{4gfs};$$

$$t = \frac{v}{2gf} = \frac{2s}{v} = \sqrt{\frac{s}{gf}};$$

$$f = \frac{v}{2gt} = \frac{s}{gt^2} = \frac{v^2}{4gs}.$$

Variable FORCES are ſuch as are continually changing

ing in their effect and intenſity; ſuch as the Force of gravity at different diſtances from the centre of the earth, which decreaſes in proportion as the ſquare of the diſtance increaſes. In variable Forces, theorems ſimilar to thoſe above may be exhibited by uſing the fluxions of quantities, and afterwards taking the fluents of the given fluxional equations. And herein conſiſts one of the great excellencies of the Newtonian or modern analyſis, by which we are enabled to manage, and compute the effects of all kinds of variable Forces, whether accelerating or retarding. Thus, uſing the ſame notation as above for conſtant forces, viz, f the accelerating Force at any inſtant, t the time a body has been in motion by the action of the variable Force, v the velocity generated in that time, s the ſpace run over in that time, and $g = 16\frac{1}{12}$ feet; then is

$$\dot{s} = \frac{v\dot{v}}{2gf} = v\dot{t};$$

$$\dot{v} = \frac{2gf\dot{s}}{v} = 2gf\dot{t};$$

$$\dot{t} = \frac{\dot{s}}{v} = \frac{\dot{v}}{2gf};$$

$$f = \frac{v\dot{v}}{2g\dot{s}} = \frac{\dot{v}}{2g\dot{t}}.$$

In theſe four theorems, the Force f, though variable, is ſuppoſed to be conſtant for the indefinitely ſmall time $\dot{t}$; and they are to be uſed in all caſes of variable Forces, as the former ones in conſtant Forces; viz, from the circumſtances of the problem under conſideration, deduce a general expreſſion for the value of the force f, at any indefinite time t; then ſubſtitute it in one of theſe theorems, which ſhall be proper to the caſe in hand; and the equation thence reſulting will determine the correſponding values of the other quantities in the problem.

It is alſo to be obſerved, that the foregoing theorems equally hold good for the deſtruction of motion and velocity, by means of retarding or reſiſting Forces, as for the generation of the ſame by means of accelerating Forces.

Many applications of theſe theorems may be ſeen in my Select Exerciſes, p. 172 &c.

There are many other denominations and kinds of Forces; ſuch as attractive, central, centrifugal, &c, &c; for which ſee the reſpective words.

FORCER, in Mechanics, is properly a piſton without a valve. For, by drawing up ſuch a piſton, the air is drawn up, and the water follows; then puſhing the piſton down again, the water, being prevented from deſcending by the lower valve, is forced up to any height above, by means of a ſide branch between the two.

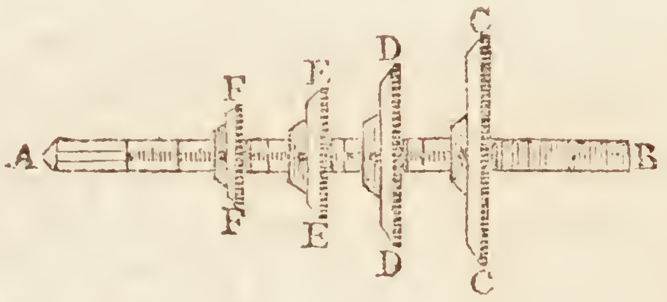
See the ways of making theſe in Deſaguliers's Exper. Philoſ. vol. 2, p. 161 &c. See alſo Clare's Motion of Fluids, p. 60.

FORCING *Pump*, one that acts, or raiſes water, by a Forcing piſton. See above.

FORELAND, or FORENESS, in Navigation, a point of land jutting out into the ſea.

FORELAND, in Fortification, is a ſmall piece of ground between the wall of a place and the moat; called alſo Berme and Liziere.

FORE-STAFF, an inſtrument uſed at ſea, for taking the altitudes of the heavenly bodies; being ſo called, becauſe the obſerver, in uſing it, turns his face forward or towards the object, in contradiſtinction to the Back-ſtaff, with which he turns his back to the object. It is alſo called the Croſs-ſtaff, becauſe it conſiſts of ſeveral pieces ſet acroſs a ſtaff.



The Fore-ſtaff is formed of a ſtraight ſquare ſtaff AB, of about 3 feet long, having each of its four ſides graduated like a line of tangents, and four croſſes, or vanes, FF, EE, DD, CC, ſliding upon it, of unequal lengths, the halves of which repreſent the radii to the lines of tangents on the different ſides of the ſtaff. The firſt or ſhorteſt of theſe vanes, FF, is called the Ten Croſs, or Ten Vane, and belongs to the 10 ſcale, or that ſide of the inſtrument on which diviſions begin at 3 degrees, and end at 10. The next longer croſs, EE, is called the 30 Croſs, belonging to that ſide of the ſtaff where the diviſions begin at 10 degrees, and end at 30, called the 30 ſcale. The third vane DD, is called the 60 Croſs, and belongs to that ſide where the diviſions begin at 20 degrees, and end at 60. The laſt or longeſt vane CC, called the 90 Croſs, belongs to the ſide where the diviſions begin at 30 degrees, and end at 90.

The chief uſe of this inſtrument, is to take the height of the ſun, and ſtars, or the diſtance between two ſtars: and the 10, 30, 60, or 90 Croſs is to be uſed, according as the altitude is more or leſs; that is, if the altitude be leſs than 10 degrees, the 10 Croſs is to be uſed; if above 10, but leſs than 30, the 30 Croſs is to be uſed; and ſo on.

To obſerve an Altitude with the Fore-ſtaff. Apply the flat end of the ſtaff to the eye, and ſlide one of the croſſes backwards and forwards upon it, till over the upper end of the Croſs be juſt ſeen the centre of the ſun or ſtar, and over the under end the extreme horizon; then the degrees and minutes cut by the croſs on the ſide of the ſtaff proper to the vane in uſe, gives the altitude above the horizon.

In like manner, for the Diſtance between two luminaries; the Staff being ſet to the eye, bring the croſs juſt to ſubtend or cover that diſtance, by having the one luminary juſt at the one end of it, and the other luminary at the other end of it; and the degrees and minutes, in the diſtance, will be cut on the proper ſide of the ſtaff, as before.

FORMULA, a theorem or general rule, or expreſſion, for reſolving certain particular caſes of ſome problem, &c. So $\frac{1}{2}s + \frac{1}{2}d$ is a general Formula for the greater of two quantities whoſe ſum is s and difference d; and $\frac{1}{2}s - \frac{1}{2}d$ is the Formula, or general value, for the leſs quantity. Alſo $\sqrt{dx - x^2}$ is the Formula, or general value, of the ordinate to a circle, whoſe diameter is d, and abſciſs x.

FORT, a little caſtle or fortreſs; or a place of ſmall extent, fortified either by nature or art.

The Fort is uſually encompaſſed with a moat, rampart, and parapet, to ſecure ſome high ground, or paſſage of a river; to make good or ſtrengthen an advantageous poſt; or to fortify the lines and quarters of a ſiege.

Field FORT, otherwiſe called *Fortin*, or *Fortlet*, and ſometimes *Sconce*, is a ſmall Fort, built in haſte, for the defence of a paſs or poſt; but particularly conſtructed for the defence of a camp in the time of a ſiege, where the principal quarters are uſually joined, or made to communicate with each other, by lines defended by fortins and redoubts. Their figure and ſize are various, according to the nature of the ſituation, and the importance of the ſervice for which they are intended; but they are moſt commonly made ſquare, each ſide about 100 toiſes, the perpendicular 10, and the faces 25; the ditch about this Fort may be 10 or 12 toiſes wide; the parapet is made of turf, and fraiſed, and the ditch palliſadoed when dry. There may be made a covert-way about this Fort, or elſe a row of palliſades might be placed on the outſide of the ditch. Some of theſe are fortified with baſtions, and ſome with demi-baſtions.——A Fort differs from a citadel, as this laſt is erected to command and guard ſome town; and from a redoubt, as it is cloſed on all ſides, while the redoubt is open on one ſide.

Royal FORT, is one whoſe line of defence is at leaſt 26 fathoms long.

Star FORT, is a ſconce or redoubt, conſtituted by re-entering and ſaliant angles, having commonly from five to eight points, and the ſides flanking each other.

Forts are ſometimes made triangular, only with half baſtions; or of various other figures, regular or irregular, and ſometimes in the form of a ſemicircle, eſpecially when they are ſituated near a river, or the ſea, as at the entrance of a harbour, for the convenience of firing at ſhips quite around them on that ſide. In the conſtruction of all Forts, it ſhould be remembered, that the figure of feweſt ſides and baſtions, that can probably anſwer the propoſed defence, is always to be preferred; as works on ſuch a plan are ſooner executed, and with leſs expence; beſides, fewer troops will ſerve, and they are more readily brought together in caſe of neceſſity.

FORTIFICATION, called alſo Military Architecture, is the art of fortifying or ſtrengthening a town or other place, by making certain works around it, to ſecure it and defend it from the attacks of enemies.

Fortification has been undoubtedly practiſed by all nations, and in all ages; being at firſt doubtleſs very rude and ſimple, and varying in its nature and manner, according to the mode of attack, and the weapons made uſe of. Thus, when villages and towns were firſt formed, it was found neceſſary, for the common ſafety, to encompaſs them with walls and ditches, to prevent all violence and ſudden ſurpriſes from their neighbours. When offenſive and miſſive weapons came to be uſed, walls were made as a defence againſt the aſſailants, and look holes or loop holes made through the walls to annoy the enemy, by ſhooting arrows &c through them. But finding that as ſoon as the enemy got cloſe to the walls, they could no longer be ſeen nor annoyed by the beſieged, theſe added ſquare towers along the wall, at proper diſtances from each other, ſo that all the intervening parts of the wall might be ſeen and defended from the adjacent ſides of the towers. However, this manner of incloſing towns was found to be rather imperfect, becauſe there remained ſtill the outer face of the towers which fronted the field, that could not be ſeen and defended from any other part. To remedy this imperfection, they next made the towers round inſtead of ſquare, as ſeeming better adapted both for ſtrength to reſiſt the battering engines, and for being defended from the other parts of the wall. Nevertheleſs, a ſmall part of theſe towers ſtill remained unſeen, and incapable of being defended, for which reaſon they were again changed for ſquare ones, as before, but with this difference, that now they preſented an angle of the ſquare outwards to the field, inſtead of a face or ſide; and thus ſuch a diſpoſition of the works was obtained, as that no part could be approached by the enemy without being ſeen and attacked.

Since the uſe of gun-powder, it has been found neceſſary to add thick ramparts of earth to the walls, and the towers have been enlarged into baſtions, as well as many other things added, that have given a new appearance to the whole art of defence, and the name of Fortification, on account of the ſtrength afforded by it, which was about the year 1500, when the round towers were changed into baſtions.

But notwithſtanding all the improvements made in this art ſince the invention of gun-powder, that of attacking is ſtill ſuperior to it: the ſuperiority of the beſiegers' fire, together with the greater number of men, ſooner or later compels the beſieged to ſubmit. A ſpecial advantage was added to the art of attacking by M. Vauban, at the ſiege of Ath, in the year 1697, viz, in the uſe of ricochet firing, or at a low elevation of the gun, by which the ſhot was made to run and roll a great way along the inſide of the works, to the great annoyance of the beſieged.

The chief authors who have treated of Fortification, ſince it has been conſidered as a particular art, are the following, and moſtly in the order of time: viz, La Treille, Alghiſi, Marchi, Paſino, Ramelli, Cataneo, and Speckle, who, as Mr. Robins ſays, was one of the greateſt geniuſes that has applied to this art: he was architect to the city of Straſburgh, and died in the year 1589: he publiſhed a treatiſe on Fortification in the German language, which was reprinted at Leipſic in 1736. Afterwards, Errard, who was engineer to Henry the Great of France; Stevinus, engineer to the prince of Orange; with Marolois, the chevalier de Ville, Lorini, Coehorn, the count de Pagan, and the marſhal de Vauban; which laſt two noble authors have contributed greatly to the perfection of the art; beſides Scheiter, Mallet, Belidor, Blondel, Muller, Montalambert, &c. Alſo a liſt of ſeveral works on the art of Fortification may be added, as follows: viz,

Melder's Praxis Fortificatoria: Les Fortifications du Comte de Pagan: L'Ingénieur Parfait du Sieur de Ville: Sturmy's Architectura Militaris Hypothetical.: Blondel's Nouvelle Maniere de Fortifier les Places: The Abbé de Fay's Veritable Maniere de Bien Fortifier: Vauban's Ingénieur François: Coehorn's Nouvelle Fortification tant pour un Terrain bas & humide, que ſec & elevé: Alexander de Grotte's Fortification: Donatus Roſelli's Fortification: Medrano's Ingénieur François: The chevalier de St. Julien's Architecture Militaire: Lanſberg's Nouvelle Maniere de Fortifier les Places: An anonymous treatiſe in French, called Nouvelle Maniere de Fortifier les Places, tirée des Methodes du chevalier de Ville, &c: Ozanam's Traité de Fortification: Memoires de l'Artillerie de Surirey de St. Remy: Muller's treatiſes of Elementary and Practical Fortification: and Montalambert's Fortification Perpendiculaire.

Maxims in FORTIFICATION. From the nature and circumſtances of this art, certain general rules, or maxims, have been drawn, and laid down. Theſe may indeed be multiplied to any extent, but the principal of them, are the following: viz,

1. That the manner of fortifying ſhould be accommodated to that of attacking. So that no one manner can be aſſured always to hold, unleſs it be aſſured that the manner of beſieging is incapable of being altered. Alſo, to judge of the perfection of a Fortification, the method of beſieging at the time when it was built muſt be conſidered.

2. All the parts of a Fortification ſhould be equally ſtrong on all ſides, where there is equal danger; and they ſhould be able to reſiſt the moſt powerful machines uſed in beſieging.

3. A Fortification ſhould be ſo contrived, as to be defended with the feweſt men poſſible: which conſideration, when well attended to, ſaves a great deal of expence.

4. That the defendants may be in the better condition, they muſt not be expoſed to the enemies' artillery; but the aggreſſors muſt be expoſed to theirs. Hence,

5. All the parts of a Fortification ſhould be ſo diſpoſed, as that they may defend each other. In order to this, every part ought to be flanked, i. e. ſeen ſideways, capable of being ſeen and defended from ſome other part; ſo that there be no place where an enemy can lodge himſelf, either unſeen, or under ſhelter.

6. All the campaign around muſt lie open to the defendants; ſo that no hill or eminence muſt be allowed, behind which the enemy might ſhelter himſelf from the guns of the Fortification; or from which he might annoy them with his own. Hence, the fortreſs is to command all the place round about; and conſequently the outworks muſt all be lower than the body of the place.

7. No line of defence muſt exceed the point-blank muſket-ſhot, which is from 120 to 150 fathoms.

8. The more acute the angle at the centre, the ſtronger is the place; as conſiſting of the more ſides, and conſequently more defenſible.

9. All the defences ſhould be as nearly direct as poſſible.

10. The works that are moſt remote from the centre of the place, ought always to be open to thoſe that are more near.

Theſe are the general laws and views of Fortification. As to the particular ones, or ſuch as reſpect the ſeveral members or parts of the work, they are given under thoſe articles reſpectively.

Fortification is either theoretical or practical.

Theoretical FORTIFICATION, conſiſts in tracing the plans and profiles of a work on paper, with ſcales and compaſſes; and in examining the ſyſtems propoſed by different authors, to diſcover their advantages and defects. And

Practical FORTIFICATION, conſiſts in forming a project of a work according to the nature of the ground, and other neceſſary circumſtances, tracing it on the ground, and executing the project, together with all the military buildings, ſuch as magazines, ſtorehouſes, bridges, &c.

Again, Fortification is either Defenſive or Offenſive.

Defenſive FORTIFICATION is the art of defending a town that is beſieged, with all the advantages the Fortification of it will admit. And

Offenſive FORTIFICATION is the ſame with the attack of a place, being the art of making and conducting all the different works in a ſiege, in order to gain poſſeſſion of the place.

FORTIFICATION is alſo uſed for the place fortified; or the ſeveral works raiſed to defend and flank it, and keep off the enemy.

All Fortifications conſiſt of lines and angles, which have names according to their various offices. The principal lines are thoſe of circumvallation, of contravallation, of the capital, &c. The principal angles are thoſe of the centre, the flanking angle, flanked angle, angle of the epaule, &c.

Fortifications are either durable or temporary.

Durable FORTIFICATION, is that which is built and intended to remain a long time. Such are the uſual Fortifications of cities, frontier places, &c. And a

Temporary FORTIFICATION, is that which is erected on ſome emergent occaſion, and only for a ſhort time. Such are field-works, thrown up for the ſeizing and maintaining a poſt, or paſſage; thoſe about camps, or in ſieges; as circumvallations, contravallations, redoubts, trenches, batteries, &c.

Again, Fortifications are either regular or irregular. A

Regular FORTIFICATION, is that in which the baſtions are all equal; or which is built in a regular polygon, the ſides and angles of which are uſually about a muſket ſhot from each other. A Regular Fortification, having the parts all equal, has the advantage of being equally defenſible; ſo that there are no weak places. And an

Irregular FORTIFICATION, is that in which the baſtions are unequal, and unlike; or the ſides and angles not all equal, and equidiſtant.

In an Irregular Fortification, the defence and ſtrength being unequal, it is neceſſary to reduce the irregular ſhape of the ground, as near as may be, to a regular figure: i. e. by inſcribing it in an oval, inſtead of a circle; ſo that one half may be ſimilar and equal to the other half.

Marine **Fortification**, is ſometimes uſed for the art of raiſing works on the ſea coaſt &c, to defend harbours againſt the attacks of ſhipping.——See a neat treatiſe on Marine Fortification, at the end of Robertſon's Elements of Navigation.

There are many modes of Fortification that have been much eſteemed and uſed; a ſmall ſpecimen of a comparative view of the principal of theſe, is repreſented in plate x, viz, thoſe of Count Pagan, and Meſſ. Vauban, Coehorn, Belidor, and Blondel; the explanation of which is as follows:

1. *Pagan's Syſtem.*

A	Half Baſtions.
B	Ravelin and Counterguard.
C	Counterguards before the baſtions.
D	The Ditch.
E	The Glacis.
G	The place of Arms.
H	Retired Flanks.
a	Line of Defence.

2. *Vauban's Syſtem.*

b	Angle of the Baſtion, or Flanked angle.
c	Angle of the Shoulder.
d	Angle of the Flank.
e	Saliant Angle.
f	Face of the baſtion.
g	The Flank.
h	The Curtain.
i	Tenailles.
k	Traverſes in the covert way.

3. *Coehorn's Syſtem.*

1	Concave Flanks.
2	The Curtains.
3	Redoubts in the re-entering angles.
4	Traverſes.
5	Stone lodgments.
6	Round Flanks.
7	Redoubt.
8	Coffers planked on the ſides, and above covered overhead with a foot of earth.

4 *Belidor's Syſtem.*

I	Cavaliers.
K	Rams-horns, or Tenailles.
L	Retrenchments within the detached baſtions.
M	Circular Curtain.
N	The Ravelin.
P	Lunettes with retired batteries.
Q	Redoubt.
R	Detached Redoubt.
S	An Arrow.
P	Small Traverſes.

5. *Blondel's Syſtem.*

l	Retired Battery.
m	Lunettes.
n	Ravelin, with Retired Baſtion.
o	Orillons.

Another, or new method of Fortification has lately been propoſed by M. Montalambert, called *Fortification Perpendiculaire*, becauſe the faces of the works are made by a ſeries of lines running zigzag perpendicular to one another.

Profile of a **Fortification**, is a repreſentation of a vertical ſection of a work; ſerving to ſhew thoſe dimenſions which cannot be repreſented in plans, and are neceſſary in the building of a Fortification. The names and dimenſions of the principal parts are as follow (ſee fig. 8, pl. vii), where the numbers or dimenſions are all expreſſed in feet.

AB The level of the ground plane,
$AC = 27$,
$CD = 18$, and $CW = 16\frac{1}{2}$; alſo DN is paral. to AB.
$DE = 30$,
$EF = 2$, $FG = 3$, $GH = 3$, $HI = 4\frac{1}{2}$, $IL = 1\frac{1}{2}$,
$LK = 18$, $KM = 2\frac{1}{2}$, $NP = 36$, $NO = 5$, $PR = 7$,
$RS = 1$, $ST = 12$ or 18, $OV = 9$, $Pn = 120$,
$mz = 3$, $nu = 3$, $mc = 30$, $cd = 2$,
$de = 3$, $ef = 3$, $fl = 4\frac{1}{2}$, $rg = 120$, $lh = 1$.

AW	the interior talus or ſlope of the rampart,
WE, or DE	the terre-plein of ditto,
FG	the talus, and GH the upper part, of the banquette,
HL	the interior ſide of the parapet,
LM	the upper part of ditto,
N	the cordon of 1 foot radius,
NP	the depth, and P*n* the breadth of the ditch,
OQ	interior ſide of revetement,
NR	the ſcarp or exterior ſide of ditto,
ST	the depth of the foundation,
YZ	revetement of the parapet,
nu	the counterſcarp,
mc	the covert-way,
ce	talus of the banquette,
ef	the upper part of ditto,
fh	parapet of the covert-way,
hg	the glacis.

Other ſections are at fig. 2, pl. 12.

Fortified *Place*, a Fortreſs, or Fortification, i. e. a place well flanked, and ſheltered with works.

Fortin, or *Fortlet*, a diminitive of the word Fort, meaning a little fort, or ſconce, called alſo Field Fort.

Star **Fortin**, is that whoſe ſides flank each other, &c.

FORTRESS, the ſame as Fort, or a Fortification.

FOSTER (Samuel), an Engliſh mathematician, and aſtronomy profeſſor of Greſham-college, was born in Northamptonſhire, and admitted a Sizer at Emanuel-college Cambridge in 1616. He took the degree of bachelor of arts in 1619, and of maſter in 1623. He applied early to the mathematics, and attained a great proficiency in it, of which he gave the firſt ſpecimen in 1624, in a treatiſe on *The Uſe of the Quadrant*.

On the death of Mr. Gellibrand, he was choſen to ſucceed him, in 1636, as aſtronomy profeſſor in Greſham-college, London. He quitted it again however the ſame year, though for what reaſon does not appear, and was ſucceeded by Mr. Mungo Murray, profeſſor of philoſophy at St. Andrews in Scotland. But this gentleman marrying, the profeſſorſhip again became vacant, and Mr. Foſter was re-elected in 1641.

Mr. Foſter was one of thoſe gentlemen who held private meetings for cultivating philoſophy and uſeful knowledge, which afterwards gave riſe to the Royal Society. In 1646, Dr. Wallis, who was one of thoſe aſſociating gentlemen, received from Mr. Foſter a theorem *de triangulo ſphærico*, which he publiſhed in his *Mechanics*. Neither was it only in this branch of ſcience that he excelled, but he was likewiſe well verſed in the ancient languages; as appears from his reviſing and correcting the *Lemmata* of Archimedes, which had been tranſlated into Latin from an Arabic manuſcript by Mr. John Greaves. He made alſo ſeveral curious obſervations upon eclipſes of the ſun and moon, in various places; and was particularly noted for inventing, as well as improving, aſtronomical and other mathematical inſtruments. After a long declining ſtate of health, he died at Greſham-college in 1652.

His printed works are as follow; of which the firſt two articles were publiſhed before his death, and the reſt of them after it.

1. *The Deſcription and Uſe of a ſmall Portable Quadrant, for the eaſy finding the hour of Azimuth*; 4to, 1624. Originally publiſhed at the end of Gunter's Deſcription of the Croſs-Staffe, as an appendix to it.
2. *The Art of Dialling*; 4to, 1638. Reprinted, with additions, in 1675.
3. *Poſthuma Foſteri*; by Wingate; 4to, 1652.
4. *Four Treatiſes of Dialling*; 4to, 1654.
5. *Miſcellanies, or Mathematical Lucubrations.* Publiſhed by John Twyſden, with additions of his own; and an appendix by Leybourne; folio, 1659.
6. *The Sector altered, and other Scales added, &c.* Publiſhed by Leybourne in 1661, in 4to.

There have been two other perſons of the ſame name, who have publiſhed ſome mathematical pieces. The firſt was,

WILLIAM FOSTER, who was a diſciple of Mr. Oughtred, and afterward a teacher of the Mathematics in London. He diſtinguiſhed himſelf by a book, which he dedicated to Sir Kenelm Digby, entitled, *The Circles of Proportion, and the Horizontal Inſtrument*, &c; 4to, 1633.—The other was

MARK FOSTER, who lived later in point of time than either of the other two, and publiſhed a treatiſe entitled, *Arithmetical Trigonometry: being the ſolution of all the uſual Caſes in Plain Trigonometry by Common Arithmetic, without any tables whatſoever.* 12mo, 1690.

FOUNDATION, that part of a building which is underground: or the maſs which ſupports a building, and upon which it ſtands: or it is the coffer or bed dug below the level of the ground, to raiſe a building upon.

FOUNTAIN, in Philoſophy, a ſpring or ſource of water riſing out of the Ground. See SPRING.

FOUNTAIN, or *Artificial* **FOUNTAIN**, in Hydraulics, a machine or contrivance by which water is violently ſpouted or darted up; called alſo a Jet d'eau.

There are various kinds of Artificial Fountains, but all formed by a preſſure of one ſort or another upon the water &c, viz, either the preſſure or weight of a head of water, or the preſſure ariſing from the ſpring and elaſticity of the air, &c. When theſe are formed by the preſſure of a head of water, or any other fluid of the ſame kind with the Fountain, or jet, then will this ſpout up nearly to the ſame height as that head, abating only a little for the reſiſtance of the air, with that of the adjutage &c, in the fluid's ruſhing through; but, when the Fountain is produced by any other force than the preſſure of a column of the ſame fluid with itſelf, it will riſe to ſuch a height as may be nearly equal to the altitude of a column of the ſame fluid whoſe preſſure is equal to the given force that produces the fountain.

To Conſtruct an Artificial Fountain, playing by the preſſure of the water. This is to be effected by making a cloſe connection between a head or elevated piece of water, and the lower place, where the Fountain is to play; which may be done in this manner: Having a head of water, naturally, or, for want of ſuch, make an artificial one, raiſing the water by pumps, or other machinery: from this head convey the water in cloſe pipes, in any direction, down to the place where the Fountain is to play; and there let it iſſue through an adjutage, or ſmall hole, turned upwards, by which means it will ſpout up nearly as high as the head of the water comes from, as above mentioned.

To Conſtruct an Artificial Fountain, playing by the ſpring or elaſticity of the air. A veſſel proper for a reſervoir, as AB, fig. 4, plate viii, is provided either of metal, or glaſs, or the like, ending in a ſmall neck *c*, at the top: through this neck is put a tube *cd*, till the lower end come near the bottom of the veſſel, this being about half full of water. The neck is ſo contrived, as that a ſyringe, or condenſing pipe, may be ſcrewed upon the tube; by means of which a large quantity of air may be intruded through the tube into the water, out of which it will diſengage itſelf, and emerge into the vacant part of the veſſel, and lie over the ſurface of the water CD.

Now, the water in the veſſel being thus preſſed by the air, which is, for ex. double the denſity of the external air; and the elaſtic force of air being proportional to its denſity, or to its gravitating force, the effect will be the ſame, as if the weight of the column of air, over the ſurface of the water, were double that of the column preſſing in the tube; ſo that the water muſt be forced to ſpout up through, when the ſyringe is removed, with a force equal to the exceſs of preſſure of the included air, above that of the external, that is, in this inſtance, with a force equal to the preſſure of an entire column of the atmoſphere; which being equal to the preſſure of a column of 33 or 34 feet of water, it follows that the Fountain will play to nearly 33 or 34 feet high.

Theſe aereal or aquatic Fountains may be applied in different ways, ſo as to exhibit various appearances; and from theſe alone ariſes the greateſt part of our artificial water-works: even the engine for extinguiſhing fire, is a Fountain playing by the force of confined air.

A Fountain ſpouting the water in various directions. Suppoſe AB the vertical tube, or ſpout, in which the water riſes, (fig. 5, pl. viii): into this let ſeveral other tubes be fitted; ſome horizontal, others oblique, or inclining, or reclining, &c, as at E, H, L, N, P, &c. Then, as all water retains the direction of the aperture through which it comes, that iſſuing through A will riſe perpendicularly; and the reſt will tend different ways, deſcribing arches of different magnitudes.

Or thus: Suppoſe the vertical tube AB (fig. 6) through which the water riſes, to be ſtopped at the top, as in A; and, inſtead of pipes or cocks, let it be only perforated with little holes all around or only half round its ſurface: then will the water ſpin out, in all directions, through the little holes, to different diſtances.

And hence, if the tube AB be about the height of a man, and having a turn-cock at C; upon opening this cock, the ſpectators will be ſprinkled unexpectedly with a ſhower.

Fountain *playing by drawing the breath.* Suppoſe AB (fig. 8, plate vi), a globe of glaſs, or metal, in which is fitted a tube CD, having a ſmall orifice in C, and reaching almoſt to D, the bottom of the globe. Then if the air be ſucked, or drawn with the mouth, out of the tube CD, and the orifice C be immediately immerged under cold water, the water will aſcend through the tube into the ſphere. Thus proceeding, by repeated exſuctions, till the veſſel be above half full of water; then applying the mouth to C, and blowing air into the tube, upon removing the mouth, the water will ſpout forth.

Or, if the globe be put into hot water, the air being thus rarefied, will make the water ſpout as before.

And this kind of Fountain is called Pila Heronis, or Hero's Ball, from the name of its inventor.

Fountain, *whoſe ſtream raiſes and plays a braſs ball.* Provide a hollow braſs ball A (fig. 9, pl. vi), made very thin, that its weight may not be too great for the force of the water; and let the tube BC, through which the water riſes, be exactly perpendicular to the horizon. Then the ball, being laid in the bottom of the cup or baſon B, will be taken up in the ſtream, and ſuſtained at a conſiderable height, playing a little up and down.

Fountain *which ſpouts water in form of a ſhower.* To the tube in which the water is to riſe, fit a ſpherical, or lenticular head AB (fig. 1, pl. xi) made of a plate of metal, and perforated at the top with a great number of little holes. The water riſing violently towards AB, will be there divided into innumerable little threads, and afterwards broken, and diſperſed into the fineſt drops.

Fountain *which ſpreads the water in form of a table-cloth.* To the tube AB (fig. 2, pl. xi) ſolder two ſpherical ſegments, C and D, almoſt touching each other, with a ſcrew E, to contract or amplify the interſtice or chink, at pleaſure. Some chooſe to make a ſmooth and even cleft, in a ſpherical or lenticular head, fitted upon the tube. The water ſpouting through this chink or cleft will expand itſelf like a cloth.—And thus, the fountain may be made to ſpout out in the figure of men, or other animals.

Fountain, *which, when it has done ſpouting, may be turned like an hour-glaſs.* Provide two glaſſes, A and B, (fig. 3, pl. xi) to be ſo much the larger as the fountain is to play the longer, and placed at ſo much the greater diſtance from each other as the water is deſired to ſpout the higher. Let CDE be a crooked tube, furniſhed in E with a jet; and GHI another bent tube, furniſhed with a jet in I: GF and KL are to be other leſſer tubes, open at both ends, and reaching near the bottom of the veſſels A and B, to which the tubes CD and GH are likewiſe to reach.

Now if the veſſel A be filled with water, it will deſcend through the tube CD; and it will ſpout up through the jet E, by the preſſure of the column of water CD. But unleſs the pipe GF were open at G, to let the air run up to F, and preſs at the top of the ſurface of the water in the cavity A, the water could not run down and ſpout at E. After its fall again, it will ſink through the little tube KL, into the veſſel B, and expel the air through the tube GI. At length, when all the water is emptied out of the veſſel A, by turning the machine upſide down, the veſſel B will be the reſervoir, and make the water ſpout up through the jet I, the pipe KL ſupplying B with air to let the water deſcend in the direction GHI.

Hence, if the veſſels A and B contain juſt as much water as will be ſpouted up in an hour's time, we ſhall have a ſpouting clipſydra, or water-clock; which may be graduated or divided into quarters, minutes, &c.

Fountain *of Command.* This depends on the ſame principles with thoſe of the former: CAE (fig. 4, pl. xi) is a veſſel of water ſecured againſt the entrance of the air, except through the pipe GF, when the cock C, by which it is filled, is ſhut. There is another pipe EDHB, going from the bottom of the water to the jet B in the baſon DB; but this is ſtopped by the cock H. At the loweſt part of the baſon DB, there is a ſmall hole at I, to let the water of the baſon DB run into the baſon GH under it; there is alſo a ſmall triangular hole or notch, in the bottom of the pipe FG, at G. Turn the cock H, and the fountain will play for ſome time, then ſtop, then play again alternately for ſeveral times together. When thoſe times of playing and ſtopping are known before hand, you may command the Fountain to play or ſtop; whence its name. The cauſe of this phenomenon is as follows: the water coming down the pipe EDHB, would not come out at B, if the air S*s*, above the water, were not ſupplied as it dilated: now it is ſupplied by the pipe GF, which takes it in at the notch G, and delivers it out at F; but after ſome time the water, which was ſpouted out at B, falling down into the baſon DB, riſes high enough to come above the notch G, which ſtops the paſſage of the air; ſo that the air S*s*, above the water in the veſſel CAE, wanting a ſupply, cannot ſufficiently preſs, and the Fountain ceaſes playing: But when the water of DB has run down into the lower baſon GH, through the hole I, till it falls below the top of the notch G, the air runs up into the upper receptacle, and ſupplies that at S*s*, and the Fountain plays again. This is ſeen a little before hand, by a ſkin of water on the notch G, before the air finds a paſſage, and then you may command the Fountain to play. It is evident that the hole I muſt be leſs than the hole of the jet, or elſe all the water would run out into the lower baſon, without riſing high enough to ſtop the notch G.

Fountain *that begins to play upon the lighting of candles, and ceaſes as they go out.* Provide two cylindrical veſſels, AB and CD, (fig. 7, pl. xi.) connect them by tubes, open at both ends, KL, BE, &c, ſo that the air may deſcend out of the higher into the lower:

to the tubes ſolder candleſticks, H, &c, and to the hollow cover of the lower veſſel CE, fit a little tube or jet, FG, furniſhed with a cock G, and reaching almoſt to the bottom of the veſſel. In G let there be an aperture, furniſhed with a ſcrew, by which water may be poured into CD. Then, upon lighting the candles H, &c, the air in the contiguous tubes becoming rarefied by the heat, the water will begin to ſpout through GF.

By the ſame contrivance may a ſtatue be made to ſhed tears upon the preſence of the ſun, or on the lighting of a candle, &c: all that is here required, being only to lay tubes from the cavity where the air is rarefied, to ſome other cavities placed near the eyes, full of water.

A FOUNTAIN *by the Rarefaction of the air*, may be made in the following manner: Let AB and CD, fig. 5, pl. xi, be two pipes fixed to a braſs head C, made to ſcrew into a glaſs veſſel E, which having a little water in it, is inverted till the pipes are ſcrewed on; then reverting it ſuddenly, ſo as to put A, the lower end of the ſpouting pipe AB, into a jar of water A, and the lower end of the deſcending pipe CD, into a receiving veſſel D, the water will ſpout up from the jar A into the tall glaſs veſſel E, from which it will go down at the mouth C, through CD, into the veſſel D, till the water is wholly emptied out of A, making a Fountain in E, into D. The reaſon of the play of the Fountain is this: the pipe CD, being 2 feet 9 inches long, lets down a column of water, which rarefies the air 1-12th part in the veſſel E, where it preſſes againſt the water ſpouting at B with 1-12th of the force by which the water is puſhed up at the hole A, by the preſſure of the common air on the water in the veſſel A; ſo that the water ſpouts up into E, when the air is rarefied 1-12th, with the difference of the preſſure of the atmoſphere, and the forementioned rarefied air; i. e. of 33 to $2\frac{3}{4}$, or of 12 to 1. This would raiſe the water 2 feet 9 inches; but the length of the pipe A, of 9 inches, being deducted, the jet will only riſe 2 feet. This, ſays Deſaguliers, may be called a ſyphon Fountain, where AB is the driving leg, and CD the iſſuing leg.

FOUNTAIN *of Hero of Alexandria*, ſo called, becauſe it was contrived by him. In the ſecond Fountain above deſcribed, the air is compreſſed by a ſyringe; in this, (ſee fig. 6, pl. xi) the air, being only compreſſed by the concealed fall of water, makes a jet, which, after ſome continuance, is conſidered by the ignorant as a perpetual motion; becauſe they imagine that the ſame water which fell from the jet riſes again. The boxes CE and DYX, being cloſe, we ſee only the baſon ABW, with a hole at W, into which the water ſpouting at B falls; but that water does not come up again; for it runs down through the pipe WX into the box DYX, from whence it drives out the air, through the aſcending pipe YZ, into the cavity of the box CE, where, preſſing upon the water that is in it, it forces it out through the ſpouting pipe OB, as long as there is any water in CE; ſo that this whole play is only whilſt the water contained in CE, having ſpouted out, falls down through the pipe WX, or of the boxes CE and DY above one another: the height of the water, meaſured from the baſon ABW to the ſurface of the water in the lower box DYX, is always equal to the height meaſured from the top of the jet to the ſurface of the water in the middle cavity at CE. Now, ſince the ſurface CE is always falling, and the water in DY always riſing, the height of the jet muſt continually decreaſe, till it is ſhorter by the depth of the cavity CE, which is emptying, added to the depth of the cavity DY, which is always filling; and when the jet is fallen ſo low, it immediately ceaſes. The air is repreſented by the points in this figure.

To prepare this Fountain for playing, which ſhould be done unobſerved, pour in water at W, till the cavity DXY is filled; then invert the Fountain, and the water will run from the cavity DXY into the cavity CE, which may be known to be full, when the water runs out at B held down. Set the Fountain up again, and, to make it play, pour in about a pint of water into the baſon ABW; and as ſoon as it has filled the pipe WX it will begin to play, and continue as long as there is any water in CE. You may then pour back the water left in the baſon ABW, into any veſſel, and invert the Fountain, which, being ſet upright again, will be made to play, by putting back the water poured out into ABW; and ſo on as often as you pleaſe.

Spouting FOUNTAIN, or Jet d'Eau, is any Fountain whoſe water is darted forth impetuouſly through jets, or ajutages, and returns in form of rains, nets, folds, or the like.

FOUNTAIN-*Pen*, is a pen contrived to contain a quantity of ink, and let it flow very gently, ſo as to ſupply the writer a long time without the neceſſity of taking freſh ink.

The Fountain-pen, repreſented fig. 8, pl. xi, conſiſts of divers pieces of metal, F, G, H, the middle piece F carrying the pen, which is ſcrewed into the inſide of a little pipe; and this again is ſoldered into another pipe of the ſame ſize as the lid G; in which lid is ſoldered a male ſcrew, for ſcrewing on the cover; as alſo for ſtopping a little hole at the place, and hindering the ink from paſſing through it: at the other end of the piece F is a little pipe, on the outſide of which may be ſcrewed the top cover H. A porte-craion goes in the cover, to be ſcrewed into the laſt-mentioned pipe, to ſtop the end of the pipe into which the ink is to be poured by a funnel.

To uſe the Pen, the cover G muſt be taken off, and the pen a little ſhaken, to make the ink run more freely.

FOURTH, in Muſic, one of the harmonic intervals, or concords. It conſiſts in the mixture of two ſounds, which are in the ratio of 4 to 3; i. e. of two ſounds produced by chords, whoſe lengths are to each other as 4 to 3.

FRACTION, or Broken Number, in Arithmetic and Algebra, is a part, or ſome parts, of another number or quantity conſidered as a whole, but divided into a certain number of parts; as 3-4ths, which denotes 3 parts out of 4, of any quantity.

Fractions are uſually divided into Vulgar, Decimal, Duodecimal, and Sexageſimal. For the laſt three ſorts, ſee the reſpective words.

Vulgar FRACTIONS, called alſo ſimple *Fractions*, are uſually denoted by two numbers, the one ſet under the other, with a ſmall line between them: thus $\frac{3}{4}$ denotes the Fraction three-fourths, or 3 parts out of 4, of ſome whole quantity conſidered as divided into 4 equal parts.

The lower number 4, is called the Denominator of the Fraction, ſhewing into how many parts the whole or integer is divided; and the upper number 3, is called the Numerator, and ſhews how many of thoſe equal parts are contained in the Fraction. Hence it follows, that as the numerator is to the denominator, ſo is the Fraction itſelf, to the whole of which it is a Fraction; or as the denominator is to the numerator, ſo is the whole or integer, to the Fraction: thus, the integer being denoted by 1, as $4 : 3 :: 1 : \frac{3}{4}$ the Fraction.—And hence there may be innumerable Fractions all of the ſame value, as there may be innumerable quantities all in the ſame ratio, viz, of 4 to 3; ſuch as 8 to 6, or 12 to 9, &c. So that if the two terms of any Fraction i. e. the numerator and denominator, be either both multiplied or both divided by any number, the reſulting Fraction will ſtill be of the ſame value: thus, $\frac{3}{4}$ or $\frac{6}{8}$ or $\frac{9}{12}$ or $\frac{12}{16}$ &c, are all of the ſame value with each other.

Fractional expreſſions are uſually diſtinguiſhed into Proper and Improper, Simple and Compound, and Mixt Numbers.

A Proper FRACTION, is that whoſe numerator is leſs than the denominator; and conſequently the Fraction is leſs than the whole or integer; as $\frac{3}{4}$.

Improper FRACTION, is when the numerator is either equal to, or greater than, the denominator; and conſequently the Fraction either equal to, or greater than, the whole integer, as $\frac{4}{4}$, which is equal to the whole; or $\frac{5}{4}$, which is greater than the whole.

Simple FRACTIONS, or *Single* FRACTIONS, are ſuch as conſiſt of only one numerator, and one denominator; as $\frac{3}{4}$, or $\frac{5}{4}$, or $\frac{12}{25}$.

Compound FRACTIONS are Fractions of Fractions, and conſiſt of ſeveral Fractions, connected together by the word *of*: as $\frac{2}{3}$ of $\frac{3}{4}$, or $\frac{1}{2}$ of $\frac{2}{3}$ of $\frac{3}{4}$.

A Mixt Number conſiſts of an integer and a Fraction joined together: as $1\frac{3}{4}$, or $12\frac{2}{3}$.

The arithmetic of Fractions conſiſts in the Reduction, Addition, Subtraction, Multiplication, and Diviſion of them.

Reduction of FRACTIONS is of ſeveral ſorts; as 1. *To reduce a given whole number into a Fraction of any given denominator.* Multiply the given integer by the propoſed denominator, and the product will be the numerator. Thus, it is found that $3 = \frac{6}{2}$, and $5 = \frac{20}{4}$, or $7 = \frac{35}{5}$.

If no denominator be given, or it be only propoſed to expreſs the integer Fraction-wiſe, or like a Fraction; ſet 1 beneath it, for its denominator. So $3 = \frac{3}{1}$, and $5 = \frac{5}{1}$, and $7 = \frac{7}{1}$.

2. *To reduce a given Fraction to another Fraction equal to it, that ſhall have a given denominator.* Multiply the numerator by the propoſed denominator, and divide the product by the former denominator, then the quotient ſet over the propoſed denominator will form the Fraction required. Thus, if it be propoſed to reduce $\frac{3}{4}$ to an equal Fraction whoſe denominator ſhall be 8; then $3 \times 8 = 24$, and $24 \div 4 = 6$ the numerator, ſo that $\frac{6}{8}$ is the Fraction ſought, being $= \frac{3}{4}$, and having 8 for its denominator.

3. *To Abbreviate, or reduce Fractions to lower terms.* Divide their terms, i. e. numerator and denominator, by any number that will divide them both without a remainder, ſo ſhall the quotients be the correſponding terms of a new Fraction, equal to the former, but in ſmaller numbers. In like manner abbreviate theſe new terms again, and ſo on till there be no number greater than 1 that will divide them without a remainder, and then the Fraction is ſaid to be in its leaſt terms. Thus, to abbreviate $\frac{15}{60}$; firſt divide both terms by 5, and the Fraction becomes $\frac{3}{12}$; next divide theſe by 3, and it becomes $\frac{1}{4}$: ſo that $\frac{15}{60} = \frac{3}{12} = \frac{1}{4}$, which is in its leaſt terms.

4. *To reduce Fractions to other equivalent ones of the ſame denominator.* Multiply each numerator, ſeparately taken, by all the denominators except its own, and the products will be the new numerators; then multiply all the denominators continually together, for the common denominator, to theſe numerators. Thus, $\frac{2}{3}$ and $\frac{4}{5}$ reduce to $\frac{10}{15}$ and $\frac{12}{15}$; and $\frac{2}{3}$, $\frac{3}{4}$, and $\frac{4}{5}$ reduce to $\frac{40}{60}$, $\frac{45}{60}$, and $\frac{48}{60}$.

5. *To find the value of a Fraction, in the known parts of its integer.* Multiply always the numerator by the number of parts of the next inferior denomination, and divide the products by the denominator. So, to find the value of $\frac{9}{16}$ of a pound ſterling; multiply 9 by 20 for ſhillings, and dividing by 16, gives 11 for the ſhillings; then multiply the remainder 4 by 12 pence, and dividing by 16 gives 3 for pence: ſo that 11s. 3d. is the value of $\frac{9}{16}$l. as required.

$$\begin{array}{r}
9 \\
20 \\
\hline
16)\,180\,(11s \\
16 \\
\hline
20 \\
16 \\
\hline
4 \\
12 \\
\hline
16)\,48\,(3d \\
48 \\
\hline
\end{array}$$

6. *To reduce a mixt number to an equivalent improper Fraction.* Multiply the integer by the denominator, and to the product add the numerator, for the new numerator, to be ſet over the ſame denominator as before. Thus $3\frac{5}{8}$ becomes $\frac{29}{8}$.

7. *To reduce an improper Fraction to its equivalent whole or mixt number.* Divide the numerator by the denominator; ſo ſhall the quotient be the integral part, and the remainder ſet over the denominator will form the fractional part of the equivalent mixt number. Thus $\frac{29}{8}$ reduces to $3\frac{5}{8}$, and $\frac{32}{4} = 8$.

8. *To reduce a compound Fraction to a ſimple one.* Multiply all the numerators together for the numerator, and all the denominators together for the denominator, of the ſimple Fraction ſought. Thus, $\frac{1}{2}$ of $\frac{3}{4} = \frac{3}{8}$, and $\frac{2}{3}$ of $\frac{4}{5}$ of $\frac{7}{9} = \frac{56}{135}$.

To reduce a Vulgar Fraction to a decimal. See DECIMALS. And for ſeveral other particulars concerning Reduction, as well as the other operations in Fractions; ſee my Arithmetic.

Addition of FRACTIONS. Firſt reduce the Fractions to their ſimpleſt form, and reduce them alſo to a common denominator, if their denominators are different; then add all the numerators together, and ſet the ſum over the common denominator, for the ſum of all the Fractions as required.

Thus, $\frac{2}{5} + \frac{3}{5} = \frac{5}{5} = 1$;
And $\frac{2}{7} + \frac{3}{4} = \frac{8}{28} + \frac{21}{28} = \frac{29}{28} = 1\frac{1}{28}$.

Subtraction of FRACTIONS. Reduce the Fractions the ſame as for addition; then ſubtract the one numerator from the other, and ſet the difference over the common denominator.

So $\frac{7}{5} - \frac{3}{5} = \frac{4}{5}$;
And $\frac{7}{9} - \frac{4}{7} = \frac{49}{63} - \frac{36}{63} = \frac{13}{63}$.

To Multiply FRACTIONS *together*. Reduce them all to the form of ſimple Fractions, if they are not ſo; then multiply all the numerators together for the numerator, and all the denominators together for the denominator of the product ſought.

Thus $\frac{2}{3} \times \frac{4}{5} = \frac{8}{15}$;
And $\frac{3}{8} \times \frac{5}{6} \times \frac{7}{10} = \frac{105}{480} = \frac{21}{96} = \frac{7}{32}$.

To Divide FRACTIONS. Divide the numerator by the numerator, and the denominator by the denominator, if they will exactly divide. Thus, $\frac{8}{15} \div \frac{4}{5} = \frac{2}{3}$.

But if they will not divide without a remainder, then multiply the dividend by the reciprocal of the diviſor, that is, by the Fraction obtained by inverting or changing its terms. Thus, $\frac{8}{15} \div \frac{3}{4} = \frac{8}{15} \times \frac{4}{3} = \frac{32}{45}$.

Algebraic FRACTIONS, or FRACTIONS *in Species*, are exactly ſimilar to vulgar Fractions, in numbers, and all the operations are performed exactly in the ſame way; therefore the rules need not be repeated, and it may be ſufficient here to ſet down a few examples to the foregoing rules. Thus,

1. The Fraction $\frac{aab}{bc}$ abbreviates to $\frac{aa}{c}$.

2. $\frac{6a^3 - 9ax^2}{6a^2 + 3ax} = \frac{2a^2 - 3x^2}{2a + x}$, by dividing by $3a$.

3. $\frac{a^3 - a^2x + ax^2 - x^3}{a^2 - ax} = \frac{a^2 + x^2}{a}$, by dividing by $a - x$. See COMMON *Meaſure*.

4. $\frac{a}{b}$ and $\frac{c}{d}$ become $\frac{ad}{bd}$ and $\frac{bc}{bd}$, when reduced to a common denominator.

5. $\frac{a}{b} + \frac{c}{d} = \frac{ad + bc}{bd}$.

6. $\frac{a}{x} - \frac{b}{z} = \frac{az - bx}{xz}$.

7. $\frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd}$.

8. $\frac{a}{x} \div \frac{b}{z} = \frac{a}{x} \times \frac{z}{b} = \frac{az}{bx}$.

Continued FRACTION, is uſed for a Fraction whoſe denominator is an integer with a Fraction, which latter Fraction has for its denominator an integer and a Fraction, and the ſame for this laſt Fraction again, and ſo on, to any extent, whether ſuppoſed to be infinitely continued, or broken off after any number of terms. Euler, Analyſ. Inf. vol. 1, p. 295.

As $\cfrac{1}{2 + \cfrac{1}{3}}$, or $\cfrac{1}{2 + \cfrac{1}{3 + \cfrac{1}{4}}}$,

or $\cfrac{1}{2 + \cfrac{2}{3 + \cfrac{3}{4 + \cfrac{4}{5 + \&c.}}}}$

Or, uſing letters inſtead of numbers,

$$\cfrac{1}{a + \cfrac{1}{b + \cfrac{1}{c + \cfrac{1}{d + \cfrac{1}{e + \&c.}}}}}$$

or

$$\cfrac{A}{a + \cfrac{B}{b + \cfrac{C}{c + \cfrac{D}{d + \cfrac{E}{e + \&c.}}}}}$$

When theſe ſeries are not far extended, it is not difficult to collect them by common arithmetic.

Lord Brounker, it ſeems, was the firſt who conſidered Continued Fractions, or at leaſt, who applied them to the quadrature of curves, in Wallis's Arith. Infin. prop. 191, vol. 1, p. 469 &c, where this author explains the manner of forming them, giving ſeveral numeral examples, in approximating ratios, as well as the general ſeries

$$\cfrac{a}{\alpha + \cfrac{b}{\beta + \cfrac{c}{\gamma + \cfrac{d}{\delta}}}} \text{ \&c, as he denotes it,}$$

Huygens alſo uſed it for the like purpoſe, viz, to approximate the ratios of large numbers, in his Deſcrip. Autom. Planet. in Oper. Relig. p. 173 &c, edit. Amſt. 1728. And a ſpecial treatiſe on Continued Fractions was given by Euler, in his Analyſ. Infin. vol. 1, pa. 295 &c.

This ſubject is perhaps capable of much improvement, though it has been rather neglected, as very little uſe has been made of it, except, by thoſe authors, in approximating to the value of Fractions, and ratios, that are expreſſed in large numbers; beſides a method of Goniometry by De Lagny, explained in the Introduction to my Logarithms, pa. 78; as alſo ſome uſe I have made of it in ſumming very ſlowly converging ſeries, in my Tracts, p. 38 & ſeq.

As to the reducing of common Fractions, and ratios, that are expreſſed in large numbers, to Continued Fractions, it is no more than the common method of finding the greateſt common meaſure of thoſe two numbers, by dividing the greater by the leſs, and the laſt diviſor always by the laſt remainder; for then the ſeveral quotients are the denominators of the Fractions, the numerators being always 1 or unity. Thus, to find approximating values of the Fraction $\frac{31415926535}{10000000000}$, or to the ratio of 31415926535 to 10000000000, being the ratio of the circumference of a circle to its diameter, by means of a Continued Fraction; or, to change the ſaid Common Fraction to a Continued Fraction: Dividing the greater term always by the leſs, the ſame as to find the greateſt common meaſure of the ſaid numbers or terms, the ſeveral quotients will be 3, 7, 15, 1, 292, 1, 1, &c, which, after the firſt, will be the denominators, to the common numerator 1; and therefore the ſaid Fraction will be changed into this Continued Fraction,

$$3 + \cfrac{1}{7 + \cfrac{1}{15 + \cfrac{1}{1 + \cfrac{1}{292 + \cfrac{1}{1 + \cfrac{1}{1 + \&c.}}}}}}$$

Hence, ſtopping at any part of theſe ſingle Fractions, one after another, will give ſeveral values of the propoſed ratio, all ſucceſſively nearer and nearer the truth, but alternately too great and too little. So, ſtopping at $\frac{1}{7}$, it is $3\frac{1}{7} = \frac{22}{7} = 3{\cdot}142857$ too great, or 22 to 7, the ratio of the circumference to the diameter as given by Archimedes. Again, ſtopping at $\frac{1}{15}$, it is

$$3\frac{1}{7\frac{1}{15}} = 3\frac{15}{106} = \frac{333}{106} = 3{\cdot}141509 \quad \&c, \quad \text{too little.}$$

But ſtopping at $\frac{1}{1}$,

it is $3\frac{1}{7\frac{1}{15+\frac{1}{1}}} = 3\frac{1}{7\frac{1}{16}} = 3\frac{16}{113} = \frac{355}{113}$ (the ratio of Metius) $= 3{\cdot}1415929$ &c, which is rather too great. And ſo on, always nearer and nearer, but alternately too great and too little.

And, in like manner is any algebraic Fraction thrown into a Continued Fraction. As the Fraction $\frac{a\beta\gamma\delta + a\beta d + ac\delta}{\alpha\beta\gamma\delta + \alpha\beta d + \alpha c\delta + b\gamma\delta + bd}$, which being in like manner divided, the quotients are $\frac{\alpha}{a}, \frac{\beta}{b}, \frac{\gamma}{c}, \frac{\delta}{d}$; which ſingle Fractions being conſidered as denominators to other Fractions whoſe common numerator is 1, theſe will be the reciprocals of the former, and ſo will become $\frac{a}{\alpha}, \frac{b}{\beta}, \frac{c}{\gamma}, \frac{d}{\delta}$; and hence the propoſed common Fraction is equal to this terminate Continued Fraction,

$$\cfrac{a}{\alpha + \cfrac{b}{\beta + \cfrac{c}{\gamma + \cfrac{d}{\delta}}}}.$$

On the other hand, any Continued Fraction being given, its equivalent common Fraction will be found, by beginning at the laſt denominator, or loweſt end of the given Continued Fraction, and gradually collecting the Fractions backwards, till we arrive at the firſt, when the whole will thus be collected together into one common Fraction; as was done above in collecting the Fractions

$$\tfrac{1}{7} + \tfrac{1}{15} + \tfrac{1}{1} + \tfrac{1}{292} + \&c.$$

And in like manner the Continued Fraction

$$\cfrac{a}{\alpha + \cfrac{b}{\beta + \cfrac{c}{\gamma + \cfrac{d}{\delta}}}} \text{ collects into the Fraction}$$

$$\frac{a\beta\gamma\delta + a\beta d + ac\delta}{\alpha\beta\gamma\delta + \alpha\beta d + \alpha c\delta + b\gamma\delta + bd}.$$

When the given Continued Fraction is an infinite one, collect it ſucceſſively, firſt one term, then two together, three together, &c, till the ſum is ſufficiently exact. Or, if theſe collected ſums converge too ſlowly to the true value, having collected a few of the terms into ſucceſſive ſums, theſe being alternately too great and too little, the true value will be found as near as you pleaſe by the method of arithmetical means, explained in my Tracts, vol. 1, Tract 2, pa. 11.

Vaniſhing FRACTIONS. Such Fractions as have both their numerator and denominator vaniſh, or equal to 0, at the ſame time, may be called *Vaniſhing Fractions*. We are not to conclude that ſuch Fractions are equal to nothing, or have no value; for that they have a certain determinate value, has been ſhewn by the beſt mathematicians. The idea of ſuch Fractions as theſe, firſt originated in a very ſevere conteſt among ſome French mathematicians, in which Varignon and Rolle were the two chief oppoſite combatants, concerning the then new or differential calculus, of which the latter gentleman was a ſtrenuous opponent. Among other arguments againſt it, he propoſed an example of drawing a tangent to certain curves at the point where the two parts croſs each other; and as the fractional expreſſion for the ſubtangent, by that method, had both its numerator and denominator equal to 0 at the point propoſed, Rolle looked upon it as an abſurd expreſſion, and as an argument againſt the method of ſolution itſelf. The ſeeming myſtery however was ſoon explained, and firſt of all by John Bernoulli. See an account of this affair in Montucla, Hiſt. Math. vol. 2, pa. 366.

Since that time, ſuch kind of fractions have often been contemplated by mathematicians. As, by Maclaurin, in his Fluxions, vol. 2, pa. 698: Saunderſon, in his Algebra, vol. 2, art. 469: De Moivre, in Miſcel. Anal. pa. 165: Emerſon, in his Algebra, pa. 212: and by many others. The ſame fractions have alſo proved a ſtumbling-block to more mathematicians than one, and the cauſe of more violent controverſies: witneſs that between Powell and Waring, when they were competitors for the profeſſorſhip at Cambridge. In the ſpecimen of a work publiſhed on occaſion of that competition, by Waring, was the fraction $\frac{p-p^5}{1-p}$, which he ſaid became 4 when p was $= 1$. This was ſtruck at by Powell, as abſurd, becauſe when $p = 1$, then the fraction $\frac{p-p^5}{1-p} = \frac{1-1}{1-1} = \frac{0}{0}$, which was one chief cauſe of his not ſucceeding to the profeſſorſhip. Waring replied that $\frac{p-p^5}{1-p}$ is $= p + p^2 + p^3 + p^4$ (by common diviſion) $= 1 + 1 + 1 + 1 = 4$, when p is $= 1$. See the controverſial pamphlets that paſſed between thoſe two gentlemen at that time.

There are two modes of finding the value of ſuch fractions, that have been given by the gentlemen above quoted. The one is by conſidering the terms of the fraction as two variable quantities, continually decreaſing, till they both vaniſh together; or finding the ultimate value of the ratio denoted by the fraction. In this way of conſidering the matter, it appears that, as the terms of the fraction are ſuppoſed to decreaſe till they vaniſh, or become only equal to their fluxions or their increments, the value of the fraction at that ſtate, will be equal to the fluxion or increment of the numerator divided by that of the denominator. Hence then, taking the example $\frac{x-x^5}{1-x}$ when $x = 1$; the fluxion of the numerator is $\dot{x} - 5x^4\dot{x}$, and of the denominator $-\dot{x}$; therefore

$$\frac{\dot{x} - 5x^4\dot{x}}{-\dot{x}} = \frac{1-5x^4}{-1} = \frac{5x^4-1}{1} = 5x^4 - 1 = 5 - 1 = 4,$$

the value of the fraction $\frac{x-x^5}{1-x}$ when $x = 1$.——Or, thus, becauſe $x = 1$, therefore $\frac{x-x^5}{1-x} = \frac{1-x^4}{1-x}$; then the fluxion of the numerator, $-4x^3\dot{x}$, divided by the fluxion of the denominator, or $-\dot{x}$, gives $4x^3$ or 4, the ſame as before.

The other method is by reducing the given expreſſion to another, or ſimple form, and then ſubſtituting the values of the letters. So in the above example $\frac{x-x^5}{1-x}$, or $\frac{1-x^4}{1-x}$, when $x=1$; divide the numerator by the denominator, and it becomes $1+x+x^2+x^3$, which when $x=1$, becomes 4, for the given fraction, the ſame as before.—Again, to find the value of $\frac{a\sqrt{ax-xx}}{a-\sqrt{ax}}$ when x is $=a$, in which caſe both the numerator and denominator become $=0$. Divide the numerator by the denominator, and the quotient is $\sqrt{ax}+x+x\sqrt{\frac{x}{a}}$; which when $x=a$, becomes $a+a+a=3a$, for the value of the fraction in that ſtate of it.

FRAISE, in Fortification, a kind of defence, conſiſting of pointed ſtakes, driven almoſt parallel to the horizon into the retrenchments of a camp, &c, to ward off and prevent any approach or ſcalade.

FRANKLIN (Dr. Benjamin), one of the moſt celebrated philoſophers and politicians of the 18th century, was born in Boſton in North America in the year 1706, being the youngeſt of 13 children. His father was a tallow-chandler in Boſton, and young Franklin was taken from ſchool at 10 years of age, to aſſiſt him in his buſineſs. In this ſituation he continued two years; but diſliking that occupation, he was bound apprentice to an elder brother, who was then a printer in Boſton, but had learned that buſineſs in London, and who in the year 1721 began to print a newſpaper, being the ſecond ever publiſhed in America; the copies of which our author was ſent to diſtribute, after having aſſiſted in compoſing and printing it. Upon this occaſion, our young philoſopher enjoyed the ſecret and ſingular pleaſure of being the much admired author of many eſſays in this paper; a circumſtance which he had the addreſs to keep a ſecret even from his brother himſelf; and this when he was only 15 years of age. The frequent ill uſage from his brother, induced young Franklin to quit his ſervice, which he did, at the age of 17, and went to New York. But not meeting employment here, he went forward to Philadelphia, where he worked with a printer a ſhort time; after which, at the inſtance of Sir William Keith, governor of the province, he returned to Boſton to ſolicit pecuniary aſſiſtance from his father to ſet up a printing-houſe for himſelf at Philadelphia, upon the promiſe of great encouragement from Sir William, &c. His father however thought fit to refuſe ſuch aid, alleging that he was yet too young (18 years old) to be entruſted with ſuch a concern; ſo our author again returned to Philadelphia without it. Upon this, Sir William ſaid he would advance the ſum that might be neceſſary, and our young philoſopher ſhould go to England, and purchaſe all the types and materials, for which purpoſe he would give him letters of credit. He could never however get theſe letters, yet by dint of fair promiſes of their being ſent on board the ſhip after him, he failed for England, expecting theſe letters of credit were in the governor's packet, which he was to receive upon its being opened. In this however he was cruelly deceived, and thus he was ſent to London without either money, friends, or credit, at 18 years of age.

He ſoon found employment, however, as a journeyman printer, firſt at a Mr. Palmer's, and afterward with Mr. Watts, with whom he worked a conſiderable time, and by whom he was greatly eſteemed, being alſo treated with ſuch kindneſs, that it was always moſt gratefully remembered by our philoſopher.

After a ſtay of 18 months in London, he returned to Philadelphia, viz in 1726, along with a merchant of that town, as his clerk, on a ſalary of 50 pounds a year. But his maſter dying the year after, he again engaged to direct the printing buſineſs of the ſame perſon with whom he had worked before. After continuing with him the beſt part of a year, our philoſopher, in partnerſhip with another young man, at length ſet up a printing-houſe himſelf.

A little before this time, young Franklin had gradually aſſociated a number of perſons, like himſelf, of a rational and philoſophical turn of mind, and formed them into a club or ſociety, to hold meetings, to converſe and communicate their ſentiments together, for their mutual improvement in all kinds of uſeful knowledge, which was in high repute for many years after. Among many other uſeful regulations, they agreed to bring ſuch books as they had into one place, to form a common library. This reſource being found defective, at Franklin's perſuaſion they reſolved to contribute a ſmall ſum monthly towards the purchaſe of books for their uſe from London. Thus their ſtock began to increaſe rapidly; and the inhabitants of Philadelphia, being deſirous of having a ſhare in their literary knowledge, propoſed that the books ſhould be lent out on paying a ſmall ſum for the indulgence. Thus in a few years the ſociety became rich, and poſſeſſed more books than were perhaps to be found in all the other colonies: the collection was advanced into a public library; and the other colonies, ſenſible of its advantages, began to form ſimilar plans; from whence originated the libraries at Boſton, New York, Charleſtown, &c; that of Philadelphia being now not inferior to any in Europe.

About 1728 or 1729, young Franklin ſet up a newſpaper, the ſecond in Philadelphia, which proved very profitable, and otherwiſe uſeful, as affording an opportunity of making himſelf known as a political writer, by inſerting ſeveral of his writings of that kind into it. In addition to his printing-houſe, he ſet up a ſhop to ſell books and ſtationary; and in 1730 he married his wife, who proved very uſeful in aſſiſting to manage the ſhop, &c. He afterward began to have ſome leiſure, both for reading books, and writing them, of which he gave many ſpecimens from time to time. In 1732 he began to publiſh Poor Richard's Almanac, which was continued for many years. It was always remarkable for the numerous and valuable conciſe maxims which it contained, for the œconomy of human life, all tending to exhort to induſtry and frugality: and in the almanac for the laſt year, all the maxims were collected in an addreſs to the reader, entitled, The Way to Wealth. This has been tranſlated into various languages, and inſerted in different publications. It has alſo been printed on a large ſheet, proper to be framed, and hung up in conſpicuous places in all houſes, as it very well deſerves to be. Mr. Franklin became gradually more known for his political talents, and in the year 1736, he was appointed clerk to the General Aſſembly of Pennſylvania; and was re-elected by ſucceeding aſſemblies for ſeveral years, till he was choſen a repreſentative for the city of Philadelphia; and in 1737 he was appointed poſt-maſter of that city. In 1738, he formed the firſt fire-company there, to extinguiſh and prevent fires and the burning of houſes: an example which was ſoon followed by other perſons, and other places. And ſoon after, he ſuggeſted the plan of an aſſociation for inſuring houſes and ſhips from loſſes by fire, which was adopted; and the aſſociation continues to this day. In the year 1744, during a war between France and Great Britain, ſome French and Indians made inroads upon the frontier inhabitants of the province, who were unprovided for ſuch an attack: the ſituation of the province was at this time truly alarming, being deſtitute of every means of defence. At this criſis Franklin ſtepped forth, and propoſed to a meeting of the citizens of Philadelphia, a plan of a voluntary aſſociation for the defence of the province. This was approved of, and ſigned by 1200 perſons immediately. Copies of it were circulated through the province; and in a ſhort time the number of ſigners amounted to 10,000. Franklin was choſen colonel of the Philadelphia regiment; but he did not think proper to accept of the honour.

Purſuits of a different nature now occupied the greateſt part of his attention for ſome years. Being always much addicted to the ſtudy of natural philoſophy; and the diſcovery of the Leyden experiment in electricity having rendered that ſcience an object of general curioſity; Mr. Franklin applied himſelf to it, and ſoon began to diſtinguiſh himſelf eminently in that way. He engaged in a courſe of electrical experiments with all the ardour and thirſt for diſcovery which characterized the philoſophers of that day. By theſe, he was enabled to make a number of important diſcoveries, and to propoſe theories to account for various phenomena; which have been generally adopted, and which will probably endure for ages. His obſervations he communicated, in a ſeries of letters, to his friend Mr. Collinſon; the firſt of which is dated March 28, 1747. In theſe he makes known the power of points in drawing and throwing off the electric matter, which had hitherto eſcaped the notice of electricians. He alſo made the diſcovery of a *plus* and *minus*, or of a *poſitive* and *negative* ſtate of electricity; from whence in a ſatisfactory manner he explained the phenomena of the Leyden phial, firſt obſerved by Cuneus or Muſchenbroeck, which had much perplexed philoſophers. He ſhewed that the bottle, when charged, contained no more electricity than before, but that as much was taken from one ſide as was thrown on the other; and that, to diſcharge it, it was only neceſſary to make a communication between the two ſides, by which the equilibrium might be reſtored, and that then no ſigns of electricity would remain. He afterwards demonſtrated by experiments, that the electricity did not reſide in the coating, as had been ſuppoſed, but in the pores of the glaſs itſelf. After a phial was charged, he removed the coating, and found that upon applying a new coating the ſhock might ſtill be received. In the year 1749, he firſt ſuggeſted his idea of explaining the phenomena of thunder-guſts, and of the aurora borealis, upon electrical principles. He points out many particulars in which lightning and electricity agree; and he adduces many facts, and reaſoning from facts, in ſupport of his poſitions. In the ſame year he conceived the bold and grand idea of aſcertaining the truth of his doctrine, by actually drawing down the forked lightning, by means of ſharp-pointed iron rods raiſed into the region of the clouds; from whence he derived his method of ſecuring buildings and ſhips from being damaged by lightning. It was not until the ſummer of 1752 that he was enabled to complete his grand diſcovery the experiment of the electrical kite, which being raiſed up into the clouds, brought thence the electricity or lightning down to the earth; and M. D'Alibard made the experiment about the ſame time in France, by following the track which Franklin had before pointed out. The letters which he ſent to Mr. Collinſon, it is ſaid, were refuſed a place among the papers of the Royal Society of London; and Mr. Collinſon publiſhed them in a ſeparate volume, under the title of *New Experiments and Obſervations on Electricity, made at Philadelphia, in America;* which were read with avidity, and ſoon tranſlated into different languages. His theories were at firſt oppoſed by ſeveral philoſophers, and by the members of the Royal Society of London; but in 1755, when he returned to that city, they voted him the gold medal which is annually given to the perſon who preſents the beſt paper on ſome intereſting ſubject. He was alſo admitted a member of the Society, and had the degree of doctor of laws conferred upon him by different univerſities: but at this time, by reaſon of the war which broke out between Britain and France, he returned to America, and intereſted himſelf in the public affairs of that country. Indeed he had done this long before; for although philoſophy was a principal object of Franklin's purſuit for ſeveral years, he did not confine himſelf to it alone. In the year 1747 he became a member of the General Aſſembly of Pennſylvania, as a burgeſs for the city of Philadelphia. Being a friend to the rights of man from his infancy, he ſoon diſtinguiſhed himſelf as a ſteady opponent of the unjuſt ſchemes of the proprietaries. He was ſoon looked up to as the head of the oppoſition; and to him have been attributed many of the ſpirited replies of the aſſembly, to the meſſages of the governors. His influence in the body was very great. This aroſe not from any ſuperior powers of eloquence; he ſpoke but ſeldom: and he never was known to make any thing like an elaborate harangue. His ſpeeches often conſiſted of a ſingle ſentence, or of a well-told ſtory, the moral of which was always obviouſly to the point. He never attempted the flowery fields of oratory. His manner was plain and mild. His ſtyle in ſpeaking was, like that of his writings, ſimple, unadorned, and remarkably conciſe. With this plain manner, and his penetrating and ſolid judgment, he was able to confound the moſt eloquent and ſubtle of his adverſaries, to confirm the opinions of his friends, and to make converts of the unprejudiced who had oppoſed him. With a ſingle obſervation, he has rendered of no avail a long and elegant diſcourſe, and determined the fate of a queſtion of importance.

In the year 1749, he propoſed a plan of an academy, to be erected in the city of Philadelphia, as a foundation for poſterity to erect a ſeminary of learning, more extenſive and ſuitable to future circumſtances; and in the beginning of 1750, three of the ſchools were opened, namely, the Latin and Greek ſchool, the Mathematical, and the Engliſh ſchools. This foundation ſoon after gave riſe to another more extenſive college, incorporated by charter May 27, 1755, which ſtill ſubſiſts, and in a very flouriſhing condition. In 1752, he was inſtrumental in the eſtabliſhment of the Pennſylvania Hoſpital, for the cure and relief of indigent invalids, which has proved of the greateſt uſe to that claſs of perſons. Having conducted himſelf ſo well as Poſtmaſter of Philadelphia, he was, in 1753, appointed Deputy Poſtmaſter-general for the whole Britiſh colonies.

The colonies being much expoſed to depredations in their frontier by the Indians and the French, at a meeting of commiſſioners from ſeveral of the provinces, Mr. Franklin propoſed a plan for the general defence, to eſtabliſh in the colonies a general government, to be adminiſtered by a preſident-general, appointed by the crown, and by a grand council, conſiſting of members choſen by the repreſentatives of the different colonies; a plan which was unanimouſly agreed to by the commiſſioners preſent. The plan however had a ſingular fate: It was diſapproved of by the miniſtry of Great Britain, becauſe it gave too much power to the repreſentatives of the people; and it was rejected by every aſſembly, as giving to the preſident general, who was to be the repreſentative of the crown, an influence greater than appeared to them proper, in a plan of government intended for freemen. Perhaps this rejection, on both ſides, is the ſtrongeſt proof that could be adduced of the excellence of it, as ſuited to the ſituation of Great Britain and America at that time. It appears to have ſteered exactly in the middle, between the oppoſite intereſts of both. Whether the adoption of this plan would have prevented the ſeparation of America from Great Britain, is a queſtion which might afford much room for ſpeculation.

In the year 1755, General Braddock, with ſome regiments of regular troops, and provincial levies, was ſent to diſpoſſeſs the French of the poſts upon which they had ſeized in the back ſettlements. After the men were all ready, a difficulty occurred, which had nearly prevented the expedition. This was the want of waggons. Franklin now ſtepped forward, and, with the aſſiſtance of his ſon, in a little time procured 150. After the defeat of Braddock, Franklin introduced into the aſſembly a bill for organizing a militia, and had the dexterity to get it paſſed. In conſequence of this act a very reſpectable militia was formed; and Franklin was appointed colonel of a regiment in Philadelphia, which conſiſted of 1200 men; in which capacity he acquitted himſelf with much propriety, and was of ſingular ſervice; though this militia was ſoon after diſbanded by order of the Engliſh miniſtry.

In 1757, he was ſent to England, with a petition to the king and council, againſt the proprietaries, who refuſed to bear any ſhare in the public expences and aſſeſſments; which he got ſettled to the ſatisfaction of the ſtate. After the completion of this buſineſs, Franklin remained at the court of Great Britain for ſome time, as agent for the province of Pennſylvania; and alſo for thoſe of Maſſachuſetts, Maryland, and Georgia. Soon after this, he publiſhed his Canada pamphlet, in which he pointed out, in a very forcible manner, the advantages that would reſult from the conqueſt of this province from the French. An expedition was accordingly planned, and the command given to General Wolfe; the ſucceſs of which is well known. He now divided his time indeed between philoſophy and politics, rendering many ſervices to both. Whilſt here, he invented the elegant muſical inſtrument called the *Armonica*, formed of glaſſes played on by the fingers. In the ſummer of 1762 he returned to America; on the paſſage to which he obſerved the ſingular effect produced by the agitation of a veſſel, containing oil floating on water: the upper ſurface of the oil remained ſmooth and undiſturbed, whilſt the water was agitated with the utmoſt commotion. On his return he received the thanks of the Aſſembly of Pennſylvania, which having annually elected him a member in his abſence, he again took his ſeat in this body, and continued a ſteady defender of the liberties of the people.

In 1764, by the intrigues of the proprietaries, Franklin loſt his ſeat in the aſſembly, which he had poſſeſſed for 14 years; but was immediately appointed provincial agent to England, for which country he preſently ſet out. In 1766 he was examined before the parliament relative to the ſtamp-act; which was ſoon after repealed. The ſame year he made a journey into Holland and Germany; and another into France; being everywhere received with the greateſt reſpect by the literati of all nations. In 1773 he attracted the public attention by a letter on the duel between Mr. Whately and Mr. Temple, concerning the publication of Governor Hutchinſon's letters, declaring that he was the perſon who had diſcovered thoſe letters. On the 29th of January next year, he was examined before the privy-council on a petition he had preſented long before as agent for Maſſachuſetts Bay againſt Mr. Hutchinſon: but this petition being diſagreeable to miniſtry, it was precipitately rejected, and Dr. Franklin was ſoon after removed from his office of Poſtmaſter-general for America. Finding now all efforts to reſtore harmony between Great Britain and her colonies uſeleſs, he returned to America in 1775; juſt after the commencement of hoſtilities. Being named one of the delegates to the Continental Congreſs, he had a principal ſhare in bringing about the revolution and declaration of independency on the part of the colonies. In 1776 he was deputed by Congreſs to Canada, to negociate with the people of that country, and to perſuade them to throw off the Britiſh yoke; but the Canadians had been ſo much diſguſted with the hot-headed zeal of the New Englanders, who had burnt ſome of their chapels, that they refuſed to liſten to the propoſals, though enforced by all the arguments Dr. Franklin could make uſe of. On his return to Philadelphia, Congreſs, ſenſible how much he was eſteemed in France, ſent him thither to put a finiſhing hand to the private negociations of Mr. Silas Deane; and this important commiſſion was readily accepted by the Doctor, though then in the 71ſt year of his age. The event is well known; a treaty of alliance and commerce was ſigned between France and America; and M. Le Roi aſſerts, that the Doctor had a great ſhare in the tranſaction, by ſtrongly adviſing M. Maurepas not to loſe a ſingle moment, if he wiſhed to ſecure the friendſhip of America, and to detach it from the mother-country. In 1777 he was regularly appointed plenipotentiary from Congreſs to the French court; but obtained leave of diſmiſſion in 1780. Having at length ſeen the full accompliſhment of his wiſhes by the concluſion of the peace in 1783, which gave independency to America, he became deſirous of reviſiting his native country. He therefore requeſted to be recalled; and, after repeated ſolicitations, Mr. Jefferſon was appointed in his ſtead. On the arrival of his ſucceſſor, he repaired to Havre de Grace, and croſſing the channel, landed at Newport in the Iſle of Wight; from whence, after a favourable paſſage, he arrived ſafe at Philadelphia in September 1785. He was received amidſt the acclamations of a vaſt multitude who flocked from all parts to ſee him, and who conducted him in triumph to his own houſe; where in a few days he was viſited by the members of the Congreſs and the principal inhabitants of Philadelphia. He was afterward twice choſen preſident of the Aſſembly of Philadelphia; but his increaſing infirmities obliged him to aſk permiſſion to retire, and to ſpend the remainder of his life in tranquillity; which was granted, in 1788. After this, the infirmities of age increaſed faſt upon him; he became more and more afflicted with the gout and the ſtone, till the time of his death, which happened the 17th of April 1790, about 11 at night, at 84 years of age; leaving one ſon, governor William Franklin, a zealous loyaliſt, who now reſides in London; and a daughter, married to Mr. William Bache, merchant in Philadelphia.

Doctor Franklin was author of many tracts on electricity, and other branches of natural philoſophy, as well as on politics, and miſcellaneous ſubjects. He had alſo many papers inſerted in the Philoſophical Tranſactions, from the year 1757 to 1774.

FREEZE, or Frize, in Architecture, a large flat member, being that part of the entablature of columns that ſeparates the architrave from the cornice.

FREEZING, or Congelation, the fixing of a fluid body into a firm or ſolid maſs by the action of cold: in which ſenſe the term is applied to water when it freezes into ice; to metals when they reſume their ſolid form after being melted by heat; or to glaſs, wax, pitch, tallow, &c, when they harden again after having been rendered fluid by heat. But it differs from cryſtallization, which is rather a ſeparation of the particles of a ſolid from a fluid in which it had been diſſolved more by the moiſture than the action of heat.

The proceſs of congelation is always attended with the emiſſion of heat, as is found by experiments on the freezing of water, wax, ſpermaceti, &c; for in ſuch caſes it is always found that a thermometer dipt into the fluid maſs, keeps continually deſcending as this cools, till it arrive at a certain point, being the point of freezing, which is peculiar to each fluid, where it is rather ſtationary, and then riſes for a little, while the congelation goes on. But by what means it is that fluid bodies ſhould thus be rendered ſolid by cold, or fluid by heat, or what is introduced into the bodies by either of thoſe principles, are matters the learned have never yet been able to diſcover, or to ſatisfy themſelves upon. The following phenomena however are uſually obſerved.

Water, and ſome other fluids, ſuddenly dilate and expand in the act of Freezing, ſo as to occupy a greater ſpace in the form of ice than before, in conſequence of which it is that ice is ſpecifically lighter than the ſame fluid, and floats in it. And the degree of expanſion of water, in the ſtate of ice, is by ſome authors computed at about $\frac{1}{10}$ of its volume. Oil however is an exception to this property, and quickſilver too, which ſhrinks and contracts ſtill more after Freezing. Mr. Boyle relates ſeveral experiments of veſſels made of metal, very thick and ſtrong; in which, when filled with water, cloſe ſtopped, and expoſed to the cold, the water being expanded in Freezing, and not finding either room or vent, burſt the veſſels. A ſtrong barrel of a gun, with water in it cloſe ſtopped and frozen, was rent the whole length. Huygens, to try the force with which it expands, filled a cannon with it, whoſe ſides were an inch thick, and then cloſed up the mouth and vent, ſo that none could eſcape; the whole being expoſed to a ſtrong Freezing air, the water froze in about 12 hours, and burſt the piece in two places. Mathematicians have computed the force of the ice upon this occaſion; and they ſay, that ſuch a force would raiſe a weight of 27720 pounds. Laſtly, Major Edward Williams, of the Royal Artillery, made many experiments on the force of it, at Quebec, in the years 1784 and 1785. He filled all ſizes of iron bomb-ſhells with water, then plugged the fuze hole cloſe up, and expoſed them to the ſtrong Freezing air of the winter in that climate; ſometimes driving in the iron plugs as hard as poſſible with a ſledge hammer; and yet they were always thrown out by the ſudden expanſion of the water in the act of Freezing, like a ball ſhot by gunpowder, ſometimes to the diſtance of between 400 and 500 feet, though they weighed near 3 pounds; and when the plugs were ſcrewed in, or furniſhed with hooks or barbs, to lay hold of the inſide of the ſhell by, ſo that they could not poſſibly be forced out, in this caſe the ſhell was always ſplit in two, though the thickneſs of the metal of the ſhell was about an inch and three quarters. It is farther remarkable, that through the circular crack, round about the ſhells, where they burſt, there ſtood out a thin film or ſheet of ice, like a fin; and in the caſes when the plugs were projected by Freezing water, there ſuddenly iſſued out from the fuze-hole, a bolt of ice, of the ſame diameter, and ſtood over it to the height ſometimes of 8 inches and a half. And hence we need not be ſurpriſed at the effects of ice in deſtroying the ſubſtance of vegetables and trees, and even ſplitting rocks, when the froſt is carried to exceſs.

It is alſo obſerved that water loſes of its weight by Freezing, being found lighter after thawing again, than before it was frozen. And indeed it evaporates almoſt as faſt when frozen, as when it is fluid.

It is ſaid too that water does not freeze in vacuo; requiring for that purpoſe the preſence and contiguity of the air. But this circumſtance is liable to ſome doubt, and it may be ſuſpected that the degree of cold has not been carried far enough in theſe inſtances; as it is found that mercury in thermometers has even been frozen, though it requires a vaſtly greater degree of cold to freeze mercury, than water.

That water which has been boiled freezes more readily than that which has not been boiled; and that a ſlight diſturbance of the fluid diſpoſes it to freeze more ſpeedily; having ſometimes been cooled ſeveral degrees below the Freezing point, without congealing when kept quite ſtill, but ſuddenly freezing into ice on the leaſt motion or diſturbance.

That the water, being covered over with a ſurface of oil of olives, does not freeze ſo readily as without it; and that nut oil abſolutely preſerves it under a ſtrong froſt, when olive oil would not.

That rectified ſpirit of wine, nut oil, and oil of turpentine, ſeldom freeze.

That the ſurface of the water, in Freezing, appears all wrinkled; the wrinkles being ſometimes in parallel lines, and ſometimes like rays, proceeding from a centre to the circumference.

FREEZING *Mixture*, a preparation for the artificial congelation of water, and other fluids.

According to Mr. Boyle, all kinds of ſalts, whether alkaline or acid; and even all ſpirits, as ſpirit of wine, &c; as alſo ſugar, and ſaccharum ſaturni, mixed with ſnow, are capable of Freezing moſt fluids; and the ſame effect is produced, in a very high degree, by a mixture of oil of vitriol, or ſpirit of nitre, with ſnow.

M. Homberg remarks the ſame of equal quantities of corroſive ſublimate, and ſal ammoniac, with four times the quantity of diſtilled vinegar.

Boerhaave gives a method of producing artificial froſt without either ſnow or ice: we muſt have for this purpoſe, at any ſeaſon of the year, the coldeſt water that can be procured; this is to be mixed with a proper quantity of any ſalt (ſal ammoniac will anſwer the intention beſt), at the rate of about 3 ounces to a quart of water. Another quart of water muſt be prepared in the ſame manner with the firſt; the ſalt, by being diſſolved in each, will make the water much colder than it was before. The two quarts are then to be mixed together, and this will make them colder ſtill. Two quarts more of water prepared and mixed in the ſame manner are to be mixed with theſe, which will increaſe the cold to a much higher degree in all. The whole of this operation is to be carried on in a cold cellar; and a glaſs of common water is then to be placed in the veſſel of the fluid thus artificially cooled, and it will be turned into ice in the ſpace of 12 hours.

There is alſo a method of making artificial ice by means of ſnow, without any kind of ſalt. For this purpoſe fill a ſmall pewter diſh with water, and upon that ſet a common pewter plate filled, but not heaped, with ſnow. Bring this ſimple apparatus near the fire, and ſtir the ſnow in the plate: the ſnow will diſſolve, and the ice will be formed on the back of the plate, which was ſet in the diſh of water.

Mr. Reaumur tried the effect of ſeveral ſalts, and examined the various degrees of cold by an ice thermometer, which being placed in the fluid to be frozen, ſhewed very exactly the degree of cold by the deſcent of the ſpirit.

Nitre, or ſaltpetre, uſually paſſes for a ſalt that may be very ſerviceable in theſe artificial congelations; but the experiments of this gentleman prove that this opinion is erroneous. The moſt perfectly refined ſaltpetre employed in the operation ſunk the ſpirit in the thermometer only 3 degrees and a half below the fixed point. Leſs refined nitre ſunk the thermometer lower, and gave a greater degree of cold; owing to the common or ſea-ſalt that it contains when leſs pure, which has a greater effect than the pure ſaltpetre itſelf.

Two parts of common ſalt being mixed with three parts of powdered ice in very hot weather, the ſpirit in the thermometer immediately deſcended 15 degrees, which is half a degree lower than it would have deſcended in the ſevereſt cold of our winters. Mr. Reaumur then tried the ſalts all round, determining with great regularity and exactneſs, what was the degree of cold occaſioned by each in a given doſe. Among the neutral ſalts, none produced a greater degree of cold than the common ſea ſalt. Among the alkalies, ſal ammoniac ſunk the thermometer only to 13 degrees. Pot-aſhes ſunk it juſt as low as well refined ſaltpetre.

For the common uſes of the table, the ice is not required to be very hard, or ſuch as is produced by long continuance of violent cold: it is rather deſired to be like ſnow. Saltpetre, which is no very powerful freezer, is therefore more fit for the purpoſe than a more potent ſalt. It is not neceſſary that the congelation ſhould be very ſuddenly made; but that it may retain its form as long as may be, when made, is of great importance.

If it be deſired to have ices very hard and firm, and very ſuddenly prepared, then ſea ſalt is of all others moſt to be choſen for the operation. The ices thus made will be very hard, but they will ſoon run. Pot-aſhes afford an ice of about the hardneſs that is uſually required. This forms indeed very ſlowly, but then it will preſerve a long time. And common wood aſhes will perform the buſineſs very nearly in the ſame manner as the pot-aſhes; but for this purpoſe, the wood which is burnt, ought to be freſh.

The ſtrong acid ſpirits of the neutral ſalts act much more powerfully in theſe congelations than the ſalts themſelves, or indeed than any ſimple ſalt can do. Thus, ſpirit of nitre mixed with twice its quantity of powdered ice, immediately ſinks the ſpirit in the thermometer to 19 degrees, or 4 degrees more than that obtained by means of ſea ſalt, the moſt powerful of all the ſalts in making artificial cold. A much greater degree of cold may be given to this mixture, by piling it round with more ice mixed with ſea ſalt. This gives a redoubled cold, and ſinks the thermometer to 24 degrees. If this whole matter be covered with a freſh mixture of ſpirit of nitre and ice, a ſtill greater degree of cold is produced, and ſo on; the cold being by this method of freſh additions to be increaſed almoſt without bounds: but it is to be obſerved, that every addition gives a ſmaller increaſe than the former.

It is very remarkable in the acid ſpirits, that though ſea ſalt is ſo much more powerful than nitre in ſubſtance in producing cold, yet the ſpirit of nitre is much ſtronger than that of ſea ſalt; and another not leſs wonderful phenomenon is, that ſpirit of wine, which is little elſe than liquid fire, has as powerful an effect in congelations, or very nearly ſo, as the ſpirit of nitre itſelf.

The ſeveral liquid ſubſtances which produce cold, in the ſame manner as the dry ſalts on being mixed with ice, are much more ſpeedy in their action than the ſalts: becauſe they immediately and much more intimately come into contact with the particles of the ice, than the ſalts can. Of this nature are ſpirit of nitre, ſpirit of wine, &c. To produce the expected degree of cold, it is always neceſſary that the ice and the added matter, whatever it be, ſhould both run together, and, intimately uniting, form one clear fluid. It is hence that no new cold is produced with oil, which, though it melts the ice, yet cannot mix itſelf into a homogeneous liquid with it, but muſt always remain floating on the ſurface of the water that is produced by the melting of the ice. Mem. Acad. Scienc. Par. 1734.

It has been diſcovered, that fluids ſtanding in a current of air, grow by this means much colder than before. Fahrenheit had long ſince obſerved, that a pond, which ſtands quite calm, often acquires a degree of cold much beyond what is ſufficient for Freezing, and yet no congelation enſued: but if a ſlight breath of air happens in ſuch a caſe to bruſh over the ſurface of the water, it ſtiffens the whole in an inſtant. It has alſo been diſcovered, that all ſubſtances grow colder by the evaporation of the fluids which they contain, or with which they are mixed. If both theſe methods therefore be practiſed upon the ſame body at the ſame time, they will increaſe the cold to almoſt any degree of intenſeneſs we pleaſe.

But the moſt extraordinary inſtances of artificial Freezing, have ſince been made in Ruſſia, at Hudſon's bay, and other parts, by which quickſilver was frozen into a ſolid maſs of metal. And the ſame thing had before happened from the natural cold of the atmoſphere alone, in Siberia. In the winter of 1733, Profeſſor Gmelin, with two other gentlemen of the Ruſſian Academy, were ſent by Anne Ivanouna, the new empreſs, to explore and deſcribe the different parts of her Aſiatic dominions, with the communication of Aſia and America. In the winter of 1734-5, Mr. Gmelin being at Yeneſeiſk in 58° 30′ north lat. and 92° long. eaſt from Greenwich, firſt obſerved ſuch a deſcent of the mercury, as muſt have been attended with congelation, being far below its Freezing point, now fixed at −40 of Fahrenheit's thermometer. "Here, ſays he, we firſt experienced the truth of what various travellers have related with reſpect to the extreme cold of Siberia; for, about the middle of December, ſuch ſevere weather ſet in, as we were ſure had never been known in our time at Peterſburg. The air ſeemed as if it were frozen, with the appearance of a fog, which did not ſuffer the ſmoke to aſcend as it iſſued from the chimneys. Birds fell down out of the air as dead, and froze immediately, unleſs they were brought into a warm room. Whenever the door was opened, a fog ſuddenly formed round it. During the day, ſhort as it was, parhelia and haloes round the ſun were frequently ſeen; and in the night mock moons, and haloes about the moon. Finally, our thermometer, not ſubject to the ſame deception as the ſenſes, left us no doubt of the exceſſive cold; for the quickſilver in it was reduced, on the 5th of January, old ſtyle, to −120° of Fahrenheit's ſcale, lower than it had ever been obſerved in nature."

The next inſtance of congelation happened at Yakutſk, in 62° north lat. and 150° eaſt longitude. The weather here was unuſually mild for the climate, yet the thermometer fell to −72°; and one perſon informed the profeſſor by a note, that the mercury in his barometer was frozen. He haſtened immediately to his houſe to behold ſuch a ſurpriſing phenomenon; but though he was witneſs to the fact, obſerving that the mercury did not continue in one column, but was divided in different places as into little cylinders, which appeared frozen, yet the prejudice he had entertained againſt the poſſibility of the congelation, would not allow him to believe it.

Another ſet of obſervations, in the courſe of which the mercury muſt frequently have been congealed, were made by profeſſor Gmelin at Kirenga fort, in 57½ north lat., and 108 eaſt long.; his thermometer, at different times, ſtanding at −108, −86, −100, −113, and many other intermediate degrees; in the courſe of the winter of 1737-8. On the 27th of November, after the thermometer had been ſtanding for two days at −46°, he found it ſunk at noon to −108. Suſpecting ſome miſtake, after he had noted down the obſervation, he inſtantly ran back, and found it at −102; but aſcending with ſuch rapidity, that in the ſpace of half an hour it had riſen to −19°. This phenomenon, which appeared ſo ſurpriſing, doubtleſs depended on the expanſion of the mercury frozen in the bulb of the thermometer, and which now melting, forced upwards the ſmall thread in the ſtem. And ſimilar appearances were obſerved on other days afterwards, when the thread of quickſilver in the thermometer was ſeparated about 6 degrees.

A ſecond inſtance where a natural congelation of mercury has certainly been obſerved, is recorded in the tranſactions of the Royal Academy of Sciences at Stockholm, as made by Mr. Andrew Hellant. The weather, in January 1760, was remarkably cold in Lapland; ſo that on the 5th of that month, the thermometers fell to −76, −128, or lower; on the 23d and following days they fell to −58, −79, −92, and below −238 entirely into the ball. This was obſerved at four different places in Lapland, ſituated between the 65th and 78th degrees of north lat. and the 21ſt and 28th degree of eaſt longitude.

But the congelation of quickſilver, by an artificial Freezing mixture, was firſt obſerved, and put beyond doubt, by Mr. Joſeph Adam Braun, profeſſor of philoſophy at Peterſburg. This gentleman wiſhing to try how many degrees of cold he could produce, availed himſelf of a good opportunity which offered for that purpoſe on the 14th of December 1759, when the mercury in the thermometer ſtood in the natural cold at −34, which it is now known is only 5 or 6 degrees above its point of congelation. Aſſiſting this natural cold therefore with a mixture prepared of aquafortis and pounded ice, his thermometer was ſunk to −69. Part of the quickſilver muſt now have been really congealed, but unexpected by him, and he only thought of purſuing his object of producing ſtill greater degrees of cold; and having expended all his pounded ice, he was obliged to uſe ſnow inſtead of it. With this freſh mixture the mercury ſunk to −100, −240, and −350°. Taking the thermometer out, he found it whole, but the quickſilver fixed, and it continued ſo for 12 minutes. On repeating the experiment, with another thermometer which had been graduated no lower than −220, all the mercury ſunk into the ball, and became ſolid as before, and did not re-aſcend till after a ſtill longer interval of time. Mr. Braun now ſuſpected that the quickſilver was really frozen, and prepared for making a deciſive experiment. This was accompliſhed on the 25th of the ſame month, and the bulb of the thermometer broken as ſoon as the metal was congealed; when it appeared that the mercury was changed into a ſolid and ſhining metallic maſs, which flatted and extended under the ſtrokes of a peſtle, being rather leſs hard than lead, and yielding a dull ſound like that metal. Mr. Æpinus made ſimilar experiments at the ſame time, employing as well thermometers as tubes of a larger bore; in which laſt he remarked, that the quickſilver fell ſenſibly on being frozen, aſſuming a concave ſurface, and likewiſe that the congealed pieces ſunk in fluid mercury: alſo, in their farther experiments, they invariably found that the mercury ſunk lower when the whole of it was congealed, than if any part of it remained fluid: all ſhewing that, contrary to water, mercury contracted in Freezing. It was farther obſerved, that the mercury when congealed looked like the moſt poliſhed ſilver, and when beaten flat, it was eaſily cut with a penknife, like ſoft thin ſheet lead.

The fact being thus eſtabliſhed, and fluidity no longer to be conſidered as an eſſential property of quickſilver, Mr. Braun communicated an account of his experiments to the Peterſburg academy, on the 6th of September 1760; of which a large extract was inſerted in the Philoſ. Tranſ. vol. 52, pa. 156. He afterwards declared that he never ſuffered a winter to paſs without repeating the experiment of Freezing quickſilver, and never failed of ſucceſs when the natural cold was of a ſufficient ſtrength for the purpoſe; and this degree of natural cold he ſuppoſes at −10 of Fahrenheit; though ſome commencement of the congelation might be perceived when the temperature of the air was as high as + 2.

The reſults of all his experiments were, that with the abovementioned frigorific mixtures, and once with rectified ſpirits and ſnow, when the natural cold was at −28°, he congealed the quickſilver, and diſcovered that it is a real metal that melts with a very ſmall degree of heat. However, not perceiving the neceſſary conſequence of its great contraction in Freezing, he always confounded its point of congelation with that of its greateſt contraction in Freezing, and thus marked the former a great deal too low.

In the proceſs of his obſervations, Mr. Braun found that double aquafortis was more effectual than ſpirit of nitre; but with this ſimple ſpirit, which ſeldom brings the mercury lower than −148, this metal may be frozen in the following manner: Six glaſſes being filled with ſnow as uſual, and the thermometer put in one of them, the ſpirit of nitre was poured upon it; when the mercury would fall no lower in this, the thermometer was removed to the ſecond, and ſo on to the third and fourth, in which fourth immerſion the mercury was congealed.

Mr. Æpinus gives the following direction for uſing the fuming ſpirit of nitre: Take ſome of this ſpirit, cooled as much as poſſible, and put it into a wine glaſs till it be about half full, filling it up with ſnow, and ſtirring them till the mixture become of the conſiſtence of pap; by which means you obtain, almoſt in an inſtant, the neceſſary degree of cold for the Freezing of quickſilver.

It is remarked by Mr. Braun, that by the mixture of ſnow and ſpirit of nitre, which froze the mercury, he never was able to bring thermometers, filled with the moſt highly rectified ſpirit of wine, lower than −148: ſo that the cold which will freeze mercury, will not freeze ſpirit of wine; and therefore ſpirit thermometers are the moſt fit to determine the degree of coldneſs in frigorific mixtures, till we can conſtruct ſolid metallic thermometers with ſufficient accuracy. Mr. Braun tried the effects of different Fluids in his frigorific mixtures: he always found that Glauber's ſpirit of nitre and double aquafortis were the moſt powerful; and from a number of experiments made when the temperature of the air was between 21 and 28 of Fahrenheit, he concludes, that ſpirit of ſalt pounded upon ſnow increaſed the natural cold 36°; ſpirit of ſal ammoniac, 12; oil of vitriol, 42; Glauber's ſpirit of nitre, 70; aquafortis, 48; ſimple ſpirit of nitre, 36; dulcified ſpirit of vitriol, 24; Hoffman's anodyne liquor, 38; ſpirit of hartſhorn, 12; ſpirit of ſulphur, 12; ſpirit of wine rectified, 24; camphorated ſpirit, 18; French brandy, 14; and ſeveral kinds of wine increaſed the natural cold to 7, 8, or 9 degrees.

The moſt remarkable congelation of mercury, by natural cold, that has ever been obſerved, was that related by Dr. Peter Simon Pallas, who had been ſent by the empreſs of Ruſſia, with ſome other gentlemen, on an expedition ſimilar to that of Mr. Gmelin. Being at Kraſnoyarſk in the year 1772, in north lat. 56° 30′, and eaſt long. 93°, he had an opportunity of obſerving the phenomenon we ſpeak of. On the 6th and 7th of December that year, ſays he, there happened the greated cold I have ever experienced in Siberia: the air was calm at the time, and ſeemingly thickened; ſo that, though the ſky was in other reſpects clear, the ſun appeared as through a fog. I had only one ſmall thermometer left, in which the ſcale went no lower than −7°; and on the 6th in the morning, I remarked that the quickſilver in it ſunk into the ball, except ſome ſhort columns which ſtuck faſt in the tube. When the ball of the thermometer, as it hung in the open air, was touched with the finger, the quickſilver roſe; and it could plainly be ſeen that the ſolid columns ſtuck and reſiſted a good while, and were at length puſhed upward with a ſort of violence. He alſo placed upon the gallery, on the north ſide of his houſe, ſome quickſilver in an open bowl. Within an hour he found the edges and ſurface of it frozen ſolid; and ſome minutes afterward the whole was condenſed by the natural cold into a ſoft maſs very much like tin. While the inner part was ſtill fluid, the frozen ſurface exhibited a great variety of branched wrinkles; but in general it remained pretty ſmooth in Freezing. The congealed mercury was more flexible than lead; but on being bent ſhort, it was found more brittle than tin; and when hammered out thin, it ſeemed ſomewhat granulated. When the hammer was not perfectly cooled, the quickſilver melted away under it in drops; and the ſame thing happened when the metal was touched with the finger, by which alſo the finger was immediately benumbed. When the frozen maſs was broken to pieces in the cold, the fragments adhered to each other and to the bowl in which they lay. In the warm room it thawed on its ſurface gradually, by drops, like wax on the fire, and did not melt all at once. Although the froſt ſeemed to abate a little towards night, yet the congealed quickſilver remained unaltered, and the experiment with the thermometer could ſtill be repeated. On the 7th of December he had an opportunity of making the ſame obſervations all day; but ſome hours after ſunſet, a northweſt wind ſprung up, which raiſed the thermometer to −46°, when the maſs of quickſilver began to melt.

The experiments of Mr. Braun were ſucceſsfully repeated at Gottingen, in 1774, by Mr. John Frederick Blumenbach; being encouraged to this attempt by the exceſſive cold of the winter that year, eſpecially the night of January the 11th, when he made the experiment, the thermometer ſtanding at −10 in the open air. Mr. Blumenbach at 5 in the evening, put 3 drachms of quickſilver into a ſmall ſugar glaſs, and covered it with a mixture of ſnow and Egyptian ſal ammoniac, ſetting the glaſs out in the air upon a mixture alſo of ſal ammoniac. At one the next morning, the mercury was found frozen quite ſolid, and hard to the glaſs; and did not melt again till 7 or 8 the next morning. The colour of the frozen mercury was a dull pale white with a blueiſh caſt, like zinc, very different from the natural appearance of quickſilver.

In the year 1775, by ſimilar means, quickſilver was twice frozen by Mr. Hutchins, governor of Albany fort, in Hudſon's bay, viz, in the months of January and February of that year. And the ſame was done on the 28th of January 1776, by Dr. Lambert Bicker, ſecretary of Rotterdam. The temperature of the atmoſphere was then at + 2°; and the loweſt it could reduce the thermometer by artificial cold, was −94; when, on breaking the glaſs, the mercury was found frozen.

In the beginning of the year 1780 M. Von Elterlein of Vytegra, a town of Ruſſia, in lat. 61° north, and long. 36° eaſt, froze quickſilver by natural cold. On the 4th of January 1780, the cold being increaſed to −34 that evening at Vytegra, he expoſed to the open air 3 ounces of very pure quickſilver in a china teacup, covered with paper pierced full of holes. Next day, at 8 in the morning, he found it ſolid, and looking like a piece of caſt lead, with a conſiderable depreſſion in the middle. On attempting to looſen it in the cup, his knife raiſed ſhavings from it as if it had been lead, which remained ſticking up; and at length the metal ſeparated from the bottom of the cup in one maſs. He then took it in his hand to try if it would bend: it was ſtiff like glue, and broke into two pieces; but his fingers immediately loſt all feeling, and could ſcarcely be reſtored in an hour and a half by rubbing with ſnow. At 8 o'clock the thermometer ſtood at −57; but half after 9 it was riſen to −40; and then the two pieces of mercury which lay in the cup had loſt ſo much of their hardneſs, that they could no longer be broken, or cut into ſhavings, but reſembled a thick amalgam, which, though it became fluid when preſſed by the fingers, immediately afterwards reſumed the conſiſtence of pap. With the thermometer at −39, the quickſilver became fluid. The cold was never leſs on the 5th than −28, and by 9 in the evening it had increaſed again to −33. This experiment ſeems to fix the Freezing point of mercury at −40 of Fahrenheit's thermometer, or 40 below 0; which is 72° below the Freezing point of water.

In the winter of 1781 and 82, Mr. Hutchins reſumed the ſubject of Freezing quickſilver by artificial cold, with ſuch ſucceſs, that from his experiments and thoſe of M. Von Elterlein, laſt mentioned, the Freezing point of mercury is now almoſt as well ſettled, viz at −40, as that of water is at + 32. Other philoſophers indeed had not been altogether inattentive to this ſubject. Profeſſor Braun himſelf had taken great pains to inveſtigate it; but for want of a proper attention to the difference between the contraction of the fluid mercury by cold, and that of the congealing metal by Freezing, he could not determine any thing certain concerning it.

An inſtance of the natural congelation of quickſilver alſo occurred in Jemptland, one of the provinces of Sweden, on the 1ſt of January 1782; and laſtly, on the 26th of the ſame month, Mr. Hutchins obſerved the ſame effect of the cold at Hudſon's bay; when he found that at the point of its Freezing a mercurial thermometer ſtood at −40, and a ſpirit thermometer at −30.

On this ſubject, ſee the Philoſ. Tranſ. vol. 51, pa. 672; vol. 52, pa. 156; vol. 66, pa. 174; vol. 73, pa. 303 and 325; vol. 76, pa. 241; vol. 77, pa. 285; vol. 78, pa. 43; and ſeveral others, particularly vol. 79, pa. 199, &c, being experiments on the congelation of quickſilver in England, by Mr. Richard Walker, where he proves that mercury may be frozen not only in England in ſummer, but even in the hotteſt climate, at any ſeaſon of the year, and without the uſe of ice or ſnow.

Freezing *Point*, denotes the point or degree of cold, ſhewn by a mercurial thermometer, at which certain fluids begin to freeze, or, when frozen, at which they begin to thaw again. On Fahrenheit's thermometer, this point is at + 32 for water, and at −40 for quickſilver, theſe fluids freezing at thoſe two points reſpectively. It would alſo be well if the Freezing points for other fluids were aſcertained, and the whole arranged in a table.

Freezing *Rain*, or *Raining Ice*, a very uncommon kind of ſhower which ſometimes falls, particularly one in December 1672, in the weſt of England: of which ſome accounts are given in the Philoſ. Tranſ. number 90.

This rain, as ſoon as it touched any thing above ground, as a bough, or the like, immediately ſettled into ice; and by enlarging and multiplying the icicles, it broke all down with its weight. The rain that fell on the ſnow, immediately froze into ice, without ſinking in the ſnow at all.

It made an amazing deſtruction of trees, beyond any thing in all hiſtory. "Had it concluded with ſome guſt of wind, ſays a gentleman on the ſpot, it might have been of terrible conſequence. Having weighed the ſprig of an aſh tree, the wood of which was juſt three quarters of a pound, the ice upon it amounted to 16 pounds. Some were frighted with the noiſe in the air; till they diſcerned it was the clatter of icy boughs daſhed againſt each other." Dr. Beale obſerves, that there was no conſiderable froſt perceived on the ground during the whole; from which he concludes, that a froſt may be very fierce and dangerous on the tops of ſome hills, while in other places it keeps at ſome feet above the ground; and may wander about very furious in ſome places, and be remiſs in others not far off. This rain was followed by glowing heats, and a wonderful forwardneſs of vegetation.

FRENICLE (Bernard), a celebrated French mathematician of the 17th century. He was the contemporary and companion of Des Cartes, Fermat, and the other learned mathematicians of their time. He was admitted Geometrician of the French Academy in 1666; and died in 1675.

He had many papers inſerted in the Ancient Memoirs of the Academy, of 1666, particularly in vol. 5, of that collection, viz, 1. A method of reſolving problems by Excluſions.—2. Treatiſe of right-angled Triangles in Numbers.—3. Short tract on Combinations.—4. Tables of Magic Squares.—5. General method of making Tables of Magic Squares.

FRESCO, is a ſort of painting which is made upon the plaſtering of walls before it is dry.

FRIABILITY, the property of a body that is Friable.

FRIABLE, a quality of bodies by which they are rendered tender and brittle, eaſily crumbled or reduced to powder between the fingers; their force of coheſion being ſuch as eaſily expoſes them to ſuch ſolution. Such are pumice, and all calcined ſtones, burnt allum, &c.

It is ſuppoſed that Friability ariſes from hence, that the body conſiſts wholly of dry parts irregularly combined, and which are readily ſeparated, as having nothing unctuous or glutinous to bind them together.

FRICTION, the act of rubbing or grating the ſurfaces of bodies againſt or over each other, called alſo Attrition.

The phenomena ariſing from the Friction of divers bodies, under different circumſtances, are very numerous and conſiderable. Mr. Hawkſbee gives a number of experiments of this kind; particularly of the attrition or Friction of glaſs, under various circumſtances; the reſult of which was, that it yielded light, and became electrical. Indeed all bodies by Friction are brought to conceive heat; many of them to emit light; particularly a cat's back, ſugar, beaten ſulphur, mercury, ſea water, gold, copper, &c, but above all diamonds, which when briſkly rubbed againſt glaſs, gold, or the like, yield a light equal to that of a live coal when blowed by the bellows.

Friction, in Mechanics, denotes the reſiſtance a moving body meets with from the ſurface on which it moves.

Friction ariſes from the roughneſs or aſperity of the ſurface of the body moved on, and that of the body moving: for ſuch ſurfaces conſiſting alternately of eminences and cavities, either the eminences of the one muſt be raiſed over thoſe of the other, or they muſt be both broken and worn off: but neither can happen without motion, nor can motion be produced without a force impreſſed. Hence the force applied to move the body is either wholly or partly ſpent on this effect; and conſequently there ariſes a reſiſtance, or Friction, which will be greater as the eminences are greater, and the ſubſtance the harder; and as the body, by continual Friction, becomes more and more poliſhed, the Friction diminiſhes.

As the Friction is leſs in a body that rolls, than when it ſlides, hence in machines, leſt the Friction ſhould employ a great part of the power, care is to be taken that no part of the machine ſlide along another, if it can be avoided; but rather that they roll, or turn upon each other. With this view it may be proper to lay the axes of cylinders, not in a groove or concave matrix, as uſual, but between little wheels, called Friction wheels, moveable on their reſpective axes: for by this contrivance, the Friction is transferred from the circumference of thoſe wheels to their pivots. And in like manner the Friction may be ſtill farther diminiſhed, by making the axis of thoſe wheels reſt upon other Friction wheels that turn round with them. This was long ſince recommended by P. Caſabus; and experience confirms the truth of it. Hence alſo it is, that a pulley moveable on its axis reſiſts leſs than if it were fixed, and the cord ſliding over the circumference. And the ſame may be obſerved of the wheels of coaches, and other carriages. Indeed about 20 years ago Friction balls or rollers were placed within the naves of carriage wheels by ſome perſons, particularly a Mr. Varlo; and lately Mr. Garnett had a patent for an improved manner of applying Friction wheels to any axis, as of carriages, blocks or pulleys, ſcale beams, &c, in which the incloſed wheels or rollers are kept always at the ſame diſtance by connecting rods or bars.

From theſe principles, with the aſſiſtance of the higher geometry, Olaus Roemer determined the figure of the teeth of wheels, that ſhould make the leaſt reſiſtance poſſible, which he found ſhould be epicycloids. And the ſame was afterwards demonſtrated by De la Hire, and Camus.

M. Amontons, by experiment, attempted to ſettle a foundation for the preciſe calculation of the quantity of Friction; which M. Parent endeavoured to confirm from reaſoning and geometry. M. Amontons' principle is, that the Friction of two bodies depends only on the weight or force with which they preſs each other, being always more or leſs in proportion to that preſſure; eſteeming it a vulgar error, that the quantity of Friction has any dependence on the extent of the ſurface that is rubbed, or that the Friction increaſes with the ſurface: arguing that it will require the ſame weight to draw along a plane, a piece of wood on its narrow edge, as on its broad and flat ſide; becauſe, though on the broad ſide there be 4 times the number of touching particles, yet each particle is preſſed with but $\frac{1}{4}$ of the weight bearing on thoſe of the narrow ſide; and ſince 4 times the number multiplied by $\frac{1}{4}$ of the weight is equal to $\frac{1}{4}$ of the number multiplied by 4 times the weight, it is plain that the effect, that is, the reſiſtance, is equal in both caſes, and therefore requires the ſame force to overcome it.

On the firſt propoſal of this paradox, M. De la Hire very properly had recourſe to experiments, as the beſt teſt, had they been judiciouſly performed: ſuch as they were however, they ſucceeded in favour of this ſyſtem. He laid ſeveral pieces of rough wood on a rough table; their ſizes were unequal; but he laid weights on them, ſo as to render them all equally heavy: and he found that the ſame preciſe force, or weight, applied to them by a little pulley, was required to put each in motion, notwithſtanding all the inequality of the ſurfaces. The experiment ſucceeded in the ſame manner with pieces of marble, laid on a marble table. After this, by reaſoning, M. de la Hire gave a phyſical ſolution of the effect. And M. Amontons ſettled a calculus of the value of Friction, with the loſs ſuſtained by it in machines, on the foundation of this new principle. In wood, iron, lead, and braſs, which are the chief materials uſed in machines, he makes the reſiſtance cauſed by Friction to be nearly the ſame in all, when thoſe materials are anointed with oil or fat: and the quantity of this reſiſtance, independent of the magnitude of the ſurface, he makes nearly equal to a third part of the weight of the body moved, or of the force with which the two bodies are preſſed together. Others have obſerved, that if the ſurfaces be hard and well poliſhed, the Friction will be leſs than a third part of the weight; but if the parts be ſoft or rugged, it will be much greater. It was farther obſerved, that in a cylinder moved on two ſmall gudgeons, or on a ſmall axis, the Friction would be diminiſhed in the ſame proportion as the diameter of theſe gudgeons is leſs than the diameter of the cylinder; becauſe in this caſe, the parts on which the cylinder moves and rubs, will have leſs velocity than the power which moves it in the ſame proportion, which is in effect making the Friction to be proportional to the velocity. So that, from the whole of their obſervations, this general propoſition is deduced, viz, That the reſiſtances ariſing from Friction, are to one another in a ratio compounded of the preſſures of the rubbing parts, and the velocities of their motions. Principles which, it is now known from better experiments, are both erroneous; notwithſtanding the hypotheſis of M. Amontons has been adopted, and attempted to be confirmed by Camus, Deſaguliers, and others.

M. Muſchenbroek and the abbé Nollet, however, on the other hand, have concluded from experiments, that the Friction of bodies depends on the magnitude of their ſurface, as well as on their weight. Though the former ſays, that in ſmall velocities the Friction varies very nearly as the velocity, but that in great velocities the proportion increaſes faſter: he has alſo attempted to prove, that by increaſing the weight of a body, the Friction does not always increaſe exactly in the ſame ratio. Introd. ad Phil. Nat. vol. 1, c. 9, and Lect. Phyſ. Exp. tom. 1, p. 241. Helſham and Ferguſon, from the ſame kind of experiments, have endeavoured to prove, that the Friction does not vary by changing the quantity of ſurface on which the body moves; and the latter of theſe aſſerts, that the Friction increaſes very nearly as the velocity; and that by increaſing the weight, the Friction is increaſed in the ſame ratio. Indeed there is ſcarce any ſubject of experiment, with regard to which, different perſons have formed ſuch various concluſions. Of thoſe who have written on the theory, no one has eſtabliſhed it altogether on true principles, till the experiments lately made by Mr. Vince of Cambridge: Euler, whoſe theory is extremely elegant, and would have been quite ſatisfactory had his principles been founded on good experiments, ſuppoſes the Friction to vary in proportion to the velocity of the body, and its preſſure upon the plane; neither of which is true: and others, though they have juſtly imagined that Friction is a uniformly retarding force, have yet retained the other ſuppoſition, and ſo rendered their ſolutions not at all applicable to the caſes for which they were intended.

For theſe reaſons a new and ingenious ſet of experiments was ſucceſsfully inſtituted by the rev. Samuel Vince, A. M. of Cambridge, which are publiſhed in the 75th vol. of the Philoſ. Tranſ. p. 165. The object of theſe experiments was to determine,

1ſt, Whether Friction be a uniformly retarding force.

2d, The quantity of Friction.

3d, Whether Friction varies in proportion to the preſſure or weight.

4th, Whether the Friction be the ſame on whichever of its ſurfaces a body moves.

Mr. Vince ſays, "the experiments were made with the utmoſt care and attention, and the ſeveral reſults agreed ſo very exactly with each other, that I do not ſcruple to pronounce them to be concluſive."—"A plane was adjuſted parallel to the horizon, at the extremity of which was placed a pulley, which could be elevated or depreſſed in order to render the ſtring which connected the body and the moving force parallel to the plane or horizon. A ſcale accurately divided was placed by the ſide of the pulley perpendicular to the horizon, by the ſide of which the moving force deſcended; upon the ſcale was placed a moveable ſtage, which could be adjuſted to the ſpace through which the moving force deſcended in any given time, which time was meaſured by a well regulated pendulum clock vibrating ſeconds. Every thing being thus prepared, the following experiments were made to aſcertain the law of Friction. But let me firſt obſerve, that if Friction be a uniform force, the difference between it and the given force of the moving power muſt be alſo uniform, and therefore the moving body muſt deſcend with a uniformly accelerated velocity, and conſequently the ſpaces deſcribed from the beginning of the motion muſt be as the ſquares of the times, juſt as when there was no Friction, only they will be diminiſhed on account of the Friction." Accordingly the experiments are then related, which are performed agreeably to theſe ingenious and philoſophical ideas, and from them are deduced theſe general concluſions, which may be conſidered as eſtabliſhed and certain facts or maxims, viz,

1ſt, That Friction is a uniformly retarding force in hard bodies, not ſubject to alteration by the velocity; except when the body is covered with cloth, woollen, &c, and in this caſe the Friction increaſes a little with the velocity.

2dly, Friction increaſes in a leſs ratio than the quantity of matter, or weight of the body. This increaſe however is different for the different bodies, more or leſs; nor is it yet ſufficiently known, for any one body, what proportion the increaſe of Friction bears to the increaſe of weight.

3dly, The ſmalleſt ſurface has the leaſt Friction ; the weight being the ſame. But the ratio of the Friction to the ſurface is not yet accurately known.

Mr. Vince's experiments conſiſted in determining how far the ſliding bodies would be drawn, in given times, by a weight hanging freely over a pulley. This method would both ſhew him if the Friction were a conſtant retarding force, and the other concluſions above ſtated. For as the ſpaces deſcribed by any conſtant force, in given times, are as the ſquares of the times ; and as the weight drawing the body is a conſtant force, if the Friction, which acts in oppoſition to the weight, ſhould alſo be a conſtant force, then their difference, or the force by which the body is urged, will alſo be conſtant, in which caſe the ſpaces deſcribed ought to be as the ſquares of the times; which happened accordingly in the experiments.

Mr. Vince adds ſome remarks on the nature of the experiments which have been made by others. Theſe, he obſerves, the authors " have inſtituted, To find what moving force would *juſt* put a body at reſt in motion : and they concluded from thence, that the accelerative force was then equal to the Friction ; but it is manifeſt, that any force which will put a body in motion muſt be greater than the force which oppoſes its motion, otherwiſe it could not overcome it; and hence, if there were no other objection than this, it is evident, that the Friction could not be very accurately obtained; but there is another objection, which totally deſtroys the experiment, ſo far as it tends to ſhew the quantity of Friction, which is the ſtrong coheſion of the body to the plane when it lies at reſt." This he confirms by ſeveral experiments, and then adds, " From theſe experiments therefore it appears, how very conſiderable the coheſion was in proportion to the Friction when the body was in motion ; it being, in one caſe almoſt $\frac{1}{3}$, and in another it was found to be very nearly equal to the whole Friction. All the concluſions therefore deduced from the experiments, which have been inſtituted to determine the Friction from the force neceſſary to *put* a body in motion (and I have never ſeen any deſcribed but upon ſuch a principle) have manifeſtly been totally falſe ; as ſuch experiments only ſhew the reſiſtance which ariſes from the coheſion and Friction conjointly." Philoſ. Tranſ. vol. 75, pa. 165.

Mr. Emerſon, in his Principles of Mechanics, deduces from experiments the following remarks relating to the quantity of Friction : When a cubic piece of ſoft wood of 8 pounds weight, moves upon a ſmooth plane of ſoft wood, at the rate of 3 feet per ſecond, its Friction is about $\frac{1}{3}$ of the weight ; but if it be rough, the Friction is little leſs than half the weight : on the ſame ſuppoſition, when both the pieces of wood are very ſmooth, the Friction is about $\frac{1}{4}$ of the weight : the Friction of ſoft wood on hard, or of hard wood upon ſoft, is $\frac{1}{5}$ or $\frac{1}{6}$ of the weight ; of hard wood upon hard wood, $\frac{1}{7}$ or $\frac{1}{8}$; of poliſhed ſteel moving on ſteel or pewter, $\frac{1}{4}$; moving on copper or lead, $\frac{1}{5}$ of the weight. He obſerves in general, that metals of the ſame ſort have more Friction than thoſe of different ſorts ; that lead makes much reſiſtance ; that iron or ſteel running in braſs makes the leaſt Friction of any ; and that metals oiled make the Friction leſs than when poliſhed, and twice as little as when unpoliſhed. Deſaguliers obſerves that, in M. Camus's experiments on ſmall models of ſledges in actual motion, there are more caſes in which the Friction is leſs than where it is more than $\frac{1}{3}$ of the weight. See a table, exhibiting the Friction between various ſubſtances, formed from his experiments in Deſag. Exp. Philoſ. vol. 1, p. 193 &c. alſo p. 133 to 138, and p. 182 to 254, and p. 458 to 460. On the ſubject of Friction, ſee ſeveral vols. of the Philoſ. Tranſ. as vol. 1, p. 206 ; vol. 34, p. 77 ; vol. 37, p. 394 ; vol. 53, p. 139, &c.

FRIDAY, the 6th day of the week, ſo called from Friga, or Friya, a goddeſs worſhipped by the Saxons on this day. It is a faſt-day in the church of England, in memory of our Saviour's crucifixion, unleſs Chriſtmas-day happen to fall on Friday, which is always a feſtival.

Good FRIDAY, the Friday next before Eaſter, repreſenting the day of our Saviour's crucifixion.

FRIGID *Zone*, the ſpace about either pole of the earth to which the ſun never riſes for one whole day at leaſt in their winter. Theſe two zones extend to about $23\frac{1}{2}$ degrees every way from the pole, as their centre.

FRIGORIFIC, in Phyſics, ſomething belonging to, or that occaſions cold.—Some philoſophers, as Gaſſendus, and other corpuſcularians, denying cold to be a mere privation, or abſence of heat, contend that there are actual Frigorific corpuſcles or particles, as well as fiery ones : whence proceed cold and heat. But later philoſophers allow of no other Frigorific particles beſide thoſe nitrous ſalts &c, which float in the air in cold weather, and occaſion freezing.

FRIZE, FRIEZE, or FREEZE, in Architecture, a part of the entablature of columns, between the architrave and cornice.

FRONT, in Architecture, denotes the principal face or ſide of a building ; or that preſented to their chief aſpect and view.

FRONT, in Perſpective, a projection or repreſentation of the face, or forepart of an object, or of that part directly oppoſite to the eye, called alſo more uſually *orthography*.

FRONTISPIECE, in Architecture, the portale, or principal face of a fine building.

FRONTON, in Architecture, an ornament among us more uſually called Pediment.

FROST, ſuch a ſtate of the atmoſphere as cauſes the congelation or freezing of water or other fluids into ice.

The nature and effects of Froſt in different countries, are mentioned under the articles Congelation, and Freezing. In the more northern parts of the world, even ſolid bodies are affected by Froſt, though this is only or chiefly in conſequence of the moiſture they contain, which being frozen into ice, and ſo expanding as water is known to do when frozen, it burſts and rends any thing in which it is contained, as plants, trees, ſtones, and large rocks. Some fluids expand by Froſt, as water, which expands about $\frac{1}{15}$th part, for which reaſon ice floats in water ; but others again contract, as quickſilver, and hence frozen quickſilver ſinks in the fluid metal.

Froſt, being derived from the atmoſphere, naturally proceeds from the upper parts of bodies downwards, as the water and the earth ; ſo, the longer a Froſt is continued, the thicker the ice becomes upon the water in ponds, and the deeper into the earth the ground is frozen. In about 16 or 17 days Froſt, Mr. Boyle found it had penetrated 14 inches into the ground. At Moſcow, in a hard ſeaſon, the Froſt will penetrate 2 feet deep in the ground; and Capt. James found it penetrated 10 feet deep in Charlton iſland, and the water in the ſame iſland was frozen to the depth of 6 feet. Scheffer aſſures us, that in Sweden the Froſt pierces 2 cubits, or Swediſh ells into the earth, and turns what moiſture is found there into a whitiſh ſubſtance, like ice; and ſtanding waters to 3 ells, or more. The ſame author alſo mentions ſudden cracks or rifts in the ice of the lakes of Sweden, 9 or 10 feet deep, and many leagues long; the rupture being made with a noiſe not leſs loud than if many guns were diſcharged together. By ſuch means however the fiſhes are furniſhed with air; ſo that they are rarely found dead.

The natural hiſtories of Froſts furniſh very extraordinary effects of them. The trees are often ſcorched, and burnt up, as with the moſt exceſſive heat; and ſplit or ſhattered. In the great Froſt in 1683, the trunks of oak, aſh, walnut, &c, were miſerably ſplit and cleft, ſo that they might be ſeen through, and the cracks often attended with dreadful noiſes like the exploſion of fire-arms. Philoſ. Tranſ. number 165.

The cloſe of the year 1708, and the beginning of 1709, were remarkable, throughout the greateſt part of Europe, for a ſevere Froſt. Dr. Derham ſays, it was the greateſt in degree, if not the moſt univerſal, in the memory of man; extending through moſt parts of Europe, though ſcarcely felt in Scotland or Ireland.

In very cold countries, meat may be preſerved by the Froſt 6 or 7 months, and prove tolerable good eating. See Capt. Middleton's obſervations made in Hudſon's bay, in the Philoſ. Tranſ. no. 465, ſect. 2. In that climate the Froſt ſeems never out of the ground, it having been found hard frozen in the two ſummer months. Brandy and ſpirit of wine, ſet out in the open air, freeze to ſolid ice in 3 or 4 hours. Lakes and ſtanding waters, not above 10 or 12 feet deep, are frozen to the ground in winter, and all their fiſh periſh. But in rivers, where the current of the tide is ſtrong, the ice does not reach ſo deep, and the fiſh are preſerved. Id. ib.

Some remarkable inſtances of Froſt in Europe, and chiefly in England, are recorded as below: In the year

220, Froſt in Britain that laſted 5 months.
250, The Thames frozen 9 weeks.
291, Moſt rivers in Britain frozen 6 weeks.
359, Severe Froſt in Scotland for 14 weeks.
508, The rivers in Britain frozen for 2 months.
558, The Danube quite frozen over.
695, Thames frozen 6 weeks; booths built on it.
759, Froſt from Oct. 1 till Feb. 26, 760.
827, Froſt in England for 9 weeks.
859, Carriages uſed on the Adriatic ſea.
908, Moſt rivers in England frozen 2 months.
923, The Thames frozen 13 weeks.
987, Froſt laſted 120 days: began Dec. 22.
998, The Thames frozen 5 weeks.
1035, Severe Froſt on June 24: the corn and fruits deſtroyed.
1063, The Thames frozen 14 weeks.
1076, Froſt in England from Nov. till April.
1114, Several wooden bridges carried away by ice.
1205, Froſt from Jan. 14 till March 22.
1407, Froſt that laſted 15 weeks.
1434, From Nov. 24 till Feb. 10. Thames frozen down to Graveſend.
1683, Froſt for 13 weeks.
170$\frac{8}{9}$, Severe Froſt for many weeks.
1715, The ſame for many weeks.
1739, One for 9 weeks. Began Dec. 24.
1742, Severe Froſt for many weeks.
1747, Severe Froſt in Ruſſia.
1754, Severe one in England.
1760, The ſame in Germany.
1776, The ſame in England.
1788, Thames frozen below bridge; booths on it.

Hoar **Frost**, is the dew frozen or congealed, early in cold mornings; chiefly in autumn. Though many Carteſians will have it formed of a cloud; and either congealed in the cloud, and ſo let fall; or ready to be congealed as ſoon as it arrives at the earth.

Hoar Froſt, M. Regis obſerves, conſiſts of an aſſemblage of little parcels of ice cryſtals; which are of various figures, according to the different diſpoſition of the vapours, when met and condenſed by the cold.

FRUSTUM, in Geometry, is the part of a ſolid next the baſe, left by cutting off the top, or ſegment, by a plane parallel to the baſe: as the Fruſtum of a pyramid, of a cone, of a conoid, of a ſpheroid, or of a ſphere, which is any part compriſed between two parallel circular ſections; and the Middle Fruſtum of a ſphere, is that whoſe ends are equal circles, having the centre of the ſphere in the middle of it, and equally diſtant from both ends.

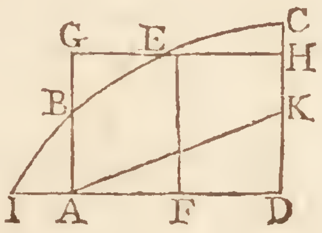
For the Solid Content of the Fruſtum of a cone, or of any pyramid, whatever figure the baſe may have. Add into one ſum, the areas of the two ends and the mean proportional between them; then $\frac{1}{3}$ of that ſum will be a mean area, or the area of an equal priſm, of the ſame altitude with the Fruſtum; and conſequently that mean area being multiplied by the height of the Fruſtum, the product will be the ſolid content of it. That is, if A denote the area of the greater end, a that of the leſs, and h the height; then $\overline{A + a + \sqrt{Aa}} \times \frac{1}{3}h$ is the ſolidity.

Other rules for pyramidal or conic Fruſtums may be ſeen in my Menſuration, p. 189, 2d edit. 1788.

The curve Surface of the Zone or Fruſtum of a ſphere, is had by multiplying the circumference of the ſphere by the height of the Fruſtum. Menſur. p. 197.

And the Solidity of the ſame Fruſtum is found, by adding together the ſquares of the radii of the two ends, and $\frac{1}{3}$ of the ſquare of the height of the Fruſtum, then multiplying the ſum by the ſaid height and by the number 1·5708. That is, $\overline{R^2 + r^2 + \frac{1}{3}h^2} \times \frac{1}{2}ph$ is the ſolid content of the ſpheric Fruſtum, whoſe height is h, and the radii of its ends R and r, p being = 3·1416. Menſur. p. 209.

For the Fruſtums of ſpheroids, and conoids, either parabolic or hyperbolic, ſee Menſur. p. 326, 328, 332, 382, 435. And in p. 486 &c, are general theorems concerning the Fruſtum of a ſphere, cone, ſpheroid, or conoid, terminated by parallel planes, when compared with a cylinder of the ſame altitude, on a baſe equal to the middle ſection of the Fruſtum made by a parallel plane. The difference between the Fruſtum and the cylinder is always the ſame quantity, in different parts of the ſame, or of ſimilar ſolids, or whatever the magnitude of the two parallel ends may be; the inclination of thoſe ends to the axis, and the altitude of the Fruſtum being given; and the ſaid conſtant difference is $\frac{1}{4}$ part of a cone of the ſame altitude with the Fruſtum, and the radius of its baſe is to that altitude, as the fixed axis is to the revolving axis of the Fruſtum. Thus, if BEC be any conic ſection, or a right line, or a circle, whoſe axis, or a part of it, is AD; AB and CD the extreme ordinates, FE the middle ordinate, AF being = FD; then taking, as AD to DK, ſo is the whole fixed axis, of which AD is a part, to its conjugate axis; and completing the parallelogram AGHD: then if the whole figure revolve about the axis AD, the line BEC will generate the Fruſtum of the cone or conoid, according as it is a right line or a conic ſection, or it will generate the whole ſolid when AB vaniſhes, or A and B meet in the ſame point; likewiſe AGHD will generate a cylinder, and ADK a cone: then is the 4th part of this cone always equal to the difference between the ſaid cylinder generated by AGHD and the ſolid or Fruſtum generated by ABECD; having all the ſame altitude or axis AD.



In the parabolic conoid, this difference and the cone vaniſh, and the Fruſtum, or whole conoid ABECD, is always equal to the cylinder AGHD, of the ſame altitude.

In the ſphere, or ſpheroid, the Fruſtum ABECD is *leſs* than the cylinder AGHD, by $\frac{1}{4}$ of the cone AKD. And

In the cone or hyperboloid, that Fruſtum is *greater* than the cylinder, by $\frac{1}{4}$ of the ſaid cone AKD, which is ſimilar to the other cone IBCD.

It may be obſerved, that the ſame relations are true, whether the ends of the Fruſtum are perpendicular or oblique to the axis. And the ſame will hold for the Fruſtum of any pyramid, whether right or oblique; and ſuch a Fruſtum of a pyramid will exceed the priſm, of the ſame altitude, and upon the middle ſection of the Fruſtum, by $\frac{1}{4}$ of the ſame cone.

It has been obſerved, that the difference, or $\frac{1}{4}$ of the cone AKD, is the ſame, or conſtant, when the altitude and inclination of the ends of the Fruſtum remain the ſame. But when the inclination of the ends varies, the altitude being conſtant; then the ſaid difference varies ſo as to be always reciprocally as the cube of the conjugate to the diameter AD. And when both the altitude and inclination of the ends vary, the differential cone is as the cube of the altitude directly, and the cube of the ſaid conjugate diameter reciprocally: but if they vary ſo, as that the altitude is always reciprocally as that diameter, then the difference is a conſtant quantity.

Another general theorem for Fruſtums, is this. In the Fruſtum of any ſolid, generated by the revolution of any conic ſection about its axis, if to the ſum of the two ends be added 4 times the middle ſection, $\frac{1}{6}$ of the laſt ſum will be a mean area, and being drawn into the altitude of the ſolid, will produce the content. That is, $\overline{AE + DC + 4FE} \times \frac{1}{6} AD$ is the content of ABCD.

And this theorem is general for all Fruſtums, as well as the complete ſolids, whether right or oblique to the axis, and not only of the ſolids generated from the circle or conic ſections, but alſo of all pyramids, cones, and in ſhort of any ſolid whoſe parallel ſections are ſimilar figures.

The ſame theorem alſo holds good for any parabolic *area* ABECD, and is very *nearly true* for the area of any other curve whatever, or for the content of any other ſolid than thoſe above mentioned.

FUGUE, in Muſic, is when the different parts of a muſical compoſition follow each other, each repeating in order what the firſt had performed.

FULCRUM, or *Prop*, in Mechanics, is the fixed point about which a lever &c turns and moves.

FULGURATING *Phoſphorus*, a term uſed by ſome Engliſh writers, to expreſs a ſubſtance of the phoſphorus kind. It was prepared both in a dry and liquid ſtate, but the preparation it ſeems was not well known to any but the inventor of it. This matter not only ſhone in the dark in both ſtates, but communicated its light to any thing it was rubbed on. When incloſed in a glaſs veſſel well ſtopped, it ſometimes would Fulgurate, or throw out little flaſhes of light, and ſometimes fill the whole phial with waves of flame. It does not need recruiting its light at the fire, or in the ſunſhine, like the phoſphorus of the Bolognian ſtone, but of itſelf continues in a ſtate of ſhining for ſeveral years together, and is ſeen as ſoon as expoſed in the dark; the ſolid or dry matter always reſembling a burning coal of fire, though not conſuming itſelf. Philoſ. Tranſ. N° 134.

FULIGINOUS, an epithet applied to thick ſmoke or vapour replete with ſoot or other craſs matter.

In the firſt fuſion of lead, there exhales a great deal of Fuliginous vapour, which being retained and collected, makes what is called Litharge. And Lampblack is what is gathered from the Fuliginous vapours of pines, and other reſinous wood, when burnt.

FULMINANT, FULMINANS, or FULMINATING, an epithet applied to ſomething that thunders, or makes a noiſe like thunder.

Aurum FULMINANS. See AURUM.

Pulvis FULMINANS, is a compoſition of 3 parts of nitre, 2 parts of ſalt of tartar, and 1 of ſulphur.—Both the Aurum and Pulvis Fulminans produce their effect chiefly downwards; in which they differ from gunpowder, which acts in orbem, or all around, but principally upwards. When the compoſition is laid in braſs ladles, and ſo ſet on fire, after fulmination, the ladles are often found perforated. It differs alſo from gunpowder in this, that it does not require to be confined, in order to fulminate, and it muſt be ſlowly and gradually heated. Some inſtants before exploſion, a light blue flame appears on its ſurface, proceeding from the vapours beginning to kindle. No more fire or flame is perceived during the fulmination, being ſuffocated and extinguiſhed by the quickneſs and violence of the commotion. Nor does the Fulminating powder generally kindle the combuſtible bodies in contact with it, becauſe the time of its inflammation is too ſhort.

Fulminating *Damp*. See Damp.

FULMINATION, or Fulguration, a vehement noiſe or ſhock reſembling thunder, cauſed by the ſudden exploſion and inflammation of divers preparations; as aurum fulminans, &c, when ſet on fire.

FUNCTION, a term uſed in analytics, for an algebraical expreſſion any how compounded of a certain letter or quantity with other quantities or numbers: and the expreſſion is ſaid to be a Function of that letter or quantity. Thus $a - 4x$, or $ax + 3x^2$, or $2x - a\sqrt{a^2 - x^2}$, or x^c, or c^x, is each of them a Function of the quantity x.

On the ſubject of Functions, their diviſions, transformations, explication by infinite ſeries, &c, ſee Euler's Analyſ. Infinitorum, c. 1, where the ſubject is fully treated.

FURLONG, an Engliſh long meaſure, containing 660 feet, or 220 yards, or 40 poles or perches, or the 8th part of a mile.

FURNITURE, in Dialling, certain additional points and lines drawn on a dial, by way of ornament. Such as the ſigns of the zodiac, length of days, parallels of declination, azimuths, points of the compaſs, meridians of chief cities, Babylonic, Jewiſh, or Italian hours, &c.

FUSAROLE, in Architecture, a ſmall round member cut in form of a collar, with oval beads, under the echinus, or quarter-round, in the Doric, Ionic, and Compoſite capitals.

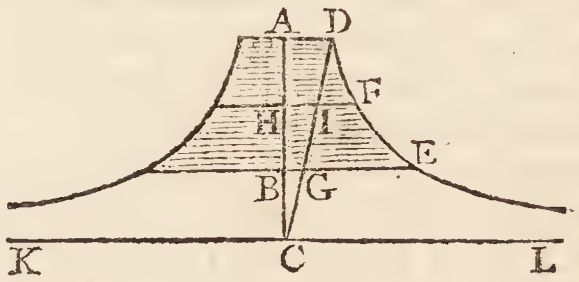
FUSEE, or Fusy, in Watch-work, is that part reſembling a low cone with its ſides a little ſunk or concave, which is drawn by the ſpring, and about which the chain or ſtring is wound.

The ſpring of a watch is the firſt mover. It is rolled up in a cylindrical box, againſt which it acts, and which it turns round in unbending itſelf. The chain, which at one end is wound about the Fuſee, and at the other faſtened to the ſpring-box, diſengages itſelf from the Fuſee in proportion as the box is turned. And hence the motion of all the other parts of the ſpring-watch. Now the effort or action of the ſpring is continually diminiſhing from firſt to laſt; and unleſs that inequality was rectified, it would draw the chain with more force, and wind a greater quantity of it upon the box, at one time than another; ſo that the movement would never keep equal time.

To correct this irregularity of the ſpring, it was very happily contrived to have the ſpring applied to the arms of levers, which are continually longer as the force of the ſpring is weaker: this foreign aſſiſtance, always increaſing as it is moſt needed, maintains the action and effect of the ſpring in an equality.

It is for this reaſon then that the Fuſee is made tapering ſomewhat conical, its radius at every point of the axis anſwering to the correſponding ſtrength of the ſpring.

Now if the action of the ſpring diminiſhed equally, as the parallels to the baſe of a triangle do; the cone, which is generated of a triangle, would be the preciſe figure required for the Fuſee; but it is certain that the weakening of the ſpring is not in that proportion; and therefore the Fuſee ſhould not be exactly conical; and in fact experience ſhews that it ſhould be a little hollowed about the middle, becauſe the action of the ſpring is not there ſufficiently diminiſhed of itſelf.



Mr. Varignon has inveſtigated the figure of the Fuſee, or the nature of the curve by whoſe revolution about its axis, ſhall be produced the ſolid whoſe figure the Fuſee is to have. This curve it may eaſily be ſhewn is an hyperbola whoſe aſymptote is the axis of the Fuſee. Thus, let DFE be the curve of the Fuſee, its axis being ABC: let AD expreſs the greateſt ſtrength of the ſpring when the watch is quite wound up, or when the ſpring acts at D, and BG the leaſt ſtrength when the watch is down, or when the ſpring acts at E; ſo as that BE : AD : : AD : BG, or BE × BG = AD²; join DG, producing it to meet the axis produced in C; then ſhall HI denote the ſtrength of the ſpring acting at the correſponding point F of the Fuſee; and the nature of it muſt be ſuch that the rectangle HI × HF be equal to a conſtant quantity, or HF muſt be reciprocally as HI,
or HI : BG : : BE : HF;
but becauſe - - AD, HI, BG,
are directly proportional to - CA, CH, CB,
theref. theſe are reciprocally propor. to AD, HF, BE;
and conſequently the curve DFE is an hyperbola, whoſe centre is C, and aſymptotes AC and KL: ſo that the figure of the Fuſee is the ſolid generated by an equilateral hyperbola revolved about its aſymptote. See alſo Martin's Mathem. Inſtit. vol. 2, p. 364.

Fusee, Fuse, or Fuze, in Artillery, is a wooden tap or tube uſed to ſet fire to the powder in a bombſhell. The bore of this tube is filled with a compoſition, of ſulphur one part, ſaltpetre 3 parts, and mealed powder 3, 4, or 5 parts. The tube is driven hard into the hole in the ſhell, having firſt cut it to the exact length anſwering to the time of the intended flight of the ſhell, ſo that, the compoſition in the Fuſe catching fire by the diſcharge of the ſhell from the mortar, it juſt burns down to its lower end, and ſo ſets fire to the powder in the ſhell, and thereby burſts it, at the moment when it arrives at the end of its range or flight.

FUST, in Architecture, the ſhaft of a column, or the part comprehended between the baſe and the capital, called alſo the Naked.

FUZE, or Fuzee. See Fusee.

G.

GABIONS, in Fortification, are large cylindrical baſkets, open at both ends, made of ozier twigs, of 3 or 4 feet in diameter, and from 3 to 6 feet high. Theſe, being filled with earth, are ſometimes uſed as merlons for the batteries, and ſometimes as a parapet for the lines of approach, when the attacks are carried on through a ſtony or rocky ground, and to advance them with extraordinary vigour. They ſerve alſo to make lodgments in ſome poſts, and to ſecure other places from the ſhot of the enemy; who, on their part, endeavour to burn and deſtroy the Gabions, by throwing pitched faggots among them.

GABLE, or GABLE-*end*, of a houſe, is the upright triangular end, from the cornice or eaves to the top of its roof.

GAGE, in Hydroſtatics, Pneumatics, &c, is an inſtrument for aſcertaining meaſures of various kinds. As

GAGE *of the Air-pump*, is adapted for ſhewing the degree to which the air is rarefied, or the receiver is exhauſted, at any time by the air-pump. This is either the common barometer-gage, both long and ſhort, or the pear gage, which at firſt was thought a great improvement, but afterwards it was diſcovered that its ſeeming accuracy was founded on a fallacy, which gave an erroneous indication of exhauſtion. See AIR-*pump*.

GAGE *of the Barometer*, is a contrivance for eſtimating the exact degree of the riſe and fall of the mercury in the tube of that inſtrument. It is well known that whilſt the mercury riſes in the tube, it ſinks in the ciſtern, and vice verſa; and conſequently the diviſions on the ſcale fixed near the top of the tube had their diſtance from the ſurface of the mercury in the ciſtern always various; from which there muſt often happen errors in determining the height of the mercury in the tube. To remedy this inconvenience, a line is cut upon a round piece of ivory, which is fixed near the ciſtern: this line is accurately placed at a given diſtance from the ſcale; for example at 27 inches; and a ſmall float of cork, with a cylindrical piece of ivory fixed to its upper ſurface, on which a line is cut at the exact diſtance of 2 inches from the under ſide of the cork, is left to play freely on the quickſilver, and the cylinder works in a groove made in the other piece. From this conſtruction it appears, that if theſe marks are made to coincide, by raiſing or lowering the ſcrew which acts on the quickſilver, then the diviſions on the ſcale will expreſs the true meaſure of the diſtance from the ſurface.

GAGE *of the Condenſer*, is a glaſs tube of a particular conſtruction, adapted to the condenſing engine, and deſigned to ſhew the exact denſity and quantity of the air contained at any time in the condenſer. See Deſaguliers's Exper. Philoſ. vol. 2, p. 394.

Sea GAGE, an inſtrument for finding the depth of the ſea. Several ſorts of theſe have been invented by Dr. Hales, Dr. Deſaguliers, and others. Formerly, the machines for this purpoſe conſiſted of two bodies, the one ſpecifically lighter, and the other ſpecifically heavier than the water, ſo joined together, that as ſoon as the heavy one came to the bottom, the lighter ſhould get looſe from it, and emerge; and the depth was to be eſtimated by the time the compound was in falling from the top to the bottom of the water, together with the time the lighter body was in riſing, reckoned from the diſappearing of the machine, till the emergent body was ſeen again; but no certain concluſion could be drawn from ſo precarious and incomplete an experiment.

But that invented by Drs. Hales and Deſaguliers was of a more exact nature, depending on the preſſure of the fluid only. For as the preſſure of fluids in all directions is the ſame at the ſame depth, a Gage which diſcovers what the preſſure is at the bottom of the ſea, will ſhew what the true depth of the ſea is in that place, whether the time of the machine's deſcent be longer or ſhorter.

Dr. Hales, in his Vegetable Statics, deſcribes his Gage for eſtimating the preſſures made in opaque veſſels; where honey being poured over the ſurface of mercury in an open veſſel, riſes upon the ſurface of the mercury as it is preſſed up into a tube whoſe lower orifice is immerſed into the honey and mercury, and whoſe top is hermetically ſealed. Now as by the preſſure, the air in the tube is condenſed, and the mercury riſes, ſo the mercury comes down again when the preſſure is taken off, and would leave no mark of the height to which it had riſen; but the honey (or treacle, which does better) which is upon the mercury, ſticking to the inſide of the tube, leaves a mark, which ſhews the height to which it had riſen, and conſequently gives the quantity of preſſure, and the height of the ſurface of the fluid.

Deſaguliers's addition to this machine, conſiſted in a contrivance to carry it down to the bottom of the ſea by means of a heavy weight, which was immediately diſengaged by ſtriking the bottom, and the Gage, made very light for the purpoſe, re-aſcended to the top.

Dr. Hales afterwards made more experiments of this ſort, and propoſed another Sea Gage for vaſt depths, which is deſcribed in the Philoſ. Tranſ. N° 405, and is to this effect. Suppoſe a pretty long tube of copper or iron, cloſe at the upper end, to be let down into the ſea, to any depth, the water will riſe in the tube to a height bearing a certain proportion to the depth of the ſea to which the machine is ſunk. And this proportion is as follows: 33 feet of ſea water being nearly equal to the mean preſſure of the atmoſphere, therefore at 33 feet deep, the air in the tube will be compreſſed into half the length of the tube, or the water will riſe and fill half way up the tube; in like manner at 66 feet deep, the water will occupy $\frac{2}{3}$ of the tube; at 99 feet deep it will fill $\frac{3}{4}$ of the tube; at 132 feet deep it will fill $\frac{4}{5}$ of the tube; and ſo on. Hence therefore, by knowing the height to which the water riſes in the tube, there will be known the conſequent depth of the ſea.

But, in very great depths, the ſcale near the top of the tube would be ſo ſmall, and the diviſions ſo cloſe, that there would be no accuracy in the experiment, unleſs the tube were of a very great length, and this again would render it both liable to be broken, and quite impracticable.

To remedy this inconvenience, he made the following contrivance: To the bottom of the tube he ſcrewed a large hollow globe of copper, with a ſmall orifice, or a ſhort pipe at bottom of the globe, to let in the water; by which means he had a very great quantity of air, and the ſcale enlarged. See alſo Deſagul. Exp. Phil. vol. 2, p. 224 and 241.

Bucket Sea Gage, is an inſtrument contrived by Dr. Hales to find the different degrees of coolneſs and ſaltneſs of the ſea at different depths. This Gage conſiſts of a common pale or bucket, with two heads: theſe heads have each a round hole in the middle, about 4 inches in diameter, covered with ſquare valves opening upward; and that they may both open and ſhut together, there is a ſmall iron rod, having one end fixed to the upper ſide of the lower valve, and the other end to the lower ſide of the upper valve. So that as the bucket deſcends with its ſinking weight into the ſea, both the valves may open by the force of the water, which by that means has a free paſſage through the bucket. But when the bucket is drawn up, then both the valves ſhut by the force of the water at the upper end of the bucket; ſo that the bucket is drawn up full of the loweſt ſea water to which it has deſcended, and immediately the mercurial thermometer, fixed within it, is examined, to ſee the degree of temperature; and the degree of ſaltneſs is afterwards examined at leiſure. Philoſ. Tranſ. numb. 9, p. 149, and numb. 24, p. 447, or Abridg. vol. 2, p. 260.

Lord Charles Cavendiſh adapted a thermometer for the temperature of the ſea water, at different depths. See Philoſ. Tranſ. vol. 50, p. 300, and Phipps's Voyage towards the North Pole, p. 142 &c.

Aqueo-mercurial Gage is the name of an apparatus contrived by Dr. Hales, and applied, in various forms, to the branches of trees, to determine the force with which they imbibe moiſture. Vegetable Statics, vol. 1, ch. 2, p. 84.

Sliding Gage, a tool uſed by mathematical inſtrument makers, for meaſuring and ſetting off diſtances; conſiſting of a beam, tooth, ſliding ſocket, and the ſhoulder of the ſocket.

Tide Gage, an inſtrument uſed for determining the height of the tides by Mr. Bayly, in the courſe of a voyage towards the ſouth pole &c, in the Reſolution and Adventure, in the years 1772, 1773, 1774, and 1775. This inſtrument conſiſts of a glaſs tube, whoſe internal diameter was 7-10ths of an inch, laſhed faſt to a 10 foot fir rod, divided into feet, inches, and parts; the rod being faſtened to a ſtrong poſt fixed firm and upright in the water. At the lower end of the tube was an exceeding ſmall aperture, through which the water was admitted. In conſequence of this conſtruction, the ſurface of the water in the tube was ſo little affected by the agitation of the ſea, that its height was not altered the 10th part of an inch when the ſwell of the ſea was 2 feet; and Mr. Bayly was certain, that with this inſtrument he could diſcern a difference of the 10th of an inch in the height of the tide.

Water Gage. See Altitude, and Hydrometer.

Wind Gage, an inſtrument for meaſuring the force of the wind upon any given ſurface. Several have been invented formerly, and one was lately invented by Dr. Lind, which is deſcribed in the Philoſ. Tranſ. vol. 65. See ſeveral alſo under the article Anemometer.

GAGER, ſee Gauger.

GAGING, ſee Gauging.

GALAXY, or *Milky-Way*, or *Via Lactea*, in Aſtronomy, that long, whitiſh, luminous track, which ſeems to encompaſs the heavens like a ſwath, ſcarf, or girdle; and which is eaſily ſeen in a clear night, eſpecially when the moon is not up. It is of a conſiderable, though unequal breadth; being alſo in ſome parts double, but in others ſingle.

The Galaxy paſſes through many of the conſtellations in its circuit round the heavens, and keeps its exact place or poſition with reſpect to them.

There have been various ſtrange and fabulous ſtories and opinions concerning the Galaxy.

The ancient poets, and even ſome of the philoſophers, ſpeak of it as the road or way by which the heroes went to heaven. But the Egyptians called it the Way of Straw, from the ſtory of its riſing from burning ſtraw, thrown behind the goddeſs Iſis in her flight from the giant Typhon. While the Greeks, who affect to derive every thing in the heavens from ſome of their own fables, have two origins for it; the one, that Juno, without perceiving it, accidentally gave ſuck to Mercury when an infant, but that as ſoon as ſhe turned her eyes upon him, ſhe threw him from her, and as the nipple was drawn from his mouth, the milk ran about for a moment; and the other, that the infant Hercules being laid by the ſide of Juno when aſleep, on waking ſhe gave him the breaſt; but ſoon perceiving who it was, ſhe threw him from her, and the heavens were marked by the waſted milk.

Some other philoſophers however gave it a different turn, and different origin: theſe eſteemed it to be a tract of liquid fire, ſpread in this manner along the ſkies: and others again, ſuppoſing a celeſtial region beyond all that was viſible, and imagining that fire, at ſome time let looſe from thence, was to conſume the world, made this a part of that celeſtial fire, and appealed to it as a preſage of what would ſurely happen. This diffuſed brightneſs they conſidered as a crack in the vault or wall of heaven, and fancied this a glimmering of the celeſtial fire through it, and that there required nothing more than the undoing of this crack by ſome accident in nature, or by the will of the Gods, to make the whole frame ſtart, and let out the fire of deſtruction.

Ariſtotle makes the Galaxy a kind of meteor, formed of a crowd of vapours, drawn into that part by certain large ſtars diſpoſed in the region of the heavens anſwering to it. Others, finding that the Galaxy was ſeen all over the globe, that it always correſponded to the ſame fixed ſtars, and that it was far above the higheſt planets, ſet Ariſtotle's opinion aſide, and placed the Galaxy in the firmament or region of the fixed ſtars; and concluded that it was nothing elſe but an aſſemblage of an infinite number of minute ſtars. And ſince the invention of teleſcopes, this opinion has been abundantly confirmed. For, by directing a good teleſcope to any part of the milky way, we perceive an innumerable multitude of very ſmall ſtars, where before we only obſerved a confuſed whiteneſs, ariſing from the aſſemblage and union of their joint light; like as any thing powdered with fine white powder, at a diſtance we only obſerve the confuſed whiteneſs, but on examining it very near we perceive all the ſmall particles of the powder ſeparately; as Milton finely expreſſes it,

> A broad and ample road, whoſe duſt is gold,
> And pavement ſtars, as ſtars to thee appear,
> Seen in the Galaxy, that milky way,
> Which nightly, as a circling zone thou ſeeſt,
> Powder'd with ſtars.

There are other ſuch marks in the heavens; as the nebulæ, or, nebulous ſtars, and certain whitiſh parts about the ſouth pole, called Magellanic clouds, which are all of the ſame nature, appearing to be vaſt cluſters of ſmall ſtars when viewed through a teleſcope, which are too faint to affect the eye ſingly.

M. le Monnier however, not being able to diſcover more ſtars in this ſpace than in other parts of the heavens, diſputes the opinion above recited as to the reaſon of the whiteneſs, and ſuppoſes that this and the nebulous ſtars are occaſioned by ſome other kind of matter. Inſt. Aſt. p. 60.

GALILEI (Galileo,) a moſt excellent philoſopher, mathematician and aſtronomer, was born at Piſa in Italy, in 1564. From his infancy he had a ſtrong propenſity to philoſophy and mathematics, and ſoon made a great progreſs in theſe ſciences. So that in 1592 he was choſen profeſſor of mathematics at Padua. While he was profeſſor there, viſiting Venice, then famous for the art of glaſs-making, he heard that in Holland a glaſs had been invented, through which very diſtant objects were ſeen diſtinctly as if near at hand. This was ſufficient for Galileo; his curioſity was raiſed, and put him upon conſidering what muſt be the form of ſuch a glaſs, and the manner of making it. The reſult of his enquiry was the invention of the teleſcope, produced from this hint, without having ſeen the Dutch glaſs. All the diſcoveries he made in aſtronomy were eaſy and natural conſequences of this invention, which opening a way, till then unknown, into the heavens, thence brought the fineſt diſcoveries. One of the firſt of theſe, was that of 4 of Jupiter's ſatellites, which he called the Medicean ſtars or planets, in honour of Coſmo the 2d, grand-duke of Tuſcany, who was of that family. Coſmo ſent for our aſtronomer from Padua, and made him profeſſor of mathematics at Piſa in 1611; and ſoon after inviting him to Florence, gave him the office and title of *principal philoſopher and mathematician to his highneſs.*

He had been but a few years at Florence, before the Inquiſition began to be very buſy with him. Having obſerved ſome ſolar ſpots in 1612, he printed that diſcovery the following year at Rome; in which, and in ſome other pieces, he ventured to aſſert the truth of the Copernican ſyſtem, and brought ſeveral new arguments to confirm it. For theſe he was cited before the Inquiſition at Rome, in 1615: after ſome months impriſonment, he was releaſed, and ſentence pronounced againſt him, that he ſhould renounce his heretical opinions, and not defend them by word or writing, or inſinuate them into the minds of any perſons. But having afterwards, in 1632, publiſhed at Florence his Dialogues of the two Great Syſtems of the World, the Ptolomaic and Copernican, he was again cited before the holy-office, and committed to the priſon of that eccleſiaſtical court at Rome. The inquiſitors convened in June that year; and in his preſence pronounced ſentence againſt him and his books, obliging him to abjure his errors in the moſt ſolemn manner; committed him to the priſon of their office during pleaſure; and enjoined him, as a ſaving penance, for three years to come, to repeat once a week the ſeven penitential pſalms: reſerving to themſelves, however, the power of moderating, changing, or taking away altogether or in part, the ſaid puniſhment and penance. On this ſentence, he was detained in priſon till 1634; and his Dialogues of the Syſtem of the World were burnt at Rome.

Galileo lived ten years after this; ſeven of which were employed in making ſtill further diſcoveries with his teleſcope. But by the continual application to that inſtrument, added to the damage his ſight received from the nocturnal air, his eyes grew gradually weaker, till he became totally blind in 1639. He bore this calamity with patience and reſignation, worthy of a great philoſopher. The loſs neither broke his ſpirit, nor ſtopped the courſe of his ſtudies. He ſupplied the defect by conſtant meditation; by which means he prepared a large quantity of materials, and began to arrange them by dictating his ideas; when, by a diſtemper of three months continuance, waſting away by degrees, he expired at Arcetri near Florence, in January 1642, being the 78th year of his age.

Galileo was in his perſon of ſmall ſtature, though of a venerable aſpect, and vigorous conſtitution. His converſation was affable and free, and full of pleaſantry. He took great delight in architecture and painting, and deſigned extremely well. He played exquiſitely on the lute; and whenever he ſpent any time in the country, he took great pleaſure in huſbandry. His learning was very extenſive; and he poſſeſſed in a high degree a clearneſs and acuteneſs of wit. From the time of Archimedes, nothing had been done in mechanical geometry, till Galileo, who being poſſeſſed of an excellent judgment, and great ſkill in the moſt abſtruſe points of geometry, firſt extended the boundaries of that ſcience, and began to reduce the reſiſtance of ſolid bodies to its laws. Beſides applying geometry to the doctrine of motion, by which philoſophy became eſtabliſhed on a ſure foundation, he made ſurpriſing diſcoveries in the heavens by means of his teleſcope. He made the evidence of the Copernican ſyſtem more ſenſible, when he ſhewed from the phaſes of Venus, like to thoſe of the moon, that Venus actually revolves about the ſun. He proved the rotation of the ſun on his axis, from his ſpots; and thence the diurnal rotation of the earth became more credible. The ſatellites that attend Jupiter in his revolution about the ſun, repreſented, in Jupiter's ſmaller ſyſtem, a juſt image of the great ſolar ſyſtem; and rendered it more eaſy to conceive how the moon might attend the earth, as a ſatellite, in her annual revolution. By diſcovering hills and cavities in the moon, and ſpots in the ſun conſtantly varying, he ſhewed that there was not ſo great a difference between the celeſtial bodies and the earth as had been vainly imagined.

He rendered no leſs ſervice to ſcience by treating, in a clear and geometrical manner, the doctrine of motion, which has juſtly been called the key of nature. The rational part of mechanics had been ſo much neglected, that hardly any improvement was made in it for almoſt 2000 years. But Galileo has given us fully the theory of equable motions, and of ſuch as are uniformly accelerated or retarded, and of theſe two compounded together. He, firſt of any, demonſtrated that the ſpaces deſcribed by heavy bodies, from the beginning of their deſcent, are as the ſquares of the times; and that a body, projected in any direction not perpendicular to the horizon, deſcribes a parabola. Theſe were the beginnings of the doctrine of the motion of heavy bodies, which has been ſince carried to ſo great a height by Newton. In geometry, he invented the cycloid, or trochoid; though the properties of it were afterwards chiefly demonſtrated by his pupil Torricelli. He invented the ſimple pendulum, and made uſe of it in his aſtronomical experiments: he had alſo thoughts of applying it to clocks; but did not execute that deſign: the glory of that invention was reſerved for his ſon Vicenzio, who made the experiment at Venice in 1649; and Huygens afterward carried this invention to perfection. Of Galileo's invention alſo, was the machine, with which the Venetians render their Laguna fluid and navigable. He alſo diſcovered the gravity of the air, and endeavoured to compare it with that of water; beſides, opening up ſeveral other enquiries in natural philoſophy. In ſhort, he was not eſteemed and followed by philoſophers only, but was honoured by perſons of the greateſt diſtinction of all nations.

Galileo had ſcholars too that were worthy of ſo great a maſter, by whom the gravitation of the atmoſphere was fully eſtabliſhed, and its varying preſſure accurately and conveniently meaſured, by the column of quickſilver of equal weight ſuſtained by it in the barometrical tube. The elaſticity of the air, by which it perpetually endeavours to expand itſelf, and, while it admits of condenſation, reſiſts in proportion to its denſity, was a phenomenon of a new kind (the common fluids having no ſuch property), and was of the utmoſt importance to philoſophy. Theſe principles opened a vaſt field of new and uſeful knowledge, and explained a great variety of phenomena, which had been accounted for before that time in a very abſurd manner. It ſeemed as if the air, the fluid in which men lived from the beginning, had been then but firſt diſcovered. Philoſophers were every where buſy enquiring into its various properties and their effects: and valuable diſcoveries rewarded their induſtry. Of the great number who diſtinguiſhed themſelves on this occaſion, may be mentioned Torricelli and Viviani in Italy, Paſcal in France, Otto Guerick in Germany, and Boyle in England.

Galileo wrote a number of treatiſes, many of which were publiſhed in his life-time. Moſt of them were alſo collected after his death, and publiſhed by Mendeſſi in 2 vols 4to, under the title of *L'Opere di Galileo Galilei Lynceo*, in 1656. Some of theſe, with others of his pieces, were tranſlated into Engliſh and publiſhed by Thomas Saliſbury, in his Mathematical Collections, in 2 vols folio. A volume alſo of his letters to ſeveral learned men, and ſolutions of ſeveral problems, were printed at Bologna in 4to. His laſt diſciple, Vincenzo Viviani, who proved a very eminent mathematician, methodized a piece of his maſter's, and publiſhed it under this title, *Quinto libro de gli Elementi d'Euclidi, &c*; at Florence in 1674, 4to. Viviani publiſhed ſome more of Galileo's things, being Extracts from his letters to a learned Frenchman, where he gives an account of the works which he intended to have publiſhed, and a paſſage from a letter of Galileo dated at Arcetri, Oct. 30, 1635, to John Camillo, a mathematician of Naples, concerning the angle of contact. Beſides all theſe, he wrote many other pieces, which were unfortunately loſt through his wife's devotion; who, ſolicited by her confeſſor, gave him leave to peruſe her huſband's manuſcripts; of which he tore and took away as many as he ſaid were not fit to be publiſhed.

GALLERY, in Architecture, a covered place in a building, much longer than broad; which is uſually placed in the wings of the building, and ſerving to walk in, and to place pictures in. It denotes a little aiſle, or walk, ſerving as a common paſſage to ſeveral rooms placed in a line, or row.

GALLERY, in Fortification, a covered walk, or paſſage, made acroſs the ditch of a beſieged town, with timbers faſtened in the ground and covered over.

GALLON, an Engliſh meaſure of capacity, for things both liquid and dry, containing 2 pottles, or 4 quarts, or 8 pints. But thoſe pints and quarts, and conſequently the Gallon itſelf, are different, according to the quality of the things meaſured: the wine Gallon, for inſtance, contains 231 cubic inches, and holds 8lb $5\frac{2}{3}$ oz, avoirdupois, of pure water; the beer and ale Gallon contains 282 cubic inches, and holds 10lb $3\frac{1}{5}$ oz of water; and the Gallon dry meaſure, for grain, meal, &c, contains $268\frac{4}{5}$ cubic inches, and holds 9lb $11\frac{1}{2}$ oz of water.

GALLOPER, in Artillery, the name of a carriage ſerving for the very ſmall guns, and having ſhafts ſo as to be drawn without a limber.

GAMING. See CHANCES, and LAWS *of* CHANCE.

GARDECAUT, or GUARD DU CORD, in a watch, is that which ſtops the fuſee, when wound up, and for that end is driven up by the ſpring. Some call it Guard-cock; others Guard du Gut.

GARRISON Guns, ſuch as are mounted and uſed in a Garriſon, conſiſting of the following weights, viz. the 42, 32, 24, 18, 12, 9, and 6 pounders; being made either of braſs or iron.

Table of the Weight and Dimenſions of Garriſon Guns.

Braſs Garriſon Guns.						Iron Garriſon Guns.					
Shot	Length		Weight.			Shot	Length		Weight		
lb	f.	in.	Cw.	qr.	lb.	lb.	f.	in.	Cw.	qr.	lb.
42	10	0	64	0	0	32	9	8	56	0	0
32	9	2	49	2	18	24	9	8	48	0	0
24	8	4	37	0	0	18	9	0	36	0	0
18	7	6	27	3	0	12	7	8	24	0	0
12	6	7	18	2	0	9	7	0	18	0	0
9	6	0	13	3	0	6	6	1	12	0	0
6	5	3	9	1	0	4	5	4	8	0	0

GASSENDI (Peter), one of the moſt celebrated philoſophers France has produced, was born at Chanterſier, about 3 miles from Digne in Provence, in the year 1592. When a child, he took great delight in gazing at the moon and ſtars whenever they appeared. This pleaſure often drew him into bye-places, that he might feaſt his eyes freely and undiſturbed; by which means his parents had him often to ſeek, not without many anxious fears and apprehenſions. In conſequence of this promiſing diſpoſition, he was ſent to the beſt ſchools, to cultivate it with the inſtructions of the firſt maſters. He profited ſo well of theſe aids, that he was invited to be profeſſor of rhetoric at Digne, before he was quite 16 years of age. After filling this office three years, upon the death of his maſter at Aix, he was appointed to ſucceed him as profeſſor of philoſophy. After a few years reſidence here, he compoſed his Paradoxical Exercitations; which coming to the hands of Nicholas Peireſc, that great patron of learning joined with Joſeph Walter, prior of Valette, in promoting him; and, having entered into holy orders, he was firſt made canon of the church of Digne and doctor of divinity, and then warden or rector of the ſame church.

Gaſſendi's fondneſs for aſtronomy grew up with his years; and his reputation daily increaſing, he was appointed the king's profeſſor of mathematics at Paris in 1645. This inſtitution being chiefly intended for aſtronomy, our author read lectures on that ſcience to a crowded audience. However, he did not long enjoy this ſituation; for a dangerous cough and inflammation of the lungs obliged him, in 1647, to return to Digne for the benefit of his native air. Having thus, and by the intermiſſion of his ſtudies, recovered his health, he again returned to Paris in 1653; where, after firſt writing and publiſhing the lives of Tycho Brahe, Copernicus, Purbach, and Regiomontanus, in 1654 he again renewed his aſtronomical labours, with the deſign of completing the ſyſtem of the heavens. But while he was thus employed, too intenſely for the feeble ſtate of his health, he relapſed into his former diſorder, under which, with the aid of too copious and numerous bleedings, by order of three phyſicians, he ſunk in the year 1655, at 63 years of age.

Gaſſendi wrote againſt the metaphyſical meditations of Des Cartes; and divided with that great man the philoſophers of his time, almoſt all of whom were either Carteſians or Gaſſendiſts. To his knowledge in philoſophy and mathematics, he joined profound erudition and deep ſkill in the languages. He wrote, 1. Three volumes on Epicurus's philoſophy; and ſix others, which contain his own philoſophy.—2. Aſtronomical Works.—3. The lives of Nicholas de Peireſc, Epicurus, Copernicus, Tycho Brahe, Purbach, and Regiomontanus.—4. Epiſtles, and other treatiſes. All his works were collected together, and printed at Lyons in 1658, in 6 volumes folio.

Gaſſendi was the firſt perſon that ſaw the tranſit of Mercury over the ſun, viz, Nov. 7, 1631; as Horrox firſt predicted and ſhewed the tranſit of Venus.—His library was large and valuable: to which he added an aſtronomical and philoſophical apparatus, which, for their accuracy and magnitude, were purchaſed by the emperor Ferdinand the 3d.—It appears by his letters, printed in the 6th volume of his works, that he was often conſulted by the moſt celebrated aſtronomers of his time, as Kepler, Longomontanus, Snell, Hevelius, Galileo, Kircher, Bulliald, and others: and he has generally been eſteemed one of the founders of the reformed philoſophy, in oppoſition to the groundleſs hypotheſes and empty ſubtleties of Ariſtotle and the ſchoolmen.

GAUGE-*Line*, a line on the common Gauging rod, uſed for the purpoſe of gauging liquids. See Gauging-*Rod*.

Gauge-*Point*, of a ſolid meaſure, is the diameter of a circle, whoſe area is expreſſed by the ſame number as the ſolid content of that meaſure. Or it is the diameter of a cylinder, whoſe altitude is 1, and its content the ſame as of that meaſure.

Thus, the ſolid content of a wine gallon being 231 cubic inches; if a circle be conceived to contain ſo many ſquare inches, its diameter will be 17·15; which is therefore the Gauge-point for wine meaſure. And an ale gallon containing 282 cubic inches; by the ſame rule, the Gauge-point for ale meaſure will be found to be 18·95. And after the ſame manner may the Gauge-point for any other meaſure be determined.

Hence it follows, that when the diameter of a cylinder in inches is equal to the Gauge-point in any meaſure, given likewiſe in inches, every inch in its length will contain an integer of the ſame meaſure. So in a cylinder whoſe diameter is 17·15 inches, every inch in height contains one entire gallon in wine meaſure; and in another, whoſe diameter is 18·95, every inch in length contains one ale gallon.

GAUGER, an officer appointed by the commiſſioners of exciſe, to Gauge, meaſure, or examine, all caſks, tuns, pipes, barrels, hogſheads, of beer, wine, oil, &c.

GAUGING, the art or act of meaſuring the capacities or contents of all kinds of veſſels, and determining the quantity of fluids, or other matters contained in them. Theſe are principally pipes, tuns, barrels, rundlets, and other caſks; alſo backs, coolers, vats, &c.

As to the ſolid contents of all priſmatical veſſels, as cubes, parallelopipedons, cylinders, &c, they are found by multiplying the area of the baſe by their altitude. And the contents of all pyramidal bodies, and cones, are equal to 1-3d of the ſame.

In Gauging, it has been uſual to divide caſks into four varieties or forms, denominated as follows, from the ſuppoſed reſemblance they bear to the fruſtums of ſolids of the ſame names: viz,

1. The middle fruſtum of a ſpheroid,
2. The middle fruſtum of a parabolic ſpindle,
3. The two equal fruſtums of a paraboloid,
4. The two equal fruſtums of a cone.

And particular rules, adapted to each of theſe forms, may be found in moſt books of Gauging, and in my Menſuration, p. 575 &c. But as the form is imaginary, and only gueſſed at, it hardly ever happens that a true ſolution is brought out in this way; beſide which, it is very troubleſome and inconvenient to have ſo many rules to put in practice. I ſhall therefore give here one rule only, from p. 592 of that book, which is not only general for all caſks that are commonly met with, but quite eaſy, and very accurate, as having been often verified and proved by filling the caſks with a true gallon meaſure.

General Rule. Add into one ſum,
39 times the ſquare of the bung diameter,
25 times the ſquare of the head diameter, and
26 times the product of thoſe diameters;
multiply the ſum by the length of the caſk, and the product by the number ·00034; then this laſt product divided by 9 will give the wine gallons, and divided by 11 will give the ale gallons.

Or, $\overline{39B^2 + 25H^2 + 26BH} \times \frac{L}{114}$ is the content in inches; which being divided by 231 for wine gallons, or by 282 for ale gallons, will be the content.

For Ex. If the length of a caſk be 40 inches, the bung diameter 32, and the head diameter 24.

Here - $32^2 \times 39 = 39936$
and - $24^2 \times 25 = 14400$
and - $32 \times 24 \times 26 = 19968$

the ſum - 74304
multiplied by - 40

and divid. by 114)2972160
gives - 26071 cubic inches;
this divided by 231 gives 112 wine gallons,
or divided by 282 gives 92 ale gallons.

But the common practice of Gauging is performed mechanically, by means of the Gauging or Diagonal Rod, or the Gauging Sliding Rule, the deſcription and uſe of which here follow.

GAUGING, or *Diagonal, Rod*, is a rod or rule adapted for determining the contents of caſks, by meaſuring the diagonal only, viz the diagonal from the bung to the extremity of the oppoſite ſtave next the head. It is a ſquare rule, having 4 ſides or faces, being uſually 4 feet long, and folding together by means of joints.

Upon one face of the rule is a ſcale of inches, for taking the meaſure of the diagonal; to theſe are adapted the areas, in ale gallons, of circles to the correſponding diameters, like the lines on the under ſides of the three ſlides in the ſliding rule, deſcribed below. And upon the oppoſite face are two ſcales, of ale and wine gallons, expreſſing the contents of caſks having the correſponding diagonals; and theſe are the lines which chiefly conſtitute the difference between this inſtrument and the ſliding rule; for all the other lines upon it are the ſame with thoſe in that inſtrument, and are to be uſed in the ſame manner.

To uſe the Diagonal Rod. Unfold the rod ſtraight out, and put it in at the bung hole of the caſk to be gauged, till its end arrive at the interſection of the head and oppoſite ſtave, or to the fartheſt poſſible diſtance from the bung-hole, and note the inches and parts cut by the middle of the bung; then draw out the rod, and look for the ſame inches and parts on the oppoſite face of it, and annexed to them are found the contents of the caſk, both in ale and wine gallons.

For Ex. Let it be required to find, by this rod, the content of a caſk whoſe diagonal meaſures 34·4 inches; which anſwers to the caſk in the foregoing example, whoſe head and bung diameters are 32 and 24, and length 40 inches; for if to the ſquare of 20, half the length, be added the ſquare of 28, half the ſum of the diameters, the ſquare root of the ſum will be 34·4 nearly.

Now, to this diagonal 34·4, correſponds, upon the rule, the content 91 ale gallons, or 111 wine gallons; which are but 1 leſs than the content brought out by the former general rule above given.

GAUGING *Rule*, or *Sliding Rule*, is a ſliding rule particularly adapted to the purpoſes of Gauging. It is a ſquare rule, of four faces or ſides, three of which are furniſhed with ſliding pieces running in grooves. The lines upon them are moſtly logarithmic ones, or diſtances which are proportional to the logarithms of the numbers placed at the ends of them; which kind of lines was placed upon rulers, by Mr. Edmund Gunter, for expeditiouſly performing arithmetical operations, uſing a pair of compaſſes for taking off and applying the ſeveral logarithmic diſtances: but inſtead of the compaſſes, ſliding pieces were added, by Mr. Thomas Everard, as more certain and convenient in practice, from whom this ſliding rule is often called Everard's Rule. For the more particular deſcription and uſes of this rule, ſee my Menſuration, p. 564, 2d edition.

The writers on Gauging are, Beyer, Kepler, Dechales, Hunt, Everard, Dougherty, Shettleworth, Shirtcliffe, Leadbetter, &c.

GAZONS, in Fortification, turfs, or pieces of freſh earth covered with graſs, cut in form of a wedge, about a foot long, and half a foot thick, to line or face the outſide of works made of earth, to keep them up, and prevent their mouldering.

GELLIBRAND (HENRY), profeſſor of aſtronomy at Greſham-college, was born in London the 27th of Nov. 1597. He was ſent to Trinity-college, Oxford, in 1615, and took his degree in arts 1619. He then entered into orders, and became curate of Chiddingſtone in Kent. Afterwards, taking a great fancy to mathematics, by happening to hear one of Sir Henry Saville's lectures in that ſcience, he immediately ſet himſelf to the cloſe ſtudy of that noble ſcience, and relinquiſhed his fair proſpects in the church. Contenting himſelf therefore

fore with his private patrimony, which was now come into his hands by the death of his father, the ſame year he entered again a ſtudent at Oxford, making mathematics his ſole employment. He made ſuch proficiency in this ſcience before he proceeded A. M., which was in 1623, that he drew the attention and intimate friendſhip of Mr. Henry Briggs, then lately removed from the geometry profeſſorſhip in Greſham-college to that of Savilian profeſſor of geometry at Oxford, by the founder Sir Henry Savile, and who, upon the death of Mr. Gunter, procured for our author the profeſſorſhip of aſtronomy in Greſham-college, to which he was elected in the beginning of the year 1627. His friend Mr. Briggs dying in 1630, before he had finiſhed the introduction to his Trigonometria Britannica, he recommended the completing and publiſhing of that work to our author. Gellibrand accordingly added a preface, and the application of the logarithms to plane and ſpherical trigonometry, &c, and the whole was printed at Gouda, under the care of Adrian Vlacq, in 1633.

While Mr. Gellibrand was preparing that work, he was brought into trouble in the high-commiſſion court, by Dr. Laud, then biſhop of London, on account of an almanac, publiſhed by William Beale, ſervant to Mr. Gellibrand, for the year 1631, with the approbation of his maſter. In this almanac, the popiſh ſaints, then uſually put into calendars, were omitted, and the names of other ſaints and martyrs, mentioned in the Book of Martyrs, were placed in their ſtead, as they ſtand in Fox's calendars. This it ſeems gave offence to the biſhop, and occaſioned the proſecution. But when the cauſe came to be heard, it appeared that other almanacs of the ſame kind had formerly been printed; upon which, both maſter and man were acquitted by Abp. Abbot and the whole court, Laud only excepted; which was afterward made one of the articles againſt him on his own trial.

It ſeems Gellibrand was ſtrongly attached to the old Ptolomaic ſyſtem. For when he went over to Holland, about the printing of Briggs's book abovementioned, he had ſome diſcourſe with Lanſberg, an eminent brother aſtronomer in Zealand, who affirming that he was fully perſuaded of the truth of the Copernican ſyſtem; our author obſerves, "that this ſo ſtyled a truth he ſhould "receive as an hypotheſis; and ſo be eaſily led on to "the conſideration of the imbecility of man's appre-"henſion, as not able rightly to conceive of this ad-"mirable opifice of God, or frame of the world, with-"out falling foul of ſo great an abſurdity:" ſo firmly was he fixed in his adherence to the Ptolomaic ſyſtem. Gellibrand wrote ſeveral things after this, chiefly tending to the improvement of navigation, which would probably have been further advanced by him, had his life been continued longer; but he was untimely carried off by a fever, in 1636, at 39 years of age.

The character of Mr. Gellibrand is that of a plain, plodding, induſtrious, well-intentioned man, with little invention or genius. His writings are chiefly as below:

1. *Trigonometria Britannica*; or the Doctrine of Triangles, being the 2d part of Briggs's work abovementioned.

2. A ſmall Tract concerning the longitude.

3. A Diſcourſe on the *Variation of the Magnetic Needle*; annexed to Wright's Errors in Navigation detected.

4. *Inſtitution Trigonometrical*, with its application to aſtronomy and navigation; 8vo, 1635.

5. *Epitome of Navigation*, with the neceſſary tables; 8vo.

6. Several manuſcripts never publiſhed; as, The Doctrine of Eclipſes.—A Treatiſe of Lunar Aſtronomy.—A Treatiſe of Ship-building, &c.

GEMINI, a conſtellation of the northern hemiſphere, one of the 48 old conſtellations, and the 3d in order of the zodiacal ſigns, Aries, Taurus, Gemini, &c. This conſtellation conſiſts of two children, twins, called Caſtor and Pollux, and denoted by the mark, ♊, being a rude drawing of the ſame.

This conſtellation was, more anciently, depicted by a couple of young kids, by the Egyptians and eaſtern nations, as denoting that part of the ſpring when theſe animals appear; but the Greeks altered them to two children, which ſome of them make to be Caſtor and Pollux, ſome of them again Hercules and Apollo, and others Triptolemus and Jaſion; but the Arabians afterwards changed the figures into two peacocks, their religion not allowing them to paint or draw any human figure. Sir Iſaac Newton thinks the figures had ſome reference to the Argonautic expedition.

The ancients attributed to every ſign of the zodiac one of the principal deities for its tutelary power. Phœbus had the care of Gemini, and hence all the jargon of aſtrologers about the agreement of the ſun and this conſtellation.

The ſtars in the ſign Gemini are, in Ptolomy's catalogue 25, in Tycho's 25, in Hevelius's 38, and in the Britannic catalogue 85.

GENERATED, is uſed by ſome mathematical writers for whatever is produced by arithmetical operation, or in geometry by the motion of other magnitudes. Thus 20 is the product Generated of 4 and 5; ab that of a and b, 4, 8, 16, &c, the powers generated of or from the root 2, and a^2, a^3, a^4, &c, thoſe from the root a. So alſo, a circle is Generated by the revolution of a line about one of its extremities, a cone by the rotation of a right-angled triangle about its perpendicular, a cylinder by the rotation of a rectangle about one of its ſides, or, otherwiſe, by the motion of a circle in the direction of a right line, and keeping always parallel to itſelf.

GENERATING *Line* or *Figure*, in Geometry, is that which, by any kind of ſuppoſed motion, may generate, or produce, any other figure, plane, or ſolid.

Thus a line, according to Euclid, generates a circle; or a right-angled triangle, a cone; &c; and thus alſo Archimedes ſuppoſes his ſpirals to be generated by the motions of Generating points and lines; the figure thus generated, is called the *Generant*.

It is a general theorem in geometry, that the meaſure of any generant, or figure produced by any kind of motion of any other figure, or Generating quantity, is equal to the product of this Generating quantity drawn into the length of the path deſcribed by its centre of gravity, whatever the kind of motion may be, whether rotatory, or direct, &c.

In the modern analyſis, or fluxions, all ſorts of quantities are conſidered as Generated by ſome ſuch motion, and the quantity hereby generated is called a Fluent.

GENERATION, in Mathematics, is uſed for the formation or production of any geometrical figure, or other quantities. Such as of the figures mentioned in the foregoing articles, or the Generation of equations, curves, ſolids, &c.

GENESIS, in Geometry, means much the ſame as Generation mentioned above, being the formation of a line, ſurface, or ſolid, by the motion or flux of a point, line, or ſurface; as of a globe by the rotation of a ſemi-circle about its diameter, &c.

In the Geneſis of figures, the line or ſurface which moves, is called the *Deſcribent;* and the line round which, or according to which, the revolution or other motion is made, the *Dirigent.*

GENETHLIACI, in Aſtrology, are perſons who erect horoſcopes, or pretend to foretell what ſhall befall a perſon, by means of the ſtars which preſided at his nativity.

The ancients called them Chaldæi, and by the general name Mathematici: accordingly, the ſeveral civil and canon laws, which we find made againſt the mathematicians, only reſpect the Genethliaci, or aſtrologers.

They were expelled Rome, by a formal decree of the ſenate; and yet found ſo much protection from the credulity of the people, that they remained in the city unmoleſted.

Antipater and Archinapolus have ſhewn that Genethliology ſhould rather be founded on the time of the conception than on that of the birth.

GEOCENTRIC, is ſaid of a planet or its orbit, to denote its having the earth for its centre. The moon alone is properly geocentric. And yet the motions of all the planets may be conſidered in reſpect of the earth, or as they appear from the earth, and thence called their Geocentric motions.—Hence alſo the terms Geocentric place, or latitude, or longitude, &c, as explained below.

GEOCENTRIC *Place*, of a planet, is the place where it appears to us, from the earth; or it is a point in the ecliptic, to which a planet, ſeen from the earth, is referred.

GEOCENTRIC *Latitude*, of a planet, is its latitude as ſeen from the earth; or the inclination of a line, connecting the planet and the earth, to the plane of the earth's (or true) ecliptic. Or it is the angle which the ſaid line (connecting the planet and the earth) makes with a line drawn to meet a perpendicular let fall from the planet to the plane of the ecliptic.

GEOCENTRIC *Longitude*, of a planet, is the diſtance meaſured on the ecliptic, in the order of the ſigns, between the Geocentric place and the firſt point of Aries.

GEODESIA, is properly that part of practical geometry that teaches how to divide or lay out lands and fields, among ſeveral owners.

GEODESIA is alſo applied, by ſome writers, to all meaſurements in the field, and as ſynonymous with ſurveying.

GEODESIA is defined by Vitalis, as the art of meaſuring ſurfaces and ſolids, not by imaginary lines, as is done in geometry, but by ſenſible and viſible things; or by the ſun's rays, &c.

GEOGRAPHER, a perſon ſkilled in Geography.

GEOGRAPHICAL, ſomething relating to Geography, as

GEOGRAPHICAL *Mile*, which is the ſea-mile or minute, being the 60th part of a degree of a great circle.

GEOGRAPHICAL *Table*. See MAP.

GEOGRAPHY, the ſcience that teaches and explains the nature and properties of the earth, as to its figure, place, magnitude, motions, celeſtial appearances, &c, with the various lines, real or imaginary, on its ſurface.

Geography is diſtinguiſhed from Coſmography, as a part from the whole; this latter conſidering the whole viſible world, both heaven and earth. And from Topography and Chorography, it is diſtinguiſhed, as the whole from a part.

Golnitz conſiders Geography as either exterior or interior: but Varenius more juſtly divides it into General and Special; or Univerſal and Particular.

General or *Univerſal* GEOGRAPHY, is that which conſiders the earth in general, without any regard to particular countries, or the affections common to the whole globe: as its figure, magnitude, motion, land, ſea, &c.

Special or *Particular* GEOGRAPHY, is that which contemplates the conſtitution of the ſeveral particular regions, or countries; their bounds, figure, climate, ſeaſons, weather, inhabitants, arts, cuſtoms, language, &c.

Hiſtory of GEOGRAPHY. The ſtudy and practice of Geography muſt have commenced at very early ages of the world. By the accounts we have remaining, it ſeems this ſcience was in uſe among the Babylonians and Egyptians, from whom it paſſed to the Greeks firſt of any Europeans, and from theſe ſucceſſively to the Romans, the Arabians, and the weſtern nations of Europe. Herodotus ſays the Greeks firſt learned the pole, the gnomon, and the 12 diviſions of the day, from the Babylonians. But Pliny and Diogenes Laertius aſſert, that Thales of Miletus, in the 6th century before Chriſt, firſt found out the paſſage of the ſun from tropic to tropic, and it is ſaid was the author of two books, the one on the tropic, and the other on the equinox; both probably determined by means of the gnomon; whence he was led to the diſcovery of the four ſeaſons of the year, which are determined by the equinoxes and ſolſtices; all which however it is likely he learned of the Egyptians, as well as his diviſion of the year into 365 days. This it is ſaid was invented by the ſecond Mercury, ſurnamed Triſmegiſtus, who, according to Euſebius, lived about 50 years after the Exodus. Pliny expreſsly ſays that this diſcovery was made by obſerving when the ſhadow returned to its marks; a clear proof that it was done by the gnomon. It is farther ſaid that Thales conſtructed a globe, and repreſented the land and ſea upon a table of braſs. Farther that Anaximander, a diſciple of Thales, firſt drew the figure of the earth upon a globe; and that Hecate, Democritus, Eudoxus, and others, formed Geographical maps, and brought them into common uſe in Greece.

Meton and Euctemon obſerved the ſummer ſolſtice at Athens, on the 27th of June 432 years before Chriſt, by watching narrowly the ſhadow of the gnomon, with the deſign of fixing the beginning of their cycle of 19 years.

Timocharis and Ariſtillus, who began their obſervations about 295 B. C., it ſeems firſt attempted to fix the latitudes and longitudes of the fixed ſtars, by conſidering their diſtances from the equator, &c. One of their obſervations gave riſe to the diſcovery of the preceſſion of the equinoxes, which was firſt remarked by Hipparchus about 150 years after: who alſo made uſe of their method, for delineating the parallels of latitude and the meridians, on the ſurface of the earth; thus laying the foundation of this ſcience as it now appears.

The latitudes and longitudes, thus introduced by Hipparchus, were not however much attended to till Ptolomy's time. Strabo, Vitruvius, and Pliny, have all of them entered into a minute geographical deſcription of the ſituation of places, according to the length of the ſhadows of the gnomon, without noticing the longitudes and latitudes.

Maps at firſt were little more than rude outlines, and topographical ſketches of different countries. The earlieſt on record were thoſe of Seſoſtris, mentioned by Euſtathius; who ſays, that "this Egyptian king, having traverſed great part of the earth, recorded his march in maps, and gave copies of them not only to the Egyptians, but to the Scythians, to their great aſtoniſhment." Some have imagined with much probability, that the Jews made a map of the Holy Land, when they gave the different portions to the nine tribes at Shiloh: for Joſhua tells us that they were ſent to walk through the land, and that they *deſcribed it in ſeven parts in a book*; and Joſephus relates that when Joſhua ſent out people from the different tribes to meaſure the land, he gave them, as companions, perſons well ſkilled in geometry, who could not be miſtaken in the truth.

The firſt Grecian map on record, was that of Anaximander, mentioned by Strabo, lib. 1, p. 7, ſuppoſed to be the one referred to by Hipparchus under the deſignation of the *ancient map*. Herodotus minutely deſcribes a map made by Ariſtagoras tyrant of Miletus, which will ſerve to give ſome idea of the maps of thoſe times. He relates, that Ariſtagoras ſhewed it to Cleomenes king of Sparta, to induce him to attack the king of Perſia at Suſa, in order to reſtore the Ionians to their ancient liberty. It was traced upon braſs or copper, and ſeems to have been a mere itinerary, containing the route through the intermediate countries which were to be traverſed in that march, with the rivers Halys, the Euphrates, and Tigris, which Herodotus mentions as neceſſary to be croſſed in that expedition. It contained one ſtraight line called the *Royal Road* or *Highway*, which took in all the ſtations or places of encampment from Sardis to Suſa; being 111 in the whole journey, and containing 13,500 ſtadia, or 1687½ Roman miles of 5000 feet each.

Theſe itinerary maps of the places of encampment were indiſpenſably neceſſary in all armies and marches; and indeed war and navigation ſeem to be the two grand cauſes of the improvements both in Geography and aſtronomy. Athenæus quotes Bæton as author of a work intitled, *The encampments of Alexander's march*; and likewiſe Amyntas to the ſame purpoſe. Pliny obſerves that Diognetus and Bæton were the ſurveyors of Alexander's marches, and then quotes the exact number of miles according to their menſuration; which he afterwards confirms by the letters of Alexander himſelf. The ſame author alſo remarks that a copy of this great monarch's ſurveys was given by Xenocles his treaſurer to Patrocles the geographer, who was admiral of the fleets of Seleucus and Antiochus. His book on geography is often quoted both by Strabo and Pliny; and it ſeems that this author furniſhed Eratoſthenes with the principal materials for conſtructing his map of the oriental part of the world.

Eratoſthenes firſt attempted to reduce Geography to a regular ſyſtem, and introduced a regular parallel of latitude, which began at the ſtraits of Gibraltar, paſſed eaſtwards through the iſle of Rhodes, and ſo on to the mountains of India, noting all the intermediate places through which it paſſed. In drawing this line, he was not regulated by the ſame latitude, but by obſerving where the longeſt day was 14 hours and a half, which Hipparchus afterwards determined was the latitude of 36 degrees.

This firſt parallel through Rhodes was ever after conſidered with a degree of preference, in conſtructing all the ancient maps; and the longitude of the then known world was often attempted to be meaſured in ſtadia and miles, according to the extent of that line, by many ſucceeding geographers.

Eratoſthenes ſoon after attempted not only to draw other parallels of latitude, but alſo to trace a meridian at right angles to theſe, paſſing through Rhodes and Alexandria, down to Syene and Meroë; and at length he undertook the arduous taſk of determining the circumference of the globe, by an actual meaſurement of a ſegment of one of its great circles. To find the magnitude of the earth, is indeed a problem which has engaged the attention of aſtronomers and geographers ever ſince the ſpherical figure of it was known. It ſeems Anaximander was the firſt among the Greeks who wrote upon this ſubject. Archytas of Tarentum, a Pythagorean, famous for his ſkill in mathematics and mechanics, alſo made ſome attempts in this way; and Dr. Long conjectures that theſe are the authors of the moſt ancient opinion that the circumference of the earth is 400,000 ſtadia: and Archimedes makes mention of the ancients who eſtimated the circumference of the earth at only 30,000 ſtadia.

As to the methods of meaſuring the circumference of the earth, it would ſeem, from what Ariſtotle ſays in his treatiſe De Cœlo, that they were much the ſame as thoſe uſed by the moderns, deficient only in the accuracy of the inſtruments. That philoſopher there ſays, that different ſtars paſs through our zenith, according as our ſituation is more or leſs northerly; and that in the ſouthern parts of the earth ſtars come above our horizon, which are no longer viſible if we go northward. Hence it appears that there are two ways of meaſuring the circumference of the earth; one by obſerving ſtars which paſs through the zenith of one place, and do not paſs through that of another; the other, by obſerving ſome ſtars which come above the horizon of one place, and are obſerved at the ſame time to be in the horizon of another. The former of theſe methods, which is the beſt, was followed by Eratoſthenes at Alexandria in Egypt, 250 years before Chriſt. He knew that at the ſummer ſolſtice, the ſun was vertical to the inhabitants of Syene, a town on the confines of Ethiopia, under the tropic of cancer, where they had a well made to obſerve it, at the bottom of which the rays of the ſun fell perpendicularly the day of the ſummer ſolſtice: he obſerved by the ſhadow of a wire ſet perpendicularly in an hemiſpherical baſon, how far the ſun was on that day at noon diſtant from the zenith of Alexandria; when he found that diſtance was equal to the 50th part of a great circle in the heavens. Then ſuppoſing Syene and Alexandria under the ſame meridian, he inferred that the diſtance between them was the 50th part of a great circle upon the earth; and this diſtance being by meaſure 5000 ſtadia, he concluded that the whole circumference of the earth was 250,000 ſtadia. But as this number divided by 360 would give 694$\frac{4}{9}$ ſtadia to a degree, either Eratoſthenes himſelf or ſome of his followers aſſigned the round number 700 ſtadia to a degree; which multiplied by 360, makes the circumference of the earth 252,000 ſtadia; whence both theſe meaſures are given by different authors, as that of Eratoſthenes.

In the time of Pompey the Great, Poſidonius determined the meaſure of the circumference of the earth by the 2d method above hinted by Ariſtotle, viz, the horizontal obſervations. Knowing that the ſtar called Canopus was but juſt viſible in the horizon of Rhodes, and at Alexandria finding its meridian height was the 48th part of a great circle in the heavens, or 7½ deg., anſwering to the like quantity of a circle on the earth: Then, ſuppoſing theſe two places under the ſame meridian, and the diſtance between them 5000 ſtadia, the circumference of the earth will be 240,000 ſtadia; which is the firſt meaſure of Poſidonius. But according to Strabo, Poſidonius made the meaſure of the earth to be 180,000 ſtadia, at the rate of 500 ſtadia to a degree. The reaſon of this difference is thought to be, that Eratoſthenes meaſured the diſtance between Rhodes and Alexandria, and found it only 3750 ſtadia: taking this for a 48th part of the earth's circumference, which is the meaſure of Poſidonius, the whole circumference will be 180,000 ſtadia. This meaſure was received by Marinus of Tyre, and is uſually aſcribed to Ptolomy. But this meaſurement is ſubject to great uncertainty, both on account of the great refraction of the ſtars near the horizon, the difficulty of meaſuring the diſtance at ſea between Rhodes and Alexandria, and by ſuppoſing thoſe places under the ſame meridian, when they are really very different.

Several geographers afterwards made uſe of the different heights of the pole in diſtant places under the ſame meridian, to find the dimenſions of the earth. About the year 800, the khalif Almamun had the diſtance meaſured between two places that were 2 degrees aſunder, and under the ſame meridian in the plains of Sinjar near the Red Sea. And the reſult was, that the degree at one time was found equal to 56 miles, and at another 56⅓ or 56⅔ miles.

The next attempt to find out the circumference of the earth, was in 1525, by Fernelius, a learned philoſopher of France. For this purpoſe, he took the height of the pole at Paris, going from thence directly northwards, till he came to the place where the height of the pole was one degree more than at that city. The length of the way was meaſured by the number of revolutions made by one of the wheels of his carriage; and after proper allowances for the declivities and turnings of the road, he concluded that 68 Italian miles were equal to a degree on the earth.

According to theſe methods many other meaſurements of the earth's circumference have ſince that time been made, with much greater accuracy: a particular account of which is given under the article DEGREE.

Though the maps of Eratoſthenes were the beſt of his time, they were yet very imperfect and inaccurate. They contained little more than the ſtates of Greece, and the dominions of the ſucceſſors of Alexander, digeſted according to the ſurveys abovementioned. He had indeed ſeen, and has quoted, the voyages of Pythias into the great Atlantic ocean, which gave him ſome faint idea of the weſtern parts of Europe; but ſo imperfect, that they could not be realized into the outlines of a chart. Strabo ſays he was very ignorant of Gaul, Spain, Germany, and Britain; and he was equally ignorant of Italy, the coaſts of the Adriatic, Pontus, and all the countries towards the north.

Such was the ſtate of Geography, and the nature of the maps, before the time of Hipparchus. He made a cloſer connection between Geography and aſtronomy, by determining the latitudes and longitudes from celeſtial obſervations.

War has uſually been the occaſion of making or improving the maps of countries; and accordingly Geography made great advances from the progreſs of the Roman arms. In all the provinces occupied by that people, camps were every where conſtructed at proper intervals, and good roads made for communication between them; and thus civilization and ſurveying were carried on according to ſyſtem, through the whole extent of that large empire. Every new war produced a new ſurvey and itinerary of the countries where the ſcenes of action paſſed; ſo that the materials of Geography were accumulated by every additional conqueſt. Polybius ſays, that at the beginning of the ſecond Punic war, when Hannibal was preparing his expedition againſt Rome, the countries through which he was to paſs were carefully meaſured by the Romans. And Julius Cæſar cauſed a general ſurvey of the Roman Empire to be made, by a decree of the ſenate. Three ſurveyors had this taſk aſſigned them, which they completed in 25 years. The Roman itineraries that are ſtill extant, alſo ſhew what care and pains they had been at in making ſurveys in all the different provinces of their empire; and Pliny has filled the 3d, 4th, and 5th books of his Natural Hiſtory with the Geographical diſtances that were thus meaſured. Other maps are alſo ſtill preſerved, known by the name of the Pentingerian Tables, publiſhed by Welſer and Bertius, which give a good ſpecimen of what Vegetius calls the *Itinera Picta*, for the better direction of their armies in their march.

The Roman empire had been enlarged to its greateſt extent, and all its provinces well known and ſurveyed, when Ptolomy, about 150 years after Chriſt, compoſed his ſyſtem of Geography. The chief materials he employed in compoſing this work, were the proportions of the gnomon to its ſhadow, taken by different aſtronomers at the times of the equinoxes and ſolſtices; calculations founded on the length of the longeſt days; the meaſured or computed diſtances of the principal roads contained in their ſurveys and itineraries; and the various reports of travellers and navigators. All theſe were compared together, and digeſted into one uniform body or ſyſtem; and afterwards were tranſlated by him into a new mathematical language, expreſſing the different degrees of latitude and longitude, after the invention of Hipparchus, which had been neglected for 250 years.

Ptolomy's ſyſtem of Geography, notwithſtanding it was ſtill very imperfect, continued in vogue till the laſt three or four centuries, within which time the great improvements in aſtronomy, the many diſcoveries of new countries by voyagers, and the progreſs of war and arms, have contributed to bring it to a very conſiderable degree of perfection; the particulars of which will be found treated under their reſpective articles in this work.

Among the moderns, the chief authors on the ſubject of Geography are Johannes de Sacroboſco, or John Hallifax, who wrote a treatiſe on the ſphere; Sebaſtian Munſter, in his Coſmographia Univerſalis, in 1559; Clavius, on the ſphere of Sacroboſco; Piccioli's Geographia et Hydrographia Reformata; Weigelius's Speculum Terræ; De Chales's Geography, in his Mundus Mathematicus; Cellarius's Geography; Cluverius's Introductio in Univerſam Geographiam; Leibnecht's Elementa Geographiæ generalis; Stevenius's Compendium Geographicum; Wolſius's Geographia, in his Elementa Matheſeos; Buſching's New Syſtem of Geography; Gordon's, Salmon's, and Guthrie's Grammars; and above all, Varenius's Geographia generalis, with Jurin's additions, the moſt ſcientific and ſyſtematical of any.

GEOMETER, more properly GEOMETRICIAN; which ſee.

GEOMETRICAL, ſomething that has a relation to Geometry, or done after the manner, or by the means of Geometry. As, a Geometrical conſtruction, a Geometrical curve, a Geometrical demonſtration, genius, line, method, Geometrical ſtrictneſs, &c.

GEOMETRICAL *Conſtruction*, of an equation, is the drawing of lines and figures, ſo as to expreſs by them the ſame general property and relation, as are denoted by the algebraical equation. See CONSTRUCTION *of Equations*.

GEOMETRICAL *Curve* or *Line*, called alſo an *Algebraical one*, is that in which the relations between the abſciſſes and ordinates may be expreſſed by a finite algebraical equation. See ALGEBRAICAL *Curves*.

GEOMETRICAL *Lines*, as obſerved by Newton, are diſtinguiſhed into claſſes, orders, or genera, according to the number of the dimenſions of the equation that expreſſes the relation between the ordinates and abſciſſes; or, which comes to the ſame thing, according to the number of points in which they may be cut by a right line.

Thus, a line of the firſt order, is a right line, ſince it can be only once cut by another right line, and is expreſſed by the ſimple equation $y + ax + b = 0$: thoſe of the 2d, or quadratic order, will be the circle, and the conic ſections, ſince all of theſe may be cut in two points by a right line, and expreſſed by the equation $y^2 + \overline{ax + b} \,.\, y + cx^2 + dx + e = 0$: thoſe of the 3d or cubic order, will be ſuch as may be cut in 3 points by a right line, whoſe moſt general equation is $y^3 + \overline{ax + b} \,.\, y^2 + \overline{cx^2 + dx + e} \,.\, y + fx^3 + gx^2 + hx + i = 0$; as the cubical and Neilian parabola, the ciſſoid, &c. And a line of an infinite order, is that which a right line may cut in infinite points; as the ſpiral, the cycloid, the quadratrix, and every line that is generated by the infinite revolutions of a radius, or circle, or wheel, &c.

In each of thoſe equations, x is the abſciſs, y its correſponding ordinate, making any given angle with it; and a, b, c, &c, are given or conſtant quantities, affected with their ſigns + and −, of which one or more may vaniſh, be wanting or equal to nothing, provided that by ſuch defect the line or equation does not become one of an inferior order.

It is to be noted that a curve of any kind is denominated by a number next leſs than the line of the ſame kind: thus, a curve of the 1ſt order, (becauſe the right line cannot be reckoned among curves) is the ſame with a line of the 2d order; and a curve of the 2d kind, the ſame with a line of the 3d order; &c.

It is to be obſerved alſo, that it is not ſo much the equation, as the conſtruction or deſcription, that makes any curve, geometrical, or not. Thus, the circle is a geometrical line, not becauſe it may be expreſſed by an equation, but becauſe its deſcription is a poſtulate: and it is not the ſimplicity of the equation, but the eaſineſs of the deſcription, that is to determine the choice of the lines for the conſtruction of a problem. The equation that expreſſes a parabola, is more ſimple than that which expreſſes a circle; and yet the circle, by reaſon of its more ſimple conſtruction, is admitted before it. Again, the circle and the conic ſections, with reſpect to the dimenſions of the equations, are of the ſame order; and yet the circle is not numbered with them in the conſtruction of problems, but by reaſon of its ſimple deſcription is depreſſed to a lower order, viz, that of a right line; ſo that it is not improper to expreſs that by a circle, which may be expreſſed by a right line; but it is a fault to conſtruct that by the conic ſections, which may be conſtructed by a circle.

Either, therefore, the law muſt be taken from the dimenſions of equations, as obſerved in a circle, and ſo the diſtinction be taken away between plane and ſolid problems: or the law muſt be allowed not to be ſtrictly obſerved in lines of ſuperior kinds, but that ſome, by reaſon of their more ſimple deſcription, may be preferred to others of the ſame order, and be numbered with lines of inferior orders.

In conſtructions that are equally Geometrical, the moſt ſimple are always to be preferred: and this law is ſo univerſal as to be without exception. But algebraical expreſſions add nothing to the ſimplicity of the conſtruction; the bare deſcriptions of the lines here are only to be conſidered; and theſe alone were conſidered by thoſe geometricians who joined a circle with a right line,

line. And as theſe are eaſy or hard, the conſtruction becomes eaſy or hard: and therefore it is foreign to the nature of the thing, from any other circumſtance to eſtabliſh laws relating to conſtructions.

Either, therefore, with the ancients, we muſt exclude all lines beſide the circle, and perhaps the conic ſections, out of geometry; or admit all, according to the ſimplicity of the deſcription. If the trochoid were admitted into geometry, by means of it we might divide an angle in any given ratio; would it be right therefore to blame thoſe who would make uſe of this line to divide an angle in the ratio of one number to another; and contend, that you muſt make uſe only of ſuch lines as are defined by equations, and therefore not of this line, which is not ſo defined? If, when an angle is propoſed to be divided, for inſtance, into 10001 parts, we ſhould be obliged to bring a curve defined by an equation of more than 100 dimenſions to do the buſineſs; which nobody could deſcribe, much leſs underſtand; and ſhould prefer this to the trochoid, which is a line well known, and eaſily deſcribed by the motion of a wheel, or circle: who would not ſee the abſurdity?

Either therefore the trochoid is not to be admitted at all in geometry; or elſe, in the conſtruction of problems, it is to be preferred to all lines of a more difficult deſcription; and the reaſon is the ſame for other curves. Hence the triſections of an angle by a conchoid, which Archimedes in his Lemmas, and Pappus in his Collections, have preferred to the inventions of all others in this caſe, muſt be allowed as good; becauſe we muſt either exclude all lines, beſide the circle and right line, out of geometry, or admit them according to the ſimplicity of their deſcriptions; in which caſe the conchoid yields to none, except the circle. Equations are expreſſions of arithmetical computation, and properly have no place in geometry, excepting ſo far as quantities truly Geometrical (that is, lines, ſurfaces, ſolids, and proportions) may be ſaid to be ſome equal to others. Multiplications, diviſions, and ſuch like computations, are newly received into Geometry, and that unwarily, and contrary to the firſt deſign of this ſcience. For whoever conſiders the conſtruction of problems by a right line and a circle, found out by the firſt geometricians, will eaſily perceive that geometry was introduced, that by drawing lines, we might eaſily avoid the tediouſneſs of computation. For which reaſon the two ſciences ought not to be confounded together: the ancients ſo carefully diſtinguiſhed between them, that they never introduced arithmetical terms into geometry; and the moderns, by confounding them, have loſt the ſimplicity in which all the elegance of geometry conſiſts. In ſhort, that is arithmetically more ſimple, which is determined by the more ſimple equations; but that is Geometrically more ſimple, which is determined by the more ſimple drawing of lines; and in geometry that ought to be reckoned beſt, which is geometrically moſt ſimple. Newton's Arith. Univerſ. appendix. See CURVES.

GEOMETRICAL *Locus*, or *Place*, called alſo ſimply *Locus*, is the path or track of ſome certain Geometrical determination, in which it always falls. See LOCUS.

GEOMETRICAL *Medium*. See MEDIUM.

GEOMETRICAL *Method of the Ancients*. The ancients eſtabliſhed the higher parts of their geometry on the ſame principles as the elements of that ſcience, by demonſtrations of the ſame kind: and they were careful not to ſuppoſe any thing done, till by a previous problem they had ſhewn that it could be done by actually performing it. Much leſs did they ſuppoſe any thing to be done that cannot be conceived; ſuch as a line or ſeries to be actually continued to infinity, or a magnitude diminiſhed till it become infinitely leſs than what it is. The elements into which they reſolved magnitudes were finite, and ſuch as might be conceived to be real. Unbounded liberties have of late been introduced; by which geometry, which ought to be perfectly clear, is filled with myſteries. Maclaurin's Fluxions, Introd. p. 39.

GEOMETRICAL *Pace*, is a meaſure of 5 feet long.

GEOMETRICAL *Plan*, in Architecture. See PLAN.

GEOMETRICAL *Plane*. See PLANE.

GEOMETRICAL *Progreſſion*, a progreſſion in which the terms have all ſucceſſively the ſame ratio: as 1, 2, 4, 8, 16, &c, where the common ratio is 2.

The general and common property of a Geometrical progreſſion is, that the product of any two terms, or the ſquare of any one ſingle term, is equal to the product of every other two terms that are taken at an equal diſtance on both ſides from the former. So of theſe terms,

$$1,\ 2,\ 4,\ 8,\ 16,\ 32,\ 64,\ \&c,$$

$$1 \times 64 = 2 \times 32 = 4 \times 16 = 8 \times 8 = 64.$$

In any Geometrical Progreſſion, if

a denote the leaſt term,

z the greateſt term,

r the common ratio,

n the number of the terms,

s the ſum of the ſeries, or all the terms;

then any of theſe quantities may be found from the others, by means of theſe general values, or equations, viz,

$$r = \sqrt[n-1]{\frac{z}{a}},$$

$$z = a \times r^{n-1},$$

$$a = z \div r^{n-1},$$

$$n = \frac{\log.\ \frac{rz}{a}}{\log.\ r} = \frac{\log.\ r + \log.\ z - \log.\ a}{\log.\ r},$$

$$s = \frac{r^n - 1}{r - 1} \times a = \frac{r^n - 1}{r - 1} \times \frac{z}{r^{n-1}} = \frac{rz - a}{r - 1} = \frac{z^{\frac{n}{n-1}} - a^{\frac{n}{n-1}}}{z^{\frac{1}{n-1}} - a^{\frac{1}{n-1}}}.$$

When the ſeries is infinite, then the leaſt term a is nothing, and the ſum $s = \frac{rz}{r-1}$. See alſo PROGRESSION.

GEOMETRICAL *Proportion*, called alſo ſimply *Proportion*, is the ſimilitude or equality of ratios.

Thus, if $a : b :: c : d$, or $a : b = c : d$, the terms a, b, c, d are in Geometrical Proportion; alſo 6, 3, 14, 7, are in Geometrical Proportion, becauſe 6 : 3 :: 14 : 7, or 6 : 3 = 14 : 7.

In a Geometrical Proportion, the product of the extremes, or 1ſt and 4th terms, is equal to the product of the means, or the 2d and 3d terms: ſo $ad = bc$, and $6 \times 7 = 3 \times 14 = 42$. See PROPORTION.

GEOMETRICAL *Solution*, of a problem, is when the problem is directly reſolved according to the ſtrict rules and principles of geometry, and by lines that are truly Geometrical. This expreſſion is uſed in contradiſtinction to an arithmetical, or a mechanical, or inſtrumental ſolution; the problem being reſolved only by a ruler and compaſſes.

The ſame term is likewiſe uſed in oppoſition to all indirect and inadequate kinds of ſolutions, as by approximation, infinite ſeries, &c. So, we have no Geometrical way of finding the quadrature of the circle, the duplicature of the cube, or two mean proportionals; though there are mechanical ways, and others, by infinite ſeries, &c.

Pappus informs us, that the ancients endeavoured in vain to triſect an angle, and to find out two mean proportionals, by means of the right line and circle. Afterwards they began to conſider the properties of ſeveral other lines; as the conchoid, the ciſſoid, and the conic ſections; and by ſome of theſe they endeavoured to reſolve ſome of thoſe problems. At length, having more thoroughly examined the matter, and the conic ſections being received into geometry, they diſtinguiſhed Geometrical problems and ſolutions into three kinds; viz,

1. *Plane* ones, which, deriving their origin from lines on a plane, may be properly reſolved by a right line and a circle.—2. *Solid* ones, which are reſolved by lines deriving their original from the conſideration of a ſolid; that is, of a cone.—3. *Linear* ones, to the ſolution of which are required lines more compounded.

According to this diſtinction, we are not to reſolve ſolid problems by other lines than the conic ſections; eſpecially if no other lines beſide the right line, circle, and the conic ſections, muſt be received into geometry.

But the moderns, advancing much farther, have received into geometry all lines that can be expreſſed by equations; and have diſtinguiſhed, according to the dimenſions of the equations, thoſe lines into claſſes or orders; and have laid it down as a law, not to conſtruct a problem by a line of a higher order, that may be conſtructed by one of a lower.

GEOMETRICIAN, a perſon ſkilled in Geometry.

GEOMETRY, the ſcience or doctrine of local extenſion, as of lines, ſurfaces, and ſolids, with that of ratios, &c.

The name Geometry literally ſignifies meaſuring of the earth, as it was the neceſſity of meaſuring the land that firſt gave occaſion to contemplate the principles and rules of this art; which has ſince been extended to numberleſs other ſpeculations; inſomuch that together with arithmetic, Geometry forms now the chief foundation of all the mathematics.

Herodotus (lib. 2), Diodorus (lib 1), Strabo (lib. 17), and Proclus, aſcribe the invention of Geometry to the Egyptians, and aſſert that the annual inundations of the Nile gave occaſion to it; for thoſe waters bearing away the bounds and land-marks of eſtates and farms, covering the face of the ground uniformly with mud, the people, ſay they, were obliged every year to diſtinguiſh and lay out their lands by the conſideration of their figure and quantity; and thus by experience and habit they formed a method or art, which was the origin of Geometry. A farther contemplation of the draughts of figures of fields thus laid down, and plotted in proportion, might naturally lead them to the diſcovery of ſome of their excellent and wonderful properties; which ſpeculation continually improving, the art continually gained ground, and made advances more and more towards perfection.

Joſephus however ſeems to aſcribe the invention to the Hebrews: and others of the ancients make Mercury the inventor. Polyd. Virgil, de Invent. Rer. lib. 1, cap. 18.

From Egypt, this ſcience paſſed into Greece, being carried thither by Thales; where it was much cultivated and improved by himſelf, as alſo by Pythagoras, Anaxagoras of Clazomene, Hippocrates of Chios, and Plato, who teſtified his conviction of the neceſſity and importance of Geometry to the ſucceſsful ſtudy of philoſophy, by this inſcription over the door of his academy, *Let no one ignorant of Geometry enter here.* Plato thought the word Geometry too mean a name for this ſcience, and ſubſtituted inſtead of it the more extenſive name of *Menſuration*; and after him others gave it the title of *Pantometry*. But even theſe are now become too ſcanty in their import, fully to comprehend its extent; for it not only inquires into, and demonſtrates the quantities of magnitudes, but alſo their qualities, as the ſpecies, figures, ratios, poſitions, transformations, deſcriptions, diviſions, the finding of their centres, diameters, tangents, aſymptotes, curvature, &c. Some again define it as the ſcience of inquiring, inventing, and demonſtrating all the affections of magnitude. And Proclus calls it the knowledge of magnitudes and figures, with their limitations; as alſo of their ratios, affections, poſitions, and motions of every kind.

About 50 years after Plato, lived Euclid, who collected together all thoſe theorems which had been invented by his predeceſſors in Egypt and Greece, and digeſted them into 15 books, called the Elements of Geometry; demonſtrating and arranging the whole in a very accurate and perfect manner. The next to Euclid, of thoſe ancient writers whoſe works are extant, is Apollonius Pergæus, who flouriſhed in the time of Ptolomy Euergetes, about 230 years before Chriſt, and about 100 years after Euclid. He was author of the firſt and principal work on Conic Sections; on account of which, and his other accurate and ingenious geometrical writings, he acquired from his patron the emphatical appellation of *The Great Geometrician.* Contemporary with Apollonius, or perhaps a few years before him, flouriſhed Archimedes, celebrated for his mechanical inventions at the ſiege of Syracuſe, and not leſs ſo for his very many ingenious Geometrical compoſitions.

We can only mention Eudoxus of Cnidus, Archytas of Tarentum, Philolaus, Eratoſthenes, Ariſtarchus of Samos, Dinoſtratus, the inventor of the quadratrix, Menechmus, his brother and the diſciple of Plato, the two Ariſteus's, Conon, Thracidius, Nicoteles, Leon, Theudius, Hermotimus, Hero, and Nicomedes the inventor of the conchoid : beſides whom, there are many other ancient geometricians, to whom this ſcience has been indebted.

The Greeks continued their attention to it, even after they were ſubdued by the Romans. Whereas the Romans themſelves were ſo little acquainted with it, even in the moſt flouriſhing time of their republic, that Tacitus informs us they gave the name of mathematicians to thoſe who purſued the chimeras of divination and judicial aſtrology. Nor does it appear they were more diſpoſed to cultivate Geometry during the decline, and after the fall of the Roman empire. But the caſe was different with the Greeks ; among whom are found many excellent Geometricians ſince the commencement of the Chriſtian era, and after the tranſlation of the Roman empire. Ptolomy lived under Marcus Aurelius ; and we have ſtill extant the works of Pappus of Alexandria, who lived in the time of Theodoſius ; the commentary of Eutocius, the Aſcalonite, who lived about the year of Chriſt 540, on Archimedes's menſuration of the circle ; and the commentary on Euclid, by Proclus, who lived under the empire of Anaſtaſius.

The conſequent inundation of ignorance and barbariſm was unfavourable to Geometry, as well as to the other ſciences ; and the few who applied themſelves to this ſcience, were calumniated as magicians. However, in thoſe times of European darkneſs, the Arabians were diſtinguiſhed as the guardians and promoters of ſcience ; and from the 9th to the 14th century, they produced many aſtronomers, geometricians, geographers, &c ; from whom the mathematical ſciences were again received into Spain, Italy, and the reſt of Europe, ſomewhat before the year 1400. Some of the earlieſt writers after this period, are Leonardus Piſanus, Lucas Paciolus or De Burgo, and others between 1400 and 1500. And after this appeared many editions of Euclid, or commentaries upon him : thus, Orontius Finæus, in 1530, publiſhed a commentary on the firſt 6 books ; as did James Peletarius, in 1557 ; and about the ſame time Nicholas Tartaglia publiſhed a commentary on the whole 15 books. There have been alſo the editions, or commentaries, of Commandine, Clavius, Billingſly, Scheubelius, Herlinus, Daſypodius, Ramus, Herigon, Stevinus, Saville, Barrow, Taquet, Dechales, Furnier, Scarborough, Keill, Stone, and many others ; but the completeſt edition of all the works of Euclid, is that of Dr. Gregory, printed at Oxford 1703, in Greek and Latin : the edition of Euclid, by Dr. Robert Simſon of Glaſgow, containing the firſt 6 books, with the 11th and 12th, is much eſteemed for its correctneſs. The principal other elementary writers, beſides the editors of Euclid, are Borelli, Pardies, Marchetti, Wolfius, Simpſon, &c. And among thoſe who have gone beyond Euclid in the nature of the Elementary parts of Geometry, may be chiefly reckoned, Apollonius, in his Conics, his Loci Plani, De Sectione Determinata, his Tangencies, Inclinations, Section of a Ratio, Section of a Space, &c ; Archimedes, in his treatiſes of the Sphere and Cylinder, the Dimenſion of the Circle, of Conoids and Spheroids, of Spirals, and the Quadrature of the Parabola ; Theodoſius, in his Spherics ; Serenius, in his Sections of the Cone and Cylinder ; Kepler's Nova Stereometria ; Cavalerius's Geometria Indiviſibilium ; Torricelli's Opera Geometrica ; Viviani, in his Divinationes Geometricæ, Exercitatio Mathematica, De Locis Solidis, De Maximis & Minimis, &c ; Vieta, in his Effectio Geometrica, Supplement. Geometriæ, Sectiones Angulares, Reſponſum ad Problema, Apollonius Gallus, &c ; Gregory St. Vincent's Quadratura Circuli ; Fermat's Varia Opera Mathematica ; Dr. Barrow's Lectiones Geometricæ ; Bulliald de Lineis Spiralibus ; Cavalerius ; Schooten and Gregory's Exercitationes Geometricæ, and Gregory's Pars Univerſalis, &c ; De Billy's treatiſe De Proportione Harmonica ; La Lovera's Geometria veterum promota ; Sluſius's Meſolabium, Problemata Solida, &c ; Wallis, in his treatiſes De Cycloide, Ciſſoide, &c ; De Proportionibus, De Sectionibus Conicis, Arithmetica Infinitorum, De Centro Gravitatis, De Sectionibus Angularibus, De Angulo Contactus, Cuno-Cuneus, &c, &c ; Hugo De Omerique, in his Analyſis Geometrica ; Paſcal on the Cycloid ; Step. Angeli's Problemata Geometrica ; Alex. Anderſon's Suppl. Apollonii Redivivi, Variorum Problematum Practice, &c ; Baronius's Geomet. Prob. &c ; Guido Grandi Geometr. Demonſtr. &c ; Ghetaldi Apollonius Redivivus, &c ; Ludolph van Colen or a Ceulen, de Circulo et Adſcriptis, &c ; Snell's Apollonius Batavus, Cyclometricus, &c ; Herberſtein's Diotome Circulorum ; Palma's Exercit. in Geometriam ; Guldini Centro-Baryca ; with ſeveral others equally eminent, of more modern date, as Dr. Rob. Simſon, Dr. Mat. Stewart, Mr. Tho. Simpſon, &c. Since the introduction of the new Geometry, or the Geometry of Curve Lines, as expreſſed by algebraical equations, in this part of Geometry, the following names, among many others, are more eſpecially to be reſpected, viz, Des Cartes, Schooten, Newton, Maclaurin, Brackenridge, Cramer, Cotes, Waring, &c, &c.

As to the ſubject of Practical Geometry, the chief writers are Beyer, Kepler, Ramus, Clavius, Mallet, Tacquet, Ozanam, Wolfius, Gregory, with innumerable others.

Geometry is diſtinguiſhed into Theoretical or Speculative, and Practical.

Theoretical or *Speculative* GEOMETRY, treats of the various properties and relations in magnitudes, demonſtrating the theorems, &c. And

Practical GEOMETRY, is that which applies thoſe ſpeculations and theorems to particular uſes in the ſolution of problems, and in the meaſurements in the ordinary concerns of life.

Speculative Geometry again may be divided into Elementary and Sublime.

Elementary or *Common* GEOMETRY, is that which is employed in the conſideration of right lines and plane ſurfaces, with the ſolids generated from them. And the

Higher or *Sublime* GEOMETRY, is that which is employed in the conſideration of curve lines, conic ſections, and the bodies formed of them. This part has been chiefly cultivated by the moderns, by help of the improved ſtate of Algebra, and the modern analyſis or Fluxions.

GIBBOUS, is uſed for the ſhape of one ſtate of the enlightened part of the moon, being that in which ſhe appears more than half full or enlightened, which is the time between the firſt quarter and the full moon, and from the full moon to the laſt quarter; appearing then Gibbous, that is, bunched out, or convex on both ſides of the enlightened part; as contradiſtinguiſhed from the ſtate when ſhe is leſs than half full, when ſhe is ſaid to be horned, or a creſcent.

GIMBOLS, are the braſs rings by which a ſea-compaſs is ſuſpended in its box that uſually ſtands in the binacle.

GIN, in Artillery and Mechanics, is a machine for raiſing great weights, uſually compoſed of three long legs, &c.

GIRDERS, in Architecture, are the largeſt beams or pieces of timber ſupporting the floors. Their ends are uſually faſtened into the ſummers, or breaſt-ſummers; and the joiſts are framed in at one end to the Girders. By the ſtatute for rebuilding London, no Girder is to lie leſs than 10 inches into the wall, and their ends to be always laid in loam, &c. The ſhorter bearings a Girder has, and the oftener it is ſupported by the internal or partition walls, ſo much the better. The eſtabliſhed breadth and depth of a Girder, according to its length of bearing, are as in the following tablet:

	From	to	muſt be in	
			Breadth	Depth
Girders and Summers in length	10 feet	15 ft.	11 inc.	8 inc.
	15	18	13	9
	18	21	14	10
	21	24	16	12
	24	26	17	14

GIRT, in Timber-meaſuring, is the circumference of a tree; though ſome uſe this word for the quarter or 4th part of the circumference only, on account of the great uſe that is made of it; for the ſquare of this 4th part is eſteemed and uſed as equal to the area of the ſection of the tree; which ſquare therefore multiplied by the length of the tree, is accounted the ſolid content. This content however is always about one-fourth part leſs than the true quantity; being nearly equal to what this will be after the tree is hewed ſquare in the uſual way: ſo that it ſeems intended to make an allowance for the ſquaring of the tree.

GIRT-*Line*, is a line on the common or carpenter's ſliding rule, employed in caſting up the contents of trees by means of their Girt.

GIVEN, *Datum*, a term very often uſed in mathematics, and ſignifies ſomething that is ſuppoſed to be known.

Thus, if a magnitude be known, or if we can find another equal to it, it is ſaid to be Given in magnitude. Or when the poſition of any thing is known, it is ſaid to be Given in poſition. And when the diameter of a circle is known, the circle is Given in magnitude. Or the circle is Given in poſition when its centre is Given in poſition. When the kind or ſpecies of a figure is known, or remains the ſame, it is Given in ſpecie. And ſo on.

Euclid wrote a book of *Data*, or concerning things Given, in 95 propoſitions, uſually accompanying his Elements, in the beſt editions, and which Pappus reckons as one of the beſt ſpecimens of the analytical works of the ancients.

GLACIS, in Fortification, a ſloping bank reaching from the parapet of the counterſcarp, or covered-way, to the level ſide of the field, commonly at the diſtance of about 40 yards.

GLOBE, a round or ſpherical body, more uſually called a ſphere, bounded by one uniform convex ſurface, every point of which is equally diſtant from a point within called its centre. Euclid defines the Globe, or ſphere, to be a ſolid figure deſcribed by the revolution of a ſemi-circle about its diameter, which remains unmoved. Alſo, its axis is the fixed line or diameter about which the ſemi-circle revolves; and its centre is the ſame with that of the revolving ſemicircle, a diameter of it being any right line that paſſes through the centre, and terminated both ways by the ſuperficies of the ſphere. Elem. 11. def. 14, 15, 16, 17.

Euclid, at the end of the 12th book, ſhews that ſpheres are to one another in the triplicate ratio of their diameters, that is, their ſolidities are to one another as the cubes of their diameters. And Archimedes determines the real magnitudes and meaſures of the ſurfaces and ſolidities of ſpheres and their ſegments, in his treatiſe de Sphæra et Cylindro: viz, 1, That the ſuperficies of any Globe is equal to 4 times a great circle of it.—2, That any ſphere is equal to $\frac{2}{3}$ of its circumſcribing cylinder, or of the cylinder of the ſame diameter and altitude.—3, That the curve ſurface of the ſegment of a globe, is equal to the circle whoſe radius is the line drawn from the vertex of the ſegment to the circumference of the baſe.—4, That the content of a ſolid ſector of the Globe, is equal to a cone whoſe altitude is the radius of the Globe, and its baſe equal to the curve ſuperficies or baſe of the ſector. With many other properties. And from hence are eaſily deduced theſe practical rules for the ſurfaces and ſolidities of Globes and their ſegments; viz,

1. *For the Surface of a Globe*, multiply the ſquare of the diameter by 3·1416; or multiply the diameter by the circumference.

2. *For the Solidity of a Globe*, multiply the cube of the diameter by ·5236 (viz $\frac{1}{6}$ of 3·1416); or multiply the ſurface by $\frac{1}{6}$ of the diameter.

3. *For the Surface of a Segment*, multiply the diameter of the Globe by the altitude of the ſegment and the product again by 3·1416.

4. *For the Solidity of a Segment*, multiply the ſquare of the diameter of the Globe by the difference between 3 times that diameter and 2 times the altitude of the ſegment, and the product again by ·5236, or $\frac{1}{6}$ of 3·1416.

Hence, if d denote the diameter of the Globe,
c the circumference,
a the altitude of any ſegment, and
$p = 3{\cdot}1416$; then

	The ſurface.	The ſolidity
In the Globe	$pd^2 = cd$	$\frac{1}{6}pd^3$
In the Segt.	pad	$\frac{1}{6}pa^2 \times \overline{3d-2a}$

See the art. SPHERE, and my Menſuration, p. 197 &c, 2d edit.

The GLOBE, or *Terraqueous* GLOBE, is the body or maſs of the earth and water together, which is nearly globular.

GLOBE, or *Artificial* GLOBE, is more particularly uſed for a Globe of metal, plaiſter, paper, paſteboard, &c, on the ſurface of which is drawn a map, or repreſentation of either the heavens or the earth, with the ſeveral circles conceived upon them. And hence

GLOBES are of two kinds, Terreſtrial, and Celeſtial; which are of conſiderable uſe in geography and aſtronomy, by ſerving to give a lively repreſentation of their principal objects, and for performing and illuſtrating many of their operations in a manner eaſy to be perceived by the ſenſes, and ſo as to be conceived even without any knowledge of the mathematical grounds of thoſe ſciences.

Deſcription of the Globes.

The fundamental parts that are common to both Globes, are an axis, repreſenting the axis of the world, paſſing through the two poles of a ſpherical ſhell, repreſenting thoſe of the world, which ſhell makes the body of the Globe, upon the external ſurface of which is drawn the repreſentation of the whole ſurface of the earth, ſea, rivers, iſlands, &c, for the Terreſtrial Globe, and the ſtars and conſtellations of the heavens, for the Celeſtial one; beſides the equinoctial and ecliptic lines, the zodiac, the two tropics and polar circles, and a number of meridian lines. There is next a brazen meridian, being a ſtrong circle of braſs, circumſcribing the Globe, at a ſmall diſtance from it quite round, in which the globe is hung by its two poles, upon which it turns round within this circle, which is divided into 4 times 90 degrees, beginning at the equator on both ſides, and ending with 90 at the two poles. There are alſo two ſmall hour circles, of braſs, divided into twice 12 hours, and fitted on the meridian round the poles, which carry an index pointing to the hour. The whole is ſet in a wooden ring, placed parallel to, and repreſenting the horizon, in which the Globe ſlides by the braſs meridian, elevating or depreſſing the pole according to any propoſed latitude. There is alſo a thin ſlip of braſs, called a Quadrant of Altitude, made to fit on occaſionally upon the braſs meridian, at the higheſt or vertical point, to meaſure the altitude of any thing above the horizon. A magnetic compaſs is ſometimes ſet underneath. See the figure of the Globes ſo mounted, at fig. 1, plate xii.

Such is the plain and ſimple conſtruction of the artificial Globe, whether celeſtial or terreſtrial, as adapted to the time only for which it is made. But as the angle formed by the equator and ecliptic, as well as their point of interſection, is always changing; to remedy theſe inconveniences, ſeveral contrivances have been made, ſo as to adapt the ſame Globes to any other time, either paſt or to come; as well as other contrivances to anſwer particular purpoſes.

Thus, Mr. Senex, a celebrated maker of Globes, had a contrivance which, by means of a nut and ſcrew, cauſed the pole of the equator to revolve about the pole of the ecliptic, by any quantity anſwering to the preceſſion of the equinoxes, ſince the time for which the Globe was made. Philoſ. Tranſ. number 447, or Abr. vol. 8, p. 217, alſo Philoſ. Tranſ. vol. 46, p. 290.

Mr. Joſeph Harris, late aſſay-maſter of the Mint, made ſome contrivances to ſhew the effects of the earth's motions. He fixed two horary circles under the braſs meridian, to the axis, one at each pole, ſo as to turn round with the Globe, and that meridian ſerved as an index to cut the horary diviſions. The Globe in this ſtate ſerves equally for reſolving problems in both north and ſouth latitudes, as alſo in places near the equator; whereas, in the common conſtruction, the axis and horary circle prevent the braſs meridian from being moveable quite round in the horizon. This Globe is alſo adapted for ſhewing how the viciſſitudes of day and night, and the alteration of their lengths, are really occaſioned by the motion of the earth: for this purpoſe, he divides the braſs meridian, at one of the poles, into months and days, according to the ſun's declination, reckoning from the pole. Therefore, by bringing the day of the month to the horizon, and rectifying the Globe according to the time of the day, the horizon will repreſent the circle ſeparating light and darkneſs, and the upper half of the Globe the illuminated hemiſphere, the ſun being in the zenith. Mr. Harris alſo gives an account of a cheap machine for ſhewing how the annual motion of the earth in its orbit cauſes the change of the ſun's declination, without the great expence of an orrery. Philoſ. Tranſ. number 456, or Abr. vol. 8, p. 352.

The late Mr. George Adams made alſo ſome uſeful improvements in the conſtruction of the Globes. Beſides what is uſual, his Globes have a thin braſs ſemicircle moveable about the poles, with a ſmall thin ſliding circle upon it. On the terreſtrial Globe, the former of theſe is a moveable meridian, and the latter is the viſible horizon of any particular place to which it is ſet. But on the celeſtial Globe, the ſemi-circle is a moveable circle of declination, and its ſmall annexed circle an artificial ſun or planet. Each Globe has a braſs wire circle, placed at the limits of the twilight. The terreſtrial Globe has many additional circles, as well as the rhumb-lines, for reſolving all the neceſſary geographical and nautical problems: and on the celeſtial Globe are drawn, on each ſide of the ecliptic, 8 parallel circles, at the diſtance of one degree from each other, including the zodiac; which are croſſed at right-angles by ſegments of great circles at every 5th degree of the ecliptic, for the more readily noting the place of the moon or of any planet upon the Globe. On the ſtrong braſs circle of the terreſtrial Globe, and about 23½ degrees on each ſide of the north pole, the days of each month are laid down according to the ſun's declination: and this braſs circle is ſo contrived, that the Globe may be placed with the north and ſouth poles in the plane of the horizon, and with the ſouth pole elevated above it. The equator, on the ſurface of either Globe, ſerves the purpoſe of the horary circle, by means of a ſemi-circular wire placed in the plane of the equator, carrying two indices, one of which is occaſionally to be uſed to point out the time. For a farther account of theſe Globes, with the method of uſing them, ſee Mr. Adams's treatiſe on their conſtruction and uſe.

There are alſo what are called Patent Globes, made by Mr. Neale; by means of which he reſolves ſeveral aſtronomical problems, which do not admit of ſolution by the common Globes.

Mr. Ferguſon likewiſe made ſeveral improvements of the Globes, particularly one for conſtructing dials, and another called a planetary Globe. See Philoſ. Tranſ. vol. 44, p. 535, and Ferguſon's Aſtron. p. 291, and 292.

Laſtly, in the Philoſ. Tranſ. for 1789, vol. 79, p. 1, Mr. Smeaton has propoſed ſome improvements of the celeſtial Globe, eſpecially with reſpect to the quadrant of altitude, for the reſolution of problems relating to the azimuth and altitude. The difficulty, he obſerves, that has occurred in fixing a ſemicircle, ſo as to have a centre in the zenith and nadir points of the Globe, at the ſame time that the meridian is left at liberty to raiſe the pole to its deſired elevation, I ſuppoſe, has induced the Globe-makers to be contented with the ſtrip of thin flexible braſs, called the quadrant of altitude; and it is well known how imperfectly it performs its office. The improvement I have attempted, is in the application of a quadrant of altitude of a more ſolid conſtruction; which being affixed to a braſs ſocket of ſome length, and this ground, and made to turn upon an upright ſteel ſpindle, fixed in the zenith, ſteadily directs the quadrant, or rather arc, of altitude to its true azimuth, without being at liberty to deviate from a vertical circle to the right hand or left: by which means the azimuth and altitude are given with the ſame exactneſs as the meaſure of any other of the great circles. For a more particular deſcription of this improvement, illuſtrated with figures, ſee the place above quoted.

The uſe of the Terreſtrial GLOBE.

PROB. I. *To find the latitude and longitude of any place.*—Bring the place to the graduated ſide of the firſt meridian: then the degree of the meridian it cuts is the latitude ſought; and the degree of the equator then under the meridian is the longitude.

II. *To find a place, having a given latitude and longitude.*—Find the degree of longitude on the equator, and bring it to the braſs meridian; then find the degree of latitude on the meridian, either north or ſouth of the equator, as the given latitude is north or ſouth; then the point of the Globe juſt under that degree of latitude is the place required.

III. *To find all the places on the Globe that have the ſame latitude, and the ſame longitude, or hour, with a given place, as ſuppoſe London.*—Bring the given place London to the meridian, and obſerve what places are juſt under the edge of it, from north to ſouth; and all thoſe places have the ſame longitude and hour with it. Then turn the Globe quite round; and all thoſe places which paſs juſt under the given degree of latitude on the meridian, have the ſame latitude with the given place.

IV. *To find the Antœci, Periœci and Antipodes, of any given place, ſuppoſe London.*—Bring the given place London to the meridian, then count 51½ the ſame degree of latitude ſouthward, or towards the other pole, and the point thus arrived at will be the Antœci, or where the hour of the day or night is always the ſame at both places at the ſame time, and where the ſeaſons and lengths of days and nights are alſo equal, but at half a year diſtance from each other, becauſe their ſeaſons are oppoſite or contrary. London being ſtill under the meridian, ſet the hour index to 12 at noon, or pointing towards London; then turn the Globe juſt half round, or till the index point to the oppoſite hour, or 12 at night; and the place that comes under the ſame degree of the meridian where London was, ſhews where the Periœci dwell, or thoſe people that have the ſame ſeaſons and at the ſame time as London, as alſo the ſame length of days and nights &c at that time, but only their time or hour is juſt oppoſite, or 12 hours diſtant, being day with one when night with the other, &c. Laſtly, as the Globe ſtands, count down by the meridian the ſame degree of latitude ſouth, and that will give the place of the Antipodes of London, being diametrically under or oppoſite to it; and ſo having all its times, both hours and ſeaſons oppoſite, being day with the one when night with the other, and ſummer with the one when winter with the other.

V. *To find the Diſtance of two places on the Globe.*—If the two places be either both on the equator, or both on the ſame meridian, the number of degrees in the diſtance between them, reduced into miles, at the rate of 70 Engliſh miles to the degree, (or more exact 69⅓), will give the diſtance nearly. But in any other ſituations of the two places, lay the quadrant of altitude over them, and the degrees counted upon it, from the one place to the other, and turned into miles as above, will give the diſtance in this caſe.

VI. *To find the Difference in the Time of the day at any two given places, and thence the Difference of Longitude.*—Bring one of the places to the meridian, and ſet the hour index to 12 at noon; then turn the Globe till the other place comes to the meridian, and the index will point out the difference of time; then by allowing 15° to every hour, or 1° to 4 minutes of time, the difference of longitude will be known.—Or the difference of longitude may be found without the time, thus:

Firſt bring the one place to the meridian, and note the degree of longitude on the equator cut by it; then do the ſame by the other place; which gives the longitudes of the two places; then ſubtracting the one number of degrees from the other, gives the difference of longitude ſought.

VII. *The time being known at any given place, as ſuppoſe London, to find what hour it is in any other part of the world.*—Bring the given place, London, to the meridian, and ſet the index to the given hour; then turn the Globe till the other place come to the meridian, and look at what hour the index points, which will be the time ſought.

VIII. *To find the Sun's place in the ecliptic, and alſo on the Globe, at any given time.*—Look into the calendar on the wooden horizon for the month and day of the month propoſed, and immediately oppoſite ſtands the ſign and degree which the ſun is in on that day. Then in the ecliptic drawn upon the Globe, look for the ſame ſign and degree, and that will be the place of the ſun required.

IX. *To find at what place on the earth the ſun is vertical, at a given moment of time at another place, as ſuppoſe London.*—Find the ſun's place on the Globe by the laſt problem, and turn the Globe about till that

that place come to the meridian, and note the degree of the meridian juſt over it. Then turn the Globe till the given place, London, come to the meridian, and ſet the index to the given moment of time. Laſtly, turn the Globe till the index points to 12 at noon; then the place of the earth, or Globe, which ſtands under the before noted degree, has the ſun at that moment in the zenith.

X. *To find how long the ſun ſhines without ſetting, in any given place in the frigid zones.*——Subtract the degrees of latitude of the given place from 90, which gives the complement of the latitude, and count the number of this complement upon the meridian from the equator towards the pole, marking that point of the meridian; then turn the Globe round, and carefully obſerve what two degrees of the ecliptic paſs exactly under the point marked on the meridian. Then look for the ſame degrees of the ecliptic on the wooden horizon, and juſt oppoſite to them ſtand the months and days of the months correſponding, and between which two days the ſun never ſets in that latitude.

If the beginning and end of the longeſt night be required, or the period of time in which the ſun never riſes at that place; count the ſame complement of latitude towards the ſouth or fartheſt pole, and then the reſt of the work will be the ſame in all reſpects as above.

Note, that this ſolution is independent of the horizontal refraction of the ſun, which raiſes him rather more than half a degree higher, by that means making the day ſo much longer, and the night the ſhorter; therefore in this caſe, ſet the mark on the meridian half a degree higher up towards the north pole, than what the complement of latitude gives; then proceed with it as before, and the more exact time and length of the longeſt day and night will be found.

XI. *A place being given in the torrid zone, to find on what two days of the year the ſun is vertical at that place.*——Turn the Globe about till the given place come to the meridian, and note the degree of the meridian it comes under. Next turn the Globe round again, and note the two points of the ecliptic paſſing under that degree of the meridian. Laſtly, by the wooden horizon, find on what days the ſun is in thoſe two points of the ecliptic; and on theſe days he will be vertical to the given place.

XII. *To find thoſe places in the torrid zone to which the ſun is vertical on a given day.*——Having found the ſun's place in the ecliptic, as in the 8th problem, turn the Globe to bring the ſame point of the ecliptic on the Globe to the meridian; then again turn the Globe round, and note all the places which paſs under that point of the meridian; which will be the places ſought.

After the ſame manner may be found what people are Aſcii for any given day. And alſo to what place of the earth, the moon, or any other planet, is vertical on a given day; finding the place of the planet on the globe by means of its right aſcenſion and declination, like finding a place from its longitude and latitude given.

XIII. *To rectify the Globe for the latitude of any place.*——By ſliding the braſs meridian in its groove, elevate the pole as far above the horizon as is equal to the latitude of the place; ſo for London, raiſe the north pole 51½ degrees above the wooden horizon: then turn the Globe on its axis till the place, as London, come to the meridian, and there ſet the index to 12 at noon. Then is the place exactly on the vertex, or top point of the Globe, at 90° every way round from the wooden horizon, which repreſents the horizon of the place. And if the frame of the Globe be turned about till the compaſs needle point to 22½ degrees, or two points weſt of the north point (becauſe the variation of the magnetic needle is nearly 22½ degrees weſt), ſo ſhall the Globe then ſtand in the exact poſition of the earth, with its axis pointing to the north pole.

XIV. *To find the length of the day or night, or the ſun's riſing or ſetting, in any latitude; having the day of the month given.*——Rectify the Globe for the latitude of the place; then bring the ſun's place on the globe to the meridian, and ſet the index to 12 at noon, or the upper 12, and then the Globe is in the proper poſition for noon day. Next turn the Globe about towards the eaſt till the ſun's place come juſt to the wooden horizon, and the index will then point to the hour of ſunriſe; alſo turn the Globe as far to the weſt ſide, or till the ſun's place come juſt to the horizon on the weſt ſide, and then the index will point to the hour of ſunſet. Theſe being now known, double the hour of ſetting will be the length of the day, and double the riſing will be the length of the night.—And thus alſo may the length of the longeſt day, or the ſhorteſt day, be found for any latitude.

XV. *To find the beginning and end of Twilight on any day of the year, for any latitude.*——It is twilight all the time from ſunſet till the ſun is 18° below the horizon, and the ſame in the morning from the time the ſun is 18° below the horizon till the moment of his riſe. Therefore, rectify the Globe for the latitude of the place, and for noon by ſetting the index to 12, and ſcrew on the quadrant of altitude. Then take the point of the ecliptic oppoſite the ſun's place, and turn the Globe on its axis weſtward, as alſo the quadrant of altitude, till that point cut this quadrant in the 18th degree below the horizon, then the index will ſhew the time of dawning in the morning; next turn the Globe and quadrant of altitude towards the eaſt, till the ſaid point oppoſite the ſun's place meet this quadrant in the ſame 18th degree, and then the index will ſhew the time when twilight ends in the evening.

XVI. *At any given day, and hour of the day, to find all thoſe places on the Globe where the ſun then riſes, or ſets, as alſo where it is noon day, where it is day light, and where it is in darkneſs.*——Find what place the ſun is vertical to, at that time; and elevate the Globe according to the latitude of that place, and bring the place alſo to the meridian; in which ſtate it will alſo be in the zenith of the Globe. Then is all the upper hemiſphere, above the wooden horizon, enlightened, or in day light; while all the lower one, below the horizon, is in darkneſs, or night: thoſe places by the edge of the meridian, in the upper hemiſphere, have noon day, or 12 o'clock; and thoſe by the meridian below, have it midnight: laſtly, all thoſe places by the eaſtern ſide of the horizon, have the ſun juſt ſetting,

ting, and thoſe by the weſtern horizon have him juſt riſing.

Hence, as in the middle of a lunar eclipſe the moon is in that degree of the ecliptic oppoſite to the ſun's place; by the preſent problem it may be ſhewn what places of the earth then ſee the middle of the eclipſe, and what the beginning or ending; by uſing the moon's place inſtead of the ſun's place in the problem.

XVII. *To find the bearing of one place from another, and their angle of poſition.*——Bring the one place to the zenith, by rectifying the Globe for its latitude, and turning the Globe till that place come to the meridian; then ſcrew the quadrant of altitude upon the meridian at the zenith, and make it revolve till it come to the other place on the Globe; then look on the wooden horizon for the point of the compaſs, or number of degrees from the ſouth, where the quadrant of altitude cuts it, and that will be the bearing of the latter place from the former, or the angle of poſition ſought.

The Uſe of the Celeſtial GLOBE.

The Celeſtial Globe differs from the terreſtrial only in this; inſtead of the ſeveral parts of the earth, the images of the ſtars and conſtellations are deſigned. The meridian circle drawn through the two poles and through the point Cancer, repreſents the ſolſtitial colure; but that through the point Aries, repreſents the equinoctial colure.

PROB. XVIII. *To exhibit the true repreſentation of the face of the heavens at any given time and place.*——Rectify for the lat. of the place, by prob. 13, ſetting the Globe with its pole pointing to the pole of the world, by means of a compaſs. Find the ſun's place in the ecliptic, and turn the Globe to bring it to the meridian, and there ſet the index to 12 at noon. Again revolve the Globe on its axis, till the index point to the given hour of the day or night: ſo ſhall the Globe in this poſition exactly repreſent the face of the heavens as it appears at that time, every conſtellation and ſtar, in the heavens, anſwering in poſition to thoſe on the Globe; ſo that, by examining the Globe, it will immediately appear which ſtars are above or below the horizon, which on the eaſt or weſtern parts of the heavens, which lately riſen, and which going to ſet, &c. And thus the poſitions of the ſeveral planets, or comets, may alſo be exhibited; having marked the places of the Globe where they are, by means of their declination and right aſcenſion.

XIX. *To find the Declination and Right-aſcenſion of any ſtar upon the Globe.*——Turn the Globe till the ſtar come to the meridian: then the number of degrees on the meridian, between the equator and the ſtar, is its declination; and the degree of the equator cut by the meridian, is the right-aſcenſion of the ſtar.——In like manner are found the declination and right-aſcenſion of the ſun, or any other point.

XX. *To find the Latitude and Longitude of any ſtar drawn upon the Globe.*——Bring the ſolſtitial colure to the meridian, and there fix the quadrant of altitude over the pole of the ecliptic in the ſame hemiſphere with the ſtar, and bring its graduated edge to the ſtar: then the degree on the quadrant cut by the ſtar is its latitude, counted from the ecliptic; and the degree of the ecliptic cut by the quadrant its longitude.

XXI. *To find the place of a ſtar, planet, comet, &c. on the Globe; its declination and right-aſcenſion being known.*——Find the given point of right-aſcenſion on the equinoctial, and bring it to the meridian; then count the degrees of declination upon the meridian from the equinoctial, and there make a mark on the Globe, which will be the place of the planet, &c, ſought.

XXII. *To find the place of a ſtar, planet, comet, or other object on the Globe; its latitude and longitude being given.*——Bring the pole of the ecliptic to the meridian, and there fix the quadrant of altitude, which turn round till its edge cut the given longitude on the ecliptic; then count the given latitude, from the ecliptic, upon the quadrant of altitude, and there make a mark on the Globe, which will be the place of the planet, &c, ſought.——The place on the Globe, of any ſuch planet, &c, being found by this or the foregoing problem, its riſing, or ſetting, or any other circumſtance concerning it, may then be found, the ſame as the ſun, by the proper problems.

XXIII. *To find the riſing, ſetting, and culminating of a ſtar, planet, ſun, &c; with its continuance above the horizon, for any place and day; as alſo its oblique aſcenſion and deſcenſion, with its eaſtern and weſtern amplitude and azimuth.*——Adjuſt the Globe to the ſtate of the heavens at 12 o'clock that day. Bring the ſtar, &c, to the eaſtern ſide of the horizon: which will give its eaſtern amplitude and azimuth, and the time of riſing, as for the ſun. Again, turn the Globe to bring the ſame ſtar to the weſtern ſide of the horizon: ſo will the weſtern amplitude and azimuth, with the time of ſetting, be found. Then, the time of riſing, ſubtracted from that of ſetting, leaves the continuance of the ſtar above the horizon: this continuance above the horizon taken from 24 hours, leaves the time it is below the horizon. Laſtly, bring the ſtar to the meridian, and the hour to which the index then points is the time of its culmination, or ſouthing.

XXIV. *To find the altitude of the ſun, or ſtar, &c, for any given hour of the day or night.*——Adjuſt the Globe to the poſition of the heavens, and turn it till the index point at the given hour. Then fix on the quadrant of altitude, at 90 degrees from the horizon, and turn it to the place of the ſun or ſtar: ſo ſhall the degrees of the quadrant, intercepted between the horizon and the ſun or ſtar, be the altitude ſought.

XXV. *Given the altitude of the ſun by day, or of a ſtar by night, to find the hour of the day or night.*——Rectify the Globe as in the foregoing problem; and turn the Globe and quadrant, till ſuch time as the ſtar or degree of the ecliptic the ſun is in, cut the quadrant in the given degree of altitude; then will the index point at the hour required.

XXVI. *Given the azimuth of the ſun or a ſtar, to find the time of the day or night.*——Rectify the Globe, and bring the quadrant to the given azimuth in the horizon; then turn the Globe till the ſun or ſtar come to the quadrant, and the index will then ſhew the time of the day or night.

GLOBULAR, relating to, or partaking the property or ſhape of, the Globe. As Globular chart, Globular projection, or Globular ſailing, &c.

GLOBULAR *Chart*, is a repreſentation of the ſurface, or part of the ſurface, of the terraqueous Globe upon a plane ; in which the parallels of latitude are circles nearly concentric ; and the meridians are curves bending towards the poles ; the rhumb-lines being curves alſo.

The merits of this chart conſiſt in theſe particulars, viz, that the diſtances between places on the ſame rhumb are all meaſured by the ſame ſcale of equal parts ; and the diſtance of any two places in the arch of a great circle, is nearly repreſented in this chart by a ſtraight line.

Land maps alſo made according to this projection would have great advantages over thoſe made in any other way. But for ſea charts for the uſe of navigation, Mercator's are preferable, as both the meridians and parallels, as alſo the rhumbs, are all ſtraight lines.

This projection is not new, though not much noticed till of late. It is mentioned by Ptolomy, in his Geography ; and alſo by Blundeville, in his Exerciſes.

For Globular projection of maps or charts, ſee Map.

Globular *Sailing*, is the method of reſolving the caſes of ſailing upon principles deduced from the ſpherical figure of the earth. Such as Mercator's ſailing, or Great-circle ſailing ; which ſee.

GLOSSOCOMON, in Mechanics, is a name given by Heron to a machine compoſed of divers dented wheels with pinions, ſerving to raiſe huge weights.

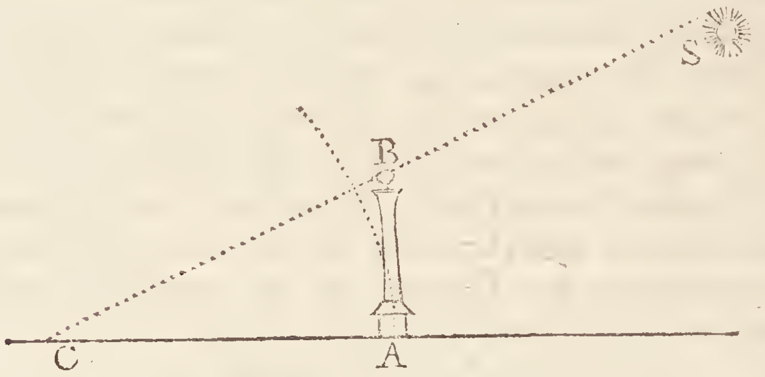
GNOMON, in Aſtronomy, is an inſtrument or apparatus for meaſuring the altitudes, declinations, &c, of the ſun and ſtars. The Gnomon is uſually a pillar, or column, or pyramid, erected upon level ground, or a pavement. For making the more conſiderable obſervations, both the ancients and moderns have made great uſe of it, eſpecially the former ; and many have preferred it to the ſmaller quadrants, both as more accurate, eaſier made, and more eaſily applied.

The moſt ancient obſervation of this kind extant, is that made by Pytheas, in the time of Alexander the Great, at Marſeilles, where he found the height of the Gnomon was in proportion to the meridian ſhadow at the ſummer ſolſtice, as 213⅛ to 600 ; juſt the ſame as Gaſſendi found it to be, by an obſervation made at the ſame place, almoſt 2000 years after, viz, in the year 1636. Ricciol. Almag. vol. 1, lib. 3, cap. 14.

Ulugh Beigh, king of Parthia, &c, uſed a Gnomon in the year 1437, which was 180 Roman feet high. That erected by Ignatius Dante, in the church of St. Petronius, at Bologna, in the year 1576, was 67 feet high. M. Caſſini erected another of 20 feet high, in the ſame church, in the year 1655.

The Egyptian obeliſks were alſo uſed as Gnomons ; and it is thought by ſome modern travellers that this was the very uſe they were deſigned and built for ; it has alſo been found that their four ſides ſtand exactly facing the four cardinal points of the compaſs. It may be added, that the Spaniards in their conqueſt of Peru, found pillars of curious and coſtly workmanſhip, ſet up in ſeveral places, by the meridian ſhadows of which their amatas or philoſophers had, by long experience and repeated obſervations, learned to determine the times of the equinoxes ; which ſeaſons of the year were celebrated with great feſtivity and rich offerings, in honour of the ſun. Garcillaſſo de la Vega, Hiſt. Peru. lib. 2, cap. 22.

Use *of the* Gnomon, *in taking the meridian altitude of the Sun, and thence finding the Latitude of the place.*——A meridian line being drawn through the centre of the Gnomon, note the point where the ſhadow of the Gnomon terminates when projected along the meridian line, and meaſure the diſtance of that extreme point from the centre of the Gnomon, which will be the length of its ſhadow. Then having the height of the Gnomon, and the length of the ſhadow, the ſun's altitude is thence eaſily found.

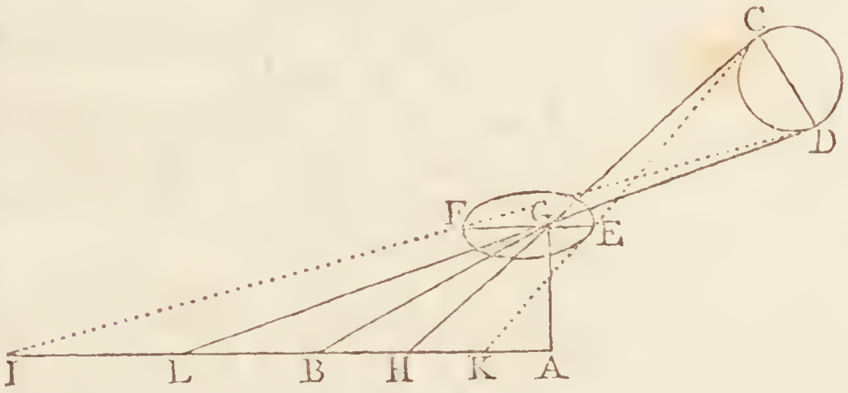


Suppoſe, ex. gr. AB the Gnomon, and AC the length of the ſhadow. Here in the right-angled triangle ABC, are given the baſe AC, and the perpendicular AB, to find the angle C, or the ſun's altitude, which will be found by this analogy, as CA : AB : : radius : the tang. of ∠ C, that is, as the length of the ſhadow is to the height of the Gnomon, ſo is radius to the tangent of the ſun's altitude above the horizon.

The following example will ſerve to illuſtrate this propoſition : Pliny ſays, Nat. Hiſt. lib. 2, cap. 72, that at Rome, at the time of the equinoxes, the ſhadow is to the Gnomon as 8 to 9 ; therefore as 8 : 9 : : 1 or radius : $\frac{9}{8} = 1\cdot125$ a tangent, to which anſwers the angle 48° 22′, which is the height of the equator at Rome, and its complement 41° 38′ is therefore the height of the pole, or the latitude of the place.

Riccioli remarks the following defects in the obſervations of the ſun's height, made with the Gnomon by the ancients, and ſome of the moderns : viz, that they neglected the ſun's parallax, which makes his apparent altitude leſs, by the quantity of the parallax, than it would be, if the Gnomon were placed at the centre of the earth : 2d, they neglected alſo the refraction, by which the apparent height of the ſun is a little increaſed: and 3dly, they made the calculations from the length of the ſhadow, as if it were terminated by a ray coming from the centre of the ſun's diſc, whereas the ſhadow is really terminated by a ray coming from the upper edge of the ſun's diſc ; ſo that, inſtead of the height of the ſun's centre, their calculations gave the height of the upper edge of his diſc. And therefore, to the altitude of the ſun found by the Gnomon, the ſun's parallax muſt be added, and from the ſum muſt be ſubtracted the ſun's ſemidiameter, and refraction, which is different at different altitudes ; which being done, the correct height of the equator at Rome will be 48° 4′ 13″, the complement of which, or 41° 55′ 46″, is the latitude. Ricciol. Geogr. Refor. lib. 7, cap. 4.

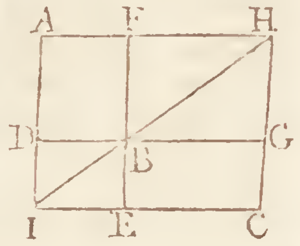
The preceding problem may be reſolved more accurately by means of a ray of light let in through a ſmall hole, than by a ſhadow, thus : Make a circular perforation in a braſs plate, to tranſmit enough of the ſun's rays to exhibit his image on the floor, or a ſtage ; fix the plate parallel to the horizon in a high place, proper for obſervation, the height of which above the floor let be accurately meaſured with a plummet. Let the floor, or ſtage, be perfectly plane and horizontal, and coloured over with ſome white ſubſtance, to ſhew the ſun more diſtinctly. Upon this horizontal plane draw a meridian line paſſing through the foot or centre of the Gnomon, i. e. the point upon which the plummet falls from the centre of the hole ; and upon this line note the extreme points I and K of the ſun's image or diameter, and from each end ſubtract the image of half the diameter of the aperture, viz KH and LI : then will HL be the image of the ſun's diameter, which, when biſected in B, gives the point on which the rays fall from the centre of the ſun.



Now having given the line AB, and the altitude of the Gnomon AG, beſide the right angle A, the angle B, or the apparent altitude of the ſun's centre, is eaſily found, thus : as AB : AG : : radius : tang. angle B.

Gnomon, in Dialling, is the ſtyle, pin, or cock of a dial, the ſhadow of which points out the hours. This is always ſuppoſed to repreſent the axis of the world, to which it is therefore parallel, or coincident, the two ends of it pointing ſtraight to the north and ſouth poles of the world.

Gnomon, in Geometry, is a figure formed of the two complements, in a parallelogram, together with either of the parallelograms about the diameter. Thus the parallelogram AC being divided into four parallelograms by the two lines DG, EF parallel to the ſides, forming the two complements AB and BC, with the two DE, FG about the diameter HI : then the two Gnomons are AB + BC + DE, and AB + BC + FG.



Gnomonic *Projection of the Sphere*, is the repreſentation of the circles of an hemiſphere upon a plane touching it in the vertex, by the eye in the centre, or by lines or rays iſſuing from the centre of the hemiſphere, to all the points in the ſurface.

In this projection of the ſphere, all the great circles are projected into right lines, on the plane, of an indefinite length ; and all leſſer circles that are parallel to the plane, into circles ; but if oblique to the plane, then are they projected either into ellipſes or hyperbolas, according to their different obliquity. It has its name from Gnomonics, or Dialling, becauſe the lines on the face of every dial are from a projection of this ſort : for if the ſphere be projected on any plane, and upon that ſide of it on which the ſun is to ſhine ; alſo the projected pole be made the centre of the dial, and the axis of the globe the ſtyle or Gnomon, and the radius of projection its height ; you will have a dial drawn with all its furniture. See Emerſon's Projection of the Sphere.

GNOMONICS, the ſame as Dialling ; or the art of drawing ſun and moon dials, on any given plane ; being ſo called, becauſe it ſhews how to find the hour of the day or night by the ſhadow of a Gnomon or ſtyle.

GOLDEN *Number*, is the particular year of the Metonic or Lunar Cycle. See *Lunar* Cycle.

To find the Golden Number : Add 1 to the given year, and divide the ſum by 19, and what remains will be the Golden Number ; unleſs 0 remain, for then 19 is the Golden Number.

Thus, the Golden Number for the year 1791, is 6 ; as by the operation in the margin.

1791
1
19)1792(94
171
82
76
Golden No. 6

Golden *Rule*, a rule ſo called on account of its excellent uſe, in arithmetic, and eſpecially in ordinary calculations, by which numbers are found in certain proportions, viz, having three numbers given, to find a 4th number, that ſhall have the ſame proportion to the 3d as the 2d hath to the 1ſt. On this account, it is otherwiſe called *The Rule of Three*, and *The Rule of Proportion*. See Rule *of Three*.

Having ſtated, or ſet down in a line, the three terms, in the order in which they are proportional, multiply the 2d and 3d together, and divide the product by the 1ſt, ſo ſhall the quotient be the anſwer, or the 4th term ſought.

Thus, if 3 yards of cloth coſt a guinea or 21 ſhillings, what will 20 yards coſt. Here the two prices or values muſt bear the ſame proportion to each other as the two quantities, or number of yards of cloth, i. e. 3 muſt bear the ſame proportion to 20, as 21*s*, the value of the former, muſt bear to the value of the latter : and therefore the ſtating and operation of the numbers will be thus,

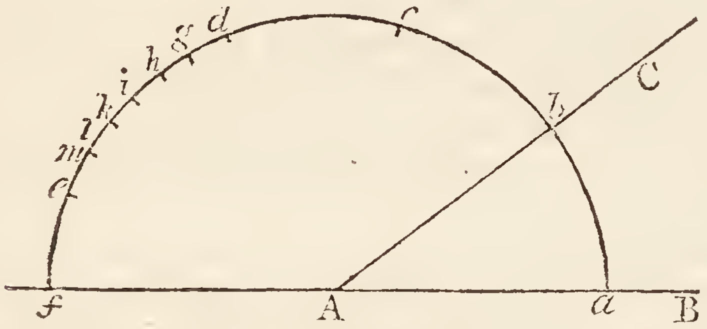
Then multiplying the 2d and 3d together, and dividing the product by the 1ſt, it gives 140*s*. or 7*l*. for the anſwer, being the coſt of 20 yards.

3 : 20 : : 21 : 140*s*.
20
3)420
Quot. 140*s*.
or 7*l*.

GONIOMETRICAL *Lines*, are lines uſed for meaſuring or determining the quantity of angles : ſuch as ſines, tangents, ſecants, verſed ſines, &c.

Mr. Jones, in the Philoſ. Tranſ. number 483, ſect. 26, gave a paper, containing a commodious diſpoſition of equations for exhibiting the relations of Goniometrical Lines ; from which a multitude of curious theorems may be derived. See alſo Robertſon's Elem. of Navigation, vol. 1, p 181, Edit. 4.

GONIOMETRY, a method of meaſuring angles, ſo called by M. de Lagny, who gave ſeveral papers, on this method, in the Memoirs of the Royal Acad. an. 1724, 1725, 1729. M. de Lagny's method of Goniometry conſiſts in meaſuring the angles with a pair of compaſſes, and that without any ſcale whatever, except an undivided ſemicircle. Thus, having any angle drawn upon paper, to be meaſured; produce one of the ſides of the angle backwards behind the angular point; then with a pair of fine compaſſes deſcribe a pretty large ſemicircle from the angular point as a centre, cutting the ſides of the propoſed angle, which will intercept a part of the ſemicircle. Take then this intercepted part very exactly between the points of the compaſſes, and turn them ſucceſſively over upon the arc of the ſemicircle, to find how often it is contained in it, after which there is commonly ſome remainder: then take this remainder in the compaſſes, and in like manner find how often it is contained in the laſt of the integral parts of the 1ſt arc, with again ſome remainder: find in like manner how often this laſt remainder is contained in the former; and ſo on continually, till the remainder become too ſmall to be taken and applied as a meaſure. By this means he obtains a ſeries of quotients, or fractional parts, one of another, which being properly reduced into one fraction, give the ratio of the firſt arc to the ſemicircle, or of the propoſed angle to two right angles, or 180 degrees, and conſequently that angle itſelf in degrees and minutes.



Thus, ſuppoſe the angle BAC be propoſed to be meaſured. Produce BA out towards *f*; and from the centre A deſcribe the ſemicircle *abcf*, in which *ab* is the meaſure of the propoſed angle. Take *ab* in the compaſſes, and apply it 4 times on the ſemicircle, as at *b*, *c*, *d*, and *e*; then take the remainder *fe*, and apply it back upon *ed*, which is but once, viz at *g*; again take the remainder *gd*, and apply it 5 times on *ge*, as at *h*, *i*, *k*, *l*, and *m*; laſtly, take the remainder *me*, and it is contained juſt 2 times in *ml*. Hence the ſeries of quotients is 4, 1, 5, 2; conſequently the 4th or laſt arc *em* is $\frac{1}{2}$ the third *ml* or *gd*, and therefore the 3d arc *gd* is $\frac{1}{5\frac{1}{2}}$ or $\frac{2}{11}$ of the 2d arc *ef*; and therefore again this 2d arc *ef* is $\frac{1}{1\frac{2}{11}}$ or $\frac{11}{13}$ of the 1ſt arc *ab*; and conſequently this 1ſt arc *ab* is $\frac{1}{4\frac{11}{13}}$ or $\frac{13}{63}$ of the whole ſemicircle *af*. But $\frac{13}{63}$ of 180° are $37\frac{1}{7}$ degrees, or 37° 8′ 34″$\frac{2}{7}$, which therefore is the meaſure of the angle ſought. When the operation is nicely performed, this angle may be within 2 or 3 minutes of the truth; though M. de Lagny pretends to meaſure much nearer than that.

It may be added, that the ſeries of fractions forms what is called a continued fraction. Thus, in the example above, the continued fraction, and its reduction, will be as follow:

$$\cfrac{1}{4+\cfrac{1}{1+\cfrac{1}{5\frac{1}{2}}}} = \cfrac{1}{4+\cfrac{1}{1\frac{2}{11}}} = \frac{1}{4\frac{11}{13}} = \frac{13}{63};$$

the quotients being the ſucceſſive denominators, and 1 always for each numerator.

GORGE, or *Neck*, in Architecture, is the narroweſt part of the Tuſcan or Doric capitals, lying above the ſhaft of the pillar, between the aſtragal and the annulets.

It is alſo a kind of concave moulding, ſerving for compartments &c, larger than a ſcotia, but not ſo deep.

GORGE, in Fortification, is the entrance into a baſtion, or a ravelin, or other out-work.

The GORGE *of a Baſtion*, is what remains of the ſides of the polygon of a place, after cutting off the curtains; in which caſe it makes an angle in the centre of the baſtion, viz, the angle made by two adjacent curtains produced to meet within the baſtion.

In flat baſtions, the Gorge is a right line on the curtain, reaching between the two flanks.

GORGE *of a Half-moon*, or *of a Ravelin*, is the ſpace between the two ends of their faces next the place.

GORGE of the other out-works, is the interval between their ſides next the great ditch.

All the Gorges are to be made without parapets: otherwiſe the beſiegers, having taken poſſeſſion of a work, might make uſe of them to defend themſelves from the ſhot of the place. So that they are only fortified with palliſadoes, to prevent a ſurprize.

The *Demi*-GORGE, or *Half* the GORGE, is that part of the polygon between the flank and the centre of the baſtion.

GOTHIC *Architecture*, is that which deviates from the manner, character, proportions, &c, of the antique; having its ornaments wild and chimerical, and its profiles incorrect. This manner of building came originally from the North, whence it was brought, in the 5th century, by the Goths into Germany, and has ſince been introduced into other countries. The firſt or moſt ancient ſtyle of Gothic building was very ſolid, heavy, maſſive and ſimple, with ſemicircular arches, &c: but the more modern ſtyle of the Gothic is exceedingly rich, light, and delicate; having an abundance of little whimſical ornaments, with ſharp-pointed arches formed by the interſections of different circular ſegments; alſo lofty and light ſpires and ſteeples, large ramified windows, cluſtered pillars, &c. Of this kind are our Engliſh cathedrals, and many other old buildings.

GRADUATION, is uſed for the act of Graduating, or dividing any thing into degrees.

For an account of the various methods of Graduating mathematical and aſtronomical inſtruments, by ſtraight and circular diagonals, and by concentric arcs, &c; ſee *Plain* SCALE, *Nonius*, and *Vernier*. And for an account of Mr. Bird's improved method of dividing aſtronomical inſtruments, ſee MURAL *Arch*.

Mr. Ramſden, an ingenious mathematical inſtrument-maker of London, has lately publiſhed, by encouragement of the commiſſioners of longitude, an explanation and deſcription of an engine contrived by him for dividing mathematical inſtruments, accompanied with proper drawings; in conſideration of which, the ſaid commiſſioners have granted to him the ſum of 615*l*. See his book, 4to, 1777.

On the ſubject of dividing a foot into many thouſand parts, for mathematical purpoſes, ſee Philoſ. Tranſ. vol. 2, p. 457, 459, 541, or Abr. vol. 1, pa. 218, 220, &c. And for an account of various other methods and Graduations, ſee a paper of Mr. Smeaton's in the Philoſ. Tranſ. vol. 76, for the year 1786, p. 1; being "Obſervations on the Graduation of aſtronomical inſtruments; with an explanation of the method invented by the late Mr. Henry Hindley, of York, clockmaker, to divide circles into any given number of parts."

GRAHAM (George), clock and watch maker, the moſt ingenious and accurate artiſt in his time, was born at Gratwick, a village in the north of Cumberland, in 1675. In 1688 he came up to London, and was put apprentice to a perſon in that profeſſion; but after being ſome time with his maſter, he was received, purely on account of his merit, into the family of the celebrated Mr. Tompion, who treated him with a kind of parental affection as long as he lived. That Mr. Graham was, without competition, the moſt eminent of his profeſſion, is but a ſmall part of his character: he was the beſt general mechanic of his time, and had a complete knowledge of practical aſtronomy; ſo that he not only gave to various movements for meaſuring time a degree of perfection which had never before been attained, but invented ſeveral aſtronomical inſtruments, by which conſiderable advances have been made in that ſcience: he made great improvements in thoſe which had before been in uſe; and, by a wonderful manual dexterity, conſtructed them with greater preciſion and accuracy than any other perſon in the world.

A great mural arch in the obſervatory at Greenwich was made for Dr. Halley, under Mr. Graham's immediate inſpection, and divided by his own hand: and from this incomparable original, the beſt foreign inſtruments of the kind are copies made by Engliſh artiſts. The ſector by which Dr. Bradley firſt diſcovered two new motions in the fixed ſtars, was of his invention and fabric. He compriſed the whole planetary ſyſtem within the compaſs of a ſmall cabinet; from which, as a model, all the modern orreries have been conſtructed. And when the French Academicians were ſent to the north, to make obſervations for aſcertaining the figure of the earth, Mr. Graham was thought the fitteſt perſon in Europe to ſupply them with inſtruments; by which means they finiſhed their operations in one year; while thoſe who went to the ſouth, not being ſo well furniſhed, were very much embarraſſed and retarded in their operations.

Mr. Graham was many years a member of the Royal Society, to which he communicated ſeveral ingenious and important diſcoveries, viz, from the 31ſt to the 42d volume of the Philoſ. Tranſactions, chiefly on aſtronomical and philoſophical ſubjects; particularly a kind of horary alteration of the magnetic needle; a quickſilver pendulum, and many curious particulars relating to the true length of the ſimple pendulum, upon which he continued to make experiments till almoſt the year of his death, which happened in 1751, at 76 years of age.

His temper was not leſs communicative than his genius was penetrating; and his principal view was the advancement of ſcience, and the benefit of mankind. As he was perfectly ſincere, he was above ſuſpicion; and as he was above envy, he was candid.

GRANADO, in Artillery, is a little ſhell or hollow globe of iron, or other matter, which, being filled with powder, is fired by means of a ſmall fuſee, and thrown either by the hand, or a piece of ordnance. As ſoon as it is kindled, the caſe flies in pieces, to the great danger of all that ſtand near it. Granadoes ſerve to ſet fire to cloſe and narrow paſſages, and are often thrown with the hand among the ſoldiers, to diſorder their ranks: more eſpecially in thoſe poſts where they ſtand thickeſt, as in trenches, redoubts, lodgments, &c.

GRAVESANDE (William James), a very celebrated Dutch mathematician and philoſopher, was born at Bois-le-duc, Sept. 27, 1688. He ſtudied the civil law at Leyden, but mathematical learning was his favourite amuſement. When he had taken his doctor's degree, in 1707, he went and ſettled at the Hague, where he practiſed at the bar, and cultivated an acquaintance with learned men; with a Society of whom he publiſhed a periodical Review, entitled *Le Journal Litteraire*, which was continued without interruption from the year 1713 to the year 1722. The parts of it written or extracted by Graveſande were chiefly thoſe relating to geometry and phyſics. But he enriched it alſo with ſeveral original pieces entirely of his own compoſition; viz, Remarks on the Conſtruction of Pneumatical Engines: A Moral Eſſay on Lying: And a celebrated Eſſay on the Colliſion of Bodies; which, as it oppoſed the Newtonian philoſophy, was attacked by Dr. Clarke, and many other learned men.

In 1715, when the States ſent to congratulate George the 1ſt, on his acceſſion to the throne, Dr. Graveſande was appointed ſecretary to the embaſſy. During his ſtay in England he was admitted a member of the Royal Society, and became intimately acquainted with Sir Iſaac Newton. On his return to Holland, he was choſen profeſſor of mathematics and aſtronomy at Leyden; where he had the honour of firſt teaching the Newtonian philoſophy, which was then in its infancy. He died in 1742, at 54 years of age.

Graveſande was a man amiable in his private character, and reſpectable in his public one; for few men of letters have rendered more eminent ſervices to their country. The miniſters of the republic conſulted him on all occaſions when his talents were requiſite to aſſiſt them, which his ſkill in calculation often enabled him to do in money matters. He was of great ſervice as a decypherer, in detecting the ſecret correſpondence of their enemies. And in his own profeſſion none ever applied the powers of nature with more ſucceſs, or to more uſeful purpoſes.

Of his publications, the principal are,

1. An Introduction to the Newtonian Philoſophy; or, a Treatiſe on the Elements of Phyſics, confirmed by Experiments. This performance, being only a more

perfect copy of his public lectures, was firſt printed in 1720; and hath ſince gone through many editions, with conſiderable improvements: the 6th edit. is in Engliſh, in 2 large vols. 4to, by Dr. Deſaguliers, in 1747, under the title of Mathematical Elements of Natural Philoſophy, confirmed by Experiments.

2. A treatiſe on the Elements of Algebra, for the uſe of Young Students; to which is added a Specimen of a Commentary on Newton's Univerſal Arithmetic; as alſo, A New Rule for determining the Form of an Aſſumed Infinite Series.

3. An Eſſay on Perſpective. This was written at 19 years of age.

4. A New Theory of the Colliſion of Bodies.

5. A Courſe of Logic and Metaphyſics.

With ſeveral ſmaller pieces.

His whole mathematical and philoſophical works, except the firſt article above, were collected and publiſhed at Amſterdam, in 2 vols. 4to, to which is prefixed a critical account of his life and writings, by Profeſſor Allamand.

GRAVITATION, the exerciſe of gravity, or the preſſure a body exerts on another body beneath it by the power of gravity.

This is ſometimes diſtinguiſhed from gravity. Thus M. Maupertuis, in his *Figure de la Terre*, takes gravity for that force by which a body would fall to the earth ſuppoſed at reſt; and Gravitation for the ſame, but diminiſhed by the centrifugal force. It is only Gravitation, or gravity thus blended with the centrifugal force, that we can uſually meaſure by our experiments. Methods however have been found to diſtinguiſh what remains of the primitive gravity, and what has been deſtroyed by the centrifugal force.

It is one of the laws of nature, diſcovered by Newton, and now received by all philoſophers, that every particle of matter in nature gravitates towards every other particle; which law is the main principle in the Newtonian philoſophy. But what is called Gravitation with reſpect to the gravitating body, is uſually called attraction with reſpect to the body gravitated to. The planets, both primary and ſecondary, as alſo the comets, do all gravitate towards the ſun, and towards each other; as well as the ſun towards them; and that in proportion to the quantity of matter in each of them.

The Peripatetics &c hold, that bodies only gravitate or weigh when out of their natural places, and that Gravitation ceaſes when they are reſtored to the ſame, the purpoſe of nature being then fulfilled: and they maintain that the final cauſe of this faculty is only to bring elementary bodies to their proper place, where they may reſt. But the moderns ſhew that bodies exerciſe gravity even when at reſt, and in their proper places. This is particularly ſhewn of fluids; and it is one of the laws of hydroſtatics, demonſtrated by Boyle and others, that fluids gravitate in proprio loco, the upper parts preſſing on the lower, &c.

For the laws of Gravitation of bodies in fluids ſpecifically lighter or heavier than themſelves, ſee SPECIFIC GRAVITY. Alſo for the centre or line or plane of Gravitation, ſee CENTRE, LINE, or PLANE.

GRAVITY, in Phyſics, the natural tendency or inclination of bodies towards the centre. And in this ſenſe Gravity agrees with Centripetal force.

Gravity however is, by ſome, defined more generally as the natural tendency of one body towards another; and again by others ſtill more generally as the mutual tendency of each body, and each particle of a body, towards all others: in which ſenſe the word anſwers to what is more uſually called Attraction. Indeed the terms Gravity, weight, centripetal force, and attraction, denote in effect all the ſame thing, only in different views and relations; all which however it is very common to confound, and uſe promiſcuouſly. But, in propriety, when a body is conſidered as tending towards the earth, the force with which it ſo tends is called Gravity, Force of Gravity, or Gravitating Force; when the body is conſidered as immediately tending to the centre of the earth, it is called Centripetal Force; but when we conſider the earth, or maſs to which the body tends, it is called Attraction, or Attractive Force; and when it is conſidered in reſpect of an obſtacle or another body in the way of its tendency, upon which it acts, it is called Weight.

Philoſophers think differently on the ſubject of Gravity. Some conſider it as an inactive property or innate power in bodies, by which they endeavour to join their centre. Others hold Gravity in this ſenſe to be an occult quality, and to be exploded as ſuch out of all ſound philoſophy. Newton, though he often calls it a vis, power, or property in bodies, yet explains himſelf, that he means nothing more by the word but the effect or phenomenon: he does not conſider the principle, the cauſe by which bodies tend downwards, but the tendency itſelf, which is no occult quality, but a ſenſible phenomenon, be its cauſes what they may; whether a property eſſential to body, as ſome make it, or ſuperadded to it, as others; or even an impulſe of ſome body from without, as others.

It is a law of nature long obſerved, that all bodies near the earth have a Gravity or weight, or a tendency towards its centre, or at leaſt perpendicular to its ſurface; which law the moderns, and eſpecially Sir I. Newton, from certain obſervations have found to be much more extenſive, and holding univerſally with reſpect to all known bodies and matter in nature. It is therefore at preſent acknowledged as a principle or law of nature, that all bodies, and all the particles of all bodies, mutually gravitate towards each other: from which ſingle principle it is that Newton has happily deduced all the great phenomena of nature.

Hence Gravity may be diſtinguiſhed into Particular and General.

Particular GRAVITY, is that which reſpects the earth, or by which bodies deſcend, or tend towards the centre of the earth; the phenomena or properties of which are as follow:

1. All circumterreſtrial bodies do hereby tend towards a point, which is either accurately or very nearly the centre of magnitude of the terraqueous globe. Not that it is meant that there is really any virtue or charm in the point called the centre, by which it attracts bodies; but becauſe this is the reſult of the gravitation of bodies towards all the parts of which the earth conſiſts.

2. This point or centre is fixed within the earth, or at leaſt has been ſo far as any authentic hiſtory reaches. For a conſequence of its ſhifting, though ever ſo little,

would be the overflowing of the low lands on that ſide of the globe towards which it ſhould approach. Dr. Halley ſuggeſts, that it would well account for the univerſal deluge, to have the centre of gravitation removed for a time towards the middle of the then inhabited world; for the change of its place but the 2000th part of the radius of the earth, or about 2 miles, would be ſufficient to lay the tops of the higheſt hills under water.

3. In all places equidiſtant from the centre of the earth, the force of Gravity is nearly equal. Indeed all parts of the earth's ſurface are not at equal diſtances from the centre, becauſe the equatorial parts are higher than the polar parts by about 17 miles; as has been proved by the neceſſity of making the pendulum ſhorter in thoſe places, before it will ſwing ſeconds. In the new Peterſburg Tranſactions, vol. 6 and 7, M. Krafft gives a formula for the proportion of Gravity in different latitudes on the earth's ſurface, which is this:

$$y = (1 + 0\cdot0052848 \text{ ſine } {}^2\lambda)\, g\,;$$

where g denotes the Gravity at the equator, and y the Gravity under any other latitude λ. On this ſubject, ſee alſo the articles DEGREE, and EARTH.

4. Gravity equally affects all bodies, without regard either to their bulk, figure, or matter: ſo that, abſtracting from the reſiſtance of the medium, the moſt compact and looſe, the greateſt and ſmalleſt bodies would all deſcend through an equal ſpace in the ſame time; as appears from the quick deſcent of very light bodies in an exhauſted receiver. The ſpace which bodies do actually fall, in vacuo, is $16\frac{1}{12}$ feet in the firſt ſecond of time, in the latitude of London; and for other times, either greater or leſs than that, the ſpaces deſcended from reſt are directly proportional to the ſquares of the times, while the falling body is not far from the earth's ſurface.

5. This power is the greateſt at the earth's ſurface, from whence it decreaſes both upwards and downwards, but not both ways in the ſame proportion; for upwards the force of Gravity is leſs, or decreaſes, as the ſquare of the diſtance from the centre increaſes, ſo that at a double diſtance from the centre, above the ſurface, the force would be only 1-4th of what it is at the ſurface; but below the ſurface, the power decreaſes in ſuch ſort that its intenſity is in the direct ratio of the diſtance from the centre; ſo that at the diſtance of half a ſemidiameter from the centre, the force would be but half what it is at the ſurface; at $\frac{1}{3}$ of a ſemidiameter the force would be $\frac{1}{3}$, and ſo on.

6. As all bodies gravitate towards the earth, ſo does the earth equally gravitate towards all bodies; as well as all bodies towards particular parts of the earth, as hills, &c, which has been proved by the attraction a hill has upon a plumb line, inſenſibly drawing it aſide.—Hence the gravitating force of entire bodies conſiſts of thoſe of all their parts: for by adding or taking away any part of the matter of a body, its Gravity is increaſed or decreaſed in the proportion of the quantity of ſuch portion to the whole maſs. Hence alſo the gravitating powers of bodies, at the ſame diſtance from the centre, are proportional to the quantities of matter in the bodies.

General or Univerſal GRAVITY, is that by which all the planets tend to one another, and indeed by which all the bodies and particles of matter in the univerſe tend towards one another.

The exiſtence of the ſame principle of Gravitation in the ſuperior regions of the heavens, as on the earth, is one of the great diſcoveries of Newton, who made the proof of it as eaſy as that on the earth. At firſt it would ſeem this was only conjecture with him: he obſerved that all bodies near the earth, and in its atmoſphere, had the property of tending directly towards it; he ſoon conjectured that it probably extended much higher than any diſtance to which we could reach, or make experiments; and ſo on, from one diſtance to another, till he at length ſaw no reaſon why it might not extend as far as to the moon, by means of which ſhe might be retained in her orbit as a ſtone in a ſling is retained by the hand; and if ſo, he next inferred why might not a ſimilar principle exiſt in the other great bodies in the univerſe, the ſun and all the other planets, both primary and ſecondary, which might all be retained in their orbits, and perform their revolutions, by means of the ſame univerſal principle of gravitation.

Theſe conjectures he ſoon realized and verified by mathematical proofs. Kepler had found out, by contemplating the motions of the planets about the ſun, that the area deſcribed by a line connecting the ſun and planet, as this revolved in its orbit, was always proportional to the time of its deſcription, or that it deſcribed equal areas in equal times, in whatever part of its orbit the planet might be, moving always ſo much the quicker as its diſtance from the ſun was leſs. And it is alſo found that the ſatellites, or ſecondary planets, reſpect the ſame law in revolving about their primaries. But it was ſoon proved by Newton, that all bodies moving in any curve line deſcribed on a plane, and which, by radii drawn to any certain point, deſcribe areas about the point proportional to the times, are impelled or acted on by ſome power tending towards that point. Conſequently the power by which all theſe planets revolve, and are retained in their orbits, is directed to the centre about which they move, viz, the primary planets to the ſun, and the ſatellites to their ſeveral primaries.

Again, Newton demonſtrated, that if ſeveral bodies revolve with an equable motion in ſeveral circles about the ſame centre, and that if the ſquares of their periodical times be in the ſame proportion as the cubes of their diſtances from the common centre, then the centripetal forces of the revolving bodies, by which they tend to their central body, will be in the reciprocal or inverſe ratio of the ſquares of the diſtances. Or if bodies revolve in orbits approaching to circles, and the apſes of thoſe orbits be at reſt, then alſo the centripetal forces of the revolving bodies will be reciprocally proportional to the ſquares of the diſtances. But it had been agreed on by the aſtronomers, and particularly Kepler, that both theſe caſes obtain in all the planets. And therefore he inferred that the centripetal forces of all the planets are reciprocally proportional to the ſquares of the diſtances from the centres of their orbits.

Upon the whole it appears, that the planets are retained in their orbits by ſome power which is continually acting upon them: that this power is directed towards the centre of their orbits: that the intenſity or efficacy of this power increaſes upon an approach towards the centre, and diminiſhes on receding from the ſame, and that in the reciprocal duplicate ratio of the diſtances: and that, by comparing this centripetal force of the planets with the force of gravity on the earth, they are found to be perfectly alike, as may eaſily be ſhewn in various inſtances. For example, in the caſe of the moon, the neareſt of all the planets. The rectilinear ſpaces deſcribed in any given time by a falling body, urged by any powers, reckoning from the beginning of its deſcent, are proportional to thoſe powers. Conſequently the centripetal force of the moon revolving in her orbit, will be to the force of Gravity on the ſurface of the earth, as the ſpace which the moon would deſcribe in falling during any ſmall time, by her centripetal force towards the earth, if ſhe had no circular motion at all, to the ſpace a body near the earth would deſcribe in falling by its Gravity towards the ſame.

Now by an eaſy calculation of thoſe two ſpaces, it appears that the former force is to the latter, as the ſquare of the ſemi-diameter of the earth is to the ſquare of that of the moon's orbit. The moon's centripetal force therefore is equal to the force of Gravity; and conſequently theſe forces are not different, but they are one and the ſame: for if they were different, bodies acted on by the two powers conjointly would fall towards the earth with a velocity double to that ariſing from the ſole power of Gravity.

It is evident therefore that the moon's centripetal force, by which ſhe is retained in her orbit, and prevented from running off in tangents, is the very power of Gravity of the earth extended thither. See Newton's Princip. lib. 1, prop. 45, cor. 2, and lib. 3, prop. 3; where the numeral calculation may be ſeen at full length.

The moon therefore gravitates towards the earth, and reciprocally the earth towards the moon. And this is alſo farther confirmed by the phenomena of the tides.

The like reaſoning may alſo be applied to the other planets. For as the revolutions of the primary planets round the ſun, and thoſe of the ſatellites of Jupiter and Saturn round their primaries, are phenomena of the ſame kind with the revolution of the moon about the earth; and as the centripetal powers of the primary are directed towards the centre of the ſun, and thoſe of the ſatellites towards the centres of their primaries; and laſtly as all theſe powers are reciprocally as the ſquares of the diſtances from the centres, it may ſafely be concluded that the power and cauſe are the ſame in all.

Therefore, as the moon gravitates towards the earth, and the earth towards the moon; ſo do all the ſecondaries to their primaries, and theſe to their ſecondaries; and ſo alſo do the primaries to the ſun, and the ſun to the primaries. Newton's Princip. lib. 3, prop. 4, 5, 6; Greg. Aſtron. lib. 1, ſect. 7, prop. 46 and 47.

The laws of Univerſal Gravity are the ſame as thoſe of bodies gravitating towards the earth, before laid down.

Cauſe of GRAVITY. Various theories have been advanced by the philoſophers of different ages to account for this grand principle of Gravitation. The ancients, who were only acquainted with particular Gravity, or the tendency of ſublunar bodies towards the earth, aimed no farther than a ſyſtem that might anſwer the more obvious phenomena of it. However, ſome hints are found concerning the Gravitation of celeſtial bodies in the account given of the doctrine of Thales and his ſucceſſors; and it would ſeem that Pythagoras was ſtill better acquainted with it, to which it is ſuppoſed he had a view in what he taught concerning the Harmony of the Spheres.

Ariſtotle and the Peripatetics content themſelves with referring Gravity or weight to a native inclination in heavy bodies to be in their proper place or ſphere, the centre of the earth. And Copernicus aſcribes it to an innate principle in all parts of matter, by which, when ſeparated from their wholes, they endeavour to return to them again the neareſt way. In anſwer to Ariſtotle and his followers, who conſidered the centre of the earth as the centre of the univerſe, he obſerved that it was reaſonable to think there was nothing peculiar to the earth in this principle of Gravity; that the parts of the ſun, moon, and ſtars tended likewiſe to each other, and that their ſpherical figure was preſerved in their various motions by this power. Copern. Revol. lib. 1, cap. 9. But neither of theſe ſyſtems aſſigns any phyſical cauſe of this great effect: they only amount to this, that bodies deſcend becauſe they are inclined to deſcend.

Kepler, in his preface to the commentaries concerning the planet Mars, ſpeaks of Gravity as of a power that was mutual between bodies, and ſays that the earth and moon tend towards each other, and would meet in a point ſo many times nearer to the earth than to the moon, as the earth is greater than the moon, if their motions did not hinder it. He adds, that the tides ariſe from the Gravity of the waters towards the moon. To him we alſo owe the important diſcovery of the analogy between the diſtances of the ſeveral planets from the ſun, and the periods in which they complete their revolutions, viz, that the ſquares of their periodic times are always in the ſame proportion as the cubes of their mean diſtances from the ſun. However, Kepler, Gaſſendi, Gilbert, and others, aſcribe Gravity to a certain magnetic attraction of the earth; conceiving the earth to be one great magnet continually emitting effluvia, which take hold of all bodies, and draw them towards the earth. But this is inconſiſtent with the ſeveral phenomena.

Des Cartes and his followers, Rohault &c, attribute Gravity to an external impulſe or truſion of ſome ſubtle matter. By the rotation of the earth, ſay they, all the parts and appendages of it neceſſarily endeavour to recede from the centre of rotation; but whence they cannot all actually recede, as there is no vacuum or ſpace to receive them. But this hypotheſis, founded on the ſuppoſition of a plenum, is overthrown by what has been ſince proved of the exiſtence of a vacuum.

Dr. Hook inclines to an opinion much like that of Des Cartes. Gravity he thinks deducible from the action of a moſt ſubtle medium, which eaſily pervades and penetrates the moſt ſolid bodies; and which, by ſome motion it has, detrudes all earthly bodies from it, towards the centre of the earth. Voſſius too, and many others, give partly into the Carteſian notion, and ſuppoſe Gravity to ariſe from the diurnal rotation of the earth round its axis.

Dr. Halley, deſpairing of any ſatisfactory theory, chooſes to have immediate recourſe to the agency of the Deity. So Dr. Clarke, from a view of ſeveral properties of Gravity, concludes that it is no adventitious effect of any motion, or ſubtle matter, but an original and general law impreſſed by God on all matter, and preſerved in it by ſome efficient power penetrating the very ſolid and intimate ſubſtance of it; being found always proportional, not to the ſurfaces of bodies or corpuſcles, but to their ſolid quantity and contents. It ſhould therefore be no more inquired why bodies gravitate, than how they came to be firſt put in motion. Annot. in Rohault. Phyſ. part 1, cap. 11.

Graveſande, in his Introduct. ad Philoſ. Newton. contends that the cauſe of Gravity is utterly unknown; and that we are to conſider it no otherwiſe than as a law of nature originally and immediately impreſſed by the Creator, without any dependence on any ſecond law or cauſe at all. Of this he thinks the three following conſiderations ſufficient proof. 1. That Gravity requires the preſence of the gravitating or attracting body: ſo the ſatellites of Jupiter, for ex. gravitate towards Jupiter, wherever he may be. 2. That the diſtance being ſuppoſed the ſame, the velocity with which bodies are moved by the force of Gravity, depends on the quantity of matter in the attracting body: and the velocity is not changed, whatever the maſs of the gravitating body may be. 3. That if Gravity do depend on any known law of motion, it muſt be ſome impulſe from an extraneous body; ſo that as Gravity is continual, a continual ſtroke muſt alſo be required. Now if there be any ſuch matter continually ſtriking on bodies, it muſt be fluid, and ſubtle enough to penetrate the ſubſtance of all bodies: but how ſhall a body ſubtle enough to penetrate the ſubſtance of the hardeſt bodies, and ſo rare as not ſenſibly to hinder the motion of bodies, be able to impel vaſt maſſes towards each other with ſuch force? how does this force increaſe the ratio of the maſs of the body, towards which the other body is moved? whence is it that all bodies move with the ſame velocity, the diſtance and body gravitated to being the ſame? can a fluid which only acts on the ſurface either of the bodies themſelves, or their internal particles, communicate ſuch a quantity of motion to bodies, which in all bodies ſhall exactly follow the proportion of the quantity of matter in them?

Mr. Cotes goes yet farther. Giving a view of Newton's philoſophy, he aſſerts that Gravity is to be ranked among the primary qualities of all bodies; and deemed equally eſſential to matter as extenſion, mobility, or impenetrability. Præfat. ad Newt. Princip. But Newton himſelf diſclaims this notion; and to ſhew that he does not take Gravity to be eſſential to bodies, he declares his opinion of the cauſe; chooſing to propoſe it by way of query, not being yet ſufficiently ſatisfied about its experiments. Thus, after having ſhewn that there is a medium in nature vaſtly more ſubtle than air, by whoſe vibrations ſound is propagated, by which light communicates heat to bodies, and by the different denſities of which the refraction and reflection of light are performed; he proceeds to inquire: “ Is not this medium much rarer within the denſe bodies of the ſun, ſtars, planets, and comets, than in the empty celeſtial ſpaces between them? and in paſſing from them to greater diſtances, doth it not grow denſer and denſer perpetually, and thereby cauſe the Gravity of thoſe great bodies towards one another, and of their parts towards the bodies; every body endeavouring to recede from the denſer parts of the medium towards the rarer?

For if this medium be ſuppoſed rarer within the ſun's body than at its ſurface, and rarer there than at the hundredth part of an inch from his body, and rarer there than at the fiftieth part of an inch from his body, and rarer there than at the orb of Saturn; I ſee no reaſon why the increaſe of denſity ſhould ſtop any where, and not rather be continued through all diſtances from the Sun to Saturn, and beyond.

And though this increaſe of denſity may at great diſtances be exceeding ſlow; yet if the elaſtic force of this medium be exceeding great, it may ſuffice to impel bodies from the denſer parts of the medium towards the rarer with all that power which we call Gravity.

And that the elaſtic force of this medium is exceeding great, may be gathered from the ſwiftneſs of its vibrations. Sounds move about 1140 Engliſh feet in a ſecond of time, and in ſeven or eight minutes of time, they move about one hundred Engliſh miles: light moves from the Sun to us in about ſeven or eight minutes of time, which diſtance is about 70000000 Engliſh miles, ſuppoſing the horizontal parallax of the Sun to be about twelve ſeconds; and the vibrations, or pulſes of this medium, that they may cauſe the alternate fits of eaſy tranſmiſſion, and eaſy reflection, muſt be ſwifter than light, and by conſequence above 700000 times ſwifter than ſounds; and therefore the elaſtic force of this medium, in proportion to its denſity, muſt be above 700000 × 700000 (that is, above 490000000000) times greater than the elaſtic force of the air is in proportion to its denſity: for the velocities of the pulſes of elaſtic mediums are in a ſubduplicate ratio of the elaſticities and the rarities of the mediums taken together.

As Magnetiſm is ſtronger in ſmall loadſtones than in great ones, in proportion to their bulk; and Gravity is ſtronger on the ſurface of ſmall planets, than thoſe of great ones, in proportion to their bulk; and ſmall bodies are agitated much more by electric attraction than great ones: ſo the ſmallneſs of the rays of light may contribute very much to the power of the agent by which they are refracted; and if any one ſhould ſuppoſe, that æther (like our air) may contain particles which endeavour to recede from one another (for I do not know what this æther is), and that its particles are exceedingly ſmaller than thoſe of air, or even than thoſe of light; the exceeding ſmallneſs of ſuch particles may contribute to the greatneſs of the force, by which they recede from one another, and thereby make that medium exceedingly more rare and elaſtic than air, and of conſequence, exceedingly leſs able to reſiſt the motions of projectiles, and exceedingly more able to preſs upon groſs bodies by endeavouring to expand itſelf.” Optics, p. 325 &c.

Gravity, in Mechanics, denotes the conatus or tendency of bodies towards the centre of the earth.—That part of mechanics which conſiders the equilibrium or motion of bodies ariſing from Gravity or weight, is particularly called ſtatics.

Gravity in this view is diſtinguiſhed into Abſolute and Relative.

Abſolute GRAVITY is that with which a body deſcends freely and perpendicularly through an unreſiſting medium. The laws of which ſee under DESCENT OF BODIES, ACCELERATION, MOTION, &c.

Relative GRAVITY is that with which a body deſcends on an inclined plane, or through a reſiſting medium, or as oppoſed by ſome other reſiſtance. The laws of which ſee under the articles INCLINED PLANE, DESCENT, FLUID, RESISTANCE, &c.

GRAVITY, in Hydroſtatics. The laws of bodies gravitating in Fluids make the buſineſs of Hydroſtatics.

Gravity is here divided into Abſolute and Specific.

Abſolute or *True* GRAVITY, is the whole force with which the body tends downwards.

Specific GRAVITY, is the relative, comparative, or apparent Gravity in any body, in reſpect of that of an equal bulk or magnitude of another body; denoting that Gravity or weight which is peculiar to each ſpecies or kind of body, and by which it is diſtinguiſhed from all other kinds.

In this ſenſe a body is ſaid to be Specifically Heavier than another, when under the ſame bulk it contains a greater weight than that other; and reciprocally the latter is ſaid to be Specifically Lighter than the former. Thus, if there be two equal ſpheres, each one foot in diameter; the one of lead, and the other of wood: ſince the leaden one is found heavier than the wooden one, it is ſaid to be Specifically, or in Specie, Heavier; and the wooden one Specifically Lighter.

This kind of Gravity is by ſome called Relative; in oppoſition to Abſolute Gravity, which increaſes in proportion to the quantity or maſs of the body.

Laws of the SPECIFIC GRAVITY *of bodies.*

I. If two bodies be equal in bulk, their ſpecific gravities are to each other as their weights, or as their denſities.

II. If two bodies be of the ſame ſpecific gravity or denſity, their abſolute weights will be as their magnitudes or bulks.

III. In bodies of the ſame weight, the ſpecific gravities are reciprocally as their bulks.

IV. The ſpecific gravities of all bodies are in a ratio compounded of the direct ratio of their weights, and the reciprocal ratio of their magnitudes. And hence again the ſpecific gravities are as the denſities.

V. The abſolute gravities or weights of bodies are in the compound ratio of their ſpecific gravities and magnitudes or bulks.

VI. The magnitudes of bodies are directly as their weights, and reciprocally as their ſpecific gravities.

VII. A body ſpecifically heavier than a fluid, loſes as much of its weight when immerſed in it, as is equal to the weight of a quantity of the fluid of the ſame bulk or magnitude.

Hence, ſince the Specific Gravities are as the abſolute gravities under the ſame bulk; the Specific Gravity of the fluid, will be to that of the body immerged, as the part of the weight loſt by the ſolid, is to the whole weight.

And hence the Specific Gravities of fluids are as the weights loſt by the ſame ſolid immerged in them.

VIII. *To find the Specific Gravity of a Fluid, or of a Solid.*—On one arm of a balance ſuſpend a globe of lead by a fine thread, and to the other faſten an equal weight, which may juſt balance it in the open air. Immerge the globe into the fluid, and obſerve what weight balances it then, and conſequently what weight is loſt, which is proportional to the Specific Gravity as above. And thus the proportion of the Specific Gravity of one fluid to another is determined by immerſing the globe ſucceſſively in all the fluids, and obſerving the weights loſt in each, which will be the proportions of the Specific Gravities of the fluids ſought.

This ſame operation determines alſo the Specific Gravity of the ſolid immerged, whether it be a globe or of any other ſhape or bulk, ſuppoſing that of the fluid known. For the Specific Gravity of the fluid is to that of the ſolid, as the weight loſt is to the whole weight.

Hence alſo may be found the Specific Gravity of a body that is lighter than the fluid, as follows:

IX. *To find the Specific Gravity of a Solid that is lighter than the fluid, as water, in which it is put.*—Annex to the lighter body another that is much heavier than the fluid, ſo as the compound maſs may ſink in the fluid. Weigh the heavier body and the compound maſs ſeparately, both in water and out of it; then find how much each loſes in water, by ſubtracting its weight in water from its weight in air; and ſubtract the leſs of theſe remainders from the greater.

Then, As this laſt remainder,
Is to the weight of the light body in air,
So is the Specific Gravity of the fluid,
To the Specific Gravity of that body.

X. The Specific Gravities of bodies of equal weight, are reciprocally proportional to the quantities of weight loſt in the ſame fluid. And hence is found the ratio of the Specific Gravities of ſolids, by weighing in the ſame fluids, maſſes of them that weigh equally in air, and noting the weights loſt by each.

The Specific Gravities of many kinds of bodies, both ſolid and fluid, have been determined by various authors. Marinus Ghetaldus particularly tried the Specific Gravities of various bodies, eſpecially metals; which were taken from thence by Oughtred. In the Philoſ. Tranſ. are ſeveral ample tables of them, by various authors, particularly thoſe of Mr. Davis, vol. 45, p. 416, or Abr. vol. 10, p. 206. Some tables of them were alſo publiſhed by P. Merſenne, Muſchenbroeck, Ward, Cotes, Emerſon, Martin, &c.

It will be ſufficient here to give thoſe of ſome of the moſt uſual bodies, that have been determined with the greater certainty. The numbers in this table expreſs the number of Avoirdupois ounces in a cubic foot of each body, that of common water being juſt 1000 ounces, or 62½ lb.

Table of Specific Gravities.

I. *Solids.*

Platina, pure	23000
Fine gold	19640
Standard gold	18888
Lead	11325
Fine Silver	11091
Standard Silver	10535
Copper	9000
Copper halfpence	8915
Gun metal	8784
Fine braſs	8350
Caſt braſs	8000
Steel	7850
Iron	7645
Pewter	7471
Caſt Iron	7425
Tin	7320
Lapis calaminaris	5000
Loadſtone	4930
Mean of the whole Earth	4500
Crude Antimony	4000
Diamond	3517
Granite	3500
White lead	3160
Iſland cryſtal	2720
Marble	2705
Pebble ſtone	2700
Jaſper	2666
Rock cryſtal	2650
Pearl	2630
Green glaſs	2600
Flint	2570
Onyx ſtone	2510
Common ſtone	2500
Cryſtal	2210
Clay	2160
Oyſter ſhells	2092
Brick	2000
Common earth	1984
Nitre	1900
Vitriol	1880
Alabaſter	1874
Horn	1840
Ivory	1825
Sulphur	1810
Chalk	1793
Solid gunpowder	1745
Alum	1714
Dry bone	1660
Human calculus	1542
Sand	1520
Lignum vitæ	1327
Coal	1250
Jet	1238
Ebony	1177
Pitch	1150
Roſin	1100
Mahogany	1063
Amber	1040
Brazil wood	1031
Boxwood	1030
Common water	1000
Bees wax	955
Butter	940
Oak	925
Gunpowder, ſhaken	922
Logwood	913
Ice	908
Aſh	800
Maple	755
Beech	700
Elm	600
Fir	550
Saſſafras wood	482
Charcoal	
Cork	240
New fallen ſnow	86

II. *Fluids.*

Quickſilver	13600
Oil of Vitriol	1700
Oil of Tartar	1550
Honey	1450
Spirit of Nitre	1315
Aqua Fortis	1300
Treacle	1290
Aqua Regia	1234
Human blood	1054
Urine	1032
Cow's milk	1031
Sea Water	1030
Ale	1028
Vinegar	1026
Tar	1015
Water	1000
Diſtilled Water	993
Red Wine	990
Proof Spirits	931
Olive Oil	913
Pure Spirits of Wine	866
Oil of Turpentine	800
Æther	726
Common Air	1·232
or very nearly	$1\frac{7}{50}$.

Theſe numbers being the weight of a cubic foot, or 1728 cubic inches, of each of the bodies, in Avoirdupois ounces, by proportion the quantity in any other weight, or the weight of any other quantity, may be readily known.

For ex. Required the content of an irregular block of common ſtone which weighs 1 cwt, or 112lb, or 1792 ounces. Here, as 2500 : 1792 : : 1728 : $1228\frac{4}{5}$ cubic inches the content.

Ex. 2. To find the weight of a block of granite, whoſe length is 63 feet, and breadth and thickneſs each 12 feet; being the dimenſions of one of the ſtones, of granite, in the walls of Balbeck. Here, 63 × 12 × 12 = 9072 feet is the content of the ſtone; therefore as 1 : 9072 : : 3500 oz : 31752000 oz or 885 tons 18 cwt. 3 qrs. the weight of the ſtone.

XI. A body deſcends in a fluid ſpecifically lighter, or aſcends in a fluid ſpecifically heavier, with a force equal to the difference between its weight and that of an equal bulk of the fluid.

XII. A body ſinks in a fluid ſpecifically heavier, ſo far as that the weight of the body is equal to the weight of a quantity of the fluid of the ſame bulk as the part immerſed. Hence, as the Specific Gravity of the fluid is to that of the body, ſo is the whole magnitude of the body, to the magnitude of the part immerſed.

XIII. The Specific Gravities of equal ſolids are as their parts immerged in the ſame fluid.

The ſeveral theorems here delivered, are both demonſtrable from the principles of mechanics, and are alſo equally conformable to experiment, which anſwers exactly to the calculation; as is abundantly evident from the courſes of philoſophical experiments, ſo frequently exhibited; where the laws of ſpecific gravitation are well illuſtrated.

GREAT BEAR, one of the conſtellations in the northern hemiſphere. See URSA MAJOR.

GREAT CIRCLES, of the Globe or Sphere, are thoſe whoſe planes paſs through the centre, dividing it into two equal parts or hemiſpheres, and therefore having the ſame centre and diameter with the ſphere itſelf. The principal of theſe are, the equator, the ecliptic, the horizon, the meridians, and the two colures.

GREAT-CIRCLE SAILING, is the art of conducting a ſhip along the arc of a great circle. And it is alſo that part of the theory of navigation which treats of ſailing in the arc of a great circle. See NAVIGATION.

GREAVES (JOHN), an eminent aſtronomer, antiquary and linguiſt, was born in 1602, being the eldeſt ſon of John Greaves rector of Colemore, near Alresford in Hampſhire, and maſter of a grammar ſchool, where his ſon of courſe was well grounded in the primary rules of literature. He then went to Baliol college in Oxford, in 1617; but afterward, on account of his ſkill in philoſophy and polite literature, he was the firſt of five that were elected into Merton college. Having read over all the ancient Greek and Latin writers, he applied to the ſtudy of natural philoſophy and mathematics; and having contracted an intimacy with Mr. Briggs, Savilian profeſſor of geometry at Oxford, and Dr. Bainbridge, Savilian profeſſor of aſtronomy there, he was animated by their examples to proſecute that ſtudy with the greateſt induſtry; and not content with reading the writings of Purbach, Regiomontanus, Copernicus, Tycho Brahe, Kepler, and other celebrated aſtronomers of that and the preceding age, he made the ancient Greek, Arabian, and Perſian authors familiar to him, having before gained an accurate ſkill in the oriental languages. Theſe accompliſhments procured him the profeſſorſhip of geometry in Greſham-college London, in 1630; and at the ſame time he held his fellowſhip of Merton-college.

In a journey to the Continent, in 1635, he viſited the celebrated Golius, profeſſor of Arabic at Leyden, and Claud Hardy at Paris, to converſe about the Perſian language. Hence he paſſed through Italy, and accurately ſurveyed the venerable remains of antiquity at Rome, viſiting and correſponding everywhere with the moſt learned men of every nation. After viſiting Padua, Florence, and Leghorn, he hence embarked for Conſtantinople, where he arrived in 1638. From thence he paſſed over to Rhodes, and Alexandria in Egypt, where he ſtaid four or five months, and made a great number of curious obſervations. He next went to Grand Cairo, meaſured the pyramids; and while there he adjuſted the meaſure of the foot, obſerved by all nations. From hence he returned again through Italy, and arrived in England in the year 1640, after ſtoring his mind with a variety of curious knowledge, and collecting many valuable oriental manuſcripts and ancient curioſities; and while at Rome he made a particular inquiry into the true ſtate of the ancient weights and meaſures.

On the death of Dr. John Bainbridge, in 1643, he was choſen Savilian profeſſor of aſtronomy at Oxford, and principal reader of Linacre's lecture in Merton college; an appointment for which he was eminently qualified, from his critical acquaintance with the works of the ancient and modern aſtronomers. In 1645 he propoſed a method of reforming the calendar, by omitting the intercalary day for 40 years to come: the paper relating to which, was publiſhed by Dr. Thomas Smith, in the Philoſ. Tranſ. for 1699. In 1646, he publiſhed his Pyramidographia, or a Deſcription of the Pyramids of Egypt; and, in 1647, his Diſcourſe on the Roman Foot and Denarius; from which, as from two principles, the meaſures and weights uſed by the Ancients may be deduced. He alſo publiſhed ſeveral other curious works concerning antiquities, &c.

Soon after publiſhing the laſt mentioned book, he was ejected, by the parliament viſitors, from the profeſſorſhip of aſtronomy and fellowſhip of Merton-college; and the ſoldiers committed many outrages, breaking open his cheſts, and deſtroying many of his manuſcripts; which greatly affected him. On this occaſion he retired to London, where he afterwards married, and proſecuted his ſtudies with great vigour, as appears from ſeveral of his philoſophical and theological writings. This however proved but a tranſient happineſs to him; for he died at London, the 8th of October 1652, before he was quite 50 years of age; and left his aſtronomical inſtruments to the Savilian library in Oxford, where they are depoſited.

GREEK ORDERS, in Architecture, are the Doric, Ionic and Corinthian; in contradiſtinction to the two Latin orders, viz the Tuſcan and Compoſite.

GREEN, One of the original colours of the rays of light, or of the priſmatic colours exhibited by the refraction of the rays of light.

Green is the pleaſanteſt of all the colours to the ſight. And hence it has been inferred as a proof of the wiſdom and goodneſs of the Deity, that almoſt all vegetables, cloathing the ſurface of the earth, are green; which they are when growing in the open air; though thoſe in ſubterraneous places, or places inacceſſible to freſh air, are white or yellow. See CHROMATICS, and COLOURS.

GREGORIAN CALENDAR, ſo called from Pope Gregory the 13th, is the new or reformed Calendar, ſhewing the new and full moons, with the time of Eaſter, and the other moveable feaſts depending upon it, by means of epacts diſpoſed through the ſeveral months of the Gregorian year.

GREGORIAN *Epoch*, is the epoch or time, from which the Gregorian calendar, or computation, took place. This began in the year 1582; ſo that the year 1800 is the 218th of this epoch.

Gregorian *Teleſcope*, a particular ſort of teleſcope, invented by Mr. James Gregory. See Telescope.

Gregorian *Year*, the new account, or new ſtyle, introduced upon the reformation of the calendar, by Pope Gregory the 13th, in the year 1582, and from whom it took its name. This was introduced to reform the old, or Julian year, eſtabliſhed by Julius Cæſar, which conſiſted of 365 days 6 hours, or 365 days and a quarter, that is three years of 365 days each, and the fourth year of 366 days. But as the mean tropical year conſiſts only of 365ds 5hrs 48m 57ſec. the former loſt 11min. 3ſec. every year, which in the time of Pope Gregory had amounted to 10 days, and who, by adding theſe 10 days, brought the account of time to its proper day again, and at the ſame time appointed that every century after, a day more ſhould be added, thereby making the years of the complete centuries, viz 1600, 1700, 1800, &c, to be common years of 365 days each, inſtead of leap years of 366 days, which makes the mean Gregorian year equal to 365ds 5hrs 45m. 36ſec.

This computation was not introduced into the account of time in England, till the year 1752, when the Julian account had loſt 11 days, and therefore the 3d of September was in that year, by act of parliament, accounted the 14th, thereby reſtoring the 11 days which had thus been omitted. See Year.

GREGORY (James), profeſſor of mathematics, firſt in the univerſity of St. Andrews, and afterwards in that of Edinburgh, was one of the moſt eminent mathematicians of the 17th century. He was a ſon of the Rev. Mr. John Gregory miniſter of Drumoak in the county of Aberdeen, and was born at Aberdeen in November 1638. His mother was a daughter of Mr. David Anderſon of Finzaugh, or Finſhaugh, a gentleman who poſſeſſed a ſingular turn for mathematical and mechanical knowledge. This mathematical genius was hereditary in the family of the Anderſons, and from them it ſeems to have been tranſmitted to their deſcendants of the names of Gregory, Reid, &c. Alexander Anderſon, couſin-german of the ſaid David, was profeſſor of mathematics at Paris in the beginning of the 17th century, and publiſhed there ſeveral valuable and ingenious works; as may be ſeen in the memoirs of his life and writings, under the article Anderson. The mother of James Gregory inherited the genius of her family; and obſerving in her ſon, while yet a child, a ſtrong propenſity to mathematics, ſhe inſtructed him herſelf in the elements of that ſcience. His education in the languages he received at the grammar ſchool of Aberdeen, and went through the uſual courſe of academical ſtudies in the Mariſchal college; but he was chiefly delighted with philoſophical reſearches, into which a new door had lately been opened by the key of the mathematics. Galileo, Kepler, Des Cartes, &c, were the great maſters of this new method: their works therefore became the principal ſtudy of young Gregory, who ſoon began to make improvements upon their diſcoveries in Optics. The firſt of theſe improvements was the invention of the reflecting teleſcope; the conſtruction of which inſtrument he publiſhed in his *Optica Promota*, in 1663, at 24 years of age. This diſcovery ſoon attracted the attention of the mathematicians, both of our own and of foreign countries, who immediately perceived its great importance to the ſciences of optics and aſtronomy. But the manner of placing the two ſpecula upon the ſame axis appearing to Newton to be attended with the diſadvantage of loſing the central rays of the larger ſpeculum, he propoſed an improvement on the inſtrument, by giving an oblique poſition to the ſmaller ſpeculum, and placing the eye-glaſs in the ſide of the tube. It is obſervable however, that the Newtonian conſtruction of that inſtrument was long abandoned for the original or Gregorian, which is now always uſed when the inſtrument is of a moderate ſize; though Herſchel has preferred the Newtonian form for the conſtruction of thoſe immenſe teleſcopes, which he has of late ſo ſucceſsfully employed in obſerving the heavens.

About the year 1664 or 1665, coming to London, he became acquainted with Mr. John Collins, who recommended him to the beſt optic glaſs-grinders there, to have his teleſcope executed. But as this could not be done for want of ſkill in the artiſts to grind a plate of metal for the object ſpeculum into a true parabolic concave, which the deſign required, he was much diſcouraged with the diſappointment; and after a few imperfect trials made with an ill-poliſhed ſpherical one, which did not ſucceed to his wiſh, he dropped the purſuit, and reſolved to make the tour of Italy, then the mart of mathematical learning, that he might proſecute his favourite ſtudy with greater advantage. And the univerſity of Padua being at that time in high reputation for mathematical ſtudies, Mr. Gregory fixed his reſidence there for ſome years. Here it was that he publiſhed, in 1667, *Vera Circuli et Hyperbolæ Quadratura*; in which he propounded another diſcovery of his own, the invention of an infinitely converging ſeries for the areas of the circle and hyperbola. He ſent home a copy of this work to his friend Mr. Collins, who communicated it to the Royal Society, where it met with the commendations of lord Brounker and Dr. Wallis. He reprinted it at Venice the year following, to which he added a new work, entitled *Geometriæ Pars Univerſalis, inſerviens Quantitatum Curvarum Tranſmutationi et Menſuræ*; in which he is allowed to have ſhewn, for the firſt time, a method for the tranſmutation of curves. Theſe works engaged the notice, and procured the author the correſpondence of the greateſt mathematicians of the age, Newton, Huygens, Wallis, and others. An account of this piece was alſo read by Mr. Collins before the Royal Society, of which Mr. Gregory, being returned from his travels, was choſen a member the ſame year, and communicated to them an account of a controverſy in Italy about the motion of the earth, which was denied by Riccioli and his followers.—Through this channel, in particular, he carried on a diſpute with Mr. Huygens on the occaſion of his treatiſe on the quadrature of the circle and hyperbola, to which that great man had ſtarted ſome objections; in the courſe of which our author produced ſome improvements of his ſeries. But in this diſpute it happened, as it generally does on ſuch occaſions, that the antagoniſts, though ſetting out with temper enough, yet grew too warm in the combat. This was the caſe here, eſpecially on the ſide of Gregory, whoſe defence was, at his own requeſt, inſerted in the Philoſophical Tranſactions. It is unneceſſary to enter into particulars:

ſuffice it therefore to ſay that, in the opinion of Leibnitz, who allows Mr. Gregory the higheſt merit for his genius and diſcoveries, M. Huygens has pointed out, though not errors, ſome conſiderable deficiencies in the treatiſe above mentioned, and ſhewn a much ſimpler method of attaining the ſame end.

In 1668, our author publiſhed at London another work, entitled, *Exercitationes Geometricæ*, which contributed ſtill much farther to extend his reputation. About this time he was elected profeſſor of mathematics in the univerſity of St. Andrew's, an office which he held for ſix years. During his reſidence there, he married, in 1669, Mary, the daughter of George Jameſon, the celebrated painter, whom Mr. Walpole has termed the Vandyke of Scotland, and who was fellow diſciple with that great artiſt in the ſchool of Rubens at Antwerp.

In 1672, he publiſhed "The Great and New Art of Weighing Vanity: or a Diſcovery of the Ignorance and Arrogance of the Great and New Artiſt, in his Pſeudo-philoſophical Writings. By M. Patrick Mathers, Arch-bedal to the Univerſity of St. Andrews. To which are annexed ſome Tentamina de Motu Penduli & Projectorum." Under this aſumed name, our author wrote this little piece to expoſe the ignorance of Mr. Sinclare, profeſſor at Glaſgow, in his hydroſtatical writings, and in return for ſome ill uſage of that author to a colleague of Mr. Gregory's. The ſame year, Newton, on his wonderful diſcoveries in the nature of light, having contrived a new reflecting teleſcope, and made ſeveral objections to Mr. Gregory's, this gave birth to a diſpute between thoſe two philoſophers, which was carried on during this and the following year, in the moſt amicable manner on both ſides; Mr. Gregory defending his own conſtruction, ſo far, as to give his antagoniſt the whole honour of having made the catoptric teleſcopes preferable to the dioptric; and ſhewing, that the imperfections in theſe inſtruments were not ſo much owing to a defect in the object ſpeculum, as to the different refrangibility of the rays of light. In the courſe of this diſpute, our author deſcribed a burning concave mirror, which was approved by Newton, and is ſtill in good eſteem. Several letters that paſſed in this diſpute, are printed by Dr. Deſaguliers, in an Appendix to the Engliſh edition of Dr. David Gregory's Elements of Catoptrics and Dioptrics.

In 1674, Mr. Gregory was called to Edinburgh, to fill the chair of mathematics in that univerſity. This place he had held but little more than a year, when, in October 1675, being employed in ſhewing the ſatellites of Jupiter through a teleſcope to ſome of his pupils, he was ſuddenly ſtruck with total blindneſs, and died a few days after, to the great loſs of the mathematical world, at only 37 years of age.

As to his character, Mr. James Gregory was a man of a very acute and penetrating genius. His temper ſeems to have been warm, as appears from his conduct in the diſpute with Huygens; and, conſcious perhaps of his own merits as a diſcoverer, he ſeems to have been jealous of loſing any portion of his reputation by the improvements of others upon his inventions. He poſſeſſed one of the moſt amiable characters of a true philoſopher, that of being content with his fortune in his ſituation. But the moſt brilliant part of his character is that of his mathematical genius as an inventor, which was of the firſt order; as will appear by the following liſt of his inventions and diſcoveries. Among many others may be reckoned, his Reflecting Teleſcope;—Burning Concave Mirror;—Quadrature of the Circle and Hyperbola, by an infinite converging ſeries;—his method for the Tranſformation of Curves;—a Geometrical Demonſtration of lord Brounker's ſeries for Squaring the Hyperbola—his Demonſtration that the Meridian Line is analogous to a ſcale of Logarithmic Tangents of the Half Complements of the Latitude;—he alſo invented and demonſtrated geometrically, by help of the hyperbola, a very ſimple converging ſeries for making the logarithms;—he ſent to Mr. Collins the ſolution of the famous Keplerian problem by an infinite ſeries;—he diſcovered a method of drawing Tangents to Curves geometrically, without any previous calculations;—a rule for the Direct and Inverſe method of Tangents, which ſtands upon the ſame principle (of exhauſtions) with that of fluxions, and differs not much from it in the manner of application; a Series for the length of the Arc of a Circle from the Tangent, and vice verſa; as alſo for the Secant and Logarithmic Tangent and Secant, and vice verſa:—Theſe, with others, for meaſuring the length of the elliptic and hyperbolic curves, were ſent to Mr. Collins, in return for ſome received from him of Newton's, in which he followed the elegant example of this author, in delivering his ſeries in ſimple terms, independent of each other. Theſe and other writings of our author are moſtly contained in the following works, viz.

1. *Optica Promota*; 4to, London 1663.
2. *Vera Circuli et Hyperbolæ Quadratura*; 4to, Padua 1667 and 1668.
3. *Geometriæ Pars Univerſalis*; 4to, Padua 1668.
4. *Exercitationes Geometricæ*; 4to, London 1668.
5. *The Great and New Art of Weighing Vanity*, &c. 8vo, Glaſgow 1672.

The reſt of his inventions make the ſubject of ſeveral letters and papers, printed either in the *Philoſ. Tranſ.* vol. 3; the *Commerc. Epiſtol. Joh. Collins et Aliorum*, 8vo, 1715; in the Appendix to the Engliſh edition of Dr. David Gregory's *Elements of Optics*, 8vo, 1735, by Dr. Deſaguliers; and ſome ſeries in the *Exercitatio Geometrica* of the ſame author, 4to, 1684, Edinburgh; as well as in his little piece on Practical Geometry.

Gregory (*Dr. David*), Savilian profeſſor of aſtronomy at Oxford, was nephew of the above-mentioned Mr. James Gregory, being the eldeſt ſon of his brother Mr. David Gregory of Kinardie, a gentleman who had the ſingular fortune to ſee three of his ſons all profeſſors of mathematics, at the ſame time, in three of the Britiſh Univerſities, viz. our author David at Oxford, the ſecond ſon James at Edinburgh, and the third ſon Charles at St. Andrews. Our author David, the eldeſt ſon, was born at Aberdeen in 1661, where he received the early parts of his education, but completed his ſtudies at Edinburgh; and, being poſſeſſed of the mathematical papers of his uncle, ſoon diſtinguiſhed himſelf likewiſe as the heir of his genius. In the 23d year of his age, he was elected profeſſor of mathematics in the univerſity of Edinburgh; and, in the ſame year, he publiſhed *Exercitatio Geometrica de Dimenſione Figurarum*,

Figurarum, ſive Specimen Methodi generalis Dimetiendi quaſvis Figuras, Edinb. 1684, 4to. He very ſoon perceived the excellence of the Newtonian philoſophy; and had the merit of being the firſt that introduced it into the ſchools, by his public lectures at Edinburgh. "He had (ſays Mr. Wiſton, in the Memoirs of his own Life, i. 32) already cauſed ſeveral of his ſcholars to keep acts, as we call them, upon ſeveral branches of the Newtonian philoſophy; while we at Cambridge, poor wretches, were ignominiouſly ſtudying the fictitious hypotheſis of the Carteſian."

In 1691, on the report of Dr. Bernard's intention of reſigning the Savilian profeſſorſhip of aſtronomy at Oxford, our author went to London; and being patroniſed by Newton, and warmly befriended by Mr. Flamſteed the aſtronomer royal, he obtained the vacant profeſſorſhip, though Dr. Halley was a competitor. This rivalſhip, however, inſtead of animoſity, laid the foundation of friendſhip between theſe eminent men; and Halley ſoon after became the colleague of Gregory, by obtaining the profeſſorſhip of geometry in the ſame univerſity. Soon after his arrival in London, Mr. Gregory had been elected a fellow of the Royal Society; and, previouſly to his election into the Savilian profeſſorſhip, had the degree of doctor of phyſic conferred on him by the univerſity of Oxford.

In 1693, he publiſhed in the Philoſ. Tranſ. a reſolution of the Florentine problem *de Teſtudine veliformi quadrabili;* and he continued to communicate to the public, from time to time, many ingenious mathematical papers by the ſame channel.

In 1695, he printed at Oxford, *Catoptricæ et Dioptricæ Sphæricæ Elementa;* a work which, we are informed in the preface, contains the ſubſtance of ſome of his public lectures read at Edinburgh, eleven years before. This valuable treatiſe was republiſhed in Engliſh, firſt with additions by Dr. William Brown, with the recommendation of Mr. Jones and Dr. Deſaguliers; and afterwards by the latter of theſe gentlemen, with an appendix containing an account of the Gregorian and Newtonian teleſcopes, together with Mr. Hadley's tables for the conſtruction of both thoſe inſtruments. It is not unworthy of remark, that, in the concluſion of this treatiſe, there is an obſervation which ſhews, that the conſtruction of achromatic teleſcopes, which Mr. Dollond has carried to ſuch great perfection, had occurred to the mind of David Gregory, from reflecting on the admirable contrivance of nature in combining the different humours of the eye. The paſſage is as follows: "Perhaps it would be of ſervice to make the object lens of a different medium, as we ſee done in the fabric of the eye; where the cryſtalline humour (whoſe power of refracting the rays of light differs very little from that of glaſs) is by nature, who never does any thing in vain, joined with the aqueous and vitreous humours (not differing from water as to their power of refraction) in order that the image may be painted as diſtinct as poſſible upon the bottom of the eye."

In 1702 our author publiſhed at Oxford, in folio, *Aſtronomiæ Phyſicæ et Geometricæ Elementa;* a work which is accounted his maſter-piece. It is founded on the Newtonian doctrines, and was eſteemed by Newton himſelf as a moſt excellent explanation and defence of his philoſophy. In the following year he gave to the world an edition, in folio, of the works of Euclid, in Greek and Latin; being done in proſecution of a deſign of his predeceſſor Dr. Bernard, of printing the works of all the ancient mathematicians. In this work, which contains all the treatiſes that have been attributed to Euclid, Dr. Gregory has been careful to point out ſuch as he found reaſon, from internal evidence, to believe to be the productions of ſome inferior geometrician. In proſecution of the ſame plan, Dr. Gregory engaged ſoon after, with his colleague Dr. Halley, in the publication of the Conics of Apollonius; but he had proceeded only a little way in this undertaking, when he died at Maidenhead in Berkſhire, in 1710, being the 49th year of his age only.

Beſides thoſe works publiſhed in our author's life time, as mentioned above, he had ſeveral papers inſerted in the Philoſ. Tranſ. vol. 18, 19, 21, 24, and 25, particularly a paper on the catenarian curve, firſt conſidered by our author. He left alſo in manuſcript, *A Short Treatiſe of the Nature and Arithmetic of Logarithms*, which is printed at the end of Keill's tranſlation of Commandine's Euclid; and a *Treatiſe of Practical Geometry*, which was afterwards tranſlated, and publiſhed in 1745, by Mr. Maclaurin.

Dr. David Gregory married, in 1695, Elizabeth, the daughter of Mr. Oliphant of Langtown in Scotland. By this lady he had four ſons, of whom, the eldeſt, David, was appointed regius profeſſor of modern hiſtory at Oxford by king George the 1ſt, and died at an advanced age in 1767, after enjoying for many years the dignity of dean of Chriſtchurch in that univerſity.

When David Gregory quitted Edinburgh, he was ſucceeded in the profeſſorſhip at that univerſity by his brother *James*, likewiſe an eminent mathematician; who held that office for 33 years, and, retiring in 1725, was ſucceeded by the celebrated Maclaurin. A daughter of this profeſſor James Gregory, a young lady of great beauty and accompliſhments, was the victim of an unfortunate attachment, that furniſhed the ſubject of Mallet's well known ballad of *William and Margaret*.

Another brother, Charles, was created profeſſor of mathematics at St. Andrews by Queen Anne, in 1707. This office he held with reputation and ability for 32 years; and, reſigning in 1739, was ſucceeded by his ſon, who eminently inherited the talents of his family, and died in 1763.

Some farther Particulars of the Family of the Gregorys and Anderſons, communicated by Dr. Thomas Reid, Profeſſor of Moral Philoſophy in the Univerſity of Glaſgow, a Nephew of the late Dr. David Gregory Savilian Profeſſor at Oxford.

Some account of the family of the Gregorys at Aberdeen, is given in the Life of the late Dr. John Gregory prefixed to his works, printed at Edinburgh for A. Strahan and T. Cadell, London, and W. Creech, Edinburgh, 1788, in four ſmall 8vo volumes.

Who was the author of that Life, or whence he had his information, I do not know. I have heard it aſcribed to Mr. Tytler the younger, whoſe father was appointed one of the guardians of Dr. John Gregory's children. Some additions to what is contained in it, and remarks upon it, is all I can furniſh upon this ſubject.

Page 3. I know nothing of the education of David Anderſon of Finzaugh. He ſeems to have been a ſelf-taught Engineer. Every public work which ſurpaſſed the ſkill of common artiſts, was committed to the management of David. Such a reputation he acquired by his ſucceſs in works of this kind, that with the vulgar he got the by-name of *Davie do a' thing*, that is in the Scottiſh dialect, *David who could do every thing*. By this appellation he is better known than by his proper name. He raiſed the great bells into the ſteeple of the principal church: he cut a paſſage for ſhips of burden through a ridge of rock under water, which croſſed the entrance into the harbour of Aberdeen. In a long picture gallery at Cullen Houſe, the ſeat of the earl of Findlater, the wooden ceiling is painted with ſeveral of the fables of Ovid's Metamorphoſis. The colours are ſtill bright, and the repreſentation lively. The preſent earl's grandfather told me that this painting was the work of David Anderſon my anceſtor, whom he acknowledged as a friend and relation of his family.

Such works, while they gave reputation to David, ſuited ill with his proper buſineſs, which was that of a merchant in Aberdeen. In that he ſucceeded ill; and having given up mercantile buſineſs, from a ſmall remainder of his fortune began a trade of making malt; and having inſtructed his wife in the management of it, left it to her care, and went into England to try his fortune as an engineer; an employment which in his own country he had practiſed gratuitouſly. Having in that way made a fortune which ſatisfied him, he returned to Aberdeen, where his wife had alſo made money by her malting buſineſs.

After making ſuch proviſion for their children as they thought reaſonable, they agreed that the longeſt liver of the two ſhould enjoy the remainder, and at death ſhould bequeath it to certain purpoſes in the management of the magiſtrates of Aberdeen.

The wife happened to live longeſt, and fulfilled what had been concerted with her huſband. Her legacies, well known in Aberdeen, are called after her name *Jane Guild's Mortifications*, a mortification in Scots law ſignifying a bequeathment for ſome charitable purpoſe. They conſiſt of ſums for different purpoſes. For orphans, for the education of boys and girls, for unmarried gentlewomen, and for widows; and they ſtill continue to be uſeful to many in indigent circumſtances. She was the daughter of Dr. Guild a miniſter of Aberdeen. Beſides her money, ſhe bequeathed a piece of tapeſtry, wrought by her own hand, and repreſenting the hiſtory of queen Eſther, from a drawing made by her huſband. The tapeſtry continues to ornament the wall of the principal church.

In the ſame page it is ſaid that Alexander Anderſon, profeſſor of mathematics at Paris, was the couſin-german of David above-mentioned. I know not the writer's authority for this: I have always heard that they were brothers; but for this I have only family tradition.

P. 4. It is here ſaid that James Gregory was inſtructed in the Elements of Euclid by his mother, the daughter of David Anderſon.

The account I have heard differs from this. It is, that his brother David, being ten or eleven years older, had the direction of his education after their father's death, and, when James had finiſhed his courſe of philoſophy, was at a loſs to what literary profeſſion he ſhould direct him. After ſome unſucceſsful trials, he put Euclid's Elements into his hand, and finding that he applied to Euclid with great avidity and ſucceſs, he encouraged and aſſiſted him in his mathematical ſtudies.

This tradition agrees with what James Gregory ſays in the preface to his *Optica Promota*; where after mentioning his advance to the 26th propoſition, he adds, *Ubi diu hæſi omne ſpe progrediendi orbatus, ſed continuis hortatibus et auxiliis fratris mei Davidis Gregorii, in Mathematicis non parum verſati (cui ſi quid in hiſce Scientiis præſtitero, me illud debere non inficias ibo) animatus, tandem incidi &c.* Whether David had been inſtructed in mathematics by his mother, or had any living inſtructor, I know not.

P. 5, 6. In theſe two pages I think the merit of Gregory compared with that of Newton in the invention of the catoptric teleſcope, is put in a light more unfavourable to Newton than is juſt. Gregory believing that the imperfection of the dioptric teleſcope aroſe ſolely from the ſpherical figure of the glaſſes, invented his teleſcope to remedy that imperfection. Being leſs converſant in the practice of mechanics, he did not attempt to make any model. The ſpecula of his teleſcope required a degree of poliſh and a figure which the beſt opticians of that age were unable to execute. Newton demonſtrated that the imperfection of the dioptric teleſcope aroſe chiefly from the different refrangibility of the rays of light; he demonſtrated alſo that the catopric teleſcope required a degree of poliſh far beyond what was neceſſary for the dioptric. He made a model of his teleſcope; and finding that the beſt poliſh which the opticians could give, was inſufficient, he improved the poliſh with his own hand, ſo as to make it anſwer the purpoſe, and has deſcribed moſt accurately the manner in which he did this. And, had he not given this example of the practicability of making a reflecting teleſcope, it is probable that it would have paſſed as an impracticable idea to this day.

P. 11. To what is ſaid of this James Gregory might have been added, that he was led by analogy to the true law of Refraction, not knowing that it was diſcovered by Des Cartes before (ſee Preface to *Optica Promota*); and that in 1670 having received in a letter from Collins, a Series for the Area of the Zone of a Circle, and as Newton had invented an univerſal method by which he could ſquare all Curves Geometrical and Mechanical by Infinite Series of that kind; Gregory after much thought diſcovered this univerſal method, or an equivalent one. Of this he perfectly ſatisfied Newton and the other mathematicians of that time, by a letter to Collins in Feb. 1671. He was ſtrongly ſolicited by his brother David to publiſh his Univerſal Method of Series without delay, but excuſed himſelf upon a point of honour; that as Newton was the firſt inventor, and as he had been led to it by an account of Newton's having ſuch a method, he thought himſelf bound to wait till Newton ſhould publiſh his method. I have ſeen the letters that paſſed between the brothers on this ſubject.

With regard to the controverſy between James Gregory and Huygens, I take the ſubject of that controverſy to have been, not whether J. Gregory's Quadrature of the Circle by a converging ſeries was juſt, but whether he had demonſtrated, as in one of his propoſitions he pretended to do, That it is impoſſible to expreſs perfectly the Area of a Circle in any known Algebraical form, beſides that of an infinite converging ſeries. Huygens excepted to the demonſtration of this propoſition, and Gregory defended it; neither of them convinced his antagoniſt, nor do I know that Leibnitz improved upon what Gregory had done.

P. 12. David Gregory of Kinardie deſerved a more particular account than is here given.

It is true that he ſerved an apprenticeſhip to a mercantile houſe in Holland, but he followed that profeſſion no longer than he was under authority, having a ſtronger paſſion for knowledge than for money. He returned to his own country in 1655, being about 28 years of age, and from that time led the life of a philoſopher. Having ſucceeded to the eſtate of Kinardie by the death of an elder brother, he lived there to the end of that century. There all his children were born, of whom he had thirty-two by two wives.

Kinardie is above 40 Engliſh miles north from Aberdeen, and a few miles from Bamf, upon the river Diveron. He was a jeſt among the neighbouring gentlemen for his ignorance of what was doing about his own farm, but an oracle in matters of learning and philoſophy, and particularly in medicine, which he had ſtudied for his amuſement, and begun to practiſe among his poor neighbours. He acquired ſuch a reputation in that ſcience, that he was employed by the nobility and gentlemen of that county, but took no fees. His hours of ſtudy were ſingular. Being much occupied through the day with thoſe who applied to him as a phyſician, he went early to bed, roſe about two or three in the morning, and, after applying to his ſtudies for ſome hours, went to bed again and ſlept an hour or two before breakfaſt.

He was the firſt man in that country who had a barometer; and by ſome old letters which I have ſeen, it appeared, that he had correſponded with ſome philoſophers on the continent about the changes in the barometer and in the weather, particularly with Mariotte the French philoſopher. He was once in danger of being proſecuted as a conjurer by the Preſbytery on account of his barometer. A deputation of that body having waited upon him to enquire into the ground of certain reports that had come to their ears, he ſatisfied them ſo far as to prevent the proſecution of a man known to be ſo extenſively uſeful by his knowledge of medicine.——About the beginning of this century he removed with his family to Aberdeen, and in the time of queen Anne's war employed his thoughts upon an improvement in artillery, in order to make the ſhot of great guns more deſtructive to the enemy, and executed a model of the engine he had conceived. I have converſed with a clock-maker in Aberdeen who was employed in making this model; but having made many different pieces by direction without knowing their intention, or how they were to be put together, he could give no account of the whole. After making ſome experiments with this model, which ſatisfied him, the old gentleman was ſo ſanguine in the hope of being uſeful to the allies in the war againſt France, that he ſet about preparing a field equipage with a view to make a campaign in Flanders, and in the mean time ſent his model to his ſon the Savilian profeſſor, that he might have his and Sir Iſaac Newton's opinion of it. His ſon ſhewed it to Newton, without letting him know that his own father was the inventor. Sir Iſaac was much diſpleaſed with it, ſaying, that if it tended as much to the preſervation of mankind as to their deſtruction, the inventor would have deſerved a great reward; but as it was contrived ſolely for deſtruction, and would ſoon be known by the enemy, he rather deſerved to be puniſhed, and urged the profeſſor very ſtrongly to deſtroy it, and if poſſible to ſuppreſs the invention. It is probable the profeſſor followed this advice. He died ſoon after, and the model was never found.

When the rebellion broke out in 1715, the old gentleman went a ſecond time to Holland, and returned when it was over to Aberdeen, where he died about 1720, aged 93.

He left an hiſtorical manuſcript of the Tranſactions of his own Time and Country, which my father told me he had read.

I was well acquainted with two of this gentleman's ſons, and with ſeveral of his daughters, beſides my own mother. The facts abovementioned are taken from what I have occaſionally heard from them, and from other perſons of his acquaintance.

P. 14. In confirmation of what is ſaid in this page, that the two brothers David and James were the firſt who taught the Newtonian philoſophy in the Scotch Univerſities; I have by me a *Theſis*, printed at Edinburgh in 1690, by James Gregory, who was at that time a profeſſor of philoſophy at St. Andrews, and ſucceeded his brother David in the profeſſion of mathematics at Edinburgh. In this *Theſis*, after a dedication to Viſcount Tarbet, follow the names of twenty-one of his ſcholars who were candidates for the degree of A. M. then twenty-five poſitions or *Theſes*. The firſt three relate to logic, and the abuſe of it in the Ariſtotelian and Carteſian philoſophy. He defines logic to be the art of making a proper uſe of things granted, in order to find what is ſought, and therefore admits only two *Categories* in logic, viz, *Data* and *Quæſita*. The remaining twenty-two poſitions are a compend of Newton's Principia. This *Theſis*, as was the cuſtom at that time in the Scotch univerſities, was to be defended in a public diſputation, by the candidates, previous to their taking their degree.

The famous Dr. Pitcairn was a fellow ſtudent and intimate companion of theſe two Gregories, and during the vacation of the college was wont to go north with them to Kinardie, their father's houſe.

David Gregory was appointed a preceptor to the duke of Glouceſter, queen Anne's ſon; but his entering upon that office was prevented by the death of that prince in the eleventh year of his age.

P. 19. D. Gregory's Euclid is ſaid to have been wrote in proſecution of a deſign of his predeceſſor Dr. Bernard, of printing the works of all the antient mathematicians. This deſign ought to have been aſcribed to Savile, who left in charge to the two profeſſors of his foundation, to print the mathematical works of the antients, and I think left a fund for defraying the expence. Wallis did ſomething in conſequence of this charge; Gregory and Halley did a great deal; but I think nothing has been done in this deſign by the Savilian profeſſors ſince their time.

P. 20. Beſides what is mentioned, Dr. Gregory left in manuſcript a Commentary on Newton's Principia, which Newton valued, and kept by him for many years after the author's death. It is probable that in what relates to aſtronomy, this commentary may coincide in a great meaſure with the author's aſtronomy, which indeed is an excellent Commentary upon that part of the Principia.

P. 24. This David Gregory publiſhed in Latin, a very good compend of arithmetic and algebra, with the title *Arithmeticæ et Algebræ Compendium, in Uſum Juventutis Academicæ*. Edinb. 1736. He had a deſign of publiſhing his uncle's Commentary on the Principia, with extracts from the papers left by James Gregory his grand uncle; but the expence being too great for his fortune, and he too gentle a ſolicitor of the aſſiſtance of others, the deſign was dropped. His ſon David, yet alive, was maſter of an Eaſt India ſhip.

P. 40. To the projectors of the ſociety at Aberdeen, ought to have been added John Stewart profeſſor of mathematics in the Mariſchal college at Aberdeen. He publiſhed an explanation of two treatiſes of Sir Iſaac Newton, viz, his Quadrature of Curves, and his Analyſis by Equations of an infinite number of terms. He was an intimate friend of Dr. Reid's.

Another of the firſt members of that ſociety was Dr. David Skene, who, beſides his eminence in the practice of medicine, had applied much to all parts of natural hiſtory, particularly to botany, and was a correſpondent of the celebrated Linnæus.

Dr. John Gregory and Dr. David Skene were the firſt who attempted a college of medicine at Aberdeen. The firſt gave lectures to his pupils in the theory and practice of medicine, and in chemiſtry; the laſt, in anatomy, materia medica, and midwifery, in order to prepare them for attending the medical college at Edinburgh.

T. R.

The following additional lines by Mr. James Millar, Profeſſor of Mathematics, Glaſgow.

Another inſtance of the prevalence of mathematical genius in the family of Gregory or Anderſon, whether produced by an original and inexplicable determination of the mind, or communicated by the force of example, and the conſciouſneſs of an intimate connection with a reputation already acquired in a particular line, is the celebrated Dr. Reid, profeſſor of moral philoſophy in the univerſity of Glaſgow; a nephew, by his mother, of the late Dr. David Gregory, Savilian profeſſor at Oxford.

This gentleman, well known to the public by his moral and metaphyſical writings, and remarkable for that liberality, and that ardent ſpirit of enquiry, which neither overlooks nor undervalues any branch of ſcience, is peculiarly diſtinguiſhed by his abilities and proficiency in mathematical learning. The objects of literary purſuit are often directed by accidental occurrences. And apprehenſion of the bad conſequences which might reſult from the philoſophy of the late Mr. Hume, induced Dr. Reid to combat the doctrines of that eminent author; and produced a work, which has excited univerſal attention, and ſeems to have given a new turn to ſpeculations upon that ſubject. But it is well known to Dr. Reid's literary acquaintance, that theſe exertions have not diminiſhed the original bent of his genius, nor blunted the edge of his inclination for mathematical reſearches; which, at a very advanced age, he ſtill continues to proſecute with a youthful attachment, and with unremitting aſſiduity.

It may farther be obſerved, of the extraordinary family above mentioned, that Dr. James Gregory, the preſent learned profeſſor of phyſic and medicine in the univerſity of Edinburgh, is the ſon of the late Dr. John Gregory, upon the memoirs of whoſe life the above remarks have been written by Dr. Reid; the ſaid James has lately publiſhed a moſt ingenious work, intitled, *Philoſophical and Literary Eſſays*, in 2 volumes 8vo, Edinb. 1792; and he ſeems to be another worthy inheritant of the ſingular genius of his family.

Gregory (*St. Vincent*), a very reſpectable Flemiſh geometrician, was born at Bruges in 1584, and became a Jeſuit at Rome at 20 years of age. He ſtudied mathematics under the learned Jeſuit Clavius. He afterward became a reputable profeſſor of thoſe ſciences himſelf, and his inſtructions were ſolicited by ſeveral princes: he was called to Prague by the emperor Ferdinand the 2d; and Philip the 4th, king of Spain, was deſirous of having him to teach mathematics to his ſon the young prince John of Auſtria. He was not leſs eſtimable for his virtues than his ſkill in the ſciences. His well-meant endeavours were very commendable, when his holy zeal, though for a falſe religion, led him to follow the army in Flanders one campaign, to confeſs the wounded and dying ſoldiers, in which he received ſeveral wounds himſelf. He died of an apoplexy at Ghent, in 1667, at 83 years of age.

As a writer, Gregory St. Vincent was very diffuſe and voluminous, but he was an excellent geometrician. He publiſhed, in Latin, three mathematical works, the principal of which was his *Opus Geometricum Quadraturæ Circuli, et Sectionum Coni*, Antwerp, 1647, 2 vol. folio. Although he has not demonſtrated, in this work, the Quadrature of the circle, as he pretends to have done, the book nevertheleſs contains a great number of truths and important diſcoveries; one of which is this, viz, that if one aſymptote of an hyperbola be divided into parts in geometrical progreſſion, and from the points of diviſion ordinates be drawn parallel to the other aſymptote, they will divide the ſpace between the aſymptote and curve into equal portions; from whence it was ſhewn by Merſenne, that, by taking the continual ſums of thoſe parts, there would be obtained areas in arithmetical progreſſion, adapted to abſciſſes in geometrical progreſſion, and which therefore were analogous to a ſyſtem of logarithms.

GRENADE, or Grenado. See Granade.

GRUS, the *Crane*, one of the new conſtellations, in the ſouthern hemiſphere; containing, according to Mr. Sharp's catalogue, 13 ſtars.

Grus is alſo one of the Arabian conſtellations, and anſwers to our Ophiucus, to which they changed this conſtellation, their religion prohibiting them from drawing any human figures.

GRY, a meaſure containing one-tenth of a line. A line is one-tenth of a digit, and a digit is one-tenth of a foot, and a philoſophical foot, one-third of a pendulum, whoſe diadromes, or vibrations, in the latitude of 45 degrees, are each equal to one ſecond of time, or one-ſixtieth of a minute.

GUARDS, a name that has been ſometimes applied to the two ſtars neareſt the north pole; being in the hind part of the chariot, at the tail of Urſa Minor or little bear; one of them being alſo called the pole ſtar.

GUERICKE (Otto or Otho), counſellor to the elector of Brandenbourg and burgomaſter of Magdebourg, was born in 1602, and died in 1686 at Hambourg. He was one of the greateſt philoſophers of his time. It was Guericke that invented the air-pump; the two braſs hemiſpheres, which being applied to each other, and the air exhauſted, 16 horſes were not able to draw them aſunder; the marmouſet of glaſs which deſcended in a tube in rainy weather, and roſe again on the return of ſerene weather. This laſt machine fell into diſuſe on the invention of the barometer, eſpecially after Huygens and Amontons gave theirs to the world. Guericke made uſe of his marmouſet to foretell ſtorms; from whence he was looked upon as a ſorcerer by the people; ſo that the thunder having one day fallen upon his houſe, and ſhivered to pieces ſeveral machines which he had employed in his experiments, they failed not to ſay it was a puniſhment from heaven that was angry with him.——Guericke was author of ſeveral works in natural philoſophy, the principal of which was his *Experimenta Magdeburgica*, in folio, which contains his experiments on a vacuum.

GUERITE, in Fortification, a centry-box; being a ſmall tower of wood, or ſtone, uſually placed on the point of a baſtion, or on the angles of the ſhoulder, to hold a centinel, who is to take care of the ditch, and watch againſt a ſurpriſe.

GUEULE, in Architecture. See Gula.

GUINEA, a gold coin ſtruck in England.——The value or rate of the guinea has varied. It was at firſt equal to 20 ſhillings; but by the ſcarcity of gold it was afterwards advanced to 21 *s.* 6*d.*; though it is now ſunk to 21*s.*

The pound weight troy of gold is cut into 44 parts and a half, and each part makes a guinea, which is therefore equal to $\frac{2}{89}$lb, or $\frac{24}{89}$oz, or 5dwts $9\frac{39}{89}$ gr.

This coin took its name, Guinea, from the circumſtance of the gold of which it was firſt ſtruck being brought from that part of Africa called Guinea, for which reaſon alſo it bore the impreſſion of an elephant.

GULA, Gueule, or Gola, in Architecture, a wavy member whoſe contour reſembles the letter S, commonly called an Ogee.

GULBE, in Architecture, the ſame as Gorge.

GULF, or Gulph, in Geography, a part of the ocean running up into the land through a narrow paſſage, or ſtrait, and forming a bay within. As, the Gulf of Venice, or Adriatic ſea; the Gulf of Arabia, or of Perſia, which is the Red Sea; the Gulf of Conſtantinople, or the Black Sea; the Gulf of Mexico; &c.

GUN, a fire-arm, or weapon of offence, which forcibly diſcharges a ball or other matter through a cylindrical tube, by means of inflamed gun-powder.

The word Gun now includes moſt of the ſpecies of fire-arms; mortars and piſtols being almoſt the only ones excepted from this denomination. They are divided into great and ſmall guns: the former including all that are uſually called cannon, ordnance, or artillery; and the latter includes muſquets, firelocks, carabines, muſquetoons, blunderbuſſes, fowling-pieces, &c.

It is not certainly known at what time theſe weapons were firſt invented. And though the introduction of guns into the weſtern part of the world is but of modern date, comparatively ſpeaking; yet it is certain that in ſome parts of Aſia they have been uſed for many ages, though in a very rude and imperfect manner. Philoſtratus ſpeaks of a city near the river Hyphaſis in the Indies, which was ſaid to be impregnable, and that its inhabitants were relations of the gods, becauſe they threw thunder and lightning upon their enemies; and other Greek authors, as alſo Quintus Curtius, ſpeak of the ſame thing having happened to Alexander the Great. Hence ſome have imagined that guns were uſed by the eaſtern nations in his time, while others ſuppoſe the thunder and lightning alluded to by thoſe authors, were only certain artificial fire-works, or rockets, ſuch as we know are uſed in the wars by the Indians even in the preſent day againſt the Europeans. Be this however as it may, it is aſſerted by many modern travellers, that Guns were uſed in China as far back as the year of Chriſt 85, and have continued in uſe ever ſince.

The firſt hint of the invention of Guns in Europe, is in the works of Roger Bacon, who flouriſhed in the 13th century. In a treatiſe written by him about the year 1280, he propoſes to apply the violent exploſive force of gun-powder for the deſtruction of armies. And though it is certainly known that the compoſition of gun-powder is deſcribed by Bacon in the ſaid work, yet the invention has uſually, though improperly, been aſcribed to Bartholdus Schwartz, a German monk, who it is ſaid diſcovered it only in the year 1320; and the invention is related in the following manner. Schwartz having, for ſome purpoſe, pounded nitre, ſulphur, and charcoal together, in a mortar, which he afterwards covered imperfectly with a ſtone, a ſpark of fire accidentally fell into the mortar, which ſetting the mixture on fire, the exploſion blew the ſtone to a conſiderable diſtance. Hence it is probable that Schwartz might be taught the ſimpleſt method of applying it in war; for it rather ſeems that Bacon conceived the manner of uſing it to be by the violent effort of the flame unconfined, and which is indeed capable of producing aſtoniſhing effects. (See Gunpowder.) And the figure and name of *mortars* given to a ſpecies of old artillery, and their employment, in throwing large ſtone bullets at an elevation, very much favour this conjecture.

Soon after the time of Schwartz, we find Guns commonly uſed as inſtruments of war. Great guns were firſt uſed. Theſe were originally made of iron-bars ſoldered together, and fortified with ſtrong iron hoops or rings; ſeveral of which are ſtill to be ſeen in the Tower of London, and in the Warren at Woolwich. Others were made of thin ſheets of iron rolled up together and hooped: and on particular emergencies ſome have been made of leather, and of lead, with plates of iron or copper. Theſe firſt pieces were executed in a rude and imperfect manner, like the firſt eſſays of moſt new inventions. Stone balls were thrown out of them, and a ſmall quantity of powder uſed on account of their weakneſs. They were of a cylindrical form, without ornaments, and were placed on their carriages by rings.

When, or by whom they were firſt made, is uncertain. It is known however that the Venetians uſed cannon at the ſiege of Claudia Jeſſa, now called Chioggia, in 1366, which were brought thither by two Germans, with ſome powder and leaden balls; as likewiſe in their wars with the Genoeſe in 1379. But before that, king Edward the 3d made uſe of cannon at the battle of Creſſy in 1346, and at the ſiege of Calais in 1347. Cannon were employed by the Turks at the ſiege of Conſtantinople, then in poſſeſſion of the Chriſtians, in 1394, and in that of 1452, which threw a weight of 100lb; but they commonly burſt at the 1ſt, 2d, or 3d firing. Louis the 12th had one caſt at Tours, of the ſame ſize, which threw a ball from the Baſtile to Charenton: one of theſe extraordinary cannon was taken at the ſiege of Dieu in 1546, by Don John de Caſtro, and is now in the caſtle of St. Julian da Barra, 10 miles from Liſbon: the length of is 20 feet 7 inches, its diameter at the middle 6 feet 3 inches, and it threw a ball of 100lb weight. It has neither dolphins, rings, nor button; is of an unuſual kind of metal; and it has a large Indoſtan inſcription upon it, which ſays it was caſt in 1400.

Formerly, cannon were dignified with uncommon names. Thus, Lewis the 12th, in 1503, had 12 braſs cannon caſt, of an extraordinary ſize, called after the names of the 12 peers of France. The Spaniſh and Portugueſe called them after their ſaints. The emperor Charles the 5th, when he marched againſt Tunis, founded the 12 apoſtles. At Milan there is a 70 pounder, called the Pimontelle; and one at Bois-le-duc, called the Devil. A 60 pounder at Dover-caſtle, called Queen Elizabeth's pocket-piſtol. An 80 pounder in the Tower of London, brought there from Edinburgh-caſtle, called Mounts-meg. An 80 pounder in the royal arſenal at Berlin, called the Thunderer. An 80 pounder at Malaga, called the Terrible. Two curious 60 pounders in the arſenal at Bremen, called the Meſſenger of bad news. And laſtly an uncommon 70 pounder in the caſtle of St. Angelo at Rome, made of the nails that faſtened the copper-plates which covered the ancient Pantheon, with this inſcription upon it, Ex clavis trabalibus porticus Agrippæ.

In the beginning of the 15th century theſe uncommon names were generally aboliſhed, and the following more univerſal ones took place, viz,

Names.	*Wt. of ball, Pounders.*	*Wt. of piece in cwts, about Cwt.*
Cannon royal, or carthoun,	48	90
Baſtard cannon, or ¾ carthoun,	36	79
Demi-carthoun,	24	60
Whole culverins,	18	50
Demi-culverins,	9	30
Falcon,	6	25
Sacker largeſt ſize	8	18
Sacker ordinary	6	15
Sacker loweſt ſort	5	13
Baſiliſk	48	85
Serpentine	4	8
Aſpic	2	7
Dragon	6	12
Syren	60	81
Falconet	3, 2, and 1	15, 10, 5
Rabinet	1	
Moyens	10 or 12 oz.	

Theſe curious names of beaſts and birds of prey were adopted on account of their ſwiftneſs in motion, or of their cruelty; as the falconet, falcon, ſacker, and culverin, &c, for their ſwiftneſs in flying; the baſiliſk, ſerpentine, aſpic, dragon, ſyren, &c, for their cruelty.

But, at preſent, cannon take their names from the weight of their proper ball. Thus, a piece that diſcharges a caſt-iron ball of 24 pounds, is called a 24 pounder; one that carries a ball of 12 pounds, is called a 12 pounder; and ſo of the reſt, divided into the following ſorts, viz,

Ship-guns, conſiſting in 42, 36, 32, 24, 18, 12, 9, 6, and 3 pounders.

Garriſon-guns, in 42, 32, 24, 18, 12, 9, and 6 pounders.

Battering-guns, in 24, 18, and 12 pounders.

Field-pieces, in 12, 9, 6, 3,2, 1½, 1, and ½ pounders.

Mortars, it is thought, have been at leaſt as ancient as cannon. They were employed in the wars of Italy, to throw balls of red-hot iron, ſtones, &c, long before the invention of ſhells. Theſe laſt, it is ſuppoſed, were of German invention, and the uſe of them in war ſhewn by the following accident; viz, a citizen of Venlo, at a feſtival celebrated in honour of the duke of Cleves, throwing a number of ſhells, one of them fell on a houſe and ſet it on fire, by which misfortune the greateſt part of the town was reduced to aſhes. The firſt account of ſhells uſed for military purpoſes, is in 1435, when Naples was beſieged by Charles the 8th. Hiſtory informs us, with more certainty, that ſhells were thrown out of mortars at the ſiege of Wachtendonk, in Guelderland, in 1588, by the earl of Mansfield; and Cyprian Lucar wrote upon the method of filling and throwing ſuch ſhells in his Appendix to the Colloquies of Tartaglia, printed at London in 1588; where alſo the compounding and throwing of carcaſſes and various ſorts of fire-works are ſhewn.

Mr. Malter, an Engliſh engineer, firſt taught the French the art of throwing ſhells, which they practiſed at the ſiege of Motte in 1634. The method of throwing red-hot balls out of mortars was firſt certainly put in practice at the ſiege of Stralſund in 1675 by the elector of Brandenburgh: though ſome ſay in 1653 at the ſiege of Bremen.

Another ſpecies of ordnance has been long in uſe, by the name of Howitzer, which is a kind of medium as to its length, between the cannon and the mortar, and is a very uſeful piece, for diſcharging either ſhells or large balls, which is done either at point-blanc, or at a ſmall elevation.

A new ſpecies of ordnance has lately been introduced by the Carron company, and thence called a Carronade, which is only a very ſhort howitzer, and which poſſeſſes the advantage of being very light and eaſy to work.

The ſpecies of Guns before mentioned, are now made chiefly of caſt iron; except the howitzer, which is of braſs, as well as ſome cannon and mortars.

Muſkets were firſt uſed at the ſiege of Rhege in the year 1521. The Spaniards were the firſt who armed part of their foot with theſe weapons. At firſt they were very heavy, and could not be uſed without a reſt. They had match-locks, and did execution at a great diſtance. On their march the ſoldiers carried only the reſts and ammunition, having boys to bear their muſkets after them. They were very ſlow in loading, not only by reaſon of the unwieldineſs of their pieces, and becauſe they carried the powder and ball ſeparate, but from the time it took to prepare and adjuſt the match; ſo that their fire was not near ſo briſk as ours is now. Afterwards a lighter match-lock muſket came in uſe: and they carried their ammunition in bandeliers, to which were hung ſeveral little caſes of wood covered with leather, each containing a charge of powder. The muſkets with reſts were uſed as late as the beginning of the civil wars in the time of Charles the 1ſt. The lighter kind ſucceeded them, and continued till the beginning of the preſent century, when they alſo were diſuſed, and the troops throughout Europe armed with firelocks. Theſe are uſually made of hammered iron. For the dimenſions, conſtruction, and practice of every ſpecies of Gun, &c, ſee the ſeveral articles Cannon, Mortar, &c. See alſo Gunnery.

GUNNERY, the art of charging, directing, and exploding fire-arms, as cannon, mortars, muſkets, &c, to the beſt advantage.

Gunnery is ſometimes conſidered as a part of the military art, and ſometimes as a part of pyrotechny. To the art of Gunnery too belongs the knowledge of the force and effect of gunpowder, the dimenſions of the pieces, and the proportions of the powder and ball they carry, with the methods of managing, charging, pointing, ſpunging, &c. Alſo ſome parts of Gunnery are brought under mathematical conſideration, which among mathematicians are called abſolutely by the name Gunnery, viz, the rules and method of computing the range, elevation, quantity of powder, &c, ſo as to hit a mark or object propoſed, and is more particularly called Projectiles; which ſee.

History *of* Gunnery.

Long before the invention of gunpowder, and of Gunnery, properly ſo called, the art of artillery, or projectiles, was actually in practice. For, not to mention the uſe of ſpears, javelins, or ſtones thrown with the hand, or of bows and arrows, all which are found among the moſt barbarous and ignorant people, accounts of the larger machines for throwing ſtones, darts, &c, are recorded by the moſt ancient writers. Thus, one of the kings of Judah, 800 years before the chriſtian æra, erected engines of war on the towers and bulwarks of Jeruſalem, for ſhooting arrows and great ſtones for the defence of that city. 2 Chron. xxvi. 15. Such machines were afterwards known among the Greeks and Romans by the names of Balliſta, Catapulta, &c, which produced effects by the action of a ſpring of a ſtrongly twiſted cordage, formed of tough and elaſtic animal ſubſtances, no leſs terrible than the artillery of the moderns. Such warlike inſtruments continued in uſe down to the 12th and 13th centuries, and the uſe of bows ſtill longer; nor is it probable that they were totally laid aſide till they were ſuperſeded by gunpowder and the modern ordnance.

The firſt application of gunpowder to military affairs, it ſeems, was made ſoon after the year 1300, for which the propoſal of friar Bacon, about the year 1280, for applying its enormous exploſion to the deſtruction of armies, might give the firſt hint; and Schwartz, to whom the invention of gunpowder has been erroneouſly aſcribed, on account of the accident abovementioned under the article Gun, might have been the firſt who actually applied it in this way, that is in Europe; for as to Aſia, it is probable that the Chineſe and Indians had ſomething of the kind many ages before. Thus, only to mention the prohibition of fire-arms in the code of Gentoo laws, printed by the Eaſt India Company in 1776, which ſeems to confirm the ſuſpicion ſuggeſted by a paſſage in Quintus Curtius, that Alexander the Great found ſome weapons of that kind in India: Cannon in the Shanſcrit idiom is called ſhet-aghnee, or the weapon that kills a hundred men at once.

However, the firſt pieces of artillery, which were charged with gunpowder and ſtone bullets of a prodigious ſize, were of very clumſy and inconvenient ſtructure and weight. Thus, when Mahomet the 2d beſieged Conſtantinople in 1453, he battered the walls with ſtones of this kind, and with pieces of the calibre of 1200 pounds; which could not be fired more than four times a day. It was however ſoon diſcovered that iron bullets, of much leſs weight than ſtone ones, would be more efficacious if impelled by greater quantities of ſtronger powder. This occaſioned an alteration in the matter and form of the cannon, which were now caſt of braſs. Theſe were lighter and more manageable than the former, at the ſame time that they were ſtronger in proportion to their bore. This change took place about the cloſe of the 15th century.

By this means came firſt into uſe ſuch powder as is now employed over all Europe, by varying the proportion of the materials. But this change of the proportion was not the only improvement it received. The practice of graining it is doubtleſs of conſiderable advantage. At firſt the powder had been always uſed in the form of fine meal, ſuch as it was reduced to by grinding the materials together. And it is doubtful whether the firſt graining of powder was intended to increaſe its ſtrength, or only to render it more convenient for filling into ſmall charges and the charging of ſmall arms, to which alone it was applied for many years, whilſt meal-powder was ſtill uſed for cannon.

But at laſt the additional ſtrength which the grained powder was found to poſſeſs, doubtleſs from the free paſſage of the air between the grains, occaſioned the meal-powder to be entirely laid aſide.

For the laſt 200 years, the formation of cannon has been very little improved; the beſt pieces of modern artillery differing little in their proportions from thoſe uſed in the time of Charles the 5th. Indeed lighter and ſhorter pieces have been often propoſed and tried; but though they have their advantages in particular caſes, it is agreed they are not ſufficient for general ſervice. Yet the ſize of the pieces has been much decreaſed; the ſame purpoſes being now accompliſhed, by ſmaller pieces than what were formerly thought neceſſary. Thus the battering cannon now approved, are thoſe that formerly were called demi cannon, carrying a ball of 24 pounds weight; this weight having been found fully ſufficient. The method alſo of making a breach, by firſt cutting off the whole wall as low as poſſible before its upper part is attempted to be beaten down, ſeems to be a conſiderable modern improvement in the practical part of gunnery. But the moſt conſiderable improvement in the practice, is the method of firing with ſmall quantities of powder, and elevating the piece but a little, ſo that the bullet may juſt go clear of the parapet of the enemy, and drop into their works, called ricochet firing: for by this means the ball, coming to the ground at a ſmall angle, and with a ſmall velocity, does not bury itſelf, but bounds or rolls along a great way, deſtroying all before it. This method was firſt practiſed by M. Vauban at the ſiege of Aeth, in the year 1692. A practice of this kind was ſucceſsfully uſed by the king of Pruſſia at the battle of Roſbach in 1757. He had ſeveral ſix-inch mortars, made with trunnions, and mounted on travelling carriages, which were fired obliquely on the enemy's lines, and among their horſe. Theſe being charged with only 8 ounces of powder, and elevated at one degree and a quarter, did great execution: for the ſhells rolling along the lines with burning fuſes made the ſtouteſt of the enemy not wait for their burſting.

The uſe of fire-arms was however long known before any theory of projectiles was formed. The Italians were the firſt people that made any attempts at the theory, which they did about the beginning of the 16th century, and amongſt them it ſeems the firſt who wrote profeſſedly on the flight of cannon ſhot, was Nicholas Tartalia, of Breſcia, the ſame author who had ſo great a ſhare in the invention of the rules for cubic equations. In 1537 he publiſhed, at Venice, his *Nova Scientia*, and in 1546 his *Queſiti & Inventioni diverſi*, in both which he treats profeſſedly on theſe motions, as well as in another work, tranſlated into Engliſh with additions by Cyprian Lucar, under the title of Colloquies concerning the Art of Shooting in great and ſmall Pieces of Artillery, and publiſhed at London in 1588. He determined, that the greateſt range of a ſhot was when diſcharged at an elevation of 45°: and he aſſerted, contrary to the opinion of his contemporaries, that no part of the path deſcribed by a ball is a right line; although the curvature in the firſt part of it is ſo ſmall, that it need not be attended to. He compared it to the ſurface of the ſea; which, though it appears to be a plane, is yet doubtleſs incurvated round the centre of the earth. He ſays he invented the gunner's quadrant, for laying a piece of ordnance at any point or degree of elevation; and though he had but little opportunity of acquiring any practical knowledge by experiments, he yet gave ſhrewd gueſſes at the event of ſome untried methods.

The philoſophers of thoſe times alſo took part in the queſtions ariſing upon this ſubject; and many diſputes on motion were held, eſpecially in Italy, which continued till the time of Galileo, and probably gave riſe to his celebrated Dialogues on Motion. Theſe were not publiſhed till the year 1638; and in the interval there were publiſhed many theories of the motion of military projectiles, as well as many tables of their comparative ranges; though for the moſt part very fallacious, and inconſiſtent with the motions of theſe bodies.

It is remarkable however that, during theſe conteſts, ſo few of thoſe who were intruſted with the care of artillery, thought it worth while to bring their theories to the teſt of experiment. Mr. Robins informs us, in the preface to his New Principles of Gunnery, that he had met with no more than four authors who had treated experimentally on this ſubject. The firſt of theſe is Collado, in 1642, who has given the ranges of a falconet, carrying a three-pound ſhot, to every point of the gunner's quadrant, each point being the 12th part, or 7° and a half. But from his numbers it is manifeſt that the piece was not charged with its uſual allotment of powder. The reſult of his trials ſhews the ranges at the point-blanc, and the ſeveral points of elevation, as below.

Collado's Experiments.

Elevation at Points.		Deg.	Range in paces.
0	or	0	268
1	-	$7\frac{1}{2}$	594
2	-	15	794
3	-	$22\frac{1}{2}$	954
4	-	30	1010
5	-	$37\frac{1}{2}$	1040
6	-	45	1053
7	-	$52\frac{1}{2}$	between the 3d and 4th
8	-	60	between the 2d and 3d
9	-	$67\frac{1}{2}$	between the 1ſt and 2d
10	-	75	between the 0 and 1ſt
11	-	$82\frac{1}{2}$	fell very near the piece.

The next was by Wm. Bourne, in 1643, in his *Art of Shooting in Great Ordnance.* His elevations were not regulated by the points of the Gunner's quadrant, but by degrees; and he gives the proportions between the ranges at different elevations and the extent of the point-blanc ſhot, thus: if the extent of the point-blanc ſhot be repreſented by 1, then the proportions of the ranges at ſeveral elevations will be as below, viz.

Bourne's Proportion of Ranges.

Elevation.		Range.
0°	-	1
5	-	$2\frac{2}{5}$
10	-	$3\frac{1}{5}$
15	-	$4\frac{1}{3}$
20	-	$4\frac{5}{6}$

and the greateſt random $5\frac{1}{2}$;

which greateſt random, he ſays, in a calm day is at 42° elevation; but according to the ſtrength of the wind, and as it favours or oppoſes the flight of the ſhot, the elevation may be from 45° to 36°.—He does not ſay with what piece he made his trials; though from his proportions it ſeems to have been a ſmall one. This however ought to have been mentioned, as the relation between the extent of different ranges varies extremely according to the velocity and denſity of the bullet.

After him, Eldred and Anderſon, both Engliſhmen, alſo publiſhed treatiſes on this ſubject. The former of theſe was many years gunner of Dover Caſtle, where moſt of his experiments were made, the earlieſt of which are dated in 1611, though his book was not publiſhed till 1646, and was intitled *The Gunner's Glaſs*. His principles were ſufficiently ſimple, and within certain limits very near the truth, though they were not rigorouſly ſo. He has given the actual ranges of different pieces of artillery at ſmall elevations, all under 10 degrees. His experiments are numerous, and appear to be made with great care and caution; and he has honeſtly ſet down ſome, which were not reconcilable to his method: upon the whole he ſeems to have taken more pains, and to have had a juſter knowledge of his buſineſs, than is to be found in moſt of his practical brethren.

Galileo printed his Dialogues on Motion in the year 1646. In theſe he pointed out the general laws obſerved by nature in the production and compoſition of motion, and was the firſt who deſcribed the action and effects of gravity on falling bodies: on theſe principles he determined, that the flight of a cannon-ſhot, or of any other projectile, would be in the curve of a parabola, unleſs ſo far as it ſhould be diverted from that track by the reſiſtance of the air. He alſo propoſed the means of examining the inequalities which ariſe from thence, and of diſcovering what ſenſible effects that reſiſtance would produce in the motion of a bullet at ſome given diſtance from the piece.

Notwithſtanding theſe determinations and hints of Galileo, it ſeems that thoſe who came after him never imagined that it was neceſſary to conſider how far the operations of Gunnery were affected by this reſiſtance. Inſtead of this, they boldly aſſerted, without making the experiment, that no great variation could ariſe from the reſiſtance of the air in the flight of ſhells or cannon ſhot. In this perſuaſion they ſupported themſelves chiefly by conſidering the extreme rarity of the air, compared with thoſe denſe and ponderous bodies; and at laſt it became an almoſt generally eſtabliſhed maxim, that the flight of theſe bodies was nearly in the curve of a parabola.

Thus, Robert Anderſon, in his *Genuine Uſe and Effects of the Gunne*, publiſhed in 1674, and again in his book, *To hit a Mark*, in 1690, relates a great many experiments; but proceeding on the principles of Galileo, he ſtrenuouſly aſſerts that the flight of all bullets is in the curve of a parabola; undertaking to anſwer all objections that could be brought to the contrary. The ſame thing was alſo undertaken by Blondel, in his *Art de jetter les Bombes*, publiſhed in 1683; where, after long diſcuſſion, he concludes, that the variations from the air's reſiſtance are ſo ſlight as not to deſerve any notice. The ſame ſubject is treated of in the Philoſ. Tranſ. N° 216, p. 68, by Dr. Halley; who alſo, ſwayed by the very great diſproportion between the denſity of the air and that of iron or lead, thought it reaſonable to believe that the oppoſition of the air to large metal-ſhot is ſcarcely diſcernible; although in ſmall and light ſhot he owns that it muſt be accounted for.

But though this hypotheſis went on ſmoothly in ſpeculation; yet Anderſon, who made a great number of trials, found it impoſſible to ſupport it without ſome new modification. For though it does not appear that he ever examined the comparative ranges of either cannon or muſket ſhot when fired with their uſual velocities, yet his experiments on the ranges of ſhells thrown with velocities that were but ſmall, in compariſon of thoſe above mentioned, convinced him that their whole track was not parabolical. But inſtead of making the proper inferences from hence, and concluding that the reſiſtance of the air was of conſiderable efficacy, he framed a new hypotheſis; which was, that the ſhell or bullet at its firſt diſcharge flew to a certain diſtance in a right line, from the end of which line only it began to deſcribe a parabola: and this right line, which he calls the line of the impulſe of the fire, he ſuppoſes is the ſame for all elevations. So that, by aſſigning a proper length to this line of impulſe, it was always in his power to reconcile any two ſhots made at any two different angles; though the ſame method could not ſucceed with three ſhots; nor indeed does he ever inform us of the event of his experiments when three ranges were tried at one time.

But after the publication of Newton's Principia, it might have been expected, that the defects of the theory would be aſcribed to their true cauſe, which is the great reſiſtance of the air to ſuch ſwift motions; as in that work he particularly conſidered the ſubject of ſuch motions, and related the reſult of experiments, made on ſlow motions at leaſt; by which it appeared, that in ſuch motions the reſiſtance increaſes as the ſquare of the velocities, and he even hints a ſuſpicion that it will increaſe above that law in ſwifter motions, as is now known to be the caſe. So far however were thoſe who treated this ſubject ſcientifically, from making a proper allowance for the reſiſtance of the atmoſphere, that they ſtill neglected it, or rather oppoſed it, and their theories ſtill differed moſt egregiouſly from the truth. Huygens alone ſeems to have attended to this principle: for in the year 1690 he publiſhed a treatiſe on gravity, in which he gave an account of ſome experiments tending to prove that the track of all projectiles, moving with very ſwift motions, was widely different from that of a parabola. The reſt of the learned generally acquieſced in the juſtneſs and ſufficiency of Galileo's doctrine, and accordingly very erroneous calculations concerning the ranges of cannon were given. Nor was any farther notice taken of theſe errors till the year 1716, at which time Mr. Reſſons, a French officer of artillery, of great merit and experience, gave in a memoir to the Royal Academy, importing that, "although it was agreed that theory joined with practice did conſtitute the perfection of every art; yet experience had taught him that theory was of very little ſervice in the uſe of mortars: That the works of M. Blondel had juſtly enough deſcribed the ſeveral parabolic lines, according to the different degrees of the elevation of the piece; but that practice had convinced him there was no theory in the effect of gunpowder; for having endeavoured, with the greateſt preciſion, to point a mortar according to theſe calculations, he had never been able to eſtabliſh any ſolid foundation upon them."—— One inſtance only occurs in which D. Bernoulli applies the doctrine of Newton to the motions of projectiles, in the Com. Acad. Petrop. tom. 2, pa. 338 &c. Beſides which nothing farther was done in this buſineſs till the time of Mr. Benjamin Robins, who publiſhed a treatiſe in 1742, intitled *New Principles of Gunnery*, in which he treated particularly, not only of the reſiſtance of the atmoſphere, but alſo of the force of gunpowder, the nature and effects of different guns, and almoſt every thing elſe relating to the flight of military projectiles; and indeed he carried the theory of gunnery nearly to its utmoſt perfection.

The firſt thing conſidered by Mr. Robins, and which is indeed the foundation of all other particulars relating to Gunnery, is the exploſive force of gunpowder. M. De la Hire, in the Hiſt. of the Acad. of Sciences for the year 1702, ſuppoſed that this force may be owing to the increaſed elaſticity of the air contained in, and between the grains, in conſequence of the heat and fire produced at the time of the exploſion: a cauſe not adequate to the 200th part of the effect. On the other hand, Mr. Robins determined, by irrefragable experiments, that this force was owing to an elaſtic fluid, ſimilar to our atmoſphere, exiſting in the powder in an extremely condenſed ſtate, which being ſuddenly freed from the powder by the combuſtion, expanded with an amazing force, and violently impelled the bullet, or whatever may oppoſe its expanſion.

The intenſity of this force of exploded gunpowder Mr. Robins aſcertained in different ways, after the example of Mr. Hawkſbee, related in the Philoſ. Tranſ. N° 295, and his Phyſico-Mechan. Exper. pa. 81. One of theſe is by firing the powder in the air thus: A ſmall quantity of the powder is placed in the upper part of a glaſs tube, and the lower part of the tube is immerged in water, the water being made to riſe ſo near the top, that only a ſmall portion of air is left in that part where the powder is placed: then in this ſituation the communication between the upper part of the tube and the external air being cloſed, the powder is fired by means of a burning glaſs, or otherwiſe; the water deſcends upon the exploſion, and ſtands lower in the tube than before, by a ſpace proportioned to the quantity of powder fired.

Another way was by firing the powder in vacuo, viz, in an exhauſted receiver, by dropping the grains of powder upon a hot iron included in the receiver. By this means a permanent elaſtic fluid was generated from the fired gunpowder, and the quantity of it was always in proportion to the quantity of powder that was uſed, as was found by the proportional ſinking of the mercurial gage annexed to the air pump. The reſult of theſe experiments was, that the weight of the elaſtic air thus generated, was equal to $\frac{3}{10}$ of the compound maſs of the gunpowder which yielded it; and that its bulk, when cold and expanded to the rarity of common atmoſpheric air, was about 240 times the bulk of the powder; and conſequently in the ſame proportion would ſuch fluid at firſt, if it were cold, exceed the force or elaſticity of the atmoſphere. But as Mr. Robins found, by another ingenious experiment, that air heated to the extreme degree of the white heat of iron, has its elaſticity quadrupled, or is 4 times as ſtrong; he thence inferred that the force of the elaſtic air generated as above, at the moment of the exploſion, is at leaſt 4 times 240, or 960, or in round numbers about 1000 times as ſtrong as the elaſticity or preſſure of the atmoſphere, on the ſame ſpace.

Having thus determined the force of the gunpowder, or intenſity of the agent by which the projectile is to be urged, Mr. Robins next proceeds to determine the effects it will produce, or the velocity with which it will impel a ſhot of a given weight from a piece of ordnance of given dimenſions; which is a problem ſtrictly limited, and perfectly ſoluble by mathematical rules, and is in general this: Given the firſt force, and the law of its variation, to determine the velocity with which it will impel a given body in paſſing through a given ſpace, which is the length of the bore of the gun.

In the ſolution of this problem, Mr. Robins aſſumes theſe two poſtulates, viz, 1, That the action of the powder on the bullet ceaſes as ſoon as the bullet is out of the piece; and 2d, That all the powder of the charge is fired and converted into elaſtic fluid before the bullet is ſenſibly moved from its place: aſſumptions which for good reaſons are found to be in many caſes very near the truth. It is to be noted alſo, that the law by which the force of the elaſtic fluid varies, is this, viz, that its intenſity is directly as its denſity, or reciprocally proportional to the ſpace it occupies, being ſo much the ſtronger as the ſpace is leſs: a principle well known, and common to all elaſtic fluids. Upon theſe principles then Mr. Robins reſolves this problem, by means of the 39th prop. of Newton's Principia in a direct way, and the reſult is equivalent to this theorem, when the quantities are expreſſed by algebraic ſymbols; viz, the velocity of the ball

$$v = 27130\sqrt{\frac{10a}{cd} \times \log. \frac{b}{a}}$$

$$\text{or} = 100\sqrt{\frac{223ad^2}{w} \times \log. \frac{b}{a}};$$

where v is the velocity of the ball,

a the length of the charge of powder,

b the whole length of the bore,

c the ſpec. grav. of the ball, or wt. of a cubic foot of the ſame matter in ounces,

d the diam. of the bore,

w the wt. of the ball in ounces.

For example, Suppoſe $a = 2\frac{5}{8}$ inc., $b = 45$ inches, $c = 11345$ oz, for a ball of lead, and $d = \frac{3}{4}$ inches;

then $v = 27130\sqrt{\frac{7}{2269} \times \log. \frac{120}{7}} = 1674$ feet per ſecond, the velocity of the ball.

Or, if the wt. of the bullet be $w = 1\frac{9}{20}$ oz $= \frac{29}{20}$ oz.

Then $v = 100\sqrt{\frac{1115 \times 189}{29 \times 32} \times \log. \frac{120}{7}} = 1674$ feet, as before.

" Having in this propoſition, ſays Mr. Robins, ſhewn how the velocity, which any bullet acquires from the force of powder, may be computed upon the principles of the theory laid down in the preceding propoſitions; we ſhall next ſhew, that the actual velocities, with which bullets of different magnitudes are impelled from different pieces, with different quantities of powder, are really the ſame with the velocities aſſigned by theſe computations; and conſequently that this theory of the force of powder, here delivered, does unqueſtionably aſcertain the true action and modification of this enormous power.

"But in order to compare the velocities communicated to bullets by the exploſion with the velocities reſulting from the theory by computation; it is neceſſary that the actual velocities with which bullets move, ſhould be capable of being diſcovered, which yet is impoſſible to be done by any methods hitherto made public. The only means hitherto practiſed by others for that purpoſe, have been either by obſerving the time of the flight of the ſhot through a given ſpace, or by meaſuring the range of the ſhot at a given elevation; and thence computing, on the parabolic hypotheſis, what velocity would produce this range. The firſt method labours under this inſurmountable difficulty, that the velocities of theſe bodies are often ſo ſwift, and conſequently the time obſerved is ſo ſhort, that an imperceptible error in that time may occaſion an error in the velocity thus found, of 2, 3, 4, 5, or 600 feet in a ſecond. The other method is ſo fallacious, by reaſon of the reſiſtance of the air (to which inequality the firſt is alſo liable), that the velocities thus aſſigned may not be perhaps the 10th part of the actual velocities ſought.

"To remedy then theſe inconveniences, I have invented a new method of finding the real velocities of bullets of all kinds; and this to ſuch a degree of exactneſs (which may be augmented too at pleaſure), that in a bullet moving with a velocity of 1700 feet in 1″, the error in the eſtimation of it need never amount to its 500th part; and this without any extraordinary nicety in the conſtruction of the machine."

Mr. Robins then gives an account of the machine by which he meaſures the velocities of the balls, which machine is ſimply this, viz, a pendulous block of wood ſuſpended freely by a horizontal axis, againſt which block are to be fired the balls whoſe velocities are to be determined.

"This inſtrument thus fitted, if the weight of the pendulum be known, and likewiſe the reſpective diſtances of its centre of gravity, and of its centre of oſcillation, from its axis of ſuſpenſion, it will thence be known what motion will be communicated to this pendulum by the percuſſion of a body of a known weight moving with a known degree of celerity, and ſtriking it in a given point; that is, if the pendulum be ſuppoſed at reſt before the percuſſion, it will be known what vibration it ought to make in conſequence of ſuch a determined blow; and, on the contrary, if the pendulum, being at reſt, is ſtruck by a body of a known weight, and the vibration, which the pendulum makes after the blow, is known, the velocity of the ſtriking body may from thence be determined.

"Hence then, if a bullet of a known weight ſtrikes the pendulum, and the vibration, which the pendulum makes in conſequence of the ſtroke, be aſcertained; the velocity with which the ball moved, is thence to be known."

Mr. Robins then explains his method of computing velocities from experiments with this machine; which method is rather troubleſome and perplexed, as well as the rules of Euler and Antoni, who followed him in this buſineſs, but a much ſimpler rule is given in my Tracts, vol. 1, p. 119, where ſuch experiments are explained at full length, and this rule is expreſſed by either of the two following formulas,

$v = 5{\cdot}6727cg \times \frac{p + b}{bir}\sqrt{o} = 614{\cdot}58cg \times \frac{p + b}{birn}$, the velocity; where v denotes the velocity of the ball when it ſtrikes the pendulum, p the weight of the pendulum, b the weight of the ball, c the chord of the arc deſcribed by the vibration to the radius r, g the diſtance below the axis of motion to the centre of gravity, o the diſtance to the centre of oſcillation, i the diſtance to the point of impact, and n the number of oſcillations the pendulum will perform in one minute, when made to oſcillate in ſmall arcs. The latter of theſe two theorems is much the eaſieſt, both becauſe it is free of radicals, and becauſe the value of the radical $\sqrt{o}$, in the former, is to be firſt computed from the number n, or number of oſcillations the pendulum is obſerved to make.

With ſuch machines Mr. Robins made a great number of experiments, with muſket barrels of different lengths, with balls of various weights, and with different charges or quantities of powder. He has ſet down the reſults of 61 of theſe experiments, which nearly agree with the correſponding velocities as computed by his theory of the force of powder, and which therefore eſtabliſh that theory on a ſure foundation.

From theſe experiments, as well as from the preceding theory, many important concluſions were deduced by Mr. Robins; and indeed by means of theſe it is obvious that every thing may be determined relative both to the true theory of projectiles, and to the practice of artillery. For, by firing a piece of ordnance, charged in a ſimilar manner, againſt ſuch a balliſtic pendulum from different diſtances, the velocity loſt by paſſing through ſuch ſpaces of air will be found, and conſequently the reſiſtance of the air, the only circumſtance that was wanting to complete the theory of Gunnery, or military projectiles; and of this kind I have ſince made a great number of experiments with cannon balls, and have thereby obtained the whole ſeries of reſiſtances to ſuch a ball when moving with every degree of velocity, from 0 up to 2000 feet per ſecond of time. In the ſtructure of artillery, they may likewiſe be of the greateſt uſe: For hence may be determined the beſt lengths of guns; the proportions of the ſhot and powder to the ſeveral lengths; the thickneſs of a piece, ſo as it may be able to confine, without burſting, any given charge of powder; as alſo the effect of wads, chambers, placing of the vent, ramming the powder, &c. For the many other curious circumſtances relating to this ſubject, and the various other improvements in the theory and practice of Gunnery, made by Mr. Robins, conſult the firſt vol. of his Tracts, collected and publiſhed by Dr. Wilſon, in the year 1761, where ample information may be found.

Soon after the firſt publication of Robins's New Principles of Gunnery, in 1742, the learned in ſeveral other nations, treading in his ſteps, repeated and farther extended the ſame ſubject, ſometimes varying and enlarging the machinery; particularly Euler in Germany, D'Antoni in Italy, and Meſſrs. D'Arcy and Le Roy in France. But moſt of theſe, like Mr. Robins, with ſmall fire-arms, ſuch as muſkets, and fuſils.

But in the year 1775, in conjunction with ſeveral able officers of the Royal Artillery, and other ingenious gentlemen, I undertook a courſe of experiments with the balliſtic pendulum, in which we ventured to extend the machinery to cannon ſhot of 1, 2, and 3 pounds weight. An account of theſe experiments was publiſhed in the Philoſ. Tranſ. for 1778, and for which the Royal Society honoured me with the prize of the gold medal. "Theſe were the only experiments that I know of which had been made with cannon balls for this purpoſe, although the concluſions to be deduced from ſuch, are of the greateſt importance to thoſe parts of natural philoſophy which are dependent on the effects of fired gunpowder; nor do I know of any other practical method of aſcertaining the initial velocities within any tolerable degree of the truth. The knowledge of this velocity is of the utmoſt conſequence in Gunnery: by means of it, together with the law of the reſiſtance of the medium, every thing is determinable relative to that buſineſs; for, beſides its being an excellent method of trying the ſtrength of different ſorts of powder, it gives us the law relative to the different quantities of powder, to the different weights of ſhot, and to the different lengths and ſizes of guns. Beſides theſe, there does not ſeem to be any thing wanting to anſwer any inquiry that can be made concerning the flight and ranges of ſhot, except the effects ariſing from the reſiſtance of the medium. In theſe experiments the weights of the pendulums employed were from 300 to near 600 pounds. In that paper is deſcribed the method of conſtructing the machinery, of finding the centres of gravity and oſcillation of the pendulum, and of making the experiments, which are all ſet down in the form of a journal, with all the minute and concomitant circumſtances; as alſo the inveſtigation of the new and eaſy rule, ſet down juſt above, for computing the velocity of the ball from the experiments. The charges of powder were varied from 2 to 8 ounces, and the ſhot from 1 to near 3 pounds. And from the whole were clearly deduced theſe principal inferences, viz,

"1. Firſt, That gunpowder fires almoſt inſtantaneouſly.—2. That the velocities communicated to balls or ſhot, of the ſame weight, by different quantities of powder, are nearly in the ſubduplicate ratio of thoſe quantities: a ſmall variation, in defect, taking place when the quantities of powder became great.—3. And when ſhot of different weights are employed, with the ſame quantity of powder, the velocities communicated to them, are nearly in the reciprocal ſubduplicate ratio of their weights.—4. So that, univerſally, ſhot which are of different weights, and impelled by the firing of different quantities of powder, acquire velocities which are directly as the ſquare roots of the quantities of powder, and inverſely as the ſquare roots of the weights of the ſhot, nearly.—5. It would therefore be a great improvement in artillery, to make uſe of ſhot of a long form, or of heavier matter; for thus the momentum of a ſhot, when fired with the ſame weight of powder, would be increaſed in the ratio of the ſquare root of the weight of the ſhot.—6. It would alſo be an improvement to diminiſh the windage; for by ſo doing, one-third or more of the quantity of powder might be ſaved.—7. When the improvements mentioned in the laſt two articles are conſidered as both taking place, it is evident that about half the quantity of powder might be ſaved, which is a very conſiderable object. But important as this ſaving may be, it ſeems to be ſtill exceeded by that of the article of the guns; for thus a ſmall gun may be made to have the effect and execution of another of two or three times its ſize in the preſent mode, by diſcharging a ſhot of two or three times the weight of its natural ball or round ſhot. And thus a ſmall ſhip might diſcharge ſhot as heavy as thoſe of the greateſt now made uſe of.

"Finally, as the above experiments exhibit the regulations with regard to the weights of powder and balls, when fired from the ſame piece of ordnance, &c; ſo by making ſimilar experiments with a gun, varied in its length, by cutting off from it a certain part before each courſe of experiments, the effects and general rules for the different lengths of guns may be certainly determined by them. In ſhort, the principles on which theſe experiments were made, are ſo fruitful in conſequences, that, in conjunction with the effects reſulting from the reſiſtance of the medium, they ſeem to be ſufficient for anſwering all the enquiries of the ſpeculative philoſopher, as well as thoſe of the practical artilleriſt.

In the year 1786 was publiſhed the firſt volume of my Tracts, in which is detailed, at great length, another very extenſive courſe of experiments which were carried on at Woolwich in the years 1783, 1784, and 1785, by order of the Duke of Richmond, Maſter General of the Ordnance. The objects of this courſe were very numerous, but the principal of them were the following:

"1. The velocities with which balls are projected by equal charges of powder, from pieces of the ſame weight and calibre, but of different lengths.

"2. The velocities with different charges of powder, the weight and length of the gun being the ſame.

"3. The greateſt velocity due to the different lengths of guns, to be obtained by increaſing the charge as far as the reſiſtance of the piece is capable of ſuſtaining.

"4. The effect of varying the weight of the piece; every thing elſe being the ſame.

"5. The penetration of balls into blocks of wood.

"6. The ranges and times of flight of balls; to compare them with their initial velocities for determining the reſiſtance of the medium.

"7. The effect of wads;
of different degrees of ramming;
of different degrees of windage;
of different poſitions of the vent;
of chambers, and trunnions, and every other circumſtance neceſſary to be known for the improvement of artillery."

All theſe objects were obtained in a very perfect and accurate manner; excepting only the article of ranges, which were not quite ſo regular and uniform as might be wiſhed. The balls too were moſt of them of one pound weight; but the powder was increaſed from 1 ounce, up till the bore was quite full; and the pendulum was from 600 to 800lb. weight. The concluſions from the whole were as follow:

" 1. That the former law, between the charge and velocity of ball, is again confirmed, viz, that the velocity is directly as the ſquare root of the weight of powder, as far as to about the charge of 8 ounces: and ſo it would continue for all charges, were the guns of an indefinite length. But as the length of the charge is increaſed, and bears a more conſiderable proportion to the length of the bore, the velocity falls the more ſhort of that proportion.

" 2. That the velocity of the ball increaſes with the charge to a certain point, which is peculiar to each gun, where it is greateſt; and that by farther increaſing the charge, the velocity gradually diminiſhes, till the bore is quite full of powder. That this charge for the greateſt velocity is greater as the gun is longer, but not greater however in ſo high a proportion as the length of the gun is; ſo that the part of the bore filled with powder bears a leſs proportion to the whole in the long guns, than it does in the ſhort ones; the part of the whole which is filled being indeed nearly in the reciprocal ſubduplicate ratio of the length of the empty part. And the other circumſtances are as in this table.

TABLE of Charges producing the Greateſt Velocity.

Gun Num.	Length of the bore.	Length filled.	Part of the whole.	Wt. of the powder.
	inches.	inches.		oz.
1	28·2	8·2	$\frac{3}{10}$	12
2	38·1	9·5	$\frac{3}{12}$	14
3	57·4	10·7	$\frac{3}{16}$	16
4	79·9	12·1	$\frac{3}{20}$	18

" 3. It appears that the velocity continually increaſes as the gun is longer, though the increaſe in velocity is but very ſmall in reſpect of the increaſe in length, the velocities being in a ratio ſomewhat leſs than that of the ſquare roots of the length of the bore, but ſomewhat greater than that of the cube roots of the length, and is indeed nearly in the middle ratio between the two.

" 4. The range increaſes in a much leſs ratio than the velocity, and indeed is nearly as the ſquare root of the velocity, the gun and elevation being the ſame. And when this is compared with the property of the velocity and length of gun in the foregoing paragraph, we perceive that very little is gained in the range by a great increaſe in the length of the gun, the charge being the ſame. And indeed the range is nearly as the 5th root of the length of the bore; which is ſo ſmall an increaſe, as to amount only to about $\frac{1}{7}$th part more range for a double length of gun.

" 5. It alſo appears that the time of the ball's flight is nearly as the range; the gun and elevation being the ſame.

" 6. It appears that there is no ſenſible difference cauſed in the velocity or range, by varying the weight of the gun, nor by the uſe of wads, nor by different degrees of ramming, nor by firing the charge of powder in different parts of it.

" 7. But a great difference in the velocity ariſes from a ſmall degree of windage. Indeed with the uſual eſtabliſhed windage only, namely, about $\frac{1}{20}$th of the caliber, no leſs than between $\frac{1}{3}$ and $\frac{1}{4}$ of the powder eſcapes and is loſt. And as the balls are often ſmaller than that ſize, it frequently happens that half the powder is loſt by unneceſſary windage.

" 8. It appears that the reſiſting force of wood, to balls fired into it, is not conſtant. And that the depths penetrated by different velocities or charges, are nearly as the logarithms of the charges, inſtead of being as the charges themſelves, or, which is the ſame thing, as the ſquare of the velocity.

" 9. Theſe, and moſt other experiments, ſhew that balls are greatly deflected from the direction they are projected in; and that ſo much as 300 or 400 yards in a range of a mile, or almoſt $\frac{1}{4}$th of the range, which is nearly a deflection of an angle of 15 degrees.

" 10. Finally, theſe experiments furniſh us with the following concomitant data, to a tolerable degree of accuracy, namely, the dimenſions and elevation of the gun, the weight and dimenſions of the powder and ſhot, with the range and time of flight, and the firſt velocity of the ball. From which it is to be hoped that the meaſure of the reſiſtance of the air to projectiles, may be determined, and thereby lay the foundation for a true and practical ſyſtem of Gunnery, which may be as well uſeful in ſervice as in theory."

Since the publication of thoſe Tracts, we have proſecuted the experiments ſtill farther, from year to year, gradually extending our aim to more objects, and enlarging the guns and machinery, till we have arrived at experiments with the 6 pounder guns, and pendulums of 1800 pounds weight. One of the new objects of enquiry, was the reſiſtance the atmoſphere makes to military projectiles; to obtain which, the guns have been placed at many different diſtances from the pendulum, againſt which they are fired, to get the velocity loſt in paſſing through thoſe ſpaces of air; by which, and the uſe of the whirling machine, deſcribed near the end of the 1ſt vol. of Robins's Tracts, for the ſlower motions, I have inveſtigated the reſiſtance of the air to given balls moving with all degrees of velocity, from 0 up to 2000 feet per ſecond; as well as the reſiſtance for many degrees of velocity, to planes and figures of other ſhapes, and inclined to their path in all varieties of angles; from which I have deduced general laws and formulas for all ſuch motions.

Mr. Robins made alſo ſimilar experiments on the reſiſtance of the air; but being only with muſket bullets, on account of their ſmallneſs, and of their change of figure by the exploſion of the powder, I find they are very inaccurate, and conſiderably different from thoſe above mentioned, which were accurately made with pretty conſiderable cannon balls, of iron. For which reaſon we may omit here the rules and theory deduced from them by Mr. Robins, till others more correct ſhall have been eſtabliſhed. All theſe experiments indeed agree in evincing the very enormous reſiſtance the air makes to the ſwift motions of military projectiles, amounting in ſome caſes to 20 or 30 times the weight of the ball itſelf; on which account the common rules for projectiles, deduced from the parabolic theory, are of little or no uſe in real practice; for, from theſe experiments it is clearly proved, that the track deſcribed by the flight even of the heavieſt ſhot, is neither a parabola, nor yet approaching any thing near it, except when they are projected with very ſmall velocities; in ſo much that ſome balls, which in the air range only to the diſtance of one mile, would in vacuo, when projected with the ſame velocity, range above 10 or 20 times as far. For the common rules of the parabolic theory, ſee Projectiles. And for a ſmall ſpecimen of my experiments on reſiſtances, ſee the 2d vol. of the Edinburgh Philoſ. Tranſ.; as alſo my Conic Sections and Select Exerciſes, at the end, alſo the articles Force, and Resistance, in this Dictionary.

Mr. Benjamin Thompſon inſtituted a very conſiderable courſe of experiments of the ſame kind as thoſe of Mr. Robins, with muſket barrels, which was publiſhed in the Philoſ. Tranſ. vol. 71, for the year 1781. In theſe experiments, the concluſions of Mr. Robins are generally confirmed, and ſeveral other curious circumſtances in this buſineſs are remarked by Mr. Thompſon. This gentleman alſo purſues a hint thrown out by Mr. Robins relative to the determining the velocity of a ball from the recoil of the pendulous gun itſelf. Mr. Robins, in prop. 11, remarks that the effect of the exploded powder upon the recoil of the gun, is the ſame, whether the gun is charged with a ball, or without one; and that the chord, or velocity, of recoil with the powder alone, being ſubtracted from that of the recoil when charged with both powder and ball, leaves the velocity which is due to the ball alone. From whence Mr. Thompſon obſerves, that the inference is obvious, viz, that the momentum thus communicated to the gun by the ball alone, being equal to the momentum of the ball, this becomes known; and therefore being divided by the known weight of the ball, the quotient will be its velocity. Mr. Thompſon ſets a great value on this new rule, the velocities by means of which, he found to agree nearly with ſeveral of thoſe deduced from the motion of the pendulum; and in the other caſes in which they differed greatly from theſe, he very inconſiſtently ſuppoſes that theſe latter ones are erroneous. In the experiments however contained in my Tracts, a great multitude of thoſe caſes are compared together, and the inaccuracy of that new rule is fully proved.

Having in the 9th prop. compared together a number of computed and experimented velocities of balls, to verify his theory: in the 10th prop. Mr. Robins aſſigns the changes in the force of powder, which ariſe from the different ſtate of the atmoſphere, as to heat and moiſture, both which he finds have ſome effect on it, but eſpecially the latter. In prop. 11 he inveſtigates the velocity which the flame of gunpowder acquires by expanding itſelf, ſuppoſing it fired in a given piece of artillery, without either a bullet or any other body before it. This velocity he finds is upwards of 7000 feet per ſecond. But the celebrated Euler, in his commentary on this part of Mr. Robins's book, thinks it may be ſtill much greater. And in this prop. too it is that Mr. Robins declares his opinion, above alluded to, viz, that the effect of the powder upon the recoil of the gun is the ſame, in all caſes, whether fired with a ball, or without one.—In prop. 12 he aſcertains the manner in which the flame of powder impels a ball which is laid at a conſiderable diſtance from the charge; ſhewing here that the ſudden accumulation and denſity of the fluid againſt the ball, is the reaſon that the barrel is ſo often burſt in thoſe caſes.—In prop. 13 he enumerates the various kinds of powder, and deſcribes the propereſt methods of examining its goodneſs. He here ſhews that the beſt proportion of the ingredients, is when the ſaltpétre is ¾ of the whole compound maſs of the powder, and the ſulphur and charcoal the other ¼ between them, in equal quantities. In this prop. Mr. Robins takes occaſion to remark upon the uſe of eprouvettes, or methods of trying powder; condemning the practice of the Engliſh in uſing what is called the vertical eprouvette; as well as that of the French, in uſing a ſmall mortar, with a very large ball, and a ſmall charge of powder: and inſtead of theſe, he ſtrongly recommends the uſe of his balliſtic pendulum, for its great accuracy: But for ſtill more diſpatch, he ſays he ſhould uſe another method, which however he reſerves to himſelf, without giving any particular deſcription of it. From what has been done by Mr. Robins upon this head, ſeveral perſons have introduced his method of ſuſpending the gun as a pendulum, and noting the quantity of its oſcillating recoil when fired with a certain quantity of powder; and of this kind I have contrived a machine, which poſſeſſes ſeveral advantages over all others, being extremely ſimple, accurate, and expeditious; ſo much ſo indeed, that the weighing out of the powder is the chief part of the trouble. See Gunpowder, and Eprouvette.

The other or 2d chapter of Mr. Robins's work, in 8 propoſitions, treats "of the reſiſtance of the air, and of the track deſcribed by the flight of ſhot and ſhells." And of theſe, prop. 1 deſcribes the general principles of the reſiſtance of fluids to ſolid bodies moving in them. Here Mr. Robins diſcriminates between continued and compreſſed fluids, which immediately ruſh into the ſpace quitted by a body moving in them, and whoſe parts yield to the impulſe of the body without condenſing and accumulating before it; and ſuch fluids as are imperfectly compreſſed, ruſhing into a void ſpace with a limited velocity, as in the caſe of our atmoſphere, which condenſes more and more before the ball as this moves quicker, and alſo preſſes the leſs behind it, by following it always with only a given velocity: hence it happens that the former fluid will reſiſt moving bodies in proportion to the ſquare of the velocity, while the latter reſiſts in a higher proportion.—Propoſition 2 is "to determine the reſiſtance of the air to projectiles by experiments." One of the methods for this purpoſe, is by the balliſtic pendulum, placing the gun at different diſtances from it, by which he finds the velocity loſt in paſſing through certain ſpaces of air, and conſequently the force of reſiſtance to ſuch velocities as the body moves with in the ſeveral parts of its path. And another way was by firing balls, with a known given velocity, over a large piece of water, in which the fall and plunge of the ball could be ſeen, and conſequently the ſpace it paſſed over in a given time. By theſe means Mr. Robins determined the reſiſtances of the air to ſeveral different velocities, all which ſhewed that there was a gradual increaſe of the reſiſtance, over the law of the ſquare of the velocity, as the body moved quicker.—In the remaining propoſitions of this chapter, he proceeds a little farther in this ſubject of the reſiſtance of the air; in which he lays down a rule for the proportion of the reſiſtance between two aſſigned velocities; and he ſhews that when a 24 pound ball, fired with its full charge of powder, firſt iſſues from the piece, the reſiſtance it meets with from the air is more than 20 times its weight. He farther ſhews that "the track deſcribed by the flight of ſhot or ſhells is neither a parabola, nor nearly a parabola, unleſs they are projected with ſmall velocities;" and that "bullets in their flight are not only depreſſed beneath their original direction by the action of gravity, but are alſo frequently driven to the right or left of that direction by the action of ſome other force: and in the 8th or laſt propoſition, he pretends to ſhew that the depths of penetration of balls into firm ſubſtances, are as the ſquares of the velocities. But this is a miſtake; for neither does it appear that his trials were ſufficiently numerous or various, nor were his ſmall leaden balls fit for this purpoſe; and I have found, from a number of trials with iron cannon balls, that the penetrations are in a much lower proportion, and that the reſiſting force of wood is not uniform. See my TRACTS.

In the following ſmall tracts, added to the principles, in this volume, Mr. Robins proſecutes the ſubject of the reſiſtance of the air much farther, and lays down rules for computing ranges made in the air. But theſe muſt be far from accurate, as they are founded on the two following principles, which I know, from numerous experiments, are erroneous: viz, 1ſt, "That till the velocity of the projectile ſurpaſſes that of 1100 feet in a ſecond, the reſiſtance may be eſteemed to be in the duplicate proportion of the velocity. 2d, That if the velocity be greater than that of 11 or 1200 feet in a ſecond, then the abſolute quantity of that reſiſtance in theſe greater velocities will be near 3 times as great, as it ſhould be by a compariſon with the ſmaller velocities." For, inſtead of leaping at once from the law of the ſquare of the velocities, and ever after being about 3 times as much, my experiments prove that the increaſe of the reſiſtance above the law of the ſquare of the velocity, takes place at firſt in the ſmalleſt motions, and increaſes gradually more and more, to a certain point, but never riſes ſo high as to be 3 times that quantity, after which it decreaſes again. To render this evident, I have inſerted the following table of the actual quantities of reſiſtances, which are deduced from accurate experiments, and which ſhew alſo the nature of the law of the variations, by means of the columns of differences annexed; reſerving the detail of the experiments themſelves to another occaſion. Theſe reſiſtances are, upon a ball of 1·965 inc. diameter, in avoirdupois ounces, and are for all velocities, from 0, up to that of 2000 feet per ſecond of time.

The quantity of the reſiſtance of the air to a ball of 1·965 inc. diameter.

Veloc. in feet	Reſiſt. in ounces	1ſt Differences	2d Differences
0	0·000		
5	0·006		
10	0·025		
15	0·054		
20	0·100		
25	0·155		
30	0·23		
40	0·42		
50	0·67		
100	2¾		
200	11	8¼	5¾
300	25	14	6
400	45	20	7
500	72	27	8
600	107	35	9
700	151	44	10
800	205	54	12
900	271	66	13
1000	350	79	13
1100	442	92	12
1200	546	104	11
1300	661	115	9
1400	785	124	7
1500	916	131	4
1600	1051	135	0
1700	1186	135	2
1800	1319	133	5
1900	1447	128	6
2000	1569	122	

The additional tracts of Mr. Robins, in the latter part of this volume, which contain many uſeful and important matters, are numbered and titled as follows, viz, Number 1, "Of the reſiſtance of the air. Number 2, Of the reſiſtance of the air; together with the method of computing the motions of bodies projected in that medium. Number 3, An account of the experiments, relating to the reſiſtance of the air, exhibited at different times before the Royal Society, in the year 1746. Number 4, Of the force of fired gunpowder, together with the computation of the velocities thereby communicated to military projectiles. Number 5, A compariſon of the experimental ranges of cannon and mortars with the theory contained in the preceding papers.—Practical Maxims relating to the effects and management of artillery, and the flight of ſhells and ſhot.—A propoſal for increaſing the ſtrength of the Britiſh navy, by changing all the guns, from the 18 pounders downwards, into others of equal weight, but of a greater bore." With ſeveral letters, and other papers, "On pointing, or the directing of cannon to ſtrike diſtant objects; Of the nature and advantage of rifled barrel pieces," &c.

I have dwelt thus long on Mr. Robins's New Principles of Gunnery, becauſe it is the firſt work that can be conſidered as attempting to eſtabliſh a practical ſyſtem of gunnery, and projectiles, on good experiments, on the force of gunpowder, on the reſiſtance of the air, and on the effects of different pieces of artillery. Thoſe experiments are however not ſufficiently perfect, both on account of the ſmallneſs of the bullets, and for want of good ranges, to form a proper theory upon. I have ſupplied ſome of the neceſſary deſiderata for this purpoſe, viz, the reſiſtance of the air to cannon balls moving with all degrees of velocity, and the velocities communicated by given charges of powder to different balls, and from different pieces of artillery. But there are ſtill wanting good experiments with different pieces of ordnance, giving the ranges and times of flight, with all varieties of charges, and at all different angles of elevation. A few however of thoſe I have obtained, as in the following ſmall table, which are derived from experiments made with a medium one-pounder gun, the iron ball being nearly 2 inches in diameter.

Powder	Elev. of gun	Veloc. of ball	Range	Time of flight
oz	°	feet	feet	″
2	15	860	4100	9
4	15	1230	5100	12
8	15	1640	6000	14½
12	15	1680	6700	15½
2	45	860	5100	21

The celebrated Mr. Euler added many excellent diſſertations on the ſubject of Gunnery, in his tranſlation of Robins's Gunnery into the German language ; which were again farther improved in Brown's tranſlation of the ſame into Engliſh in the year 1777. See alſo Antoni's *Examen de la Poudre ;* the experiments of MM. D'Arcy and Le Roy, in the Memoirs of the Academy in 1751 ; and D'Arcy's *Eſſai d'une theorie d'artillerie* in 1760 : my Tracts ; and paper on the force of fired gunpowder in the Philoſ. Tranſ. for 1778 : and Thompſon's paper on the ſame ſubject in 1781. Of the common or parabolic theory of Gunnery, Mr. Simpſon gave a very neat and conciſe treatiſe in his Select Exerciſes. And other authors on this part, are Starrat, Gray, Williams, Glenie, &c.

GUNPOWDER, a compoſition of nitre, ſulphur, and charcoal, mixed together, and uſually granulated. This eaſily takes fire ; and when fired, it rarefies or expands with great vehemence, by means of its elaſtic force.—It is to this powder that we owe all the effect and action of guns, and ordnance of all ſorts. So that fortification, with the modern military art, &c, in a great meaſure depends upon it. The above definition however is not general, for inſtead of the nitre, it has lately been diſcovered that the marine acid anſwers much better.

The invention of Gunpowder is aſcribed, by Polydore Virgil, to a chemiſt ; who having accidentally put ſome of this compoſition in a mortar, and covered it with a ſtone, it happened to take fire, and blew up the ſtone. Thevet ſays that the perſon here ſpoken of was a monk of Fribourg, named Conſtantine Anelzen ; but Belleforet and other authors, with more probability, hold it to be Bartholdus Schwartz, or the black, who diſcovered it, as ſome ſay, about the year 1320 ; and the firſt uſe of it is aſcribed to the Venetians, in the year 1380, during the war with the Genoeſe. But there are earlier accounts of its uſe, after the accident of Schwartz, as well as before it. For Peter Mexia, in his Various Readings, mentions that the Moors being beſieged in 1343, by Alphonſus the 11th, king of Caſtile, diſcharged a kind of iron mortars upon them, which made a noiſe like thunder ; and this is ſeconded by what is related by Don Pedro, biſhop of Leon, in his chronicle of king Alphonſus, who reduced Toledo, viz, that in a ſea-combat between the king of Tunis and the Mooriſh king of Seville, about that time, thoſe of Tunis had certain iron tubs or barrels, with which they threw thunderbolts of fire.

Du-Cange adds, that there is mention made of gunpowder in the regiſters of the chambers of accounts in France as early as the year 1338.

But it appears that Roger Bacon knew of Gunpowder near 100 years before Schwartz was born. He tells us, in his Treatiſe De Secretis Operibus Artis & Naturæ, & de Nullitate Magiæ, cap. 6, (which is ſuppoſed by ſome to have been publiſhed at Oxford in 1216, and which was undoubtedly written before his Opus Majus, in 1267), " that from ſaltpetre, and other ingredients, we are able to make a fire that ſhall burn at what diſtance we pleaſe." And Dr. Plott, in his Hiſtory of Oxfordſhire, pa. 236, aſſures us that theſe " other ingredients were explained in a MS. copy of the ſame treatiſe, in the hands of Dr. G. Langbain, and ſeen by Dr. Wallis, to be ſulphur and wood coal." Farther, in the life of Friar Bacon in Biographia Britannica, vol. 1, we are told that Bacon himſelf has divulged the ſecret of this compoſition in a cypher, by tranſpoſing the letters of the two words in chap. xi. of the ſaid treatiſe ; where it is thus expreſſed : *ſed tamen ſalis petræ* LURA MOPE CAN UBRE (i. e. *carbonum pulvere) et ſulphuris ; et ſic facies tonitrum & corruſcationem, ſi ſcias artificium :* and from hence the biographer apprehends the words *carbonum pulvere* were transferred to the 6th chapter of Langbain's MS. In this ſame chapter Bacon expreſsly ſays that ſounds like thunder, and corruſcations, may be formed in the air, much more horrible than thoſe that happen naturally. And farther adds, that there are many ways of doing this, by which a city or an army might be deſtroyed : and he ſuppoſes that by an artifice of this kind Gideon defeated the Midianites with only 300 men : Judges, chap. 7. There is alſo another paſſage to the ſame purpoſe, in the treatiſe De Scientia Experimentali. See Dr. Jebb's edition of the Opus Majus, p. 474.

Mr. Robins, in the preface to his Gunnery, apprehends that Bacon deſcribes Gunpowder not as a new compoſition firſt propoſed by himſelf, but as the application of an old one to military purpoſes, and that it was known long before his time.

But Mr. Dutens carries the antiquity of Gunpowder ſtill much higher, and refers to the writings of the ancients themſelves for the proof of it. "Virgil, ſays he, and his Commentator Servius (Æneid, lib. 6, v. 585), Hyginus (Fabul. 61 and 650), Euſtathius (ad Odyſſ, λ 234, pa. 1682, lib. 1), La Cerda (in Virgil. loc. cit.), Valerius Flaccus (lib. i. 662), and many other authors (as Raphael Volatarran. in Commentar. Cornelius Agrippa poſter. Oper. de Verbo Dei, c. 100, p. 237.—Gruteri Fax Artium Liberal tom. 2, p. 1236), ſpeak in ſuch a manner of Salmoneus's attempts to imitate thunder, as ſuggeſt to us that this prince uſed for that purpoſe a compoſition of the nature of gunpowder. Euſtathius in particular ſpeaks of him on this occaſion, as being ſo very expert in mechanics, that he formed machines, which imitated the noiſe of thunder; and the writers of fable, whoſe ſurpriſe in this reſpect may be compared to that of the Mexicans when they firſt beheld the fire-arms of the Spaniards, give out that Jupiter, incenſed at the audacity of this prince, ſlew him with lightning, as he was employing himſelf in launching his thunder. But it is much more natural to ſuppoſe that this unfortunate prince, the inventor of Gunpowder, gave riſe to theſe fables, by having accidentally fallen a victim to his own experiments. Dion (Hiſt. Rom. in Caligula, p. 662) and Joannes Antiochenus (in Chronico, &c. a Valeſio edita, Paris 1634, p. 804), report the very ſame thing of Caligula, aſſuring us that this emperor imitated thunder and lightning by means of certain machines, which at the ſame time emitted ſtones. Themiſtius informs us that the Brachmans encountered one another with thunder and lightning, which they had the art of launching from on high at a conſiderable diſtance; (Themiſt. Oratio 27, p. 337). And in another place he relates, that Hercules and Bacchus, attempting to aſſail them in a fort where they were entrenched, were ſo roughly received by reiterated ſtrokes of thunder and lightning, launched upon them from on high by the beſieged, that they were obliged to retire, leaving behind them an everlaſting monument of the raſhneſs of their enterpriſe. Agathias the hiſtorian reports of Anthemius Tralienſis, that having fallen out with his neighbour Zeno the rhetorician, he ſet fire to his houſe with thunder and lightning. It appears from all theſe paſſages, that the effects aſcribed to theſe engines of war, eſpecially thoſe of Caligula, Anthemius, and the Indians, could be only brought about by Gunpowder. And what is ſtill more, we find in Julius Africanus a receipt for an ingenious compoſition to be thrown upon an enemy, which very nearly reſembles that powder. But what places this beyond all doubt, is a clear and poſitive paſſage of an author called Marcus Græcus, whoſe work in manuſcript is in the royal library at Paris, intitled Liber Ignium. Dr. Mead had the ſame alſo in manuſcript, and a copy of that is now in my hands. (See above). The author deſcribes ſeveral ways of encountering an enemy, by launching fire upon him; and among others gives the following. Mix together one pound of live ſulphur, two of charcoal of willow, and 6 of ſaltpetre; reducing them to a very fine powder in a marble mortar. He adds, that a certain quantity of this is to be put into a long, narrow, and well compacted cover, and ſo diſcharged into the air. Here we have the deſcription of a rocket. The cover with which thunder is imitated, he repreſents as ſhort, thick, but half-filled, and ſtrongly bound with packthread; which is exactly the form of a cracker. He then treats of different methods of preparing the match, and how one ſquib may ſet fire to another in the air, by having it incloſed within it. In ſhort, he ſpeaks as clearly of the compoſition and effects of Gunpowder, as any perſon in our times could do. I own I have not yet been able preciſely to determine when this author lived, but probably it was before the time of the Arabian phyſician Meſue, who ſpeaks of him, and who flouriſhed in the beginning of the 9th century. Nay, there is reaſon to believe that he is the ſame of whom Galen ſpeaks; in which caſe he will be of antiquity ſufficient to ſupport what I advance." It appears too, from many authors, and many circumſtances, that this compoſition has been known to the Chineſe and Indians for thouſands of years. See what is ſaid on this head under the article GUN.

To this hiſtory of Gunpowder it may be added, that it has lately been diſcovered that ſaltpetre or nitre is not eſſential to this compoſition, but that its place may be ſupplied by other ſubſtances; for new Gunpowder, of double the ſtrength of the old, has lately been made in France, by the chemiſts in that country, without any nitre at all; and in the year 1790 I tried ſome of this new powder, that was made at Woolwich, with my eprouvette, when I found it about double the ſtrength of the ordinary ſort. This is effected by ſubſtituting, inſtead of the nitre, the like quantity of the marine acid.

But perhaps this new compoſition may not come into common and general uſe; both becauſe of the great expence in procuring or making the acid, and of the trouble and danger of preventing it from taking fire by the heat of making it; for it is found to catch fire and explode from a very ſmall degree of heat, and without the aid of a ſpark.

As to the Preparation of GUNPOWDER; there are divers compoſitions of it, with reſpect to the proportions of the three ingredients, to be met with in pyrotechnical writings; but the proceſs of making it up is much the ſame in all.

For ſome time after the invention of artillery, Gunpowder was of a much weaker compoſition than that now in uſe, or that deſcribed by Marcus Græcus; which was chiefly owing to the weakneſs of their firſt pieces. See GUN and CANNON. Of 23 different compoſitions, uſed at different times, and mentioned by Tartaglia in his Queſ. and Inv. lib. 3, queſ. 5, the firſt, which was the oldeſt, contained equal parts of the three ingredients. But when guns of modern ſtructure were introduced, Gunpowder of the ſame compoſition as the preſent came alſo into uſe. In the time of Tartaglia the cannon powder was made of 4 parts of nitre, one of ſulphur, and one of charcoal; and the muſket powder of 48 parts of nitre, 7 parts of ſulphur, and 8 parts of charcoal; or of 18 parts of nitre, 2 parts of ſulphur, and 3 parts of charcoal. But the modern compoſition is 6 parts of nitre, to one of each of the other two ingredients. Though Mr. Napier ſays, he finds the ſtrength commonly to be greateſt when the proportions are, nitre 3lb, charcoal about 9oz, and ſulphur about 3oz. See his paper on Gunpowder in the Tranſactions of the Royal Iriſh Academy, vol. 2. The cannon powder was in meal, and the muſket powder grained. And it is certain that the graining of powder, which is a very conſiderable advantage, is a modern improvement. See the preface to Robins's Math. Tracts, pa. 32.

To make Gunpowder duly, regard is to be had to the purity or goodneſs of the ingredients, as well as the proportions of them; for the ſtrength of the powder depends much on that circumſtance, and alſo on the due working or mixing of them together.

To purify the nitre, by taking away the fixt or common ſalt, and earthy part. Diſſolve it in a quantity of hot water over the fire; then filtrate it through a flannel bag, into an open veſſel, and ſet it aſide to cool, and to cryſtallize. Theſe cryſtals may in like manner be diſſolved and cryſtallized again; and ſo on, till they become quite pure and white. Then put the cryſtals into a dry kettle over a moderate fire, which gradually increaſe till it begins to ſmoke, evaporate, loſe its humidity, and grow very white: it muſt be kept continually ſtirring with a ladle, leſt it ſhould return to its former figure, by which its greaſineſs would be taken away: after that, ſo much water is to be poured into the kettle as will cover the nitre; and when it is diſſolved, and reduced to the conſiſtency of a thick liquor, it muſt be continually ſtirred with a ladle till all the moiſture is again evaporated, and it be reduced to a dry and white meal.

The like regard is to be had to the ſulphur; chooſing that which is in large lumps, clear and perfectly yellow; not very hard, nor compact, but porous; nor yet too much ſhining; and if, when ſet on fire, it freely burns all away, it is a ſign of its goodneſs: ſo likewiſe, if it be preſſed between two iron plates that are hot enough to make it run, and in the running appear yellow, and that which remains of a reddiſh colour, it is then fit for the purpoſe. But in caſe it be foul, it may be purified in this manner: melt the ſulphur in a large iron ladle, or pot, over a very gentle coal fire, well kindled, but not flaming; then ſcum off all that riſes on the top, and ſwims upon the ſulphur; take it preſently after from the fire, and ſtrain it through a double linen cloth, letting it paſs leiſurely; ſo will it be pure, the groſs matter remaining behind in the cloth.

For the charcoal, the third ingredient, ſuch ſhould be choſen as is large, clear, and free from knots, well burnt, and cleaving. The charcoal of light woods is moſtly preferred, as of willow, and that of the branches or twigs of a moderate thickneſs, as of an inch or two in diameter. Dogwood is now much eſteemed for this purpoſe. And a method of charring the wood in a large iron cylinder has lately been recommended, and indeed proved, as yielding better charcoal than formerly.

The charcoal not only concurs with the ſulphur in ſupplying the inflammable matter, which cauſes the detonation of the nitre, but alſo greatly adds to the exploſive power of it by the quantity of elaſtic vapour expelled during its combuſtion.

Theſe three ingredients, in their pureſt ſtate, being procured, long experience has ſhewn that they are then to be mixed together in the proportion before mentioned, to have the beſt effect, viz, three-quarters of the compoſition to be nitre, and the other quarter made up of equal parts of the other two ingredients; or, which is the ſame thing, 6 parts nitre, 1 part ſulphur, and 1 part charcoal.

But it is not the due proportion of the materials only, which is neceſſary to the making of good powder; another circumſtance, not leſs eſſential, is the mixing them well together: if this be not effectually done, ſome parts of the compoſition will have too much nitre in them, and others too little; and in either caſe there will be a defect of ſtrength in the powder. Robins, pa. 119.

After the materials have been reduced to fine duſt, they are mixed together, and moiſtened with water, or vinegar, or urine, or ſpirit of wine, &c, and then beaten together with wooden peſtles for 24 hours, either by hand, or by mills, and afterwards preſſed into a hard, firm, and ſolid cake. When dry, it is grained or corned; which is done by breaking the cake of powder into ſmall pieces, and ſo running it through a ſieve; by which means the grains may have any ſize given them, according to the nature of the ſieve employed, either finer or coarſer; and thus alſo the duſt is ſeparated from the grains, and again mixed with other manufacturing powder, or worked up into cakes again.

Powder is ſmoothed, or glazed, as it is called, for ſmall arms, by the following operation: a hollow cylinder or caſk is mounted on an axis, turned by a wheel; this caſk is half filled with powder, and turned for 6 hours; and thus by the mutual friction of the grains of powder it is ſmoothed, or glazed. The fine mealy part, thus ſeparated or worn off from the reſt, is again granulated.

The Nature, Effects, &c, of Powder.—When the powder is prepared as above, if the leaſt ſpark be ſtruck upon it from a ſteel and flint, the whole will immediately inflame, and burſt out with extreme violence.—The effect is not hard to account for: the charcoal part of the grain upon which the ſpark falls, catching fire like tinder, the ſulphur and nitre are readily melted, and the former alſo breaks into flame; the contiguous grains at the ſame time undergoing the ſame fate.

Sir Iſaac Newton reaſons thus upon the point: The charcoal and ſulphur in Gunpowder eaſily take fire, and kindle the nitre; and the ſpirit of the nitre, being thereby rarefied into vapour, ruſhes out with an exploſion much after the manner that the vapour of water ruſhes out of an eolipile; the ſulphur alſo, being volatile, is converted into vapour, and augments the exploſion: add, that the acid vapour of the ſulphur, namely that which diſtils under a bell into oil of ſulphur, entering violently into the fixt body of the nitre, lets looſe the ſpirit of the nitre, and excites a greater fermentation, by which the heat is farther augmented, and the fixt body of the nitre is alſo rarefied into fume; and the exploſion is thereby made more vehement and quick.

For if ſalt of tartar be mixed with Gunpowder, and that mixture be warmed till it takes fire, the exploſion will be far more violent and quick than that of Gunpowder alone; which cannot proceed from any other cauſe, than the action of the vapour of the Gunpowder upon the ſalt of tartar, by which that ſalt is rarefied.

The exploſion of Gunpowder therefore ariſes from the violent action, by which all the mixture being quickly and vehemently heated, is rarefied and converted into fume and vapour; which vapour, by the violence of that action becoming ſo hot as to ſhine, appears in the form of a flame.

M. De la Hire, in the Hiſtory of the French Academy for 1702, aſcribes all the force and effect of Gunpowder to the ſpring or elaſticity of the air incloſed in the ſeveral grains of it, and in the intervals or ſpaces between the grains: the powder being kindled, ſets the ſprings of ſo many little parcels of air a-playing, and dilates them all at once, whence the effect; the powder itſelf only ſerving to light a fire which may put the air in action; after which the whole is done by the air alone.

But it appears from the experiments and obſervations of Mr. Robins, that if this air be in its natural ſtate at the time when the powder is fired, the greateſt addition its elaſticity could acquire from the flame of the exploſion, would not amount to five times its uſual quantity, and therefore could not ſuffice for the 200th part of the effort which is exerted by fired powder.

To underſtand the force of Gunpowder, it muſt be conſidered that, whether it be fired in a vacuum or in air, it produces by its exploſion a permanent elaſtic fluid. See Philoſ. Tranſ. number 295; alſo Haukſbee's Phyſ. Mechan. Exp. p. 81. It alſo appears from experiment, that the elaſticity or preſſure of the fluid produced by the firing of Gunpowder, is, cæteris paribus, directly as its denſity.

To determine the elaſticity and quantity of this elaſtic fluid, produced from the exploſion of a given quantity of Gunpowder, Mr. Robins premiſes, that the elaſticity of this fluid increaſes by heat, and diminiſhes by cold, in the ſame manner as that of the air; and that the denſity of this fluid, and conſequently its weight, is the ſame with the weight of an equal bulk of air, having the ſame elaſticity and the ſame temperature. From theſe principles, and from the experiments by which they are eſtabliſhed (for a detail of which we muſt refer to the book itſelf, ſo often cited in theſe articles), he concludes that the fluid produced by the firing of Gunpowder is nearly $\frac{3}{10}$ of the weight of the generating powder itſelf; and that the volume or bulk of this air or fluid, when expanded to the rarity of common atmoſpheric air, is about 244 times the bulk of the ſaid generating powder.—Count Saluce, in his Miſcel. Phil. Mathem. Soc. Priv. Taurin. p. 125, makes the proportion as 222 to 1; which he ſays agrees with the computation of Meſſ. Haukſbee, Amontons, and Belidor.

Hence it appears, that any quantity of powder fired in any confined ſpace, which it adequately fills, exerts at the inſtant of its exploſion againſt the ſides of the veſſel containing it, and the bodies it impels before it, a force at leaſt 244 times greater than the elaſticity of common air, or, which is the ſame thing, than the preſſure of the atmoſphere; and this without conſidering the great addition ariſing from the violent degree of heat with which it is endued at that time; the quantity of which augmentation is the next head of Mr. Robins's enquiry. He determines that the elaſticity of the air is augmented in a proportion ſomewhat greater than that of 4 to 1, when heated to the extremeſt heat of red hot iron; and ſuppoſing that the flame of fired Gunpowder is not of a leſs degree of heat, increaſing the former number a little more than 4 times, makes nearly 1000; which ſhews that the elaſticity of the flame, at the moment of exploſion, is about 1000 times ſtronger than the elaſticity of common air, or than the preſſure of the atmoſphere. But, from the height of the barometer, it is known that the preſſure of the atmoſphere upon every ſquare inch, is on a medium $14\frac{3}{4}$ lb; and therefore 1000 times this, or 14750lb, is the force or preſſure of the flame of Gunpowder, at the moment of exploſion, upon a ſquare inch, which is very nearly equivalent to 6 tons and a half.

This great force however diminiſhes as the fluid dilates itſelf, and in that proportion, viz, in proportion to the ſpace it occupies, it being only half the ſtrength when it occupies a double ſpace, one third the ſtrength when triple the ſpace, and ſo on.

Mr. Robins farther ſuppoſes the degree of heat above mentioned to be a kind of medium heat; but that in the caſe of large quantities of powder the heat will be higher, and in very ſmall quantities lower; and that therefore in the former caſe the force will be ſomewhat more, and in the latter ſomewhat leſs, than 1000 times the force of the atmoſphere.

He farther found that the ſtrength of powder is the ſame in all variations in the denſity of the atmoſphere. But that the moiſture of the air has a great effect upon it; for the ſame quantity which in a dry ſeaſon would diſcharge a bullet with a velocity of 1700 feet in one ſecond, will not in damp weather give it a velocity of more than 12 or 1300 feet in a ſecond, or even leſs, if the powder be bad, and negligently kept. Robins's Tracts, vol. 1, p. 101, &c. Farther, as there is a certain quantity of water which, when mixed with powder, will prevent its firing at all, it cannot be doubted but every degree of moiſture muſt abate the violence of the exploſion; and hence the effects of damp powder are not difficult to account for.

It is to be obſerved, that the moiſture imbibed by powder does not render it leſs active when dried again. Indeed, if powder be expoſed to very great damps without any caution, or when common ſalt abounds in it, as often happens through negligence in refining the nitre, in ſuch caſes the moiſture it imbibes may perhaps be ſufficient to diſſolve ſome part of the nitre: which is a permanent damage that no drying can retrieve. But when tolerable care is taken in preſerving powder, and the nitre it is compoſed of has been well purged from common ſalt, it will retain its force for a long time; and it is ſaid that powder has been known to have been preſerved for 50 years without any apparent damage from its age.

The velocity of expanſion of the flame of Gunpowder, when fired in a piece of artillery, without either bullet or other body before it, is prodigiouſly great, viz, 7000 feet per ſecond, or upwards, as appears from the experiments of Mr. Robins. But Mr. Bernoulli and Mr. Euler ſuſpect it is ſtill much greater. And I ſuſpect it may not be leſs, at the moment of exploſion, than 4 times as much.

It is this prodigious celerity of expanſion of the flame of fired Gunpowder, which is its peculiar excellence, and the circumſtance in which it ſo eminently ſurpaſſes all other inventions, either ancient or modern: for as to the momentum of theſe projectiles only, many of the warlike machines of the ancients produced this in a degree far ſurpaſſing that of our heavieſt cannon ſhot or ſhells; but the great celerity given to theſe bodies cannot be in the leaſt approached by any other means but the flame of powder.

To prove Gunpowder. There are ſeveral ways of doing this. 1, By ſight: thus, if it be too black, it is a ſign that it is moiſt, or elſe that it has too much charcoal in it; ſo alſo, if rubbed upon white paper, it blackens it more than good powder does: but if it be of a kind of azure colour, ſomewhat inclining to red, it is a ſign of good powder. 2, By touching: for if in cruſhing it with the fingers ends, the grains break eaſily, and turn into duſt, without feeling hard, it has too much coal in it; or if, in preſſing it under the fingers upon a ſmooth hard board, ſome grains feel harder than the reſt, it is a ſign the ſulphur is not well mixed with the nitre. Alſo by thruſting the hand into the parcel of powder, and graſping it, as if to take out a handful, you will feel if it is dry and equal grained, by its evading the graſp, and running moſtly out of the hand. 3, By burning; and here the method moſt commonly followed for this purpoſe with us, ſays Mr. Robins, is to fire a ſmall heap of it on a clean board, and to attend nicely to the flame and ſmoke it produces, and to the marks it leaves behind on the board: but beſides this uncertain method, there are other contrivances made uſe of, ſuch as powder-triers acting by a ſpring, commonly ſold at the ſhops, and others again that move a great weight, throwing it upwards, which is a very bad ſort of eprouvette. But theſe machines, ſays Mr. Robins, though more perfect than the common powder-triers, are yet liable to great irregularities; for as they are all moved by the inſtantaneous ſtroke of the flame, and not by its continued preſſure, they do not determine the force of the fired powder with ſufficient certainty and uniformity. Another method is to judge from the range given to a large ſolid ball, thrown from a very ſhort mortar, charged with a ſmall quantity of powder; which is alſo an uncertain way, both on account of the great diſproportion between the weight of the ball and powder, and the unequal reſiſtance of the air; not to mention that it is too tedious to prove large quantities of powder in this way; for, "if each barrel of powder was to be proved in this manner, the trouble of charging the mortar, and bringing back the ball each time, would be intolerable, and the delay ſo great, that no buſineſs of this kind could ever be finiſhed; and if a number of barrels are received on the merit of a few, it is great odds, but ſome bad ones would be amongſt them, which may prove a great diſappointment in time of ſervice." Theſe exceptions do "noways hold, continues Mr. Robins, againſt the method by which I have tried the comparative ſtrength of different kinds of powder, which has been by the actual velocity given to a bullet, by ſuch a quantity of powder as is uſually eſteemed a proper charge for the piece: and as this velocity, however great, is eaſily diſcovered by the motion which the pendulum acquires from the ſtroke of the bullet, it might ſeem a good amendment to the method uſed by the French (viz, that of the ſmall mortar above mentioned) to introduce this trial by the pendulum inſtead of it. But though I am ſatisfied, that this would be much more accurate, leſs laborious, and readier than the other, yet, as there is ſome little attention and caution required in this practice, which might render it of leſs diſpatch than might be convenient, when a great number of barrels were to be ſeparately tried, I ſhould myſelf chooſe to practiſe another method not leſs certain, but prodigiouſly more expeditious; ſo that I could engage, that the weighing out of a ſmall parcel of powder from each barrel ſhould be the greateſt part of the labour; and, doubtleſs, three or four hands could, by this means, examine 500 barrels in a morning: beſides, the machines for this purpoſe, as they might be made of caſt iron, would be ſo very cheap, that they might be multiplied at pleaſure." Robins, page 123. It is not certainly known what might be the particular conſtruction of the eprouvette here hinted at, but it was probably a piece of ordnance ſuſpended like a pendulum, as he had made ſeveral experiments with a barrel in that manner. Be this however as it may, ſeveral perſons, from thoſe ideas and experiments of Mr. Robins, have made eprouvettes on this principle, which ſeems to be the beſt of any; and on this idea alſo I have lately made a machine for this purpoſe, which has ſeveral peculiar contrivances, and advantages over all others, both in the nature of its motion, and the diviſions on its arc, &c. It is a ſmall cannon, the bore of which is about one inch in diameter, and is uſually charged with 2 ounces of powder, and with powder only, as a ball is not neceſſary, and the ſtrength of the powder is accurately ſhewn by the arc of the gun's recoil. The whole machine is ſo ſimple, eaſy, and expeditious, that, as Mr. Robins obſerved above, the weighing of the powder is the chief part of the trouble; and ſo accurate and uniform, that the ſucceſſive repetitions or firings with the ſame quantity of the ſame ſort of powder, hardly ever yield a difference in the recoil of the 100th part of itſelf.

To recover damaged Powder. The method of the powder merchants is this; they put part of the powder on a ſail-cloth, to which they add an equal weight of what is really good; then with a ſhovel they mingle it well together, dry it in the ſun, and barrel it up, keeping it in a dry and proper place.

Others again, if it be very bad, reſtore it by moiſtening it with vinegar, water, urine, or brandy; then they beat it fine, ſift it, and to every pound of powder add an ounce, or an ounce and a half, or two ounces (according as it is decayed), of melted nitre; and afterwards theſe ingredients are to be moiſtened and well mixed, ſo that nothing may be diſcerned in the compoſition; which may be known by cutting the maſs, and then they granulate it as uſeful.

In caſe the powder be quite ſpoiled, the only way is to extract the ſaltpetre with water, in the uſual way, by boiling, filtrating, evaporating, and cryſtallizing; and then, with freſh ſulphur and charcoal, to make it up afreſh.

On the ſubject of Gunpowder, ſee alſo Euler on Robins's Gunnery, Antoni Examen de la Poudre, Baumé's Chemiſtry, and Thompſon's Experiments in the Philoſ. Tranſ. for 1781.

GUNTER (Edmund), an excellent Engliſh mathematician,

thematician, was born in Hertfordſhire in 1581. He was educated at Weſtminſter ſchool under Dr. Buſby, and from thence was elected to Chriſt-church College, Oxford, in 1599, where he took the degree of maſter of arts in 1606, and afterwards entered into holy orders; and in 1615 he took the degree of bachelor of divinity. But being particularly diſtinguiſhed for his mathematical talents, when Mr. Williams reſigned the profeſſorſhip of aſtronomy in Greſham College, London, Mr. Gunter was choſen to ſucceed him, the 6th of March, 1619; where he greatly diſtinguiſhed himſelf by his lectures and writings, and where he died in 1626, at only 45 years of age, to the great loſs of the mathematical world.

Mr. Gunter was the author of many uſeful inventions and works. About the year 1606, he merited the title of an inventor, by the new projection of his Sector, which he then deſcribed in a Latin treatiſe, not printed however till ſome time afterwards.—In 1618 he had invented a ſmall portable quadrant, for the more eaſy finding the hour and azimuth, and other uſeful purpoſes in aſtronomy.—And in 1620 or 1623, he publiſhed his *Canon Triangulorum*, or Table of Artificial Sines and Tangents, to the radius, 10,000,000 parts, to every minute of the quadrant, being the firſt tables of this kind publiſhed; together with the firſt 1000 of Briggs's logarithms of common numbers, which were in later editions extended to 10,000 numbers.—In 1622, he diſcovered, by experiment made at Deptford, the variation or changeable declination of the magnetic needle; his experiment ſhewing that the declination had changed by 5 degrees in the ſpace of 42 years; and the ſame was confirmed and eſtabliſhed by his ſucceſſor Mr. Gellibrand.—He applied the logarithms of numbers, and of ſines and tangents, to ſtraight lines drawn on a ſcale or rule; with which proportions in common numbers and trigonometry were reſolved by the mere application of a pair of compaſſes; a method founded on this property, that the logarithms of the terms of equal ratios are equidifferent. This was called Gunter's Proportion, and Gunter's Line; and the inſtrument, in the form of a two-foot ſcale, is now in common uſe for navigation and other purpoſes, and is commonly called the Gunter. He alſo greatly improved the Sector and other inſtruments for the ſame uſes; the deſcription of all which he publiſhed in 1624.—He introduced the common meaſuring chain, now conſtantly uſed in land-ſurveying, which is thence called Gunter's Chain.—Mr. Gunter drew the lines on the dials in Whitehall-garden, and wrote the deſcription and uſe of them, by the direction of prince Charles, in a ſmall tract; which he afterwards printed at the deſire of king James, in 1624.—He was the firſt who uſed the word *co-ſine*, for the ſine of the complement of an arc. He alſo introduced the uſe of Arithmetical Complements into the logarithmical arithmetic, as is witneſſed by Briggs, cap. 15, Arith. Log. And it has been ſaid that he firſt ſtarted the idea of the Logarithmic Curve, which was ſo called, becauſe the ſegments of its axis are the logarithms of the correſponding ordinates.

His works have been collected, and various editions of them have been publiſhed; the 5th is by Mr. William Leybourn, in 1673, containing the Deſcription and Uſe of the Sector, Croſs-ſtaff, Bow, Quadrant, and other inſtruments; with ſeveral pieces added by Samuel Foſter, Henry Bond, and William Leybourn.

Gunter's Chain, the chain in common uſe for meaſuring land, according to true or ſtatute meaſure; ſo called from Mr. Gunter its reputed inventor.

The length of the chain is 66 feet, or 22 yards, or 4 poles of 5½ yards each; and it is divided into 100 links, of 7·92 inches each.

This Chain is the moſt convenient of any thing for meaſuring land, becauſe the contents thence computed are ſo eaſily turned into acres. The reaſon of which is, that an acre of land is juſt equal to 10 ſquare chains, or 10 chains in length and 1 in breadth, or equal to 100000 ſquare links. Hence, the dimenſions being taken in chains, and multiplied together, it gives the content in ſquare chains; which therefore being divided by 10, or a figure cut off for decimals, brings the content to acres; after which the decimals are reduced to roods and perches, by multiplying by 4 and 40. But the better way is to ſet the dimenſions down in links as integers, conſidering each chain as 100 links; then, having multiplied the dimenſions together, producing ſquare links, divide theſe by 100000, that is, cut off five places for decimals, the reſt are acres, and the decimals are reduced to roods and perches, as before.

Ex. Suppoſe in meaſuring a rectangular piece of ground, its length be 795 links,
and its breadth 480 links.

63600
3180
Ac. 3·81600
4
Ro. 3·264
40
Per. 10·560

So the content is 3 ac. 3 roods 10 perches.

Gunter's Line, a Logarithmic line, uſually graduated upon ſcales, ſectors, &c; and ſo called from its inventor Mr. Gunter.

This is otherwiſe called the *line of lines*, or *line of numbers*, and conſiſts of the logarithms transferred upon a ruler, &c, from the tables, by means of a ſcale of equal parts, which therefore ſerves to reſolve problems inſtrumentally, in the ſame manner as logarithms do arithmetically. For, whereas logarithms reſolve proportions, or perform multiplication and diviſion, by only addition and ſubtraction, the ſame are performed on this line by turning a pair of compaſſes over this way or that, or by ſliding one ſlip of wood by the ſide of another, &c.

This line has been contrived various ways, for the advantage of having it as long as poſſible. As, firſt, on the two-feet ruler or ſcale, by Gunter. Then, in 1627 the logarithms were drawn by Wingate, on two ſeparate rulers, ſliding againſt each other, to ſave the uſe of compaſſes in reſolving proportions. They were alſo in 1627 applied to concentric circles by Oughtred. Then in a ſpiral form by Mr. Milburne of Yorkſhire, about the year 1650. Alſo, in 1657, on the preſent common ſliding rule, by Seth Partridge.

Laſtly, Mr. William Nicholſon has propoſed another diſpoſition of them, on concentric circles, in the Philoſ. Tranſ. an. 1787, pa. 251. His inſtrument is equivalent

to

to a ſtraight rule of 28½ inches long. It conſiſts of three concentric circles, engraved and graduated on a plate of about 1½ inch in diameter. From the centre proceed two legs, having right-lined edges in the direction of radii; which are moveable either ſingly, or together. To uſe this inſtrument; place the edge of one leg at the antecedent of any proportion, and the other at the conſequent, and fix them to that angle: the two legs being then moved together, and the antecedent leg placed at any other number, the other leg gives its conſequent in the like poſition or ſituation on the lines.

The whole length of the line is divided into two equal intervals, or radii, of 9 larger diviſions in each radius, which are numbered from 1 to 10, the 1 ſtanding at the beginning of the line, becauſe the logarithm of 1 is 0, and the 10 at the end of each radius; alſo each of theſe 9 ſpaces is ſubdivided into 10 other parts, unequal according to the logarithms of numbers; the ſmaller diviſions being always 10ths of the larger; thus, if the large diviſions be units or ones, the ſmaller are tenth-parts; if the larger be tens, the ſmaller are ones; and if the larger be 100's, the ſmaller are 10's; &c.

Uſe of Gunter's Line. 1. *To find the product of two numbers.* Extend the compaſſes from 1 to either of the numbers, and that extent will reach the ſame way from the other number to the product. Thus, to multiply 7 and 5 together; extend the compaſſes from 1 to 5, and that extent will reach from 7 to 35, which is the product.

2. *To divide one number by another.* Extend the compaſſes from the diviſor to 1, and that extent will reach the ſame way from the dividend to the quotient. Thus, to divide 35 by 5; extend the compaſſes from 5 to 1, and that extent will reach from 35 to 7, which is the quotient.

3. *To find a 4th Proportional to three given Numbers;* as ſuppoſe to 6, 9, and 10. Extend from 6 to 9, and that extent will reach from 10 to 15, which is the 4th proportional ſought. And the ſame way a 3d proportional is found to two given terms, extending from the 1ſt to the 2d, and then from the 2d to the 3d.

4. *To find a Mean Proportional between two given numbers,* as ſuppoſe between 7 and 28. Extend from 7 to 28, and biſect that extent; then its half will reach from 7 forward, or from 28 backward, to 14, the mean proportional between them.—Alſo, to extract the ſquare root, as of 25, which is only finding a mean proportional between 1 and the given ſquare 25, biſect the diſtance between 1 and 25, and the half will reach from 1 to 5, the root ſought.—In like manner the cubic or 3d root, or the 4th, 5th, or any higher root, is found, by taking the extent between 1 and the given power; then take ſuch part of it as is denoted by the index of the root, viz, the 3d part for the cube root, the 4th part for the 4th root, and ſo on, and that part will reach from 1 to the root ſought.

If the Line on the Scale or Ruler have a ſlider, this is to be uſed inſtead of the compaſſes.

Gunter's Quadrant, is a quadrant made of wood, braſs, or ſome other ſubſtance; being a kind of ſtereographic projection on the plane of the equinoctial, the eye being ſuppoſed in one of the poles: ſo that the tropic, ecliptic, and horizon, form the arches of circles, but the hour circles other curves, drawn by means of ſeveral altitudes of the ſun, for ſome particular latitude every day in the year.

The uſe of this inſtrument, is to find the hour of the day, the ſun's azimuth, &c, and other common problems of the ſphere or globe; as alſo to take the altitude of an object in degrees. See Quadrant.

Gunter's Scale, uſually called by ſeamen *the Gunter*, is a large plain ſcale, having various lines upon it, of great uſe in working the caſes or queſtions in Navigation.

This Scale is uſually 2 feet long, and about an inch and a half broad, with various lines upon it, both natural and logarithmic, relating to trigonometry, navigation, &c.

On the one ſide are the natural lines, and on the other the artificial or logarithmic ones. The former ſide is firſt divided into inches and tenths, and numbered from 1 to 24 inches, running the whole length near one edge. One half the length of this ſide conſiſts of two plane diagonal ſcales, for taking off dimenſions to three places of figures. On the other half or foot of this ſide, are contained various lines relating to trigonometry, in the natural numbers, and marked thus, viz,

Rumb,	the rumbs or points of the compaſs,
Chord,	the line of chords,
Sine,	the line of ſines,
Tang.	the tangents,
S. T.	the ſemitangents,

and at the other end of this half are

Leag.	leagues, or equal parts,
Rumb.	another line of rumbs,
M. L.	miles of longitude,
Chor.	another line of chords.

Alſo in the middle of this foot are *L.* and *P.* two other lines of equal parts. And all theſe lines on this ſide of the ſcale ſerve for drawing or laying down the figures to the caſes in trigonometry and navigation.

On the other ſide of the ſcale are the following artificial or logarithmic lines, which ſerve for working or reſolving thoſe caſes; viz,

S. R.	the ſine rumbs,
T. R.	the tangent rumbs,
Numb.	line of numbers,
Sine,	Sines,
V. S.	the verſed ſines,
Tang.	the tangents,
Meri.	Meridional parts.
E. P.	Equal parts.

The late Mr. John Robertſon, librarian to the Royal Society, greatly improved this ſcale, both as to ſize and accuracy, for the uſe of mariners. He extended it to 30 inches long, 2 inches broad, and half an inch thick; upon which the ſeveral lines are very accurately laid down by Meſſrs. Nairne and Blunt, ingenious inſtrument makers. Mr. Robertſon died before his improved ſcales were publiſhed; but the account and deſcription of them were ſupplied and drawn up by his friend Mr. William Mountaine, and publiſhed in 1778.

GUTTÆ, or *Drops*, in Architecture, are ornaments in form of little bells or cones, uſed in the Doric order, on the architrave, below the tryglyphs. There are uſually ſix of them.

H.

HADLEY's *Quadrant*, *Sextant*, &c, an excellent inſtrument ſo called from its inventor John Hadley, Eſq. See its deſcription and uſe under the article QUADRANT.

HAIL, or HAILSTONES, an aqueous concretion, uſually in form of white or pellucid ſpherules, deſcending out of the atmoſphere.

Hailſtones aſſume various ſhapes, being ſometimes round, at other times pyramidal, crenated, angular, thin, and flat, and ſometimes ſtellated, with ſix radii like the ſmall cryſtals of ſnow.

It is very difficult to account for the phenomena of hail in a ſatisfactory manner; and there are various opinions upon this head. It is uſually conceived that hail is formed of drops of rain, frozen in their paſſage through the middle region. Others, as the Carteſians, take it for the fragments of a frozen cloud, half melted, and thus precipitated and congealed again. Signior Beccaria ſuppoſes, that it is formed in the higher regions of the air, where the cold is intenſe, and where the electric matter is very copious. In theſe circumſtances, a great number of particles of water are brought near together, where they are frozen, and in their deſcent they collect other particles; ſo that the denſity of the ſubſtance of the Hailſtone grows leſs and leſs from the centre; this being formed firſt in the higher regions, and the ſurface being collected in the lower. Accordingly, in mountains, Hailſtones as well as drops of rain, are very ſmall; and both agree in this circumſtance, that the more intenſe is the electricity that forms them, the larger they are.

It is frequently obſerved that Hail attends thunder and lightning; and hence Beccaria obſerves, that as motion promotes freezing, ſo the rapid motion of the electrified clouds may promote that effect in the air.

Natural hiſtories furniſh us with a great variety of curious inſtances of extraordinary ſhowers of Hail. See the Philoſ. Tranſ. number 203, 229; and Hiſt. de France, tom. 2, pa. 339.

HALF-MOON, in Fortification, is an outwork having only two faces, forming together a ſaliant angle, which is flanked by ſome part of the place, and of the other baſtions. See DEMILUNE and RAVELIN.

HALF-TANGENTS, are the tangents of the half arcs. See SCALE and SEMITANGENTS.

HALLEY (Dr. EDMUND), a moſt eminent Engliſh mathematician, philoſopher and aſtronomer, was born in the pariſh of St. Leonard, Shoreditch, near London, Oct. 29, 1656. His father, a wealthy citizen and ſoap-boiler, reſolving to improve the promiſing diſpoſition obſerved in his ſon, put him firſt to St. Paul's ſchool, where he ſoon excelled in all parts of claſſical learning, and made beſides a conſiderable advance in the mathematics; ſo that, as Wood obſerves, he had perfectly learnt the uſe of the celeſtial globe, and could make a complete dial; and we are informed by Halley himſelf, that he obſerved the change of the variation of the magnetic needle at London in 1672, one year before he left ſchool. In 1673 he was ſent to Oxford, where he chiefly applied himſelf to mathematics and aſtronomy, in which he was greatly aſſiſted by a curious apparatus of inſtruments, which his father, willing to encourage his ſon's genius, had purchaſed for him. At 19 years of age he began to oblige the world with new obſervations and diſcoveries (which he continued to do to the end of a very long life), by publiſhing “A Direct and Geometrical method of finding the Aphelia and Excentricity of the planets.” Beſides various particular obſervations, made from time to time upon the celeſtial phenomena; he had, from his firſt admiſſion into college, purſued a general ſcheme for aſcertaining the true places of the fixed ſtars, and ſo to correct the errors of Tycho Brahe. His original view in this was, to carry on the deſign of that firſt reſtorer of aſtronomy, by completing the catalogue of thoſe ſtars from his own obſervations; but upon farther enquiry, finding this province taken up by Hevelius and Flamſteed, he dropped that purſuit, and formed another; which was, to perfect the whole ſcheme of the heavens, by the addition of the ſtars which lie ſo near the ſouth pole, that they could not be obſerved by thoſe aſtronomers, as never riſing above the horizon either at Dantzick or at Greenwich. With this view he left the Univerſity, before he had taken any degree, and embarked for the iſland of St. Helena in Nov. 1676, when he was only 20 years of age, and arrived there after a voyage of three months. He immediately ſet about his taſk with ſuch diligence, that he completed his catalogue, and, returning home, landed in England in Nov. 1678, after an abſence of two years only. The univerſity of Oxford immediately conferred upon him the degree of A. M. and the Royal Society of London elected him one of their members.

In 1679 he was pitched upon by the Royal Society to go to Dantzick, to endeavour to adjuſt a diſpute between Hevelius and Mr. Hooke, concerning the preference as to plain and glaſs ſights in aſtroſcopical inſtruments. He arrived at Dantzick the 26th of May, when he immediately, in conjunction with Hevelius, ſet about their aſtronomical obſervations, which they cloſely continued till the 18th of July, when Halley left Dantzick, and returned to England.

In the year 1680 he undertook what is called the grand tour, accompanied by his friend the celebrated Mr. Nelſon. In the way from Calais to Paris, Mr. Halley had a ſight of a remarkable comet, as it then appeared a ſecond time that year, in its return from the ſun. He had the November before ſeen it in its deſcent; and he now haſtened to complete his obſervations upon it, by viewing it from the royal obſervatory of France. His deſign in this part of his tour was, to ſettle a friendly correſpondence between the two royal aſtronomers of Greenwich and Paris; and in the mean time to improve himſelf under ſo great a maſter as Caſſini. From thence he went to Italy, where he ſpent great part of the year 1681; but his affairs calling him home, he then returned to England.

Soon after his return, he married the daughter of Mr. Tooke, auditor of the exchequer, and took up his reſidence at Iſlington, where he ſet up his tube and ſextant, and eagerly purſued his favourite ſtudy: in the ſociety of this amiable lady he lived happily for five-and-fifty years. In 1683 he publiſhed his *Theory of the Variation of the Magnetical Compaſs*; in which he ſuppoſes the whole globe of the earth to be one great magnet, having four magnetical poles or points of attraction, &c. The ſame year alſo he entered upon a new method of finding out the longitude, by an accurate obſervation of the moon's motion. His purſuits it ſeems were now a little interrupted by the death of his father, who having ſuffered greatly by the fire of London, as well as by a ſecond marriage, into which he had imprudently entered, was found to have waſted his fortunes. Our author ſoon reſumed his purſuits however; for in the beginning of 1684 he turned his thoughts to the ſubject of Kepler's ſeſqui-alterate proportion; when, after ſome meditation, he concluded from it, that the centripetal force muſt decreaſe in proportion to the ſquare of the diſtance reciprocally. He found himſelf unable to make it out in any geometrical way; and therefore, after applying in vain for aſſiſtance to Mr. Hooke and Sir Chriſtopher Wren, he went to Cambridge to Mr. Newton, who fully ſupplied him with what he ſo ardently ſought. But Halley having now found an immenſe treaſure in Newton, could not reſt, till he had prevailed with the owner to enrich the public with it; and to this interview the world is in ſome meaſure indebted for the *Principia Mathematica Philoſophiæ Naturalis*. That great work was publiſhed in 1686; and Halley, who had the whole care of the impreſſion, prefixed to it a diſcourſe of his own, giving a general account of the aſtronomical part of the book; and alſo a very elegant copy of verſes in Latin.

In 1687 he undertook to explain the cauſe of a natural phenomenon, which had till then baffled the reſearches of the ableſt geographers. It is obſerved that the Mediterranean ſea never ſwells in the leaſt, although there is no viſible diſcharge of the prodigious quantity of water that runs into it from nine large rivers, beſides ſeveral ſmall ones, and the conſtant ſetting in of the current at the mouth of the Streights. His ſolution of this difficulty gave ſo much ſatisfaction to the Society, that he was requeſted to proſecute theſe enquiries. He did ſo; and having ſhewn, by accurate experiments, how that vaſt acceſſion of water was actually carried off in vapours raiſed by the action of the ſun and wind upon its ſurface, he proceeded with the like ſucceſs to point out the method uſed by nature to return the ſaid vapours into the ſea. This circulation he ſuppoſes to be carried on by the winds driving theſe vapours to the mountains; where being collected, they form ſprings, which uniting become rivulets or brooks, and many of theſe again meeting in the valleys, grow into large rivers, emptying themſelves at laſt into the ſea: thus demonſtrating, in the moſt beautiful manner, the way in which the equilibrium of receipt and expence is continually preſerved in the univerſal ocean.

He next ranged in the field of ſpeculative geometry, where, obſerving ſome imperfections in the methods before laid down for conſtructing ſolid problems, or equations of the 3d and 4th powers, he furniſhed new rules, which were both more eaſy and more elegant than any of the former; together with a new method of finding the number of roots of ſuch equations, and the limits of the ſame.

Mr. Halley next undertook to publiſh a more correct Ephemeris for the year 1688, there being then great want of proper ephemerides of any tolerable exactneſs, the common ones being juſtly complained of by Mr. Flamſteed.—In 1691 he publiſhed exact tables of the conjunctions of Venus and Mercury; and he afterwards ſhewed one extraordinary uſe to be made of thoſe tables, viz, for diſcovering the ſun's parallax, and thence the true diſtance of the earth from the ſun.—In 1692, our author produced his tables for ſhewing the value of annuities on lives, calculated from bills of mortality; and his univerſal theorem for finding the foci of optic glaſſes.

But it would be endleſs to enumerate all his valuable diſcoveries now communicated to the Royal Society, and publiſhed in the Philoſ. Tranſ. of which, for many years, his pieces were the chief ornament and ſupport. Their various merit is thrown into one view by the writer of his eloge in the Paris Memoirs; who, having mentioned his Hiſtory of the Trade-winds and Monſoons, proceeds in theſe terms: “This was immediately followed by his eſtimation of the quantity of vapours which the ſun raiſes from the ſea; the circulation of vapours; the origin of fountains; queſtions on the nature of light and tranſparent bodies; a determination of the degrees of mortality, in order to adjuſt the valuation of annuities on lives; and many other works, all the ſciences relating to aſtronomy, geometry, and algebra, optics and dioptrics, baliſtics and artillery, ſpeculative and experimental philoſophy, natural hiſtory, antiquities, philology, and criticiſm; being about 25 or 30 diſſertations, which he produced during the 9 or 10 years of his reſidence at London; and all abounding with ideas new, ſingular, and uſeful.”

In 1691, the Savilian profeſſorſhip of aſtronomy at Oxford being vacant, he applied for that office, but without ſucceſs. Whiſton, in the Memoirs of his own Life, tells us from Dr. Bentley, that Halley “being thought of for ſucceſſor to the mathematical chair at

Oxford, biſhop Stillingfleet was deſired to recommend him at court; but hearing that he was a ſceptic and a banterer of religion, the biſhop ſcrupled to be concerned, till his chaplain Bentley ſhould talk with him about it, which he did. But Halley was ſo ſincere in his infidelity, that he would not ſo much as pretend to believe the chriſtian religion, though he thereby was likely to loſe a profeſſorſhip; which he did accordingly, and it was then given to Dr. Gregory."

Halley had publiſhed his Theory of the Variation of the Magnetical Compaſs, as has been already obſerved, in 1683; which, though it was well received both at home and abroad, he found, upon a review, liable to great and inſuperable objections. Yet the phenomena of the variation of the needle, upon which it is raiſed, being ſo many certain and indiſputed facts, he ſpared no pains to poſſeſs himſelf of all the obſervations relating to it he could poſſibly come at. To this end he procured an application to be made to king William, who appointed him commander of the Paramour pink, with orders to ſearch out by obſervations the diſcovery of the rule of variations, and to lay down the longitudes and latitudes of the Engliſh ſettlements in America.—He ſet out on this attempt on the 24th of November, 1698: but having croſſed the line, his men grew ſickly; and his firſt lieutenant mutinying, he returned home in June 1699. Having got the lieutenant tried and caſhiered, he ſet ſail a ſecond time in September following, with the ſame ſhip, and another of leſs bulk, of which he had alſo the command. He now traverſed the vaſt Atlantic ocean from one hemiſphere to the other, as far as the ice would permit him to go; and having made his obſervations at St. Helena, Brazil, Cape Verde, Barbadoes, the Madeiras, the Canaries, the coaſt of Barbary, and many other latitudes, he arrived in England in September 1700; and the next year publiſhed a general chart, ſhewing at one view the variation of the compaſs in all thoſe places.

Captain Halley, as he was now called, had been at home little more than half a year, when he was ſent by the king, to obſerve the courſe of the tides, with the longitude and latitude of the principal head-lands in the Britiſh channel; which having executed with his uſual expedition and accuracy, he publiſhed a large map of the channel.

Soon after, the emperor of Germany reſolving to make a convenient harbour for ſhipping in the Adriatic, captain Halley was ſent by queen Anne to view the two ports on the coaſt of Dalmatia. He embarked on the 22d of November 1702; paſſed over to Holland; and going through Germany to Vienna, he proceeded to Iſtria: but the Dutch oppoſing the deſign, it was laid aſide; yet the emperor made him a preſent of a rich diamond ring from his finger, and honoured him with a letter of recommendation, written with his own hand, to queen Anne. Preſently after his return, he was ſent again on the ſame buſineſs; when paſſing through Hanover, he ſupped with the electoral prince, who was afterward king George the 1ſt, and his ſiſter the queen of Pruſſia. On his arrival at Vienna, he was the ſame evening preſented to the emperor, who ſent his chief engineer to attend him to Iſtria, where they repaired the fortifications of Trieſte, and added new ones.

Mr. Halley returned to England in Nov. 1703; and the ſame year he was made profeſſor of geometry in the univerſity of Oxford, inſtead of Dr. Wallis then juſt deceaſed, and he was at the ſame time honoured by the univerſity with the degree of doctor of laws. He was ſcarcely ſettled in Oxford, when he began to tranſlate into Latin, from the Arabic, *Apollonius de Sectione Rationis*; and to reſtore the two books *De Sectione Spatii* of the ſame author, which are loſt, from the account given of them by Pappus; and he publiſhed the whole work in 1706. He afterwards had a ſhare in preparing for the preſs Apollonius's Conics; and ventured to ſupply the whole 8th book, the original of which is alſo loſt. To this work he added Serenus on the Section of the Cylinder and Cone, printed from the original Greek, with a Latin tranſlation, and publiſhed the whole in folio 1710. Beſide theſe, the *Miſcellanea Curioſa*, in 3 volumes 8vo, had come out under his direction in 1708.

In 1713, he ſucceeded Doctor, afterwards Sir, Hans Sloane, in the office of Secretary to the Royal Society. And, upon the death of Mr. Flamſteed in 1719, he was appointed to ſucceed him at Greenwich as Aſtronomer Royal; upon which occaſion, that he might be more at leiſure to attend the duties of this office, he reſigned that of ſecretary to the Royal Society in 1721. Although he was 63 or 64 years of age when he entered upon his office at Greenwich, for the ſpace of 18 years he watched the heavens with the cloſeſt attention, hardly ever miſſing an obſervation during all that time, and, without any aſſiſtant, performed the whole buſineſs of the obſervatory himſelf.

Upon the acceſſion of the late king, his conſort queen Caroline made a viſit at the Royal Obſervatory; and being pleaſed with every thing ſhe ſaw, took notice that Dr. Halley had formerly ſerved the crown as a captain in the navy: and ſhe ſoon after obtained a grant of his half-pay for that commiſſion, which he accordingly enjoyed from that time during his life. An offer was alſo made him of being appointed mathematical preceptor to the duke of Cumberland; but he declined that honour, on account of his advanced age, and the duties of his office. In 1729 he was choſen a foreign member of the Academy of Sciences at Paris.

About 1737 he was ſeized with a paralytic diſorder in his right hand, which, it is ſaid, was the firſt attack he ever felt upon his conſtitution: however, he came as uſual once a week, till within a very ſhort time of his death, to meet his friends in town on Thurſdays, before the meeting of the Royal Society, at what is yet called Dr. Halley's club. His paralytic diſorder increaſing, his ſtrength gradually wore away, till he expired Jan. 14, 1742, in the 86th year of his age; and his corps was interred in the church-yard of Lee near Blackheath.——Beſide the works before mentioned, his principal publications are, 1. *Catalogus Stellarum Auſtralium*. 2. *Tabulæ Aſtronomicæ*. 3. The Aſtronomy of Comets. With a great multitude of Papers in the Philoſ. Tranſ. from vol. 11 to vol. 60.

HALIFAX (John). See Sacrobosco.

HALO, or Corona, a coloured circle appearing round the body of the ſun, moon, or any of the larger ſtars.

Naturaliſts conceive the Halo to ariſe from a refraction of the rays of light in paſſing through the fine rare veſiculæ of a thin vapour towards the top of the atmoſphere.

Des Cartes obſerves, that the Halo never appears when it rains; whence he concludes that this phenomenon is occaſioned by the refraction of light in the round particles of ice, which are then floating in the atmoſphere; and to the different protuberance of theſe particles he aſcribes the variation in the diameter of the Halo. Gaſſendi ſuppoſes, that a Halo is occaſioned in the ſame manner as the rainbow; the rays of light being, in both caſes, twice refracted and once reflected within each drop of rain or vapour, and that the difference between them is wholly owing to their different ſituation with reſpect to the obſerver. Dechales alſo endeavours to ſhew that the generation of the Halo is ſimilar to that of the rainbow; and that the reaſon why the colours of the Halo are more dilute than thoſe of the rainbow, is owing chiefly to their being formed, not in large drops of rain, but in very ſmall vapour. But the moſt conſiderable and generally received theory, relating to the generation of Halos, is that of Mr. Huygens. This celebrated author ſuppoſes Halos, or circles round the ſun, to be formed by ſmall round grains of hail, compoſed of two different parts, the one of which is tranſparent, incloſing the other, which is opaque; which is the general ſtructure actually obſerved in hail. He farther ſuppoſes that the grains or globules, that form theſe Halos, conſiſted at firſt of ſoft ſnow, and that they have been rounded by a continual agitation in the air, and thawed on their outſide by the heat of the ſun, &c. And he illuſtrates his ideas of their formation by geometrical figures.

Mr. Weidler endeavours to refute Huygens's manner of accounting for Halos, by a vaſt number of ſmall vapours, each with a ſnowy nucleus, coated round with a tranſparent covering. He ſays, that when the ſun paints its image in the atmoſphere, and by the force of its rays puts the vapours in motion, and drives them toward the ſurface, till they are collected in ſuch a quantity, and at ſuch a diſtance from the ſun on each ſide, that its rays are twice refracted, and twice reflected, when they reach the eye they exhibit the appearance of a Halo, adorned with the colours of the rainbow; which may happen in globular pellucid vapours without ſnowy nuclei, as appears by the experiment of hollow glaſs ſpheres filled with water: therefore, whenever thoſe ſpherical vapours are ſituated as before mentioned, the refractions and reflections will happen every where alike, and the figure of a circular crown, with the uſual order of colours, will be the conſequence. Philoſ. Tranſ. number 458.

Newton's theory of Halos may be ſeen in his Optics, p. 155. And this curious theory was confirmed by actual obſervation in June 1692, when the author ſaw by reflection, in a veſſel of ſtagnated water, three Halos, crowns, or rings of colours, about the ſun, like three little rainbows concentric to his body. Theſe crowns incloſed one another immediately, ſo that their colours proceeded in this continual order from the ſun outward: blue, white, red; purple, blue, green, pale yellow, and red; pale blue, pale red. The like crowns ſometimes appear about the moon. The more equal the globules of water or ice are to one another, the more crowns of colours will appear, and the colours will be the more lively. Optics, p. 288.

There are ſeveral ways of exhibiting phenomena ſimilar to theſe. The flame of a candle, placed in the midſt of a ſteam in cold weather, or placed at the diſtance of ſome feet from a glaſs window that has been breathed upon, while the ſpectator is alſo at the diſtance of ſome feet from another part of the window, or placed behind a glaſs receiver, when air is admitted into the vacuum within it to a certain denſity, in each of theſe circumſtances will appear to be encompaſſed by a coloured Halo. Alſo, a quantity of water being thrown up againſt the ſun, as it breaks and diſperſes into drops, forms a kind of Halo or iris, exhibiting the colours of the natural rainbow. Muſſchenbroek obſerved, that when the glaſs windows of his room were covered with a thin plate of ice on the inſide, the moon ſeen through it was ſurrounded with a large and variouſly coloured Halo; which, upon opening the window, he found aroſe entirely from that thin plate of ice, becauſe none was ſeen except through this plate. Muſſchenbroek concludes his account of coronas with obſerving, that ſome denſity of vapour, or ſome thickneſs of the plates of ice, divides the light in its tranſmiſſion either through the ſmall globules or their interſtices, into its ſeparate colours; but what that denſity is, or what the ſize of the particles which compoſe the vapour, he does not pretend to determine. Introd. ad Phil. Nat. p. 1037.

It has often been obſerved that a Halo about the ſun or moon, does not appear circular and concentric to the luminary, but oval and excentric, with its longeſt diameter perpendicular to the horizon, and extended from the moon farther downward than upward. Dr. Smith aſcribes this phenomenon to the apparent concave of the ſky being leſs than a hemiſphere. When the angle which the diameter of a Halo ſubtends at the eye is 45° or 46°, and the bottom of the Halo is near the horizon, and conſequently its apparent figure is moſt oval, the apparent vertical diameter is divided by the moon in the proportion of about 2 to 3 or 4, and is to the horizontal diameter drawn through the moon, as 4 to 3, nearly.—See farther on the ſubject of this article, Prieſtley's Hiſt. of Diſcoveries relating to Viſion, p. 596—613; and Smith's Optics, art. 167, 513, 526, 527, &c.

HAMEL (John Baptiste du), a very learned French philoſopher and writer, in the 17th century, was born in lower Normandy in 1614. At 18 years of age he publiſhed a treatiſe, in which he explained, in a very ſimple manner, and by one or two figures, Theodoſius's 3 books upon Spherics; to which he added a tract upon trigonometry extremely perſpicuous, and deſigned as an introduction to aſtronomy. He publiſhed afterwards various other works on aſtronomy and philoſophy. Natural philoſophy, as it was then taught, was only a collection of vague, knotty, and barren queſtions; when our author undertook to eſtabliſh it upon right principles, and publiſhed his *Aſtronomia Phyſica*.

In 1666 M. Colbert propoſed to Louis the 14th a ſcheme, which was approved of, for eſtabliſhing a royal academy of ſciences; and appointed our author ſecretary of it. In 1678, his *Philoſophia Vetus & Nova* was printed at Paris in 4 vols, 12mo; and in 1681 it was enlarged and printed there in 6 vols. He wrote ſeveral other pieces; and his works in this way were collected and publiſhed at Nuremberg 1681, in 4 volumes 4to, under the title of *Opera Philoſophica & Aſtronomica.* Theſe were highly valued then, though the improvements in philoſophy ſince that time have rendered them of little or no uſe now.

In 1697 he reſigned his place of ſecretary of the Royal Academy of Sciences; in which he was ſucceeded by M. Fontenelle. However, he publiſhed, in 1698, *Regiæ Scientiarum Academiæ Hiſtoria*, 4to, in four books; which being much liked, he afterwards augmented with two books more. This work contains an account of the foundation of the Royal Academy of Sciences, and its tranſactions, from 1666 to 1700, and is now the moſt uſeful of all his works. He was Regius Profeſſor of Philoſophy, in which office he was ſucceeded by M. Varignon, at his death, which happened Aug. 6, 1706, in the 93d year of his age.

HANCES, HANCHES, or HANSES, in architecture, are certain ſmall intermediate parts of arches between the key or crown and the ſpring at the bottom, being perhaps about one-third of the arch, and ſituated nearer the bottom than the top or crown; and are otherwiſe called the *ſpandrels*.

HANDSPIKE, or HANDSPEC, a lever or piece of aſh, elm, or other ſtrong wood, for raiſing by the hand great weights, &c. It is 5 or 6 feet long, cut thin and crooked at the lower end, that it may get the eaſier between things that are to be ſeparated, or under any thing that is to be raiſed. It is better than a crow of iron, becauſe its length allows a better poiſe.

HARD *Bodies*, are ſuch as are abſolutely inflexible to any preſſure or percuſſion whatever: differing from ſoft bodies, whoſe parts yield and are eaſily moved amongſt one another, without reſtoring themſelves again; and from elaſtic bodies, the parts of which alſo yield and give way, but preſently reſtore themſelves again to their former ſtate and ſituation. Hence, hard bodies do not bend, or indent, but break. It is probable however there are no bodies in nature that are abſolutely or perfectly either hard, ſoft, or elaſtic; but all poſſeſſing theſe qualities, more or leſs, in ſome degree. M. Bernoulli goes ſo far as to ſay that Hardneſs, in the common ſenſe, is abſolutely impoſſible, being contrary to the law of continuity.

The laws of motion for hard bodies are the ſame as for ſoft ones, both being ſuppoſed to adhere together on their impact. And theſe two ſorts of bodies might be comprized under the common name of Unelaſtic.

HARDENING, the giving a greater degree of hardneſs to bodies than they had before.

There are ſeveral ways of Hardening iron and ſteel; as by hammering them, quenching them in cold water, &c.

Caſe-HARDENING, is a ſuperficial converſion of iron into ſteel, as if it were caſing it, or covering it with a thin coat of harder matter. It is thus performed: Take cow horn or hoof, dry it well in an oven, and beat it to powder; put equal quantities of this powder and of bay ſalt into ſtale urine, or white wine vinegar, and mix them well together; cover the iron or ſteel all over with this mixture, and wrap it up in loam, or plate iron, ſo as the mixture touch every part of the work; then put it in the fire, and blow the coals to it, till the whole lump have a blood red heat, but no higher; laſtly, take it out, and quench it.—See STEEL, under which article are deſcribed other proceſſes for this purpoſe.

HARDNESS, or Rigidity, that quality in bodies by which their parts ſo cohere as not to yield inward, or give way to an external impulſe, without inſtantly going beyond the diſtance of their mutual attraction; and therefore are not ſubject to any motion in reſpect of each other, without breaking the body.

There were many fanciful opinions among the ancients concerning the cauſe of hardneſs; ſuch as, heat, cold, dryneſs, the hooked figure of the particles of matter. The Carteſians make the Hardneſs of bodies to conſiſt in reſt, as that of ſoft and fluid ones in the motion of their particles.

Newton ſhews that the primary particles of all bodies, whether ſolid or fluid, are perfectly hard; and are not capable of being broken or divided by any power in nature. Theſe particles, he maintains, are connected together by an attractive power; and according to the circumſtances of this attraction, the body is either hard, or ſoft, or even fluid. If the particles be ſo diſpoſed or fitted for each other, as to touch in large ſurfaces, the body will be hard; and the more ſo as thoſe ſurfaces are the larger. If, on the contrary, they only touch in ſmall ſurfaces, the body, by the weakneſs of the attraction, will remain ſoft.

At preſent, many philoſophers think that Hardneſs conſiſts in the abſence or want of the action of the univerſal fluid, or elementary fire, among the particles of the body, or a deficiency of what is called latent heat; while on the contrary, fluidity, according to them, conſiſts in the motion of the particles, in conſequence of the action of that elementary fire.

Hardneſs appears to diminiſh the coheſion of bodies, in ſome degree, though their frangibility or brittleneſs does not by any means keep pace with their hardneſs. Thus, though glaſs be very hard and very brittle; yet flint is ſtill harder, though leſs brittle. Among the metals, theſe two properties ſeem to be more connected, though even here the connection is by no means complete: for though ſteel be both the hardeſt and moſt brittle of all the metals; yet lead, which is the ſofteſt, is not the moſt ductile. Neither is Hardneſs connected with the ſpecific gravity of bodies; for a diamond, the hardeſt ſubſtance in nature, is little more than half the weight of the lighteſt metal. And as little is it connected with the coldneſs, or electrical properties, or any other quality with which we are acquainted. Some bodies are rendered hard by cold, and others by different degrees of heat.

Mr. Quiſt and others have conſtructed tables of the Hardneſs of different ſubſtances. And the manner of conſtructing theſe tables, was by obſerving the order in which they were able to cut or make any impreſſion upon one another. The following table, extracted from Magellan's edition of Cronſtedt's Mineralogy, was taken from Quiſt, Bergman, and Kirwan. The firſt column ſhews the Hardneſs, and the ſecond the ſpecific gravity.

	Hard-neſs.	*Spec. Grav.*
Diamond from Ormus	20	3·7
Pink diamond	19	3·4
Blueiſh diamond	19	3·3
Yellowiſh diamond	19	3·3
Cubic diamond	18	3·2
Ruby	17	4·2
Pale blue ſapphire	17	3·8
Pale ruby from Brazil	16	3·5
Deep blue ſapphire	16	3·8
Topaz	15	4·2
Whitiſh ditto	14	3·5
Ruby ſpinell	13	3·4
Emerald	12	2·8
Garnet	12	4·4
Agate	12	2·6
Onyx	12	2·6
Sardonyx	12	2·6
Bohemian topaz	11	2·8
Occid. amethyſt	11	2·7
Cryſtal	11	2·6
Carnelian	11	2.7
Green jaſper	11	2·7
Schoerl	10	3·6
Tourmaline	10	3·0
Quartz	10	2·7
Opal	10	2·6
Chryſolite	10	3·7
Reddiſh yellow jaſper	9	2·6
Zeolyte	8	2·1
Fluor	7	3·5
Calcareous ſpar	6	2·7
Gypſum	5	2·3
Chalk	3	2·7

HARMONICA, Harmonics, a branch or diviſion of the ancient muſic; being that part which conſiders the differences and proportions of ſounds, with reſpect to acute and grave; as diſtinguiſhed from Rhythmica, and Metrica.

Mr. Malcolm has made a very induſtrious and learned enquiry into the Harmonica, or harmonic principles, of the ancients.

Harmonica, the name of a muſical inſtrument invented by Dr. Franklin, conſiſting of the glaſſes, called muſical glaſſes.

It is ſaid that the firſt hint of muſical glaſſes is to be found in an old Engliſh book, in which a number of various amuſements were deſcribed. That author directs his pupil to chooſe half a dozen drinking-glaſſes; to fill each of them with water in proportion to the gravity or acuteneſs of the ſound which he intended it to give; and having thus adjuſted them one to another, he might entertain the company with a church tune. Theſe were perhaps the rude hints which Mr. Puckeridge, an Iriſh gentleman, afterwards improved, and after him, Mr. E. Delaval, an ingenious member of the Royal Society; and finally brought to perfection by the celebrated Franklin. See the hiſtory and deſcription in his Letters, particularly in that to Beccaria.

HARMONICAL, or Harmonic, ſomething relating to Harmony. Thus,

Harmonical *Arithmetic*, is ſo much of the theory and doctrine of numbers, as relates to making the compariſons and reductions of muſical intervals, which are expreſſed by numbers, for finding their mutual relations, compoſitions, and reſolutions.

Harmonical *Compoſition*, in its general ſenſe, includes the compoſition both of harmony and melody; i. e. of muſic, or ſong, both in a ſingle part, and in ſeveral parts.

Harmonical *Interval*, the difference between two ſounds, in reſpect of acute and grave: or that imaginary ſpace terminated with two ſounds differing in acuteneſs or gravity.

Harmonical *Proportion*, or *Muſical Proportion*, is that in which the firſt term is to the third, as the difference of the firſt and ſecond is to the difference of the 2d and 3d; or when the firſt, the third, and the ſaid two differences, are in geometrical proportion. Or, four terms are in Harmonical proportion, when the 1ſt is to the 4th, as the difference of the 1ſt and 2d is to the difference of the 3d and 4th. Thus, 2, 3, 6, are in harmonical proportion, becauſe 2 : 6 :: 1 : 3. And the four terms 9, 12, 16, 24 are in harmonical proportion, becauſe 9 : 24 :: 3 : 8.—If the proportional terms be continued in the former caſe, they will form an harmonical progreſſion, or ſeries.

1. The reciprocals of an arithmetical progreſſion are in Harmonical progreſſion; and, converſely, the reciprocals of Harmonicals are arithmeticals. Thus, the reciprocals of the Harmonicals 2, 3, 6, are $\frac{1}{2}, \frac{1}{3}, \frac{1}{6}$, which are arithmeticals; for $\frac{1}{2} - \frac{1}{3} = \frac{1}{6}$, and $\frac{1}{3} - \frac{1}{6} = \frac{1}{6}$ alſo: and the reciprocals of the arithmeticals 1, 2, 3, 4, &c, are $\frac{1}{1}, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}$, &c, which are Harmonicals; for $\frac{1}{1} : \frac{1}{3} :: \frac{1}{1} - \frac{1}{2} : \frac{1}{2} - \frac{1}{3}$; and ſo on. And, in general, the reciprocals of the arithmeticals a, $a + d$, $a + 2d$, $a + 3d$, &c, viz, $\frac{1}{a}, \frac{1}{a + d}, \frac{1}{a + 2d}, \frac{1}{a + 3d}$, &c, are Harmonicals; et e contra.

2. If three or four numbers in Harmonical proportion be either multiplied or divided by ſome number, the products, or the quotients, will ſtill be in Harmonical proportion. Thus,

the Harmonicals	6,	8,	12,
multiplied by 2 give	12,	16,	24,
or divided by 2 give	3,	4,	6,

which are alſo Harmonicals.

3. To find a Harmonical mean proportional between two terms: Divide double their product by their ſum.

4. To find a 3d term in Harmonical proportion to two given terms: Divide their product by the difference between double the 1ſt term and the 2d term.

5. To find a 4th term in Harmonical proportion to three terms given: Divide the product of the 1ſt and 3d by the difference between double the 1ſt and the 2d term.

Hence, of the two terms a and b;

the Harmonical mean is $\frac{2ab}{a+b}$,

the 3d Harmonical propor. is $\frac{ab}{2a-b}$,

alſo to a, b, c, the 4th Harm. is $\frac{ac}{2a-b}$.

6. If there be taken an arithmetical mean, and a Harmonical mean, between any two terms, the four terms will be in geometrical proportion. Thus, between 2 and 6,

the arithmetical mean is 4, and
the Harmonical mean is 3;
and hence 2 : 3 : : 4 : 6.

Alſo, between a and b,

the arithmetical mean is $\frac{a+b}{2}$, and

the Harmonical mean is $\frac{2ab}{a+b}$;

but $a : \frac{2ab}{a+b} :: \frac{a+b}{2} : b$.

HARMONY, in Muſic, the agreeable reſult of an union of ſeveral muſical ſounds, heard at one and the ſame time; or the mixture of divers ſounds, which together have an agreeable effect on the ear.

As a continued ſucceſſion of muſical ſounds produces melody, ſo a continued combination of them produces Harmony.

Among the ancients however, as ſometimes alſo among the moderns, Harmony is uſed in the ſtrict ſenſe of conſonance; and ſo is equivalent to the ſymphony.

The words *concord* and Harmony do really ſignify the ſame thing; though cuſtom has made a little difference between them. Concord is the agreeable effect of two ſounds in conſonance; and Harmony the effect of any greater number of agreeable ſounds in conſonance.

Again, Harmony always implies conſonance; but concord is alſo applied to ſounds in ſucceſſion; though never but where the terms can ſtand agreeably in conſonance. The effect of an agreeable ſucceſſion of ſeveral ſounds, is called *melody*; as that of an agreeable conſonance is called Harmony.

Harmony is well defined, the ſum or reſult of the combination of two or more concords; that is, of three or more ſimple ſounds ſtriking the ear all together; and different compoſitions of concords make different Harmony.

The ancients ſeem to have been entirely unacquainted with Harmony, the ſoul of the modern muſic. In all their explications of the melopœia, they ſay not one word of the concert or Harmony of parts. We have inſtances, indeed, of their joining ſeveral voices, or inſtruments, in conſonance; but then theſe were not ſo joined, as that each had a diſtinct and proper melody, ſo making a ſucceſſion of various concords; but they were either uniſons, or octaves, in every note; and ſo all performed the ſame individual melody, and conſtituted one ſong.

When the parts differ, not in the tenſion of the whole, but in the different relations of the ſucceſſive notes, it is this that conſtitutes the modern art of Harmony.

HARMONY *of the Spheres*, or *Celeſtial Harmony*, a kind of muſic much ſpoken of by many of the ancient philoſophers and fathers, ſuppoſed to be produced by the ſweetly tuned motions of the ſtars and planets. This Harmony they attributed to the various proportionate impreſſions of the heavenly bodies upon one another, acting at proper intervals. They think it impoſſible that ſuch prodigious bodies, moving with ſuch rapidity, ſhould be ſilent: on the contrary, the atmoſphere, continually impelled by them, muſt yield a ſet of ſounds proportionate to the impreſſion it receives; and that conſequently, as they run all in different circuits, and with various degrees of velocity, the different tones ariſing from the diverſity of motions, directed by the hand of the Almighty, muſt form an agreeable ſymphony or concert.

They therefore ſuppoſed, that the moon, as being the loweſt of the planets, correſponded to *mi*; Mercury, to *fa*; Venus, to *ſol*; the Sun, to *la*; Mars, to *ſi*; Jupiter, to *ut*; Saturn, to *re*; and the orb of the fixed ſtars, as being the higheſt of all, to *mi*, or the octave.

It is thought that Pythagoras had a view to the gravitation of celeſtial bodies, in what he taught concerning the Harmony of the ſpheres.

A muſical chord gives the ſame note as one double in length, when the tenſion or force with which the latter is ſtretched is quadruple; and the gravity of a planet is quadruple of the gravity of a planet at a double diſtance. In general, that any muſical chord may become uniſon to a leſſer chord of the ſame kind, its tenſion muſt be increaſed in the ſame proportion as the ſquare of its length is greater; and that the gravity of a planet may become equal to the gravity of another planet nearer the ſun, it muſt be increaſed in proportion as the ſquare of its diſtance from the ſun is greater. If therefore we ſhould ſuppoſe muſical chords extended from the ſun to each planet, that all theſe chords might become uniſon, it would be requiſite to increaſe or diminiſh their tenſions in the ſame proportions as would be ſufficient to render the gravities of the planets equal; and from the ſimilitude of thoſe proportions, the celebrated doctrine of the Harmony of the ſpheres is ſuppoſed to have been derived.

Kepler wrote a large work, in folio, on the Harmonies of the world, and particularly of that of the celeſtial bodies. He firſt endeavoured to find out ſome relation between the dimenſions of the five regular ſolids and the intervals of the planetary ſpheres; and imagining that a cube, inſcribed in the ſphere of Saturn, would touch by its ſix planes the ſphere of Jupiter, and that the other four regular ſolids in like manner fitted the intervals that are between the ſpheres of the other planets, he became perſuaded that this was the true reaſon why the primary planets were preciſely ſix in number, and that the author of the world had determined their diſtances from the ſun, the centre of the ſyſtem, from a regard to this analogy. But afterwards finding that the diſpoſition of the five regular ſolids amongſt the planetary ſpheres, was not agreeable to the intervals between their orbits, he endeavoured to diſcover other ſchemes of Harmony. For this purpoſe he compared the motions of the ſame planet at its greateſt and leaſt diſtances, and of the different planets in their ſeveral orbits, as they would appear viewed from the ſun; and here he fancied that he found a ſimilitude to the diviſions of the octave in muſic. Laſtly, he imagined that if lines were drawn from the earth, to each of the planets, and the planets appended to them, or ſtretched by weights proportional to the planets, theſe lines would then ſound all the notes in the octave of a muſical chord.

See his Harmonics; alſo Plin. lib. 2, cap. 22; Macrob. in Somn. Scip. lib. 2, cap. 1; Plutarch de Animal. Procreatione, è Timæo; and Maclaurin's View of Newton's Diſcov. book 1, chap. 2.

HARQUEBUSS, a hand-gun, or a fire-arm of a proper length and weight to be borne in the arm. Hanzelet preſcribes its proper length to be 40 calibres, or diameters of its bore; and the weight of its ball 1 oz. and $\frac{7}{8}$; its charge of powder as much.

HARRIOT (Thomas), a very eminent Engliſh mathematician and aſtronomer, was born at Oxford in 1560, and died at London July 2, 1621, in the 61ſt year of his age. Harriot has hitherto been known to the world only as an algebraiſt, though a very eminent one; but from his manuſcript papers, that have been but lately diſcovered by Dr. Zach, aſtronomer to the duke of Saxe-Gotha, it appears that he was not leſs eminent as an aſtronomer and geometrician. Dr. Zach has printed an account of thoſe papers, in the Aſtronomical Ephemeris of the Royal Academy of Sciences at Berlin, for the year 1788; of which, as it is very curious, and contains a great deal of information, I ſhall here give a tranſlation, to ſerve as memoirs concerning the life and writings of this eminent man; afterwards adding only ſome neceſſary remarks of my own.

"I here preſent to the world (ſays Dr. Zach), a ſhort account of ſome valuable and curious manuſcripts, which I found in the year 1784, at the ſeat of the earl of Egremont, at Petworth in Suſſex, in hopes that this learned and inquiſitive age will either think my endeavours about them worthy of its aſſiſtance, or elſe will be thereby induced to attempt ſome other means of publiſhing them. The only undeniable proof I can now produce of the uſefulneſs of ſuch an undertaking, is by giving a ſuccinct report of the contents of theſe materials, and briefly ſhewing what may be effected by them. And although I come to the performance of ſuch an enterprize with much leſs abilities than the different parts of it require, yet I truſt that my love for truth, my deſign and zeal to vindicate the honour due to an Engliſhman, the author of theſe manuſcripts, which are the chief reaſons that have influenced me in this undertaking, will ſerve as my excuſe.

"A predeceſſor of the family of lord Egremont, viz, that noble and generous earl of Northumberland, named Henry Percy, was not only a generous favourer of all good learning, but alſo a patron and Mæcenas of the learned men of his age. Thomas Harriot, the author of the ſaid manuſcripts, Robert Hues (well known by his Treatiſe upon the Globes), and Walter Warner, all three eminent mathematicians, who were known to the earl, received from him yearly penſions; ſo that when the earl was committed priſoner to the Tower of London in the year 1606, our author, with Hues and Warner, were his conſtant companions; and were uſually called the earl of Northumberland's three Magi.

"Thomas Harriot is a known and celebrated mathematician among the learned of all nations, by his excellent work, *Artis Analyticæ Praxis, ad æquationes algebraicas nova expedita & generali methodo, reſolvendas, Tractatus poſthumus*; Lond. 1631: dedicated to Henry earl of Northumberland; publiſhed after his death by Walter Warner. It is remarkable, that the fame and the honour of this truly great man were conſtantly attacked by the French mathematicians; who could not endure that Harriot ſhould in any way diminiſh the fame of their Vieta and Des Cartes, eſpecially the latter, who was openly accuſed of plagiariſm from our author. [*See Montucla's Hiſtoire des Mathematiques, part* 3, *p.* 485 *& ſeq.—Lettres de M. Des Cartes, tom.* 3, *pa.* 457, *edit. Paris* 1667, *in* 4*to.—Dictionnaire de Moreri, word* Harriot.—*Encyclopedie, word* Algebra.—*Lettres de M. de Voltaire, ſur la nation Angloiſe, lettre* 14.—*Memoire de l'Abbé de Gua dans les Mem. de l'Acad. des Sciences de Paris pour* 1741.—*Jer. Collier's great Hiſtorical Dictionary, word* Harriot.—*Dr. Wallis's preface to his Algebra.—To which may be added the article* Algebra, *in this dictionary.*]

"Des Cartes publiſhed his Geometry 6 years after Harriot's work appeared, viz, in the year 1637. Sir Charles Cavendiſh, then ambaſſador at the French court at Paris, when Des Cartes's Geometry made its firſt appearance in public, obſerved to the famous geometrician Roberval, that theſe improvements in Analyſis had been already made theſe 6 years in England, and ſhewed him afterwards Harriot's *Artis Analyticæ Praxis*, which as Roberval was looking over, at every page he cried out, *Oui! oui! il l'a vu! Yes! yes! he has ſeen it!* Des Cartes had alſo been in England before Harriot's death, and had heard of his new improvements and inventions in Analyſis. A critical life of this man, which his papers would enable me to publiſh, will ſhew more clearly what to think upon this matter, which I hope may be diſcuſſed to the due honour of our author.

"Now all this relates to Harriot the celebrated analyſt; but it has not hitherto been known that Harriot was an eminent aſtronomer, both theoretical and practical, which firſt appears by theſe manuſcripts; among which, the moſt remarkable are 199 obſervations of the Sun's Spots, with their drawings, calculations and determinations of the ſun's rotation about his axis. There is the greateſt probability that Harriot was the firſt diſcoverer of theſe ſpots, even before either Galileo or Scheiner. The earlieſt intelligence we have of the firſt diſcovered ſolar ſpots, is of one Joh. Fabricius Phryſius, who in the year 1611 publiſhed at Wittemberg a ſmall treatiſe, intitled, *De Maculis in Sole obſervatis & apparente eorum cum Sole converſione narratio.* Galileo, who is commonly accounted the firſt diſcoverer of the Solar Spots, publiſhed his book, *Iſtoria e Dimonſtrazioni intorne alle Machie Solare e loro accidenti*, at Rome, in the year 1613. His firſt obſervation in this work, is dated June 2, 1612. Angelo de Filiis, the editor of Galileo's work, who wrote the dedication and preface to it, mentions, pa. 3, that Galileo had not only diſcovered theſe ſpots in the month of April in the year 1611, at Rome, in the Quirinal Garden, but had ſhewn them ſeveral months before *(molti meſi innanzi)* to his friends in Florence. And that the obſervations of the diſguiſed Apelles (the Jeſuit Scheiner, a pretender

tender to this firſt diſcovery) were not later than the month of October in the ſame year; by which the epoch of this diſcovery was fixed to the beginning of the year 1611. But a paſſage in the firſt letter of Galileo's works, pa. 11, gives a more preciſe term to this diſcovery. Galileo there ſays in plain terms, that he had obſerved the Spots in the Sun 18 months before. The date of this letter is May 24, 1612; which brings the true epoch of this diſcovery to the month of November, 1610. However, Galileo's firſt produced obſervations are only from June 2, 1612, and thoſe of father Scheiner of the month of October, in the ſame year. But now it appears from Harriot's manuſcripts, that his firſt obſervations of theſe Spots are of Dec. 8, 1610. It is not likely that Harriot could have this notice from Galileo, for I do not find this mathematician's name ever quoted in Harriot's papers. But I find him mentioning Joſephus a Coſta's book 1, chap. 2, of his *Natural and Moral Hiſtory of the Weſt Indies*, in which he relates that in Peru there are Spots to be ſeen in the Sun which are not to be ſeen in Europe. It rather ſeems that Harriot had taken the hint from thence. Beſides, it is very likely that Harriot, who lived with ſo generous a patron to all good learning and improvements, had got the new invention of teleſcopes in Holland much ſooner in England, than they could reach Galileo, who at that time lived at Venice. Harriot's very careful and exact obſervations of theſe Spots, ſhew alſo that he was in poſſeſſion of the beſt and moſt improved teleſcopes of that time; for it appears he had ſome with magnifying powers of 10, 20, and 30 times. At leaſt there are no earlier obſervations of the Solar Spots extant than his: they run from December 8, 1610, till January 18, 1613. I compared the correſponding ones with theſe obſerved by Galileo, between which I found an exact agreement. Had Harriot had any notion about Galileo's diſcoveries, he certainly would have alſo known ſomething about the phaſes of Venus and Mercury, and eſpecially about the ſingular ſhape of Saturn, firſt diſcovered by Galileo; but I find not a word in all his papers concerning the particular figure of that planet.

“ *Of Jupiter's Satellites.* I found among his papers a large ſet of obſervations, with their drawing, poſition, and calculations of their revolutions and periods. His firſt obſervation of thoſe diſcovered Satellites, I find to be of January 16, 1610; and they go till February 26, 1612. Galileo pretends to have diſcovered them January 7, 1610; ſo that it is not improbable that Harriot was likewiſe the firſt diſcoverer of theſe attendants of Jupiter.

“ Among his other obſervations of the Moon, of ſome Eclipſes, of the planet Mars, of Solſtices, of Refraction, of the Declination of the Needle; there are moſt remarkable ones of the noted Comets of 1607, and of 1618, the latter, for there were two this year (*ſee Kepler de Cometis, pa.* 49). They were all obſerved with a croſs-ſtaff, by meaſuring their diſtances from fixed ſtars; whence theſe obſervations are the more valuable, as comets had before been but groſsly obſerved: Kepler himſelf obſerved the comet of 1607 only with the naked eye, pointing out its place by a coarſe eſtimation, without the aid of an inſtrument; and the elements of their orbits could, in defect of better obſervations, be only calculated by them. The obſervations of the comet of the year 1607, are of the more importance, even now for modern aſtronomy, as this is the ſame comet that fulfilled Dr. Halley's prediction of its return in the year 1759. That prediction was only grounded upon the elements afforded him by theſe coarſe obſervations; for which reaſon he only aſſigned the term of its return to the ſpace of a year. The very intricate calculations of the perturbations of this comet, afterwards made by M. Clairaut, reduced the limits to a month's ſpace. But a greater light may now be thrown upon this matter by the more accurate obſervations on this comet by Mr. Harriot. In the month of October 1785, when I converſed upon the ſubject of Harriot's papers, and eſpecially on this comet, with the very celebrated mathematician M. de la Grange, director of the Royal Academy of Sciences at Berlin; he then ſuggeſted to me an idea, which, if brought into execution, will clear up an important point in aſtronomy. It is well known to aſtronomers how difficult a matter it is, to determine the maſs, or quantity of matter, in the planet Saturn; and how little ſatisfactory the notions of it are, that have hitherto been formed. The whole theory of the perturbations of comets depending upon this uncertain datum, ſeveral attempts and trials have been made towards a more exact determination of it by the moſt eminent geometricians of this age, and particularly by la Grange himſelf; but never having been ſatisfied with the few and uncertain data heretofore obtained for the reſolution of this problem, he thought that Harriot's obſervations on the comet of 1607, and the modern ones of the ſame comet in 1759, would ſuggeſt a way of reſolving the problem *a poſteriori*; that of determining by them the elements of its ellipſis. The retardation of the comet compared to its period, may clearly be laid to the account of the attraction and perturbation it has ſuffered in the region of Jupiter and Saturn; and as the part of it belonging to Jupiter is very well known, the remainder muſt be the ſhare which is due to Saturn; from whence the maſs of the latter may be inferred. In conſequence of this conſideration I have already begun to reduce moſt of Harriot's obſervations of this comet, in order to calculate by them the true elements of its orbit on an elliptical hypotheſis, to complete M. la Grange's idea upon this matter.

“ I forbear to mention here any more of Harriot's analytical papers, which I found in a very great number. They contain ſeveral elegant ſolutions of quadratic, cubic, and biquadratic equations; with ſome other ſolutions and *loca geometrica*, that ſhew his eminent qualifications, and will ſerve to vindicate them againſt the attacks of ſeveral French writers, who refuſe him the juſtice due to his ſkill and accompliſhments, merely to ſave Des Cartes's honour, who yet, by ſome impartial men of his own nation, was accuſed of public plagiariſm.

“ Thomas Harriot was born at Oxford, in the year 1560. After being inſtructed in the rudiments of languages, he became a commoner of St. Mary's-Hall, where he took the degree of bachelor of arts in 1579. He had then ſo diſtinguiſhed himſelf by his uncommon ſkill in mathematics, as to be recommended ſoon after to Sir Walter Raleigh, as a proper preceptor to him in that ſcience. Accordingly that noble knight became his firſt patron, took him into his family, and allowed him a handſome penſion. In 1584 he went over with Sir Walter's firſt colony to Virginia; where he was employed in diſcovering and ſurveying the country, &c; maps of which I have found (ſays Dr. Zach) very neatly done among his papers. After his return he publiſhed *A Brief and True Report of the Newfoundland of Virginia, of the Commodities there found to be raiſed*, &c; Lond. 1588. This was reprinted in the 3d volume of Hakluyt's Voyages: it was alſo tranſlated into Latin, and printed at Frankfort in the year 1590. Sir Walter introduced him to the acquaintance of the earl of Northumberland, who allowed him a yearly penſion; Wood ſays, of 120l. only; but by ſome of his receipts, which I have found among his papers, it appears that he had 300l, which indeed was a very large ſum at that time. Wood, in his *Athen. Oxon.* mentions nothing of Harriot's papers, except a manuſcript in the library at Sion College, London, entitled *Ephemeris Chyrometrica.* I got acceſs to this library and manuſcripts, and was indeed in hopes of finding ſomething more of Harriot's; for moſt of his obſervations are dated from Sion College; but I could not find any thing from Harriot himſelf. I found indeed ſome other papers of his friends: he mentions, in his obſervations, one Mr. Standiſh, at Oxford, and Nicol. Torperly, who alſo was of the acquaintance of the earl of Northumberland, and had a yearly penſion: from the former I found two obſervations of the ſame comet of 1618, made in Oxford, which he communicated to Mr. Harriot.

"Thomas Harriot died July 2, 1621. His diſeaſe was a cancerous ulcer in the lip, which ſome pretend he got by a cuſtom he had of holding the mathematical braſs inſtruments, when working, in his mouth. I found ſeveral of his letters, and anſwers to them, from his phyſician Dr. Alexander Rhead, who in his treatiſe mentions Harriot's diſeaſe. His body was conveyed to St. Chriſtopher's church, in London. Over his grave was ſoon after erected a monument, with a large inſcription, which was deſtroyed with the church itſelf by the dreadful fire of September 1666. He was but 60 years of age."

The peculiar nature and merits of Harriot's Algebra, we have ſpoken largely and particularly of, under the art. *Algebra*, page 89. As to his manuſcripts lately diſcovered by Dr. Zach, as above mentioned, it is with pleaſure I can announce, that they are in a fair train to be publiſhed: they have been preſented to the univerſity of Oxford, on condition of their printing them; with a view to which, they have been lately put into the hands of an ingenious member of that learned body, to arrange and prepare them for the preſs.

HARRISON (John), a moſt accurate mechanic, the celebrated inventor of the famous *time-keeper* for aſcertaining the longitude at ſea, and alſo of the compound or *gridiron-pendulum*; was born at Foulby, near Pontefract in Yorkſhire, in 1693. His father was a carpenter, in which profeſſion the ſon aſſiſted; occaſionally alſo, according to the miſcellaneous practice of country artiſts, ſurveying land, and repairing clocks and watches; and young Harriſon always was, from his early childhood, greatly attached to any machinery moving by wheels. In 1700 he removed with his father to Barrow in Lincolnſhire; where, though his opportunities of acquiring knowledge were very few, he eagerly improved every incident from which he might collect information; frequently employing all or great part of his nights in writing or drawing: and he always acknowledged his obligations to a clergyman who came every Sunday to officiate in the neighbourhood, who lent him a MS. copy of profeſſor Sanderſon's lectures; which he carefully and neatly tranſcribed, with all the diagrams. His native genius exerted itſelf ſuperior to theſe ſolitary diſadvantages; for in the year 1726, he had conſtructed two clocks, moſtly of wood, in which he applied the eſcapement and compound pendulum of his own invention: theſe ſurpaſſed every thing then made, ſcarcely erring a ſecond in a month. In 1728, he came up to London with the drawings of a machine for determining the longitude at ſea, in expectation of being enabled to execute one by the Board of Longitude. Upon application to Dr. Halley, the aſtronomer royal, he referred him to Mr. George Graham; who adviſed him to make his machine, before applying to that Board. He accordingly returned home to perform his taſk; and in 1735 came to London again with his firſt machine; with which he was ſent to Liſbon the next year, to make trial of it. In this ſhort voyage, he corrected the dead reckoning about a degree and a half; a ſucceſs which procured him both public and private encouragement. About the year 1739 he completed his ſecond machine, of a conſtruction much more ſimple than the former, and which anſwered much better: this, though not ſent to ſea, recommended Mr. Harriſon yet ſtronger to the patronage of his friends and the public. His third machine, which he produced in 1749, was ſtill leſs complicated than the ſecond, and more accurate, as erring only 3 or 4 ſeconds in a week. This he conceived to be the *ne plus ultra* of his attempts; but by endeavouring to improve pocket-watches, he found the principles he applied to ſurpaſs his expectations, ſo much, as to encourage him to make his fourth time-keeper, which is in the form of a pocket-watch, about 6 inches diameter. With this time-keeper his ſon made two voyages, the one to Jamaica, and the other to Barbadoes; in which experiments it corrected the longitude within the neareſt limits required by the act of the 12th of queen Anne; and the inventor had therefore, at different times, more than the propoſed reward, receiving from the Board of Longitude at different times almoſt 24,000l. beſides a few hundreds from the Eaſt India Company, &c. Theſe four machines were given up to the Board of Longitude. The three former were not of any uſe, as all the advantages gained by making them, were comprehended in the laſt: being worthy however of preſervation, as mechanical curioſities, they are depoſited in the Royal Obſervatory at Greenwich. The fourth machine, emphatically diſtinguiſhed by the name of *The time-keeper*, was copied by the ingenious Mr. Kendal; and that duplicate, during a three years circumnavigation of the globe in the ſouthern hemiſphere by captain Cook, anſwered as well as the original.

The latter part of Mr. Harriſon's life was employed in making a fifth improved time-keeper, on the ſame principles with the preceding one; which, after a ten-weeks trial, in 1772, at the king's private obſervatory at Richmond, erred only $4\frac{1}{2}$ ſeconds. Within a few years of his death, his conſtitution viſibly declined; and he had frequent fits of the gout, a diſorder that never attacked him before his 77th year. His conſtitution at laſt yielding to the infirmities of old age, he died at his houſe in Red-Lion Square, in 1776, at 83 years of age.

Like many other mere mechanics, Mr. Harriſon found a difficulty in delivering his ſentiments in writing (at leaſt in the latter periods of his life, when his faculties were much impaired) in which he adhered to a peculiar and uncouth phraſeology. This was but too evident in his *Deſcription concerning ſuch Mechaniſm as will afford a nice or true Menſuration of Time*, &c. 8vo, 1775. This ſmall work includes alſo an account of his new muſical ſcale; being a mechanical diviſion of the octave, according to the proportion which the radius and diameter of the circle have reſpectively to the circumference. He had in his youth been the leader of a band of church-ſingers; had a very delicate ear for muſic; and his experiments on ſound, with a curious monochord of his own improvement, it has been ſaid were not leſs accurate than thoſe he was engaged in for the menſuration of time.

HAUTEFEUILLE (John), an ingenious mechanic, born at Orleans in 1674. He made a great progreſs in mechanics in general, but had a particular taſte for clock-work, and made ſeveral diſcoveries in it that were of ſingular uſe. It was he it ſeems who found out the ſecret of moderating the vibration of the balance by means of a ſmall ſteel-ſpring, which has ſince been made uſe of. This diſcovery he laid before the members of the Academy of Sciences in 1694; and theſe watches are, by way of eminence, called *pendulum-watches;* not that they have real pendulums, but becauſe they nearly approach to the juſtneſs of pendulums. M. Huygens perfected this happy invention; but having declared himſelf the inventor and obtained a patent for making watches with ſpiral ſprings, the abbé Feuille oppoſed the regiſtering of it, and publiſhed a piece on the ſubject againſt Huygens. He died in 1724, at 50 years of age. Beſides the above,

He wrote a great many other pieces, moſt of which are ſmall pamphlets, but very curious: as, 1. His Perpetual Pendulum. 2. New Inventions. 3. The Art of Breathing under Water, and the means of preſerving a Flame ſhut up in a Small Place. 4. Reflections on machines for Raiſing Water. 5. His Opinion on the different Sentiments of Mallebranche and Regis, relating to the Appearance of the Moon when ſeen in the Horizon. 6. The Magnetic Balance. 7. A Placet to the king on the Longitude. 8. Letter on the Secret of the Longitude. 9. A New Syſtem on the Flux and Reflux of the Sea. 10. The means of making Senſible Experiments that prove the Motion of the Earth: and many other pieces.

HAYES (Charles, Eſq.), a very ſingular perſon, whoſe great erudition was ſo concealed by his modeſty, that his name is known to very few, though his publications are many. He was born in 1678, and died in 1760, at 82 years of age. He became diſtinguiſhed in 1704 by a Treatiſe of Fluxions, in folio, being we believe the firſt treatiſe on that ſcience ever publiſhed in the Engliſh language; and the only work to which he ever ſet his name. In 1710 came out a ſmall 4to pamphlet, in 19 pages, intitled, A New and eaſy Method to find out the Longitude from obſerving the Altitudes of the Celeſtial bodies. Alſo, in 1723, he publiſhed, The Moon, a Philoſophical Dialogue; tending to ſhew, that the moon is not an opaque body, but has native light of her own.

To a ſkill in the Greek and Latin, as well as the modern languages, he added the knowledge of the Hebrew: and he publiſhed ſeveral pieces relating to the tranſlation and chronology of the ſcriptures. During a long courſe of years he had the chief management of the late African company, being annually elected ſub-governor. But on the diſſolution of that company in 1752, he retired to Down in Kent, where he gave himſelf up to ſtudy; from whence however he returned in 1758, to chambers in Gray's inn, London, where he died in 1760, as mentioned above.

He left a poſthumous work, that was publiſhed in 8vo, under the title of *Chronographia Aſiatica et Ægyptiaca &c.*

HEAT, the oppoſite to cold, being a relative term denoting the property of fire, or of thoſe bodies we denominate hot; being in us a ſenſation excited by the action of fire.

Heat, as it exiſts in the hot body, or that which conſtitutes and denominates a body hot, and enables it to produce ſuch effects on our organs, is variouſly conſidered by the philoſophers: ſome making it a quality, others a ſubſtance, and others only a mechanical affection. The former principle is laid down by Ariſtotle and the Peripatetics. While the Epicureans, and other corpuſcularians, define Heat not as an accident of fire, but as an eſſential power or property of it, the ſame in reality with it, and only diſtinguiſhed from it in the manner of our conception. So that Heat, on their principles, is no other than the volatile ſubſtance of fire itſelf, reduced into atoms, and emitted in a continual ſtream from ignited bodies; ſo as not only to warm the objects within its reach, but alſo, if they be inflammable, to kindle them, turn them into fire, and conſpire with them to make flame. In effect, theſe corpuſcles, ſay they, flying off from the ignited body, conſtitute fire while yet contained within the ſphere of its flame; but when fled, or got beyond the ſame, and diſperſed every way, ſo as to eſcape the apprehenſion of the eye, and only to be perceived by the feeling, they take the denomination of Heat, inaſmuch as they excite in us that ſenſation.

The Carteſians, improving on this doctrine, aſſert that Heat conſiſts in a certain motion of the inſenſible particles of a body, reſembling the motion by which the ſeveral parts of our body are agitated by the motion of the heart and blood.

Our lateſt and beſt writers of mechanical, experimental, and chemical philoſophy, differ very conſiderably about Heat. The chief difference is, whether it be a peculiar property of one certain immutable body, called fire, or phlogiſton, or electricity; or whether it may be produced mechanically in other bodies, by inducing an alteration in their particles. The former tenet, which is as ancient as Democritus, and the ſyſtem of atoms, had given way to that of the Carteſians, and other mechaniſts; but is now with great addreſs retrieved, and improved on, by ſome of the lateſt writers, particularly Homberg, the younger Lemery, Graveſande, Boerhaave in his lectures on fire, Black, Crawford, and other chemical philoſophers.

The thing called fire, according to Boerhaave, is a body ſui generis, created ſuch ab origine, unalterable in its nature and properties, and not either producible de novo from any other body, nor capable of being reduced into any other body, or of ceaſing to be fire. This fire, he contends, is diffuſed equably every where, and exiſts alike, or in equal quantity, in all the parts of ſpace, whether void, or poſſeſſed by bodies; but that naturally, and in itſelf, it is perfectly latent and imperceptible; being only diſcovered by certain effects which it produces, and which are cognizable by our ſenſes. Theſe effects are Heat, light, colour, rarefaction, and burning, which are all indications of fire, as being none of them producible by any other cauſe: ſo that whereever we obſerve any of theſe, we may ſafely infer the action and preſence of fire. But though the effect cannot be without the cauſe, yet the fire may remain without any of theſe effects; any, we mean, groſs enough to affect our ſenſes, or become objects of them: and this, he adds, is the ordinary caſe; there being a concurrence of other circumſtances, which are often wanting, neceſſary to the production of ſuch ſenſible effects.

The mechanical philoſophers, particularly Bacon, Boyle, and Newton, conceive otherwiſe of Heat; conſidering it not as an original inherent property of any particular ſort of body; but as mechanically producible in any body. The former, in an expreſs treatiſe De Forma Calidi, from a particular enumeration of the ſeveral phenomena and effects of Heat, deduces ſeveral general properties of it; and hence he defines Heat, an expanſive undulatory motion in the minute particles of the body; by which they tend, with ſome rapidity, towards the circumference, and at the ſame time incline a little upwards.

Mr. Boyle, in a Treatiſe on the Mechanical Origin of Heat and Cold, ſtrongly ſupports the doctrine of the producibility of Heat, with new obſervations and experiments; as in the inſtance of a ſmith briſkly hammering a ſmall piece of iron, which, though cold before, ſoon becomes exceedingly hot.

This ſyſtem is alſo farther ſupported by Newton, who does not conceive fire as any particular ſpecies of body, originally endued with ſuch and ſuch properties. Fire, according to him, is only a body much ignited, that is heated hot, ſo as to emit light copiouſly: what elſe, ſays he, is red-hot iron but fire? and what elſe is a burning charcoal but red-hot wood? or flame itſelf, but red-hot ſmoke? It is certain that flame is only the volatile part of the fuel heated red-hot, i. e. ſo hot as to ſhine; and hence only ſuch bodies as are volatile, that is, ſuch as emit a copious fume, will flame; nor will they flame longer than they have fume to burn. In diſtilling hot ſpirits, if the head of the ſtill be taken off, the aſcending vapours will catch fire from a candle, and turn into a flame. And in the ſame manner ſeveral bodies, much heated by motion, attrition, fermentation, or the like, will emit lucid fumes, which, if they be copious enough, and the heat ſufficiently great, will be flame; and the reaſon why fuſed metals do not flame, is the ſmallneſs of their fume; this is evident, becauſe ſpilter, which fumes moſt copiouſly, does likewiſe flame. Add, that all flaming bodies, as oil, tallow, wax, wood, pitch, ſulphur, &c, by flaming, waſte and vaniſh into burning ſmoke. And do not all fixed bodies, when heated beyond a certain degree, emit light, and ſhine? and is not this emiſſion performed by the vibrating motion of their parts? and do not all bodies, which abound with terreſtrial and ſulphureous parts, emit light as often as thoſe parts are ſufficiently agitated, whether that agitation be made by external fire, or by friction, or percuſſion, or putrefaction, or by any other cauſe? Thus, ſea water, in a ſtorm; quickſilver agitated in vacuo; the back of a cat, or the neck of a horſe, obliquely rubbed in a dark place; wood, fleſh, and fiſh, while they putrefy; vapours from putrefying waters, uſually called ignes fatui; ſtacks of moiſt hay or corn; glow-worms; amber and diamonds by rubbing; fragments of ſteel ſtruck off with a flint, &c, all emit light. Are not groſs bodies and light convertible into one another? and may not bodies receive much of their activity from the particles of light which enter their compoſition? I know no body leſs apt to ſhine than water; and yet water, by frequent diſtillations, changes into fixed earth, which, by a ſufficient Heat, may be brought to ſhine like other bodies.

Add, that the ſun and ſtars, according to Newton's conjecture, are no other than great earths vehemently heated: for large bodies, he obſerves, preſerve their Heat the longeſt, their parts heating one another; and why may not great, denſe, and fixed bodies, when heated beyond a certain degree, emit light ſo copiouſly, as by the emiſſion and reaction of it, and the reflections and refractions of the rays within the pores, to grow ſtill hotter, till they arrive at ſuch a period of Heat as is that of the ſun? Their parts alſo may be farther preſerved from fuming away, not only by their fixity, but by the vaſt weight and denſity of their atmoſpheres incumbent on them, thus ſtrongly compreſſing them, and condenſing the vapours and exhalations ariſing from them. Hence we ſee warm water, in an exhauſted receiver, ſhall boil as vehemently as the hotteſt water open to the air; the weight of the incumbent atmoſphere, in this latter caſe, keeping down the vapours, and hindering the ebullition, till it has conceived its utmoſt degree of Heat. So alſo a mixture of tin and lead, put on a red-hot iron in vacuo, emits a fume and flame; but the ſame mixture in the open air, by reaſon of the incumbent atmoſphere, does not emit the leaſt ſenſible flame.

Thus much for the ſyſtem of the producibility of Heat.

On the other hand, M. Homberg, in his Eſſai du Soufre Principe, holds, that the chemical principle or element, ſulphur, which is ſuppoſed one of the ſimple, primary, pre-exiſtent ingredients of all natural bodies, is real fire; and conſequently that fire is co-eval with body. Mem. de l'Acad. an. 1705.

Dr. Graveſande goes upon much the ſame principle. According to him, fire enters the compoſition of all bodies, is contained in all bodies, and may be ſeparated or procured from all bodies, by rubbing them againſt each other, and thus putting their fire in motion. But fire, he adds, is by no means generated by ſuch motion. Elem. Phyſ. tom. 2, cap. 1. Heat, in the hot body, he ſays, is an agitation of the parts of the body, made by means of the fire contained in it; by ſuch agitation a motion is produced in our bodies, which excites the idea of Heat in our minds: ſo that Heat, in reſpect of us, is nothing but that idea, and in the hot body nothing but motion. If ſuch motion expel the fire in right lines, it may give us the idea of light; if in a various and irregular motion, only of Heat.

Lemery, the younger, agrees with theſe two authors, in aſſerting this abſolute and ingenerable nature of fire; but he extends it farther. Not contented with confining it as an element to bodies, he endeavours to ſhew, that it is equably diffuſed through all ſpace; that it is preſent in all places, even in the void ſpaces between the bodies, as well as in the inſenſible interſtices between their parts. And this laſt ſentiment falls in with that of Boerhaave above delivered. Mem. de l'Acad. an. 1713.

Philoſophers have lately diſtinguiſhed Heat into Abſolute, and Senſible. By Abſolute Heat, or fire, they mean that power or element which, when it is in a certain degree, excites in animals the ſenſation of Heat; and by Senſible Heat, the ſame power conſidered in its relation to the effects which it produces: thus, two bodies are ſaid to have equal quantities of ſenſible Heat, when they produce equal effects upon the mercury in the thermometer; but as bodies of different kinds have different capacities for containing Heat, the abſolute Heat in ſuch bodies will be different, though the ſenſible Heat be the ſame. Thus, a pound of water and a pound of antimony, of the ſame temperature, have equal ſenſible Heat; but the former contains a much greater quantity of abſolute Heat than the latter.

M. De Luc has evinced, by a variety of experiments, that the expanſions of mercury between the freezing and boiling points of water, correſpond preciſely to the quantities of abſolute Heat applied, and that its contractions are proportionable to the diminution of this element within theſe limits. And from hence it may be inferred, that if the mercury were to retain its fluid form, its contractions would be proportionable to the decrements of the abſolute Heat, though the diminution were continued to the point of total privation. But the comparative quantities of abſolute Heat, which are communicated to different bodies, or ſeparated from them, cannot be determined in a direct manner by the thermometer.

Some philoſophers have apprehended that the quantities of abſolute Heat in bodies, are in proportion to their denſities. While others, as Boerhaave, imagined that Heat is equally diffuſed through all bodies, the denſeſt as well as the rareſt, and therefore that the quantities of Heat in bodies are in proportion to their bulk or magnitude: and, at his deſire, Fahrenheit attempted to determine the fact by experiment. For this purpoſe, he took equal quantities of the ſame fluid, and gave them different degrees of Heat, then upon mixing them intimately together, he found that the temperature of the mixture was a juſt medium, or arithmetical mean, between the two. But if this experiment be made with water and mercury, in the ſame circumſtances, viz in equal bulks, the reſult will be different, as the temperature of the mixture will not be a mean between the two, but always nearer to that of the water than to the quickſilver; ſo that, when the water is the hotter, the temperature of the mixture is above the mean, and below it when the water is the colder. And from experiments of this kind it has been inferred, that the comparative quantities of the abſolute Heats of theſe fluids, are reciprocally proportional to the changes which are produced in their ſenſible Heats, when they are mixed together at different temperatures: and this fact has been publicly taught, for ſeveral years, by Dr. Black, and Dr. Irvine, in the univerſities of Edinburgh and Glaſgow. This rule however does not apply to thoſe ſubſtances which, in mixture, excite ſenſible Heat by chemical action.

From the experiments and reaſoning employed by Dr. Crawford, it more fully appears, that the quantities of abſolute Heat in different bodies, are not as their denſities; or that equal weights of heterogeneous ſubſtances, as air and water, having the ſame temperature, may contain unequal quantities of abſolute Heat: he alſo ſhews, that if phlogiſton be added to a body, a quantity of the abſolute Heat of that body will be extricated; and if the phlogiſton be ſeparated again, an equal quantity of Heat will be abſorbed. So that Heat and phlogiſton appear to be two oppoſite principles in nature. But this ingenious writer has not preſumed abſolutely to decide the queſtion that has been long agitated, whether Heat be a ſubſtance or a quality.— He inclines to the former opinion however, and obſerves, that if we adopt the opinion, that Heat is a diſtinct ſubſtance, or an element ſui generis, the phenomena will be found to admit of a ſimple and obvious interpretation, and to be perfectly agreeable to the analogy of nature. See Crawford's Experiments and Obſervations on Animal Heat and the Inflammation of Combuſtible Bodies.

Animal HEAT. The Heat of animals is very various, both according to the variety of their kinds, and the difference of the ſeaſons: accordingly, zoologiſts have divided them into hot and cold blooded, reckoning thoſe to be hot that are near or above our own temperature, and all others cold whoſe Heat is below ours, and conſequently affect us with the ſenſe of cold; thus making the human ſpecies a medium between the hot and cold blooded animals, or at leaſt the loweſt order of the hot blooded.

The Heat of the human body, in its natural ſtate, according to Dr. Boerhaave, is ſuch as to raiſe the mercury in the thermometer to 92° or at moſt to 94°; and Dr. Pitcairn makes the heat of the human ſkin the ſame. Indeed it is evident that different parts of the human body, and its different ſtates, as well as the different ſeaſons, will make it ſhew of different temperatures. Thus, by various experiments at different times, the Heat of the human body is made various by the following authors:

Boerhaave and Pitcairn	92°
Amontons	91, 92, or 93
Sir Iſaac Newton	95½
Fahrenheit and Muſſchenbroek, the blood,	96
Dr. Martine, the ſkin	97 or 98
————, the urine	99
Dr. Hales, the ſkin	97
————, the urine	103
Mr. John Hunter, under his tongue,	97
————, in his rectum	98½
————, his urethra at 1 inch,	92
at 2 inches,	93
at 4 inches,	94
the ball of the thermom, at the bulb of the urethra	97

For the powers of animals to bear various degrees of Heat, ſee the Philoſ. Tranſ. vol. 65, 68, &c.

There is hardly any ſubject of philoſophical inveſtigation that has afforded a greater variety of hypotheſes, conjectures, and experiments, than the cauſe of animal Heat. The firſt opinion which has very generally obtained, is, that the Heat of animal bodies is owing to the attrition between the arteries and the blood. All the obſervations and reaſoning brought in favour of this opinion however, only ſhew that the Heat and the motion of the arteries are generally proportional to each other; without ſhewing which is the cauſe, and which the effect; or indeed that either is the cauſe or effect of the other, ſince both may be the effects of ſome other cauſe.

Dr. Douglas, in his Eſſay on the Generation of Heat in animals, aſcribes it ſolely to the friction of the globules of blood in their circulation through the capillary veſſels.

Another opinion is, that the lungs are the fountain of Heat in the human body: and this opinion is ſupported by much the ſame ſort of arguments as the former, and ſeemingly to little better purpoſe.

A third opinion is, that the cauſe of animal Heat is owing to the action of the ſolid parts upon one another. And as the heart and arteries move moſt, it has been thought natural to expect that the Heat ſhould be owing to this motion. But even this does not ſeem very plauſible, from the following conſiderations: 1ſt, The moving parts, however we term them ſolid, are neither hard nor dry; which two conditions are abſolutely requiſite to make them fit to generate Heat by attrition. 2d, None of their motions are ſwift enough to promiſe Heat in this way. 3d, They have but little change of ſurface in their attritions. And 4thly, The moveable fibres have fat, mucilage, or liquors everyway ſurrounding them, to prevent their being deſtroyed, or heated by attrition.

A fourth cauſe aſſigned for the Heat of our bodies, is that proceſs by which our aliment and fluids are perpetually undergoing ſome alteration. And this opinion is chiefly ſupported by Dr. Stevenſon, in the Edinburgh Medical Eſſays, vol. 5, art. 77.

The late ingenious Dr. Franklin inclines to this opinion, when he ſays, that the fluid fire, as well as the fluid air, is attracted by plants in their growth, and becomes conſolidated with the other materials of which they are formed, and makes a great part of their ſubſtance; that when they come to be digeſted, and to undergo a kind of fermentation in the veſſels, part of the fire, as well as part of the air, recovers its fluid active ſtate again, and diffuſes itſelf on the body digeſting and ſeparating it; &c. Exper. and Obſ. on Electricity, p. 346.

Dr. Mortimer thinks the Heat of animals explicable from the phoſphorus and air they contain. Phoſphorus exiſts, at leaſt in a dormant ſtate, in animal fluids; and it is alſo known that they all contain air; it is therefore only neceſſary to bring the phoſphoreal and aereal particles into contact, and Heat muſt of conſequence be generated; and were it not for the quantity of aqueous humours in animals, fatal accenſions would frequently happen. Philoſ. Tranſ. number 476.

Dr. Black ſuppoſes, that animal Heat is generated altogether in the lungs, by the action of the air on the principle of inflammability, and is thence diffuſed over the reſt of the body by means of the circulation. But Dr. Leſlie urges ſeveral arguments againſt this hypotheſis, tending to ſhew that it is repugnant to the known laws of the animal machine; and he advances another hypotheſis inſtead of it, viz. that the ſubtle principle by chemiſts termed phlogiſton, which enters into the compoſition of natural bodies, is in conſequence of the action of the vaſcular ſyſtem gradually evolved through every part of the animal machine, and that during this evolution Heat is generated. This opinion, he candidly acknowledges, was firſt delivered by Dr. Duncan of Edinburgh; and that ſomething ſimilar to it is to be found in Dr. Franklin's works, and in a paper of Dr. Mortimer's in the Philoſ. Tranſ.

The laſt hypotheſis we ſhall mention, is the very plauſible one of Dr. Crawford, lately publiſhed in his Experiments and Obſervations on Animal Heat. This ingenious gentleman has inferred, from a variety of experiments, that Heat and phlogiſton, ſo far from being connected, as moſt philoſophers have imagined, act in ſome meaſure in oppoſition to each other. By the action of Heat on bodies, the force of their attraction of phlogiſton is diminiſhed, and by the action of phlogiſton, a part of their abſolute Heat is expelled. He has alſo demonſtrated, that atmoſpherical air contains a greater quantity of abſolute Heat than the air which is expired from the lungs of animals: he makes the proportion of the abſolute Heat of atmoſpherical air to that of fixed air, as 67 to 1; and the Heat of dephlogiſticated air to that of atmoſpherical air as 4·6 to 1; and obſerving that Dr. Prieſtley has proved, that the power of this dephlogiſticated air in ſupporting animal life, is 5 times as great as that of atmoſpherical air, he concludes that the quantity of abſolute Heat contained in any kind of air fit for reſpiration, is very nearly in proportion to its purity, or to its power of ſupporting animal life; and ſince the air exhaled by reſpiration, is found to contain only the 67th part of the Heat which was contained in the atmoſpherical air, previous to inſpiration, it is very reaſonably inferred, that the latter muſt neceſſarily depoſit a very great proportion of its abſolute Heat in the lungs. Dr. Crawford has alſo ſhewn, that the blood which paſſes from the lungs to the heart, by the pulmonary vein, contains more abſolute Heat than that which paſſes from the heart to the lungs, by the pulmonary artery; the abſolute Heat of florid arterial blood being to that of venous blood, as 11½ to 10: therefore, ſince the blood which is returned by the pulmonary vein to the heart has the quantity of its abſolute Heat increaſed, it muſt have acquired this Heat in its paſſage through the lungs; ſo that in the proceſs of reſpiration a quantity of abſolute Heat is ſeparated from the air, and abſorbed by the blood. Dr. Prieſtley has alſo proved, that in reſpiration, phlogiſton is ſeparated from the blood, and combined with air.

This theory however has been conteſted and diſputed, and, it has been ſaid, Dr. Crawford's experiments repeated, with contrary reſults. Though no regular and ſyſtematical theory has yet been formed in its ſtead.

HEAT *of Combuſtible and Inflammable Bodies.* Dr. Crawford's theory with reſpect to the inflammation of combuſtible bodies, is founded on the ſame principles as his doctrine concerning the Heat of animals. According to him, the Heat which is produced by combuſtion, is derived from the air, and not from the inflammable body. Inflammable bodies, he ſays, abound with phlogiſton, and contain little abſolute Heat: the atmoſphere, on the contrary, abounds with abſolute Heat, and contains little phlogiſton. In the proceſs of inflammation, the phlogiſton is ſeparated from the inflammable body, and combined with the air; the air is phlogiſticated, and gives off a great proportion of its abſolute Heat, which, when extricated ſuddenly, burſts forth into flame, and produces an intenſe degree of ſenſible Heat. And ſince it appears by calculation, that the Heat produced by converting atmoſpherical into fixed air, is ſuch, if it were not diſſipated, as would be ſufficient to raiſe the air ſo changed, to more than 12 times the Heat of red-hot iron, it follows, that in the proceſs of inflammation a very great quantity of Heat is derived from the air. But, on the contrary, no part of the Heat can be derived from the combuſtible body; becauſe this body, during the inflammation, being deprived of its phlogiſton, undergoes a change ſimilar to that of the blood by the proceſs of reſpiration, in conſequence of which its capacity of containing Heat is increaſed; and therefore it will not give off any part of its abſolute Heat, but, like the blood in its paſſage through the lungs, it will abſorb Heat.

A ſimilar theory of Heat has lately been publiſhed by Mr. Elliot. See his Philoſophical Obſervations on the ſenſes of Viſion and Hearing; to which is added an Eſſay on Combuſtion and Animal Heat. 8vo, 1780.

Heat, *in Geography*, is that which relates to the earth. There is a great variety in the Heat of different places and ſeaſons. Naturaliſts have commonly laid it down, that the nearer any place is to the centre of the earth, the hotter it is found; but this does not hold ſtrictly true. And if it were, the effect might be otherwiſe accounted for, and more ſatisfactorily, than from their imagined central fire.

Mr. Boyle, who had been at the bottom of ſome mines himſelf, with more probability ſuſpects that this degree of Heat, at leaſt in ſome of them, may ariſe from the peculiar nature of the minerals there produced. And he inſtances a mineral of the vitriolic kind, dug up in large quantities, in ſeveral parts of England, which, by the bare effuſion of common water, will grow ſo hot as almoſt to take fire. To which may be added, that ſuch places, in the bowels of the earth, uſually feel hot, from the confined and ſtagnant ſtate of the air in them, in which the heat is retained, through the want of a current or change of air to carry the Heat off.

On the other hand, on aſcending high mountains, the air grows more and more cold and piercing. Thus, the tops of the Pike of Teneriffe, the Alps, and ſeveral other mountains, even in the moſt ſultry countries, are found always inveſted with ſnow and ice, which the Heat is never ſufficient to thaw. In ſome of the mountains of Peru there is no ſuch thing as running water, but all ice: plants vegetate a little about the bottom of the mountains, but near the top no vegetable can live, for the intenſeneſs of the cold. This effect is attributed to the thinneſs of the air, and the little ſurface of the earth there is to reflect the rays, as well as the great diſtance of the general ſurface of the earth which reflects the rays back into the atmoſphere.

As to the diverſity in the Heat of different climes and ſeaſons, it ariſes from the different angles under which the ſun's rays ſtrike upon the ſurface of the earth. In the Philoſ. Tranſ. Abr. vol. 2, p. 165, Dr. Halley has given a computation of this Heat, on the principle, that the ſimple action of the ſun's rays, like other impulſes or ſtrokes, is more or leſs forcible, according to the ſines of the angles of incidence, or to the ſines of the ſun's altitudes, at different times or places.

Hence it follows, that, the time of continuance, or the ſun's ſhining on any place, being taken for a baſis, and the ſines of the ſun's altitudes perpendicularly erected upon it, and a curve line drawn through the extremities of thoſe perpendiculars, the area thus comprehended will be proportional to the collection of all the Heat of the ſun's beams in that ſpace of time.

Hence it will likewiſe follow, that at the pole, the collection of all the Heat of a tropical day, is proportional to the rectangle or product of the ſine of 23½ degrees into 24 hours, or the circumference of a circle, or as $\frac{8}{10}$ into 12 hours, the ſine of 23½ degrees being nearly $\frac{4}{10}$ of radius. Or the polar Heat will be equal to that of the ſun continuing 12 hours above the horizon at 53 degrees height; and the ſun is not 5 hours more elevated than this under the equinoctial.

But as it is the nature of Heat to remain in the ſubject, after the luminary is removed, and particularly in the air, under the equinoctial the 12 hours abſence of the ſun abates but little from the effect of his Heat in the day; but under the pole, the long abſence of the ſun for 6 months has ſo chilled the air, that it is in a manner frozen, and after the ſun has riſen upon the pole again, it is long before his beams can make any impreſſion, being obſtructed by thick clouds and fogs.

From the foregoing principle Dr. Halley computes the following table, exhibiting the Heat to every 10th degree of latitude, for the equinoctial and tropical ſun, and from which an eſtimate may eaſily be made for the intermediate degrees.

Lat.	Sign that the Sun is in.		
	♈ or ♎	♋	♑
0	20000	18341	18341
10	19696	20290	15834
20	18794	21737	13166
30	17321	22651	10124
40	15321	23048	6944
50	12855	22991	3798
60	10000	22773	1075
70	6840	23543	0
80	3473	24673	0
90	0	25055	0

From the ſame principles, and table, alſo are deduced the following corollaries, viz, 1, That the equatorial Heat, when the ſun becomes vertical, is as twice the ſquare of the radius.—2, That at the equator the Heat is as the ſine of the ſun's declination.—3, That in the frigid zones, when the ſun ſets not, the Heat is as the circumference of a circle into the ſine of the altitude at 6: And conſequently that in the ſame latitude, theſe aggregates of Heat are as the ſines of the ſun's declination; and at the ſame declination of the ſun, they are as the ſines of the latitudes; and generally they are as the ſines of the latitudes into the ſines of declination.—4, That the equatorial day's Heat is everywhere as the coſine of the latitude.—5, In all places where the ſun ſets, the difference between the ſummer and winter Heats, when the declinations are contrary, is equal to a circle into the ſine of the altitude at 6, in the ſummer parallel; and conſequently thoſe differences are as the rectangles of the ſines of the latitude and declination.—6, The tropical ſun has the leaſt force of any at the equator; and at the pole it is greateſt of all.

Many objections have been urged againſt this theory of Dr. Halley. Some have objected, that the effect of the ſun's Heat is not in the ſimple, but in the duplicate ratio of the ſines of the angles of incidence; like the law of the impulſe of fluids. And indeed, the quantity of the ſun's direct rays received at any place, being evidently as the ſine of the angle of incidence, or of the ſun's altitude, *if* the Heat be alſo proportional to the force with which a ray ſtrikes, like the mechanical action or impulſe of any body, then it will follow that the Heat muſt be in the compound ratio of both, that is, as the ſquare of the ſine of the ſun's altitude. But this laſt principle is here only aſſumed gratis, as we do not know a priori that the Heat is proportional to the force of a ſtriking body; and it is only experiment that can determine this point.

It is certain that Heat communicated by the ſun to bodies on the earth, depends alſo much upon other circumſtances beſide the direct force of his rays. Theſe muſt be modified by our atmoſphere, and variouſly reflected and combined by the action of the earth's ſurface itſelf, to produce any remarkable effects of Heat. So that if it were not for theſe additional circumſtances, it is probable the naked Heat of the ſun would not be very ſenſible.

Dr. Halley himſelf was well appriſed, that many other circumſtances, beſides the direct force of the ſun's rays, contributed to augment or diminiſh the effect of this, and the Heat reſulting from it, in different climates; and therefore no calculation, formed on the preceding theory, can be ſuppoſed to correſpond exactly with obſervation and experiment. It has alſo been objected that, according to the foregoing theory, the greateſt Heat in the ſame place ſhould be at the ſummer ſolſtice, and the moſt extreme cold at the winter ſolſtice; which is contrary to experience. To this objection it may be replied, that Heat is not produced in bodies by the ſun inſtantaneouſly, nor do the effects of his Heat ceaſe immediately when his rays are withdrawn; and therefore thoſe parts which are once heated, retain the Heat for ſome time; which, with the additional Heat daily imparted, makes it continue to increaſe, though the ſun declines from us: and this is the reaſon why July is hotter than June, although the ſun has withdrawn from the ſummer tropic: as we alſo find it is generally hotter at one, two, or three in the afternoon, when the ſun has declined towards the weſt, than at noon, when he is on the meridian. As long as the heating particles, which are conſtantly received, are more numerous than thoſe which fly away or loſe their force, the Heat of bodies muſt continually increaſe. So, after the ſun has left the tropic, the number of particles, which Heat our atmoſphere and earth, conſtantly increaſes, becauſe we receive more in the day than we loſe at night, and therefore our Heat muſt alſo increaſe. But as the days decreaſe again, and the action of the ſun becomes weaker, more particles will fly off in the night time than are received in the day, by which means the earth and air will gradually cool. Farther, thoſe places which are well cooled, require time to be heated again; and therefore January is moſtly colder than December, although the ſun has withdrawn from the winter tropic, and begun to emit his rays more perpendicularly upon us.

But the chief cauſe of the difference between the Heat of ſummer and winter is, that in ſummer the rays fall more perpendicularly, and paſs through a leſs denſe part of the atmoſphere; and therefore with greater force, or at leaſt in greater number in the ſame place: and beſides, by their long continuance, a much greater degree of Heat is imparted by day than can fly off by night.

For the calculations and opinions of ſeveral other philoſophers on this head, ſee Keill's Aſtron. Lect. 8; Ferguſon's Aſtron. chap. 10; Long's Aſtron. § 777; Memo. Acad. Scienc. 1719.

As to the temperature or Heat of our atmoſphere, it may be obſerved that the mercury ſeldom falls under 16° in Fahrenheit's thermometer; but we are apt to reckon it very cold at 24°, and it continues coldiſh to 40° and a little above. However, ſuch colds have been often known as bring it down to 0°, the beginning of the ſcale, or nearly the cold produced by a mixture of ſnow and ſalt, often near it, and in ſome places below it. Thus, the degree of the thermometer has been obſerved at various times and places as follows:

Places	Latit.	Year	Thermom.
Penſylvania	40° 0′	1732	5°
Paris	48 50	1709 & 1710	8
Leyden	52 10	1729	5
Utrecht	52 8	——	4
London	51 31	1709 & 1710	0
Copenhagen	55 43	1709	0
Upſal	59 56	1732	−1
Peterſburg	59 56	——	−28
Torneo	65 51	1736-7	−33
Hudſon's Bay	52 24	1775	−37

The middle temperature of our atmoſphere is about 48°, being nearly a medium of all the ſeaſons. The French make it ſomewhat higher, reckoning it equal to the cave of their royal obſervatory, or 53°. In cold countries, the air is found agreeable enough to the inhabitants while it is between 40 and 50°. In our climate we are beſt pleaſed with the heat of the air from 50 to 60°; while in the hot countries the air is generally at a medium about 70°. With us, the air is not reckoned warm till it arrives at about 64°, and it is very warm and ſultry at 80°. It is to be noted that the foregoing obſervations are to be underſtood of the ſtate of the air in the ſhade; for as to the Heat of bodies acted upon by the direct rays of the ſun, it is much greater: thus, Dr. Martine found dry earth heated to above 120°; but Dr. Hales found a very hot ſun-ſhine Heat in 1727 to be about 140°; and Muſſchenbroek once obſerved it ſo high as 150°; but at Montpelier the ſun was ſo very hot, on one day in the year 1705, as to raiſe M. Amontons's thermometer to the mark of boiling water itſelf, which is our 212°.

It appears from the regiſter of the thermometer kept at London by Dr. Heberden for 9 years, viz, from the end of 1763 to the end of 1772, that the mean Heat at 8 in the morning was 47°·4; and by another regiſter kept at Hawkhill, near Edinburgh, that the mean Heat in that place, during the ſame period of time, was 46°. Alſo by regiſters kept in London and at Hawkhill, for the three years 1772, 1773, 1774, it appears, that the mean Heat of theſe three years in London, at 8 in the morning, was 48°·5, and at 2 in the afternoon 56°, but the mean of both morning and afternoon 52°·2; while the mean Heat at Hawkhill for the ſame time,

at 8 in the morning was	45°·4
and at 2 in the afternoon	50·1
and the mean of both	47·7.

The mean Heat of ſprings near Edinburgh ſeems to be 47°, and at London 51°. Philoſ. Tranſ. vol. 65, art. 44.

Laſtly, from the meteorological journals of the Royal Society, publiſhed in the Philoſ. Tranſ. it appears that the mean heights of the thermometer, for the whole years, kept without and within the houſe, are as below:

	Therm. Without	Therm. Within
For 1775	51·5	52·7
1776	51·1	52·9
1777	51·0	53·0
1778	52·0	53·1
mean of all	51·4	52·9

HEAVEN, an azure tranſparent orb inveſting our earth, where the celeſtial bodies perform their motions. It is of divers denominations, as the higheſt or empyrean Heaven, the etherial or ſtarry Heaven, the planetary Heaven, &c.

Formerly the Heavens were conſidered as ſolid ſubſtances, or elſe as ſpaces full of ſolid matter; but Newton has abundantly ſhewn that the Heavens are void of almoſt all reſiſtance, and conſequently of almoſt all matter: this he proves from the phenomena of the celeſtial bodies; from the planets perſiſting in their motions, without any ſenſible diminution of their velocity; and the comets freely paſſing in all directions towards all parts of the Heavens.

Heaven, taken in this general ſenſe, or the whole expanſe between our earth and the remoteſt regions of the fixed ſtars, may be divided into two very unequal parts, according to the matter occupying them; viz, the atmoſphere or aereal Heaven, poſſeſſed by air; and the ethereal Heaven, poſſeſſed by a thin and unreſiſting medium, called ether.

Heaven is more particularly uſed, in Aſtronomy, for an orb, or circular region, of the ethereal Heaven.

The ancient aſtronomers aſſumed as many different Heavens as they obſerved different celeſtial motions. All theſe they made ſolid, thinking they could not otherwiſe ſuſtain the bodies fixed in them; and of a ſpherical form, as being the moſt proper for motion. Thus they had ſeven Heavens for the ſeven planets; viz, the Heavens of the Moon, Mercury, Venus, the Sun, Mars, Jupiter, and Saturn. The 8th was for the fixed ſtars, which they particularly called the firmament. Ptolomy added a 9th Heaven, which he called the primum mobile. After him two cryſtalline Heavens were added by king Alphonſus, &c, to account for ſome irregularities in the motions of the other Heavens. And laſtly an empyrean Heaven was drawn over the whole for the reſidence of the Deity; which made the number 12.

But others admitted many other Heavens, according as their different views and hypotheſes required. Eudoxus ſuppoſed 23, Calippus 30, Regiomontanus 33, Ariſtotle 47, and Fracaſtor no leſs than 70.

The aſtronomers however did not much concern themſelves whether the Heavens they thus allowed, were real or not; provided they ſerved a purpoſe in accounting for any of the celeſtial motions, and agreed with the phenomena.

HEAVINESS, the ſame as Gravity, which ſee.

Heavy bodies do not tend preciſely to the very centre of the earth, except at the poles, and the equator, on account of the ſpheroidal figure of the earth; their direction being every where perpendicular to the ſurface of the ſpheroid.

HEGIRA, a term in Chronology, ſignifying the epoch, or account of time, uſed by the Mahomedans, who begin their computation from the day that Mahomet was forced to make his eſcape from the city of Mecca, which happened on Friday the 16th of July 622.

The years of the Hegira are lunar ones, conſiſting only of 354 days. Hence, to reduce theſe years to the Julian calendar, that is, to find what Julian year a given year of the Hegira anſwers to: reduce the year of the Hegira into days, by multiplping by 354, divide the product by $365\frac{1}{4}$, and to the quotient add 622, the year the Hegira commenced.

HEIGHT, the third dimenſion of a body, conſidered with regard to its elevation above the ground.

Height, of a figure, the ſame as its altitude, being the perpendicular from its vertex to the baſe.

Height of the Pole, &c. See Altitude of the Pole, &c.

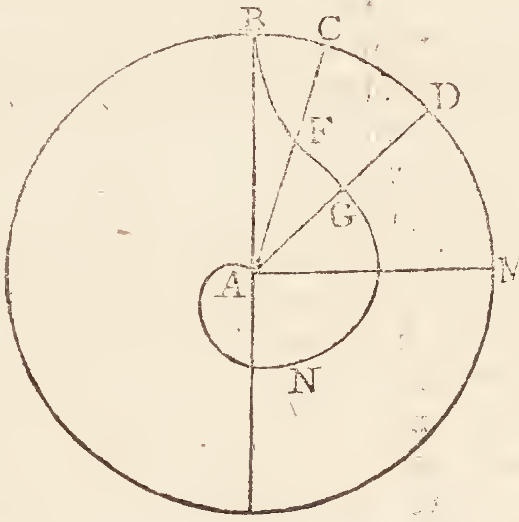
HELIACAL, ſomething relating to the ſun. Thus,

Heliacal *Riſing*, of a ſtar or planet, is when it riſes with, or at the ſame time, as the ſun. And Heliacal ſetting, the ſame as the ſetting with the ſun.

Or, a ſtar riſes Heliacally, when, after it has been in conjunction with the ſun, and ſo inviſible, it gets at ſuch a diſtance from him as to be ſeen in the morning before the ſun's riſing. And it is ſaid to ſet Heliacally, when it approaches ſo near the ſun as to be hid by his beams. So that, in ſtrictneſs, the Heliacal riſing and ſetting are only an apparition and occultation.

HELICE *Major* and *Minor*: the ſame as Urſa Major and Minor.

HELICOID *Parabola*, or the *Parabolic Spiral*, is a curve ariſing from the ſuppoſition that the common or Apollonian parabola is bent or twiſted, till the axis come into the periphery of a circle, the ordinates ſtill retaining their places and perpendicular poſitions with reſpect to the circle, all theſe lines ſtill remaining in the ſame plane. Thus, the axis of a parabola being bent into the circumference BCDM, and the ordinates CF, DG, &c, ſtill perpendicular to it, then the parabola itſelf, paſſing through the extremities of the ordinates, is twiſted into the curve BFG, &c, called the Helicoid, or Parabolic Spiral.



Hence all the ordinates CF, DG, &c, tend to the centre of the circle, being perpendicular to the circumference.

Alſo, the equation of the curve remains the ſame as when it was a parabola; viz, putting $x =$ any circular abſciſs BC, and $y =$ CF the correſponding ordinate, then is $px = y^2$, where p is the parameter of the parabola.

HELIOCENTRIC *Place of a Planet*, is the place in which a planet would appear to be when viewed from the ſun; or the point of the ecliptic, in which a planet viewed from the ſun would appear to be. And therefore the Heliocentric place coincides with the longitude of a planet viewed from the ſun.

HELIOCENTRIC *Latitude of a Planet*, is the inclination of the line drawn between the centre of the ſun and the centre of a planet, to the plane of the ecliptic. The greateſt Heliocentric Latitude is equal to the inclination of the planet's orbit to the plane of the ecliptic.

HELIOCOMETES, Comet of the Sun, a phenomenon ſometimes obſerved at the ſetting of the ſun; thus denominated by Sturmius and Pylen, who had ſeen it, becauſe it ſeems to make a comet of the ſun, being a large tail, or column of light, fixed or hung to that luminary, and dragging after it, at its ſetting, like the tail of a comet.

HELIOMETER, or ASTROMETER; an inſtrument for meaſuring, with particular exactneſs, the diameters of the ſun, moon, and ſtars.

This inſtrument was invented by M. Bouguer in 1747, and is a kind of teleſcope, conſiſting of two object glaſſes of equal focal diſtance, placed by the ſide of each other, ſo that the ſame eye-glaſs ſerves for both. The tube of this inſtrument is of a conical form, larger at the upper end, which receives the two object-glaſſes, than at the lower, which is furniſhed with an eye-glaſs and micrometer. By the conſtruction of this inſtrument, two diſtinct images of an object are formed in the focus of the eye-glaſs, whoſe diſtance, depending on that of the two object-glaſſes from one another, may be meaſured with great accuracy. Mem. Acad. Sci. 1748.

Mr. Servington Savery diſcovered a ſimilar method of improving the micrometer, which was communicated to the Royal Society in 1743.

HELIOSCOPE, a kind of teleſcope peculiarly adapted for viewing and obſerving the ſun without hurting the eye.

There are various kinds of this inſtrument, uſually made by employing coloured glaſs for the object or eye-glaſs, or both; and ſometimes only uſing an eye-glaſs blacked by holding it over the ſmoke or flame of a lamp or candle, which is Huygens's way.—See Dr Hooke's treatiſe on Helioſcopes.

HELIOSTATA, an inſtrument invented by Dr. Graveſande, and ſo called from its property of fixing the ſun-beam in one poſition, viz, in a horizontal direction acroſs the dark chamber while it is uſed. See Graveſande's Phyſices Element. Mathematica, tom. 2, p. 715 ed. 3tia 1742, for an account of the principles, conſtruction and uſe of this inſtrument.

HELISPHERICAL *Line*, is the Rhumb-line in Navigation; being ſo called, becauſe on the globe it winds round the pole helically or ſpirally, coming ſtill nearer and nearer to it.

HELIX, a Spiral line. See SPIRAL.

HEMISPHERE, the half of a ſphere or globe, when divided in two by a plane paſſing through its centre.

Hemiſphere is alſo uſed for a map or projection of half the terreſtrial globe, or of half the celeſtial ſphere, on a plane; being more frequently called a planiſphere.

The centre of gravity of a Hemiſphere, is five-eighths of the radius diſtant from the vertex.

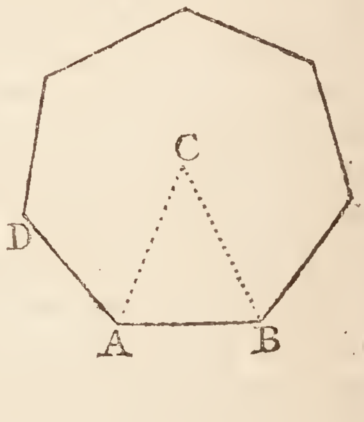
A glaſs Hemiſphere unites the parallel rays at the diſtance of four-thirds of a diameter from the pole of the glaſs.

HEMITONE, in Muſic, a half note.

HENDECAGON, a figure of eleven ſides, or the Endecagon; which ſee.

HENIOCHAS, or HENIOCHUS, a northern conſtellation, the ſame as Auriga, which ſee.

HEPTAGON, in Geometry, a figure of ſeven ſides and ſeven angles.—When thoſe ſides and angles are all equal, the Heptagon is ſaid to be regular, otherwiſe it is irregular.



In a regular Heptagon, the angle C at the centre is $= 51^\circ\frac{3}{7}$, the angle DAB of the polygon is $= 128^\circ\frac{4}{7}$, and its half $\text{CAB} = 64^\circ\frac{2}{7}$. Alſo the area is = the ſquare of the ſide $\text{AB}^2 \times 3{\cdot}6339124$ or $= \text{AB}^2 \times \frac{7}{4}t$, where t is the tangent of the angle CAB of $64^\circ\frac{2}{7}$ to the radius 1; or t is the root of the equation;

$$t^{12} - 26t^{10} + 143t^8 - 245t^6 + 143t^4 - 26t^2 + 1 = 0; \text{ or}$$

$$t = \sqrt{\frac{1+x}{1-x}} = \frac{1+\sqrt{1-y^2}}{y} = \frac{y}{1-\sqrt{1-y^2}},$$

where the value of x and y are the roots of the equations

$$x^6 - \frac{5}{4}x^4 + \frac{3}{8}x^2 - \frac{1}{46} = 0,$$
$$y^6 = \frac{7}{4}y^4 + \frac{7}{8}y^2 - \frac{7}{64} = 0.$$

See my Menſuration, p. 21, 114, and 116, 2d edition.

HEPTAGON, in Fortification, a place fortified or ſtrengthened with ſeven baſtions for its defence.

HEPTAGONAL *Numbers*, are a kind of polygonal numbers in which the difference of the terms of the correſponding arithmetical progreſſion is 5. Thus,

Arithmeticals, 1, 6, 11, 16, 21, 26, &c.
Heptagonals, 1, 7, 18, 34, 55, 81, &c.

where the Heptagonals are formed by adding continually the terms of the arithmeticals, above them, whoſe common difference is 5.

One property, among many others, of theſe Heptagonal numbers is, that if any one of them be multiplied by 40, and to the product add 9, the ſum will be a ſquare number.

Thus $1 \times 40 + 9 = 49 = 7^2$;
and $7 \times 40 + 9 = 289 = 17^2$;
and $18 \times 40 + 9 = 729 = 27^2$;
and $34 \times 40 + 9 = 1369 = 37^2$; &c.

Where it is remarkable that the ſeries of ſquares ſo formed is 7^2, 17^2, 27^2, 37^2, &c, the common difference of whoſe roots is 10, the double of the common difference of the arithmetical ſeries from which the Heptagonals are formed.—See POLYGONALS.

HEPTANGULAR *Figure*, in Geometry, is one that has ſeven angles; and therefore alſo ſeven ſides.

HERCULES, in Aſtronomy, a conſtellation of the northern hemiſphere, and one of the 48 old conſtellations mentioned by ancient writers.

It is not known by what name it was diſtinguiſhed by the Egyptians and others before the Greeks. Theſe latter, perhaps not knowing its real name, firſt called it ſimply the kneeling man, becauſe he is drawn in that poſture; but they afterwards ſucceſſively aſcribed it to, and called it by the names of, Cetheus, Theſeus, and laſtly Hercules, which it ſtill retains.

The ſtars in this conſtellation, in Ptolomy's catalogue, are 29; in Tycho's 28; and in the Britannic catalogue, 113.

HERISSON, in Fortification, a beam armed with iron ſpikes, having their points turned outward. It is ſupported in the middle by a ſtake, having a pivot on which it turns; and ſerves as a barrier to block up a paſſage.—Heriſſons are frequently placed before gates, eſpecially the poſterns of a town or fortreſs, to ſecure thoſe paſſages which muſt of neceſſity be often opened.

HERMANN (JAMES), a learned mathematician of the Academy of Berlin, and member of the Academy of Sciences at Paris, was born at Baſil in 1678. He was a great traveller; and for 6 years was profeſſor of mathematics at Padua. He afterwards went to Ruſſia, being invited thither by the Czar in 1724, as well as his compatriot Daniel Bernoulli. On his return to his native country, he was appointed profeſſor of morality and natural law at Baſil; where he died in 1733, at 55 years of age.

He wrote ſeveral mathematical and philoſophical pieces, in the Memoirs of different Academies, and elſewhere; but his principal work, is the *Phoronomia*, or two Books on the Forces and Motions of both Solid and Fluid bodies; 4to, 1716: a very learned work on the new mathematical phyſics.

HERMETIC, or HERMETICAL *Art*, a name given to chemiſtry, on a ſuppoſition that Hermes Triſmegiſtus was its inventor.

HERMETICAL *Philoſophy*, is that which undertakes to ſolve and explain all the phenomena of nature, from the three chemical principles, ſalt, ſulphur, and mercury.——A conſiderable addition was made to the ancient Hermetical Philoſophy, by the modern doctrine of alcali and acid.

HERMETICAL *Seal*, or *Hermetical Sealing*, a manner of ſtopping or cloſing glaſs veſſels, for chemical and other operations, ſo very cloſely, that no ſubſtance can poſſibly exhale or eſcape. This is uſually done by heating the neck of the veſſel in the flame of a lamp, with a blow-pipe, till it be ready to melt, and then with a pair of hot pincers twiſting it cloſe together.

HERSCHEL, the name by which the French, and moſt other European nations, call the new planet, diſcovered by Dr. Herſchel in the year 1781. Its mark or character is ♅. The Italians call it Ouranos, or Urania, but the Engliſh, the GEORGIAN *Planet*, which ſee.

HERSE, in Fortification, a lattice or portculliſe, in the form of a harrow, beſet with iron ſpikes, to block up a gate way, &c.

HERSILLON, or little Herſe, in Fortification, is a plank armed with iron ſpikes, for the ſame uſe as the Herſe, and alſo to impede the march of the infantry or cavalry.

HESSE (WILLIAM Prince of), rendered his name immortal by his encouragement of learning, by his ſtudies, and by his obſervations, for many years, of the celeſtial bodies. For this purpoſe, he erected an obſervatory at Caſſel, and furniſhed it with good inſtruments, well adapted to that deſign; calling alſo to his aſſiſtance two eminent artiſts, Chriſtopher Rothmann and Juſte Byrge. His obſervations, which are of a very curious nature, were publiſhed at Leyden, in the year 1618, by Willebrord Snell; and are in part mentioned by Tycho Brahe, as well in his epiſtles as in the 2d volume of his *Progymnaſmata*; a ſignal example to all princely and heroic minds, to undertake the promoting the arts of peace, and advancing this truly noble and celeſtial ſcience. This prince died in the year 1597.

HETERODROMUS *Vectis*, or *Lever*, in Mechanics, a lever in which the fulcrum, or point of ſuſpenſion, is between the weight and the power; being the ſame as what is otherwiſe called a lever of the firſt kind.

HETEROGENEAL, the ſame as HETEROGENEOUS; which ſee.

HETEROGENEOUS, literally imports things of different natures, or ſomething that conſiſts of parts of different or diſſimilar kinds; in oppoſition to Homogeneous. Thus,

HETEROGENEOUS *Bodies*, are ſuch as have their parts of unequal denſity.

HETEROGENEOUS *Line*, is that which conſiſts of parts or rays of different refrangibility, reflexibility, and colour.

HETEROGENEOUS *Numbers*, are mixed numbers, conſiſting of integers and fractions.

HETEROGENEOUS *Particles*, are ſuch as are of different kinds, natures, and qualities; of which generally all bodies conſiſt.

HETEROGENEOUS *Quantities*, in Mathematics, are thoſe which cannot have proportion, or be compared together as to greater and leſs; being of ſuch different kind and conſideration, as that one of them taken any number of times, never equals or exceeds the other. As lines, ſurfaces, and ſolids in geometry.

HETEROGENEOUS *Surds*, are ſuch as have different radical ſigns; as $\sqrt{a}$ and $\sqrt[3]{b^2}$; or $\sqrt[5]{10}$ and $\sqrt[7]{20}$.

HETEROSCII, in Geography, are ſuch inhabitants of the earth as have their ſhadows at noon projected always the ſame way with regard to themſelves, or always contrary ways with reſpect to each other. Thus, all the inhabitants without the torrid zone are Heteroſcii, with regard to themſelves, ſince any one ſuch inhabitant has his ſhadow at noon always the ſame way, viz, always north of him in north latitude, and always ſouth of him in ſouth latitude; or theſe two ſituations are Heteroſcii to each other, having ſuch ſhadows projected contrary ways at all times of the year.

HEVELIUS (JOHN), a very celebrated aſtronomer, and a burgomaſter of Dantzick, was born in that city in 1611. He ſtudied mathematics under Peter Cruger, in which he made a wonderful progreſs. He afterwards ſpent ſeveral years on his travels through Holland, England, Germany, and France, for his improvement in the ſciences. On his return, he conſtructed excellent teleſcopes himſelf, and began diligently to obſerve the heavens, an employment he cloſely followed during the courſe of a long life, which was terminated only in 1687, at 76 years of age. Hevelius was author of ſeveral notable diſcoveries in the heavens. He was the firſt that obſerved the phenomenon called the libration of the moon, and made ſeveral other important obſervations on the other planets. He alſo diſcovered ſeveral fixed ſtars, which he named the *firmament of Sobieſki*, in honour of John the 3d, king of Poland. He framed a large catalogue of the ſtars, and collected multitudes of the unformed ones into new conſtellations of his own framing. His wife was alſo well ſkilled in aſtronomy, and made a part of the obſervations that were publiſhed by her huſband. His principal publications are, his *Selenographia*, or an exact deſcription of the moon; in which he has engraved all her phaſes, and remarkable parts, diſtinguiſhed by names, and aſcertained their reſpective bounds by the help of teleſcopes; containing alſo a delineation of the ſeveral viſible ſpots, with the various motions, changes, and appearances, diſcovered by the teleſcope, as alſo in the ſun and other planets, 1647.——In 1654, two epiſtles; one to the celebrated aſtronomer Riccioli, concerning the Libration of the Moon; and the other to Bulliald, on the Eclipſes of both luminaries.—In 1656, a Diſſertation *De Natura Saturni faciei, &c.*—In 1668, his *Cometogrophia*, repreſenting the whole nature of comets, their ſituation, parallaxes, diſtances, diverſe appearances, and ſurpriſing motions, with a hiſtory of all the comets from the beginning of the world down to the preſent time; being enriched with curious ſculpture of his own execution: to which he added a treatiſe on the planet Mercury, ſeen in the ſun at Dantzick, May 3, 1661; with the hiſtory of a new ſtar appearing in the neck of Cetus, and another in the beak of Cygnus; beſides an Illuſtration of ſome aſtronomical diſcoveries of the late Mr. Horrox, in his treatiſe on Venus ſeen in the ſun, Nov. 24, 1639; with a diſcourſe of ſome curious Paraſelena and Parhelia obſerved at Dantzick. He ſent copies of this work to ſeveral members of the Royal Society at London, and among them to Mr. Hooke, in return for which, this gentleman ſent to Hevelius a deſcription of the Dioptric Teleſcope, with an account of the manner of uſing it; and recommending it to him, as much preferable to teleſcopes with plain ſights. This gave riſe to a diſpute between them, viz, “whether diſtances and altitudes could be taken with plain ſights any nearer than to a minute.” Hooke aſſerted that they could not; but that, with an inſtrument of a ſpan radius, by the help of a teleſcope, they might be determined to the exactneſs of a ſecond. Hevelius, on the other hand, inſiſted that, by the advantage of a good eye and long practice, he was able with his inſtruments to come up even to that exactneſs; and, appealing to experience and facts, ſent by way of challenge 8 diſtances, each between two different ſtars, to be examined by Hooke. Thus the affair reſted for ſome time with outward decency, but not without ſome inward grudge between the parties.

In 1673, Hevelius publiſhed the firſt part of his *Machina Cœleſtis*, as a ſpecimen of the exactneſs both of his inſtruments and obſervations; and ſent ſeveral copies as preſents to his friends in England, but omitting Mr. Hooke. This, it is ſuppoſed, occaſioned that gentleman to print, in 1674, *Animadverſions on the Firſt Part of the Machina Cœleſtis*; in which he treated Hevelius with a very magiſterial air, and threw out ſeveral unhandſome reflections, which were greatly reſented; and the diſpute grew aftewards to ſuch a height, and became ſo notorious, that in 1679 Dr. Halley went, at the requeſt of the Royal Society, to examine both the inſtruments and the obſervations made with them. Of both theſe, Halley gave a favourable account, in a letter to Hevelius; and Hooke managed the controverſy ſo ill, that he was univerſally condemned, though the preference has ſince been given to teleſcopic ſights. However, Hevelius could not be prevailed on to make uſe of them: whether he thought himſelf too experienced to be informed by a young aſtronomer, as he conſidered Hooke; or whether, having made ſo many obſervations with plain ſights, he was unwilling to alter his method, leſt he might bring their exactneſs into queſtion; or whether, being by long practice accuſtomed to the uſe of them, and not thoroughly apprehending the uſe of the other, nor well underſtanding the difference, is uncertain. Beſides Halley's letter, Hevelius received many others in his favour, which he took the opportunity of inſerting among the aſtronomical obſervations in his *Annus Climactericus*, printed in 1685. In a long preface to this work, he ſpeaks with more confidence and greater indignation than he had done before, and particularly exclaimed againſt Hooke's dogmatical and magiſterial manner of aſſuming a kind of dictatorſhip over him. This revived the conteſt, and occaſioned ſeveral learned men to engage in it. The book itſelf being ſent to the Royal Society, at their requeſt an account of it was given by Dr. Wallis; who, among other things, took notice, that “Hevelius's obſervations had been miſrepreſented, ſince it appeared from this book, that he could diſtinguiſh by plain ſights to a ſmall part of a minute.” About the ſame time Mr. Molyneux alſo wrote a letter to the ſociety, in vindication of Hevelius, againſt Hooke's animadverſions. To all which, Hooke drew up a letter in anſwer, which was read before the ſociety, containing many qualifying and accommodating expreſſions, but ſtill at leaſt expreſſing the ſuperiority of teleſcopic ſights over plain ones, excellent as the obſervations were that had been made with theſe.

In 1679, Hevelius had publiſhed the ſecond part of his *Machina Cœleſtis*; but the ſame year, while he was at a ſeat in the country, he had the misfortune to have his houſe at Dantzic burnt down. By this calamity it is ſaid he ſuſtained ſeveral thouſand pounds damage; having not only his obſervatory and all his valuable inſtruments and aſtronomical apparatus deſtroyed, but alſo a great many copies of his *Machina Cœleſtis*, an accident which has made this ſecond part very ſcarce, and conſequently very dear.

In 1690, were publiſhed a deſcription of the heavens, called, *Firmamentum Sobieſcianum*, in honour of John the 3d, king of Poland, as above mentioned; and alſo *Prodromus Aſtronomiæ, &c. Novæ Tabulæ Solares, una cum Catalogo Fixarum*; in which he lays down the neceſſary preliminaries for taking an exact catalogue of the ſtars.

But both theſe works were poſthumous; for Hevelius died the 28th of January 1687, exactly 76 years of age, as above ſaid, and univerſally admired and reſpected; abundant evidence of which appears in a collection of letters between him and many other perſons, that was printed at Dantzic in 1683.

HEXACHORD, a certain interval or muſical concord, uſually called a ſixth.

HEXAEDRON, or HEXAHEDRON, one of the five regular or Platonic bodies; being indeed the ſame as the cube; and is ſo called from its having 6 faces.—The ſquare of the ſide or edge of a Hexahedron, is one-third of the ſquare of the diameter of the circumſcribing ſphere: and hence the diameter of a ſphere is to the ſide of its inſcribed Hexahedron, as $\surd 3$ to 1.

In general, if A, B, and C be put to denote reſpectively the linear ſide, the ſurface, and the ſolidity of a Hexahedron or cube, alſo r the radius of the inſcribed ſphere, and R the radius of the circumſcribed one; then we have theſe general equations or relations:

1. $A = 2r = \frac{2}{3}R\surd 3 = \surd\frac{1}{6}B = \sqrt[3]{C}$.
2. $B = 24r^2 = 8R^2 = 6A^2 = 6\sqrt[3]{C^2}$.
3. $C = 8r^3 = \frac{8}{9}R^3\surd 3 = A^3 = \frac{1}{6}B\surd\frac{1}{6}B$.
4. $R. = r\surd 3 = \frac{1}{2}A\surd 3 = \frac{1}{2}\surd\frac{1}{2}B = \frac{1}{2}\surd 3 \times \sqrt[3]{C}$.
5. $r = \frac{1}{3}R\surd 3 = \frac{1}{2}A = \frac{1}{2}\surd\frac{1}{6}B = \frac{1}{2}\sqrt[3]{C}$.

HEXAGON, in Geometry, a figure of ſix ſides, and conſequently of as many angles. When theſe are equal, it is a regular Hexagon.—The angles of a Hexagon are each equal to 120°, and its ſides are each equal to the radius of its circumſcribing circle. Hence a regular Hexagon is inſcribed in a circle, by ſetting the radius off 6 times upon the periphery. And hence alſo, to deſcribe a regular Hexagon upon a given line, deſcribe an equilateral triangle upon it, the vertex of which will be the centre of the circumſcribing circle.

The ſide of a Hexagon being s, its area will be $= 2{\cdot}5980762s^2 = \frac{3}{2}s^2 \times \text{tang. } 60^\circ = \frac{3}{2}s^2\surd 3$.

HEXASTYLE, in the Ancient Architecture, a building with 6 columns in front.

HIERO's *Crown*, in Hydroſtatics. The hiſtory of this crown, and of the important hydroſtatical propoſition which it gave occaſion to, is as follows: Hiero, king of Syracuſe, having furniſhed a workman with a quantity of gold for making a crown, ſuſpected that he had been cheated, by the workman uſing a greater alloy of ſilver than was neceſſary in making it; and he applied to Archimedes to diſcover the fraud, without defacing the crown.

This celebrated mathematician was led by chance to a method of detecting the impoſture, and of determining preciſely the quantities of gold and ſilver compoſing the crown: for he obſerved, when bathing in a tub of water, that the water ran over as his body entered it, and he preſently concluded that the quantity ſo running over was equal to the bulk of his body that was immerſed. He was ſo pleaſed with the diſcovery, that it is ſaid he ran about naked crying out, ευρηκα, εὑρηκα, *I have found it*; and ſome affirm that he offered a hetacomb to Jupiter for having inſpired him with the thought.

On this principle he procured a ball or maſs of gold, and another of ſilver, exactly of the ſame weight with the crown; conſidering, that, if the crown were of pure gold, it would be of equal bulk and expel an equal quantity of water as the golden ball; and if it were of ſilver, then it would be of equal bulk and expel an equal quantity of water with the ball of ſilver; but of intermediate quantity, if it conſiſted of a mixture of the two, gold and ſilver; which, upon trial, he found to be the caſe; and hence, by a compariſon of the quantities of water diſplaced by the three maſſes, he diſcovered the exact portions of gold and ſilver in the crown.

Now, ſuppoſe, for example, that each of the three maſſes weighed 100 ounces; and that on immerſing them ſeverally in water, there were diſplaced 5 ounces of water by the golden ball, 9 ounces by the ſilver, and 6 ounces by the compound, or crown; that is, their reſpective or comparative bulks are as 5, 9, and 6, the ſum of which is 20.

Then the method of operation is this:

From 9 6
Take 6 5
— —
rem. 3 1, whoſe ſum is 4.
Therefore 4 : 100 :: 3 : 75 oz. of gold,
and 4 : 100 :: 1 : 25 oz. of ſilver.

That is, the crown conſiſted of 75 ounces of gold, and 25 ounces of ſilver.

See Cotes Hydroſ. Lect. p. 81; or Martin's Philoſ. Britan, vol. 1, p. 305, &c. See alſo SPECIFIC *Gravity*.

HIGH-*Water*, that ſtate of the tides when they have flowed to their greateſt height, or have ceaſed to flow or riſe. At High-water the motion commonly ceaſes for a quarter or half an hour, before it begin to ebb again. The times of High-water of every day of the moon's age, is uſually computed from that which is obſerved on the day of the full or change; viz, by taking 4-5ths of the moon's age on any day of the month, and adding it to the time of High-water on the day of the full or change; then is the ſum nearly equal to the time of High-water on the day of the month propoſed. And as to the times of High-water, on the day of the full and change of the moon, at many different places; they have been obſerved as they are ſet down in the following table.

Table of the Times of High-water on the Days of the New and Full Moons, at many different Places.

Names of Places.	Countries.	High-w.
Aberdeen.	Scotland	0h 45m
Aldborough	England	9 45
Alderney I.	England	12 0
Amazons River	South America	6 0
Amſterdam	Holland	3 0
Amſterdam I. of	South Seas	8 30
Andrew's St.	Scotland	2 15
Anholt I.	Denmark	0 0
Antwerp	Flanders	6 0
Archangel	Ruſſia	6 0
Arran I.	Ireland	11 0
Aſhley Riv.	Carolina	0 45
Auguſtine St.	Florida	7 30
Bajador Ca.	Negroland	0 0
Baltimore	Ireland	4 30
Barfleur Ca.	France	7 30
Bayonne	France	3 30
Beachy-head	England	0 0
N. and S. Bear	Labradore	12 0
Belfaſt	Ireland	10 0
Belliſle	France	3 30
Bermudas I.	Bahama I.	7 0
Berwick	England	2 30
Blackney	England	6 0
Blanco Cap.	Negroland	9 45
Blavet	France	3 0
Bourdeaux	France	3 0
Boulogne	France	10 30
Bremen	Germany	6 0
Breſt	France	3 45
Bridlington B.	England	3 45
Brill	Holland	1 30
Briſtol	England	6 45
Buchaneſs	Scotland	3 0
Button's Iſles	New Brit.	6 50
Cadiz	Spain	4 30
Caen	France	9 0
Calais	France	11 30
Canaria I.	Canaries	3 0
C. Cantin	Barbary	0 0
Cape Town	Caffers	2 30
Caſkets	Guernſey	8 15
Cathneſs Po.	Scotland	9 0
Charles Town	Carolina	3 0
Q. Charlotte's S.	New Zealand	9 0
Cherbourg	France	7 30
Churchill R.	Hudſon's Bay	7 20
Ca. Cleare	Ireland	4 30
Concarneau	France	3 0
Conquet	France	2 15
Coquet Iſle	England	3 0
Corke	Ireland	6 30
C. Corſe	Guinea	3 30
Cromer	England	7 0
Dartmouth	England	6 30
St. David's H.	Wales	6 0
Dieppe	France	10 30
Dort	Holland	3h 0m
Dover	England	11 30
Downs	England	1 15
Dublin	Ireland	9 15
Dunbar	Scotland	2 30
Dundee	Scotland	2 15
Dungarvan	Ireland	4 30
Dungeneſs	England	9 45
Dunkirk	France	0 0
Dunnoſe	I. of Wight	9 45
Duſky Bay	N. Zealand	10 57
Eaſter Iſle	Chili	2 0
Edyſtone	Engliſh Channel	5 30
Elbe R.	Germany	0 0
Embden	Germany	0 0
Eſtaples	France	11 0
Falmouth	England	5 30
Flamborough H.	England	4 0
C. Florida	Florida	7 30
Fluſhing	Holland	0 45
N. Foreland	England	9 45
Foulneſs	England	6 45
Fowey	England	5 15
Fayal Iſl.	Azores	2 20
Garonne R.	France	3 0
Gibraltar	Spain	0 0
C. Good Hope	Caffers	3 0
Goree (Iſle)	Atlantic Ocean	1 30
Granville	France	7 0
Gravelines	Flanders	0 0
Graveſend	England	1 30
Groin	Spain	3 3
Guernſey I.	Engliſh Channel	1 30
Hague	Holland	8 15
Halifax	Nova Scotia	7 30
Hamburgh	Germany	6 0
Hare Iſle	Canada	3 30
Harlem	Holland	9 0
Hartlepool	England	3 0
Harwich	England	11 15
Havre de Grace	France	9 0
Holy Head	Wales	1 30
Honfleur	France	9 0
Hull	England	6 0
Humber R.	England	5 13
St. John's	Newfoundland	6 0
St. Julian (Port)	Patagonia	4 45
Kentiſhnock	Engliſh coaſt	0 0
Kinſale	Ireland	5 15
Land's End	England	7 30
Leith	Scotland	4 30
Leoſtoff	England	9 45
Liſbon	Portugal	2 15
Liverpool	England	11 15
Lizard	England	7 30
Loire (Riv.)	France	3 0
London	England	3 0

Names of Places.	Countries.	High-w.	Names of Places.	Countries.	High-w.
Lundy (Iſle)	England	5h 15m	Sandwich	England	11h 30m
Madeira	Atl. Ocean	12 4	Scarborough H.	England	3 45
St. Maloes	France	6 0	Scilly Iſles	England	3 45
Iſle of Man	England	9 0	Senegal	Negroland	10 30
Margate	England	11 15	Severn, (Mouth.)	England	6 0
St. Mary's (Iſle)	Scilly Iſles	3 45	Sheerneſs	England	0 0
Milford	Wales	5 15	Sierra Leona	Guinea	8 15
Mount's Bay	England	4 30	Shetland I.	Scotland	3 0
Nantes	France	3 0	Iſle of Sky	Scotland	5 30
Naze	Norway	11 15	Spurn	England	5 15
Needles	England	10 15	Start Point	England	6 45
Newcaſtle	England	3 15	Stockton	England	5 15
Nieuport	Flanders	12 0	Sunderland	England	3 20
Nore	England	0 0	Tanna	Pacific Ocean	3 0
North Cape	Lapland	3 0	Teneriff	Canaries	3 0
Orfordneſs	England	9 45	Texel (Iſle)	Holland	7 30
Orkneys	Scotland	3 0	Thames Mouth	England	1 30
Oſtend	Flanders	12 0	Tinmouth	England	3 0
Placentia	Newfoundland	9 0	Torbay	England	5 15
Plymouth	England	6 0	St. Valery	France	10 30
Portland	England	8 15	Vannes	France	3 45
Porto Praya	Cape Verdes	11 0	Uſhant	France	4 30
Portſmouth	England	11 15	Waterford	Ireland	6 30
Quebec	Canada	7 30	Wells	England	6 0
Rhée (Iſle)	France	3 0	Weymouth	England	7 20
Reſolution (Bay)	Ohitahoo	2 30	Whitby	England	3 0
Robin Hood's B.	England	3 0	Iſle of Wight	England	0 0
Rochefort	France	4 15	Winchelſea	England	0 45
Rochelle	France	3 45	Wintertoneſs	England	9 0
Rocheſter	England	0 45	Yarmouth	England	9 45
Rotterdam	Holland	3 0	New York	America	3 0
Rouen	France	1 15	Youghall	Ireland	4 30
Rye	England	11 15	Zuric Sea	Holland	3 0

HIPS, in Architecture, are thoſe pieces of timber placed at the corners of a roof. Theſe are much longer than the rafters, becauſe of their oblique poſition.

Hip means alſo the angle formed by two parts of the roof, when it riſes outwards.

Hip-*Roof*, called alſo Italian Roof, is one in which two parts of the roof meet in an angle, riſing outwards: the ſame angle being called a valley when it ſinks inwards.

HIPPARCHUS, a celebrated aſtronomer among the ancients, was born at Nice in Bithynia, and flouriſhed between the 154th and the 163d olympiads; that is, between 60 and 135 years before Chriſt; for in this ſpace of time it is that his obſervations are dated. He is accounted the firſt, who from vague and ſcattered obſervations, reduced aſtronomy into a ſcience, and proſecuted the ſtudy of it ſyſtematically. Pliny often mentions him, and always with great commendation. He was the firſt, he tells us, who attempted to count the number of the fixed ſtars; and his catalogue is preſerved in Ptolomy's Almageſt, where they are all noted according to their longitudes and apparent magnitudes. Pliny places him among thoſe men of a ſublime genius, who, by foretelling the eclipſes, taught mankind, that they ought not to be frightened at theſe phenomena. Thales was the firſt among the Greeks, who could diſcover when there was to be an eclipſe. Sulpitius Gallus among the Romans began to ſucceed in this kind of prediction; and he gave an eſſay of his ſkill very ſeaſonably, the day before a battle was fought. After theſe two, Hipparchus improved that ſcience very much; making ephemerides, or catalogues of eclipſes, for 600 years. He admires him for making a review of all the ſtars, acquainting us with their ſituations and magnitudes; for by theſe means, ſays he, poſterity will be able to diſcover, not only whether they are born and die, but alſo whether they change their places, and whether they increaſe or decreaſe. He mentioned a new ſtar which was produced in his days; and by its motion, at its firſt appearance, he began to doubt whether this did not frequently happen, and whether thoſe ſtars, which we call fixed, do not likewiſe move. Hipparchus is alſo memorable for being the firſt who diſcovered the preceſſion of the equinoxes, or a very ſlow apparent motion of the fixed ſtars from eaſt to weſt, by which in a great number of years they will ſeem to have performed a complete revolution. He endeavoured alſo to reduce to rule the many diſcoveries he made, and invented new inſtruments, by which he marked their magnitudes and places in the heavens; ſo that by means of them it might be eaſily obſerved, not only whether they appear and diſappear, but likewiſe whether they paſs by one another, or move, and whether they increaſe or decreaſe.

The firſt obſervations he made, were in the iſle of

Rhodes; whence he got the name Rhodius; but afterwards he cultivated this ſcience in Bithynia and Alexandria only. One of his works is ſtill extant, viz, his Commentary upon Aratus's Phenomena. He compoſed ſeveral other works; and upon the whole it is agreed, that aſtronomy is greatly indebted to him, for laying that rational and ſolid foundation, upon which all ſucceeding aſtronomers have ſince built their ſuperſtructure.

HIRCUS, in Aſtronomy, a fixed ſtar of the firſt magnitude, the ſame with Capella.

HIRCUS is alſo uſed by ſome writers for a comet, encompaſſed as it were with a mane, ſeemingly rough and hairy.

HIRE (PHILIP DE LA), an eminent French mathematician and aſtronomer, was born at Paris in 1640. His father, who was painter to the king, intending him for the ſame occupation, taught him drawing and ſuch branches of mathematics as relate to it: but died when the ſon was only 17 years of age. Three years after this, he travelled into Italy for improvement in that art, where he ſpent 4 years. He applied himſelf alſo to mathematics, which gradually engroſſed all his attention. On his return to Paris, he continued his mathematical ſtudies with great eagerneſs, and he afterwards publiſhed ſome works, which gained him ſo much reputation, that he was named a member of the Academy of Sciences in 1678.

The miniſter Colbert having formed a deſign for a better chart or map of France than any former ones, De la Hire was appointed, with Picard, to make the neceſſary obſervations for that purpoſe. This occupied him ſome years in ſeveral of the provinces; and, beſide the main object of his peregrinations, he was not unmindful of other branches of knowledge, but philoſophized upon every thing that occurred, and particularly upon the variations of the magnetic needle, upon refractions, and upon the height of mountains, as determined by the barometer.

In 1683, de la Hire was employed in continuing the meridian line, which Picard had begun in 1669. He continued it from Paris northward, while Caſſini carried it on to the ſouth: but Colbert dying the ſame year, the work was dropped before it was finiſhed. De la Hire was next employed, with other members of the academy, in taking the neceſſary levels for the grand aqueducts, which Louis the 14th was about to make.

The great number of works publiſhed by our author, together with his continual employments, as profeſſor of the Royal College and of the Academy of Architecture, give us ſome idea of the great labours he underwent. His days were always ſpent in ſtudy; his nights very often in aſtronomical obſervations; ſeldom ſeeking any other relief from his labours, than a change of one for another. In his manner, he had the exterior politeneſs, circumſpection, and prudence of Italy; on which account he appeared too reſerved in the eyes of his countrymen; though he was always eſteemed as a very honeſt diſintereſted man. He died in 1718, at 78 years of age.

Of the numerous works which he publiſhed, the principal are, 1. Traité de Mechanique; 1665.—2. Nouvelle Methode en Geometrie pour les Sections des Superficies Coniques & Cylindriques; 1673, 4to.—3. De Cycloide; 1677, 12mo.—4. Nouveaux Elemens des Sectiones Coniques: les Lieux Geometriques: la Conſtruction, ou Effection des Equations; 1678, 12mo.—5. La Gnomonique, &c; 1682. 12mo.—6. Traité du Nivellement de M. Picard, avec des additions; 1684.—7. Sectiones Conicæ in novem libros diſtributæ; 1685, folio. This was conſidered as an original work, and gained the author great reputation all over Europe.—8. Traité du Mouvement des Eaux, &c; 1686.—9. Tabulæ Aſtronomicæ; 1687 and 1702, 4to.—10. Ecole des Arpenteurs; 1689.—11. Veterum Mathematicorum Opera, Græcè & Latinè, pleraque nunc primùm edita; 1693, folio. This edition had been begun by Thevenot; who dying, the care of finiſhing it was committed to de la Hire. It ſhews that our author's ſtrong application to mathematical and aſtronomical ſtudies had not hindered him from acquiring a very competent knowledge of the Greek tongue. Beſide theſe, and other ſmaller works, there are a vaſt number of his pieces ſcattered up and down in Journals, and particularly in the Memoirs of the Academy of Sciences, viz, from 1666 till the year 1718.

HOBBES (THOMAS), a famous writer and philopher, was born at Malmſbury in Wiltſhire, in 1588, being the ſon of a clergyman of that place. He completed his ſtudies at Oxford, and was afterwards governor to the eldeſt ſon of William Cavendiſh earl of Devonſhire, with whom he travelled through France and Italy, applying himſelf cloſely to the ſtudy of polite literature. In 1626 his patron the earl of Devonſhire died; and 1628 his ſon alſo; the ſame year Mr. Hobbes publiſhed his tranſlation of Thucydides in Engliſh. He ſoon after went abroad a ſecond time as governor to the ſon of Sir Gervaſe Clifton; but ſhortly after returned, to reſume his concern for the hopes of the Devonſhire family, to whom he had ſo early attached himſelf; the counteſs dowager having deſired to put the young earl under his care, then about 13 years of age. This charge was very agreeable to Mr. Hobbes's inclinations, and he diſcharged the truſt with great diligence and fidelity. In 1634 he accompanied his young pupil to Paris, where he employed his own vacant hours in the ſtudy of natural philoſophy, frequently converſing with Father Merſenne, Gaſſendi, and other eminent philoſophers there. From Paris he attended his pupil into Italy, where he became acquainted with the celebrated Galileo, who freely communicated his notions to him; and from hence he returned with his ward into England. But afterwards, foreſeeing the civil wars, he went to ſeek a retreat at Paris; where he was ſoon made acquainted with Des Cartes and the other learned philoſophers there, with whom he afterwards held a correſpondence upon ſeveral mathematical ſubjects, as appears from the letters of Mr. Hobbes publiſhed in the works of Des Cartes.

In 1642, Mr. Hobbs printed his famous book *De Cive*, which raiſed him many adverſaries, who charged him with inſtilling principles of a dangerous tendency. Among many illuſtrious perſons who, from the troubles in England, retired to France for ſafety, was Sir Charles Cavendiſh, brother to the Duke of Newcaſtle: and this gentleman, being well ſkilled in the mathematics, proved a conſtant friend and patron to Mr. Hobbes; who, by embarking in 1645 in a controverſy about ſquaring the circle, was grown ſo famous by it, that in 1647 he was recommended to inſtruct Charles prince of Wales, afterwards king Charles the 2d, in mathematical learning. During this he employed his vacant time in compoſing his Leviathan, which was publiſhed in England in 1651. After the publication of this work, he returned to England, and paſſed the remainder of his long life in a very retired and ſtudious manner, in the houſe of the Earl of Devonſhire, moſtly at his ſeat in Derbyſhire, but accompanying the earl always to London, fearing to be left out of his immediate protection, leſt he ſhould be ſeized by officers from the parliament or government, on account of the freedom of his opinions in politics and religion. He received great marks of reſpect from king Charles the 2d at the reſtoration in 1660, with a penſion of 100l. a year. From that time, till his death, he applied himſelf to his ſtudies, and in oppoſing the attacks of his adverſaries, who were very numerous: in mathematical ſubjects diſputes roſe to a great height between him and Dr. Wallis, on account of his pretended Quadrature of the Circle, Cubature of the Sphere, and Duplication of the Cube, which he obſtinately defended without ever acknowledging his error.

His long life was that of a perfectly honeſt man; a lover of his country, a good friend, charitable and obliging. He accuſtomed himſelf much more to thinking, than reading; and was fond of a well-ſelected, rather than a large library. He had a hatred to the clergy, having been perſecuted by them, on account of the freedom of his doctrine, and having a very indifferent opinion of their knowledge and their principles. In his laſt ſickneſs he was very anxious to know whether his diſeaſe was curable; and when intimations were given, that he might have eaſe, but no remedy, he ſaid, ‘ I ſhall be glad to find a hole to creep out of the world at.’ He died the 4th of Dec. 1679, at 91 years of age.

His chief publications were,

1. An Engliſh tranſlation of Thucydides's Hiſtory of the Grecian war.
2. De Mirabilibus Pecci, and Memoirs of his own Life, both in Latin verſe.
3. Elements of Philoſophy.
4. Anſwer to Sir William Davenant's Epiſtle, or Preface to Gondibert.
5. Human Nature, or the Fundamental Elements of Policy.
6. Elements of Law.
7. Leviathan; or the Matter, Form, and Power of a Commonwealth.
8. A Compendium of Ariſtotle's Rhetoric.
9. A Letter on Liberty and Neceſſity.
10. The Queſtions, concerning Neceſſity and Chance, ſtated.
11. Six Leſſons to the Profeſſors of Mathematics, of the Inſtitution of Sir Henry Saville.
12. The marks of Abſurd Geometry, &c.
13. Dialogues of Natural Philoſophy.

Beſides many other pieces on Polity, Theology, Mathematics, and other miſcellaneous ſubjects, to the number of 41.

HOBITS, in Gunnery. See Howitz.

HOGSHEAD, a meaſure, or veſſel, of wine or oil; containing the 4th part of a tun, the half of a pipe, or 63 gallons.

HOLDER (William), a learned and philoſophical Engliſhman, was born in Nottinghamſhire, educated at Cambridge, and in 1642 became rector of Blechingdon in Oxfordſhire. In 1660 he proceeded D. D. he became afterwards canon of Ely, Fellow of the Royal Society, canon of St. Paul's, ſub-dean of the royal chapel, and ſub-almoner to the king. He was a general ſcholar, a very accompliſhed perſon, and a great virtuoſo.

Dr. Holder greatly diſtinguiſhed himſelf, by giving ſpeech to a young gentleman of the name of Popham, who was born deaf. This was effected at his own houſe at Blechingdon in 1659; but the young man loſing what he had been taught by Holder after he was called home to his friends, he was ſent to Dr. Wallis, who brought him to his ſpeech again. Holder publiſhed a book, intitled " the Elements of Speech; an eſſay or inquiry into the natural Production of Letters: with an appendix, concerning perſons that are deaf and dumb, 1669," 8vo. In the appendix he relates how ſoon, and by what methods, he brought young Popham to ſpeak. In the Philoſ. Tranſ. for July 1670, was inſerted a letter from Dr. Wallis, in which he claims to himſelf the honour of bringing that gentleman to ſpeak. By way of anſwer to which, in 1678, Dr. Holder publiſhed in 4to, " A Supplement to the Philoſ. Tranſ. of July 1670, with ſome reflections on Dr. Wallis's letter there inſerted." Upon which the latter ſoon after publiſhed " A Defence of the Royal Society, and the Philoſophical Tranſactions, particularly thoſe of July 1670, in anſwer to the cavils of Dr. William Holder, 1678," 4to.

Dr. Holder's accompliſhments were very general. He was ſkilled in the theory and practice of muſic, and wrote " A Treatiſe of the Natural Grounds and Principles of Harmony, 1694," 8vo. He wrote alſo " A Treatiſe concerning Time, with applications of the Natural Day, Lunar Month, and Solar Year, &c, 1694," 8vo. He died at Amen Corner in London, Jan. 24, 1697, and was buried in St. Paul's.

HOLLOW, in Architecture, a concave moulding, about a quarter of a circle, by ſome called a Caſement, by others an Abacus.

Hollow-*Tower*, in Fortification, is a rounding made of the remainder of two briſures, to join the curtin to the crillon, where the ſmall ſhot are played, that they may not be ſo much expoſed to the view of the enemy.

HOLY *Thurſday*, otherwiſe called Aſcenſion day, being the 39th day after Eaſter Sunday, and kept in commemoration of Chriſt's aſcenſion up into heaven.

Holy *Rood*, or *Holy Croſs*, a feſtival kept on the 14th of September, in memory of the exaltation of our Saviour's croſs.

Holy *Week*, is the laſt week of Lent, called alſo Paſſion Week,

HOLYWOOD (John), or Halifax, or *Sacroboſco*, was, according to Leland, Bale, and Pitts, born at Halifax in Yorkſhire: according to Stainhurſt, at Holywood near Dublin; and according to Dempſter and Mackenzie, in Nithſdale in Scotland. Though there may perhaps have been more than one of the name. Mackenzie informs us, that having finiſhed his ſtudies, he entered into orders, and became a canon regular of the order of St. Auguſtin in the famous monaſtery of Holywood in Nithſdale. The Engliſh biographers, on the contrary, tell us that he was educated at Oxford. They all agree however in aſſerting, that he ſpent moſt of his life at Paris; where, ſays Mackenzie, he was admitted a member of the univerſity, June 5, 1221, under the ſyndics of the Scotch nation; and ſoon after was elected profeſſor of mathematics, which he taught with applauſe for many years. According to the ſame author, he died in 1256, as appears from the inſcription on his monument in the cloiſters of the convent of St. Maturine at Paris.

Holywood was contemporary with Roger Bacon, but probably older by about 20 years. He was certainly the firſt mathematician of his time; and he wrote, 1. *De Sphæra Mundi;* a work often reprinted, and illuſtrated by various commentators.—2. *De Anni Ratione, ſeu de Computo Eccleſiaſtico.*—3. *De Algoriſmo,* printed with *Comm. Petri Cirvilli Hiſp:* Paris, 1498.

HOMOCENTRIC, the ſame as Concentric.

HOMODROMUS *Vectis,* or *Lever,* in Mechanics, is a lever in which the weight and power are both on the ſame ſide of the fulcrum, as in the lever of the 2d and 3d kind; being ſo called becauſe here the weight and power move both in the ſame direction, whereas in the Heterodromus they move in oppoſite directions.

HOMOGENEAL, or HOMOGENEOUS, conſiſting of ſimilar parts, or of the ſame kind and nature, in contradiſtinction from heterogeneous, where the parts are of different kinds.——Natural bodies are uſually compoſed of Homogeneous parts, as a diamond, a metal, &c. But artificial bodies, on the contrary, are aſſemblages of heterogeneous parts, or parts of different kinds; as a building, of ſtone, wood, &c.

HOMOGENEAL *Light,* is that whoſe rays are all of one and the ſame colour, refrangibility, &c.

HOMOGENEAL *Numbers,* are thoſe of the ſame kind and nature.

HOMOGENEAL *Surds,* are ſuch as have one common radical ſign; as $\sqrt{27}$ and $\sqrt{30}$, or $\sqrt{a}$ and $\sqrt{b}$, or $2\sqrt[3]{c}$ and $\sqrt[3]{d}$.

HOMOGENEUM *Adfectionis,* a name given by Vieta to the ſecond term of a compound or affected equation, being that which makes it adfected.

HOMOGENEUM *Comparationis,* in Algebra, a name given by Vieta to the abſolute known number or term in a compound or affected equation. This he places on the right-hand ſide of the equation, and all the other terms on the left.

HOMOLOGOUS, in Geometry, is applied to the correſponding ſides of ſimilar figures, or thoſe that are oppoſite to equal or correſponding angles, and are ſo called becauſe they are proportional to each other. For all ſimilar figures have their like ſides Homologus, or proportional to one another, alſo their areas or ſurfaces are Homologous or proportional to the ſquares of the like ſides, and their ſolid contents Homologous or proportional to the cubes of the ſame.

HOOKE (ROBERT), a very eminent mathematician and philoſopher, was born, 1635, at Freſhwater in the Iſle of Wight, where his father was miniſter. He was intended for the church; but being of a weakly conſtitution, and very ſubject to the head-ach, all thoughts of that nature were laid aſide. Thus left to himſelf, the boy followed the bent of his genius, which was turned to mechanics; and employed his time in making little toys, which he did with wonderful art and dexterity. He had alſo a good turn for drawing; for which reaſon, after his father's death, which happened in 1648, he was placed with Sir Peter Lely; but the ſmell of the oil-colours increaſing his head-ach, he quitted painting in a very ſhort time. He was afterwards kindly taken by Dr. Buſby into his houſe, and ſupported there, while he attended Weſtminſter-ſchool; where he not only acquired a competent ſhare of Greek and Latin, together with an inſight into Hebrew and ſome other Oriental languages, but alſo made himſelf maſter of a good part of Euclid's Elements; and, as Wood aſſerts, invented 30 different ways of flying.

About the year 1653 he went to Chriſt-church in Oxford; and in 1655 was introduced to the Philoſophical Society there; where, diſcovering his mechanic genius, he was firſt employed to aſſiſt Dr. Willis in his chemical operations, and was afterwards recommended to Mr. Robert Boyle, whom he ſerved ſeveral years in the ſame capacity. He was alſo inſtructed in aſtronomy about this time by Dr. Seth Ward, Savilian Profeſſor of that ſcience; and from henceforward diſtinguiſhed himſelf by many noble inventions and improvements of the mechanic kind. He alſo invented ſeveral aſtronomical inſtruments, for making obſervations both at ſea and land, and was particularly ſerviceable to Mr. Boyle in completing the invention of the air-pump. In 1662 he was appointed Curator of Experiments to the Royal Society; and when that body was eſtabliſhed by royal charter, he was in the liſt of thoſe, who were firſt named by the council in May 20, 1663; and he was admitted accordingly June 3, with a peculiar exemption from all payments. Sept. 28 of the ſame year, he was named by lord Clarendon, chancellor of Oxford, for the degree of M. A.; and Oct. 19 it was ordered that the repoſitory of the Royal Society ſhould be committed to his care: the white gallery in Greſham-college being appropriated to that uſe. In May 1664, he began to read the aſtronomy lecture at Greſham college for the profeſſor Dr. Pope, then in Italy; and the ſame year he was made Profeſſor of Mechanics to the Royal Society by Sir John Cutler, with a ſalary of 50l. per annum, which that gentleman, the founder, ſettled upon him for life. Jan. 11, 1665, that ſociety granted a ſalary alſo of 30l. a year, for his office of Curator of Experiments for life; and the month of March the ſame year he was elected profeſſor of geometry in Greſham-college.

In 1665 too, he publiſhed in folio, his "Micrographia, or ſome Philoſophical Deſcriptions of Minute Bodies, made by Magnifying Glaſſes, with Obſervations and Enquiries thereupon." And the ſame year, during the receſs of the Royal Society on account of the plague, he attended Dr. Wilkins and other ingenious gentlemen into Surry, where they made ſeveral experiments. In 1666 he produced to the Royal Society a model for rebuilding the city of London, then deſtroyed by the great fire, with which the Society was well pleaſed; and the Lord Mayor and Aldermen preferred it to that of the city ſurveyor, though it happened not to be carried into execution. The rebuilding of the city according to the act of parliament requiring able perſons to ſet out the ground for the proprietors, Mr. Hooke was appointed one of the ſurveyors; an employment in which he got moſt part of his eſtate, as appeared from a large iron cheſt of money found after his death, locked down with a key in it, and a date of the time, which ſhewed it to have been ſo ſhut up above 30 years. From 1668 he was engaged for many years in a warm conteſt with Hevelius, concerning the difference in accuracy between obſerving with aſtronomical inſtruments with plain and teleſcopic ſights; in which diſpute many learned men afterwards engaged, and in which Hooke managed ſo ill, as to be univerſally condemned, though it has ſince been agreed that he had the better ſide of the queſtion.—In 1771 he attacked Newton's "New Theory of Light and Colours;" where, though he was obliged to ſubmit in reſpect to the argument, it is ſaid he came off with more credit. The Royal Society having commenced their meetings at Greſham-college, November 1674, the Committee in December allowed him 40l. to erect a turret over part of his lodgings, for trying his inſtruments, and making aſtronomical obſervations: and the year following he publiſhed "A Deſcription of Teleſcopes, and ſome other inſtruments made by R. H. with a Poſtſcript," complaining of ſome injuſtice done him by their ſecretary Mr. Oldenburg, who publiſhed the Philoſophical Tranſactions, in regard to his invention of pendulum watches. This charge drew him into a diſpute with that gentleman, which ended in a declaration of the Royal Society in their ſecretary's favour.—Mr. Oldenburg dying in 1677, Mr. Hooke was appointed to ſupply his place, and began to take minutes at the meeting in October, but did not publiſh the Tranſactions.—Soon after this, he grew more reſerved than formerly; and though he read his Cutlerian Lectures, often made experiments, and ſhewed new inventions before the Royal Society, yet he ſeldom left any account of them to be entered in their regiſters; deſigning, as he ſaid, to publiſh them himſelf, which however he never performed.—In 1686, when Newton's work the Principia was publiſhed, Hooke laid claim to his diſcovery concerning the force and action of gravity, which was warmly reſented by that great philoſopher. Hooke, though a great inventor and diſcoverer himſelf, was yet ſo envious and ambitious, that he would fain have been thought the only man who could invent and diſcover. This made him often lay claim to the inventions and diſcoveries of other perſons; on which occaſions however, as well as in the preſent caſe, the thing was generally carried againſt him.

In the beginning of the year 1687, his brother's daughter, Mrs. Grace Hooke, who had lived with him ſeveral years, died: and he was ſo affected with grief at her death, that he hardly ever recovered it, but was obſerved from that time to become leſs active, more melancholy, and more cynical than ever. At the ſame time, a chancery ſuit in which he was concerned with Sir John Cutler, on account of his ſalary for reading the Cutlerian Lectures, made him uneaſy, and increaſed his diſorder.—In 1691, he was employed in forming the plan of the hoſpital near Hoxton, founded by Robert Aſk, alderman of London, who appointed archbiſhop Tillotſon one of his executors; and in December the ſame year, Hooke was created M. D. by a warrant from that prelate. In July 1696, the chancery ſuit with Sir John Cutler was determined in his favour, to his inexpreſſible ſatisfaction. His joy on that occaſion was found in his diary thus expreſſed; DOMSHLGISSA; that is, *Deo, Optimo, Maximo, ſit honor, laus, gloria, in ſæcula ſæculorum, Amen.* "I was born on this day of July 1635, and God hath given me a new birth: may I never forget his mercies to me! while he gives me breath may I praiſe him!"—In the ſame year 1696, an order was granted to him for repeating moſt of his experiments at the expence of the Royal Society, upon a promiſe of his finiſhing the accounts, obſervations, and deductions from them, and of perfecting the deſcription of all the inſtruments contrived by him: but his increaſing illneſs and general decay rendered him unable to perform it. He continued ſome years in this waſting condition; and thus languiſhing till he was quite emaciated, he died March 3, 1702, in his 67th year, at his lodgings in Greſham college, and was buried in St. Helen's church, Biſhopſgate ſtreet; his corps being attended by all the members of the Royal Society then in London.

As to Mr. Hooke's character, it is not in all reſpects one of the moſt amiable. In his perſon he made rather a deſpicable figure, being but of a ſhort ſtature, very crooked, pale, lean, and of a meagre aſpect, with dark-brown hair, very long, and hanging over his face lank and uncut. Suitable to his perſon, his temper was penurious, melancholy, and miſtruſtful: and, though poſſeſſed of great philoſophical knowledge, he had ſo much ambition, that he would be thought the only man who could invent or diſcover; and hence he often laid claim to the inventions and diſcoveries of others, while he boaſted of many of his own which he never communicated. In the religious part of his character, he was ſo exemplary, that he always expreſſed a great veneration for the Deity; and ſeldom received any remarkable benefit in life, or made any conſiderable diſcovery in nature, or invented any uſeful contrivance, or found out any difficult problem, without ſetting down his acknowledgment to God, as many places in his diary plainly ſhew.—His chief publications are,

1. *Lectiones Cutlerianæ*, or the Cutlerian Lectures.
2. *Micrographia*, or Deſcriptions of Minute Bodies made by Magnifying Glaſſes.
3. A Deſcription of Helioſcopes.
4. A Deſcription of ſome Mechanical Improvements of Lamps and Water-poiſes.
5. Philoſophical Collections.
6. Poſthumous Works, collected from his papers by Richard Waller ſecretary to the Royal Society. Beſides a number of papers in the Philoſ. Tranſ. volumes 1, 2, 3, 5, 6, 9, 16, 17, 22.

HORARY, ſomething relating to Hours. As,

HORARY *Circles*, hour lines or circles, marking the hours, or drawn at the diſtance of hours from one another.

HORARY *Motion*, is the motion or ſpace moved in an hour. Thus, the Horary motion of the earth on her axis, is 15°; for, completing her revolution of 360°, in 24 hours, therefore the motion in one hour will be the 24th part of 360°, which is 15 degrees.

HORIZON, in Aſtronomy, a great circle of the ſphere, dividing the world into two parts, or hemiſpheres; the one upper, and viſible; the other lower, and hid.

The Horizon is either Rational or Senſible.

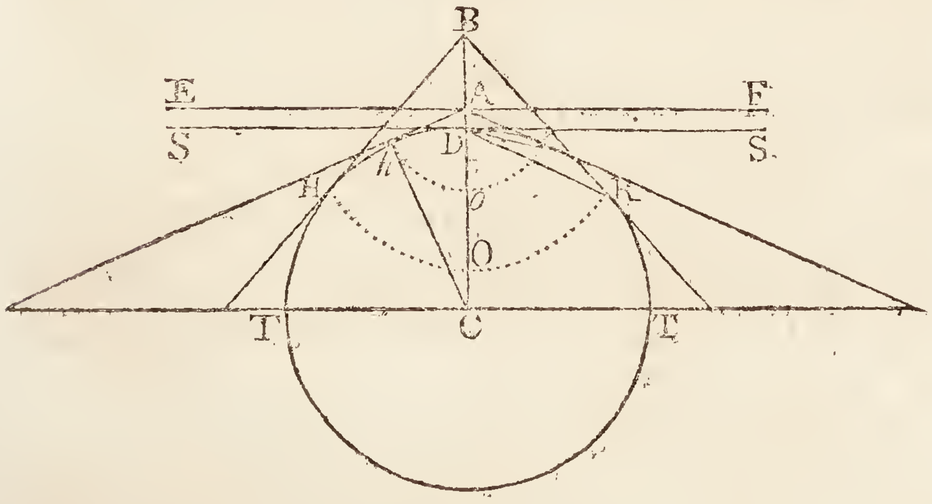
HORIZON, *Rational*, *True*, or *Aſtronomical*, called alſo ſimply and abſolutely the Horizon, is a great circle having its plane paſſing through the centre of the earth, and its poles are the zenith and nadir. Hence all the points of the Horizon, quite around, are at a quadrant diſtance from the zenith and nadir. Alſo the meridian and vertical circles cut the Horizon at right angles, and into two equal parts.

HORIZON, *Apparent*, *Senſible*, or *Viſible*, is a leſſer circle of the ſphere, parallel to the rational Horizon, dividing the viſible part of the ſphere from the inviſible, and whoſe plane touches the ſpherical ſurface of the earth.

The ſenſible Horizon is divided into Eaſtern and Weſtern; the Eaſtern or Ortive being that in which the heavenly bodies riſe; and the Weſtern, or Occidual, being that in which they ſet.

HORIZON, in Geography, is a circle dividing the viſible part of the earth and heavens from that which is inviſible. This is peculiarly called Senſible or Apparent Horizon, to diſtinguiſh it from the Rational or True, which paſſes through the centre of the earth; as already obſerved. Theſe two Horizons, though diſtant from one another by the ſemidiameter of the earth, will appear to coincide when continued to the ſphere of the fixed ſtars; becauſe the earth compared with this ſphere is but a point.

By Senſible Horizon is alſo often meant a circle which determines the ſegment of the ſurface of the earth, over which the eye can reach; called alſo the Phyſical Horizon. And in this ſenſe we ſay, a ſpacious Horizon, a narrow or ſcanty Horizon, &c; depending chiefly on the height the eye is elevated above the earth.



For, it is evident that the higher the eye is placed, the farther is the viſible Horizon extended. Thus, if the eye be at A, at the height AD above the earth; draw the two tangents A*h*, A*r*; and let one of theſe lines A*h*, be moved round the point A, and in its revolution always touch the ſurface of the earth; then the other point *h* will deſcribe the viſible Horizon *h o r*, &c. But if the eye be placed higher as at B, the tangents BH and BR will reach farther, and the viſible Horizon HOR will be larger.

The viſible Horizon is moſt accurately obſerved at ſea, and is therefore ſometimes called the Horizon of the ſea. In obſerving this Horizon, the viſual rays A*h* and A*r* will, on account of the curve ſurface of the ſea, always point a little below the true ſenſible Horizon SS or EF, and conſequently below the rational Horizon TT, which is parallel to it.

To find the Depreſſion of the Horizon of the ſea below the true Horizon, which varies with the height of the eye, and in a ſmall degree with the variation of the refractive power of the atmoſphere, ſee DEPRESSION.

As to the right-lined diſtance, or tangent E*h*, it may be found thus; as radius : ſin. $\angle$ C : : CA : A*h*,
or thus; as radius : tan. $\angle$ C : : C*h* : A*h*,
either of which will be nearly the ſame as the arc or curved diſtance D*h*. Or, without finding the angle C, thus; the ſquare of A*h* is equal to the difference of the ſquares of CA and C*h*, i. e. $Ah^2 = CA^2 - Ch^2 = \overline{CA + Ch} \times \overline{CA - Ch} = \overline{CA + Ch} \times AD$, and hence $Ah = \sqrt{\overline{CA + Ch} \times AD}$, which is alſo equal to A*h* nearly.

The diſtance on a perfect globe, if the viſual rays came to the eye in a ſtraight line, would be as above ſtated: but by means of the refraction of the atmoſphere, diſtant objects on the Horizon appear higher than they really are, or appear leſs depreſſed below the true Horizon SS, and may be ſeen at a greater diſtance, eſpecially on the ſea. M. Legendre, in his Memoir on Meaſurements of the Earth, in the Mem. Acad. Sci. for the year 1787, ſays that, from ſeveral experiments, he is induced to allow for refraction a 14th part of the diſtance of the place obſerved, expreſſed in degrees and minutes of a great circle. Thus, if the diſtance be 14000 toiſes, the refraction will be 1000 toiſes, equal to the 57th part of a degree, or 1′ 3″.

HORIZON *of the Globe*, a broad wooden circle. See GLOBE.

HORIZONTAL, ſomething that relates to the Horizon, or that is taken in the Horizon, or on a level with or parallel to it. Thus, we ſay, a Horizontal plane, Horizontal line, Horizontal diſtance, &c.

HORIZONTAL *Dial*, is one drawn on a plane parallel to the horizon; having its gnomon or ſtyle elevated according to the altitude of the pole of the place it is deſigned for.

HORIZONTAL *Diſtance*, is that eſtimated in the direction of the horizon.

HORIZONTAL *Line*, in Perſpective, is a right line drawn through the principal point, parallel to the horizon; or it is the interſection of the Horizontal and perſpective planes.

HORIZONTAL *Line*, or baſe of a hill, in Surveying, a line drawn on the Horizontal plane of the hill, or that on which it ſtands.

HORIZONTAL *Moon*. See *Apparent* MAGNITUDE.

HORIZONTAL *Parallax*. See PARALLAX.

HORIZONTAL *Plane*, is that which is parallel to the horizon of the place, or not inclined to it.

Horizontal *Plane*, in Perſpective. See Plane.

Horizontal *Projection*. See Projection, and Map.

Horizontal *Range*, of a piece of ordnance, is the diſtance at which it falls on, or ſtrikes the horizon, or on a Horizontal plane, whatever be the angle of elevation or direction of the piece. When the piece is pointed parallel to the horizon, the range is then called the point-blank or point-blanc range.

The greateſt Horizontal range, in the parabolic theory, or in a vacuum, is that made with the piece elevated to 45 degrees, and is equal to double the height from which a heavy body muſt freely fall to acquire the velocity with which the ſhot is diſcharged. Thus, a ſhot being diſcharged with the velocity of v feet per ſecond; becauſe gravity generates the velocity $2g$ or $32\frac{1}{6}$ feet in the firſt ſecond of time, by falling $16\frac{1}{12}$ or g feet, and becauſe the ſpaces deſcended are as the ſquares of the velocities, therefore as $4g^2 : v^2 :: g : \frac{v^2}{4g}$ the ſpace a body muſt deſcend to acquire the velocity v of the ſhot or the ſpace due to the velocity v; conſequently the double of this, or $\frac{v^2}{2g} = \frac{v^2}{32\frac{1}{6}}$ is the greateſt Horizontal range with the velocity v, or at an elevation of 45 degrees; which is nearly half the ſquare of a quarter of the velocity.

In other elevations, the Horizontal range is as the ſine of double the angle of elevation; ſo that, any other elevation being e, it will be,

as radius $1 : \text{ſin. } 2e :: \frac{v^2}{32\frac{1}{6}} : \frac{v^2}{32\frac{1}{6}} \times \text{ſin. } 2e$, the range at the elevation e, with the velocity v.

But in a reſiſting medium, like the atmoſphere, the actual ranges fall far ſhort of the above theorems, in ſo much that with the great velocities, the actual or real ranges may be leſs than the 10th part of the potential ranges; ſo that ſome balls, which actually range but a mile or two, would in vacuo range 20 or 30 miles. And hence alſo it happens that the elevation of the piece, to ſhoot fartheſt in the reſiſting medium, is always below 45°, and gradually the more below it as the velocity is greater, ſo that the greater velocities with which balls are diſcharged from cannon with gunpowder, require an elevation of the gun equal to but about 30°, or even leſs. And the leſs the ſize of the balls is too, the leſs muſt this angle of elevation be, to ſhoot the fartheſt with a given velocity. See Projectile, and Gunnery.

Horizontal *Refraction*. See Refraction.

Horizontal *Speculum*, one to find a horizon at Sea, &c, when the atmoſphere is hazy near the horizon, by which the ſight of it is prevented.

A ſpeculum of this kind was invented by a Mr. Serſon, on the principle of a top ſpinning, which always keeps its upright poſition, notwithſtanding the motion of the ſubſtance it ſpins upon. This curious inſtrument, as it has ſince been improved by Mr. Smeaton, conſiſts of a well poliſhed metal ſpeculum, of about 3 inches and a half in diameter, incloſed within a circular rim of braſs; ſo fitted that the centre of gravity of the whole ſhall fall near the point on which it ſpins. This is the end of a ſteel axis running through the centre of the ſpeculum, above which it finiſhes in a ſquare, for the conveniency of fitting a roller on it, which ſets it in motion by means of a piece of tape wound about the roller.

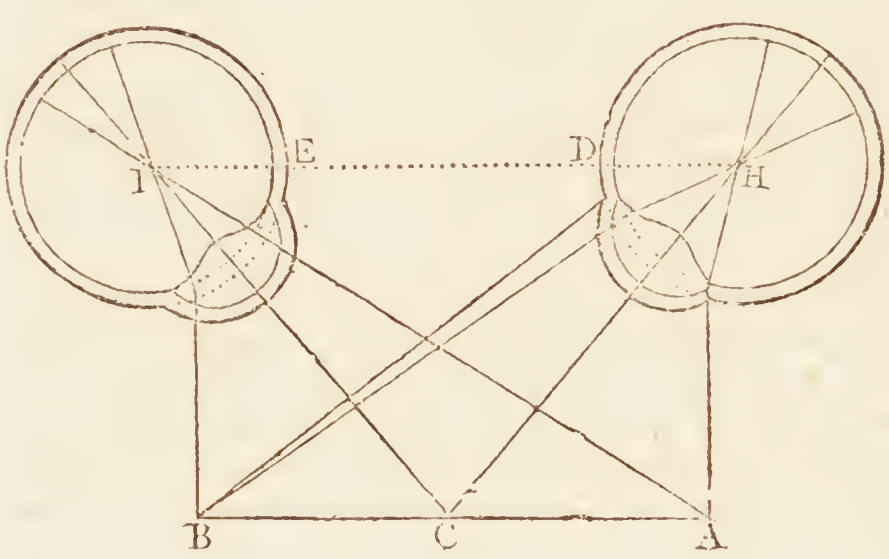
Various other contrivances to form artificial horizons have been invented by different perſons, as glaſs planes floating on mercury, &c. See Hadley's Quadrant, and ſeveral inventions of this ſort in the Philoſ. Tranſ. by Elton, Halley, Leigh, &c. vol. xxxvii, p. 273, vol. xxxviii, p. 167, vol. xl, p. 413, 417, &c.

HORN-Work, in Fortification, a ſort of out-work, advancing toward the field, to cover and defend a curtin, baſtion, or other place, ſuſpected to be weaker than the reſt; as alſo to poſſeſs a height; carrying in the fore-part, or head, two demi-baſtions, reſembling horns: theſe horns, epaulments, or ſhoulderings, being joined by a curtin, ſhut up on the ſide by two wings, parallel to one another, are terminated at the gorge of the work, and ſo preſent themſelves to the enemy.

HOROGRAPHY, the art of making or conſtructing dials; called alſo Dialling, Horologiography, Gnomonica, Sciatherica, Photoſciatherica, &c.

HOROLOGIUM, a common name, among ancient writers, for any inſtrument or machine for meaſuring the hours. See Clock, Watch, Sun-Dial, Chronometer, Clepsydra, &c.

HOROMETRY, the art of meaſuring or dividing time by hours, and keeping the account of time.



HOROPTER, in Optics, is a right line drawn through the point where the two optic axes meet, parallel to that which joins the centres of the two eyes, or the two pupils. As the line AB drawn through C the point of concourſe of the optic axes of the eyes, and parallel to HI joining the centres of the eyes.— This line is called the Horopter, becauſe it is found to be the limit of diſtinct viſion. It has ſeveral properties in Optics, which are deſcribed at large in Aguillonius, Opt. lib. 2, diſs. 10.

HOROSCOPE, in Aſtrology, is the aſcendant or firſt houſe, being that part of the zodiac which is juſt riſing in the eaſtern ſide of the horizon at any propoſed time, when a ſcheme is to be ſet or calculated, or a prediction made of any event. See Ascendant.

Horoscope is alſo uſed for a ſcheme or figure of the 12 celeſtial houſes; i. e. the 12 ſigns of the zodiac, in which is marked the diſpoſition of the heavens for any given time. Thus it is ſaid, To draw or conſtruct a Horoſcope or ſcheme, &c. And it is more peculiarly called, Calculating a nativity, when the life and fortune of a perſon are the ſubject of prediction.

Lunar Horoscope, is the point the moon iſſues out of, when the ſun is in the aſcending point of the eaſt; and is alſo called the Part of Fortune.

Horoscope was alſo a mathematical inſtrument, in manner of a planiſphere; but now diſuſed. It was invented by J. Paduanus, who wrote a ſpecial treatiſe upon it.

HORROR *of a Vacuum*, an imaginary principle among the more ancient philoſophers, to which they aſcribed the aſcent of water in pumps, and other ſimilar phenomena, which are now known to be occaſioned by the weight of the air.

HORROX (Jeremiah), an eminent Engliſh aſtronomer, was born at Toxteth in Lancaſhire, about the year 1619. From a grammar ſchool in the country, he was ſent to Cambridge, where he ſpent ſome time in academical ſtudies. About 1633 he began to apply himſelf to the ſtudy of aſtronomy: but living at that time with his father at Toxteth, in very moderate circumſtances, and being deſtitute of books and other aſſiſtances for ſuch ſtudies, he could not make any conſiderable progreſs in it. About the year 1636, he formed an acquaintance with Mr. William Crabtree, of Broughton near Mancheſter, who was engaged in the ſame ſtudies, with whom a mutual correſpondence was carried on till his death; ſometimes communicating their improvements to Mr. Samuel Foſter, profeſſor of geometry at Greſham College in London. Having now obtained a companion in his ſtudies, Mr. Horrox aſſumed new vigour, procured other inſtruments and books, and was purſuing his ſtudies and obſervations with great aſſiduity, when he was ſuddenly cut off by death, the 3d of January 1640, in the 22d year of his age.

What we have of his writings is ſufficient to ſhew how great a loſs the world had by his death. He had juſt finiſhed his *Venus in Sole viſa*, 1639, a little before, as appears by ſome of the letters to his friend Mr. Crabtree, by which alſo it appears that he made his obſervations on that phenomenon at Hool near Liverpool. This tract, of Venus ſeen in the Sun, was publiſhed at Dantzick in 1668, by Hevelius, together with his own *Mercurius in Sole viſus* May 3, 1661. His other poſthumous works, or rather his imperfect papers, were publiſhed by Dr. Wallis, in 1673, 4to, with ſome account of his life; in which we find he firſt aſſerts and promotes the Keplerian aſtronomy againſt the hypotheſis of Lanſberg; which he proves to be inconſiſtent with itſelf, and neither agreeing with obſervations nor theory. He likewiſe reaſons very juſtly concerning the celeſtial bodies and their motions, vindicates Tycho Brahe from ſome objections made to his hypothesis, and gives a new theory of the moon: to which are added the Lunar Numbers of Mr. Flamſteed. There are alſo extracts from ſeveral letters between him and Mr. Crabtree, upon various aſtronomical ſubjects; with a catalogue of aſtronomical obſervations.

There are two things particularly which will perpetuate the memory of this very extraordinary young man. The one is, that he was the firſt that ever predicted or ſaw the planet Venus in the ſun; for we do not find that any perſons, beſides himſelf and Mr. Crabtree, ever beheld ſuch a phenomenon. Though he was not appriſed of the great uſe that was to be made of it, in diſcovering the parallax and diſtance of the ſun and planets, yet he made from it many uſeful obſervations, corrections, and improvements in the theory of the motions of Venus.—Secondly, his New Theory of Lunar Motions, which Newton himſelf made the ground work of all his aſtronomy, relative to the moon, who always ſpoke of our author as a genius of the firſt rank.

HORSE-Shoe, in Fortification, is a work ſometimes of a round, ſometimes of an oval figure, incloſed with a parapet, raiſed in the ditch of a marſhy place, or in low grounds; ſometimes alſo to cover a gate; or to ſerve as a lodgment for ſoldiers, to prevent ſurpriſes, or relieve an over-tedious defence.

HOSPITAL, *or* Hopital (William-Francis-Anthony, *marquis of)*, a celebrated French mathematician, was born of an ancient family in 1661. He was a mathematician almoſt from his infancy; for being one day at the duke of Rohan's, where ſome able mathematicians were ſpeaking of a problem of Paſcal's, which appeared to them very difficult, he ventured to ſay, that he believed he could reſolve it. They were ſurpriſed at ſuch preſumption in a boy of 15, for he was then no more; however, in a few days he ſent them the ſolution.

M. l'Hoſpital entered early into the army, and was a captain of horſe; but being very ſhort-ſighted, and on that account expoſed to perpetual inconveniences and errors, he at length quitted the army, and applied himſelf entirely to his favourite amuſement.—He contracted a friendſhip with Malbranche, and took his opinion upon all occaſions.—In 1699 he was received an honorary member of the Academy of Sciences at Paris.

He was the firſt perſon in France who wrote upon Newton's analyſis, and on this account was regarded almoſt as a prodigy. His work was entitled *l'Analyſe des Infinimens Petits*, 1696. He engaged afterwards in another mathematical work, in which he included *Les Sections Coniques, les Lieux Géometriques, la Conſtruction des Equations, et une Théorie des Courbes Mechaniques*: but, a little before he had finiſhed it he was ſeized with a fever, which carried him off, the 2d of February 1704, at 43 years of age. The work was publiſhed after his death, viz, in 1707. There are alſo ſix of his pieces inſerted in different volumes of the Memoirs of the Academy of Sciences.

HOUR, in Chronology, an aliquot part of a natural day, uſually the 24th, but ſometimes a 12th part. With us, it is the 24th part of the earth's diurnal rotation, or the time from noon to noon, and therefore it anſwers to 15 degrees of the whole circle of longitude, or of 360°. The hour is divided by 60ths, viz, firſt into 60 minutes, then each minute into 60 ſeconds, &c.

The diviſion of the day into Hours is very ancient; as is ſhewn by Kircher, Oedip. Ægypt. tom. 2, par. 2, claſs 7, cap. 8. The moſt ancient Hour is that of the 12th part of the day. Herodotus, lib. 2, obſerves, that the Greeks learnt from the Egyptians, among other things, the method of dividing the day into 12 parts. And the aſtronomers of Cathaya, &c, ſtill retain this diviſion.

The diviſion of the day into 24 hours, was not known to the Romans before the Punic war. Till that time they only regulated their days by the riſing and ſetting of the ſun. They divided the 12 hours of their day into four; viz, Prime, which commenced at 6 o'clock, Third at 9, Sixth at 12, and None at 3. They alſo divided the night into four watches, each containing 3 Hours.

There are various kinds of Hours, uſed by chronologers, aſtronomers, dialiſts, &c. Sometimes too,

Hours are divided into Equal and Unequal.

Equal Hours, are the 24th parts of a day and night preciſely; that is, the time in which the 15 degrees of the equator paſs the meridian. Theſe are alſo called Equinoctial Hours, becauſe meaſured on the equinoctial; and Aſtronomical, becauſe uſed by aſtronomers.

Aſtronomical Hours, are equal Hours, reckoned from noon to noon, in a continued ſeries of 24.

Babyloniſh Hours, are equal Hours, reckoned from ſun-riſe in a continued ſeries of 24.

European Hours, uſed in civil computation, are equal Hours, reckoned from midnight; 12 from thence till noon, and 12 more from noon till midnight.

Jewiſh, or *Planetary*, or *Ancient* Hours, are 12th parts of the artificial day and night. They are called Ancient or Jewiſh Hours, becauſe uſed by the ancients, and ſtill among the Jews. They are called Planetary Hours, becauſe the aſtrologers pretend, that a new planet comes to predominate every Hour; and that the day takes its denomination from that which predominates the firſt Hour of it; as Monday from the moon, &c.

Italian Hours, are equal Hours, reckoned from ſunſet, in a continued ſeries of 24.

Unequal or *Temporary* Hours, are 12th parts of the artificial day and night. The obliquity of the ſphere renders theſe more or leſs unequal at different times; ſo that they only agree with the equal Hours at the times of the equinoxes.

Hour-*Circles*, or Horary-*Circles*, are great Circles, meeting in the poles of the globe or world, and croſſing the equinoctial or equator at right angles; the ſame as meridians. They are ſuppoſed to be drawn through every 15th degree of the equinoctial and equator, each anſwering to an hour, and dividing them into 24 equal parts; and on both globes they are ſupplied by the meridian Hour-circle and index.

Hour-*Glaſs*, a popular kind of chronometer or clepſydra, ſerving to meaſure time by the deſcent or running of ſand, water, &c, out of one glaſs veſſel into another.—The beſt, it is ſaid, are ſuch as, inſtead of ſand, have egg-ſhells, well dried in the oven, then beaten fine and ſifted.

Hour-*Lines*, on a Dial, are lines which ariſe from the interſections of the plane of the Dial, with the ſeveral planes of the Hour-circles of the ſphere; and therefore muſt be all right lines on a plane Dial.

Hour-*Scale*, a divided line on the edge of Collins's quadrant, being only two lines of tangents of 45 degrees each, ſet together in the middle. Its uſe, together with the lines of latitude, is to draw the Hour-lines of Dials that have centres, by means of an equilateral triangle, drawn on the dial-planes.

HOWITZ, or Howitzer, in Artillery, a kind of mortar, or ſomething between a cannon and mortar, partaking of the nature of both, being either a very ſhort gun or a long mortar. It is of German invention, and is mounted upon a carriage like a travelling gun-carriage, with its trunnions placed nearly in the middle. The Howitz is one of the moſt uſeful kinds of ordnance, as it can be employed occaſionally either as a cannon or mortar, diſcharging either ſhells or grape ſhot, as well as balls, and ſo doing great execution. They are alſo very eaſily travelled about from place to place.

HUMIDITY, or moiſture, the power or quality of wetting or moiſtening other bodies, and adhering to them.

Fluids are moiſt to ſome bodies, and not to others. Thus, quickſilver is not moiſt in reſpect to our hands or clothes, and other things, which it will not ſtick to; but it may be called Humid in reference to gold, tin, or lead, to the ſurfaces of which it will preſently adhere, and render them ſoft and moiſt. Even water itſelf, which wets almoſt every thing, and is the great ſtandard of moiſture and Humidity, is not capable of wetting all things; for it ſtands or runs off in globular drops from any thing greaſed or oiled, or the leaves of cabbages, and many other planets; and it will not wet the feathers of ducks, geeſe, ſwans, and other water-fowl.

HUNDRED, the number of ten times ten, or the ſquare of 10. The place of Hundreds makes the third in order in the Arabic or modern numeration, being denoted thus 100. In the Roman notation it is denoted by the letter C, being the initial of its name, Centum.

Hundred *Weight*, or the great Hundred, contains 112 pounds weight. It is ſubdivided into 4 quarters, and each quarter into 28 lbs.

HURTERS, in Fortification, denote pieces of timber, about 6 inches ſquare, placed at the lower end of the platform, next to the parapet, to prevent the wheels of the gun-carriages from damaging the parapet.

HUYGENS (Christian), a very eminent aſtronomer and mathematician, was born at the Hague in Holland, in 1629, being the ſon of Conſtantine Huygens, lord of Zuylichem, who had ſerved three ſucceſſive princes of Orange in the quality of ſecretary. He ſpent his whole life in cultivating the mathematics; and not in the ſpeculative way only, but alſo in making them ſubſervient to the uſes of life. From his infancy he diſcovered an extraordinary fondneſs for the mathematics; in a ſhort time made a great progreſs in them; and perfected himſelf in thoſe ſtudies under profeſſor Schooten, at Leyden. In 1649 he went to Holſtein and Denmark, in the retinue of Henry count of Naſſau; and was extremely deſirous of going to Sweden, to viſit Des Cartes who was then in that country with the queen Chriſtina, but the count's ſhort ſtay in Denmark would not permit him.—In 1651 he gave the world a ſpecimen of his genius for mathematics, in a treatiſe intitled, *Theoremata de Quadratura Hyperboles, Ellipſis, & Circuli, ex dato Portionum Gravitatis Centro*; in which he clearly ſhewed what might be expected from him afterwards.——In 1655 he travelled into France, and took the degree of LL.D. at Angers.—In 1658 he publiſhed his *Horologium Oſcillatorium, ſive de Motu Pendulorum*, &c, at the Hague. He had exhibited in a former work, intitled, *Brevis Inſtitutio de Uſu Horologiorum ad inveniendas Longitudines*, a model of a new invented pendulum; but as ſome perſons, envious of his reputation, were labouring to deprive him of the honour of the invention, he wrote this book to explain the conſtruction of it; and to ſhew that it was very different from the pendulum of aſtronomers invented by Galileo.—In 1659 he publiſhed his *Syſtema Saturninum*, &c; in which he firſt of any one explained the ring of Saturn, and diſcovered alſo one of the ſatellites belonging to that planet, which had hitherto eſcaped the eyes of aſtronomers: new diſcoveries, made with glaſſes of his own forming, which gained him a high rank among the aſtronomers of his time.

In 1660, he took a ſecond journey into France, and the year after paſſed over into England, where he communicated his art of poliſhing glaſſes for teleſcopes, and was made Fellow of the Royal Society. About this time the air-pump was invented, which received conſiderable improvements from him. This year alſo he diſcovered the laws of the colliſion of elaſtic bodies; as did alſo about this time Wallis and Wren, with whom he had a diſpute about the honour of this diſcovery. Upon his return to France, in 1663, the miniſter Colbert, being informed of his great merit, ſettled a conſiderable penſion upon him, to engage him to fix at Paris; to which Mr. Huygens conſented, and ſtaid there from the year 1666 to 1681, where he was admitted a member of the Academy of Sciences. All this time he ſpent in mathematical purſuits, wrote ſeveral books, which were publiſhed from time to time, and invented and perfected ſeveral uſeful inſtruments and machines: particularly he had a diſpute, about the year 1668, with Mr. James Gregory, concerning the Quadrature of the Circle and Hyperbola of the latter, then juſt publiſhed, in which Huygens it ſeems had the better ſide of the queſtion. But continual application gradually impaired his health; and though he had viſited his native country twice, viz, in 1670 and 1675, for the recovery of it, he was now obliged to betake himſelf to it altogether. Accordingly he left Paris in 1681, and retired to his own country, where he ſpent the remainder of his life in the ſame purſuits and employments. He died at the Hague, June 8, 1695, in the 67th year of his age, while his *Coſmotheoros*, or treatiſe concerning a plurality of worlds, was printing; ſo that this work did not appear till 1698.

Mr. Huygens loved a quiet and ſtudious manner of life, and frequently retired into the country to avoid interruption, but did not contract that moroſeneſs which is ſo commonly the effect of ſolitude and retirement. He was one of the pureſt and moſt ingenious mathematicians of his age, and indeed of any other; and made many valuable diſcoveries. He was the firſt who diſcovered Saturn's ring, and a third ſatellite of that planet, as mentioned above. He invented the means of rendering clocks exact, by applying the pendulum, and of rendering all its vibrations equal, by the cycloid. He brought teleſcopes to perfection, and made many other uſeful diſcoveries.

He was the author of many excellent works. The principal of theſe are now contained in two collections, of 2 volumes each, printed in 4to, under the care of profeſſor Graveſande. The firſt was at Leyden in 1682, under the title of *Opera Varia*; and the ſecond at Amſterdam, in 1728, entitled *Opera Reliqua*.

HYADES, a cluſter of 5 ſtars in the face of the conſtellation Taurus, or the Bull.

HYALOIDES, the vitreous humour of the eye contained between the tunica-retina and the uvea.

HYBERNAL *Occident*. See *Occident*.

Hybernal *Orient*. See *Orient*.

HYDATOIDES, the watery humour of the eye contained between the cornea and the uvea.

HYDRA, a ſouthern conſtellation, conſiſting of a number of ſtars, imagined to repreſent a water ſerpent. The ſtars in the conſtellation Hydra, in Ptolomy's catalogue, are 27; in Tycho's, 19; in Hevelius's, 31; and in the Britannic catalogue, 60.

HYDRAULICS, the ſcience of the motion of water and other fluids, with its application in artificial water-works of all ſorts.—As to what reſpects merely the equilibrium of fluids, or their gravitation or action at reſt, belongs to Hydroſtatics. Upon removing or deſtroying that equilibrium, motion enſues; and here Hydraulics commence. Hydraulics therefore ſuppoſe Hydroſtatics; and many writers, from the near relation between them, like mechanics and ſtatics, join the two together, and treat of them conjointly as one ſcience.

The laws of Hydraulics are given under the word Fluid. And the art of raiſing water, with the ſeveral machines employed for that purpoſe, are deſcribed under their ſeveral names, Fountain, Hydrocaniſterium Pump, Siphon, Syringe, &c.

The principal writers who have cultivated and improved Hydraulics and Hydroſtatics, are Archimedes, in his Libris de Inſidentibus Humido; Hero of Alexandria, in his Liber Spiritualium; Marinus Ghetaldus, in his Archimedes promotus; Mr. Oughtred; Jo. Ceva, in his Geometria Motus; Jo. Bap. Balianus, De Motu Naturali Gravium, Solidorum et Liquidorum; Mariotte, in his treatiſe of the Motion of Water and other fluids; Boyle, in his Hydroſtatical Paradoxes; Fran. Tertius de Lanis, in his Magiſterium Naturæ et Artis; Lamy, in his Traité de l'Equilibre des Liqueurs; Rohault; Dr. Wallis, in his Mechanics; Dechales; Newton, in his Principia; Gulielmeni, in his Menſura Aquarum Fluentium; Herman; Wolfius; Graveſande; Muſſchenbroek; Leopold; Schottus, in his Mechanica Hydraulico-Pneumatica; Geo. Andr. Bockler, in his Architectura Curioſa Germanica; Auguſt. Rammilleis; Lucas Antonius Portius; Sturmy, in his treatiſe on the Conſtruction of Mills; Switzer's Hydroſtatics; Varignon, in the Mem. Acad. Sci.; Jurin; Belidor; Bernoulli; Deſaguliers; Clare; Emerſon; Ferguſon; Ximenes; Boſſu; D'Alembert; Buat; &c, &c.

HYDRAULICO-Pneumatical, a term applied by ſome authors to ſuch engines as raiſe water by means of the weight or ſpring of the air.

HYDROGRAPHICAL *Charts* or *Maps*, more uſually called ſea-charts, are projections of ſome part of the ſea, or coaſt, for the uſe of navigation.

In theſe are laid down all the rhumbs or points of the compaſs, the meridians, parallels, &c, with the coaſts, capes, iſlands, rocks, ſhoals, ſhallows, &c, in their proper places, and proportions.

The making and ſelling theſe charts was for ſome time the employment of Columbus, the firſt diſcoverer of America. The ſtory goes, that happening to be heir to the memoirs and journals of one Alonzo Sanchez de Huelva, a noted pilot and captain of a ſhip, who by chance had been driven by a ſtorm to the iſland of St. Domingo, and dying at Columbus's houſe ſoon after his return, this gave Columbus the firſt hint to attempt a diſcovery of the Weſt Indies.

For the conſtruction and uſe of the ſeveral kinds of Hydrographical Maps, ſee CHART, and SAILING.

HYDROLOGY, is that part of natural hiſtory which examines and explains the nature and properties of water in general.

HYDROMANCY, the act or art of divining or foretelling future events by means of water.

This is one of the four general kinds of divination: the other three reſpecting the other elements, fire, air, and earth, are denominated reſpectively pyromancy, aeromancy, and geomancy.

Varro mentions the Perſians as the firſt inventors of Hydromancy, adding, that Numa Pompilius and Pythagoras made uſe of it.

The writers on optics furniſh us with divers Hydromantic machines, veſſels, &c. For example,

To conſtruct an Hydromantic machine, by means of which an image or object ſhall be removed out of the ſight of the ſpectator, and reſtored again at pleaſure, without altering the poſition, either of the one or the other. Provide two veſſels, ABF, CGLK (Plate 12, fig. 3), the uppermoſt filled with water, and ſupported by three little pillars, one of which BC is hollow, and furniſhed with a cock B. Let the lower veſſel CL be divided by a partition HI into two parts, the lower of which may be opened or cloſed by means of a cock at P. Upon the partition place an object, or image, which the ſpectator at O cannot ſee by a direct ray GL.

If now the cock B be opened, the water deſcending into the cavity CI, the ray GL will be refracted from the perpendicular GR to O; ſo that the ſpectator will now ſee the object by the refracted ray OG. And again, ſhutting the cock B, and opening the other P, the water will deſcend into the lower cavity HL; where, the refraction ceaſing, no rays will now come from the object to the eye: but upon ſhutting the cock P, and opening the other B, the water will fill the cavity again, and bring the object in ſight of O afreſh.

To make an Hydromantic veſſel, which ſhall exhibit the images of external objects as if ſwimming in water. Provide a cylindrical veſſel ABCD (fig. 4, pl. 12.) divided into two cavities by a glaſs EF, not perfectly poliſhed: in G apply a lens convex on both ſides; and in H incline a plane mirror, of an elliptic figure, to an angle of 45 degrees; and let IH and HG be ſomething leſs than the diſtance of the focus of the lens G; ſo that the place of the images of an object radiating through the ſame may fall within the cavity of the upper veſſel: let the inner cavity be blacked, and the upper filled with clear water.—If now the veſſel be diſpoſed in a dark place, ſo as the lens be turned towards an object illuminated by the ſun, its image will be ſeen as ſwimming in the water.

HYDROMETER, an inſtrument for meaſuring the properties and effects of water, as its denſity, gravity, force, velocity, &c.

That with which the ſpecific gravity of water is determined, is often called an aerometer, or water-poiſe.

The general principle on which the conſtruction and uſe of the Hydrometer depends, has been illuſtrated under the article *Specific* GRAVITY; where it is ſhewn that a body ſpecifically lighter than ſeveral fluids, will ſerve to find out their ſpecific gravities; becauſe it will ſink deepeſt in the fluids whoſe ſpecific gravity is the leaſt. So if AB (fig. 5, pl. 12) be a ſmall even glaſs tube, hermetically ſealed, having a ſcale of equal diviſions marked upon it, with a hollow ball of about an inch in diameter at bottom, and a ſmaller ball C under it, communicating with the firſt; into the little ball is put mercury or ſmall ſhot, before the tube is ſealed, ſo that it may ſink in water below the ball, and float or ſtand upright, the diviſions on the ſtem ſhewing how far it ſinks.—If this inſtrument be dipped in common water, and ſink to D, it will ſink only to ſome lower point E in ſalt water; but in port wine it will ſink to ſome higher point F, and in brandy perhaps to B.

It is evident that an Hydrometer of this kind will only ſhew that one liquid is ſpecifically heavier than another; but the true ſpecific weight of any liquid cannot be determined without a calculation for this particular inſtrument, the tube of which ſhould be truly cylindrical. Beſides, theſe inſtruments will not ſerve for fluids whoſe denſities are much different.

Mr. Clarke conſtructed a new Hydrometer, ſhewing whether any ſpirits be proof, or above or below proof, and in what degree. This inſtrument was made of a ball of copper (becauſe ivory imbibes ſpirituous liquors, and glaſs is apt to break), to which is ſoldered a braſs wire about a quarter of an inch thick; upon this wire is marked the point to which it exactly ſinks in proof ſpirits; as alſo two other marks, one above and one below the former, exactly anſwering to one-tenth above proof and one-tenth below proof. There are alſo a number of ſmall weights made to add to it, ſo as to anſwer to the other degrees of ſtrength beſides thoſe above, and for determining the ſpecific gravities of different fluids. Philoſ. Tranſ. Abr. vol. vi, p. 326.

Dr. Deſaguliers contrived an Hydrometer for determining the ſpecific gravities of different waters, to ſuch a degree of nicety, that it would ſhew when one kind of water was but the 40,000th part heavier than another. It conſiſts of a hollow glaſs ball of about 3 inches in diameter, charged with ſhot to a proper degree, and having fixed in it a long and very ſlender wire, of only the 40th part of an inch in diameter, and divided into tenths of inches, each tenth anſwering to the 40,000th part, as above. See his Exper. Philoſ. vol. 2, p. 234.

Mr. Quin and other perſons have alſo conſtructed Hydrometers, with other and various contrivances, and with different degrees of accuracy; but all nearly on the ſame general principles.

But there is one circumſtance which deſerves particular attention in the conſtruction and graduation of Hydrometers, for determining the preciſe ſtrength of different brandies, and other ſpirituous liquors. Mr. Reaumur diſcovered, in making his ſpirit thermometers, that when rectified ſpirit and water, or phlegm, the other conſtituent part of brandy, are mixed together, there appears to be a mutual penetration of the two liquors, and not merely juxtapoſition of parts; ſo that a part of the one fluid ſeems to be received into the pores of the other; by which it happens, that if a pint of rectified ſpirit be added to a pint of water, the mixture will be ſenſibly leſs than a quart. The variations hence produced in the bulk of the mixed fluid render the Hydrometer, when graduated in the uſual way by equal diviſions, an erroneous meaſure of its ſtrength; becauſe the ſpecific gravity of the compound is found not to correſpond to the mean gravity of the two ingredients. M. Montigny conſtructed a ſcale for this inſtrument in the manner before ſuggeſted by Dr. Lewis, on actual obſervation of the ſinking or riſing of the Hydrometer in various mixtures of alcohol and water, made in certain known proportions. Hiſt. de l'Acad. Roy. des Sci. 1768; alſo Neumann's Chem. by Lewis, p. 450, note *r*.

M. De Luc has lately publiſhed a ſcheme for the conſtruction of a comparable Hydrometer, ſo that a workman, after having conſtructed one upon his principles, may make all others ſimilar to each other, and capable of indicating the ſame degree on the ſcale, when immerſed in the ſame liquor of the ſame temperature. This inſtrument is propoſed to be conſtructed of a ball of flint glaſs, communicating with a ſmall hollow cylinder, containing ſuch a quantity of quickſilver for a ballaſt, that the inſtrument may ſink nearly to the top, in the moſt ſpirituous liquor, made as hot as poſſible; to which is alſo attached a thin ſilvered tube, for a ſcale, &c. The whole deſcription may be ſeen at large in the Philoſ. Tranſ. vol. 68, p. 500.

M. Le Roi alſo publiſhed a propoſal for conſtructing comparable Hydrometers. See Hiſt. de l'Acad. des Scien. for 1770, Mem. 7.

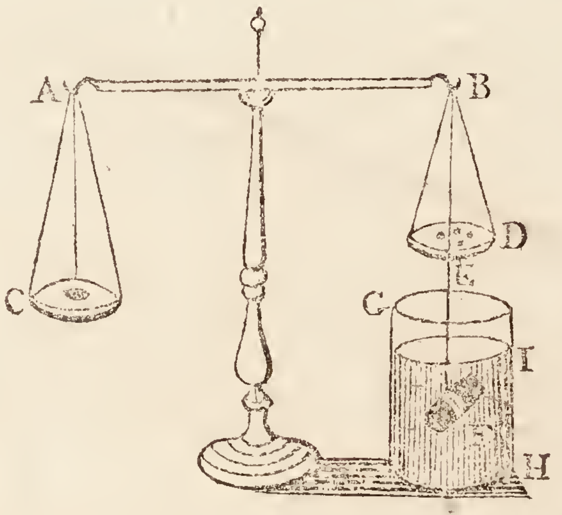
HYDROMETRIA, HYDROMETRY, the menſuration of water and other fluid bodies, their gravity, force, velocity, quantity, &c; including both hydroſtatics and hydraulics.

HYDROSCOPE, an inſtrument anciently uſed for the meaſure of time. It was a kind of water-clock, conſiſting of a cylindrical tube, conical at bottom: the cylinder was graduated with diviſions, to which the top of the water becoming ſucceſſively contiguous, as it trickled out of the vertex of the cone, pointed out the hour.

HYDROSTATICAL *Balance*, a kind of balance contrived for the exact and eaſy finding the ſpecific gravities of bodies, both ſolid and fluid, and thereby of eſtimating the degree of purity of bodies of all kinds, with the quality and richneſs of metals, ores, minerals, &c, and the proportions in any mixture, adulteration, or the like.

This is effected by weighing the body both in water, or other fluid, and out of it; and for this purpoſe one of the ſcales has uſually a hook at the bottom, for ſuſpending the body by ſome very fine thread. And the uſe of the inſtrument is founded on this theorem of Archimedes, that any body weighed in water, loſes as much of its weight as is equal to the weight of the ſame bulk of the water. Thus then is known the proportion of the ſpecific gravities of the ſolid and fluid, or the proportion of their weights under the ſame bulk, viz, the proportion of the weight of the body weighed out of water, to the difference between the ſame and its weight in water. Hence alſo, by doing the ſame thing for ſeveral different ſolids, with the ſame fluid, or different fluids with the ſame ſolid, all their ſpecific gravities become known.

The inſtrument needs but little deſcription. AB is a nice balance beam, with its ſcales C and D, turning with the ſmall part of a grain, the one of them, D, having a hook in the bottom, to receive the loop of a horſe hair &c, E, by which the body F is ſuſpended. GH is a jar of water, in which the body is immerſed when weighing.



The pieces in the ſcale C denote the weight of the body out of water; then, upon immerging it, put weights in the ſcale D to reſtore the balance again, and they will ſhew the ſpecific gravity of the body.

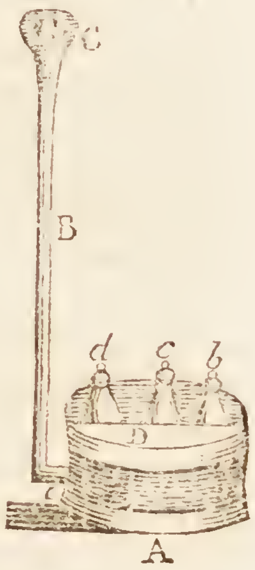
There have been various kinds of the Hydroſtatical balance, and improvements made on it, by different perſons. Thus, Dr. Deſaguliers ſet three ſcrews in the foot of the ſtand, to move any ſide higher or lower, till the ſtem be quite upright, which is known by a plummet hanging over a fixed point in the pedeſtal. Deſag. Exp. Philoſ. vol. 2, p. 196. And for ſundry other conſtructions of this inſtrument, deſigned for greater accuracy than the common ſort, ſee Martin's Phil. Britan. or Graveſande's Phyſices Elem. Math. tom. 1, lib. 3, cap. 3, &c.

The ſpecific gravities of ſmall weights may be determined by ſuſpending them in loops of horſe hair, or fine ſilken threads, to the hook at the bottom of the ſcale. Thus, if a guinea ſuſpended in air weigh 129 grains, and upon being immerſed in water require $7\frac{1}{5}$ grains to be put in the ſcale over it, to reſtore the equilibrium; we thus find that a quantity of water of equal bulk with the guinea, weighs $7\frac{1}{5}$ grains, or 7·2; therefore dividing the 129 by the 7·2, the quotient 17·88 ſhews that the guinea is ſo many times heavier than its bulk of water. Whence, if any piece of gold be tried, by weighing it firſt in air, then in water, and if, upon dividing the weight in air by the loſs in water, the quotient be 17·88, the gold is good; if the quotient be 18 or more, the gold is more fine; but if it be leſs than 17·88, the gold is too much alloyed with other metal. If ſilver be tried in the ſame manner, and found to be 11 times heavier than water, it is very fine; if it be $10\frac{1}{2}$ times heavier, it is ſtandard; but if leſs, it is mixed with ſome lighter metal, ſuch as tin.

When the body, whoſe ſpecific gravity is ſought, is lighter than water, ſo that it will not quite ſink; annex to it a piece of another body heavier than water, ſo that the maſs compounded of the two may ſink together. Weigh the denſer body, and the compound maſs, ſeparately, both in water and out of it, thereby finding how much each loſes in water; and ſubtract the leſs of theſe two loſſes from the greater; then ſay,

As the remainder
is to the weight of the light body in air,
ſo is the ſpecific gravity of water
to the ſpecific gravity of the light body.

HYDROSTATICAL *Bellows*, a machine for ſhewing the upward preſſure of fluids and the Hydroſtatical paradox. It conſiſts of two thick boards, A, D, each about 16 or 18 inches diameter, more or leſs, covered or connected firmly with leather round the edges, to open and ſhut like a common bellows, but without valves; only a pipe, B, about 3 feet high is fixed into the bellows at *e*, Now let water be poured into the pipe at C, and it will run into the bellows, gradually ſeparating the boards, by raiſing the upper one. Then if ſeveral weights, as three hundred weights, be laid upon the upper board, by pouring the water in at the pipe till it be full, it will ſuſtain all the weights, though the water in the pipe ſhould not weigh a quarter of a pound; for the pipe or tube may be as ſmall as we pleaſe, provided it be but long enough, the whole effect depending upon the height, and not at all on the width of the pipe: for the proportion is always this,

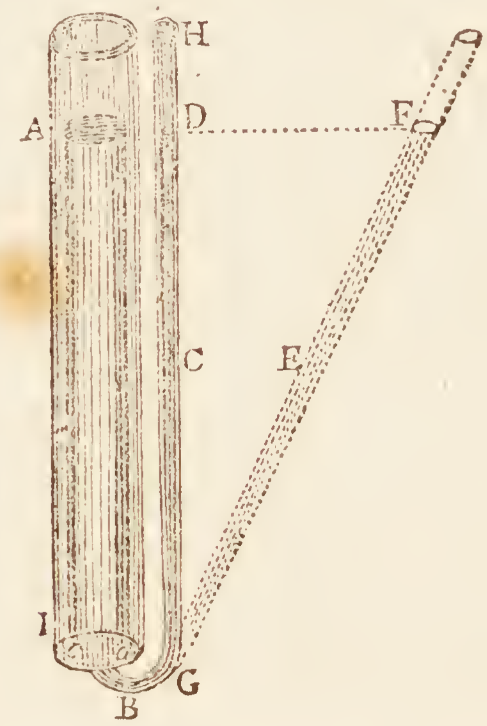


As the area of the orifice of the pipe
is to the area of the bellows board,
ſo is the weight of water in the pipe
to the weight it will ſuſtain on the board.

Hence if a man ſtand upon the upper board, and blow into the pipe B, he will raiſe himſelf upon the board; and the ſmaller the pipe, the eaſier he will be able to raiſe himſelf; and then by putting his finger upon the top of the pipe, he can ſupport himſelf as long as he pleaſes, provided the bellows be air-tight.

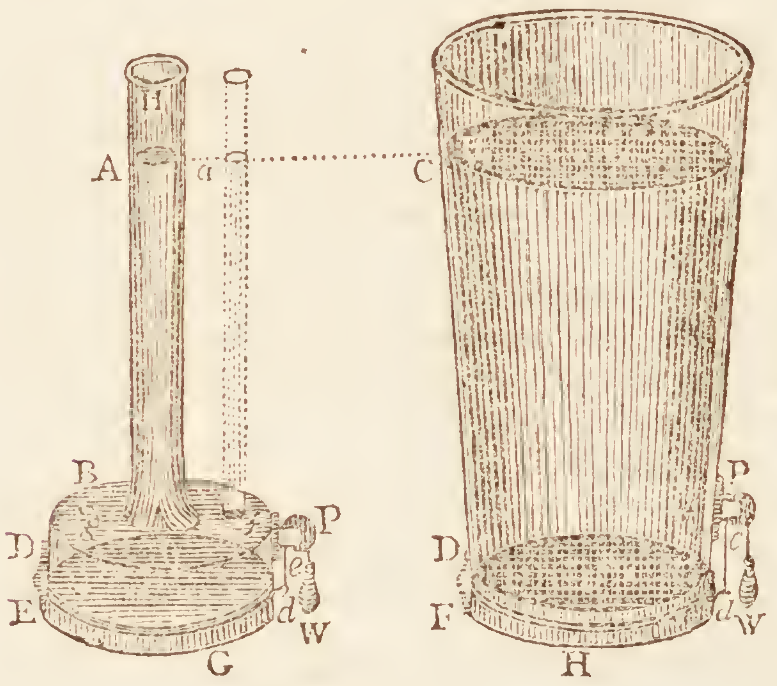
Mr. Ferguſon has deſcribed another machine, which may be ſubſtituted inſtead of this common Hydroſtatical bellows. It is however on the ſame principle of the Hydroſtatical paradox; and may be ſeen in the Supplement to his Lectures, p. 19.

HYDROSTATICAL *Paradox*, is a principle in Hydroſtatics, ſo called becauſe it has a paradoxical appearance at firſt view, and it is this; that any quantity of water, or other fluid, how ſmall ſoever, may be made to balance and ſupport any quantity, or any weight, how great ſoever. This is partly illuſtrated in the laſt article, on the Hydroſtatical bellows, where it appears that any weight whatever may be blown up and ſupported by the breath from a perſon's mouth. And the principle may be explained as follows: It is well-known that water in a pipe or canal, open at both ends, always riſes to the ſame height at both ends, whether thoſe ends be wide or narrow, equal or unequal. Thus, the ſmall pipe GH being cloſe joined to another open veſſel AI, of any ſize whatever; then pouring water into the one of theſe, it will riſe up in the other, and ſtand at the ſame height, or horizontal line DF in both of them, and that whether they are upright, or inclined in any poſition. So that all the water that is in the large veſſel from A to I, is ſupported by that which is in the ſmall veſſel from D to I only. And as there is no limit to this latter one, but that it may be made as fine even as a hair, it hence evidently appears that any quantity of water may be thus ſupported by any other the ſmalleſt quantity.



Since then the preſſure of fluids is directly as their perpendicular heights, without any regard to their quantities, it appears that whatever the figure or ſize of the veſſels may be, if they are but of equal heights, and the areas of their bottoms equal, the preſſures of equal heights of water are equal upon the bottoms of theſe veſſels; even though the one ſhould contain a thouſand or ten thouſand times as much as the other.

Mr. Ferguſon confirms and illuſtrates this paradox by the following experiment.



Let two veſſels be prepared of equal heights, but very unequal contents, ſuch as AB and CD; each veſſel being open at both ends, and their bottoms E and F of equal widths. Let a braſs bottom G and H be exactly fitted to each veſſel, not to go into it, but for it to ſtand upon; and let a piece of wet leather be put between each veſſel and its braſs bottom, for the ſake of cloſeneſs. Join each bottom to its veſſel by a hinge D, ſo that it may open like the lid of a box; and let each bottom be kept up to its veſſel by equal weights W, hung to lines which go over the pulleys P, whoſe blocks are fixed to the ſides of the veſſel at *f*, and the lines tied to hooks at *d*, fixed in the braſs bottoms oppoſite to the hinges D. Things being thus prepared and fitted, hold one veſſel upright in the hands over a baſon on a table, and cauſe water to be poured ſlowly into it, till the preſſure of the water bears down its bottom at the ſide *d*, and raiſes the weight E; and then part of the water will run out at *d*. Mark the height at which the ſurface H of the water ſtood in the veſſel, when the bottom began to give way at *d*; and then, holding up the other veſſel in the ſame manner, cauſe water to be poured into it; and it will be ſeen that when the water riſes in this veſſel juſt as high as it did in the former, its bottom will alſo give way at *d*, and it will loſe part of the water.

The natural reaſon of this ſurpriſing phenomenon is, that ſince all parts of a fluid at equal depths below the ſurface, are equally preſſed in all manner of directions, the water immediately below the fixed part B*f* will be preſſed as much upward againſt its lower ſurface within the veſſel, by the action of the column A*g*, as it would be by a column of the ſame height, and of any diameter whatever; and therefore, ſince action and reaction are equal and contrary to each other, the water immediately below the ſurface B*f* will be preſſed as much downward by it, as if it were immediately touched and preſſed by a column of the height A*g*, and of the diameter B*f*; and therefore the water in the cavity BD*df* will be preſſed as much downward upon its bottom G, as the bottom of the other veſſel is preſſed by all the water above it. Lectures, p. 105.

HYDROSTATICS, is the ſcience which treats of the nature, gravity, preſſure, and equilibrium of fluids; and of the weighing of ſolids in them.

That part of the ſcience of fluids which treats of their motions, being included under the head of Hydraulics.

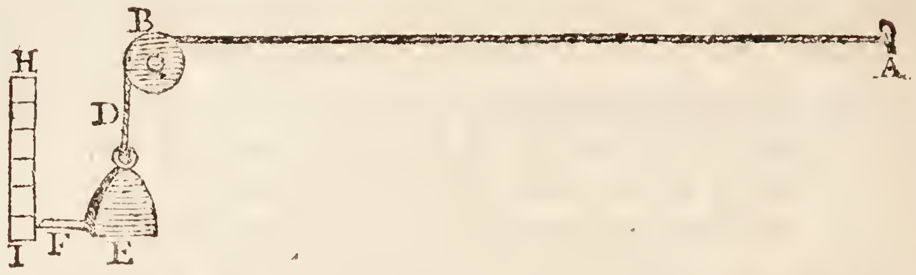
Hydroſtatics and hydraulics together conſtitute a branch of philoſophy that is juſtly conſidered as one of the moſt curious, ingenious, and uſeful of any; affording theorems and phenomena not only of the firſt uſe and importance, but alſo ſurpriſingly amuſing and pleaſant; as appears in the numberleſs writings upon the ſubject; the principal points of which may be found under the ſeveral particular articles of this work; and the chief writings on this ſcience may be ſeen under the article Hydraulics.

HYDRUS, or *Water Serpent*, one of the new ſouthern conſtellations, including only ten ſtars.

HYEMAL *Solſtice*, the ſame with Winter Solſtice. See SOLSTICE.

HYGROMETER, or HYGROSCOPE, or NOTIOMETER, an inſtrument for meaſuring the degrees of moiſture in the air.

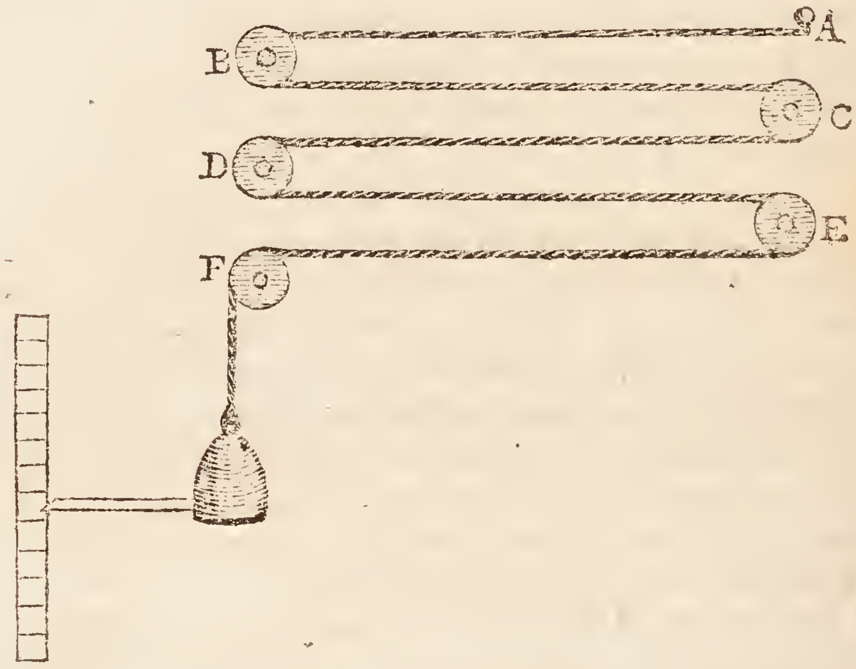
There are various kinds of Hygrometers; for whatever body either ſwells by moiſture, or ſhrinks by dryneſs, is capable of being formed into an Hygrometer. Such are woods of moſt kinds, particularly deal, aſh, poplar, &c. Such alſo is catgut, the beard of a wild oat, and twiſted cord, &c. The beſt and moſt uſual contrivances for this purpoſe are as follow.



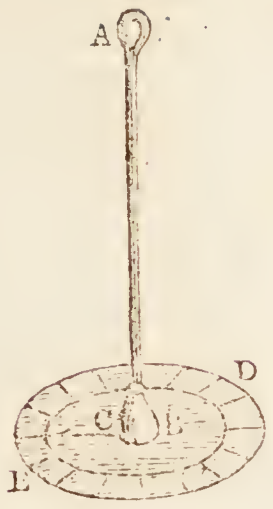
1. Stretch a common cord, or a fiddle-ſtring, ABD along a wall, paſſing it over a pulley B; fixing it at one end A, and to the other end hanging a weight E, carrying a ſtyle or index F. Againſt the ſame wall fit a plate of metal HI, graduated, or divided into any number of equal parts; and the Hygrometer is complete.

For it is matter of conſtant obſervation, that moiſture ſenſibly ſhortens cords and ſtrings; and that, as the moiſture evaporates, they return to their former length again. The like may be ſaid of a fiddle-ſtring: and from hence it happens that ſuch ſtrings are apt to break in damp weather, if they are not ſlackened by the ſcrews of the violin. Hence it follows, that the weight E will aſcend when the air is more moiſt, and deſcend again when it becomes drier. By which means the index F will be carried up and down, and, by pointing to the ſeveral diviſions on the ſcale, will ſhew the degrees of moiſture or dryneſs.

2. Or thus, for a more ſenſible and accurate Hygrometer: ſtrain a whipcord, or catgut, over ſeveral pulleys B, C, D, E, F; and proceed as before for the reſt of the conſtruction. Nor does it matter whether the ſeveral parts of the cord be parallel to the horizon, as expreſſed in the annexed figure, or perpendicular to the ſame, or in any other poſition; the advantage of this, over the former method, being merely the having a greater length of cord in the ſame compaſs; for the longer the cord, the greater is the contraction and dilatation, and conſequently the degrees of variation of the index over the ſcale, for any given change of moiſture in the air.

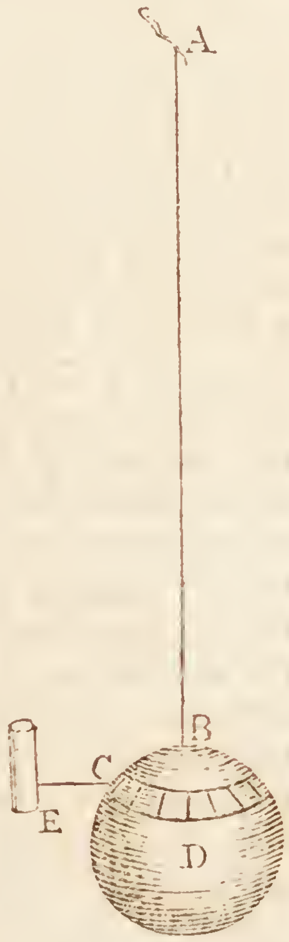


3. Or thus: Faſten a twiſted cord, or fiddle-ſtring, AB, by one end at A, ſuſtaining a weight at B, carrying an index C round a circular ſcale DE deſcribed on a horizontal board or table.——For a cord or catgut twiſts itſelf as it moiſtens, and untwiſts again as it dries. Hence, upon an increaſe or decreaſe of the humidity of the air, the index will ſhew the quantity of twiſting or untwiſting, and conſequently the increaſe or decreaſe of moiſture or dryneſs.

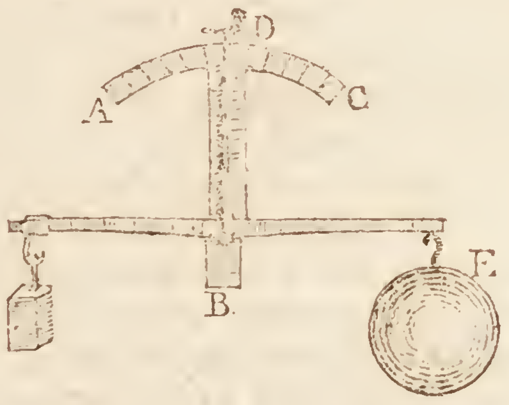


4. Thoſe Dutch toys, called weather houſes, where a ſmall image of a man, and one of a woman, are fixed upon the ends of an index, are conſtructed upon this principle. For the index, being ſuſtained by a cord or twiſted catgut, turns backwards and forwards, bringing out the man in wet weather, and the woman in dry.

5. Or thus: Faſten one end of a cord, or catgut, AB, to a hook at A; and to the other end a ball D of about one pound weight; upon which draw two concentric circles, and divide them into any number of equal parts, for a ſcale; then fit a ſtyle or index EC into a proper ſupport at E, ſo as the extremity C may almoſt touch the diviſions of the ball.——Here the cord twiſting or untwiſting, as in the former caſe, will indicate the change of moiſture, by the ſucceſſive application of the diviſions of the circular ſcale, as the ball turns round, to the index C.



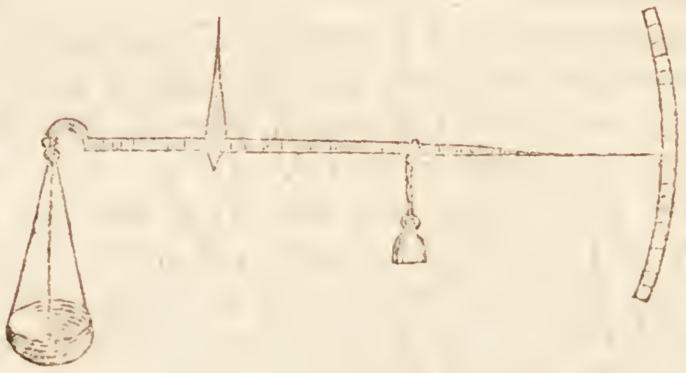
6. Or an Hygrometer may be made of the thin boards of aſh or fir, by their ſwelling or contracting. But this, and all the other kinds of this inſtrument, above deſcribed, become in time ſenſibly leſs and leſs accurate; till at laſt they loſe their effect entirely, and ſuffer no alteration from the weather. But the following ſort is much more durable, ſerving for many years with tolerable accuracy.



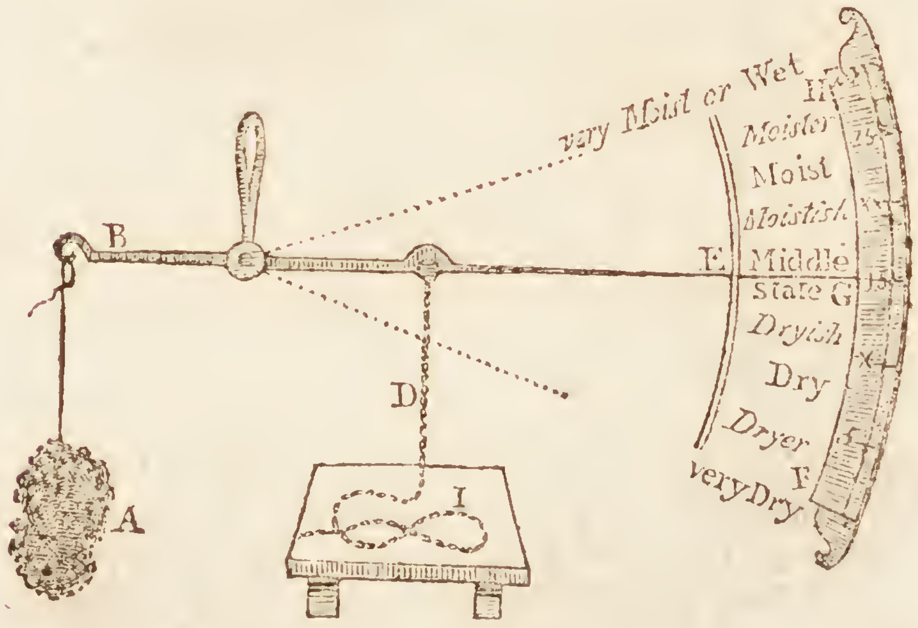
7. Take the Manoſcope, deſcribed under that article, and inſtead of the exhauſted ball E, ſubſtitute a ſponge, or other body, that eaſily imbibes moiſture. To prepare the ſponge, it may be proper firſt to waſh it in water very clean; and, when dry again, in water or vinegar in which there has been diſſolved ſal ammoniac, or ſalt of tartar; after which let it dry again.——Now, if the air become moiſt, the ſponge will imbibe it and grow heavier, and conſequently will preponderate, and turn the index towards C; on the contrary, when the air becomes drier, the ſponge becomes lighter, and the index turns towards A; and thus ſhewing the ſtate of the air.

8. In the laſt mentioned Hygrometer, Mr. Gould, in the Philoſ. Tranſ. inſtead of a ſponge, recommends oil of vitriol, which grows ſenſibly lighter or heavier from the degrees of moiſture in the air; ſo that being ſaturated in the moiſteſt weather, it afterwards retains or loſes its acquired weight, as the air proves more or leſs moiſt. The alteration in this liquor is ſo great, that in the ſpace of 57 days it has been known to change its weight from 3 drachms to 9; and has ſhifted a tongue or index of a balance 30 degrees. So that in this way a pair of ſcales may afford a very nice Hygrometer. The ſame author ſuggeſts, that oil of ſulphur or campanam, or oil of tartar per deliquium, or the liquor of fixed nitre, might be uſed inſtead of the oil of vitriol.

9. This balance may be contrived in two ways; by either having the pin in the middle of the beam, with a ſlender tongue a foot and a half long, pointing to the diviſions on an arched plate, as repreſented in the laſt figure above. Or the ſcale with the liquor may be hung to the point of the beam near the pin, and the other extremity made ſo long, as to deſcribe a large arch on a board placed for the purpoſe; as in the figure here annexed.



10. Mr. Arderon has propoſed ſome improvement in the ſponge Hygrometer. He directs the ſponge A to be ſo cut, as to contain as large a ſuperficies as poſſible, and to hang by a fine thread of ſilk upon the beam of a



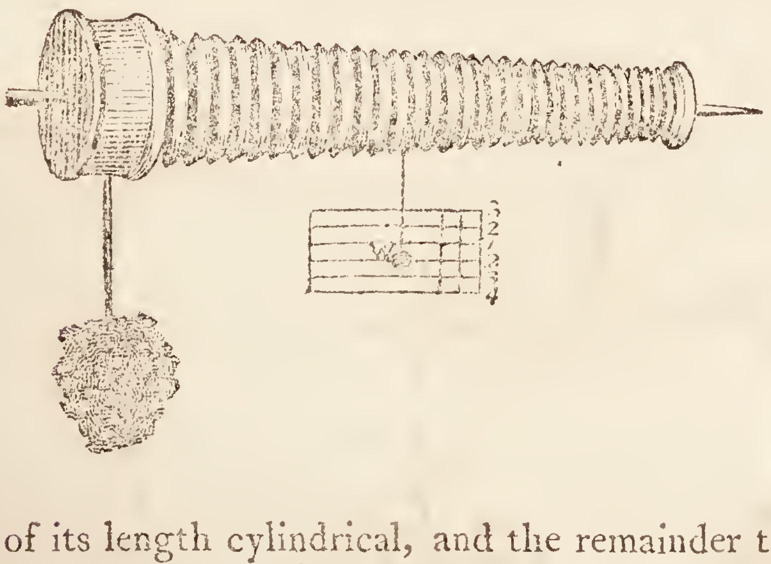
balance B, and exactly balanced on the other ſide by another thread of ſilk at D, ſtrung with the ſmalleſt lead ſhot, at equal diſtances, ſo adjuſted as to cauſe an index E to point at G, the middle of a graduated arch FGH, when the air is in a middle ſtate between the greateſt moiſture and the greateſt dryneſs. Under this ſilk ſo ſtrung with ſhot, is placed a little table or ſhelf I, for that part of the ſilk or ſhot to reſt upon which is not ſuſpended. When the moiſture imbibed by the ſponge increaſes its weight, it will raiſe the index, with part of the ſhot, from the table, and vice verſa when the air is dry. Philoſ. Tranſ. vol. 44, p. 96.

11. From a ſeries of Hygroſcopical obſervations, made with an apparatus of deal wood, deſcribed in the Philoſ. Tranſ. number 480, Mr. Coniers concludes, 1ſt, That the wood ſhrinks moſt in ſummer, and ſwells moſt in winter, but is moſt liable to change in the ſpring and fall. 2d, That this motion happens chiefly in the day time, there being ſcarce any variation in the night. 3d, That there is a motion even in dry weather, the wood ſwelling in the morning, and ſhrinking in the afternoon. 4th, That the wood, by night as well as by day, uſually ſhrinks when the wind is in the north, north-eaſt, and eaſt, both in ſummer and winter. 5th, That by conſtant obſervation of the motion and reſt of the wood, with the help of a thermometer, the direction of the wind may be told nearly without a weather-cock. He adds, that even the time of the year may be known by it; for in ſpring it moves more and quicker than in winter; in ſummer it is more ſhrunk than in ſpring; and has leſs motion in autumn than in ſummer.

See an account of a method of conſtructing theſe and other Hygrometers, in Phil. Tranſ. Abr. vol. 2, p. 30, &c, and plate 1 annexed. See alſo Philoſ. Tranſ. vol. 11. p. 647 and 715, vol. 15, p. 1032, vol. 43, p. 6, vol. 44, p. 95, 169 and 184, vol. 54, p. 259, vol. 61, p. 198, vol. 63, p. 404, &c.

12. Dr. Hook's Hygrometer was made of the beard of a wild oat, ſet in a ſmall box, with a dial plate and an index. See his Micrographia, p. 150.

13. The Doctors Hales and Deſaguliers both contrived another form of ſponge Hygrometer, on this principle. They made an horizontal axis, having a ſmall part of its length cylindrical, and the remainder tapering conically with a ſpiral thread cut in it, after the manner of the fuzee of a watch. The ſponge is ſuſpended by a fine ſilk thread to the cylindrical part of the axis, upon which it winds. This is balanced by a ſmall weight W, ſuſpended alſo by a thread, which winds upon the ſpiral fuzee. Then when the ſponge grows heavier, in moiſt weather; it deſcends and turns the axis, and ſo draws up the weight, which coming to a thicker part of the axis it becomes a balance to the ſponge, and its motion is ſhewn by an attached ſcale. And vice verſa when the air becomes drier.——Salt of tartar, or any other ſalt, or pot aſhes, may be put into the ſcale of a balance, and uſed inſtead of the ſponge. Deſag. Exper. Philoſ. vol. 2, p. 300.



14. Mr. Ferguſon made an Hygrometer of a thin deal pannel; and to enlarge the ſcale, and ſo render its variations more ſenſible, he employed a wheel and axle, making one cord paſs over the axle, which turned a wheel ten times as large, over which paſſed a line with a weight at the end of it, whoſe motion was therefore ten times as much as that of the deal pannel. The board ſhould be changed in 3 or 4 years. See Philoſ. Tranſ. vol. 54, art. 47.

15. Mr. Smeaton gave alſo an ingenious and elaborate conſtruction of an Hygrometer; which may be ſeen in the Philoſ. Tranſ. vol. 61, art. 24.

16. Mr. De Luc's contrivance for an Hygrometer is very ingenious, and on this principle. Finding that even ivory ſwells with moiſture, and contracts with dryneſs, he made a ſmall and very thin hollow cylinder of ivory, open only at the upper end, into which is fitted the under or open end of a very fine long glaſs tube, like that of a thermometer. Into theſe is introduced ſome quickſilver, filling the ivory cylinder, and a ſmall part of the length up the glaſs tube. The conſequence is this: when moiſture ſwells the ivory cylinder, its bore or capacity grows larger, and conſequently the mercury ſinks in the fine glaſs tube; and vice verſa, when the air is drier, the ivory contracts, and forces the mercury higher up the tube of glaſs. It is evident that an inſtrument thus conſtructed is in fact alſo a thermometer, and muſt neceſſarily be affected by the viciſſitudes of heat and cold, as well as by thoſe of dryneſs and moiſture; or that it muſt act as a thermometer as well as an Hygrometer. The ingenious contrivances in the ſtructure and mounting of this inſtrument may be ſeen in the Philoſ. Tranſ. vol. 63, art. 38; where it may be ſeen how the above imperfection is corrected by ſome ſimple and ingenious expedients, employed in the original conſtruction and ſubſequent uſe of the inſtrument; in conſequence of which, the variations in the temperature of the air, though they produce their full effects on the inſtrument, as a thermometer, do not interfere with or embarraſs its indications as an Hygrometer.

17. In the Philoſ. Tranſ. for 1791, Mr. De Luc has given a ſecond paper on Hygrometry. This has been chiefly occaſioned by a Memoir of M. de Sauſſure on the ſame ſubject, entitled Eſſais ſur l'Hygrometrie, in 4to, 1783. In this work M. de S. deſcribes a new Hygrometer of his conſtruction, on the following principle. It is a known fact that a hair will ſtretch when it is moiſtened, and contract when dried: and M. de Sauſſure found, by repeated experiments, that the difference between the greateſt extenſion and contraction, when the hair is properly prepared, and has a weight of about 3 grains ſuſpended by it, is nearly one 40th of its whole length, or one inch in 40. This circumſtance ſuggeſted the idea of a new Hygrometer. To

render theſe ſmall variations of the length of the hair perceptible, an apparatus was contrived, in which one of the extremities of the hair is fixed, and the other, bearing the counterpoiſe abovementioned, ſurrounds the circumference of a cylinder, which turns upon an axis to which a hand is adapted, marking upon a dial in large diviſions the almoſt inſenſible motion of this axis. About 12 inches high is recommended as the moſt convenient and uſeful: and to render them portable, a contrivance is added, by which the hand and the counterpoiſe can be occaſionally fixed.

But M. de Luc, in his Idées ſur la Meteorologie, vol. 1, anno 1786, ſhews that hairs, and all the other animal or vegetable hygroſcopic ſubſtances, taken lengthwiſe, or in the direction of their fibres, undergo contrary changes from different variations of humidity; that when immerſed in water, they lengthen at firſt, and afterwards ſhorten; that when they are near the greateſt degree of humidity, if the moiſture be increaſed, they ſhorten themſelves; if it be diminiſhed, they lengthen themſelves firſt before they contract again. Theſe irregularities, which render them incapable of being true meaſures of humidity, he ſhews to be the neceſſary conſequence of their organic reticular ſtructure De Sauſſure takes his point of extreme moiſture from the vapours of water under a glaſs bell, keeping the ſides of the bell continually moiſtened; and affirms, that the humidity is, there, conſtantly the ſame in all temperatures; the vapours even of boiling water having no other effect than thoſe of cold. De Luc, on the contrary, ſhews that the differences in humidity under the bell are very great, though De Sauſſure's Hygrometer was not capable of diſcovering them; and that the real undecompoſed vapour of boiling water has the directly oppoſite effect to that of cold, the effect of extreme dryneſs; and on this point he mentions an intereſting fact, communicated to him by Mr. Watt, viz, that wood cannot be employed in the ſteam engine, for any of thoſe parts where the vapour of the boiling water is confined, becauſe it dries ſo as to crack as if expoſed to the fire.

To theſe charges of M. De Luc, a reply is made by M. De Sauſſure, in his Defence of the Hair Hygrometer, in 1788; where he attributes the general diſagreement between the two inſtruments, to irregularities of M. De Luc's; and aſſigns ſome aberrations of his own Hygrometer, which could not have proceeded from the above cauſe, but to its having been out of order; &c.

This has drawn from M. De Luc a ſecond paper on Hygrometry, publiſhed in the Philoſ. Tranſ. for 1791, p. 1, and 389. This author here reſumes the four fundamental principles which he had ſketched out in the former paper, viz, 1ſt, That fire is a ſure, and the only ſure means of obtaining extreme dryneſs. 2d, That water, in its liquid ſtate, is a ſure, and the only ſure means of determining the point of extreme moiſture. 3d, There is no reaſon, a priori, to expect, from any hygroſcopic ſubſtance, that the meaſurable effects, produced in it by moiſture, are proportional to the intenſities of that cauſe.—But, 4th, perhaps the comparative changes of the dimenſions of a ſubſtance, and of the weight of the ſame or other ſubſtances, by the ſame variations of moiſture, may lead to ſome diſcovery in that reſpect. On theſe heads M. De Luc expatiates at large in this paper, ſhewing the imperfections of M. De Sauſſure's principles of Hygrometry, and particularly as to a hair, or any ſuch ſubſtance when extended lengthwiſe, being properly uſed as an Hygrometer. On the other hand, he ſhews that the expanſion of ſubſtances acroſs the fibres, or grain, renders them, in that reſpect, by far the moſt proper for this purpoſe. He chooſes ſuch as can be made very thin, as ivory, or deal ſhavings, but over all he finds whalebone to be far the beſt of any. But, for all the reaſonings of theſe ingenious philoſophers on this intereſting ſubject, and complete information, ſee the publications above quoted, as alſo the Monthly Review, vol. 51, p. 224, vol. 71, p. 213, vol. 76, p. 316, vol. 78, p. 236, and vol. 6, of the new ſeries for the year 1791, p. 133.

HYGROMETRY, the ſcience of the meaſurement of the moiſture of the atmoſphere. The chief writings on this ſcience are thoſe of M. De Luc and M. De Sauſſure, for which ſee the laſt article on Hygrometers.

HYGROSCOPE, is commonly uſed in the ſame ſenſe with Hygrometer. Wolfius, however, regarding the etymology of the word, makes ſome difference. According to him, the Hygroſcope only ſhews the alterations of the air in reſpect of humidity and dryneſs, but the hygrometer meaſures them. A Hygroſcope therefore is only an indefinite or leſs accurate hygrometer.

HYPATIA, a very learned and beautiful lady, was born at Alexandria, about the end of the 4th century, as ſhe flouriſhed about the year of Chriſt 430. She was the daughter of Theon, a celebrated philoſopher and mathematician, and preſident of the famous Alexandrian ſchool. Her father, encouraged by her extraordinary genius, had her not only educated in all the ordinary qualifications of her ſex, but inſtructed in the moſt abſtruſe ſciences. She made ſuch great progreſs in philoſophy, geometry, aſtronomy, and the mathematics in general, that ſhe paſſed for the moſt learned perſon of her time. She publiſhed commentaries on Apollonius's Conics, on Diophantus's Arithmetic, and other works. At length ſhe was thought worthy to ſucceed her father in that diſtinguiſhed and important employment, the government of the ſchool of Alexandria; and to deliver inſtructions out of that chair where Ammonius, Hierocles, and many other great men, had taught before; and this at a time too when men of great learning abounded both at Alexandria and in many other parts of the Roman empire. Her fame being ſo extenſive, and her worth ſo univerſally acknowledged, it was no wonder that ſhe had a crowded auditory. "She explained to her hearers (ſays Socrates, an eccleſiaſtical hiſtorian of the 5th century, born at Conſtantinople) the ſeveral ſciences that go under the general name of philoſophy; for which reaſon there was a confluence to her, from all parts, of thoſe who made philoſophy their delight and ſtudy."

Her ſcholars were not leſs eminent than they were numerous. One of them was the celebrated Syneſius, who was afterwards biſhop of Ptolomais. This ancient Chriſtian Platoniſt always expreſſes the ſtrongeſt, as well as the moſt grateful, teſtimony of the virtue of his tutoreſs; never mentioning her without the moſt profound reſpect, and ſometimes in terms of affection but little ſhort of adoration. But it was not Syneſius only, and the diſciples of the Alexandrian ſchool, who admired Hypatia for her virtue and learning: never was woman more careſſed by the public, and yet never had woman a more unſpotted character. She was held as an oracle for her wiſdom, for which ſhe was conſulted by the magiſtrates in all important caſes; a circumſtance which often drew her among the greateſt concourſe of men, without the leaſt cenſure of her manners. In ſhort, when Nicephorus intended to paſs the higheſt compliment on the princeſs Eudocia, he thought he could not do it better than by calling her another Hypatia.

While Hypatia thus reigned the brighteſt ornament of Alexandria, Oreſtes was governor of that place for the emperor Theodoſius, and Cyril was biſhop or patriarch. Oreſtes, having had a liberal education, could not but admire Hypatia; and as a wiſe governor often conſulted her. This, together with an averſion which Cyril had againſt Oreſtes, proved fatal to the lady. About 500 monks aſſembling, attacked the governor one day, and would have killed him, had he not been reſcued by the townſmen; and the reſpect which Oreſtes had for Hypatia cauſing her to be traduced among the Chriſtian multitude, they dragged her from her chair, tore her in pieces, and burnt her limbs.

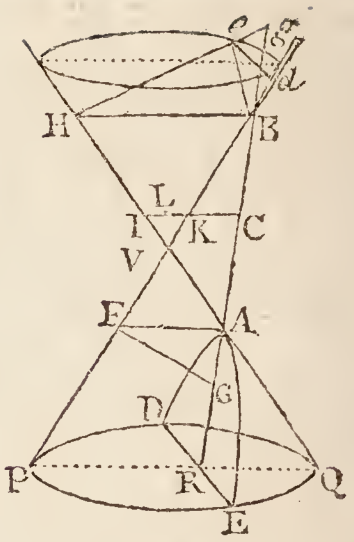
Cyril is ſtrongly ſuſpected of having fomented this tragedy. Cave indeed endeavours to remove the imputation of ſo horrid an action from the patriarch; and lays it upon the Alexandrian mob in general, whom he calls "a very trifling inconſtant people." But though Cyril ſhould be allowed neither to have been the perpetrator, nor even the contriver, of it, yet it is much to be ſuſpected that he did not diſcountenance it in the manner he ought to have done: a ſuſpicion which muſt needs be greatly confirmed by reflecting, that he was ſo far from blaming the outrage committed by the monks upon Oreſtes, that he afterwards received the dead body of Ammonius, one of the moſt forward in that outrage, who had grievouſly wounded the governor, and who was juſtly puniſhed with death. Upon this riotous ruffian Cyril made a panegyric in the church where he was laid, in which he extolled his courage and conſtancy, as one that had contended for the truth; and changing his name to Thaumaſius, or the Admirable, ordered him to be conſidered as a martyr. "However (continues Socrates), the wiſeſt part of Chriſtians did not approve the zeal which Cyril ſhewed on this man's behalf, being convinced that Ammonius had juſtly ſuffered for his deſperate attempt."

HYPERBOLA, one of the conic ſections, being that which is made by a plane cutting a cone ſo, that, entering one ſide of the cone, and not being parallel to the oppoſite ſide, it may cut the circular baſe when the oppoſite ſide is ever ſo far produced below the vertex, or ſhall cut the oppoſite ſide of the cone produced above the vertex, or ſhall make a greater angle with the baſe than the oppoſite ſide of the cone makes; all theſe three circumſtances amounting to the ſame thing, but in other words.

1. Thus, the figure DAE is an Hyperbola, made by a plane entering the ſide VQ of a cone PVQ at A, and either cutting the baſe PEQ when the plane is not parallel to VP, and this is ever ſo far produced; or when the angle ARQ is greater than the angle VPQ; or when the plane cuts the oppoſite ſide in B above the vertex.

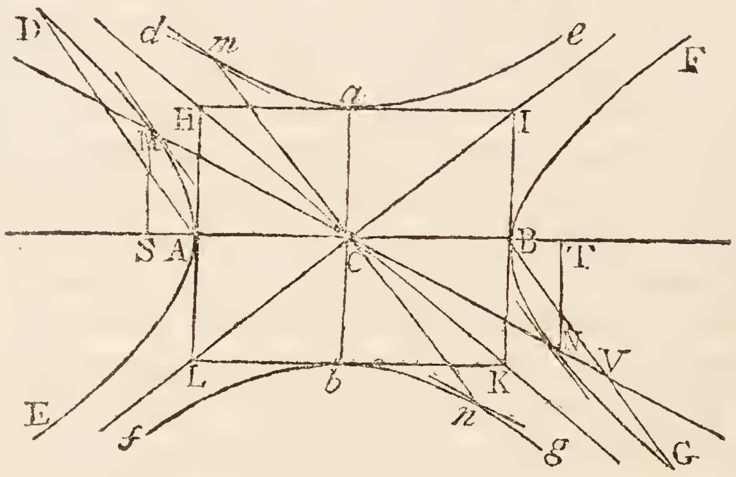
2. By the Hyperbola is ſometimes meant the whole plane of the ſection, and ſometimes only the curve line of the ſection.

3. Hence, the cutting plane meets the oppoſite cone in B, and there forms another Hyperbola *d* B *e*, equal to the former one, and having the ſame tranſverſe axis AB; and the ſame vertices A and B. Alſo the two are called Oppoſite Hyperbolas.



4. The centre C is the middle point of the tranverſe axis.

5. The ſemi-conjugate axis is CL, a mean proportional between CI and CK, the diſtances to the ſides of the oppoſite cone, when CI is drawn parallel to the diameter PQ of the baſe of the cone. Or the whole conjugate axis is a mean proportional between AF and BH, which are drawn parallel to the baſe of the cone.



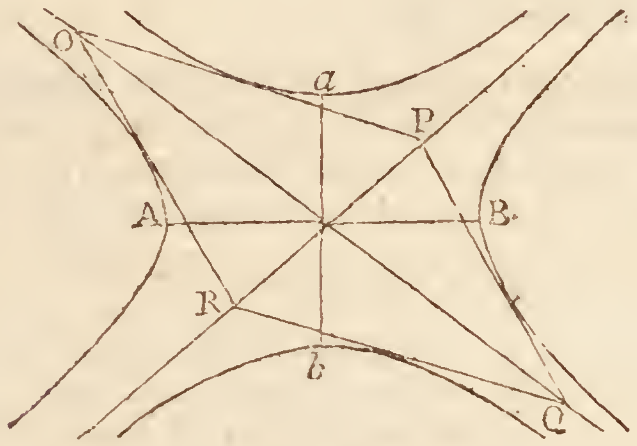
6. If DAE and FBG be two oppoſite Hyperbolas, having the ſame tranſverſe and conjugate axes AB and *a b*, perpendicularly biſecting each other; and if *d a e* and *f b g* be two other oppoſite Hyperbolas, having the ſame axes with the two former, but in the contrary order, viz, having *a b* for their firſt or tranſverſe axis, and AB for their ſecond or conjugate axis: then any two adjacent curves are called Conjugate Hyperbolas, and the whole figure formed by all the four curves, the Figure of the Conjugate Hyperbolas. And if the rectangle HIKL be inſcribed within the four conjugate Hyperbolas, touching the vertices A, B, *a*, *b*, and having their ſides parallel and equal to the two axes; and if then the two diagonals HCK, ICL, of the parallelogram be drawn, theſe diagonals are the aſymptotes of the curves, being lines that continually approach nearer and nearer to the curves, without meeting them, except at an infinite diſtance, where each aſymptote and the two adjacent ſides of the two conjugate Hyperbolas may be ſuppoſed all to meet; the aſymptote being there a common tangent to them both, viz, at that infinite diſtance.

7. Hence the four Hyperbolas, meeting and running into each other at the infinite diſtance, may be conſidered as the four parts of one entire curve, having the ſame axes, tangents, and other properties.

8. A Diameter in general, is any line, as MN, drawn through the centre C, and meeting, or termi-

nate

nated by the oppoſite legs of the oppoſite Hyperbolas. And if parallel to this diameter there be drawn two tangents, at m and n, to the oppoſite legs of the other two oppoſite Hyperbolas, the line mCn joining the points of contact, is the conjugate diameter to MN, and the two mutually conjugates to each other. Or, if to the points M or N there be drawn a tangent, and through the centre C the line mn parallel to it, that line will be the conjugate to MN. The points where each of theſe meet the curves, as M, N, m, n, are the vertices of the diameters; and the tangents to the curves at the two vertices of any diameter, are parallel to each other, and alſo to the other or conjugate diameter.



9. Moreover, if thoſe tangents to the four Hyperbolas, at the vertices of two conjugate diameters, be produced till they meet, they will form a parallelogram OPQR; and the diagonals OQ and PR of the parallelogram will be the aſymptotes of the curves; which therefore paſs through the oppoſite angles of all the parallelograms ſo inſcribed between the curves. Alſo it is a property of theſe parallelograms, that they are all equal to each other, and therefore equal to the rectangle of the two axes; as will be farther noticed below. Farther, if theſe diagonals or aſymptotes make a right angle between them, or if the inſcribed parallelogram be a ſquare, or if the two axes be equal to each other, then the Hyperbola is called a right-angled or an equilateral one.

10. An Ordinate to any diameter, is a line drawn parallel to its conjugate, or to the tangent at its vertex, and terminated by the diameter produced and the curve. So MS and TN are ordinates to the axis AB; alſo AD and BG are ordinates to the diameter MN (laſt fig. but one). Hence the ordinates to the axis are perpendicular to it; but ordinates to the other diameters are oblique to them.

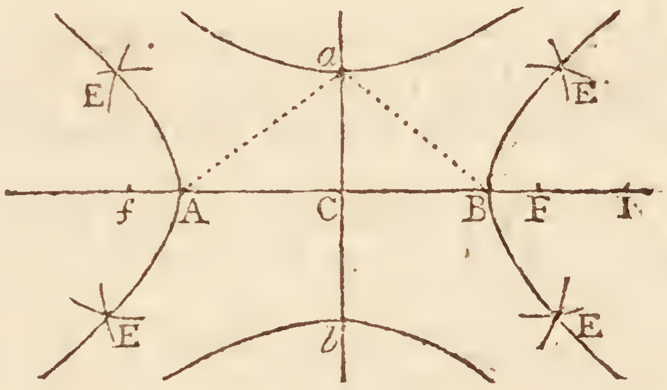
11. An abſciſs is a part of any diameter, contained between its vertex and an ordinate to it; and every ordinate has two abſciſſes: as AT and BT, or MV and NV.

12. The Parameter of any diameter, is a third proportional to the diameter and its conjugate.—The Parameter of the axis is alſo equal to the line AG or Bg (fig. 1), if FG be drawn to make the angle AFG = the angle BAV, or the line Hg to make the angle BHg = the angle ABV.

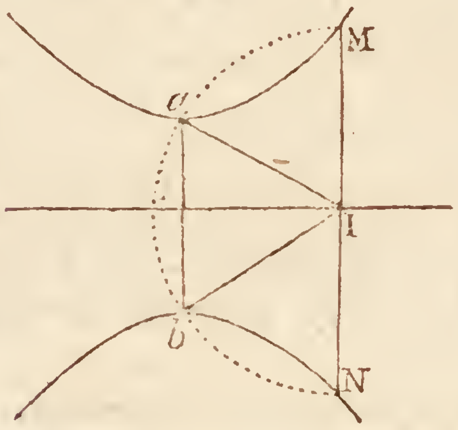
13. The Focus is the point in the axis where the ordinate is equal to half the parameter of the axis; as S and T (fig. 2) if MS and TN be half the parameter, or the 3d proportional to CA and Ca. Hence there are two Foci, one on each ſide the vertex, or one for each of the oppoſite Hyperbolas. Theſe two points in the axis are called Foci, or burning points, becauſe it is found by opticians that rays of light iſſuing from one of them, and falling upon the curve of the Hyperbola, are reflected into lines that verge towards the other point or Focus.

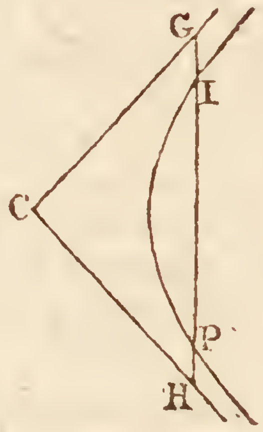
To deſcribe an Hyperbola, in various ways.

14. (1ſt *Way by points.*)—In the tranſverſe axis AF produced, take the foci F and f, by making CF and Cf = Aa or Ba, aſſume any point I: Then with the radii AI, BI, and centres F, f, deſcribe arcs interſecting in E, which will give four points in the curves. In like manner, aſſuming other points I, as many other points will be found in the curve. Then, with a ſteady hand, draw the curve line through all the points of interſection E.—In the ſame manner are to be conſtructed the other pair of oppoſite Hyperbolas, uſing the axis ab inſtead of AB.

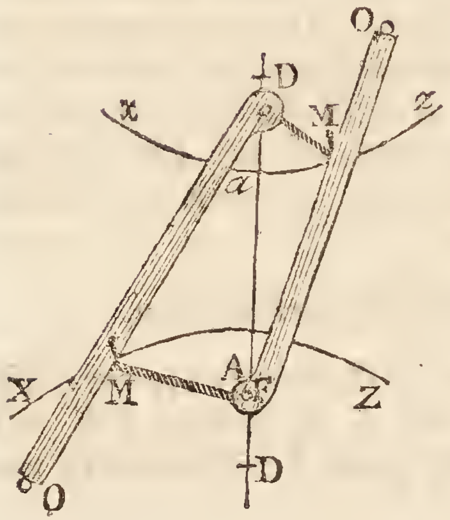


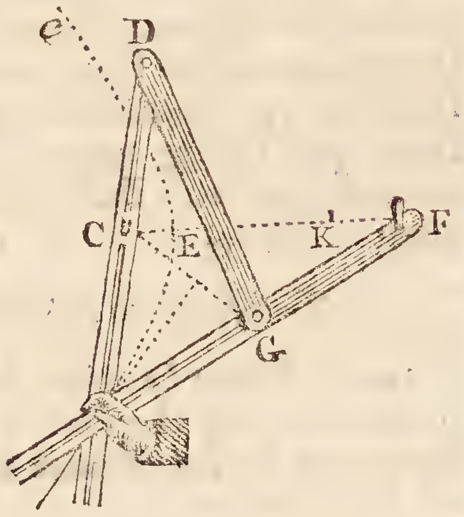
15. (2d *Way by points, for a Right-angled Hyperbola only.*)—On the axis produced if neceſſary, take any point I, through which draw a perpendicular line, upon which ſet off IM and IN equal to the diſtance Ia or Ib from I to the extremities of the other axis; and M and N will be points in the curve.





16. (3d *Way by points, to deſcribe the curve through a given point.*)—CG and CH being the aſymptotes, and P the given point of the curve; through the point P draw any line GPH between the aſymptotes, upon which take GI = PH, ſo ſhall I be another point of the curve. And in this manner may any number of points be found, drawing as many lines through the given point P.

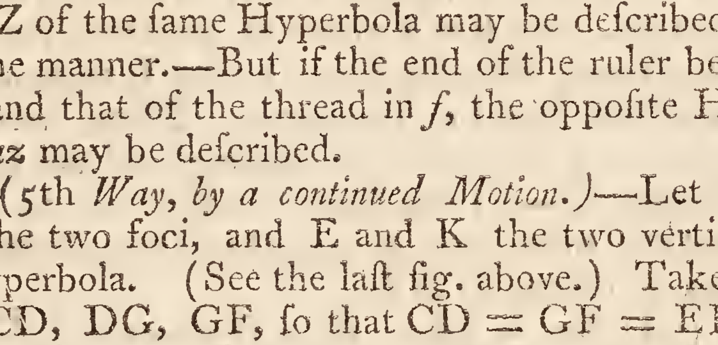




17. (4th *Way by a continued Motion.*)—If one end of a long ruler fMO be faſtened at the point f, by a pin on a plane, ſo as to turn freely about that point as a centre. Then take a thread FMO, ſhorter than the ruler, and fix one end of it in F, and the other to the end O of the ruler. Then if the ruler fMO be turned about the fixed point f, at the ſame time keeping the thread OMF always tight, and its part MO cloſe to the ſide of the ruler, by means of the pin M; the curve line AX deſcribed by the motion of the pin M is one part of an Hyperbola. And if the ruler be turned, and move on the other ſide of the fixed point F, the other part AZ of the ſame Hyperbola may be deſcribed after the ſame manner.—But if the end of the ruler be fixed in F, and that of the thread in f, the oppoſite Hyperbola xaz may be deſcribed.

18. (5th *Way, by a continued Motion.*)—Let C and F be the two foci, and E and K the two vertices of the Hyperbola. (See the laſt fig. above.) Take three rulers CD, DG, GF, ſo that CD = GF = EK, and DG = CF; the rulers CD and GF being of an indefinite length beyond C and G, and having ſlits in them for a pin to move in; and the rulers having holes in them at C and F, to faſten them to the foci C and F by means of pins, and at the points D and G they are to be joined by the ruler DG. Then, if a pin be put in the ſlits, viz, the common interſection of the rulers CD and GF, and moved along, cauſing the two rulers GF, CD, to turn about the foci C and F, that pin will deſcribe the portion Ee of an Hyperbola.—The foregoing are a few among various ways given by ſeveral authors.

Some of the chief Properties of the Hyperbola.



19. (1ſt) The ſquares of the ordinates, of any diameter, are to each other, as the rectangles of their abſciſſes; i. e. $DE^2 : GH^2 :: AD \cdot BD : AG \cdot BG$.

20. As the ſquare of any diameter, is to the ſquare of its conjugate; ſo is the rectangle of two abſciſſes, to the ſquare of their ordinate. That is, $AB^2 : ab^2 :: AD \cdot BD : DE^2$.

Or, becauſe the rectangle AD . BD is = the difference of the ſquares $CD^2 - CB^2$, the ſame property is,

$$\text{As } AB^2 : ab^2 :: CD^2 - CB^2 : DE^2,$$

$$\text{Or } AB : \frac{ab^2}{AB} :: CD^2 - CB^2 : DE^2,$$

$$\text{That is } AB : p :: CD^2 - CB^2 : DE^2,$$

where p is the parameter of the diameter AB, or the 3d proportional $\frac{ab^2}{AB}$.

And hence is deduced the common equation of the Hyperbola, by which its general nature is expreſſed. Thus, putting d = the ſemidiameter CA or CB,

c = its ſemiconjugate Ca or Cb,

p = its parameter or $\frac{2d^2}{c}$,

x = the abſciſs BD from the vertex,

y = the ordinate DE, and

v = the abſciſs CD from the centre:

$$\text{Then is } d^2 : c^2 :: \overline{2d + x} \cdot x : y^2,$$

$$\text{or } d^2 : c^2 :: \overline{d + v} \cdot x : y^2,$$

$$\text{or } d^2 : c^2 :: v^2 - d^2 : y^2,$$

$$\text{or } d : p :: v^2 - d^2 : y^2; \text{ ſo that}$$

$$y^2 = \frac{2dx + x^2}{d^2} \cdot c^2 = \frac{v^2 - d^2}{d^2} \cdot c^2 = \frac{v^2 - d^2}{d} \cdot p;$$

any of which equations or proportions expreſs the nature of the curve. And hence ariſes the name Hyperbola, ſignifying to exceed, becauſe the ratio of d^2 to c^2, or of d to p, exceeds that of $2dx$ to y^2; that ratio being equal in the parabola, and defective in the ellipſe, from which circumſtances alſo theſe take their names.

21. The diſtance between the centre and the focus, is equal to the diſtance between the extremities of the tranſverſe and conjugate axes. That is, CF = Aa or Ab, where F is the focus.

22. The conjugate ſemi-axis is a mean proportional between the diſtances of the focus from both vertices of the tranſverſe. That is, Ca is a mean between AF and BF, or AF : Ca : : Ca : BF, or $AF \cdot BF = Ca^2$.

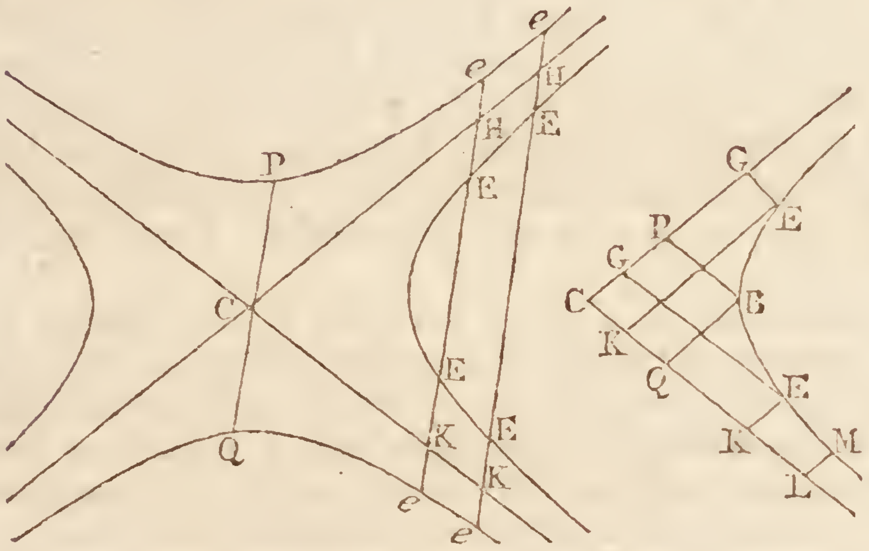
23. The difference of two lines drawn from the foci, to meet in any point of the curve, is equal to the tranſverſe axis. That is, fE − FE = AB, where F and f are the two foci.

24. All the parallelograms inſcribed between the four conjugate Hyperbolas are equal to one another, and each equal to the rectangle of the two axes. That is, the parallelogram OPQR = AB . ab (fig. to art. 9).

25. The difference of the ſquares of every pair of conjugate diameters, is equal to the ſame conſtant quantity, viz, the difference of the ſquares of the two axes. That is, $MN^2 - mn^2 = AB^2 - ab^2$, (fig. to art. 6); where MN and mn are any two conjugate diameters.

26. The rectangles of the parts of two parallel lines, terminated by the curve, are to one another, as the rectangles of the parts of any other two parallel lines, any where cutting the former. Or the rectangles of the parts of two interſecting lines, are as the ſquares of their parallel diameters, or ſquares of their parallel tangents.

27. All the rectangles are equal which are made of the ſegments of any parallel lines, cut by the curve, and limited by the aſymptotes, and each equal to the ſquare of their parallel diameter. That is, HE . EK or $He . eK = CQ^2$ or CP^2.



28. All the parallelograms are equal, which are formed between the aſymptotes and curve, by lines parallel to the aſymptotes. That is, the paral. CGEK = CPBQ.—Hence is obtained another method of expreſſing the nature of the curve by an equation, involving the abſciſs taken on one aſymptote, and ordinate parallel to the other aſymptote. Thus, if $x =$ CK, $y =$ KE, $a =$ CQ, and $b =$ BQ the ordinate at the vertex B of the curve; then, by the property in this article, $ab = xy$, or $a : x :: y : b$; that is, the rectangle of the abſciſs and ordinate is every where of the ſame magnitude, or any ordinate is reciprocally as its abſciſs.

29. If the abſciſſes CQ, CK, CL, &c, taken on the one aſymptote, be in geometrical progreſſion increaſing; then ſhall the ordinates QB, KE, LM, &c, parallel to the other aſymptote, be a like geometrical progreſſion in the ſame ratio, but decreaſing; and all the rectangles are equal, under every abſciſs and its ordinate, viz, CQ . QB = CK . KE = CL . LM, &c.

30. The abſciſſes CQ, CK, CL, &c, being taken in geometrical progreſſion; the ſpaces or aſymptotic areas BQKE, EKLM, &c, will be all equal; or, the ſpaces BQKE, BQLM, &c, will be in arithmetical progreſſion; and therefore theſe ſpaces are the hyperbolic logarithms of thoſe abſciſſes.

Theſe, and many other curious properties of the Hyperbola, may be ſeen demonſtrated in my Treatiſe on Conic Sections, and ſeveral others. See alſo CONIC SECTIONS.

Acute HYPERBOLA, one whoſe aſymptotes make an acute angle.

Ambigenal HYPERBOLA, is that which has one of its infinite legs falling within an angle formed by the aſymptotes, and the other falling without that angle. This is one of Newton's triple Hyperbolas of the 2d order. See his Enumeratio Lin. tert. Ord. See alſo AMBIGENAL.

Common, or *Conic* HYPERBOLA, is that which ariſes from the ſection of a cone by a plane; called alſo the Apollonian Hyperbola, being that kind treated on by the firſt and chief author Apollonius.

Conjugate HYPERBOLAS, are thoſe formed or lying together, and having the ſame axes, but in a contrary order, viz, the tranſverſe of each equal the conjugate of the other; as the two Conjugate Hyperbolas *Pee* and EEE in the laſt figure but one.

Equilateral, or *Rectanglar* HYPERBOLA, is that whoſe two axes are equal to each other, or whoſe aſymptotes make a right angle.—Hence, the property or equation of the equilateral Hyperbola, is $y^2 = ax + x^2$, where a is the axis, x the abſciſs, and y its ordinate; which is ſimilar to the equation of the circle, viz, $y^2 = ax - x^2$, differing only in the ſign of the ſecond term, and where a is the diameter of the circle.

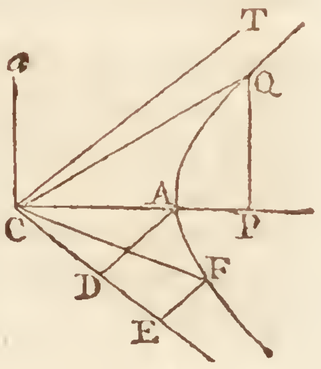
Infinite HYPERBOLAS, or HYPERBOLAS *of the higher kinds*, are expreſſed or defined by general equations ſimilar to that of the conic or common Hyperbola, but having general exponents, inſtead of the particular numeral ones, but ſo as that the ſum of thoſe on one ſide of the equation, is equal to the ſum of thoſe on the other ſide. Such as, $ay^{m+n} = bx^m(d + x)^n$, where x and y are the abſciſs and ordinate to the axis or diameter of the curve; or $x^m y^n = a^{m+n}$, where the abſciſs x is taken on one aſymptote, and the ordinate y parallel to the other.

As the Hyperbola of the firſt kind, or order, viz the conic Hyperbola, has two aſymptotes; that of the 2d kind or order has three; that of the 3d kind, four; and ſo on.

Obtuſe HYPERBOLA, is that whoſe aſymptotes form an obtuſe angle.

Rectangular HYPERBOLA, the ſame as Equilateral Hyperbola.

HYPERBOLIC *Arc*, is the arc of an Hyperbola.



Put $a =$ CA the ſemitranſverſe axe, $c = Ca$ the ſemiconjugate, $y =$ an ordinate PQ to the axe drawn from the end Q of the arc AQ, beginning at the vertex A: then putting $q = \frac{aa + cc}{c^4}$,

$$A = \text{the hyp. log. of } \frac{y + \sqrt{cc + yy}}{c}$$

$$= 2{\cdot}302585093 \times \text{common log. of } \frac{y + \sqrt{cc + yy}}{c},$$

$$B = \frac{y\sqrt{cc + yy} - ccA}{2}, \quad C = \frac{y^3\sqrt{cc + yy} - 3ccB}{4},$$

$$D = \frac{y^5\sqrt{cc + yy} - 5ccC}{6}, \text{ \&c;}$$

then is the length of the arc AQ expreſſed by

$$c \times (A + \frac{q}{2}B - \frac{q^2}{2.4}C + \frac{1.3q^3}{2.4.6}D - \frac{1.3.5q^4}{2.4.6.8}E \text{ \&c;}$$

or by $\frac{15c^2t + (21c^2 + 19t^2)x}{15c^2t + (21c^2 + 9t^2)x} \times y$, nearly; where t is the whole tranſverſe axe 2CA, $c =$ 2Ca the conjugate, $x =$ AP the abſciſs, and $y =$ PQ the ordinate.

Theſe and other rules may be ſeen demonſtrated in my Menſuration, p. 408, &c, 2d edit.

Hyperbolic *Area*, or *Space*, the area or ſpace included by the Hyperbolic curve and other lines.

Putting $a =$ CA the ſemitranſverſe, $c =$ Ca the ſemiconjugate, $y =$ PQ the ordinate, and $v =$ CP its diſtance from the centre; then is the

$$\text{area APQ} = \tfrac{1}{2}vy - \tfrac{1}{2}ac \times \text{hyp. log. of } \frac{ay + cv}{ac};$$

$$\text{ſector CAQ} = \tfrac{1}{2}ac \times \text{hyp. log. of } \frac{ay + cv}{ac};$$

$$\text{area APQ} = 2xy \times \left(\tfrac{1}{3} - \frac{z^1}{1\cdot3\cdot5} - \frac{z^2}{3\cdot5\cdot7} - \frac{z^3}{5\cdot7\cdot9} - \frac{z^4}{7\cdot9\cdot11}\ \&c\right),$$

where $x =$ AP, and $z = \frac{x}{2a + x}$; or

$$\text{APQ} = \frac{2cx}{15a} \times \left(\sqrt{2ax} + 4\sqrt{2ax + \tfrac{3}{4}xx}\right) \text{ nearly.}$$

Let CT and CE be the two aſymptotes, and the ordinates DA, EF parallel to the other aſymptote CT; then the aſymptotic ſpace ADEF or ſector CAF is

$$= \text{CD} \times \text{DA} \times \text{hyp. log. of } \frac{\text{CE}}{\text{CD}} \text{ or}$$

$$= \text{CD} \times \text{DA} \times \text{hyp. log. } \frac{\text{DA}}{\text{EF}} \text{ or}$$

$$= \text{CD} \times \text{DA} \times \frac{\text{DE}}{\text{CD}} - \frac{\text{DE}^2}{2\text{CD}^2} + \frac{\text{DE}^3}{3\text{CD}^3} - \frac{\text{DE}^4}{4\text{CD}^4}\ \&c;$$

and this laſt ſeries was firſt given by Mercator in his Logarithmotechnia.

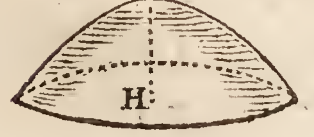
See my Menſuration, p. 413, &c, 2d edit.

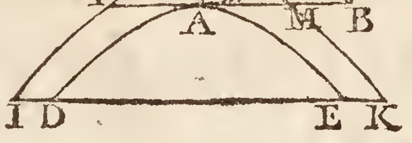
Generally, if $x^m y^n = a^{m+n}$ be an equation expreſſing an Hyperbola of any order; then its aſymptotic area will be $\frac{n}{n - m}xy$; which ſpace therefore is always quadrable, in all the orders of Hyperbolas, except the firſt or common Hyperbola only, in which m and n being each 1, the denominator $n - m$ becomes 0 or nothing.

Hyperbolic *Conoid*, a ſolid formed by the revolution of an Hyperbola about its axis, otherwiſe called an Hyperboloid.

To find the Solid Content of an Hyperboloid.

Let AC be the ſemitranſverſe of the generating Hyperbola, and AH the height of the ſolid; then as 2AC + AH is to 3AC + AH, ſo is the cone of the ſame baſe and altitude, to the content of the Conoid.





To find the Curve Surface of an Hyperboloid.

Let AC be the ſemitranſverſe, and AB perpendicular to it, and equal to the ſemiconjugate of ADE the generating Hyperbola, or ſection through the axis of the ſolid. Join CB; make CF = CA, and on CA let fall the perpendicular FG; then with the ſemitranſverſe CG, and ſemiconjugate GH = AB, deſcribe the Hyperbola GIK; then as the diameter of a circle is to its circumference, ſo is the Hyperbolic fruſtum ILAMK to the curve ſurface of the Conoid generated by DAE. See my Menſur. p. 429, &c, 2d edit.

Hyperbolic *Cylindroid*, a ſolid formed by the revolution of an Hyperbola about its conjugate axis, or line through the centre perpendicular to the tranſverſe axis. This ſolid is treated of in the Philoſ. Tranſ. by Sir Chriſtopher Wren, where he ſhews ſome of its properties, and applies it to the grinding of Hyperbolical Glaſſes; affirming that they muſt be formed this way, or not at all. See Philoſ. Tranſ. vol. 4, pa. 961.

Hyperbolic *Leg*, of a curve, is that having an aſymptote, or tangent at an infinite diſtance.—Newton reduces all curves, both of the firſt and higher kinds, into Hyperbolic and parabolic legs, i. e. ſuch as have aſymptotes, and ſuch as have not, or ſuch as have tangents at an infinite diſtance, and ſuch as have not.

Hyperbolic *Line*, is uſed by ſome authors for what is more commonly called the Hyperbola itſelf, being the curve line of that figure; in which ſenſe the ſurface terminated by it is called the Hyperbola.

Hyperbolic *Logarithm*, a logarithm ſo called as being ſimilar to the aſymptotic ſpaces of the Hyperbola. The Hyperbolic logarithm of a number, is to the common logarithm, as 2·3025850929940457 to 1, or as 1 to ·4342944819032518. The firſt invented logarithms, by Napier, are of the Hyperbolic kind; and ſo are Kepler's. See Logarithm.

Hyperbolic *Mirror*, is one ground into that ſhape.

Hyperbolic *Space*, that contained by the curve of the Hyperbola, and certain other lines. See Hyperbolic Area.

HYPERBOLICUM *Acutum*, a ſolid made by the revolution of the infinite area or ſpace contained between the curve of the Hyperbola, and its aſymptote. This produces a ſolid, which though infinitely long and generated by an infinite area, is nevertheleſs equal to a finite ſolid body; as is demonſtrated by Torricelli, who gave it this name.

HYPERBOLIFORM *Figures*, are ſuch curves as approach, in their properties, to the nature of the Hyperbola; called alſo Hyperboloides.

HYPERBOLOIDS, are Hyperbolas of the higher kind, whoſe nature is expreſſed by this equation, $ay^{m+n} = bx^m(a+x)^n$. See Hyperbola. It alſo means the Hyperbolic Conoid. See that article.

HYPERBOREANS, the moſt northern nations, or regions, as dwelling beyond or about the wind Boreas: as the Siberians, Samoieds, &c.

HYPERTHYRON, in Architecture, a ſort of table, uſually placed over gates or doors of the Doric order, above the chambranle, in form of a frize.

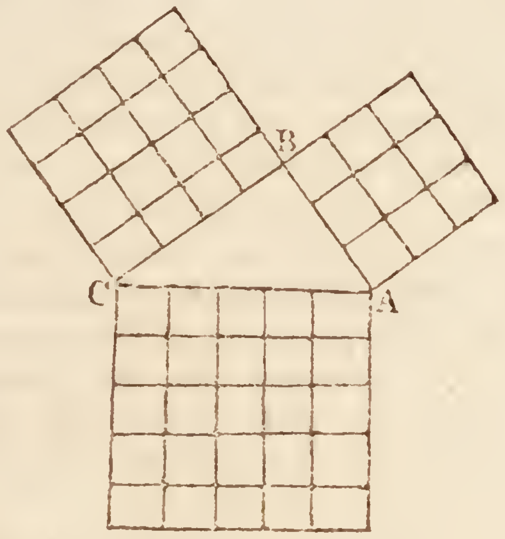
HYPETHRE, in Ancient Architecture, two rows of pillars ſurrounding, and ten at each face of a temple, &c, with a periſtyle within of ſix columns.

HYPOGEUM, in the ancient Architecture, a name common to all the parts of a building that are under ground; as the cellars, butteries, &c.

Hypogeum, in Aſtrology, a name given to the celeſtial houſes that are below the horizon; and eſpecially the imum cœli, or bottom point of the heavens.

HYPOMOCHLION, the fulcrum or prop of a lever; or the point which ſuſtains its preſſure, when employed either in raiſing or lowering bodies. The Hypomochlion is frequently a roller ſet under the lever; or under ſtones or pieces of timber, &c, that they may be the more eaſily lifted up, or removed.

HYPOTENUSE, or Hypothenuse, in a right-angled triangle, is the ſide which ſubtends, or is oppoſite to the right angle, and is always the longeſt of the three ſides; as the ſide AC, oppoſite to the right angle B.



It is a celebrated theorem in Plane Geometry, being the 47th prop. of the 1ſt book of Euclid, that in every right-angled triangle ABC, the ſquare formed upon the Hypothenuſe AC, is equal to both the two ſquares formed upon the other two ſides AB and BC; or that $AC^2 = AB^2 + BC^2$. This is particularly called the Pythagorean theorem, from its reputed inventor Pythagoras, who it is ſaid ſacrificed a whole hecatomb to the muſes, in gratitude for the diſcovery. But the ſame thing is true of circles or any other ſimilar figures, viz, that any figure deſcribed on the Hypotenuſe, is equal to the ſum of the two ſimilar figures deſcribed on both the other two ſides.

HYPOTHENUSE. See Hypotenuse.

HYPOTHESIS, in Geometry, or Mathematics, means much the ſame thing with ſuppoſition, being a ſuppoſition or an aſſumption of ſomething as a condition, upon which to raiſe a demonſtration, or from which to draw an inference.

Dr. Barrow ſays, Hypotheſes, or poſtulatums, are propoſitions aſſuming or affirming ſome evidently poſſible mode, action, or motion of a thing, and that there is the ſame affinity between hypotheſes and problems, as between axioms and theorems: a problem ſhewing the manner, and demonſtrating the poſſibility of ſome ſtructure, and an Hypotheſis aſſuming ſome conſtruction which is manifeſtly poſſible.

Hypothesis, in Philoſophy, denotes a kind of ſyſtem laid down from our own imagination, by which to account for ſome phenomenon or appearance of nature. Thus there are Hypotheſes to account for the tides, for gravity, for magnetiſm, for the deluge, &c.

The real and ſcientific cauſes of natural things generally lie very deep: obſervation and experiment, the proper means of arriving at them, are in moſt caſes extremely ſlow; and the human mind is very impatient: hence we are often induced to feign or invent ſomething that may ſeem like the cauſe, and which is calculated to anſwer the ſeveral phenomena, ſo that it may poſſibly be the true cauſe.

Philoſophers are divided as to the uſe of ſuch fictions or Hypotheſes, which are much leſs current now than they were formerly. The lateſt and beſt writers are for excluding Hypotheſes, and ſtanding intirely on obſervation and experiment. Whatever is not deduced from phenomena, ſays Newton, is an Hypotheſis; and Hypotheſes, whether metaphyſical, or phyſical, or mechanical, or of occult qualities, have no place in experimental philoſophy. Phil. Nat. Prin. Math. in Calce.

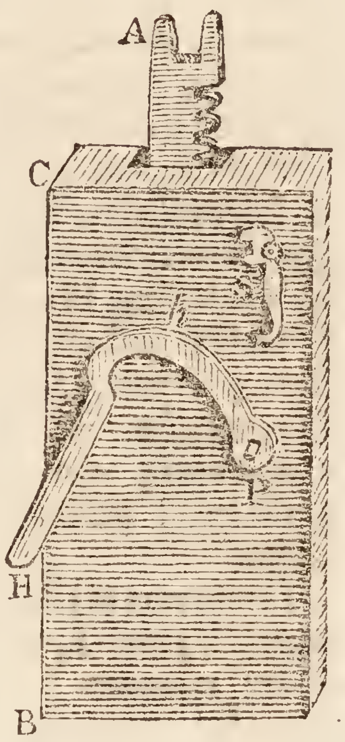
Hypothesis is more particularly applied, in Aſtronomy, to the ſeveral ſyſtems of the heavens; or the divers manners in which different aſtronomers have ſuppoſed the heavenly bodies to be ranged, or moved. The principal Hypotheſes are the Ptolomaic, the Tychonic, and the Copernican. This laſt is now ſo generally received, and ſo well eſtabliſhed and warranted by obſervation, that it is thought derogatory to it to call it an Hypotheſis.

HYPOTRACHELION, in Architecture, is uſed for a little frize in the Tuſcan and Doric capital, between the aſtragal and annulets; called alſo the colerin and gorgerin.

The word is applied by ſome authors in a more general ſenſe, to the neck of any column, or that part of its capital below the aſtragal.

I. J.

JACK, in Mechanics, is an inſtrument in common uſe for raiſing heavy timber, or very great weights of any kind; being a certain very powerful combination of teeth and pinions, and the whole incloſed in a ſtrong wooden ſtock or frame BC, and moved by a winch or handle HP; the outſide appearing as in fig. 1, here annexed.



In fig. 6, pl. 12, the wheel or rack-work is ſhewn, being the view of the inſide when the ſtock is removed. Though it is not drawn in the juſt proportions and dimenſions, for the rack AB muſt be ſuppoſed at leaſt four times as long in proportion to the wheel Q, as the figure repreſents it; and the teeth, which will be then four times more in number to have about 3 in the inch. Now if the handle HP be 7 inches long, the circumference of this radius will be 44 inches, which is the diſtance or ſpace the power moves through in one revolution of the handle: but as the pinion of the handle has but 4 leaves, and the wheel Q ſuppoſe 20 teeth, or 5 times the number, therefore to make one revolution of the wheel Q, it requires 5 turns of the handle, in which caſe it paſſes through 5 times 44 or 220 inches: but the wheel having a pinion R of 3 leaves, theſe will raiſe the rack 3 teeth, or one inch, in the ſame ſpace. Hence then, the handle or power moving 220 times as faſt as the weight, will raiſe or balance a weight of 220 times its own power. And if this be the hand of a man, who can ſuſtain 100 pounds weight, he will, by help of this Jack, be able to raiſe, or ſuſtain a weight or force of 22000lb, or about 10 tons weight.

This machine is ſometimes open behind from the bottom almoſt up to the wheel Q, to let the lower claw, which in that caſe is turned up as at B, draw up any weight. When the weight is drawn or puſhed ſufficiently high, it is kept from going back by hanging the end of the hook S, fixed to a ſtaple, over the curved part of the handle at *h*.

JACK is alſo the name of a well-known engine in the kitchen, uſed for turning a ſpit. Here the weight is the power applied, acting by a ſet of pulleys; the friction of the parts, and the weight with which the ſpit is charged, make the force to be overcome; and a ſteady uniform motion is maintained by means of the fly.

See the fig. of this machine, pl. 12, fig. 7.

Smoke JACK, is an engine uſed for the ſame purpoſe with the common Jack, and is ſo called from its being moved by means of the ſmoke, or rarefied air, aſcending the chimney, and ſtriking againſt the ſails of the horizontal wheel AB (plate 12, fig. 8), which being inclined to the horizon, is moved about the axis of the wheel, together with the pinion C, which carries the wheels D and E; and E carries the chain F, which turns the ſpit. The wheel AB ſhould be placed in the narrow part of the chimney, where the motion of the ſmoke is ſwifteſt, and where alſo the greateſt part of it muſt ſtrike upon the ſails.—The force of this machine depends upon the draught of the chimney, and the ſtrength of the fire.

JACK-*arch*, in Architecture, is an arch of one brick thickneſs.

JACK-*head*, in Hydraulics, a part ſometimes annexed to the forcing pump.

JACOB's-*Staff*, a mathematical inſtrument for taking heights and diſtances; the ſame with the Croſs-ſtaff; which ſee.

JACOBUS, a gold coin, worth 25 ſhillings; ſo called from king James the firſt of England, in whoſe reign it was ſtruck. They diſtinguiſhed two kinds of the Jacobus, the old and the new; the former valued at 25 ſhillings, weighing 6 dwts 10 grs; the latter, called alſo Carolus, valued at 23 ſhillings, and weighing 5 dwts 20 grains.

JAMBS, or JAUMS, in Architecture, are the upright ſides of chimneys, from the hearth to the mantletree. Alſo door poſts, or the upright poſts at the ends of the window frames.

St JAMES's *Day*, a feſtival in the calendar, obſerved on the 25th of July, in honour of St. James the apoſtle.

JANUARY, the firſt month of the year, according to the computation now uſed in the Weſt, and containing 31 days; ſo called by the Romans from Janus, one of their divinities, to whom they gave two faces; becauſe on the one ſide, the firſt day of this month looked towards the new year, and on the other towards the old one. The name may alſo be derived from Janua, a gate; this month, being the firſt of the year, may be conſidered as the gate or entrance of it.

January and February were introduced into the year by Numa Pompilius; Romulus's year beginning with the month of March.

JAUMS. See Jambs.

ICE, a brittle tranſparent body, formed of ſome fluid, frozen or fixed by cold. The ſpecific gravity of Ice to water, is various, according to the nature and circumſtances of the water, degree of cold, &c. Dr. Irving (Phipps's Voyage towards the North Pole) found the denſeſt Ice he could meet with about a 14th part lighter than water. M. de Mairan found it, at different trials, 1-14th, 18th, or 19th lighter than water; and when the water was previouſly purged of air, only a 22d part.

The rarefaction of Ice has been ſuppoſed owing to the air-bubbles produced in Ice while freezing; theſe, being conſiderably large in proportion to the water frozen, render the Ice ſo much ſpecifically lighter. It is well known that a conſiderable quantity of air is lodged in the interſtices of water, though it has there little or no elaſtic property, on account of the diſunion of its particles; but upon theſe particles coming cloſer together, and uniting as the water freezes, light, expanſive, and elaſtic air-bubbles are thus generated, and increaſe in bulk as the cold grows ſtronger, and by their elaſtic force burſts to pieces any veſſel in which the water is cloſely contained. But ſnow-water, or any water long boiled over the fire, affords an Ice more ſolid, and with fewer bubbles. Pure water long kept in vacuo and frozen afterwards there, freezes much ſooner, on being expoſed to the ſame degree of cold, than water unpurged of its air and ſet in the open atmoſphere. And the Ice made of water thus diveſted of its air, is much harder, more ſolid and tranſparent, and heavier than common Ice.

But M. de Mairan, in a diſſertation on Ice, attributes the increaſe of the bulk of the water under this form, chiefly to a different arrangement of its parts: the icy ſkin on water being compoſed of filaments which are found to be joined conſtantly and regularly at an angle of 60°, and which, by this diſpoſition, occupy a greater volume than if they were parallel. Beſides, after Ice is formed, he found it continue to expand by cold; a piece of Ice, which was at firſt only a 14th part ſpecifically lighter than water, on being expoſed ſome days to the froſt, became a 12th part lighter; and thus he accounts for the burſting of Ice in ponds.

It appears from an experiment of Dr. Hooke, in 1663, that Ice refracts the light leſs than water; whence he infers, that the lightneſs of Ice, which cauſes it to ſwim in water, is not produced merely by the ſmall bubbles which are viſible in it, but that it ariſes from the uniform conſtitution or general texture of the whole maſs: a fact which was afterward confirmed by M. de la Hire. See Hooke's Exper. by Derham, p. 26, Acad. Per. 1693, Mem. p. 25.

Sir Robert Barker thus deſcribes the proceſs of making Ice in the Eaſt Indies, in a country where he never ſaw any natural Ice. On a large plain they dig three or four pits, each about 30 feet ſquare, and 2 feet deep; the bottoms of which are covered, about 8 or 12 inches thick, with ſugar-cane, or the ſtems of the large Indian corn, dried. On this bed are placed in rows a number of ſmall ſhallow unglazed earthen pans, formed of a very porous earth, a quarter of an inch thick, and about an inch and a quarter deep; which, at the duſk of the evening, they fill with ſoft water that has been boiled. In the morning before ſunriſe the Ice-makers attend at the pits, and collect what has been frozen in baſkets, which they convey to the place of preſervation. This is uſually prepared in ſome high and dry ſituation, by ſinking a pit 14 or 15 feet deep, which they line firſt with ſtraw, and then with a coarſe kind of blanketing. The Ice is depoſited in this pit, and beaten down with rammers, till at length its own accumulated cold again freezes it, and it forms one ſolid maſs. The mouth of the pit is well ſecured from the exterior air with ſtraw and blankets, and a thatched roof is thrown over the whole. Philoſ. Tranſ. vol. 65, p. 252.

ICHNOGRAPHY, in Architecture, is a tranſverſe or horizontal ſection of a building, exhibiting the plot of the whole edifice, and of the ſeveral rooms and apartments in any ſtory; together with the thickneſs of the walls and partitions; the dimenſions of the doors, windows, and chimneys; the projectures of the columns and piers, with every thing viſible in ſuch a ſection.

Ichnography, in Fortification, is the plan or repreſentation of the length and breadth of a fortreſs; the diſtinct parts of which are marked out, either on the ground itſelf, or upon paper.

Ichnography, in Perſpective, the view of any thing cut off by a plane parallel to the horizon, juſt by the baſe or bottom of it; being the ſame with what is otherwiſe called the plan, geometrical plan, or ground-plot, of any thing, and is oppoſed to Orthography or Elevation.

ICOSAEDRON, or Icosahedron, one of the five regular bodies or ſolids, terminated by twenty equilateral and equal triangles. It may be conſidered as conſiſting of 20 equal and ſimilar triangular pyramids, whoſe vertices meet in the centre of a ſphere conceived to circumſcribe it, and therefore having all their heights and baſes equal; therefore the ſolidity of one of thoſe pyramids multiplied by 20, the number of them, gives the ſolid content of the Icoſaedron.

To form or make the Icoſaedron.—Deſcribe upon a card paper, or ſome other ſuch like ſubſtance, 20 equilateral triangles, as in the figure at the article *Regular* Body. Cut it out by the extreme edges, and cut all the other lines half through, then fold the ſides up by theſe edges half cut through, and the ſolid will be formed.

The linear edge or ſide of the Icoſaedron being A, then will the ſurface be $5A^2\sqrt{3} = 8{\cdot}6602540A^2$, and the ſolidity =

$$\tfrac{5}{6}A^3\sqrt{\frac{7+3\sqrt{5}}{2}} = 2{\cdot}1816950A^3.$$

More generally, put A = the linear edge or ſide, B the ſurface, and C the ſolid content of the Icoſaedron, alſo r the radius of the inſcribed, and R the radius of the circumſcribing ſphere, then we have theſe general equations, viz, 1ſt, $A = r\sqrt{42 - 18\sqrt{5}} = R\sqrt{\frac{10 - 2\sqrt{5}}{5}} = \sqrt{\frac{B\sqrt{3}}{15}} = \sqrt[3]{\frac{6}{5}C\sqrt{\frac{7 - 3\sqrt{5}}{2}}}$.

2d, $B = 3r^2 \times \overline{7\sqrt{3} - 3\sqrt{15}} = 2R^2 \times \overline{5\sqrt{3} - \sqrt{15}} = 5A^2\sqrt{3} = 3\sqrt[3]{70\sqrt{3} - 30\sqrt{15}}$.

3d, $C = 10r^3 \times 7\sqrt{3} - 3\sqrt{15} = \frac{7}{3}R^3\sqrt{10 + 2\sqrt{5}} = \frac{5}{6}A^3\sqrt{\frac{7 + 3\sqrt{5}}{2}} = \frac{B}{18}\sqrt{\frac{7\sqrt{3} + 3\sqrt{15}}{10}B}$.

4th, $R = r\sqrt{15 - 6\sqrt{5}} = \frac{1}{2}A\sqrt{\frac{5 + \sqrt{5}}{2}} = \frac{1}{2}\sqrt{\frac{5\sqrt{3} + \sqrt{15}}{30}B} = \sqrt[3]{\frac{3}{4}C\sqrt{\frac{5 - \sqrt{5}}{10}}}$.

5th, $r = R\sqrt{\frac{5 + 2\sqrt{5}}{15}} = \frac{1}{2}A\sqrt{\frac{7 + 3\sqrt{5}}{6}} = \frac{1}{6}\sqrt{\frac{7\sqrt{3} + 3\sqrt{15}}{10}B} = \frac{1}{2}\sqrt[3]{\frac{7\sqrt{3} + 3\sqrt{15}}{30}C}$.

See my Menſuration, p. 258, 2d edit.

IDES, in the Roman Calendar, a name given to a ſeries of 8 days in each month; which, in the full months, March, May, July, and October, commenced on the 15th day; and in the other months, on the 13th day; from thence reckoned backward, ſo as in thoſe four months to terminate on the 8th day, and in the reſt on the 6th. Theſe came between the calends and the nones. And this way of counting is ſtill uſed in the Roman Chancery, and in the Calendar of the Breviary.

The Ides of May were conſecrated to Mercury; the Ides of March were always eſteemed unhappy, after the death of Cæſar; the time after the Ides of June was reckoned fortunate for thoſe who entered into matrimony; the Ides of Auguſt were conſecrated to Diana, and were obſerved as a feaſt by the ſlaves; on the Ides of September, auguries were taken for appointing the magiſtrates, who formerly entered into their offices on the Ides of May, and afterwards on thoſe of March.

JET D'EAU, a French word, ſignifying a fountain that throws up water to ſome height in the air.

A Jet of water is thrown up by the weight of the column of water above its ajutage, or orifice, up to its ſource or reſervoir; and therefore it would riſe to the ſame height as the head or reſervoir, if certain cauſes did not prevent it from riſing quite ſo high. For firſt, the velocity of the lower particles of the Jet being greater than that of the upper, the lower water ſtrikes that which is next above it; and as fluids preſs every way, by its impulſe it widens, and conſequently ſhortens the column. Secondly, the water at the top of the Jet does not immediately fall off, but forms a kind of ball or head, the weight of which depreſſes the Jet; but if the Jet be a little inclined, or not quite upright, it will play higher, though it will not be quite ſo beautiful. Thirdly, the friction againſt the ſides of the pipe and hole of the ajutage, will prevent the Jet from riſing quite ſo high, and a ſmall one will be more impeded than a large one. And the fourth cauſe is the reſiſtance of the air, which is proportional to the ſquare of the velocity of the water nearly; and therefore the defect in the height will be nearly in the ſame proportion, which is alſo the ſame as the proportion of the heights of the reſervoirs above the ajutage. Hence, and from experience, it is found that a Jet, properly conſtructed, will riſe to different heights according to the height of the reſervoir, as in the following table of the heights of reſervoirs and the heights of their correſponding Jets; the former in feet, and the latter in feet and tenths of a foot.

Heights of Reſervoirs and their Jets.

Ref.	Jet.	Ref.	Jet.	Ref.	Jet.	Ref.	Jet.
5	4·9	31	28·3	57	49·0	82	67·0
6	5·9	32	29·2	58	49·7	83	67·7
7	6·8	33	30·0	59	50·5	84	68·4
8	7·8	34	30·8	60	51·2	85	69·1
9	8·7	35	31·6	61	52·0	86	69·8
10	9·7	36	32·5	62	52·7	87	70·5
11	10·6	37	33·3	63	53·5	88	71·1
12	11·6	38	34·1	64	54·2	89	71·8
13	12·5	39	34·9	65	54·9	90	72·5
14	13·4	40	35·7	66	55·7	91	73·2
15	14·3	41	36·6	67	56·4	92	73·8
16	15·2	42	37·4	68	57·1	93	74·5
17	16·1	43	38·1	69	57·8	94	75·2
18	17·0	44	38·9	70	58·6	95	75·8
19	17·9	45	39·8	71	59·3	96	76·5
20	18·8	46	40·5	72	60·0	97	77·2
21	19·7	47	41·3	73	60·7	98	77·8
22	20·6	48	42·1	74	61·4	99	78·5
23	21·5	49	42·9	75	62·1	100	79·1
24	22·3	50	43·7	76	62·8	110	85·6
25	23·2	51	44·4	77	63·5	120	91·9
26	24·1	52	45·2	78	64·2	130	98·0
27	24·9	53	46·0	79	64·9	140	104
28	25·8	54	46·7	80	65·6	150	110
29	26·6	55	47·5	81	66·3	160	116
30	27·5	56	48·2				

By various experiments that have been made by Mariotte, Deſaguliers, and others, it has been found, that if the reſervoir be 5 feet high, a conduct pipe $1\frac{3}{4}$ inch diameter will admit a hole in the ajutage from $\frac{1}{4}$ to $\frac{3}{8}$ of an inch; and ſo on as in the following table:

Height Reſervoir.	Diam. of the Ajutage.	Diam. of the Conduct Pipe.
5 feet	$\frac{1}{4}$ to $\frac{3}{8}$ inch	$1\frac{3}{4}$ inch
10	$\frac{1}{4}$ to $\frac{1}{2}$	2
15	$\frac{1}{2}$	$2\frac{1}{4}$
20	$\frac{1}{2}$	$2\frac{1}{2}$
25	$\frac{1}{2}$	$2\frac{3}{4}$
30	$\frac{1}{2}$ to $\frac{3}{4}$	$3\frac{1}{4}$
40	$\frac{3}{4}$	$4\frac{1}{2}$
50	$\frac{3}{4}$	$5\frac{1}{4}$
60	1	$5\frac{3}{4}$ or 6
80	$1\frac{1}{4}$	$6\frac{1}{2}$ or 7
100	$1\frac{1}{4}$ to $1\frac{1}{2}$	7 or 8

But the ſize of the pipe will be more or leſs with the diſtance.

If it be required to keep any number of Jets of given dimenſions playing, by one common conduct-pipe; the diameter of an ajutage muſt be found that ſhall be equal to all the ſmall ones that are given, and from this its proper conduct-pipe. Thus, if there be 4 ajutages, each $\frac{3}{4}$ of an inch diameter; then the ſquare of $\frac{3}{4}$ is $\frac{9}{16}$, which multiplied by 4, the number of them, makes $\frac{36}{16}$, the ſquare root of which is $\frac{6}{4}$ or $1\frac{1}{2}$, the diameter of an ajutage equal to all the other four; to which in the table anſwers a pipe of 8 inches diameter. In general, the diameter of the conduct-pipe ſhould be about 6 times that of the ajutage.

See Mariotte's Mouvement des Eaux; Deſaguliers's Exper. Philoſ. vol. 2, p. 127, &c; Clare's Motion of Fluids, p. 109; &c.

JETTE, the border made round the ſtilts under a pier, in certain old bridges, being the ſame with ſtarling, conſiſting of a ſtrong framing of timber filled with ſtones, chalk, &c; to preſerve the foundations of the piers from injury.

IGNIS Fatuus, a common meteor, chiefly ſeen in dark nights about meadows, marſhes, and other moiſt places, as alſo in burying grounds, and near dung-hills. It is known among the people by the appellations, Will with a Wiſp, and Jack with a Lantern.

Dr. Shaw deſcribes a remarkable Ignis Fatuus, which he ſaw in the Holy Land, that was ſometimes globular, or in the form of the flame of a candle; and preſently afterward it ſpread itſelf ſo much as to involve the whole company in a pale harmleſs light, and then contract itſelf again, and ſuddenly diſappear. But in leſs than a minute it would become viſible as before; or, running along from one place to another, with a ſwift progreſſive motion, would expand itſelf at certain intervals over more than 2 or 3 acres of the adjacent mountains. The atmoſphere had been thick and hazy, and the dew on the horſes' bridles was uncommonly clammy and unctuous. In the ſame weather he obſerved thoſe luminous appearances, which ſkip about the maſts and yards of ſhips at ſea, and which the ſailors call corpuſanſe, by a corruption of the Spaniſh cuerpoſanto. Shaw's Travels, p. 363.

Newton calls it a vapour ſhining without heat; and ſuppoſed it to be of the ſame nature with the light iſſuing from putreſcent ſubſtances. Willughby and Ray were of opinion that it is occaſioned by ſhining inſects: but all the appearances of it obſerved by Derham, Beccaria, and others, ſufficiently evince that it muſt be an ignited vapour. Inflammable air has been found to be the moſt common of all the factitious airs in nature; and that it is the uſual product of the putrefaction and decompoſition of vegetable ſubſtances in water. Signor Volta writes to Dr. Prieſtley, that he fires inflammable air by the electric ſpark, even when the electricity is very moderate: and he ſuppoſes that this experiment explains the inflammation of the Ignes Fatui, provided they conſiſt of inflammable air, iſſuing from marſhy ground by help of the electricity of fogs, and by falling ſtars, which have probably an electrical origin. See Prieſtley's Obſ. on Air, vol. 3, p. 382; the Philoſ. Tranſ. Abr. vol. 7, p. 147 &c.

ILLUMINATION, the act or effect of a luminous body, or a body that emits light; ſometimes alſo the ſtate of another body that receives it.

Circle of Illumination. See Circle.

ILLUMINATIVE *Lunar Month*, the ſpace of time that the moon is viſible, between one conjunction and another.

IMAGE, in Optics, is the ſpectre or appearance of an object, made either by reflection or refraction.

In all plane mirrors, the Image is of the ſame magnitude as the object; and it appears as far behind the mirror as the object is before it. In convex mirrors, the Image appears leſs than the object; and farther diſtant from the centre of the convexity, than from the point of reflection. Mr. Molyneux gives the following rule for finding the diameter of an Image, projected in the diſtinct baſe of a convex mirror, viz, As the diſtance of the object from the mirror, is to the diſtance from the Image to the glaſs; ſo is the diameter of the object, to the diameter of the Image. See Lens, Mirror, Reflection, and Refraction.

IMAGINARY *Quantities*, or Impoſſible Quantities, in Algebra, are the even roots of negative quantities; which expreſſions are Imaginary, or impoſſible, or oppoſed to real quantities; as $\sqrt{-aa}$, or $\sqrt[4]{-a^4}$, &c. For, as every even power of any quantity whatever, whether poſitive or negative, is neceſſarily poſitive, or having the ſign +, becauſe + by +, or − by − give equally +; from hence it follows that every even power, as the ſquare for inſtance, which is negative, or having the ſign −, has no poſſible root; and therefore the even roots of ſuch powers or quantities are ſaid to be impoſſible or Imaginary. The mixt expreſſions ariſing from Imaginary quantities joined to real ones, are alſo Imaginary; as $a - \sqrt{-aa}$, or $b + \sqrt{-aa}$.

The roots of negative quantities were, perhaps, firſt treated of in Cardan's Algebra. As to the uneven roots of ſuch quantities, he ſhews that they are negative, and he aſſigns them: but the even roots of them he rejects, obſerving that they are nothing as to common uſe, being neither one thing nor another; that is, they are merely Imaginary or impoſſible. And ſince his time, it has gradually become a part of Algebra to treat of the roots of negative quantities. Albert Girard, in his *Invention Nouvelle en l'Algebre*, p. 42, gives names to the three ſorts of roots of equations, calling them, greater than nothing, leſs than nothing, and *envelopée*, as $\sqrt{-3}$: but this was ſoon after called Imaginary or impoſſible, as appears by Wallis's Algebra, p. 264, &c; where he obſerves that the ſquare root of a negative quantity, is a mean proportional between a poſitive and a negative quantity; as $\sqrt{-bc}$ is the mean proportional between $+b$ and $-c$, or between $-b$ and $+c$; and this he exemplifies by geometrical conſtructions. See alſo p. 313.

The arithmetic of theſe Imaginary quantities has not yet been generally agreed upon; viz, as to the operations of multiplication, diviſion, and involution; ſome authors giving the reſults with +, and others on the contrary with the negative ſign −. Thus, Euler, in his Algebra, p. 106 &c, makes the ſquare of $\sqrt{-3}$ to be -3, of $\sqrt{-1}$ to be -1, &c; and yet he makes the product of two impoſſibles, when they are unequal, to be poſſible and real: as $\sqrt{-2} \times \sqrt{-3} = \sqrt{6}$;

$= \sqrt{6}$; and $\sqrt{-1} \times \sqrt{-4} = \sqrt{4}$ or 2. But how can the equality or inequality of the factors cauſe any difference in the ſigns of the products?
If $\sqrt{-2} \times \sqrt{-3}$ be $= \sqrt{+6}$, how can
$\sqrt{-3} \times \sqrt{-3}$, which is the ſquare of $\sqrt{-3}$, be -3? Again, he makes $\sqrt{-3} \times \sqrt{+5} = \sqrt{-15}$. Alſo in diviſion, he makes $\sqrt{-4} \div \sqrt{-1}$ to be $= \sqrt{+4}$ or 2; and $\sqrt{+3} \div \sqrt{-3} = \sqrt{-1}$; alſo that 1 or $\sqrt{+1} \div \sqrt{-1} = \sqrt{\frac{+1}{-1}} = \sqrt{-1}$; conſequently, multiplying the quotient root $\sqrt{-1}$ by the diviſor $\sqrt{-1}$, muſt give the dividend $\sqrt{+1}$; and yet, by ſquaring, he makes the ſquare of $\sqrt{-1}$, or the product $\sqrt{-1} \times \sqrt{-1}$, equal to -1.

But Emerſon makes the product of Imaginaries to be Imaginary; and for this reaſon, that "otherwiſe a real product would be raiſed from impoſſible factors, which is abſurd. Thus,
$\sqrt{-a} \times \sqrt{-b} = \sqrt{-ab}$, and $\sqrt{-a} \times -\sqrt{-b} = -\sqrt{-ab}$, &c.
Alſo $\sqrt{-a} \times \sqrt{-a} = -a$, and $\sqrt{-a} \times -\sqrt{-a} = +a$, &c."
And thus moſt of the writers on this part of Algebra, are pretty equally divided, ſome making the product of impoſſibles real, and others Imaginary.

In the Philoſ. Tranſ. for 1778, p. 318 &c, Mr. Playfair has given an ingenious diſſertation "On the Arithmetic of Impoſſible Quantities." But this relates chiefly to the applications and uſes of them, and not to the algorithm of them, or rules for their products, quotients, ſquares, &c. From ſome operations however here performed, we learn that he makes the product of $\sqrt{-1}$ by $\sqrt{-1}$, or the ſquare of $\sqrt{-1}$, to be -1; and yet in another place he makes the product of $\sqrt{-1}$ and $\sqrt{1-z^2}$ to be $\sqrt{-1+z^2}$. Mr. Playfair concludes, "that Imaginary expreſſions are never of uſe in inveſtigations but when the ſubject is a property common to the meaſures both of ratios and of angles; but they never lead to any conſequence which might not be drawn from the affinity between thoſe meaſures; and that they are indeed no more than a particular method of tracing that affinity. The deductions into which they enter are thus reduced to an argument from analogy, but the force of them is not diminiſhed on that account. The laws to which this analogy is ſubject; the caſes in which it is perfect, in which it ſuffers certain alterations, and in which it is wholly interrupted, are capable of being preciſely aſcertained. Supported on ſo ſure a foundation, the arithmetic of impoſſible quantities will always remain an uſeful inſtrument in the diſcovery of truth, and may be of ſervice when a more rigid analyſis can hardly be applied. For this reaſon, many reſearches concerning it, which in themſelves might be deemed abſurd, are nevertheleſs not deſtitute of utility. M. Bernoulli has found, for example, that if r be the radius of a circle, the circumference is $= \frac{4 \log. \sqrt{-1}}{\sqrt{-1}} r$. Conſidered as a quadrature of the circle, this Imaginary theorem is wholly inſignificant, and would deſervedly paſs for an abuſe of calculation; at the ſame time we learn from it, that if in any equation the quantity $\frac{\log. \sqrt{-1}}{\sqrt{-1}}$ ſhould occur, it may be made to diſappear, by the ſubſtitution of a circular arch, and a property, common to both the circle and hyperbola, may be obtained. The ſame is to be obſerved of the rules which have been invented for the transformation and reduction of impoſſible quantities *; they facilitate the operations of this imaginary arithmetic, and thereby lead to the knowledge of the moſt beautiful and extenſive analogy which the doctrine of quantity has yet exhibited.

* The rules chiefly referred to, are thoſe for reducing the impoſſible roots of an equation to the form $A + B\sqrt{-1}$."

Imaginary *Roots*, of an equation, are thoſe roots or values of the unknown quantity in an equation, which contain ſome Imaginary quantity. So the roots of the equation $xx + aa = 0$, are the two Imaginary quantities $+\sqrt{-aa}$ and $-\sqrt{-aa}$, or $+a\sqrt{-1}$ and $-a\sqrt{-1}$; alſo the two roots of the equation $xx + ax + aa = 0$, are the Imaginary quantities $\pm \frac{1}{2} a - \frac{1}{2} a\sqrt{-3}$; and the three roots of the equation $x^3 - 1 = 0$, or $x^3 = 1$, are 1 and $\frac{-1+\sqrt{-3}}{2}$ and $\frac{-1-\sqrt{-3}}{2}$, the firſt real, and the two latter Imaginary. Sometimes too the real root of an equation may be expreſſed by Imaginary quantities; as in the irreducible caſe of cubic equations, when the root is expreſſed by Cardan's rule; and that happens whenever the equation has no Imaginary roots at all; but when it has two Imaginary roots, then the only real root is expreſſed by that rule in an Imaginary form. See my paper on Cubic Equations, in the Philoſ. Tranſ. for 1780, p. 406 &c.

Albert Girard firſt treated expreſsly on the impoſſible or Imaginary roots of equations, and ſhewed that every equation has as many roots, either real or Imaginary, as the index of the higheſt power denotes. Thus, the roots of the biquadratic equation $x^4 = 4x - 3$, he ſhews are two real and two Imaginary, viz, 1, 1, $-1 + \sqrt{-2}$, and $-1 - \sqrt{-2}$; and he renders the relation general, between all the roots and the coefficients of the terms of the equation. See his *Invention Nouvelle en l'Algebre, anno* 1629, theor. 2, pa. 40 &c.

M. D'Alembert demonſtrated, that every Imaginary root of any equation can always be reduced to the form $e + f\sqrt{-1}$, where e and f are real quantities. And hence it was alſo ſhewn, that if
one root of an equation be - $e + f\sqrt{-1}$,
another root of it will always be - $e - f\sqrt{-1}$:
and hence it appears that the number of the Imaginary roots in any equation is always even, if any; i. e. either none, or elſe two, or four, or ſix, &c. Memoirs of the Academy of Berlin, 1746.

To diſcover how many impoſſible roots are contained in any propoſed equation, Newton gave this rule, in his Algebra, viz, Conſtitute a ſeries of fractions, whoſe denominators are the ſeries of natural numbers 1, 2, 3, 4, 5, &c, continued to the number ſhewing the index or exponent of the higheſt term of the equations, and their numerators the ſame ſeries of numbers in the contrary order: and divide each of theſe fractions by that next before it, and place the reſulting quotients over the intermediate terms of the equation; then under each of the intermediate terms, if its ſquare multiplied by the fraction over it, be greater than the product of the terms on each ſide of it, place the ſign +; but if not, the ſign −; and under the firſt and laſt term place the ſign +. Then will the equation have as many Imaginary roots as there are changes of the underwritten ſigns from + to −, and from − to +. So for the equation $x^3 - 4x^2 + 4x - 6 = 0$, the ſeries of fractions is $\frac{3}{1}$, $\frac{2}{2}$, $\frac{1}{3}$; then the ſecond divided by the firſt gives $\frac{2}{6}$ or $\frac{1}{3}$, and the third divided by the ſecond gives $\frac{1}{3}$ alſo; hence theſe quotients placed over the intermediate terms, the whole will ſtand

$$\text{thus,}\quad \underset{+}{x^3} - \underset{+}{\overset{\frac{1}{3}}{4x^2}} + \underset{-}{\overset{\frac{1}{3}}{4x}} - \underset{+}{6}.$$

Now becauſe the ſquare of the 2d term multiplied by its ſuperſcribed fraction, is $\frac{16}{3}x^4$, which is greater than $4x^4$ the product of the two adjacent terms, therefore the ſign + is ſet below the 2d term; and becauſe the ſquare of the 3d term multiplied by its overwritten fraction, is $\frac{16}{3}x^2$, which is leſs than $24x^2$ the product of the terms on each ſide of it, therefore the ſign − is placed under that term; alſo the ſign + is ſet under the firſt and laſt terms. Hence the two changes of the underwritten ſigns + + − +, the one from + to −, and the other from − to +, ſhew that the given equation has two impoſſible roots.

When two or more terms are wanting together, under the place of the 1ſt of the deficient terms write the ſign −, under the 2d the ſign +, under the 3d −, and ſo on, always varying the ſigns, except that under the laſt of the deficient terms muſt always be ſet the ſign +, when the adjacent terms on both ſides of the deficient terms have contrary ſigns. As in the equation

$$\underset{+}{x^5} + \underset{+}{ax^4} \;\underset{-}{*}\; \underset{+}{*}\; \underset{-}{*} + \underset{+}{a^5} = 0,$$

which has four Imaginary roots.

The author remarks, that this rule will ſometimes fail of diſcovering all the impoſſible roots of an equation, for ſome equations may have more of ſuch roots than can be found by this rule, tho' this ſeldom happens.

Mr. Maclaurin has given a demonſtration of this rule of Newton's, together with one of his own, that will never fail. And the ſame has alſo been done by Mr. Campbell. See Philoſ. Tranſ. vol. 34, p. 104, and vol. 35, p. 515.

The real and imaginary roots of equations may be found from the method of fluxions, applied to the doctrine of maxima and minima, that is, to find ſuch a value of x in an equation, expreſſing the nature of a curve, made equal to y, an abſciſſa which correſponds to the greateſt and leaſt ordinate. But when the equation is above 3 dimenſions, the computation is very laborious. See Stirling's treatiſe on the lines of the 3d order, Schol. pr. 8, pa. 59, &c.

IMBIBE, is commonly uſed in the ſame ſenſe as abſorb, viz, where a dry porous body takes up another that is moiſt.

IMMENSE, that whoſe amplitude or extenſion cannot be equalled by any meaſure whatſoever, or how often ſoever repeated.

IMMERSION, the act of plunging into water, or ſome other fluid.

Immersion, in Aſtronomy, is when a ſtar, or planet comes ſo near the ſun, that it cannot be ſeen; being as it were enveloped, and hid in the rays of that luminary.

Immersion alſo denotes the beginning of an eclipſe, or of an occultation, when the body, or any part of it juſt begins to diſappear, either behind the edge of another body, or in its ſhadow. As, in an eclipſe of the moon, when ſhe begins to be darkened by entering into the ſhadow of the earth: or the beginning of an eclipſe of the ſun, when the moon's diſc juſt begins to cover him: or the beginning of the eclipſes of any of the ſatellites, as thoſe of Jupiter, by entering into his ſhadow: or, laſtly, the beginning of an occultation of any ſtar or planet, by paſſing behind the body of the moon or another planet. In all theſe caſes, the darkened body is ſaid to immerge, or to be immerged, or begin to be hid, by dipping as it were into the ſhade. In like manner, when the darkened body begins to appear again, it is ſaid to emerge, or come out of darkneſs again.

IMPACT, the ſimple or ſingle action of one body upon another to put it in motion. Point of Impact, is the place or point where a body acts.

IMPENETRABILITY, a quality by which a thing cannot be pierced or penetrated; or a property of body by which it fills up certain ſpaces, ſo that there is no room in them for any other body.

IMPENETRABLE, that cannot be penetrated.

IMPERFECT *Number*, is that whoſe aliquot parts, taken all together, do not make a ſum that is equal to the number itſelf, but either exceed it, or fall ſhort of it; being an abundant number in the former caſe, and a defective number in the latter. Thus, 12 is an abundant Imperfect number, becauſe the ſum of all its aliquot parts, 1, 2, 3, 4, 6, makes 16, which exceeds the number 12. And 10 is a defective Imperfect number, becauſe its aliquot parts, 1, 2, 5, taken all together, make only 8, which is leſs than the number 10 itſelf.

IMPERIAL *Table*, is an inſtrument made of braſs, with a box and needle, and ſtaff, &c, uſed for meaſuring of land.

IMPERVIOUS, not to be pervaded or entered either becauſe of the cloſeneſs of the pores, or the particular configuration of its parts.

IMPETUS, in Mechanics, force, momentum, motion, &c.

IMPOSSIBLE *Quantity*, or *Root*, the ſame as Imaginary ones; which ſee.

IMPOST, in Architecture, a capital or plinth, to a pillar, or pilaſter, or pier, that ſupports an arch, &c.

IMPROPER *Fraction*, is a fraction whoſe numerator is either equal to, or greater than, its denominator. As $\frac{5}{5}$ or $\frac{5}{3}$ or $\frac{19}{7}$. An Improper fraction is reduced to a whole or mixt number, by dividing the numerator by the denominator; the quotient is the integer, and the remainder ſet over the diviſor makes the fractional part of the value of the original Improper fraction. Thus $\frac{5}{5} = 1$, and $\frac{5}{3} = 1\frac{2}{3}$, and $\frac{19}{6} = 3\frac{1}{6}$. So that when the numerator is juſt equal to the denominator, the Improper fraction is exactly equal to unity or 1; but when the numerator is the greater, the fraction is greater than 1.

IMPULSE, the ſingle or momentary action or force by which a body is impelled; in contradiſtinction to continued forces; like the blow of a hammer, &c.

IMPULSIVE, a term applied to actions by impulſe.

INACCESSIBLE *Height* or *Diſtance*, is that which cannot be approached, or meaſured by actual meaſurement, by reaſon of ſome impediment in the way; as water, &c.

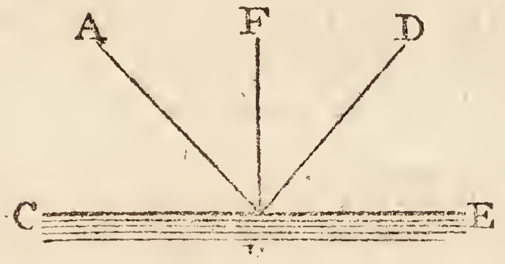
See HEIGHTS and DISTANCES.

INCEPTIVE, of *Magnitude*, a term uſed by Dr. Wallis, to expreſs ſuch moments, or firſt principles, as, though of no magnitude themſelves, are yet capable of producing ſuch as are. See INFINITE, and INDIVISIBLE. Thus, a point has no magnitude itſelf, but is inceptive of a line, which it produces by its motion. Alſo a line, though it has no breadth, is yet Inceptive of breadth; that is, it is capable, by its motion, of producing a ſurface, which has breadth.

INCH, a common Engliſh meaſure, being the 12th part of a foot, or 3 barley corns in length.

INCIDENCE, or *line of* INCIDENCE, in Mechanics, implies the direction or inclination in which one body ſtrikes or acts on another.——In the incurſions of two moving bodies, their Incidence is ſaid to be Direct or Oblique, as the directions of their motion make a ſtraight line, or an angle at the point of Impact.

Angle of INCIDENCE, by ſome writers, denotes the angle comprehended between the line of Incidence, and a perpendicular to the body acted on at the point of Incidence. Thus, ſuppoſe AB an Incident line, and BF a perpendicular to the plane CB at the incident point B; then ABF is the angle of Incidence, or of inclination.



But, according to Dr. Barrow, and ſome other writers, the Angle of Incidence is the complement of the former, or the angle made between the incident line, and the plane acted on, or a tangent at the point of Incidence; as the angle ABC.

It is demonſtrated by optical writers, 1ſt, That the Angle of Incidence, of the rays of light, is always equal to the angle of reflection; and that they lie in the ſame plane. And the ſame is proved by the writers on Mechanics, concerning the reflection of elaſtic bodies. That is, the ∠ABF = the ∠FBD, or the ∠ABC = the ∠DBE.—2d, That the ſines of the Angles of Incidence and refraction are to each other, either accurately, or very nearly, in a given or conſtant ratio.—3dly, That from air into glaſs, the ſine of the Angle of Incidence, is to the ſine of the angle of refraction, as 300 to 193, or nearly as 14 to 9: and, on the other hand, that out of glaſs into air, the ſign of the Angle of Incidence, is to the ſine of the angle of refraction, as 193 to 300, or as 9 to 14 nearly.

INCIDENCE *of Eclipſe*. See ECLIPSE and IMMERSION.

Axis of INCIDENCE, is the line FB perpendicular to the reflecting plane at the point of Incidence B.

Cathetus of INCIDENCE. See CATHETUS, and REFLECTION.

Line of INCIDENCE, in Catoptrics, denotes a right line, as AB, in which light is propagated from a radiant point A, to a point B, in the ſurface of a ſpeculum. The ſame line is alſo called an Incident ray.

Line of INCIDENCE, in Dioptrics, is a right line, as AB, in which light is propagated unrefracted, in the ſame medium, from the radiant point to the ſurface of the refracting body, CBE.

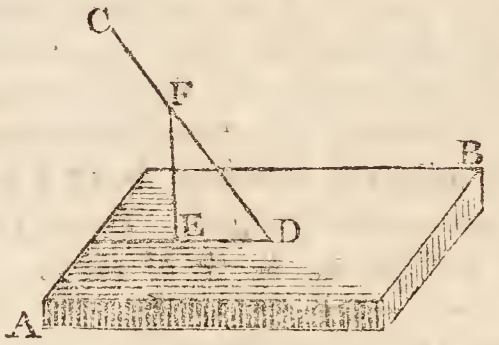
Point of INCIDENCE, is the point B on the ſurface of the reflecting or refracting medium, on which the Incident ray falls.

Scruples of INCIDENCE. See SCRUPLES.

INCIDENT *Ray*, is the line or ray AB, falling on the ſurface of any body, at B.

INCLINATION, in Geometry, Mechanics, or Phyſics, denotes the mutual tendency of two lines, planes, or bodies, towards one another; ſo that their directions make at the point of concourſe ſome certain angle.

INCLINATION *of the Axis of the Earth*, is the angle it makes with the plane of the ecliptic; or the angle between the planes of the equator and ecliptic.



INCLINATION *of a Line to a plane*, is the acute angle, as CDE, which the line CD makes with another line DE drawn in the plane through the incident point D and the foot of a perpendicular E from any point of the line upon the plane.

INCLINATION *of an Incident ray*, is the angle of inclination, or angle of incidence.

INCLINATION *of the Magnetical needle*. See DIPPING *Needle*.

INCLINATION *of Meridians*, in Dialling, is the angle that the hour-line on the globe, which is perpendicular to the dial-plane, makes with the meridian.

INCLINATION *of the Orbit of a planet*, is the angle formed by the planes of the ecliptic and of the orbit of the planet. The quantity of this Inclination for the ſeveral planets, is as follows, viz.

Mercury	-	-	6°	54′
Venus	-	-	3	20
Earth	-	-	0	0
Moon	-	-	5	18
Mars	-	-	1	52
Jupiter	-	-	1	20
Saturn	-	-	2	30
Herſchel	-	-	0	48

INCLINATION *of a Plane*, in Dialling, is the arch of a vertical, a vertical circle, perpendicular both to the plane and the horizon, and intercepted between them.

Inclination *of a Planet*, is the arch or angle comprehended between the ecliptic and the place of the planet in its orbit. The greateſt Inclination, or declination, is the ſame as the Inclination of the orbit; which ſee above.

Inclination *of a Reflected ray*, is the angle which a ray after reflection makes with the axis of Inclination; as the angle FBD, in the laſt fig. but one.

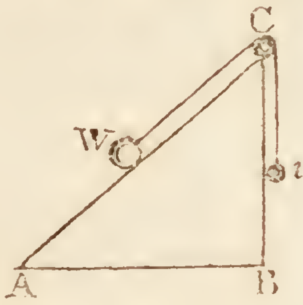
Inclination *of Two Planes*, is the angle made by two lines drawn in thoſe planes perpendicular to their common interſection, and meeting in any point of that interſection.

Angle of Inclination, is the ſame as what is otherwiſe called the angle of incidence.

Argument of Inclination. See Argument.

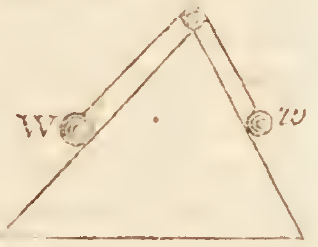
Inclined *Plane*, in Mechanics, is a plane inclined to the horizon, or making an angle with it. It is one of the ſimple mechanic powers, and the double inclined plane makes the wedge.

1. The power gained by the Inclined plane, is in proportion as the length of the plane is to its height, or as radius to the ſine of its inclination; that is, a given weight hanging freely, will balance upon the plane another weight, that ſhall be greater in that proportion. So, when the greater weight W on the plane, is balanced by the leſs weight *w* hanging perpendicularly, then is *w* : W :: BC : AC :: ſin. ∠A : radius. Or, in other words, the relative gravity of a body upon the plane, or its force in deſcending down the plane, is to its abſolute gravity or weight, in the ſame proportion of the height of the plane to its length, or of the ſine of inclination to radius.



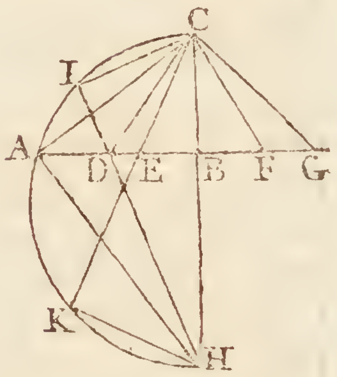
2. Hence therefore the relative gravities of the ſame body on different inclined planes, or their forces to deſcend down the planes, are to each other, as the ſines of the angles of inclination, to radius 1, or directly as the heights of the planes, and inverſely as their lengths.

3. Hence, if the planes have the ſame height, and abſolute weights of the bodies be directly proportional to the lengths of the planes, then the forces to deſcend will be equal. Conſequently, if the bodies be then connected by a ſtring acting parallel to the planes, they will exactly balance each other; as in the annexed figure.



4. The relative force of gravity upon the plane being in a conſtant ratio to the abſolute weight of the body, viz, as ſine of inclination to radius; therefore all the laws relating to the perpendicular free deſcents of bodies by gravity, hold equally true for the deſcents on inclined planes; ſuch as, that the motion is a uniformly accelerated one; that the velocities are directly as the times, and the ſpaces as the ſquare of either of them; uſing only the relative force upon the plane for the abſolute weight of the body, or inſtead of $32\frac{1}{6}$ feet, the velocity generated by gravity in the firſt ſecond of time, uſing $32\frac{1}{6}s$, where *s* is the ſine of the inclination to the radius 1.

5. The velocity acquired by a body in deſcending down an Inclined plane AC, when the body arrives at A, is the ſame as the velocity acquired by deſcending freely down the perpendicular altitude BC, when it arrives at B. But the times are very different; for the time of deſcending down the Inclined plane, is greater than down the perpendicular, in the ſame proportion as the length of the plane AC, is to the height CB: and ſo the time of deſcending from any point C to a horizontal line or plane ABG &c, down any oblique line, or Inclined plane, is directly proportional to the length of that plane, CA, or CD, or CE, or CB, or CF, &c.



6. Hence, if there be drawn AH perpendicular to AC, meeting CB produced in H; then the time of deſcending down any plane CA, is equal to the time of deſcending down the perpendicular CH. So that, if upon CH as a diameter a circle be deſcribed, the times of deſcent will be exactly equal, down every chord in the circle, beginning at C, and terminating any where in the circumference, as CI, CA, CK, CH, &c, or beginning any where in the circumference, and terminating at the loweſt point of the circle, as CH, IH, AH, KH, &c.

7. When bodies aſcend up Inclined planes, their motion is uniformly retarded; and all the former laws for deſcents, or the generation of motion, hold equally true for aſcents, or the deſtruction of as much motion.

Inclined *Towers*, are towers inclined, or leaning out of the perpendicular. See Towers.

INCLINERS, in Dialling, are inclined dials. See Dial.

INCOMMENSURABLE, Lines, or Numbers, or Quantities in general, are ſuch as have no common meaſure, or no line, number, or quantity of the ſame kind, that will meaſure or divide them both without a remainder. Thus, the numbers 15 and 16 are Incommenſurable, becauſe, though 15 can be meaſured by 3 and 5, and 16 by 2, 4, and 8, there is yet no ſingle number that will divide or meaſure them both.

Euclid demonſtrates (prop. 117, lib. 10) that the ſide of a ſquare and its diagonal are Incommenſurable to each other. And Pappus, prop. 17, lib. 4, ſpeaks of Incommenſurable angles.

Incommensurable *in Power*, is ſaid of quantities whoſe 2d powers, or ſquares, are Incommenſurable. As $\sqrt{2}$ and $\sqrt{3}$, whoſe ſquares are 2 and 3, which are Incommenſurable. It is commonly ſuppoſed that the diameter and circumference of a circle are Incommenſurable to each other; at leaſt their commenſurability has never been proved. And Dr. Barrow ſurmiſes even that they are infinitely Incommenſurable, or that all poſſible powers of them are Incommenſurable.

INCOMPOSITE *Numbers*, are the ſame with thoſe called by Euclid prime numbers, being ſuch as are not compoſed by the multiplication together of other numbers. As 3, 5, 7, 11, &c.

INCREMENT, is the ſmall increaſe of a variable quantity. Newton, in his Treatiſe on Fluxions, calls theſe by the name Moments, and obſerves that they are proportional to the velocity or rate of increaſe of the flowing or variable quantities, in an indefinitely ſmall time; he denotes them by ſubjoining a cipher o, to the flowing quantity whoſe moment or Increment it is; thus xo the moment of x. In the doctrine of Increments, by Dr. Brooke Taylor and Mr. Emerſon, they are denoted by points below the variable quantities; as $\underset{.}{x}$. Some have alſo denoted them by accents underneath the letter, as $\underset{\prime}{x}$; but it is now more uſual to expreſs them by accents over the ſame letter; as $\acute{x}$.

INCREMENTS, *Method of*, a branch of Analytics, in which a calculus is founded on the properties of the ſucceſſive values of variable quantities, and their differences, or Increments.

The inventor of the Method of Increments was the learned Dr. Taylor, who, in the year 1715, publiſhed a treatiſe upon it; and afterwards gave ſome farther account and explication of it in the Philoſ. Tranſ. as applied to the finding the ſums of ſeries. And another ingenious and eaſy treatiſe on the ſame, was publiſhed by Mr. Emerſon, in the year 1763. The method is nearly allied to Newton's Doctrine of Fluxions, and ariſes out of it. Alſo the Differential method of Mr. Stirling, which he applies to the ſummation and interpolation of ſeries, is of the ſame nature as the Method of Increments, but not ſo general and extenſive.

From the Method of Increments, Mr. Emerſon obſerves, "The principal foundation of the Method of Fluxions may be eaſily derived. For as in the Method of Increments, the Increment may be of any magnitude, ſo in the Method of Fluxions, it muſt be ſuppoſed infinitely ſmall; whence all preceding and ſucceſſive values of the variable quantity will be equal, from which equality the rules for performing the principal operations of fluxions are immediately deduced. That I may give the reader, continues he, a more perfect idea of the nature of this method: ſuppoſe the abſciſſa of a curve be divided into any number of equal parts, each part of which is called the Increment of the abſciſſa; and imagine ſo many parallelograms to be erected thereon; either circumſcribing the curvilineal figure, or inſcribed in it; then the finding the ſum of all theſe parallelograms is the buſineſs of the Method of Increments. But if the parts of the abſciſſa be taken infinitely ſmall, then theſe parallelograms degenerate into the curve; and then it is the buſineſs of the Method of Fluxions, to find the ſum of all, or the area of the curve. So that the Method of Increments finds the ſum of any number of finite quantities; and the Method of Fluxions the ſum of any infinite number of infinitely ſmall ones: and this is the eſſential difference between theſe two methods." Again, "There is ſuch a near relation between the Method of Fluxions, and that of Increments, that many of the rules for the one, with little variation, ſerve alſo for the other. And here, as in the Method of Fluxions, ſome queſtions may be ſolved, and the integrals found, in finite terms; whilſt in others we are forced to have recourſe to infinite ſeries for a ſolution. And the like difficulties will occur in the Method of Increments, as uſually happen in Fluxions. For whilſt ſome fluxionary quantities have no fluents, but what are expreſſed by ſeries; ſo ſome Increments have no integrals, but what infinite ſeries afford; which will often, as in fluxions, diverge and become uſeleſs."

By means of the Method of Increments, many curious and uſeful problems are eaſily reſolved, which ſcarcely admit of a ſolution in any other way. As, ſuppoſe ſeveral ſeries of quantities be given, whoſe terms are all formed according to ſome certain law, which is given; the Method of Increments will find out a general ſeries, which comprehends all particular caſes, and from which all of that kind may be found.

The Method of Increments is alſo of great uſe in finding any term of a ſeries propoſed: for the law being given by which the terms are formed; by means of this general law, the Method of Increments will help us to this term, either expreſſed in finite quantities, or by an infinite ſeries.

Another uſe of the Method of Increments, is to find the ſums of ſeries; which it will often do in finite terms. And when the ſum of a ſeries cannot be had in finite terms, we muſt have recourſe to infinite ſeries; for the integral being expreſſed by ſuch a ſeries, the ſum of a competent number of its terms will give the ſum of the ſeries required. This is equivalent to transforming one ſeries into another, converging quicker: and ſometimes a very few terms of this ſeries will give the ſum of the ſeries ſought.

Definitions in the Method of Increments.

1. When a quantity is conſidered as increaſing, or decreaſing, by certain ſteps or degrees, it is called an Integral.

2. The increaſe of any quantity from its preſent value, to the next ſucceeding value, is called an Increment: or, if it decreaſes, a Decrement.

3. The increaſe of any Increment, is the Second Increment; and the increaſe of the 2d Increment, is the 3d Increment; and ſo on.

4. Succeeding Values, are the ſeveral values of the integral, ſucceeding one another in regular order, from the preſent value; and Preceding Values, are ſuch as ariſe before the preſent value. All theſe are called by the general term Factors.

5. A Perfect quantity is ſuch as contains any number of ſucceſſive values without intermiſſion; and a Defective quantity, is that which wants ſome of the ſucceſſive values. Thus $x_2\,x_3\,x_4\,x_5$ is a Perfect quantity; and $x_2\,x_4\,x_5$, an Imperfect or defective one.

Notation. This, according to Mr. Emerſon's method, is as follows:

1. Simple Integral quantities are denoted by any letters whatever, as z, y, x, u, &c.

2. The ſeveral values of a ſimple integral, are denoted by the ſame letter with ſmall figures under them: ſo if z be an integral, then z, z_1, z_2, z_3, &c are the preſent value, and the 1ſt, 2d, 3d, &c, ſucceſſive values of it; and the preceding values are denoted by figures with negative ſigns, thus z_{-1}, z_{-2}, z_{-3}, z_{-4}, are the 1ſt, 2d,

3d, 4th preceding values; and the figure denoting any value, is the characteriſtic.

3. The Increments are denoted with the ſame letters, and points under them: thus, $\underset{\cdot}{x}$ is the Increment of x, and $\underset{\cdot}{z}$ is the increment of z. Alſo $\underset{1}{\underset{\cdot}{x}}$ is the Increment of $\underset{1}{x}$; and $\underset{n}{\underset{\cdot}{x}}$ of $\underset{n}{x}$, &c.

4. The 2d, 3d, and other Increments, are denoted with two, three, or more points: ſo $\underset{\cdot\cdot}{z}$ is the 2d Increment of z, and $\underset{\vdots}{z}$ is the 3d Increment of z, and ſo on. And theſe are denominated Increments of ſuch an order, according to the number of points.

5. If $\underset{\cdot}{x}$ be any Increment, then $[\underset{\cdot}{x}]$ is the integral of it; alſo ${}^{2}[\underset{\cdot}{x}]$ denotes the integral of $[\underset{\cdot}{x}]$, or the 2d integral of $\underset{\cdot}{x}$; and ${}^{3}[\underset{\cdot}{x}]$ is the 3d integral of $\underset{\cdot}{x}$, or an integral of the 3d order, &c.

6. Quantities written thus, $\underset{1}{x}\ldots\underset{5}{x}$ mean the ſame as $\underset{1}{x}\underset{2}{x}\underset{3}{x}\underset{4}{x}\underset{5}{x}$, or ſignify that the quantities are continued from the firſt to the laſt, without break or interruption.

To find the Increment of any Integral, or variable quantity.

Rule 1. If the propoſed quantity be not fractional, and be a perfect integral, conſiſting of the ſucceſſive values of the variable quantity which increaſes uniformly: Multiply the propoſed integral by the number of factors, and change the loweſt factor for an Increment. So the Increment of $a-3x+6z$ is $-3\underset{\cdot}{x}+6\underset{\cdot}{z}$; for the Increment of the conſtant quantity a is 0 or nothing. So likewiſe,

The Increment of $c\,x\underset{1}{x}\underset{2}{x}\underset{3}{x}$, is $4c\,\underset{\cdot}{x}\underset{1}{x}\underset{2}{x}\underset{3}{x}$.

The Increment of $a\underset{-3}{x}\underset{-2}{x}\underset{-1}{x}$, is $3a\underset{\cdot}{x}\underset{-2}{x}\underset{-1}{x}$.

The Increment of $\underset{-m}{x}\ldots\underset{n}{x}$, is $\overline{m+n+1}\,.\,\underset{1}{x}\underset{-m+1}{x}\ldots\underset{n}{x}$.

Rule 2. In fractional quantities, where the denominator is perfect, and the variable quantity increaſes uniformly: Multiply the propoſed integral by the number of factors, and by the conſtant Increment with a negative ſign, and take the next ſucceeding value into the denominator. Thus,

The Increment of $\dfrac{a}{x\ldots\underset{4}{x}}$, is $\dfrac{-5a\underset{\cdot}{x}}{x\ldots\underset{5}{x}}$.

The Increment of $\dfrac{c}{\underset{-3}{x}\ldots\underset{5}{x}}$, is $\dfrac{-9c\underset{\cdot}{x}}{\underset{-3}{x}\ldots\underset{6}{x}}$.

Rule 3. The Increment of any power, as x^n is $\overline{x+\underset{\cdot}{x}}|^{n}-n^{n}$; that is, the difference between the preſent value x^n and the next ſucceeding value $\overline{x+\underset{\cdot}{x}}|^{n}$. And generally, the Increment of any quantity whatever, is found by ſubtracting the preſent value, or the given quantity, from its next ſucceeding value. Alſo by expanding the compound quantity in a ſeries, and ſubtracting x^n from it, the Increment will be either

$$= nx^{n-1}\underset{\cdot}{x}+\frac{n.n-1}{2}x^{n-2}\underset{\cdot}{x}^{2}+\frac{n.n-1.n-2}{2\cdot3}x^{n-3}\underset{\cdot}{x}^{3}\ \&c,\ \text{or}$$

$$= x^n\times:\frac{n\underset{\cdot}{x}}{\underset{1}{x}}+\frac{n.n-1}{2}\cdot\frac{\underset{\cdot}{x}^{2}}{\underset{1}{x}^{2}}+\frac{n.n-1.n-2}{2\cdot3}\cdot\frac{\underset{\cdot}{x}^{3}}{\underset{1}{x}^{3}}\ \&c.$$

So the Increment of x^4, is $\overline{x+\underset{\cdot}{x}}|^{4}-x^4= 4x^3\underset{\cdot}{x}+6x^2\underset{\cdot}{x}^2+4x\underset{\cdot}{x}^3+\underset{\cdot}{x}^4$.

The Increment of $\frac{1}{x^3}$ or x^{-3}

$$\text{is}\ \frac{1}{\overline{x+\underset{\cdot}{x}}|^{3}}-\frac{1}{x^3}=\overline{x+\underset{\cdot}{x}}|^{-3}-x^{-3}$$

$$=-3x^{-4}\underset{\cdot}{x}+6x^{-5}\underset{\cdot}{x}^2-10x^{-6}\underset{\cdot}{x}^3\ \&c$$

$$=-\frac{3\underset{\cdot}{x}}{x^4}+\frac{6\underset{\cdot}{x}^2}{x^5}-\frac{10\underset{\cdot}{x}^3}{x^6}\ \&c.$$

The Increment of a^x, a being conſtant, is $a^{\underset{1}{x}}-a^x=a^{x+\underset{\cdot}{x}}-a^x=a^x.(a^{\underset{\cdot}{x}}-1)$.

The Increment of $\frac{1}{a^x}$ is $\frac{1}{a^{\underset{1}{x}}}-\frac{1}{a^x}$

$$=\frac{a^x-a^{\underset{1}{x}}}{a^x a^{\underset{1}{x}}}=\frac{a^x-a^{x+\underset{\cdot}{x}}}{a^x a^{x+\underset{\cdot}{x}}}=\frac{1-a^{\underset{\cdot}{x}}}{a^{x+\underset{\cdot}{x}}}.$$

The Increment of xz is $\underset{1}{x}\underset{1}{z}-xz= \overline{x+\underset{\cdot}{x}}\,.\,\overline{z+\underset{\cdot}{z}}-xz=z\underset{\cdot}{x}+x\underset{\cdot}{z}+\underset{\cdot}{x}\underset{\cdot}{z}$.

And ſo on for any form of Integral whatever, ſubtracting the given quantity from its next ſucceeding value. So,

The Increment of the log. of x is log. $\underset{1}{x}$ − log. x = log. $x+\underset{\cdot}{x}$ − log. x = log. $\dfrac{x+\underset{\cdot}{x}}{x}$, which, by the nature of logarithms, is

$$\frac{\underset{\cdot}{x}}{x}-\frac{\underset{\cdot}{x}^2}{2x^2}+\frac{\underset{\cdot}{x}^3}{3x^3}-\frac{\underset{\cdot}{x}^4}{4x^4}\ \&c.$$

Schol. From hence may be deduced the principles and rules of fluxions; for the method of fluxions is only a particular caſe of the method of Increments, fluxions being infinitely ſmall Increments; therefore if in any form of Increments the Increment be taken infinitely ſmall, the form or expreſſion will be changed into a fluxional one.

Thus, in $z\underset{\cdot}{x}+x\underset{\cdot}{z}+\underset{\cdot}{x}\underset{\cdot}{z}$, which is the Increment of the rectangle xz, if $\underset{\cdot}{x}$ and $\underset{\cdot}{z}$ be changed for $\dot{x}$ and $\dot{z}$, the expreſſion will become $z\dot{x}+x\dot{z}+\dot{x}\dot{z}$ for the fluxion of xz, or only $z\dot{x}+x\dot{z}$, becauſe $\dot{x}\dot{z}$ is infinitely leſs than the reſt.

So likewiſe, if $\underset{\cdot}{x}$ be changed for $\dot{x}$ in this

$$nx^{n-1}\underset{\cdot}{x}+\frac{n.n-1}{2}x^{n-2}\underset{\cdot}{x}^2+\frac{n.n-1.n.-2}{2\cdot3}x^{n-3}\underset{\cdot}{x}^3$$

&c, which is the Increment of x^n, it becomes

$$nx^{n-1}\dot{x}+\frac{n.n-1}{2}x^{n-2}\dot{x}^2+\frac{n.n-1.n-2}{2\cdot3}x^{n-3}\dot{x}^3$$

&c, or only $nx^{n-1}\dot{x}$, for the fluxion of the power x^n, as all the terms after the firſt will be nothing, becauſe $\dot{x}^2$ and $\dot{x}^3$ &c are infinitely leſs than $\dot{x}$.

And thus may all the other forms of fluxions be derived from the correſponding Increments. And in like manner, the finding of the integrals, is only a more general way of finding fluents, as appears in what follows.

To find out the Integral of any given Increment.

Rule 1. When the variable quantity increaſes uniformly, and the propoſed integral conſiſts of the ſucceſſive values of it multiplied together, or is a perfect Increment not fractional: Multiply the given Increment by the next preceding value of the variable quantity, then divide by the new number of factors, and by the conſtant Increment.

Ex. Thus, the integral of $4c\underset{\cdot}{x}x_1x_2x_3$ is $cxx_1x_2x_2$.

The integral of $3a\underset{\cdot}{x}\,x_{-2}\,x_{-1}$ is $a\underset{\cdot}{x}_{-3}\,x_{-2}\,x_{-1}$.

Rule 2. In a fractional expreſſion, where the variable quantity increaſes uniformly, and the denominator is perfect, containing the ſucceſſive values of the variable quantity: Throw out the greateſt value of the variable letter, then divide by the new number of factors, and by the conſtant Increment with a negative ſign. So,

The integral of $\frac{a\underset{\cdot}{x}}{x\,.\,.\,.\,x_5}$ is $\frac{-a}{5\underset{\cdot}{x}\,.\,.\,x_4}$.

The integral of $\frac{-c\underset{\cdot}{x}}{x_{-3}\,.\,.\,.\,.\,x_6}$ is $\frac{c}{x_{-3}\,.\,.\,.\,.\,x_5}$.

Rule 3. Various other particular rules are given, but theſe and the two foregoing are all beſt included in the following general table of the moſt uſeful forms of Increments and integrals, to be uſed in the ſame way as the ſimilar table of fluxions and fluents, to which theſe correſpond.

A Table of Increments and their Integrals.

Forms	Increments	Integrals
1	$\underset{\cdot}{x}$ when conſtant not conſtant	x, or x_1, or x_2, or x_3 &c. x only.
2	$x_{-m}\,.\,.\,.\,.\,.\,.\,.\,.\,x_n\underset{\cdot}{x}$ $\underset{\cdot}{x}$ conſtant	$\frac{x_{-m-1}\,.\,.\,.\,.\,.\,.\,.\,.\,x_n}{m+n}$
3	$\frac{a\underset{\cdot}{x}}{x_{-m}\,.\,.\,.\,.\,.\,.\,.\,.\,x_n}$ $\underset{\cdot}{x}$ conſtant	$\frac{-a}{m+n-2\,.\,x_{-m}\,.\,.\,.\,.\,x_{n-1}}$
4	$z\underset{\cdot}{x}+x_1\underset{\cdot}{z}$	xz
5	$\frac{z\underset{\cdot}{x}-x\underset{\cdot}{z}}{zz_1}$	$\frac{x}{z}$
6	a^x $\underset{\cdot}{x}$ given	$\frac{a^x}{a^{\underset{\cdot}{x}}-1}$

Integrals, when found from given Increments, are corrected in the very ſame way as fluents when found from given fluxions, viz, inſtead of every ſeveral variable quantity in the integral, ſubſtituting ſuch a determinate value of them as they are known to have in ſome particular caſe; and then ſubtracting each ſide of the reſulting equation from the correſponding ſide of the integral, the remaining equation will be the correct form of the integrals.

For an example of the uſe of the Method of Increments, ſuppoſe it were required to find the ſum of any number of terms of the ſeries $1.2+2.3+3.4+4.5$ &c.

Let x be the number of the terms,
and z the ſum of them.

Then, by the progreſſion of the ſeries, the laſt or the x term is xx_1, and the next term after that will be x_1x_2, that is $\underset{\cdot}{z}=x_1x_2$, where $\underset{\cdot}{x}=1$. Hence the integral is $z=\frac{1}{3}xx_1x_2=\frac{1}{3}x\,.\,x+1\,.\,x+2$, which is the ſum of x terms of the given ſeries. So if the number of terms x be 10, this becomes $\frac{1}{3}.10.11.12=440$, which is the ſum of 10 terms of the given ſeries $1.2+2.3+3.4$ &c. Or, when $x=100$, the ſum of 100 terms of the ſame ſeries is $\frac{1}{3}.100.101.102=100.101.34=343400$.

Again, to find the ſum z of n terms of the ſeries $\frac{1}{1.3.5}+\frac{1}{3.5.7}+\frac{1}{5.7.9}$ &c.

Here the nth term is $\frac{1}{2n-1.2n+1.2n+3}$.

Put $x=2n-1$; then is $\underset{\cdot}{x}=2\underset{\cdot}{n}=2$, and the nth term is $\frac{1}{x.x+2.x+4}=\frac{1}{xx_1x_2}$;

and the $n+1$th term or $\underset{\cdot}{z}$ is $\frac{1}{xx_1x_2}$; the general integral of which is $z=\frac{-1}{2\underset{\cdot}{x}x_1x_2}=\frac{-1}{4x_1x_2}$.

But this wants a correction; for when $n=0$ or no terms, then $x=-1$, and the ſum $z=0$, and the integral becomes z or $0=\frac{-1}{4.1.3}=\frac{-1}{12}$; that is $-\frac{1}{12}$ is the correction, and being ſubtracted, the correct ſtate of the integrals becomes

$$z=\frac{1}{12}-\frac{1}{4x_1x_2}=\frac{1}{12}-\frac{1}{4.2n+1.2n+3},$$

which is the ſum of n terms of the propoſed ſeries. And when n is infinite, the latter fraction is nothing, and the ſum of the infinite ſeries, or the infinite number of the terms, is accurately $\frac{1}{12}$.

When $n=100$, the ſum of 100 terms of the ſeries becomes $z=\frac{1}{12}-\frac{1}{4.201.203}=\frac{1}{12}-\frac{1}{163212}$.

For more ample information and application on this ſcience, ſee Emerſon's Increments, Taylor's Methodus Incrementorum, and Stirling's Summatio & Interpolatio Serierum.

INCURVATION *of the rays of Light*. See LIGHT, and REFRACTION.

INDEFINITE, *Indeterminate*, that which has no certain bounds, or to which the human mind cannot affix any. Des Cartes uſes the word, in his Philoſophy, inſtead of infinite, both in numbers and quantities, to ſignify an inconceivable number, or number ſo great, that an unit cannot be added to it; and a quantity ſo great, as not to be capable of any addition. Thus, he ſays, the ſtars, viſible and inviſible, are in number Indefinite, and not, as the Ancients held, infinite; and that quantity may be divided into an Indefinite number of parts, not an infinite number.

Indefinite is now commonly uſed for indeterminate, number or quantity, that is, a number or quantity in general, in contradiſtinction from ſome particular known and given one.

INDETERMINED, or INDETERMINATE, in Geometry, is underſtood of a quantity, which has no certain or definite bounds.

INDETERMINATE *Problem*, is that which admits of innumerable different ſolutions, and ſometimes perhaps only of a great many different anſwers; otherwiſe called an unlimited problem.

In problems of this kind the number of unknown quantities concerned, is greater than the number of the conditions or equations by which they are to be found; from which it happens that generally ſome other conditions or quantities are aſſumed, to ſupply the defect, which being taken at pleaſure, give the ſame number of anſwers as varieties in thoſe aſſumptions.

As, if it were required to find two ſquare numbers whoſe difference ſhall be a given quantity *d*. Here, if x^2 and y^2 denote the two ſquares, then will $x^2 - y^2 = d$, by the queſtion, which is only one equation, for finding two quantities. Now by aſſuming a third quantity z ſo that $z = x + y$ the ſum of the two roots; then is $x = \frac{z^2 + d}{2z}$, and $y = \frac{z^2 - d}{2z}$, which are the two roots having the difference of their ſquares equal to the given quantity *d*, and are expreſſed by means of an aſſumed quantity z; ſo that there will be as many anſwers to the queſtion, as there can be taken values of the Indeterminate quantity z, that is, innumerable.

Diophantus was the firſt writer on Indeterminate problems, viz, in his Arithmetic or Algebra, which was firſt publiſhed in 1575 by Xilander, and afterwards in 1621 by Bachet, with a large commentary, and many additions to it. His book is wholly upon this ſubject; whence it has happened, that ſuch kind of queſtions have been called by the name of Diophantine problems. Fermat, Des Cartes, Frenicle, in France, and Wallis and others in England, particularly cultivated this branch of Algebra, on which they held a correſpondence, propoſing difficult queſtions to each other; an inſtance of which are thoſe two curious ones, propoſed by M. Fermat, as a challenge to all the mathematicians of Europe, viz 1ſt, To find a cube number which added to all its aliquot parts ſhall make a ſquare number; and 2d, To find a ſquare number which added to all its aliquot parts ſhall make a cubic number; which problems were anſwered after ſeveral ways by Dr. Wallis, as well as ſome others of a different nature. See the Letters that paſſed between Dr. Wallis, the lord Brounker, Sir Kenelm Digby, &c, in the Doctor's Works; and the Works of Fermat, which were collected and publiſhed by his ſon. Moſt authors on Algebra have alſo treated more or leſs on this part of it, but more eſpecially Kerſey, Preſtet, Ozanam, Kirkby, &c. But afterwards, mathematicians ſeemed to have forgot ſuch queſtions, if they did not even deſpiſe them as uſeleſs, when Euler drew their attention by ſome excellent compoſitions, demonſtrating ſome general theorems, which had only been known by induction. M. la Grange has alſo taken up the ſubject, having reſolved very difficult problems in a general way, and diſcovered more direct methods than heretofore. The 2d volume of the French tranſlation of Euler's Algebra contains an elementary treatiſe on this branch, and, with la Grange's additions, an excellent theory of it; treating very generally of Indeterminate problems, of the firſt and ſecond degree, of ſolutions in whole numbers, of the method of Indeterminate coefficients, &c.

Finally, Mr. John Leſlie has given, in the 2d volume of the Edinburgh Philoſ. Tranſactions, an ingenious paper on the reſolution of Indeterminate problems, reſolving them by a new and general principle. "The doctrine of Indeterminate equations," ſays Mr. Leſlie, "has been ſeldom treated in a form equally ſyſtematic with the other parts of Algebra. The ſolutions commonly given are devoid of uniformity, and often require a variety of aſſumptions. The object of this paper is to reſolve the complicated expreſſions which we obtain in the ſolution of Indeterminate problems, into ſimple equations, and to do ſo, without framing a number of aſſumptions, by help of a ſingle principle, which though extremely ſimple, admits of a very extenſive application."

"Let $A \times B$ be any compound quantity equal to another, $C \times D$, and let m be any rational number aſſumed at pleaſure; it is manifeſt that, taking equimultiples, $A \times mB = C \times mD$. If, therefore, we ſuppoſe that $A = mD$, it muſt follow that $mB = C$, or $B = \frac{C}{m}$. Thus two equations of a lower dimenſion are obtained. If theſe be capable of farther decompoſition, we may aſſume the multiples n and p, and form four equations ſtill more ſimple. By the repeated application of this principle, an higher equation admitting of diviſors, will be reſolved into thoſe of the firſt order, the number of which will be one greater than that of the multiples aſſumed."

For example, reſuming the problem at firſt given, viz, to find two rational numbers, the difference of the ſquares of which ſhall be a given number. Let the given number be the product of a and b; then by hypotheſis, $x^2 - y^2 = ab$; but theſe compound quantities admit of an eaſy reſolution, for $\overline{x + y} \times \overline{x - y} = a \times b$. If therefore we ſuppoſe $x + y = ma$, we ſhall obtain $x - y = \frac{b}{m}$; where m is arbitrary, and if rational, x and y muſt alſo be rational. Hence the reſolution of theſe two equations gives the values of x and y, the numbers ſought, in terms of m; viz, $x = \frac{m^2a + b}{2m}$, and $y = \frac{m^2a - b}{2m}$.

INDEX, in Arithmetic, is the ſame with what is otherwiſe called the characteriſtic or the exponent of a logarithm; being that which ſhews of how many places the abſolute or natural number belonging to the logarithm conſiſts, and of what nature it is, whether an integer or a fraction; the Index being leſs by 1 than the number of integer figures in the natural number, and is poſitive for integer or whole numbers, but negative in fractions, or in the denominator of a fraction; and in decimals, the negative index is 1 more than the number of ciphers in the decimal, after the point, and before the firſt ſignificant figure; or, ſtill more generally, the Index ſhews how far the firſt figure of the natural number is diſtant from the place of units, either towards the left hand, as in whole numbers, or towards the right, as in decimals; theſe oppoſite caſes being marked by the correſpondent ſigns + and −, of oppoſite affections, the ſign − being ſet over the Index, and not before it, becauſe it is this Index only which is underſtood as negative, and not the decimal part of the logarithm. Thus, in this logarithm 2·4234097, the figures of whoſe natural number are 2651, the 2 is the Index, and being poſitive, it ſhews that the firſt figure of the number muſt be two places removed from the units place, or that there will be three places of integers, the number of theſe places being always 1 more than the Index; ſo that the natural number will be 265·1. But if the ſame Index be negative, thus $\bar{2}$·4234097, it ſhews that the natural number is a decimal, and that the firſt ſignificant figure of it is in the 2d place from units, or that there is one cipher at the beginning of the decimal, being 1 leſs than the negative Index; and conſequently that the natural number of the logarithm in this caſe is ·02651. Hence, by varying the natural number, with reſpect to the decimal places in it as in the former of the two columns here annexed, the Index of their logarithm will vary as in the 2d column.

Number.	Logarithm.
2651	3·4234097
265·1	2·4234097
26·51	1·4234097
2·651	0·4234097
·2651	$\bar{1}$·4234097
·02651	$\bar{2}$·4234097
·002651	$\bar{3}$·4234097

Mr. Townly introduced a peculiar way of noting theſe Indices, when they become negative, or expreſs decimal figures, which is now much in uſe, eſpecially in the log. ſines and tangents, &c, viz, by taking, inſtead of the true Index, its arithmetical complement to 10; ſo that, in this way, the logarithm $\bar{2}$·4234097 is written 8·4234097.

For the addition and ſubtraction of Indices, ſee Logarithm.

Index of a Globe, is a little ſtyle fitted on to the north-pole, and turning round with it, pointing out the diviſions of the hour-circle.

Index of a Quantity, in Arithmetic and Algebra, otherwiſe called the exponent, is the number that ſhews to what power it is underſtood to be raiſed: as in 10^3, or a^3, the figure 3 is the Index or exponent of the power, ſignifying that the root or quantity, 10 or a, is raiſed to the 3d power. See this fully treated under Exponent.

INDICTION, or *Roman* Indiction, a kind of epoch, or manner of counting time, among the Romans; containing a cycle or revolution of 15 years.

The popes have dated their acts by the year of the Indiction, which was fixed to the 1ſt of January anno Domini 313, ever ſince Charlemagne made them ſovereign; before that time, they dated them by the years of the Emperors.

At the time of reforming the calendar, the year 1582 was reckoned the 10th year of the Indiction; ſo that beginning to reckon from hence, and dividing the number of years elapſed between that time and this, by 15, the remainder, with the addition of 10, rejecting 15 if the ſum be more, will be the year of the Indiction.

But the Indiction will be eaſier found thus: Add 3 to the given year of Chriſt; divide the ſum by 15, and the remainder after the diviſion, will be the year of the Indiction: if there be no remainder, the Indiction is 15. In either of theſe ways, the Indiction for the year 1795 is 13.

INDIVISIBLES, are thoſe indefinitely ſmall elements, or principles, into which any body or figure may ultimately be divided.

A line is ſaid to conſiſt of points, a ſurface of parallel lines, and a ſolid of parallel ſurfaces: and becauſe each of theſe elements is ſuppoſed Indiviſible, if in any figure a line be drawn perpendicularly through all the elements, the number of points in that line, will be the ſame as the number of the elements.

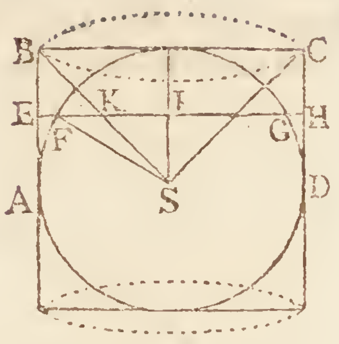
Whence it appears, that a parallelogram, or a priſm, or a cylinder, is reſolvable into elements, as Indiviſibles, all equal to each other, parallel, and like or ſimilar to the baſe; for which reaſon, one of theſe elements multiplied by the number of them, that is the baſe of the figure multiplied by its height, gives the area or content. And a triangle is reſolvable into lines parallel to the baſe, but decreaſing in arithmetical progreſſion; ſo alſo do the circles, which conſtitute the parabolic conoid, as well as thoſe which conſtitute the plane of a circle, or the ſurface of a cone. In all which caſes, as the laſt or leaſt term of the arithmetic progreſſion is 0, and the length of the figure the ſame thing as the number of the terms, therefore the greateſt term, or baſe, being multiplied by the length of the figure, half the product is the ſum of the whole, or the content of the figure.

And in any other figure or ſolid, if the law of the decreaſe of the elements be known, and thence the relation of the ſum to the greateſt term, which is the baſe, the whole number of them being the altitude of the figure, then the ſaid ſum of the elements is always the content of the figure.

A cylinder may alſo be reſolved into cylindrical curve ſurfaces, having all the ſame height, and continually decreaſing inwards, as the circles of the baſe do, on which they inſiſt.

This way of conſidering magnitudes, is called the Method of Indiviſibles, which is only the ancient method of exhauſtions, a little diſguiſed and contracted. And it is found of good uſe, both in computing the contents of figures in a very ſhort and eaſy way, as above inſtanced, and in ſhortening other demonſtrations in mathematics; an inſtance of which may here be given in that celebrated propoſition of Archimedes, that a ſphere is two-thirds of its circumſcribed cylinder. Thus,

Suppoſe a cylinder, a hemiſphere, and an inverted cone, having all the ſame baſe and altitude, and cut by an infinite number of planes all parallel to the baſe, of which EFGH is one; it is evident that the ſquare of EI, the radius of the cylinder, is every where equal



to the ſquare of SF, the radius of the ſphere; and alſo that the ſquare of EI, or of SF, is equal to the ſum of the ſquares of IF and IS, or of IF and IK, becauſe IK = IS; that is, $IE^2 = IF^2 + IK^2$, in every poſition; but IE is the radius of the cylinder, IF the correſponding radius of the ſphere, and IK that of the cone; and the circular ſections of theſe bodies, are as the ſquares of their radii; therefore the ſection of the cylinder is every where equal to the ſum of the ſections of the hemiſphere and cone; and, as the number of all thoſe ſections, which is the common height of the figures, is the ſame, therefore all the ſections, or elements, of the cylinder, will be equal to the ſum of all thoſe of the hemiſphere and cone taken together; that is, the cylinder is equal to both the hemiſphere and cone: but as the cone itſelf is equal to one-third part of the cylinder; therefore the hemiſphere is equal to the other two-thirds of it.

The Method of Indiviſibles was introduced by Cavalerius, in 1635, in his Geometria Indiviſibilium. The ſame was alſo purſued by Torricelli in his works, printed 1644: and again by Cavalerius himſelf in another treatiſe, publiſhed in 1647.

INERTIÆ *Vis*. See Vis *Inertiæ*.

INFINITE, is applied to quantities which are either greater or leſs than any aſſignable ones. In which ſenſe it differs but little from the terms Indefinite and Indeterminate. Thus, an

Infinite, or *Infinitely great* line, denotes only an indefinite or indeterminate line; or a line to which no certain bounds or limits are preſcribed.

Infinite *Quantities*. Though the idea of magnitude infinitely great, or ſuch as exceeds any aſſignable quantity, does include a negation of limits, yet all ſuch magnitudes are not equal among themſelves; but beſides Infinite length, and Infinite area, there are no leſs than three ſeveral ſorts of Infinite ſolidity; all of which are quantities *ſui generis*; and thoſe of each ſpecies are in given proportions.

Infinite length, or a line infinitely long, may be conſidered, either as beginning at a point, and ſo infinitely extended one way; or elſe both ways from the ſame point.

As to Infinite ſurface or area, any right line infinitely extended both ways on a plane infinitely extended every way, divides that plane into two equal parts, one on each ſide of the line. But if from any point in ſuch a plane, two right lines be infinitely extended, making an angle between them; the Infinite area, intercepted between theſe Infinite right lines, is to the whole Infinite plane, as that angle is to 4 right angles. And if two Infinite and parallel lines be drawn at a given diſtance on ſuch an Infinite plane, the area intercepted between them will be likewiſe Infinite; but yet it will be infinitely leſs than the whole plane; and even infinitely leſs than the angular or ſectoral ſpace, intercepted between two Infinite lines, that are inclined, though at never ſo ſmall an angle; becauſe in the one caſe, the given finite diſtance of the parallel lines diminiſhes the Infinity in one of the dimenſions; whereas in a ſector, there is Infinity in both dimenſions. And thus there are two ſpecies of Infinity in ſurfaces, the one infinitely greater than the other.

In like manner there are ſpecies of Infinites in ſolids, according as only one, or two, or as all their three dimenſions, are Infinite; which, though they be all infinitely greater than a finite ſolid, yet are they in ſucceſſion infinitely greater than each other.

Some farther properties of Infinite quantities are as follow:

The ratio between a finite and an Infinite quantity, is an Infinite ratio.

If a finite quantity be multiplied by an infinitely ſmall one, the product will be an infinitely ſmall one; but if the former be divided by the latter, the quotient will be infinitely great.

On the contrary, a finite quantity being multiplied by an infinitely great one, the product is infinitely great; but the former divided by the latter, the quotient will be infinitely little.

The product or quotient of an infinitely great or an infinitely little quantity, by a finite one, is reſpectively infinitely great, or infinitely little.

An infinitely great multiplied by an infinitely little, is a finite quantity; but the former divided by the latter, the quotient is infinitely Infinite.

The mean proportional between infinitely great, and infinitely little, is finite.

Arithmetic of Infinites. See Arithmetic. Alſo Wallis's treatiſe of this ſubject; and another by Emerſon, at the beginning of his Conic Sections; alſo Bulliald's treatiſe *Arithmetica Infinitorum*.

Infinite *Decimals*, ſuch as do not terminate, but go on without end; as $\cdot 3\dot{3}3$ &c $= \frac{1}{3}$, or $\cdot\dot{1}42857$ &c $= \frac{1}{7}$. See Repetend.

Infinitely *Infinite Fractions*, or all the powers of the fractions whoſe numerator is 1; which are all together equal to unity, as is demonſtrated by Dr. Wood, in Hook's Philoſ. Coll. N° 3, p. 45; where ſome curious properties are deduced from the ſame.

Infinite *Series*, a ſeries conſidered as infinitely continued as to the number of its terms. See Series.

INFINITESIMALS, are certain infinitely or indefinitely ſmall parts; as alſo the method of computing by them.

In the method of Infiniteſimals, the element by which any quantity increaſes or decreaſes, is ſuppoſed to be infinitely ſmall, and is generally expreſſed by two or more terms, ſome of which are infinitely leſs than the reſt, which being neglected as of no importance, the remaining terms form what is called the *difference* of the propoſed quantity. The terms that are neglected in this manner, as infinitely leſs than the other terms of the element, are the very ſame which ariſe in conſequence of the acceleration, or retardation, of the generating motion, during the infinitely ſmall time in which the element is generated; ſo that the remaining terms expreſs the element that would have been produced in that time, if the generating motion had continued uniform. Therefore, thoſe *differences* are accurately in the ſame ratio to each other, as the generating motions or fluxions. And hence, though in this method, Infiniteſimal parts of the elements are neglected, the concluſions are accurately true, without even an infinitely ſmall error, and agree preciſely with thoſe that are deduced by the method of fluxions.

But however ſafe and convenient this method may be, ſome will always ſcruple to admit infinitely little quantities, and infinite orders of Infiniteſimals, into a ſcience that boaſts of the moſt evident and accurate principles, as well as of the moſt rigid demonſtrations. In order to avoid ſuch ſuppoſitions, Newton conſiders the ſimultaneous increments of the flowing quantities as finite, and then inveſtigates the ratio which is the limit of the various proportions which thoſe increments bear to each other, while he ſuppoſes them to decreaſe together till they vaniſh; which ratio is the ſame with the ratio of the fluxions. See Maclaurin's Treatiſe of Fluxions, in the Introduc. p. 39 &c, alſo art. 495 to 502.

INFLAMMABILITY, that property of bodies by which they kindle, or catch fire.

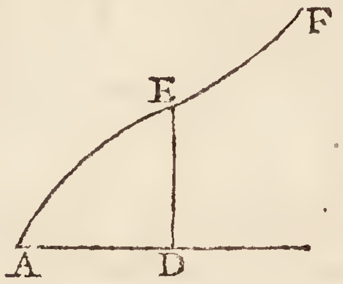
INFLECTION, in Optics, called alſo Diffraction, and Deflection of the rays of light, is a property of them, by reaſon of which, when they come within a certain diſtance of any body, they will either be bent from it, or towards it; being a kind of imperfect reflection or refraction.

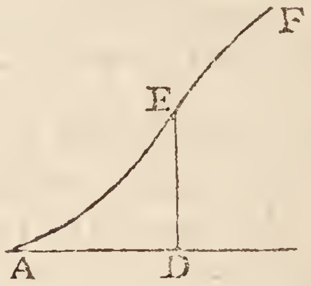
Some writers aſcribe the firſt diſcovery of this property to Grimaldi, who firſt publiſhed an account of it, in his Treatiſe De Lumine, Coloribus, & Iride, printed in 1666. But Dr. Hook alſo claims the diſcovery of it, and communicated his obſervations on this ſubject to the Royal Society, in 1672. He ſhews that this property differs both from reflection and refraction; and that it ſeems to depend on the unequal denſity of the conſtituent parts of the ray, by which the light is diſperſed from the place of condenſation, and rarefied or gradually diverged into a quadrant; and this deflection, he ſays, is made towards the ſuperficies of the opaque body perpendicularly.

Newton diſcovered, by experiments, this Inflection of the rays of light; which may be ſeen in his Optics.

M. De la Hire obſerved, that when we look at a candle, or any luminous body, with our eyes nearly ſhut, rays of light are extended from it, in ſeveral directions, to a conſiderable diſtance, like the tails of comets. The true cauſe of this phenomenon, which has exerciſed the ſagacity of Des Cartes, Rohault, and others, ſeems to be, that the light paſſing among the eyelaſhes, in this ſituation of the eye, is inflected by its near approach to them, and therefore enters the eye in a great variety of directions. He alſo obſerves, that he found that the beams of the ſtars being obſerved, in a deep valley, to paſs near the brow of a hill, are always more refracted than if there were no ſuch hill, or the obſervation was made on the top of it; as if the rays of light were bent down into a curve, by paſſing near the ſurface of the mountain.

Point of Inflection, or of *contrary flexure*, in a curve, is the point or place in the curve where it begins to bend or turn a contrary way; or which ſeparates the concave part from the convex part, and lying between the two; or where the curve changes from concave to convex, or from convex to concave, on the ſame ſide of the curve: ſuch as the point E in the annexed figures; where the former of the two is concave towards the axis AD from A to E, and convex from E to F; but, on the contrary, the latter figure is convex from A to E, and concave from E to F.





There are various ways of finding the point of Inflexion; but the following, which is new, ſeems to be the ſimpleſt and eaſieſt of all. From the nature of curvature it is evident that, while a curve is concave towards an axis, the fluxion of the ordinate decreaſes, or is in a decreaſing ratio, with regard to the fluxion of the abſciſs; but, on the contrary, that the ſaid fluxion increaſes, or is in an increaſing ratio to the fluxion of the abſciſs, where the curve is convex towards the axis; and hence it follows that thoſe two fluxions are in a conſtant ratio at the point of Inflection, where the curve is neither concave nor convex. That is, if $x = AD$ the abſciſs, and $y = DE$ the ordinate, then $\dot{x}$ is to $\dot{y}$ in a conſtant ratio, or $\frac{\dot{x}}{\dot{y}}$ or $\frac{\dot{y}}{\dot{x}}$ is a conſtant quantity. But conſtant quantities have no fluxion, or their fluxion is equal to nothing; ſo that in this caſe the fluxion of $\frac{\dot{x}}{\dot{y}}$ or of $\frac{\dot{y}}{\dot{x}}$ is equal to nothing. And hence we have this general rule: viz,

Put the given equation of the curve into fluxions; from which equation of the fluxions find either $\frac{\dot{x}}{\dot{y}}$ or $\frac{\dot{y}}{\dot{x}}$; then take the fluxion of this ratio or fraction, and put it equal to 0 or nothing; and from this laſt equation find alſo the value of the ſame $\frac{\dot{x}}{\dot{y}}$ or $\frac{\dot{y}}{\dot{x}}$: then put this latter value equal to the former, which will be an equation from whence, and the firſt given equation of the curve, x and y will be determined, being the abſciſs or ordinate anſwering to the point of Inflection in the curve.

Or, putting the fluxion of $\frac{\dot{x}}{\dot{y}}$ equal to 0, that is, $\frac{\ddot{x}\dot{y} - \dot{x}\ddot{y}}{\dot{y}^2} = 0$, or $\ddot{x}\dot{y} - \dot{x}\ddot{y} = 0$, or $\ddot{x}\dot{y} = \dot{x}\ddot{y}$, or $\dot{x} : \dot{y} :: \ddot{x} : \ddot{y}$, that is, the 2d fluxions have the ſame ratio as the 1ſt fluxions, which is a conſtant ratio; and therefore if $\dot{x}$ be conſtant, or $\ddot{x} = 0$, then ſhall $\ddot{y}$ be $= 0$ alſo; which gives another rule, viz; Take both the 1ſt and 2d fluxions of the given equation of the curve, in which make both $\ddot{x}$ and $\ddot{y} = 0$, and the reſulting equations will determine the values of x and y, or abſciſs and ordinate anſwering to the point of Inflection.

For example, if it be required to find the point of Inflection in the curve whoſe equation is

$ax^2 = a^2y + x^2y$. Now the fluxion of this is $2ax\dot{x} = a^2\dot{y} + 2xy\dot{x} + x^2\dot{y}$, which gives $\frac{\dot{x}}{\dot{y}} = \frac{a^2 + x^2}{2ax - 2xy}$. Then the fluxion of this again made = 0, gives $2x\dot{x} \cdot \overline{ax - xy} = \overline{a^2 + x^2} \cdot \overline{a\dot{x} - y\dot{x} - x\dot{y}}$; and this gives again $\frac{\dot{x}}{\dot{y}} = \frac{a^2 + x^2}{a^2 - x^2} \times \frac{x}{a - y}$. Laſtly, this value of $\frac{\dot{x}}{\dot{y}}$ put = the former, gives $\frac{a^2 + x^2}{a^2 - x^2} \cdot \frac{x}{a - y} = \frac{a^2 + x^2}{2x} \cdot \frac{1}{a - y}$; and hence $2x^2 = a^2 - x^2$, or $3x^2 = a^2$, and $x = a\sqrt{\frac{1}{3}}$, the abſciſs.

Hence alſo, from the original equation,
$y = \frac{ax^2}{a^2 + x^2} = \frac{\frac{1}{3}a^3}{\frac{4}{3}a^2} = \frac{1}{4}a$, the ordinate to the point of Inflection ſought.

When the curve has but one point of Inflection, it will be determined by a ſimple equation, as above; but when there are ſeveral points of Inflection, by the curve bending ſeveral times from the one ſide to the other, the reſulting equation will be of a degree correſponding to them, and its roots will determine the abſciſſes or ordinates to the ſame.

Other methods of determining the points of Inflection in curves, may be ſeen in moſt books on the doctrine of fluxions.

To know whether a curve be concave or convex towards any point aſſigned in the axis; find the value of $\dot{y}$ at that point; then if this value be poſitive, the curve will be convex towards the axis, but if it be negative, it will be concave.

INFORMED *Stars*, or INFORMES *Stellæ*, are ſuch ſtars as have not been reduced into any conſtellation; otherwiſe called Sporades.—There was a great number of this kind left by the ancient aſtronomers; but Hevelius and ſome others of the moderns have provided for the greater part of them, by making new conſtellations.

INGINEER. See ENGINEER.

INGRESS, in Aſtronomy, the ſun's entrance into one of the ſigns, eſpecially Aries.

INNOCENTS *Day*, a feaſt celebrated on the 28th day of December, in commemoration of the infants murdered by Herod.

INORDINATE *Proportion*, is where the order of the terms compared, is diſturbed or irregular. As, for example, in two ranks of numbers, three in each rank, viz, in one rank, - - 2, 3, 9,
and in the other rank, - - 8, 24, 36,
which are proportional, the former to the latter, but in a different order, viz, - 2 : 3 :: 24 : 36,
and - 3 : 9 :: 8 : 24.
then, caſting out the mean terms in each rank, it is concluded that - - 2 : 9 :: 8 : 36,
that is, the firſt is to the 3d in the firſt rank,
as the firſt is to the 3d in the 2d rank.

INSCRIBED *Figure*, is one that has all its angular points touching the ſides of another figure in which the former is ſaid to be inſcribed.

INSCRIBED *Hyperbola*, is one that lies wholly within the angle of its aſymptotes; as the common or conical hyperbola doth.

INSTANT, otherwiſe called a Moment, an infinitely ſmall part of duration, or in which we perceive no ſucceſſion, or which takes up the time of only one idea in our mind.

It is a maxim in mechanics, that no natural effect can be produced in an Inſtant, or without ſome definite time; alſo that the greater the time, the greater the effect. And hence may appear the reaſon, why a burthen ſeems lighter to a perſon, the faſter he carries it; and why, the faſter a perſon ſlides or ſcates on the ice, the leſs liable it is to break, or bend.

INSULATE, or INSULATED, a term applied to a column or other ediſice, which ſtands alone, or free and detached from any adjacent wall, &c, like an iſland in the ſea.

INSULATED, in Electricity, is a term applied to bodies that are ſupported by electrics, or non-conductors; ſo that their communication with the earth, by conducting ſubſtances, is interrupted.

INTACTÆ, are right lines to which curves do continually approach, and yet can never meet them more uſually called Aſymptotes.

INTEGERS, denote whole numbers: as contradiſtinguiſhed from fractions.—Integers may be conſidered as numbers which refer to unity, as a whole to a part.

INTEGRAL *Number*, an integer; not a fraction.

INTEGRAL *Calculus*, in the New Analyſis, is the reverſe of the differential calculus, and is the finding the Integral from a given differential; being ſimilar to the inverſe method of fluxions, or the finding the fluent to a given fluxion.

INTEGRANT *Parts*, in *Philoſophy*, are the ſimilar parts of a body, or parts of the ſame nature with the whole; as filings of iron are the Integrant parts of iron, having the ſame nature and properties with the bar or maſs they were filed off from.

INTENSITY, or INTENSION, in Phyſics, is the degree or rate of the power or energy of any quality; as heat, cold, &c. The Intenſity of qualities, as gravity, light, heat, &c, vary in the reciprocal ratio of the ſquares of the diſtances from the centre of the radiating quality.

INTERCALARY *Day*, denotes the odd day inſerted in the leap-year. See BISSEXTILE.

INTERCEPTED *Axis*, in Conic Sections, the ſame with what is otherwiſe called the abſciſs or abſciſſa.

INTERCOLUMNATION, or INTERCOLUMNIATION, is the ſpace between column and column.

INTEREST, is a ſum reckoned for the loan or forbearance of another ſum, or principal, lent for, or due at, a certain time, according to ſome certain rate or proportion; being eſtimated uſually at ſo much per cent. or by the 100. This forms a particular rule in Arithmetic. The higheſt legal Intereſt now allowed in England, is after the rate of 5 per cent. per annum, or the 20th part of the principal for the ſpace of a year, and ſo in proportion for other times, either greater or leſs. Except in the caſe of pawn-brokers, to whom it has lately been made legal to take a higher intereſt, for one of the worſt and moſt deſtructive purpoſes that can be ſuffered in any ſtate.

Intereſt is either Simple or Compound.

Simple INTEREST, is that which is counted and allowed upon the principal only, for the whole time of forbearance.

The ſum of the Principal and Intereſt is called the Amount.

As the Intereſt of any ſum, for any time, is directly proportional to the principal ſum and time; therefore the Intereſt of 1 pound for one year being multiplied by any propoſed principal ſum, and by the time of its forbearance, in years and parts, will be its Intereſt for that time. That is, if

r = the rate of Intereſt of 1*l.* per annum,
p = any principal ſum lent,
t = the time it is lent for, and
a = the amount, or ſum of principal and Intereſt;

then is prt = the Intereſt of the ſum p, for the time t, at the rate r; and conſequently $p + prt = p \times \overline{1 + rt} = a$, the amount of the ſame for that time. And from this general theorem, other theorems can eaſily be deduced for finding any of the quantities above mentioned; which collected all together, will be as follow:

1ſt, $a = p + prt$ the amount,

2d, $p = \dfrac{a}{1 + rt}$ the principal,

3d, $r = \dfrac{a - p}{pt}$ the rate,

4th, $t = \dfrac{a - p}{pr}$ the time.

For example, let it be required to find in what time any principal ſum will double itſelf, at any rate of Simple Intereſt. In this caſe we muſt uſe the 1ſt theorem $a = p + prt$, in which the amount a muſt be $= 2p$ or double the principal, i. e. $p + prt = 2p$; and hence $t = \dfrac{1}{r}$; where r being the intereſt of 1*l.* for one year, it follows that the time of doubling at Simple Intereſt, is equal to the quotient of any ſum divided by its Intereſt for one year. So that, if the rate of Intereſt be 5 per cent. then $\dfrac{100}{5} = 20$ is the time of doubling.

Or the 4th theorem immediately gives

$$t = \frac{a - p}{pr} = \frac{2p - p}{pr} = \frac{2 - 1}{r} = \frac{1}{r}.$$

For more readily computing the Intereſt on money, various Tables of numbers are calculated and formed; ſuch as a Table of Intereſt of 1*l.* for any number of years, and for any number of months, or weeks, or days, &c, and at various rates of Intereſt.

Another Table is the following, by which may be readily found the Intereſt of any ſum of money, from 1 to a million of pounds, for any number of days, at any rate of Intereſt.

Numb.	*l.*	*s.*	*d.*	*q.*	No.	*l.*	*s.*	*d.*	*q.*
1000000	2739	14	6	0·99	100	0	5	5	3·01
900000	2465	15	0	3·29	90	0	4	11	0·71
800000	2191	15	7	1·59	80	0	4	4	2·41
700000	1917	16	1	3·89	70	0	3	10	0·11
600000	1643	16	8	2·19	60	0	3	3	1·81
500000	1369	17	3	0·49	50	0	2	8	3·51
400000	1095	17	9	2·79	40	0	2	2	1·21
300000	821	18	4	1·10	30	0	1	7	2·90
200000	547	18	10	3·40	20	0	1	1	0·60
100000	273	19	5	1·70	10	0	0	6	2·30
90000	246	11	6	0·33	9	0	0	5	3·67
80000	219	3	6	2·96	8	0	0	5	1·04
70000	191	15	7	1·59	7	0	0	4	2·41
60000	164	7	8	0·22	6	0	0	3	3·78
50000	136	19	8	2·85	5	0	0	3	1·15
40000	109	11	9	1·48	4	0	0	2	2·55
30000	82	3	10	0·11	3	0	0	1	3·89
20000	54	15	10	2·74	2	0	0	1	1·26
10000	27	7	11	1·37	1	0	0	0	2·63
9000	24	13	1	3·23	0·9	0	0	0	2·37
8000	21	18	4	1·10	0·8	0	0	0	2·10
7000	19	3	6	2·96	0·7	0	0	0	1·84
6000	16	8	9	0·82	0·6	0	0	0	1·58
5000	13	13	11	2·68	0·5	0	0	0	1·32
4000	10	19	2	0·55	0·4	0	0	0	1·05
3000	8	4	4	2·41	0·3	0	0	0	0·79
2000	5	9	7	0·27	0·2	0	0	0	0·53
1000	2	14	9	2·14	0·1	0	0	0	0·26
900	2	9	3	3·12	0·09	0	0	0	0·24
800	2	3	10	0·11	0·08	0	0	0	0·21
700	1	18	4	1·10	0·07	0	0	0	0·18
600	1	12	10	2·08	0·06	0	0	0	0·16
500	1	7	4	3·07	0·05	0	0	0	0·13
400	1	1	11	0·05	0·04	0	0	0	0·11
300	0	16	5	1·04	0·03	0	0	0	0·08
200	0	10	11	2·03	0·02	0	0	0	0·05
100	0	5	5	3·01	0·01	0	0	0	0 03

The Rule for uſing the Table is this:

Multiply the principal by the rate, both in pounds; multiply the product by the number of days, and divide this laſt product by 100; then take from the Table the ſeveral ſums which ſtand oppoſite the ſeveral parts of the quotient, and adding them together will give the intereſt required.

Ex. What is the intereſt of 225*l.* 10*s.* for 23 days, at $4\frac{1}{2}$ per cent. per annum?

princ. 225·5
rate 4·5
1014·74
75 days 23
100)23339·25
233·3925

Then in the Table

	l.	*s.*	*d.*	*q.*
againſt 200 is	0	10	11	2·03
30 —	0	1	7	2·90
3 —	0	0	1	3·89
0·3 —	0	0	0	0·79
0·09 —	0	0	0	0·24
Anſ.	0	12	9	1·85 true

in the laſt place of decimals.

Another ingenious and general method of computing Intereſt, is by the following ſmall but comprehenſive Table.

A General Intereſt Table,
By which the Intereſt of any Sum, at any Rate, and for any Time, may be readily found.

Days.	3 per Cent.				3½ per Cent.				4 per Cent.				4½ per Cent.				5 per Cent.			
	l.	s.	d.	q.	l.	s.	d.	q.	l.	s.	d.	q.	l.	s.	d.	q.	l.	s.	d.	q.
1			1	3			2	1			2	2			3	0			3	1
2			3	3			4	2			5	1			6	0			6	2
3			5	3			6	3			7	3			8	3			9	3
4			7	3			9	0			10	2			11	3		1	1	0
5			9	3			11	2		1	1	1		1	2	3		1	4	1
6			11	3		1	1	3		1	3	3		1	5	3		1	7	2
7		1	1	3		1	4	0		1	6	1		1	8	3		1	11	0
8		1	3	3		1	6	1		1	9	0		1	11	3		2	2	1
9		1	5	3		1	8	2		1	11	2		2	2	2		2	5	2
10		1	7	2		1	11	0		2	2	1		2	5	2		2	8	3
20		3	3	1		3	10	0		4	4	2		4	11	1		5	5	3
30		4	11	0		5	9	0		6	6	3		7	4	2		8	2	2
40		6	6	3		7	8	0		8	9	0		9	10	1		10	11	2
50		8	2	2		9	7	0		10	11	2		12	3	3		13	8	1
60		9	10	1		11	6	0		13	1	3		14	9	2		16	5	1
70		11	6	0		13	5	0		15	4	0		17	3	1		19	2	0
80		13	1	3		15	4	0		17	6	1		19	8	3	1	1	11	0
90		14	9	2		17	3	0		19	8	2	1	2	2	1	1	4	7	3
100		16	5	1		19	2	0	1	1	11	0	1	4	8	0	1	7	4	3
200	1	12	10	2	1	18	4	1	2	3	10	0	2	9	3	3	2	14	9	2
300	2	9	3	3	2	17	6	1	3	5	9	0	3	13	11	3	4	2	2	1

N. B. This Table contains the intereſt of 100l. for all the ſeveral days in the 1ſt column, and at the ſeveral rates of 3, 3½, 4, 4½, and 5 per cent. in the other 5 columns.

To find the Intereſt of 100*l. for any other time*, as 1 year and 278 days, at 4½ per cent. Take the ſums for the ſeveral days as here below.

The Int. for 1 year	4	10	0	0
Againſt 200 ds. is	2	9	3	3
——— 70 ds. -	0	17	3	1
——— 8 ds. -	0	1	11	0
Intereſt required -	7	18	6	0

For any other Sum than 100*l.* Firſt find for 100l. as above, and take it ſo many times or parts as the ſum is of 100l. Thus, to find for 355l. at 4½, for 1 year and 278 days.

First, 3 times the above ſum,

(for 300l.) is -	23	15	8	1
½ (for 50l.) is -	3	19	3	1
$\frac{1}{10}$ of this (for 5l.)	0	7	11	0
So for 355 it is -	28	2	10	2

When the intereſt is required for any other rate than thoſe in the Table, it may be eaſily made out from them. So ½ of 5 is 2½, ½ of 4 is 2, ½ of 3 is 1½, ⅓ of 3 is 1, 1-6th of 3 is ½, and 1-12th of 3 is ¼. And ſo, by parts, or by adding or ſubtracting, any rate may be made out.

Compound INTEREST, called alſo *Intereſt-upon-Intereſt*, is that which is counted not only upon the principal ſum lent, but alſo for its Intereſt, as it becomes due, at the end of each ſtated time of payment.

Although it be not lawful to lend money at Compound Intereſt, yet in purchaſing annuities, penſions, &c, and taking leaſes in reverſion, it is uſual to allow Compound Intereſt to the purchaſer for his ready money; and therefore it is very neceſſary to underſtand this ſubject.

Beſides the quantities concerned in Simple Intereſt, viz, the principal p, the rate or Intereſt of 1l. for 1 year r, the amount a, and the time t, there is another quantity employed in Compound Intereſt, viz, the ratio of the rate of Intereſt, which is the amount of 1l. for 1 time of payment, and which here let be denoted by R, viz, $R = 1 + r$. Then, the particular amounts for the ſeveral times may be thus computed, viz, As 1 pound is to its amount for any time, ſo is any propoſed principal ſum to its amount for the ſame time; i. e.

1l. : R : : p : pR the 1ſt year's amount,
1l. : R : : pR : pR^2 the 2d year's amount,
1l. : R : : pR^2 : pR^3 the 3d year's amount,
and ſo on.

Therefore in general, $pR^t = a$ is the amount for the t year, or t time of payment. From whence the following general theorems are deduced:

1ſt, $a = pR^t$ the amount,

2d, $p = \frac{a}{R^t}$ the principal,

3d, $R = \sqrt[t]{\frac{a}{p}}$ the ratio,

4th, $t = \frac{\text{log. of } a - \text{log. of } p}{\text{log. of } R}$ the time.

From which any one of the quantities may be found, when the reſt are given.

For example, ſuppoſe it were required to find in how many years any principal ſum will double itſelf, at any rate of Intereſt. In this caſe we muſt employ the 4th theorem, where a will be $= 2p$, and then it

is $t = \frac{\text{l. } a - \text{l. } p}{\text{log. R}} = \frac{\text{l. } 2p - \text{l. } p}{\text{log. R}} = \frac{\text{log. } 2}{\text{log. R}}$.

So, if the rate of Intereſt be 5 per cent. per annum; then $R = 1 + {\cdot}05 = 1{\cdot}05$, and hence

$$t = \frac{\text{log. } 2}{\text{log. } 1{\cdot}05} = \frac{{\cdot}3010300}{{\cdot}0211893} = 14{\cdot}2067 \text{ nearly};$$

that is, any ſum doubles in 14⅕ years nearly, at the rate of 5 per cent. per annum Compound Intereſt.

Hence, and from the like queſtion in Simple Intereſt, above given, are deduced the times in which any ſum doubles itſelf, at ſeveral rates of Intereſt, both ſimple and compound: viz,

At		At Simp. Int. Years.	At Comp. Int. Years.
2	per cent. per an. Intereſt, 1*l.* or any other ſum will double in	50	35·0028
2½		40	28·0701
3		33⅓	23·4498
3½		28 4/7	20·1488
4		25	17·6730
4½		22 2/9	15·7473
5		20	14·2067
6		16⅔	11·8957
7		14 2/7	10·2448
8		12½	9·0065
9		11 1/9	8·0432
10		10	7·2725;

The following Table will facilitate the calculation of Compound Intereſt for any ſum, and any number of years, at various rates of Intereſt.

The Amount of 1*l. in any Number of Years.*

Yrs.	3	3½	4	4½	5	6
1	1·0300	1·0350	1·0400	1·0450	1·0500	1·0600
2	1·0609	1·0712	1·0816	1·0920	1·1025	1·1236
3	1·0927	1·1087	1·1249	1·1412	1·1576	1·1910
4	1·1255	1·1475	1·1699	1·1925	1·2155	1·2625
5	1·1593	1·1877	1·2167	1·2462	1·2763	1·3382
6	1·1941	1·2293	1·2653	1·3023	1·3401	1·4185
7	1·2299	1·2723	1·3159	1·3609	1·4071	1·5036
8	1·2668	1·3168	1·3686	1·4221	1·4775	1·5939
9	1·3048	1·3629	1·4233	1·4861	1·5513	1·6895
10	1·3439	1·4106	1·4802	1·5530	1·6289	1·7909
11	1·3842	1·4600	1·5395	1·6229	1·7103	1·8983
12	1·4258	1·5111	1·6010	1·6959	1·7959	2·0122
13	1·4685	1·5640	1·6651	1·7722	1·8856	2·1329
14	1·5126	1·6187	1·7317	1·8519	1·9799	2·2609
15	1·5580	1·6753	1·8009	1·9353	2·0789	2·3966
16	1·6047	1·7340	1·8730	2·0224	2·1829	2·5404
17	1·6528	1·7947	1·9479	2·1134	2·2920	2·6928
18	1·7024	1·8575	2·0258	2·2085	2·4066	2·8543
19	1·7535	1·9225	2·1068	2·3079	2·5270	2·0256
20	1·8061	1·9898	2·1911	2·4117	2·6533	2·2071

The uſe of this Table, which contains all the powers R^t, to the 20th power, or the amounts of 1*l.* is chiefly to calculate the Intereſt, or the amount, of any principal ſum, for any time, not more than 20 years. For example, required to find to how much 523*l.* will amount in 15 years, at the rate of 5*l.* per cent. per annum Compound Intereſt.

In the Table, on the line 15 and column 5 per cent, is the amount of 1*l.* viz. - 2·0789,
this multiplied by the principal 523,

gives the amount 1087·2647
or - - - - - 1087*l.* 5*s.* 3¼*d.*
and therefore the Intereſt is 564*l.* 5*s.* 3¼*d.*

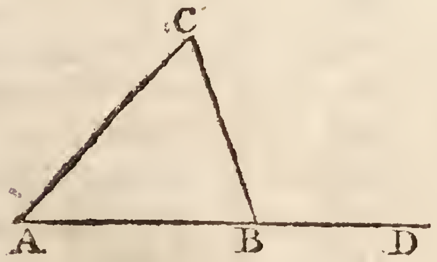
See Annuities; Diſcount; Reverſion; Smart's Tables of Intereſt; the Philoſ. Tranſ. vol. 6, p. 508; and moſt books on Arithmetic.

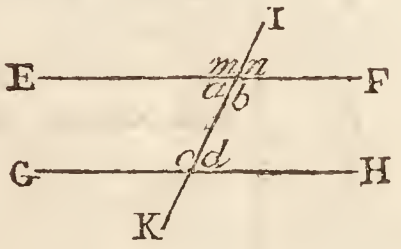
INTERIOR *Figure, Angle of.* See ANGLE.

INTERIOR *Polygon.* See POLYGON.

INTERIOR *Talus.* See TALUS.

INTERNAL *Angles*, are all angles made within any figure, by the ſides of it. In a triangle ABC, the two angles A and C are peculiarly called Internal and oppoſite, in reſpect of the external angle CBD, which is equal to them both together.





INTERNAL *Angle* is alſo applied to the two angles formed between two parallels, by a line interſecting thoſe parallels, on each ſide of the interſecting line. Such are the angles *a*, *b*, *c*, *d*, formed between the parallels EF and GH, on each ſide of the interſecting line.——The two adjacent Internal angles *a* and *b*, or *c* and *d*, are together equal to two right angles.

INTERNAL and Oppoſite Angles, is alſo applied to the two angles *a* and *b*, which are reſpectively equal to the two *n* and *m*, called the external and oppoſite angles.

Alſo the alternate Internal angles are equal to one another; viz, $a = d$, and $b = c$.

INTERPOLATION, in the modern Algebra, is uſed for finding an intermediate term of a ſeries, its place in the ſeries being given.

The Method of Interpolation was firſt invented by Mr. Briggs, and applied by him to the calculation of logarithms, &c, in his Arithmetica Logarithmica, and his Trigonometria Britannica; where he explains, and fully applies the method of Interpolation by differences. His principles were followed by Reginal and Mouton in France, and by Cotes and others in England. Wallis made uſe of the method of Interpolation in various parts of his works; as his Arithmetic of Infinites, and his Algebra, for quadratures, &c. The ſame was alſo happily applied by Newton in various ways: by it he inveſtigated his binomial theorem, and quadratures of the circle, ellipſe, and hyperbola: ſee Wallis's Algebra, chap. 85, &c. Newton alſo, in lemma 5, lib. 3 Princip. gave a moſt elegant ſolution of the problem for drawing a curve line through the extremities of any number of given ordinates; and in the ſubſequent propoſition, applied the ſolution of this problem to that of finding from certain obſerved places of a comet, its place at any given intermediate time. And Dr. Waring, who adds, that a ſolution ſtill more elegant, on ſome accounts, has been ſince diſcovered by Meſſ. Nichol and Stirling, has alſo reſolved the ſame problem, and rendered it more general, without having recourſe to finding the ſucceſſive differences. Philoſ. Tranſ. vol. 69, part 1, art. 7.

Mr. Stirling indeed purſued this branch as a diſtinct ſcience, in a ſeparate treatiſe, viz, Tractatus de Summatione et Interpolatione Serierum Infinitarum, in the year 1730.

When the 1ſt, 2d, or other ſucceſſive differences of the terms of a ſeries become at laſt equal, the Interpolation of any term of ſuch a ſeries may be found by Newton's Differential Method.

When the Algebraic equation of a ſeries is given, the term required, whether it be a primary or intermediate one, may be found by the reſolution of affected equations; but when that equation is not given, as it often happens, the value of the term ſought muſt be exhibited by a converging ſeries, or by the quadrature of curves. See Stirling, ut ſupra, p. 86. Meyer, in Act. Petr. tom. 2, p. 180.

A general theorem for Interpolating any term is as follows: Let A denote any term of an equidiſtant ſeries of terms, and *a*, *b*, *c*, &c, the firſt of the 1ſt, 2d, 3d, &c orders of differences; then the term *z*, whoſe diſtance from A is expreſſed by x, will be this, viz, *Theorem* 1,

$$z = A + xa + x \cdot \frac{x-1}{2} b + x \cdot \frac{x-1}{2} \cdot \frac{x-2}{3} c \, \&c.$$

Hence, if any of the orders of differences become equal to one another, or $= 0$, this ſeries for the interpolated term will break off, and terminate, otherwiſe it will run out in an infinite ſeries.

Ex. To find the 20th term of the ſeries of cubes 1, 8, 27, 64, 125, &c, or 1^3, 2^3, 3^3, 4^3, 5^3, &c.

Set down the ſeries in a column, and take their continual differences as here annexed, where the 4th differences, and all after it become $= 0$, alſo $A = 1$, $a = 7$, $b = 12$, $c = 6$, and $x = 19$; therefore the 20th term ſought is barely

$$\begin{array}{ccccc}
A & & & & \\
 & a & & & \\
1 & & b & & \\
 & 7 & & c & \\
8 & & 12 & & d \\
 & 19 & & 6 & \\
27 & & 18 & & 0 \\
 & 37 & & 6 & \\
64 & & 24 & & \\
 & 61 & & & \\
125 & & & &
\end{array}$$

$$z = 1 + 19 \times 7 + 19 \times \frac{18}{2} \times 12 + 19 \cdot \frac{18}{2} \cdot \frac{17}{3} \cdot 6$$
$$= 1 + 133 + 2052 + 5814 = 8000.$$

Theor. 2. In any ſeries of equidiſtant terms, a, b, c, d, &c, whoſe firſt differences are ſmall; to find any term wanting in that ſeries, having any number of terms given. Take the equation which ſtands againſt the number of given terms, in the following Table; and by reducing the equation, that term will be found.

No.	Equations.
1	$a - b = 0$
2	$a - 2b + c = 0$
3	$a - 3b + 3c - d = 0$
4	$a - 4b + 6c - 4d + e = 0$
5	$a - 5b + 10c - 10d + 5e - f = 0$
6	$a - 6b + 15c - 20d + 15e - 6f + g = 0$
&c.	&c.
n	$a - nb + n \cdot \frac{n-1}{2} c - n \cdot \frac{n-1}{2} \cdot \frac{n-2}{3} d \, \&c = 0.$

where it is evident that the coefficients in any equation, are the unciæ of a binomial $1 + 1$ raiſed to the power denoted by the number of the equation.

Ex. Given the logarithms of 101, 102, 104, and 105; to find the log. of 103.

Here are 4 quantities given; therefore we muſt take the 4th equation $a - 4b + 6c - 4d + e = 0$, in which it is the middle quantity or term c that is to be found, becauſe 103 is in the middle among the numbers 101, 102, 104, 105; then that equation gives the value of c as follows, viz $c = \frac{4 \cdot \overline{b+d} - \overline{a+e}}{6}$.

Now the logs. of the given numbers will be thus:

$$\begin{array}{rl}
 & 2\cdot0043214 = a \\
 & 2\cdot0086002 = b \\
 & 2\cdot0170333 = d \\
 & 2\cdot0211893 = e \\
\hline
 & 4\cdot0256335 = b + d \\
 & \qquad 4 \\
\hline
 & 16\cdot1025340 = 4 \cdot \overline{b+d} \\
\text{ſubtr.} & 4\cdot0255107 = \quad a + e \\
\hline
 & 6)\,12\cdot0770233 \\
 & 2\cdot0128372 \text{ the log. of } 103.
\end{array}$$

Theor. 3. When the terms a, b, c, d, &c, are at unequal diſtances from each other; to find any intermediate one of theſe terms, the reſt being given.

Let p, q, r, s, &c, be the ſeveral diſtances of thoſe terms from each other; then let

$$B = \frac{b-a}{p}, \; C = \frac{B1 - B}{p+q}, \; D = \frac{C1 - C}{p+q+r}, \; E = \frac{D1 - D}{p+q+r+s}.$$
$$B1 = \frac{c-b}{q}, \; C1 = \frac{B2 - B1}{q+r}, \; D1 = \frac{C2 - C1}{q+r+s},$$
$$B2 = \frac{d-c}{r}, \; C2 = \frac{B3 - B2}{r+s},$$
$$B3 = \frac{e-d}{s},$$
$$\&c \qquad \&c \qquad \&c$$

Then the term z, whoſe diſtance from the beginning is x, will be

$$z = a + Bx + \underline{Cx} \cdot \underline{\overline{x-p}} + \underline{D^x \cdot \overline{x-p}} \cdot \underline{\overline{x-p-q}}$$
$$+ Ex \cdot x-p \cdot x-p-q \cdot x-p-q-r + \&c;$$

to be continued to as many terms as there are terms in the given ſeries.

By this ſeries may be found the place of a comet, or the ſun, or any other object at a given time; by knowing the places of the ſame for ſeveral other given times.

Other methods of Interpolation may be found in the Philoſ. Tranſ. number 362; or Stirling's Summation and Interpolation of Series.

INTERSCENDENT, in Algebra, is applied to quantities, when the exponents of their powers are radical quantities. Thus $x^{\sqrt{2}}$, $x^{\sqrt{a}}$, &c, are interſcendent quantities. See FUNCTION.

INTERSECTION, the cutting of one line, or plane, by another; or the point or line in which two lines or two planes cut each other.——The mutual interſection of two planes is a right line. The centre of a circle, or conic ſection, &c, is in the interſection of two diameters; and the central point of a quadrangle is the Interſection of two diagonals.

INTERSTELLAR, a word uſed by ſome authors, to expreſs thoſe parts of the univerſe, that are without and beyond the limits of our ſolar ſyſtem.

In the Interſtellar regions, it is ſuppoſed there are ſeveral other ſyſtems of planets moving round the fixed ſtars, as the centres of their reſpective motions. And if it be true, as it is not improbable, that each fixed ſtar is thus a ſun to ſome habitable orbs, or earths, that move round it, the Interſtellar world will be infinitely the greateſt part of the univerſe.

INTERTIES, or INTERDUCES, in Architecture, thoſe ſmall pieces of timber which lie horizontally between the ſummers, or between them and the cell or raiſing plate.

INTERVAL, in Muſic, the difference between two ſounds, in reſpect of acute and grave. Authors diſtinguiſh ſeveral diviſions of an Interval, as firſt into Simple and Compound. The

Simple INTERVAL is that without parts, or diviſion; ſuch are the octave, and all that are within it; as the 2d, 3d, 4th, 5th, 6th, and 7th, with their varieties.

Compound INTERVAL conſiſts of ſeveral leſſer Intervals; as the 9th, 10th, 11th, 12th, &c, with their varieties.

This Simple Interval was by the ancients called a Diaſtem, and the Compound they called a Syſtem.

An Interval is alſo divided into Juſt or True, and into Falſe. The

Juſt or *True* INTERVALS, are ſuch as all thoſe above mentioned, with their varieties, whether major or minor. And the

Falſe INTERVALS, are the diminutive or ſuperfluous ones.

An Interval is alſo divided into the Conſonance and Diſſonance; which ſee.

INTESTINE *Motion of the parts of fluids*, that which is among its corpuſcles or component parts.

When the attracting corpuſcles of any fluid are elaſtic, they muſt neceſſarily produce an inteſtine motion; and that, greater or leſs, according to the degrees of their elaſticity and attractive force. For, two elaſtic particles, after meeting, will fly from each other, with the ſame degree of velocity with which they met; abſtracting from the reſiſtance of the medium. But when, in leaping back from each other, they approach other particles, their velocity will be increaſed.

INTRADOS, the interior and lower ſide, or curve, of the arch of a bridge, &c. In contradiſtinction from the extrados, or exterior curve, or line on the upper ſide of the arch. See my Treatiſe on Bridges &c. prop. 5.

INVERSE, is applied to a manner of working the rule of three, or proportion, which ſeems to go backward, i. e. reverſe or contrary to the order of the common and direct rule: So that, whereas, in the direct rule, more requires more, or leſs requires leſs; in the Inverſe rule, on the contrary, more requires leſs, or leſs requires more.

For inſtance, in the direct rule it is ſaid, If 3 yards of cloth coſt 20 ſhillings, how much will 6 yards coſt? the anſwer is 40 ſhillings: where more yards require more money, and leſs yards require leſs money. But in the Inverſe rule it is ſaid, If 20 men perform a piece of work in 4 days, in how many days will 40 men perform as much? where the anſwer is 2 days; and here the more men require the leſs time, and the fewer men the more time.

INVERSE *Method of Fluxions*, is the method of finding fluents, from the fluxions being given; and is ſimilar to what the foreign mathematicians call the Calculus Integralis. See FLUENTS.

INVERSE *Method of Tangents*, is the method of finding the curve belonging to a given tangent; as oppoſed to the direct method, or the finding the tangent to a given curve.

As, to find a curve whoſe ſubtangent is a third proportional to $r - y$ and y, or whoſe ſubtangent is equal to the ſemiordinate, or whoſe ſubnormal is a conſtant quantity.——The ſolution of this problem depends chiefly on the Inverſe method of fluxions. See TANGENT.

INVERSE *Proportion*, or INVERSE *Ratio*, is that in which more requires leſs, or leſs requires more. As for inſtance, in the caſe of light, or heat from a luminous object, the light received is leſs at a greater diſtance, and greater at a leſs diſtance; ſo that here more, as to diſtance, gives leſs, as to light, and leſs diſtance gives more light. This is uſually expreſſed by the term Inverſely, or Reciprocally; as in the caſe above, where the light is Inverſely, or Reciprocally as the ſquare of the diſtance; or in the Inverſe or Reciprocal duplicate ratio of the diſtance.

INVERSION, *Invertendo*, or *by Inverſion*, according to the 14th definition of Euclid, lib. 5, is Inverting the terms of a proportion, by changing the antecedents into conſequents, and the conſequents into antecedents. As in theſe, $a : b :: c : d$, then by Inverſion $b : a :: d : c$.

INVESTIGATION, the ſearching or finding any thing out, by means of certain ſteps, traces, or ways.

INVOLUTE *Figure* or *Curve*, is that which is traced out by the outer extremity of a ſtring as it is folded or wrapped upon another figure, or as it is unwound from off it.——The Involute of a cycloid, is alſo a cycloid equal to the former, which was firſt diſcovered by Huygens, and by means of which he contrived to make a pendulum vibrate in the curve of a cycloid, and ſo theoretically at leaſt vibrate always in equal times whether the arch of vibration were great or ſmall, which is a property of that curve.——For the doctrine and nature of Involutes and Evolutes, ſee EVOLUTE.

INVOLUTION, in Arithmetic and Algebra, is the raiſing of powers from a given root; as oppoſed to Evolution, which is the extracting, or developing of roots from given powers. So the Involution of the number 3, or its powers, are thus raiſed:

3 - or 3^1 or 3 is the 1ſt power, or root,
3×3 or 3^2 or 9 is the 2d power, or ſquare,
$3 \times 3 \times 3$ or 3^3 or 27 is the 3d power, or cube,
and ſo on.

And hence, to find any power of a given root, or quantity, let the root be multiplied by itſelf a number of times which is one leſs than the number of the index; i. e. once multiplied for the 2d root, twice for the 3d root, thrice for the 4th root, &c.

Thus, to Involve ·12 to the 3d power.
·12

·0144 ſquare, or 2d power.
·12

·001728 cube, or 3d power.

So alſo, in Algebra, to Involve the binomial $a + b$, or raiſe its powers.

$$\begin{array}{llllll}
a & +\,b & & & & \text{- • - - 1ſt. power, or root} \\
a & +\,b & & & & \\
\hline
a^2 & +\ ab & & & & \\
 & +\ ab & +\ b^2 & & & \\
\hline
a^2 & +\ 2ab & +\ b^2 & & & \text{- - 2d power} \\
a & +\ b & & & & \\
\hline
a^3 & +\ 2a^2b & +\ 2ab^2 & & & \\
 & +\ a^2b & +\ ab^2 & +\ b^3 & & \\
\hline
a^3 & +\ 3a^2b & +\ 3ab^2 & +\ b^3 & & \text{- 3d power} \\
a & +\ b & & & & \\
\hline
a^4 & +\ 3a^3b & +\ 3a^2b^2 & +\ ab^3 & & \\
 & +\ a^3b & +\ 3a^2b^2 & +\ 3ab^3 & +\ b^4 & \\
\hline
a^4 & +\ 4a^3b & +\ 6a^2b^2 & +\ 4ab^3 & +\ b^4 & \text{4th power}
\end{array}$$

And in like manner for any other quantities, whatever the number of their terms may be. But compound algebraic quantities are beſt involved by the BINOMIAL *Theorem*; which ſee.

Simple quantities are Involved, by raiſing the numeral coefficients to the given power, and the literal quantities are raiſed by multiplying their indices by that of the root; that is, the raiſing of powers is performed by the multiplication of indices, the ſame as the multiplication of logarithms. Thus,

The 2d power of a is a^2.

The 2d power of $2a^2$ is $2^2a^{2 \times 2}$ or $4a^4$.

The 3d power of $3a^2b^3$ is $27a^6b^9$.

The 3d power of $a^{\frac{1}{2}}b^{\frac{2}{3}}$ is $a^{\frac{3}{2}}b^2$.

The *n*th power of $a^m c^p$ is $a^{mn}c^{pn}$ or $\overline{a^m c^p}\,^n$.

INWARD *Flanking Angle*, in Fortification, is that made by the curtin and the razant flanking line of defence.

JOINTS, in Architecture, are the ſeparations between the ſtones or bricks; which may be filled with mortar, plaſter, or cement.

JOINT, in Carpentry, &c, is applied to ſeveral manners of aſſembling, ſetting, or fixing pieces of wood together. As by a mortiſe Joint, a dove-tail Joint, &c.

Univerſal JOINT, in Mechanics, an excellent invention of Dr. Hook, adapted to all kinds of motions and flexures; of which he has given a large account in his Cutlerian Lectures, printed in 1678. This ſeems to have given occaſion to the gimbols uſed in ſuſpending the ſea compaſſes; the mechaniſm of which is the ſame with that of Deſaguliers's rolling lamp.

JOINT-*Lives*, are ſuch as continue during the ſame time, or that exiſt together. See LIFE-*Annuities*.

JOISTS, or JOYSTS, thoſe pieces of timber framed into the girders and ſummers, and on which the boarding of floors is laid.

"JONES (WILLIAM), F. R. S. a very eminent mathematician, was born at the foot of Bodavon mountain [Mynydd Bodafon] in the pariſh of Llanfihangel tre'r Bard, in the Iſle of Angleſy, North Wales, in the year 1675. His father's name was John George, his ſurname being the proper name of his father. For it is a cuſtom in ſeveral parts of Wales for the proper name of the father to become the ſurname of his children. John George the father was commonly called Sion Siors of Llanbabo, to which place he moved, and where his children were brought up. Accordingly our author, whoſe proper name was William, took the ſurname of Jones from the proper name of his father, who was a farmer, and of a good family, being deſcended from Hwfa ap Cynddelw, one of the 15 tribes of North Wales. He gave his two ſons the common ſchool education of the country, reading, writing, and accounts, in Engliſh, and the Latin Grammar. Harry his ſecond ſon took to the farming buſineſs; but William the eldeſt, having an extraordinary turn for mathematical ſtudies, determined to try his fortune abroad from a place where the ſame was but of little ſervice to him. He accordingly came to London, accompanied by a young man, Rowland Williams, afterwards an eminent perfumer in Wych-ſtreet. The report in the country is, that Mr. Jones ſoon got into a merchant's counting houſe, and ſo gained the eſteem of his maſter, that he gave him the command of a ſhip for a Weſt India voyage; and that upon his return he ſet up a mathematical ſchool, and publiſhed his book of Navigation; and that upon the death of the merchant he married his widow: that, lord Macclesfield's ſon being his pupil, he was made ſecretary to the chancellor, and one of the deputy tellers of the exchequer:—and they have a ſtory of an Italian wedding which cauſed great diſturbance in lord Macclesfield's family, but was compromiſed by Mr. Jones; which gave riſe to a ſaying, "that Macclesfield was the making of Jones, and Jones the making of Macclesfield." The foregoing account of Mr. Jones, I found among the papers of the late Mr. John Robertſon, librarian and clerk to the Royal Society, who had been well known to Mr. Jones, and poſſeſſed many of his papers.

Mr. Jones having by his induſtry acquired a competent fortune, lived upon it as a private gentleman for many years, in the latter part of his life, in habits of intimacy with Sir Iſaac Newton and others the moſt eminent mathematicians and philoſophers of his time; and died July 3, 1749, at 74 years of age, being one of the vice-preſidents of the Royal Society; leaving at his death one daughter, and his widow with child, which proved a ſon, who is the preſent Sir William Jones, now one of the judges in India, and highly eſteemed for his great abilities, extenſive learning, and eminent patriotiſm.——Mr. Jones's publications are,

1. *A new Compendium of the Whole Art of Navigation*, &c; in ſmall 8vo, London, 1702. This is a neat little piece, and dedicated to the Rev. Mr. John Harris, the ſame I believe who was author of the *Lexicon Technicum*, or Univerſal Dictionary of Arts and Sciences, in whoſe houſe Mr. Jones ſays he compoſed his book.

2. *Synopſis Palmariorum Matheſeos*: Or a New Introduction to the Mathematics, &c; 8vo, London, 1706. Being a very neat and uſeful compendium of all the mathematical ſciences, in about 300 pages.

His papers in the Philoſ. Tranſ. are the following:

3. A Compendious Diſpoſition of Equations for exhibiting the relations of Goniometrical Lines; vol. 44, p. 560.

4. A Tract on Logarithms ; vol. 61, pa. 455.

5. Account of the perſon killed by lightning in Tottenham-Court-Chapel, and its effects on the building; vol. 62, pa. 131.

6. Properties of the Conic Sections, deduced by a compendious method ; vol. 63, pa. 340.

In all theſe works of Mr. Jones, a remarkable neatneſs, brevity, and accuracy, every where prevails. He ſeemed to delight in a very ſhort and comprehenſive mode of expreſſion and arrangement ; in ſo much that ſometimes what he has contrived to expreſs in two or three pages, would occupy a little volume in the ordinary ſtyle of writing.

Mr. Jones it is ſaid poſſeſſed the beſt mathematical library in England ; ſcarcely any book of that kind but what was there to be found. He had collected alſo a great quantity of manuſcript papers and letters of former mathematicians, which have often proved uſeful to writers of their lives, &c. After his death, theſe were diſperſed, and fell into different perſons hands ; many of them, as well as of Mr. Jones's own papers, were poſſeſſed by the late Mr. John Robertſon, before mentioned, at whoſe death I purchaſed a conſiderable quantity of them. From ſuch collections of theſe it was that Mr. Jones was enabled to give that firſt and elegant edition, in 4to, 1711, of ſeveral of Newton's papers, that might otherwiſe have been loſt, intitled, *Analyſis per quantitatum Series, Fluxiones, ac Differentias : cum Enumeratione Linearum Tertii Ordinis.*

IONIC *Column*, or *Order*, the 3d of the five orders, or columns, of Architecture. The firſt idea of this order was given by the people of Ionia; who, according to Vitruvius, formed it on the model of a young woman, dreſſed in her hair, and of an eaſy elegant ſhape, as the Doric had been formed on the model of a ſtrong robuſt man.

This column is a medium between the maſſive and the more delicate orders, the ſimple and the rich. It is diſtinguiſhed from the Compoſite, by having none of the leaves of acanthus in its capital ; and from the Tuſcan, Doric, and Corinthian, by the volutes, or rams horns, which adorn its capital ; and from the Tuſcan and Doric too, by the channels, or fluting, in its ſhaft.

The height of this column is 18 modules, or 9 diameters of the column taken at the bottom : indeed at firſt its height was but 16 modules ; but, to render it more beautiful than the Doric, its height was augmented by adding a baſe to it, which was unknown in the Doric. M. le Clerc makes its entablature to be 4 modules and 10 minutes, and its pedeſtal 6 modules; ſo that the whole order makes 28 modules 10 minutes.

JOURNAL, in Merchants Accounts, is a book into which every particular article is poſted out of the Waſte-book, according to the order of time, ſpecifying the debtor and creditor in each account and tranſaction.

JOURNAL, in Maritime Affairs, is a regiſter kept by the pilot, and others, noticing every thing that happens to the ſhip, from day to day, and from hour to hour, with regard to the winds, the rhumbs or courſes, the knots or rate of running, the rake, ſoundings, aſtronomical obſervations, for the latitudes and longitudes, &c ; to enable them to adjuſt the reckoning, and determine the place where the ſhip is.

In all ſea Journals, the day, or what is called the 24 hours, is divided into twice 12 hours, thoſe before noon marked A. M. for ante meridiem, and thoſe from noon to midnight marked P. M. poſt meridiem, or afternoon.

There are various ways of keeping a ſea Journal, according to the different notions of mariners concerning the articles that are to be entered. Some writers direct the keeping ſuch a kind of Journal as is only an abſtract of each day's tranſactions, ſpecifying the weather, what ſhips or lands were ſeen, accidents on board, the latitude, longitude, meridional diſtance, courſe, and run : theſe particulars are to be drawn from the ſhip's log-book, or from that kept by the perſon himſelf. Other authors recommend the keeping only of one account, including the log-book, and all the work of each day, with the deductions drawn from it.

JOURNAL is alſo uſed for the title of ſeveral books which come out at ſtated times ; and give accounts and abſtracts of the new books that are publiſhed, with the new improvements daily made in arts and ſciences.

The firſt Journal of this kind was, the Journal des Sçavans, printed at Paris : the deſign was ſet on foot for the eaſe of ſuch as are too buſy, or too lazy, to read the entire books themſelves. It ſeems an excellent way of ſatisfying a man's curioſity, and becoming learned upon eaſy terms : and ſo uſeful has it been found, that it has been executed in moſt other countries, though under a great variety of titles.

Of this kind are the Acta Eruditorum of Leipſic ; the Nouvelles de la Republique des lettres of Mr. Bayle, &c ; the Bibliotheque Univerſelle, Choiſie, et Ancienne et Moderne, of M. le Clerc ; the Memoirs de Trevoux, &c. In 1692, Juncker printed in Latin, An Hiſtorical Treatiſe of the Journals of the Learned, publiſhed in the ſeveral parts of Europe ; and Wolſius, Struvius, Morhoff, Fabricius, &c, have done ſomething of the ſame kind.

The Philoſophical Tranſactions of London ; the Memoirs of the Royal Academy of Sciences ; thoſe of the Academy of Belles Lettres ; the Miſcellanea Naturæ Curioſorum ; the Experiments of the Academy del Cimento, the Acta Philo-exoticorum Naturæ et Artis, which appeared from March 1686 to April 1687, and which are a hiſtory of the Academy of Breſſe ; the Miſcellanea Berolinenſia, or Memoirs of the Academy of Berlin ; the Commentaries of the Academy of Peterſburgh ; the Memoirs of the Inſtitute at Bologna ; the Acta Literaria Sueciæ ; the Memoirs of the Royal Academy of Stockholm, begun in 1740 ; the Commentarii Societatis Regiæ Gottingenſis, begun in 1750, &c, &c, are not ſo properly Journals, though they are frequently ranked in the number.

Juncker and Wolſius give the honour of the firſt invention of Journals to Photius. His Bibliotheca, however, is not altogether of the ſame nature with the modern Journals ; nor was his deſign the ſame. It conſiſts of abridgments, and extracts of books which he had read during his embaſſy in Perſia. M. Salo firſt began the Journal des Sçavans at Paris, in 1665, under the name of the Sieur de Hedonville ; but his death ſoon after interrupted the work. The abbé Gallois then took it up, and he, in the year 1674, gave way to the abbé de la Roque, who continued it nine years, and was ſucceeded by M. Couſin, who carried it on till the year 1702, when the abbé Bignon inſtituted a new Society, and committed the care of continuing the Journal to them, who improved and publiſhed it under a new form. This Society is ſtill continued, and M. de Loyer has had the inſpection of the Journal; which is no longer the work of any ſingle author, but of a great number.

The other French Journals are the Memoirs and Conferences of Arts and Sciences, by M. Dennis, during the years 1672, 1673, and 1674; New Diſcoveries in all the parts of Phyſic, by M. de Blegny; the Journal of Phyſic, begun in 1684, and ſome others, diſcontinued almoſt as ſoon as begun.

Rozier's Journal de Phyſique, begun in July 1771, and continued, till in the year 1780, there were 19 vols. quarto.

The Nouvelles de la Republique des Lettres, News from the Republic of Letters, were begun by M. Bayle in 1684, and carried on by him till the year 1687, when M. Bayle being diſabled by ſickneſs, his friends, M. Bernard and M. de la Roque, took them up, and continued them till 1699. After an interruption of nine years, M. Bernard reſumed the work, and continued it till the year 1710. The Hiſtory of the Works of the Learned, by M. Baſnage, was begun in the year 1686, and ended in 1710. The Univerſal Hiſtorical Library, by M. le Clerc, was continued to the year 1693, and contained twenty-five volumes. The Bibliotheque Choiſie of the ſame author, began in 1703. The Mercury of France is one of the moſt ancient Journals of that country, and is continued by different hands: the Memoirs of a Hiſtory of Sciences and Arts, uſually called Memoires des Trevoux, from the place where they are printed, began in 1701. The Eſſays of Literature reached but to a twelfth volume in 1702, 1703, and 1704; theſe only take notice of ancient authors. The Journal Literaire, by Father Hugo, began and ended in 1705. At Hamburgh they have made two attempts for a French Journal, but the deſign failed: an Ephemerides Sçavantes has alſo been undertaken, but that ſoon diſappeared. A Journal des Sçavans, by M. Dartis, appeared in 1694, and was dropt the year following. That of M. Chauvin, begun at Berlin in 1696, held out three years; and an eſſay of the ſame kind was made at Geneva. To theſe may be added, the Journal Literaire begun at the Hague 1715, and that of Verdun, and the Memoires Literaires de la Grande Bretagne by M. de la Roche; the Bibliotheque Angloiſe, and Journal Britannnique, which are confined to Engliſh books alone. The Italian Journals are, that of abbot Nazari, which laſted from 1668 to 1681, and was printed at Rome. That of Venice began in 1671, and ended at the ſame time with the other: the authors were Peter Moretti, and Francis Miletti. The Journal of Parma, by Roberti and Father Bacchini, was dropped in 1690, and reſumed again in 1692. The Journal of Ferrara, by the abbé de la Torre, began and ended in 1691. La Galerio di Minerva, begun in 1696, is the work of a Society of men of letters. Seignior Apoſtolo Zeno, Secretary to that Society, began another Journal in 1710, under the protection of the Grand Duke: it is printed at Venice, and ſeveral perſons of diſtinction have a hand in it.

The Faſti Eruditi della Bibliotheca Volante, were publiſhed at Parma. There has appeared ſince, in Italy, the Giornale dei Letterati.

The principal among the Latin Journals, is that of Leipſic, under the title of Acta Eruditorum, begun in 1682: P. P. Manzani began another at Parma. The Nova Literaria Maris Balthici laſted from 1698 to 1708. The Nova Literaria Germaniæ, collected at Hamburgh, began in 1703. The Acta Literaria ex Manuſcriptis, and the Bibliotheca curioſa, begun in 1705, and ended in 1707, are the work of Struvius. Meſſ. Kuſter and Sike, in 1697, began a Bibliotheca Novorum Librorum, and continued it for two years. Since that time, there have been many Latin Journals; ſuch, beſides others, is the Commentarii de Rebus in Scientia Naturali et Medicina geſtis, by M. Ludwig. The Swiſs Journal, called Nova Literaria Helvetiæ, was begun in 1702, by M. Scheuchzer; and the Acta Medica Hafnenſia, publiſhed by T. Bartholin, make five volumes from the year 1671 to 1679. There are two Low-Dutch Journals; the one under the title of Boockzal van Europe; it was begun at Rotterdam in 1692, by Peter Rabbus; and continued from 1702 to to 1708, by Sewel and Gavern: the other was done by a phyſician, called Ruiter, who began it in 1710. The German Journals of beſt note are, the Monathlichen Unterredungen, which continued from 1689 to 1698. The Bibliotheca Curioſa, began in 1704, and ended in 1707, both by M. Tenzel. The Magazin d'Hambourg, begun in 1748: the Phyſicaliſche Beluſtigunzen, or Philoſophical Amuſements, begun at Berlin in 1751. The Journal of Hanover began in 1700, and continued for two years by M. Eccard, under the direction of M. Leibnitz, and afterwards carried on by others. The Theological Journal, publiſhed by M. Loeſcher, under the title of Altes und Neues, that is, Old and New. A third at Leipſic and Francfort, the authors Meſſ. Walterck, Krauſe, and Groſchuffius; and a fourth at Hall, by M. Turk.

The Engliſh Journals are, The Hiſtory of the Works of the Learned, begun at London in 1699. Cenſura Temporum, in 1708. About the ſame time there appeared two new ones, the one under the title of Memoirs of Literature, containing little more than an Engliſh tranſlation of ſome articles in the foreign Journals, by M. de la Roche; the other a collection of looſe tracts, entitled, Bibliotheca curioſa, or a Miſcellany. Theſe, however, with ſome others, are now no more, but are ſucceeded by the Annual Regiſter, which began in 1758; the New Annual Regiſter, begun in 1780; the Monthly Review, which began in the year 1749, and gives a character of all Engliſh literary publications, with the moſt conſiderable of the foreign ones: the Critical Review, which began in 1756, and is nearly on the ſame plan: as alſo the London Review, by Dr. Kenrick, from 1775 to 1780; Maty's Review, from Feb. 1782 to Aug. 1786; the Engliſh Review begun in Jan. 1783; and the Analytical Review, begun in May 1788, and ſtill continues with much reputation. Beſides theſe, we have ſeveral monthly pamphlets, called Magazines, which, together with a chronological Series of occurrences, contain letters from correſpondents, communicating extraordinary diſcoveries in nature and art, with controverſial pieces on all ſubjects. Of theſe, the principal are thoſe called, the Gentleman's Magazine, which began with the year 1731; the London Magazine, which began a few months after, and has lately been diſcontinued; the Univerſal Magazine, which is nearly of as old a date.

IRIS, another name for the Rainbow; which ſee.

Iris alſo denotes the ſtriped variegated circle round the pupil of the eye, formed of a duplicature of the uvea.

In different ſubjects, the Iris is of ſeveral very different colours; whence the eye is called grey, or black &c. In its middle is a perforation, through which appears a ſmall black ſpeck, called the ſight, pupil, or apple of the eye, round which the Iris forms a ring.

Iris is alſo applied to thoſe changeable colours, which ſometimes appear in the glaſſes of teleſcopes, microſcopes, &c; ſo called from their ſimilitude to a rainbow.

The ſame appellation is alſo given to that coloured ſpectrum, which a triangular priſmatic glaſs will project on a wall, when placed at a proper angle in the ſun-beams.

Iris *Marina*, the *Sea-Rainbow*. This elegant appearance is generally ſeen after a violent ſtorm, in which the ſea water has been in vaſt emotions. The celeſtial rainbow however has great advantage over the marine one, in the brightneſs and variety of the colours, and in their diſtinctneſs one from the other; for in the ſea-rainbow, there are ſcarce any other colours than a duſky yellow on the part next the ſun, and a pale green on the oppoſite ſide. The other colours are not ſo bright or ſo diſtinct as to be well determined; but the ſea-rainbows are more frequent and more numerous than the others: it is not uncommon to ſee 20 or 30 of them at a time at noon-day.

IRRATIONAL *Numbers*, or *Quantities*, are the ſame as ſurds, or ſuch roots as cannot be accurately extracted, being incommenſurable to unity. See Surds.

IRREDUCIBLE *Caſe*, in Algebra, is uſed for that caſe of cubic equations where the root, according to Cardan's rule, appears under an impoſſible or imaginary form, and yet is real. Thus, in the equation $x^3 - 15x = 4$, the root, according to Cardan's rule, will be $x = \sqrt[3]{2+\sqrt{-121}} + \sqrt[3]{2-\sqrt{-121}}$, which is in the form of an impoſſible expreſſion, and yet it is equal to the quantity 4:

for $\sqrt[3]{2+\sqrt{-121}} = 2 + \sqrt{-1}$,
and $\sqrt[3]{2-\sqrt{-121}} = 2 - \sqrt{-1}$,
therefore there ſum is $x = 4$.

The other two roots of the equation are alſo real.

Algebraiſts, for almoſt three centuries, have in vain endeavoured to reſolve this caſe, and to bring it under a real form; and the problem is not leſs celebrated among them, than the ſquaring of the circle is among geometricians.

It is to be obſerved, that, as in ſome other caſes of cubic equations, the value of the root, though rational, is found under an irrational or ſurd form; becauſe the root in this caſe is compounded of two equal ſurds with contrary ſigns, which deſtroy each other; as if $x = 2 + \sqrt{3} + 2 - \sqrt{3}$, then $x = 4$. In like manner, in the Irreducible caſe, where the root is rational, there are two equal imaginary quantities, with contrary ſigns, joined to real quantities; ſo that the imaginary quantities deſtroy each other; as in the caſe above of the root of the equation $x^3 - 15x = 4$, which was ſound to be $2 + \sqrt{-1} + 2 - \sqrt{-1} = 4$.

It is remarkable that this caſe always happens, viz one root, by Cardan's rule, in an impoſſible form, whenever the equation has three real roots, and no impoſſible ones, but at no time elſe.

If we were poſſeſſed of a general rule for accurately extracting the cube root of a binomial radical quantity, it is evident we might reſolve the Irreducible caſe generally, which conſiſts of two of ſuch cubic binomial roots. But the labours of the algebraiſts, from Cardan's down to the preſent time, have not been able to remove this difficulty. Dr. Wallis thought that he had diſcovered ſuch a rule; but, like moſt others, it is merely tentative, and can only ſucceed in certain particular circumſtances.

Mr. Maſeres, curſitor baron of the exchequer, has lately deduced, by a long train of algebraical reaſoning, from Newton's celebrated binomial theorem, an infinite ſeries, which will reſolve this caſe, without any mention of either impoſſible or negative quantities. And I have alſo diſcovered ſeveral other ſeries which will do the ſame thing, in all caſes whatever; both inſerted in the Phil. Tranſ. See Cardan's Algebra; the articles Algebra, Cubic Equations; Wallis's Algebra, chap. 48; De Moivre in the Appendix to Sanderſon's Algebra, p. 744; Philoſ. Tranſ. vol. 68, part 1, art. 42, and vol. 70, p. 387.

IRREGULAR, ſomething that deviates from the common forms or rules. Thus, we ſay an Irregular fortification, an Irregular building, &c.

Irregular *Figure*, in Geometry, whether plane or ſolid, is that whoſe ſides, as well as angles, are not all equal and ſimilar among themſelves.

IRREGULARITIES *in the Moon's motion*. See Moon.

ISAGONE, in Geometry, is ſometimes uſed for a figure conſiſting of equal angles.

ISLAND, or Isle, a tract of dry land encompaſſed by water; whether by the ſea, a river, or lake, &c. In which ſenſe Iſland ſtands contradiſtinguiſhed from Continent, or terra firma; like Great Britain, Ireland, Jerſey, Sicily, Minorca, &c.

Some naturaliſts imagine that Iſlands were formed at the deluge: others think they have been rent and ſeparated from the continent by violent ſtorms, inundations, and earthquakes; while others are thrown up by volcanoes, or otherwiſe grow or emerge from the ſea.

Varenius thinks moſt of theſe opinions true in ſome inſtances, and believes that there have been Iſlands produced each of theſe ways. St. Helena, Aſcenſion, and other ſteep rocky Iſlands, he ſuppoſes have become ſo, by the ſea's overflowing their neighbouring champaigns. By the heaping up huge quantities of ſand &c, he thinks the Iſlands of Zealand, Japan, &c, were formed: Sumatra and Ceylon, and moſt of the Eaſt India Iſlands, he rather thinks were rent off from the main land, as England probably was from France. It is alſo certain that ſome have emerged from the bottom of the ſea; as Santorini formerly, and three other Iſles near it lately; the laſt in 1707, which roſe from the bottom of the ſea, after an earthquake: the ancients had a tradition that Delos roſe from the bottom of the ſea ; and Seneca obſerves that the Iſland Theraſia roſe out of the Ægean ſea in his time, of which the mariners were eye witneſſes ; as they have been within theſe 10 years, in the ſea between Norway and Iceland, where an Iſland has juſt emerged. The late circumnavigators too have made it probable, that many of the South-ſea Iſlands have had their foundations, of coral rock, gradually increaſing, and growing out of the ſea.

ISLES, or rather Ailes, in Architecture, the ſides or wings of a building.

ISOCHRONAL, or Isochronous, is applied to ſuch vibrations of a pendulum as are performed in equal times. Of which kind are all the vibrations of the ſame pendulum in a cycloidal curve, and in a circle nearly, whether the arcs it deſcribes be longer or ſhorter ; for when it deſcribes a ſhorter arc, it moves ſo much the ſlower ; and when a long one, proportionably faſter.

Isochronal *Line*, is that in which a heavy body is ſuppoſed to deſcend with a uniform velocity, or without any acceleration.

Leibnitz, in the Act. Erud. Lipſ. for April 1689, has a diſcourſe on the Linea Iſochrona, in which he ſhews, that a heavy body, with the velocity acqnired by its deſcent from any height, may deſcend from the ſame point by an infinite number of Iſochronal curves, which are all of the ſame ſpecies, differing from one another only in the magnitude of their parameters (ſuch as are all the quadratocubical paraboloids), and conſequently ſimilar to one another. He ſhews alſo, how to find a line, in which a heavy body deſcending, ſhall recede uniformly from a given point, or approach uniformly to it.

ISOMERIA, in Algebra, a term of Vieta, denoting the freeing an equation from fractions ; which is done by reducing all the fractions to one common denominator, and then multiplying each member of the equation by that common denominator, that is rejecting it out of them all.

ISOPERIMETRICAL *Figures*, are ſuch as have equal perimeters, or circumferences.

It is demonſtrated in geometry, that among Iſoperimetrical figures, that is always the greateſt which contains the moſt ſides or angles. From whence it follows, that the circle is the moſt capacious of all figures which have the ſame perimeter with it.

That of two Iſoperimetrical triangles, which have the ſame baſe, and one of them two ſides equal, and the other unequal ; that is the greater whoſe ſides are equal.

That of Iſoperimetrical figures, whoſe ſides are equal in number, that is the greateſt which is equilateral, and equiangular. And hence ariſes the ſolution of that popular problem, To make the hedging or walling, which will fence in a certain given quantity of land, alſo to fence in any other greater quantity of the ſame. For, let x be one ſide of a rectangle that will contain the quantity aa of acres ; then will $\frac{aa}{x}$ be its other ſide, and double their ſum, viz, $2x + \frac{2aa}{x}$, will be the perimeter of the rectangle : let alſo bb be any greater number of acres, in the form of a ſquare, then is b one ſide of it, and $4b$ its perimeter, which muſt be equal to that of the rectangle ; and hence the equation $2x + \frac{2aa}{x} = 4b$, or $x^2 + a^2 = 2bx$, in which quadratic equation the two roots are $x = b \pm \sqrt{b^2 - a^2}$, which are the lengths of the two dimenſions of the rectangle, viz, whoſe area b^2 is in any proportion leſs than the ſquare a^2, of the ſame perimeter. As, for example, if one ſide of a ſquare be 10, and one ſide of a rectangle be 19, but the other only 1 ; ſuch ſquare and parallelogram will be Iſoperimetrical, viz, each perimeter 40 ; yet the area of the ſquare is 100, and of the parallelogram only 19.

Iſoperimetrical lines and figures have greatly engaged the attention of mathematicians at all times. The 5th book of Pappus's Collections is chiefly upon this ſubject ; where a great variety of curious and important properties are demonſtrated, both of planes and ſolids, ſome of which were then old in his time, and many new ones of his own. Indeed it ſeems he has here brought together into this book all the properties relating to Iſoperimetrical figures then known, and their different degrees of capacity.

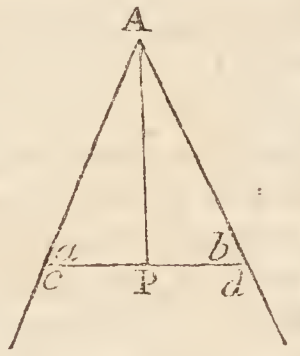
The analyſis of the general problem concerning figures that, among all thoſe of the ſame perimeter, produce maxima and minima, was given by Mr. James Bernoulli, from computations that involve 2d and 3d fluxions. And ſeveral enquiries of this nature have been ſince proſecuted in like manner, but not always with equal ſucceſs. Mr. Maclaurin, to vindicate the doctrine of fluxions from the imputation of uncertainty, or obſcurity, has illuſtrated this ſubject, which is conſidered as one of the moſt abſtruſe parts of this doctrine, by giving the reſolution and compoſition of theſe problems by firſt fluxions only ; and in a manner that ſuggeſts a ſynthetic demonſtration, ſerving to verify the ſolution. See Maclaurin's Fluxions, p. 486 ; Analyſis Magni problematis Iſoperimetrici Act. Erud. Lipſ. 1701, p. 213 ; Mem. Acad. Scienc. 1705, 1706, 1718 ; and the works of John Bernoulli, tom. 1, p. 202, 208, 424, and tom. 2, p. 235 ; where is contained what he and his brother James publiſhed on this problem. Mr. John Bernoulli, in his firſt paper, conſidered only two ſmall ſucceſſive ſides of the curve ; whereas the true method of reſolving this problem in general, requires the conſidering three ſuch ſmall ſides, as may be perceived by examining the two ſolutions.

M. Euler has alſo publiſhed, on this ſubject, many profound reſearches, in the Peterſburg commentaries ; and there was printed at Lauſanne, in 1744, a pretty large work upon it, intitled, Methodus inveniendi lineas curvas, maximi minimive proprietate gaudentes : ſive Solutio problematis Iſoperimetrici in latiſſimo ſenſu accepti.

M. Cramer too, in the Berlin Memoirs for 1752, has given a paper, in which he propoſes to demonſtrate in general, what can be demonſtrated only of regular figures in the elements of geometry, viz, that the circle is the greateſt of all Iſoperimetrical figures, regular or irregular.

On this head, ſee alſo Simpſon's Tracts, p. 98; and the Philoſ. Tranſ. vol. 49 and 50.

ISOSCELES *Triangle*, is a triangle that has two ſides equal. In the 5th prop. of Euclid's 1ſt book, which prop. is uſually called the Pons Aſinorum, or Aſſes bridge, it is demonſtrated, that the angles, *a* and *b*, at the baſe of the Iſoſceles triangle, are equal to each other; and that if the equal ſides be produced, the two angles, *c* and *d*, below the baſe, will alſo be equal. It is alſo inferred, that every equilateral triangle is alſo equiangular. Other properties of this figure are, that the perpendicular AP, from the vertex to the baſe, biſects the baſe, the vertical angle, and alſo the whole triangle. And that if the vertical angles of two Iſoſceles triangles be equal, the two triangles will be equiangular.



ISTHMUS, in Geography, a narrow neck or ſlip of land, that joins two other large tracts together, and ſeparating two ſeas, or two parts of the ſame ſea.

The moſt remarkable Iſthmuſes are, that of Panama or Straits of Darien, joining north and ſouth America; that of Suez, which connects Aſia and Africa; that of Corinth, or Peloponneſus, in the Morea; that of Crim Tartary, otherwiſe called Taurica Cherſoneſus; that of the peninſula Romania and Eriſſo, or the Iſthmus of the Thracian Cherſoneſus, 12 furlongs broad, and which Xerxes undertook to cut through. The Ancients had ſeveral deſigns of cutting the Iſthmus of Corinth, which is a rocky hillock, about 10 miles over; but without effect, the invention of ſluices being not then known. There have alſo been attempts for cutting the Iſthmus of Suez, to make a communication between the Mediterranean and the Red-ſea.

JUDICIAL, or JUDICIARY *Aſtrology*; that relating to the forming of judgments, and making prognoſtications. See ASTROLOGY.

JULIAN *Calendar*, is that depending on, and connected with the Julian year and account of time; ſo called from Julius Cæſar, by whom it was eſtabliſhed. See CALENDAR.

JULIAN *Epoch*, is that of the inſtitution of the Julian reformation of the calendar, which began the 46th year before Chriſt.

JULIAN *Period*, is a cycle of 7980 conſecutive years, invented by Julius Scaliger, from whom it was named; though ſome ſay his name was Joſeph Scaliger, and that it was called the Julian Period, becauſe he made uſe of Julian years. This period is formed by multiplying continually together the three following cycles, viz, that of the ſun of 28 years, that of the moon of 19 years, and that of the indiction of 15 years; ſo that this epoch, although but artificial or feigned, is yet of good uſe; in that every year within the period is diſtinguiſhable by a certain peculiar character; for the year of the ſun, moon, and indiction, will not be the ſame again till the whole 7980 years have revolved. Scaliger fixed the beginning of this period 764 years before the creation, or rather the period naturally reduces to that year, taking the numbers of the three given cycles as he then found them; and accounting 3950 years from the creation to the birth of Chriſt, this makes the 1ſt year of the Chriſtian era anſwer to the 4714th year of the Julian period; therefore, to find the year of this period, anſwering to any propoſed year of Chriſt, to the conſtant number 4713, add the given year of Chriſt, and the ſum will be the year of the Julian period: thus, to 4713 adding 1791, the ſum 6504 is the year of this period for the year of Chriſt 1791. Hence the firſt revolution of the Julian period will not be completed till the year of Chriſt 3267, after which a new revolution of this period will commence.

But the year of this period may be found for any time, from the numbers of the three cycles that compoſe it, without making uſe of the given year of Chriſt, thus: multiply the

$$\text{numbers} \left\{\begin{array}{l}4845\\4200\\6916\end{array}\right\} \begin{array}{l}\text{reſpectively by}\\\text{the year of the}\end{array} \left\}\begin{array}{l}\text{ſun,}\\\text{moon,}\\\text{indiction;}\end{array}\right.$$

then add the three products together, and divide the ſum by 7980, ſo ſhall the remainder after diviſion be the year of the Julian period correſponding to the given years of the other three cycles. Thus, for the year 1791, the years of the ſolar, lunar, and indiction cycles, are 8, 6, and 9; therefore multiplying by theſe, &c, according to the rule, thus

4845	4200	6916
8	6	9
38760	25200	62244
	38760	
	62244	

$$\begin{array}{rl} 7980\ ) & 126204\ (\ 15 \\ & 7980 \\ \hline & 46404 \\ & 39900 \\ \hline \text{remains} & 6504 \text{ the year of the Jul. period.} \end{array}$$

JULIAN *Year*, is the old account of the year, eſtabliſhed by Julius Cæſar, and conſiſted of $365\frac{1}{4}$ days. This year continued in uſe in all Europe till it was ſuperſeded in moſt parts by the new or Gregorian account, in the year 1582. In England however it continued to be uſed till the year 1752, when it was aboliſhed by act of parliament, and eleven days added to the account, to bring it up to the new ſtyle. In Ruſſia, the old or Julian year and ſtyle are ſtill in uſe.

JULY, the 7th month of the year, conſiſting of 31 days; about the 21ſt of which the ſun uſually enters the ſign ♌ leo. This month was ſo named by Mark Antony, from Julius Cæſar, who was born in this month.

JUPITER, ♃, one of the ſuperior planets, remarkable for its brightneſs, being the brighteſt of all, except ſometimes the planet Venus, and is much the largeſt of all the planets.

Jupiter is ſituated between Mars and Saturn, being the 5th in order of the primary planets from the ſun. His diameter is more than 10 times the diameter of the earth, and therefore his magnitude more than 1000 times. His annual revolution about the ſun, is performed in 11 years 314 days 12 hours 20 minutes 9 ſeconds, going at the rate of more than 25 thouſand miles per hour; and he revolves about his own axis in the ſhort ſpace of 9 hours 56 minutes, by which his equatorial parts are carried round at the amazing rate of 26 thouſand miles per hour, which is about 25 times faſter than the like parts of our earth revolve.

Jupiter is ſurrounded by faint ſubſtances, called zones or belts, in which ſo many changes appear, that they are generally aſcribed to clouds: for ſome of them have been firſt interrupted and broken, and then have vaniſhed entirely. They have ſometimes been obſerved of different breadths, and afterwards have all become nearly of the ſame breadth. Large ſpots have been ſeen in theſe belts; and when a belt vaniſhes, the contiguous ſpots diſappear with it. The broken ends of ſome belts have often been obſerved to revolve in the ſame time with the ſpots: only thoſe nearer the equator in ſomewhat leſs time than thoſe nearer the poles; perhaps on account of the ſun's greater heat near the equator, which is parallel to the belts and courſe of the ſpots. Several large ſpots, which appear round at one time, grow oblong by degrees, and then divide into two or three round ſpots. The periodical time of the ſpots near the equator is 9 hours 50 minutes, but of thoſe near the poles 9 hours 56 minutes. See Dr. Smith's Optics, § 1105 and 1109.

The axis of Jupiter is ſo nearly perpendicular to his orbit, that he has no ſenſible change of ſeaſons; which is a great advantage, and wiſely ordered by the Author of Nature. For, if the axis of this planet were inclined any conſiderable number of degrees, juſt ſo many degrees round each pole would in their turn be almoſt 6 years together in darkneſs. And, as each degree of a great circle on Jupiter contains about 706 miles, it is eaſy to judge what vaſt tracts of land would be rendered uninhabitable by any conſiderable inclination of his axis.

The difference between the equatorial and polar diameters of Jupiter, is upwards of 6000 miles; the former being to the latter as 13 to 12: ſo that his poles are more than 3000 miles nearer his centre than the equator is. This happens from his quick motion round his axis; for the fluids, together with the light particles, which they can carry or waſh away with them, recede from the poles which are at reſt, towards the equator where the motion is quickeſt, until there be a ſufficient number accumulated to make up the deficiency of gravity loſt by the centrifugal force, which always ariſes from a quick motion round an axis: and when the deficiency of weight or gravity of the particles is made up by a ſufficient accumulation, there is then an equilibrium, and the equatorial parts riſe no higher.

Jupiter's orbit is 1° 20′ inclined to the ecliptic. The place of his aphelion 9° 10′ of ♎, the place of his aſcending node 7° 29′ of ♋, and that of his ſouth or deſcending node 7° 29′ of ♑. The excentricity of his orbit is $\frac{1}{20}$ of his mean diſtance from the ſun.

The ſun appears to Jupiter but the 48th part ſo large as to us; and his light and heat are in the ſame ſmall proportion, but compenſated by the quick returns of them, and by 4 moons, ſome of them larger than our earth, which revolve about him; ſo that there is ſcarce any part of this huge planet but what is, during the whole night, enlightened by one or more of theſe moons, except his poles, whence only the fartheſt moons can be ſeen, and where their light is not wanted, becauſe the ſun conſtantly circulates in or near the horizon, and is very probably kept in view of both poles by the refraction of Jupiter's atmoſphere, which, if it be like ours, has certainly refractive power enough for that purpoſe. This planet ſeen from its neareſt moon, appears 1000 times as large as our moon does to us; increaſing and waneing in all her monthly ſhapes, every $42\frac{1}{2}$ hours. The periods, diſtances, in ſemidiameters of Jupiter, and angles of the orbits of theſe moons, ſeen from the earth are as follow:

No.	Periods round Jupiter.	Diſtances.	Angles of orbits.
1	$1^d\ 18^h\ 36^m$	$5\frac{2}{3}$	3′ 55″
2	3 13 15	9	6 14
3	7 3 59	$14\frac{1}{3}$	9 58
4	16 18 30	$25\frac{1}{3}$	17 30

The three neareſt moons of Jupiter fall into his ſhadow, and are eclipſed in every revolution: but the orbit of the 4th ſatellite is ſo much inclined, that it paſſeth by its oppoſition to Jupiter, without falling into his ſhadow, two years in every ſix. By theſe eclipſes, aſtronomers have not only diſcovered that the ſun's light takes up 8 minutes of time in coming to us; but have alſo by them determined the longitudes of places on this earth, with greater certainty and facility, than by any other method yet known. The outermoſt of theſe ſatellites will appear nearly as large as the moon does to us. See M. De la Place's Theory of Jupiter's Satellites, in the Memoires de l'Acad. and in the Connoiſſance des Temps for 1792, pa. 273.

Though there be 4 primary planets below Jupiter, yet an eye placed on his ſurface would never perceive any of them; unleſs perhaps as ſpots paſſing over the ſun's diſc, when they happen to come between the eye and the ſun.—The parallax of the ſun, viewed from Jupiter, will ſcarce be ſenſible, being not much above 20 ſeconds; and the ſun's apparent diameter in Jupiter, but about 6 minutes.—Dr. Gregory adds, that an aſtronomer in Jupiter would eaſily diſtinguiſh two kinds of planets, four nearer him, viz his ſatellites, and two more remote, viz the ſun and Saturn: the former however will fall vaſtly ſhort of the ſun in brightneſs, notwithſtanding the great diſproportion in the diſtances and apparent magnitude.

JURIN (Dr. James), a very diſtinguiſhed perſon in ſeveral walks of literature, particularly medicine, mathematics, and philoſophy, which he cultivated with equal ſucceſs. He was ſecretary of the Royal Society in London, as well as preſident of the College of Phyſicians there, at the time of his death, which happened March 22, 1750.

Doctor Jurin was author of ſeveral ingenious compoſitions; particularly "an Eſſay upon Diſtinct and Indiſtinct Viſion ;" printed at the end of the 2d volume of Dr. Smith's Syſtem of Optics; alſo ſeveral controverſial papers; againſt Michellotti, upon the momentum of running waters; againſt Robins, upon diſtinct viſion; and againſt the partiſans of Leibnitz, upon the forces of moving bodies; &c. His papers inſerted in the Philoſ. Tranſ. are the following:

1. On the Suſpenſion of Water in Capillary Tubes: vol. 30, p. 739.

2. Obſervations on the Motion of Running Water: p. 748.

3. On an old Roman Inſcription: p. 813.

4. A Diſcourſe on the Power of the Heart: p. 863 and 929.

5. On the Specific Gravity of Human Blood: p. 1000.

6. Defence of his Doctrine of the Power of the Heart againſt the Objections of Dr. Keill: p. 1039.

7. On the Action of Glaſs-Tubes upon Water and Quickſilver: p. 1083.

8. On the Specific Gravity of Solids when weighed in Water: vol. 31, p. 223.

9. On the Motion of Running Water, againſt Michellotti: vol. 32, p. 179.

10. Remarkable Inſtance of the Small-pox: vol. 32, p. 191.

11. Inoculated and Natural Small-pox compared: vol. 32, pa. 213.

12. On Meteorological Diaries: vol. 32, p. 422.

13. On the Meaſure and Motion of Running Water: vol. 41, p. 5 and 65.

14. Meteorological Obſervations in Charles Town: vol. 42, p. 491.

15. On the Action of Springs: vol. 43, p. 46.

16. On the Force of Bodies in Motion: p. 423.

17. Dynamic Principles, or Meteorological Principles of Mechanics: vol. 66, p. 103.

END OF VOLUME I.

DIALS.

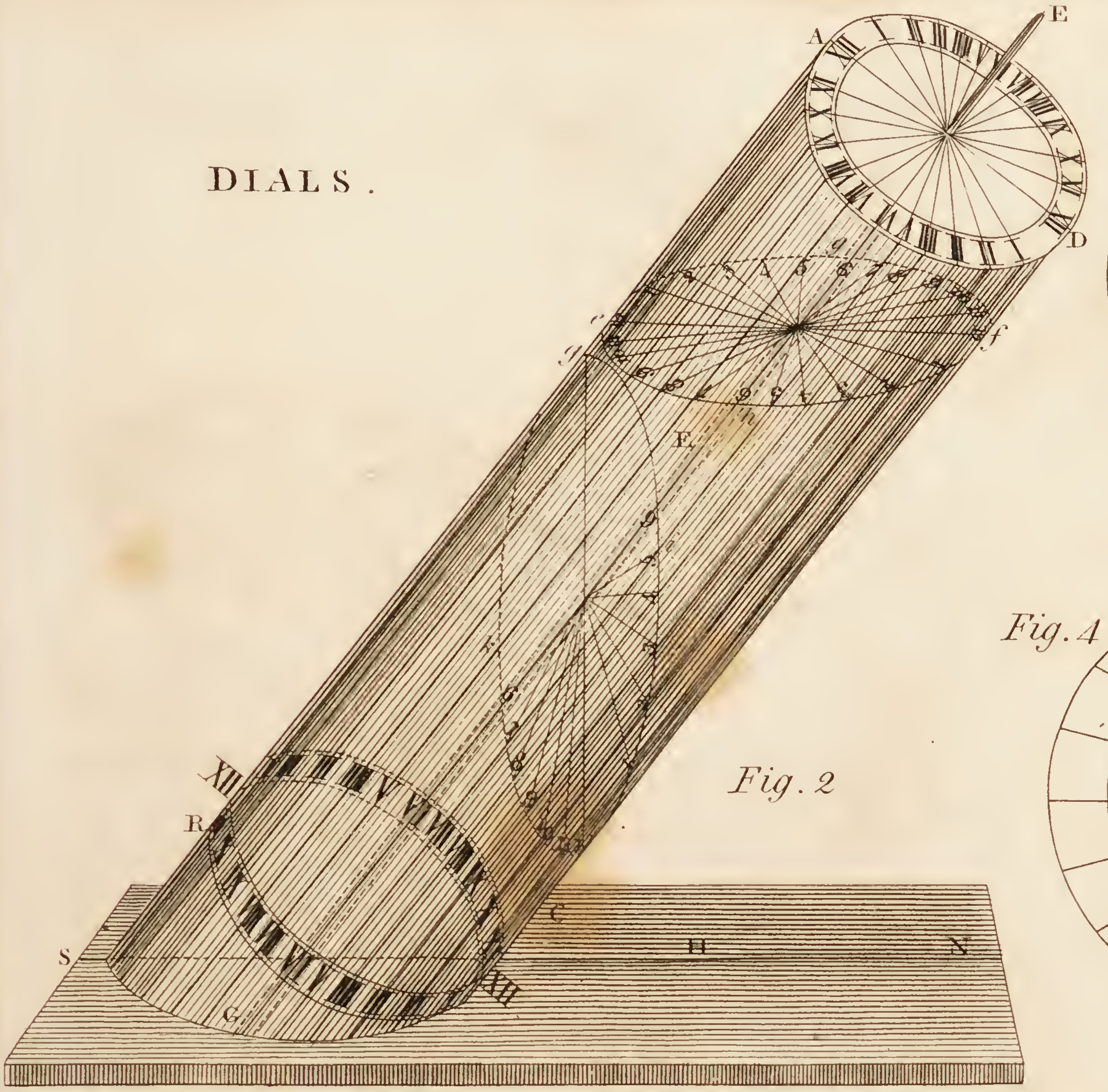


Fig. 2

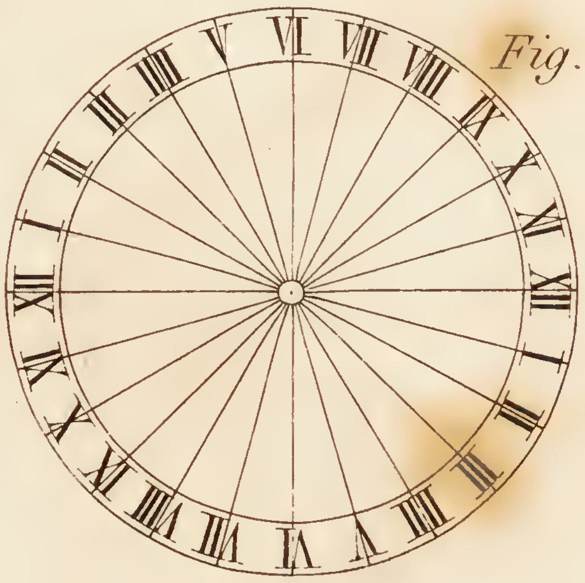


Fig. 3

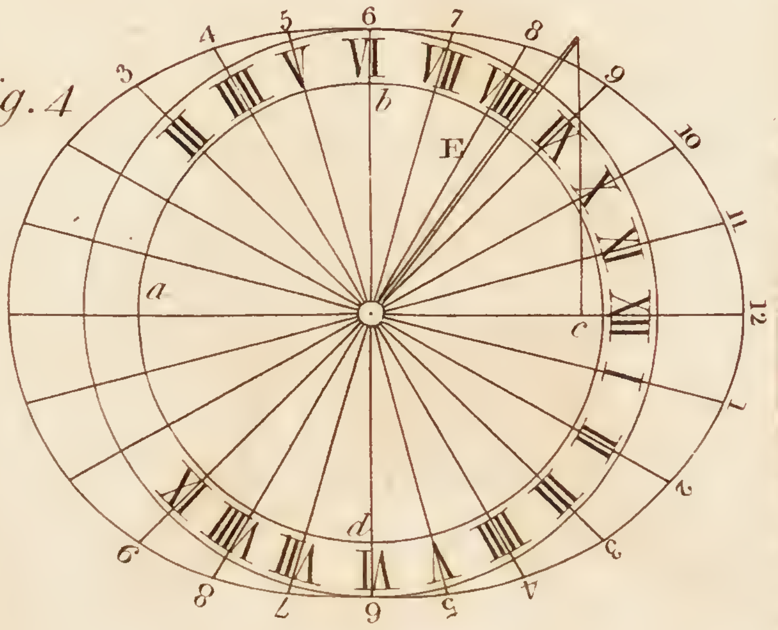


Fig. 4

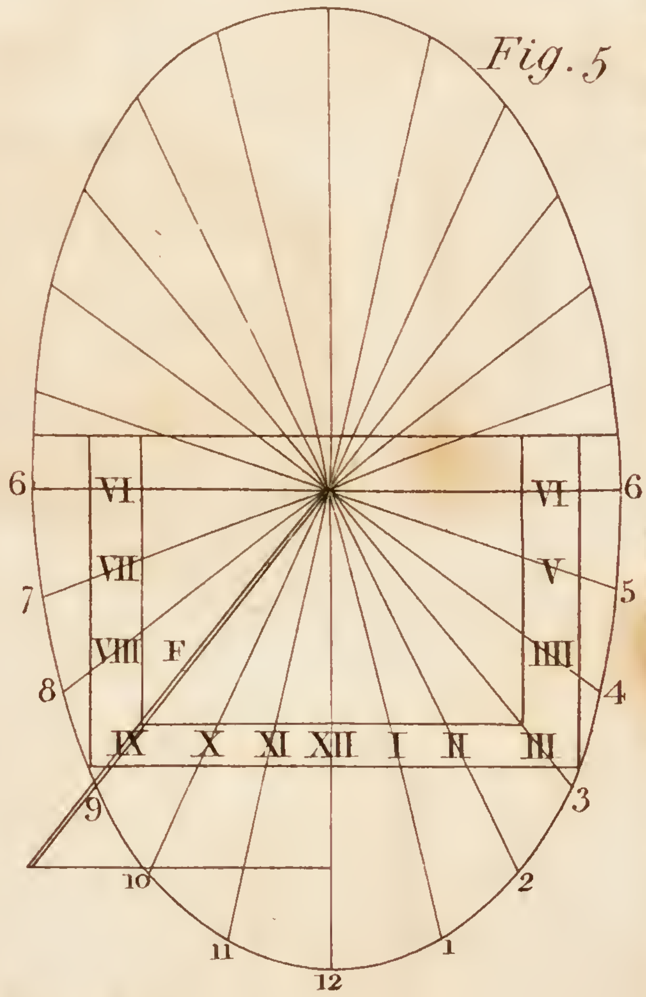


Fig. 5

Halley's DIVING BELL.

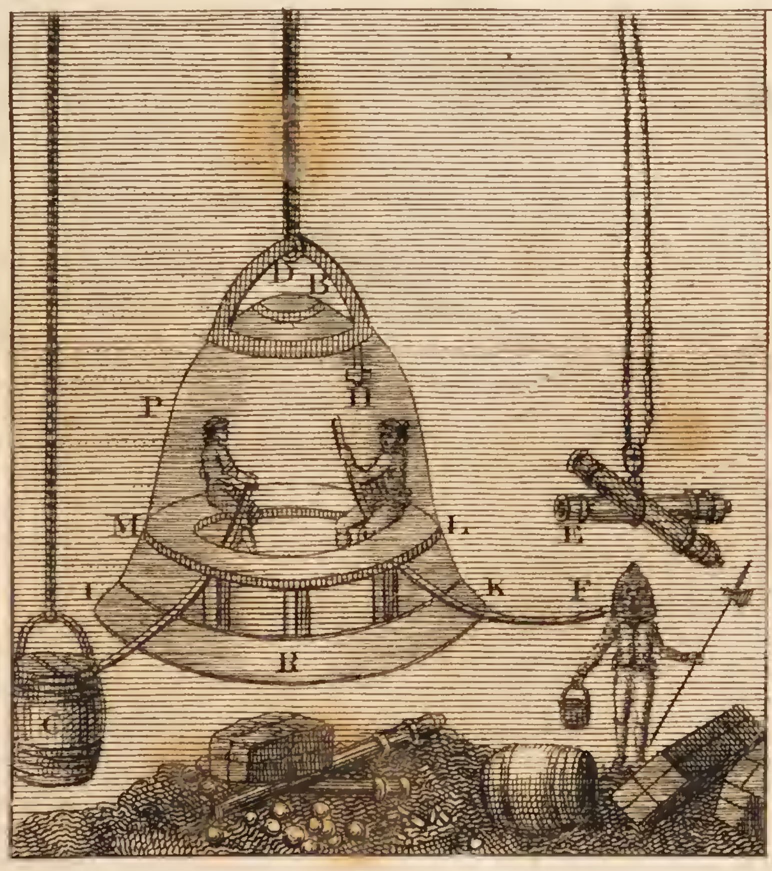


Fig. 6

Triewald's.

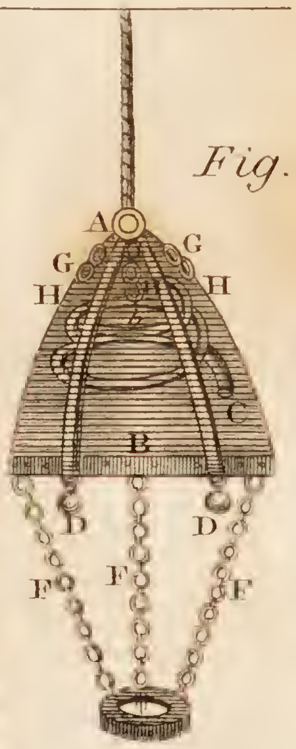


Fig. 7

Profile of a FORTIFICATION.

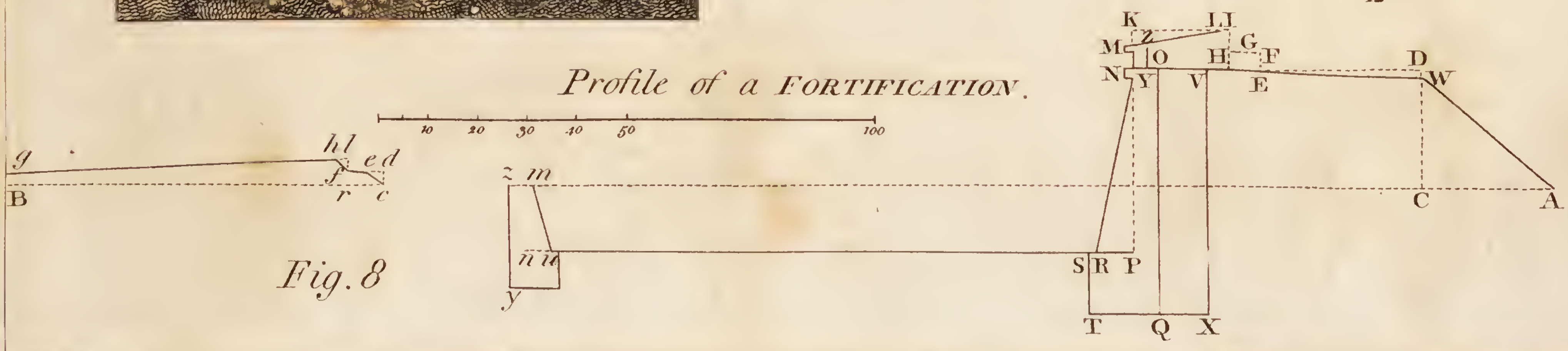


Fig. 8

Plate VIII.

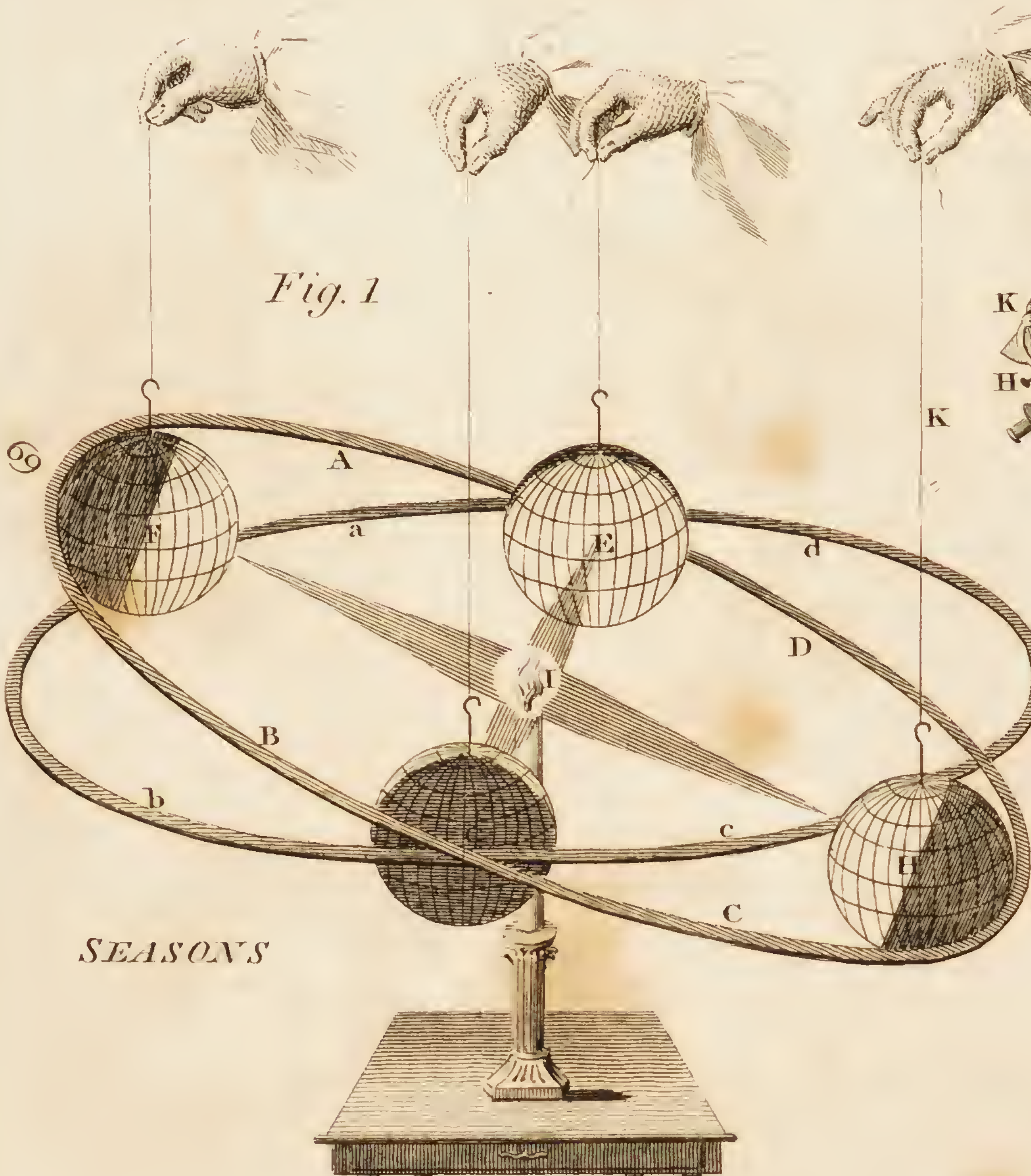


Fig. 1

SEASONS

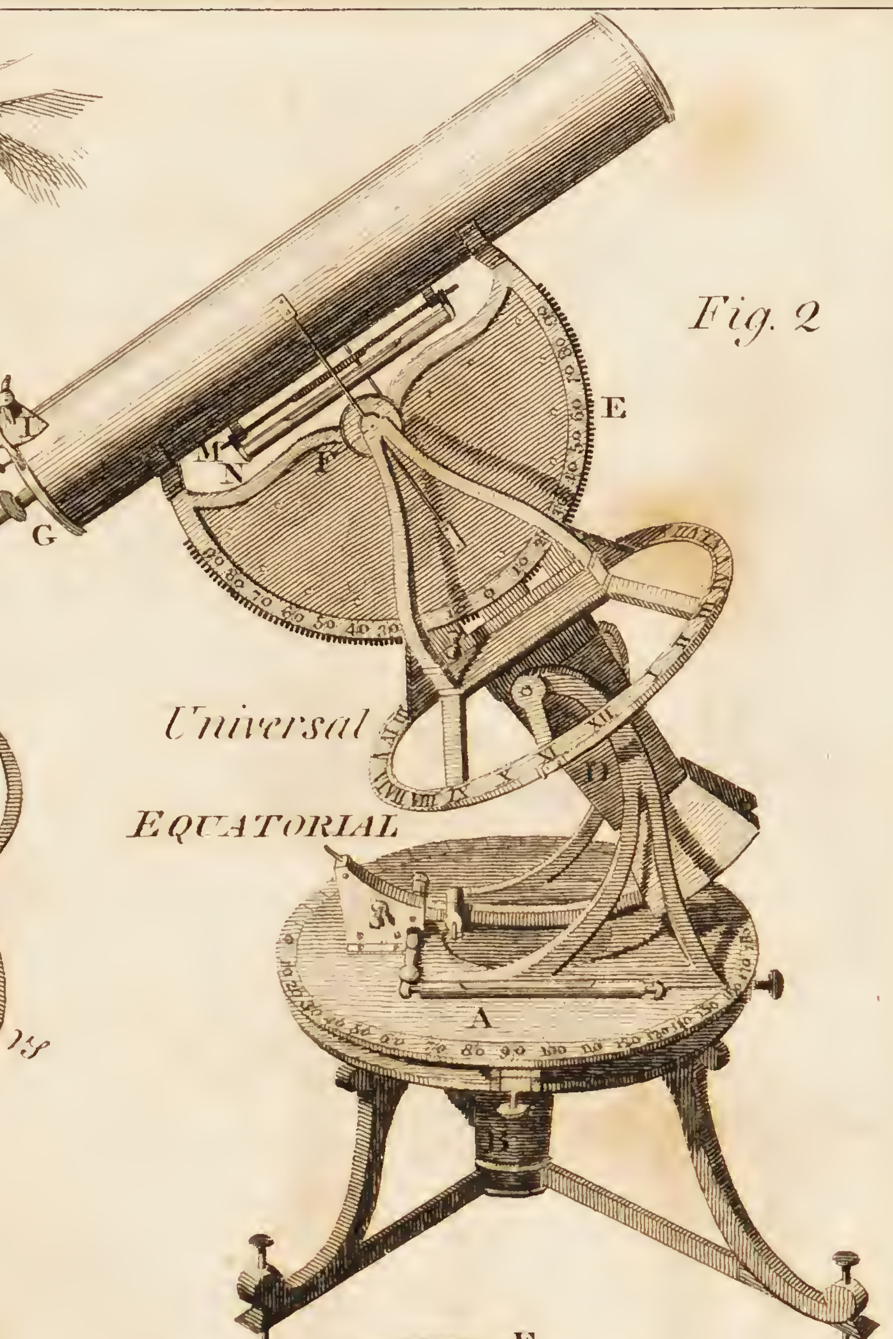
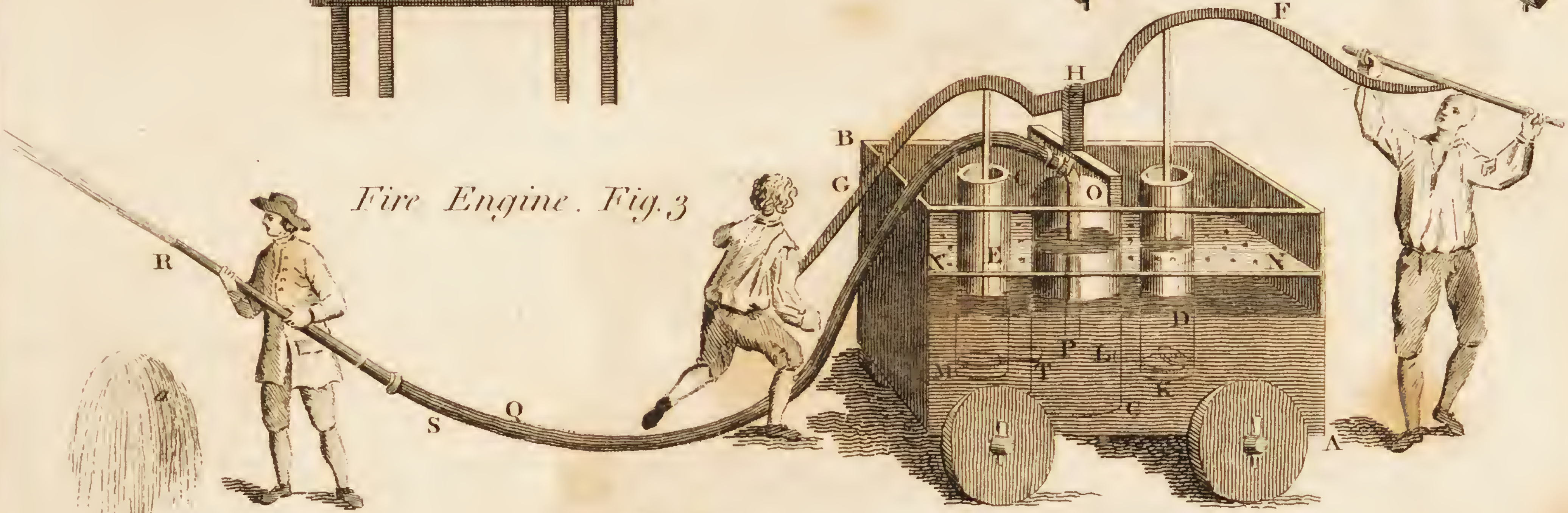


Fig. 2

Universal EQUATORIAL



Fire Engine. Fig. 3

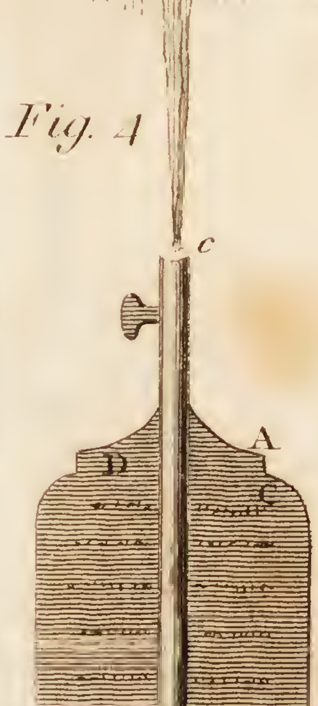


Fig. 4

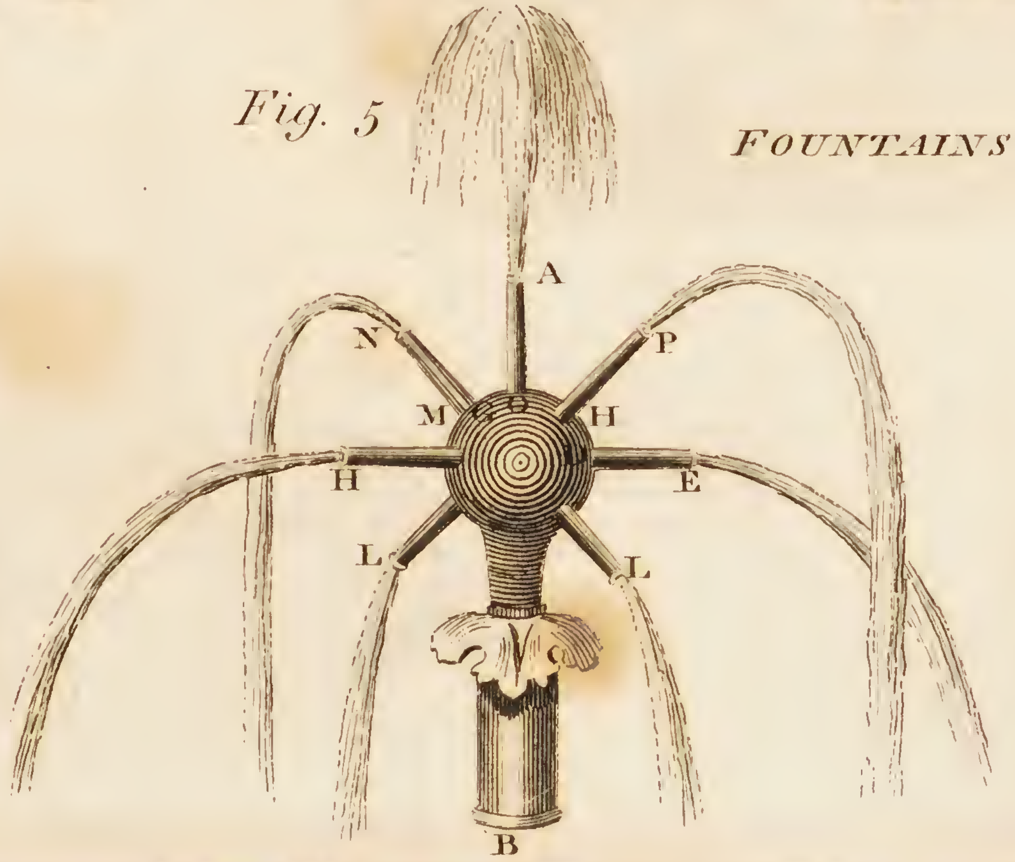


Fig. 5

FOUNTAINS

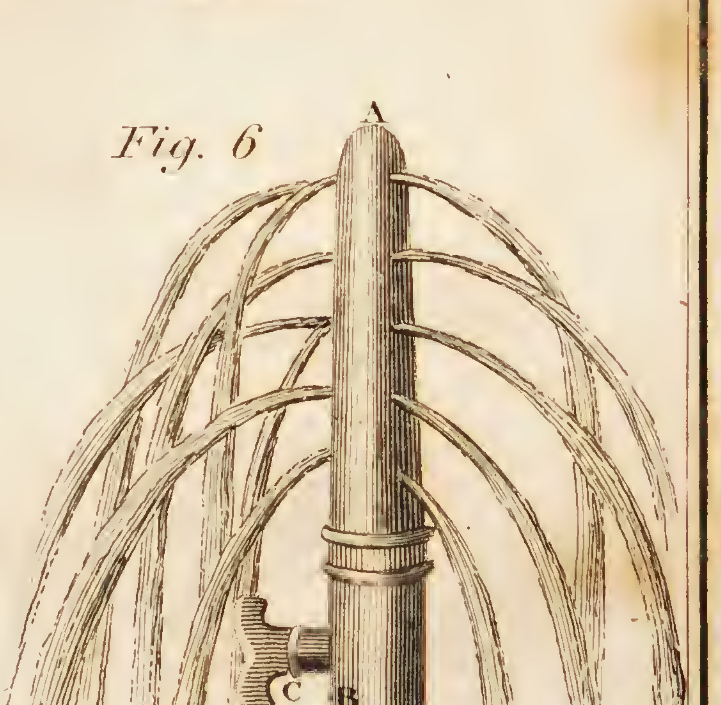


Fig. 6

ELECTRICAL MACHINES.

Fig. 1

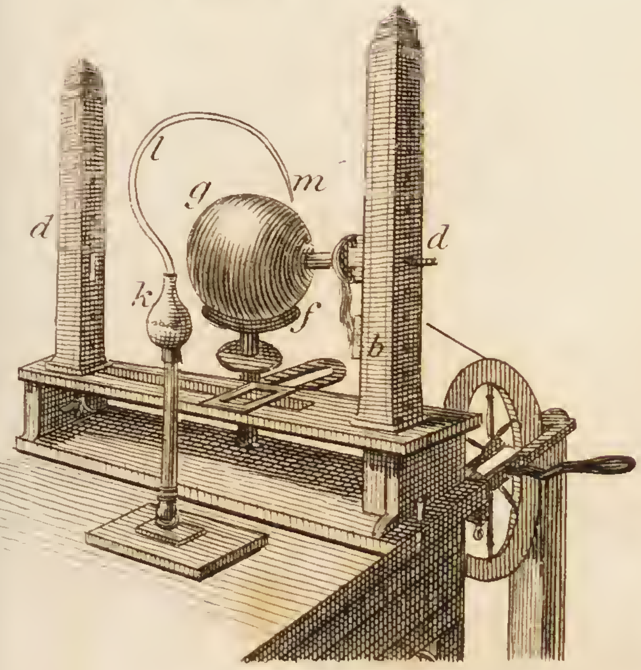


Fig. 2

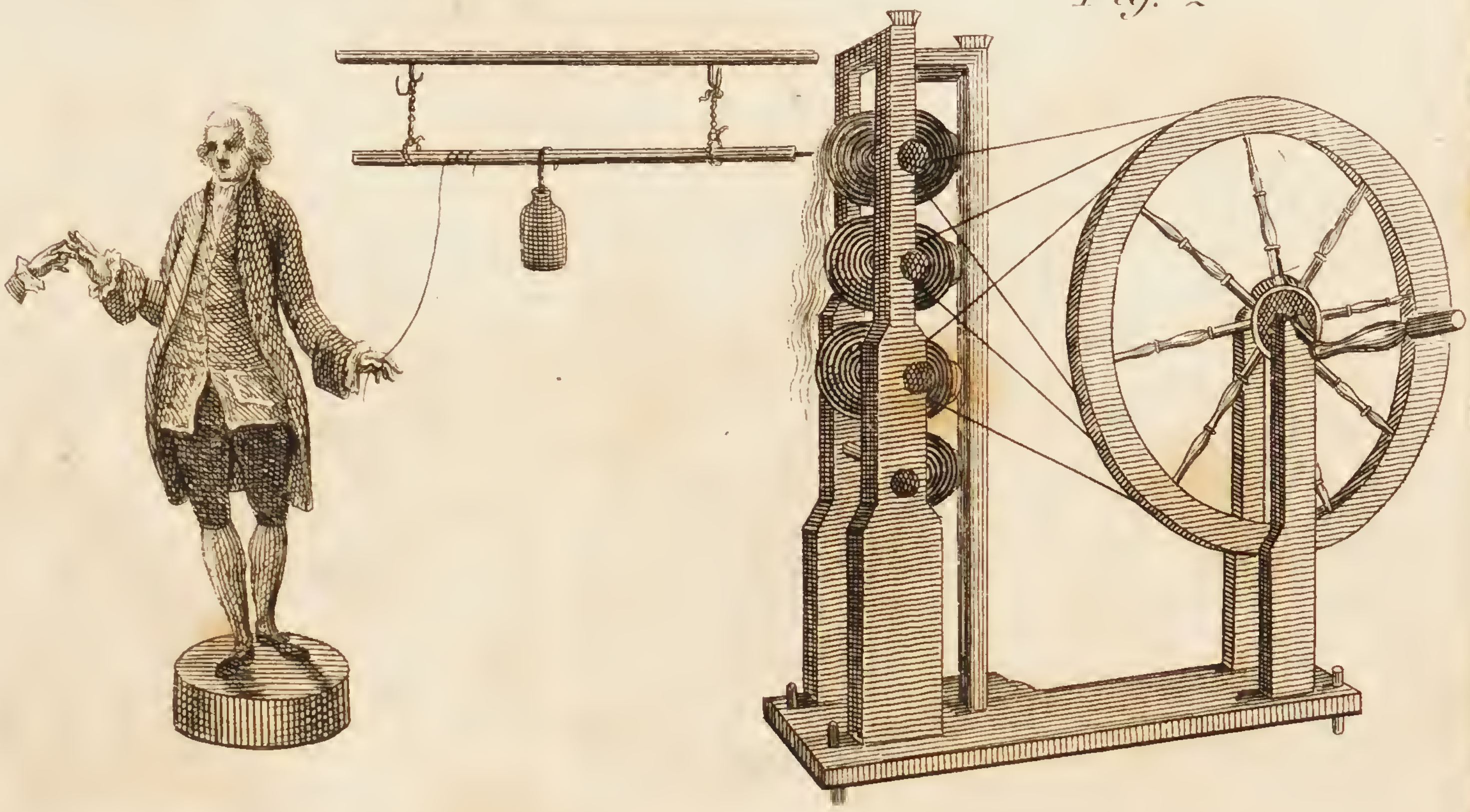


Fig. 3

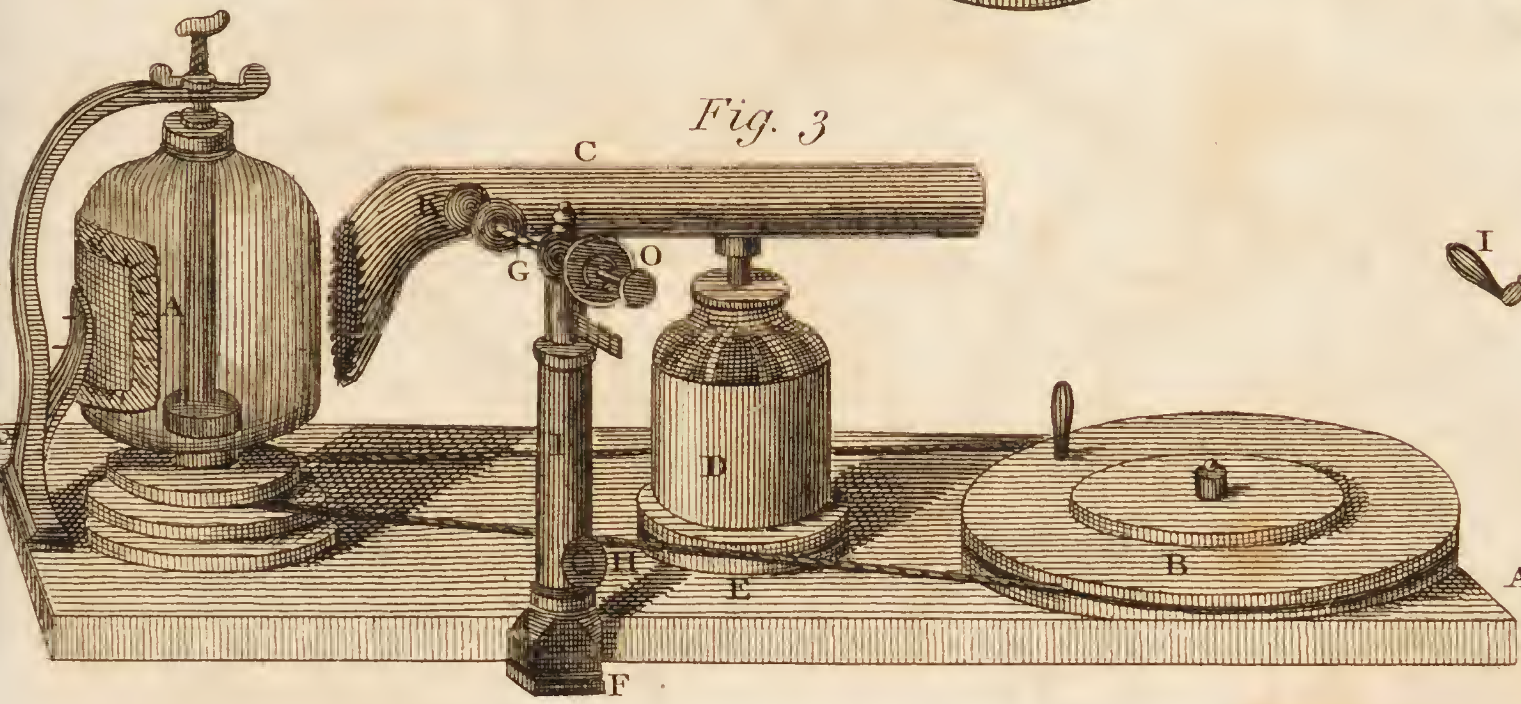


Fig. 4

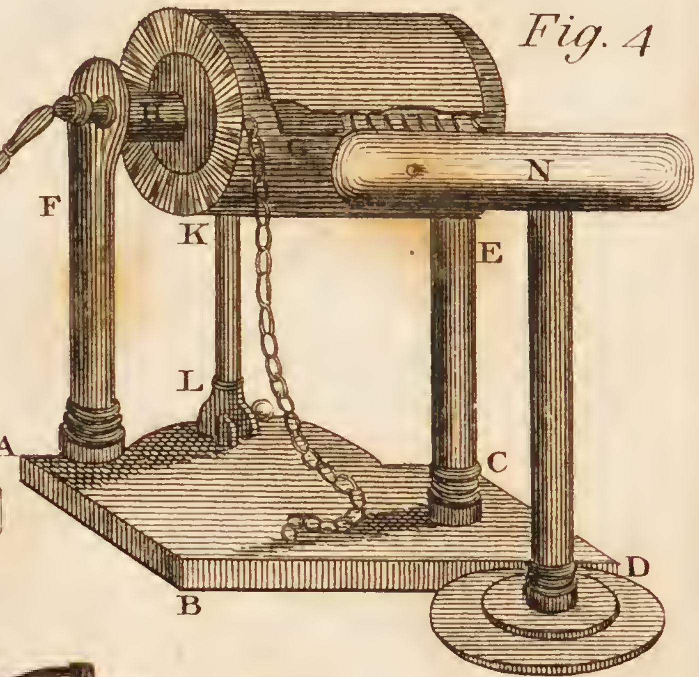
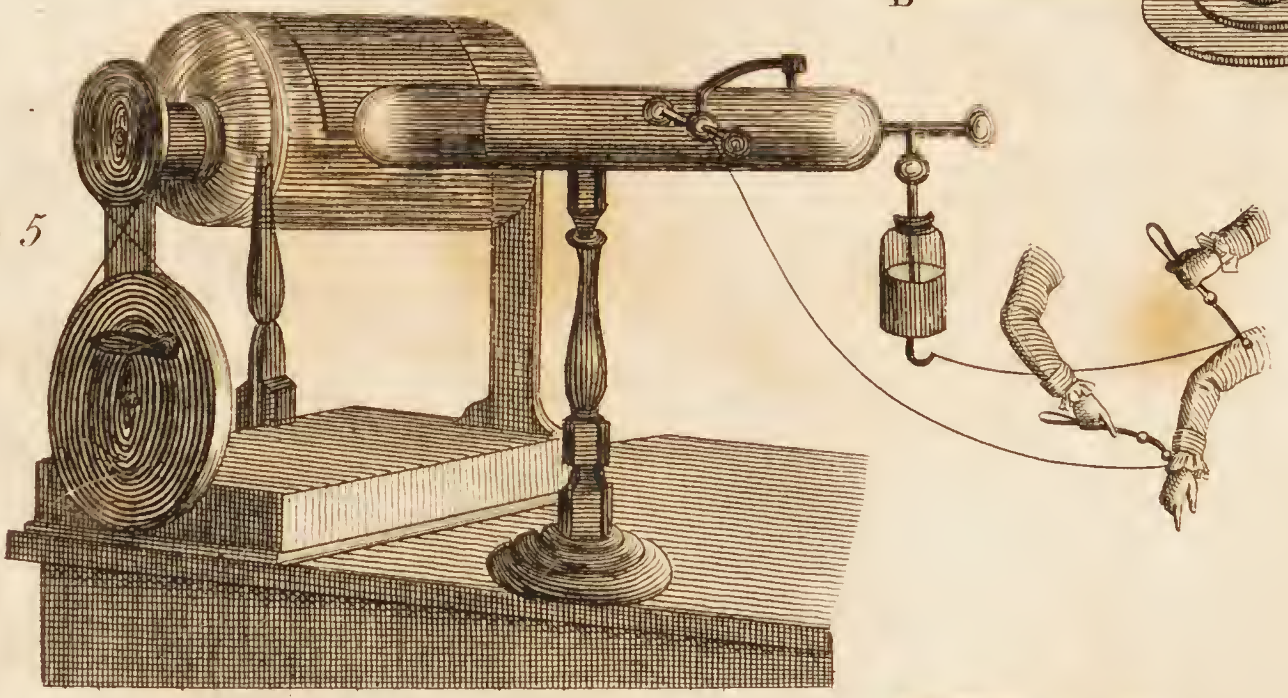


Fig. 5



Canton's
ELECTROMETER.

Fig. 7

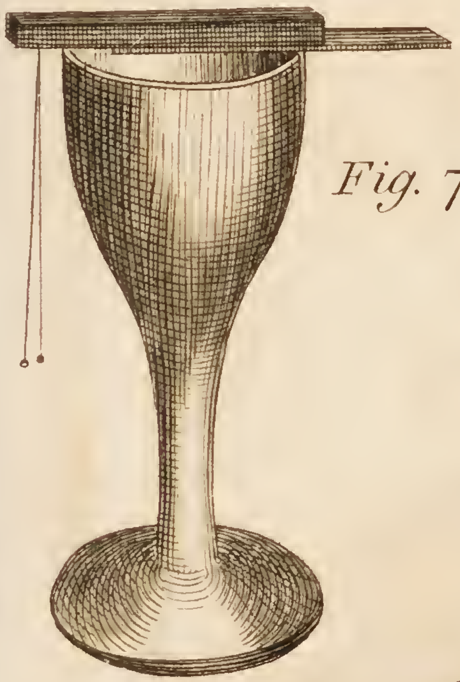


Fig. 8

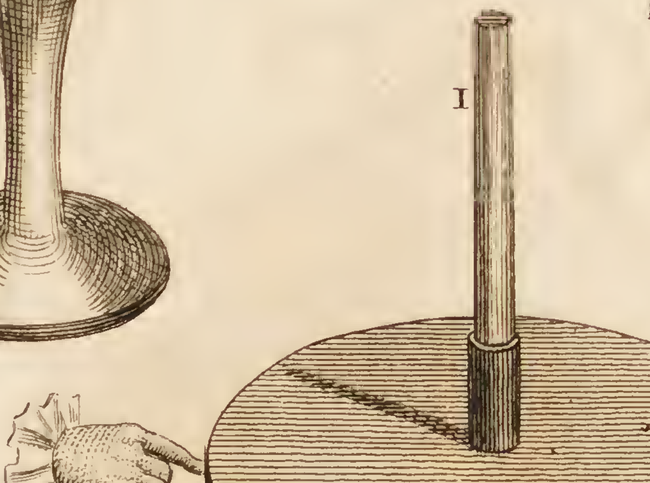
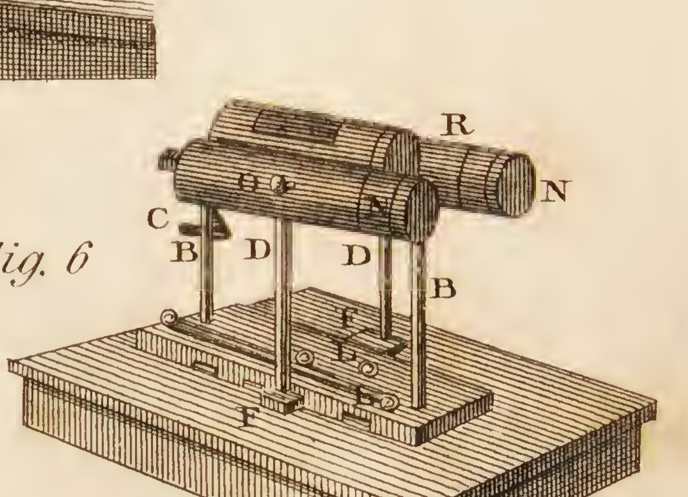
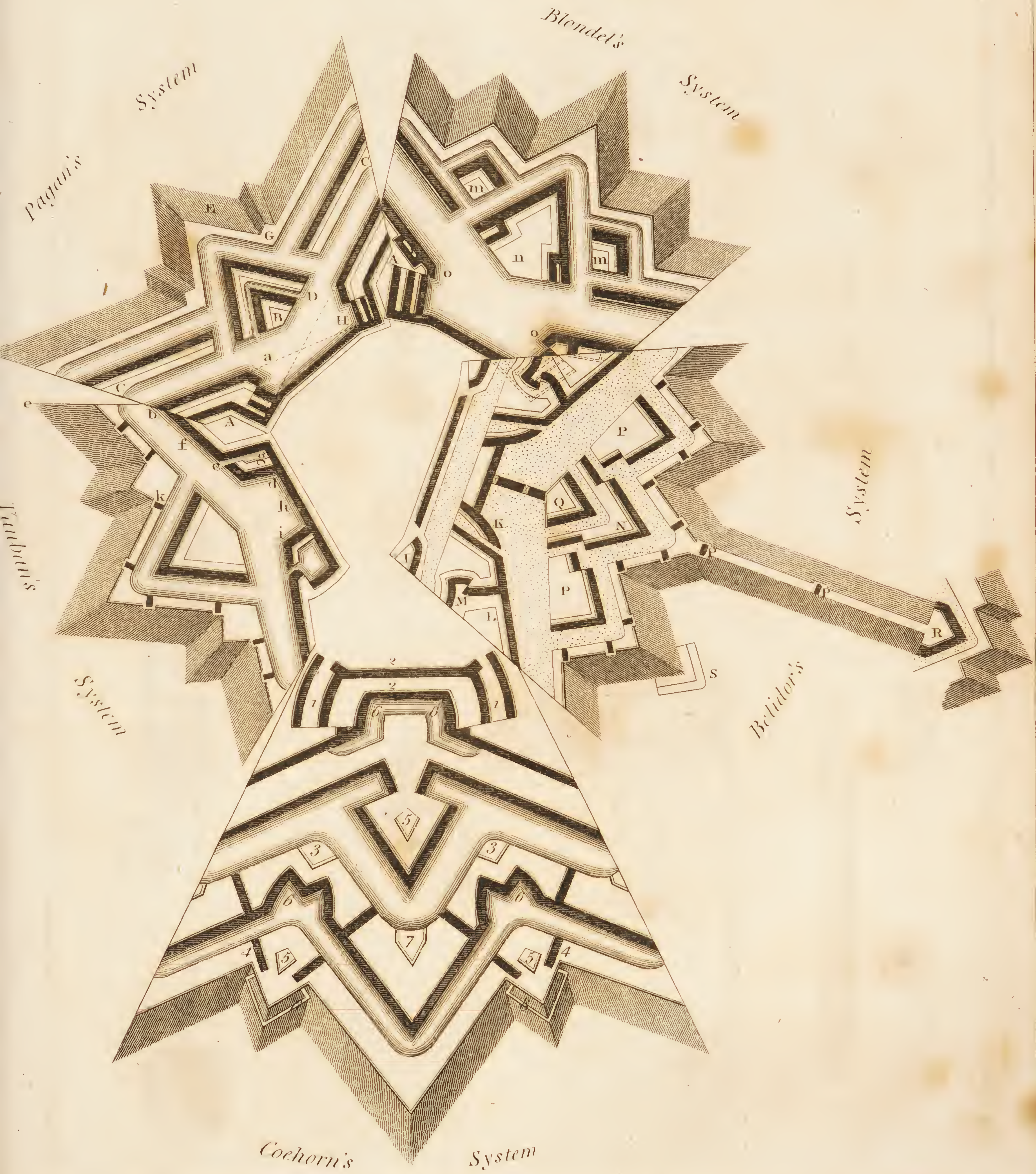


Fig. 6



General View of FORTIFICATION.



FOUNTAINS.

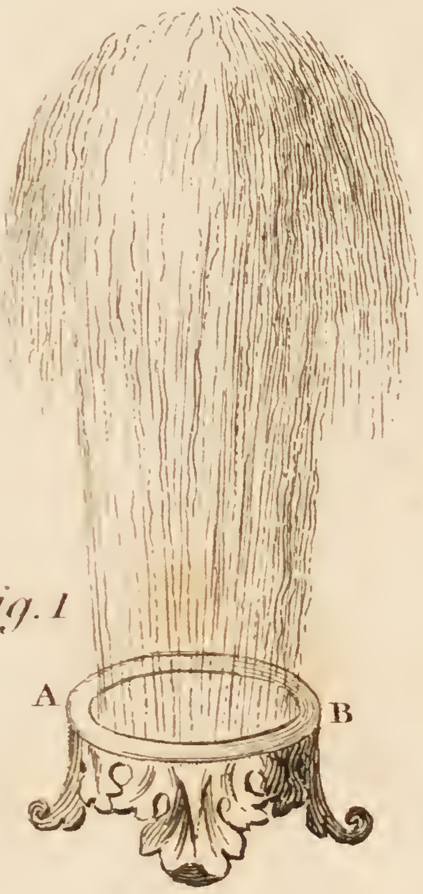


Fig. 1

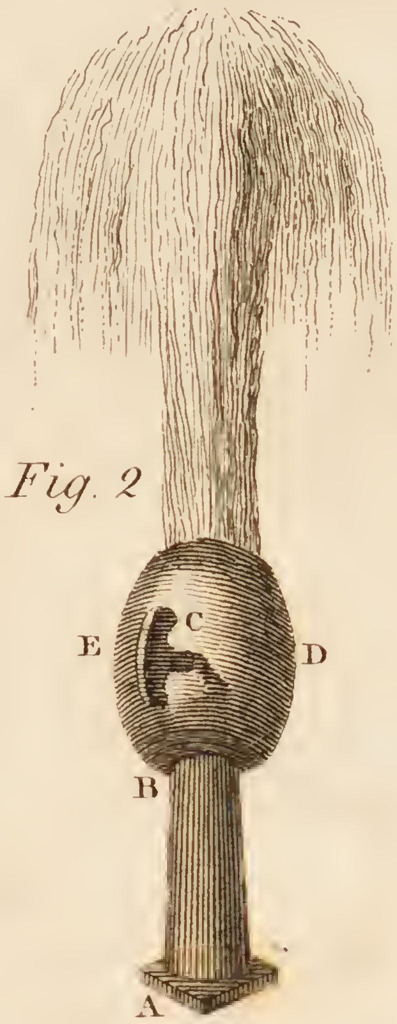


Fig. 2

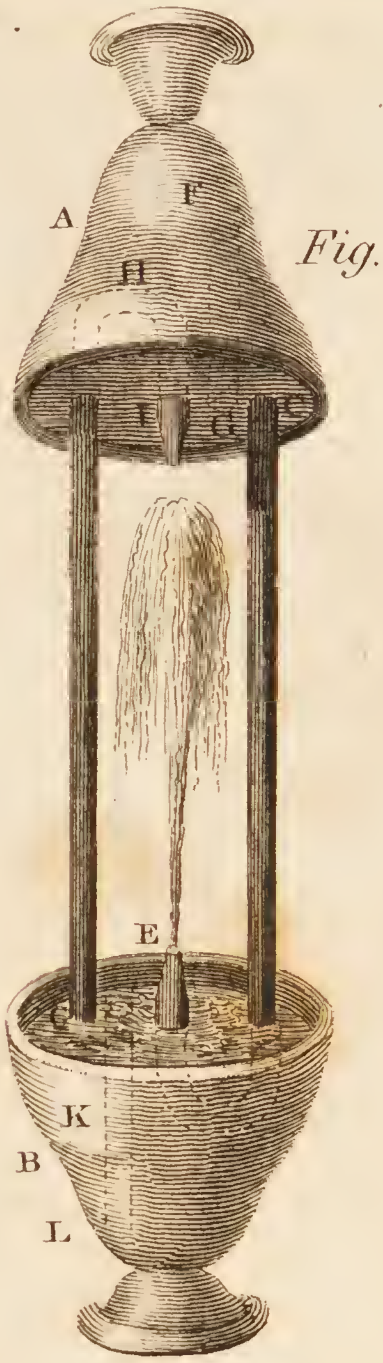


Fig. 3

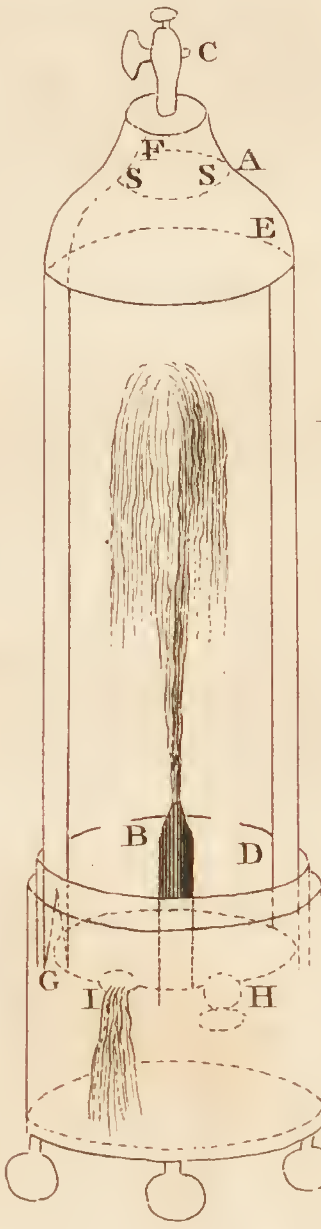


Fig. 4

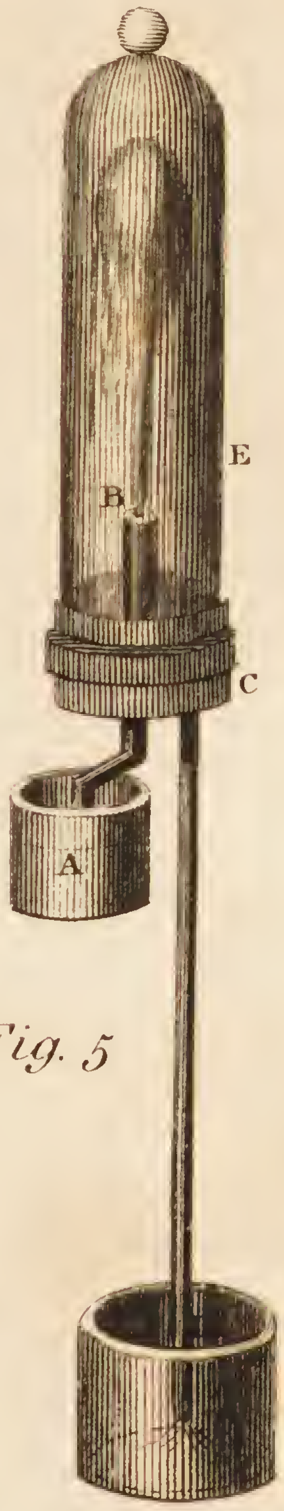


Fig. 5

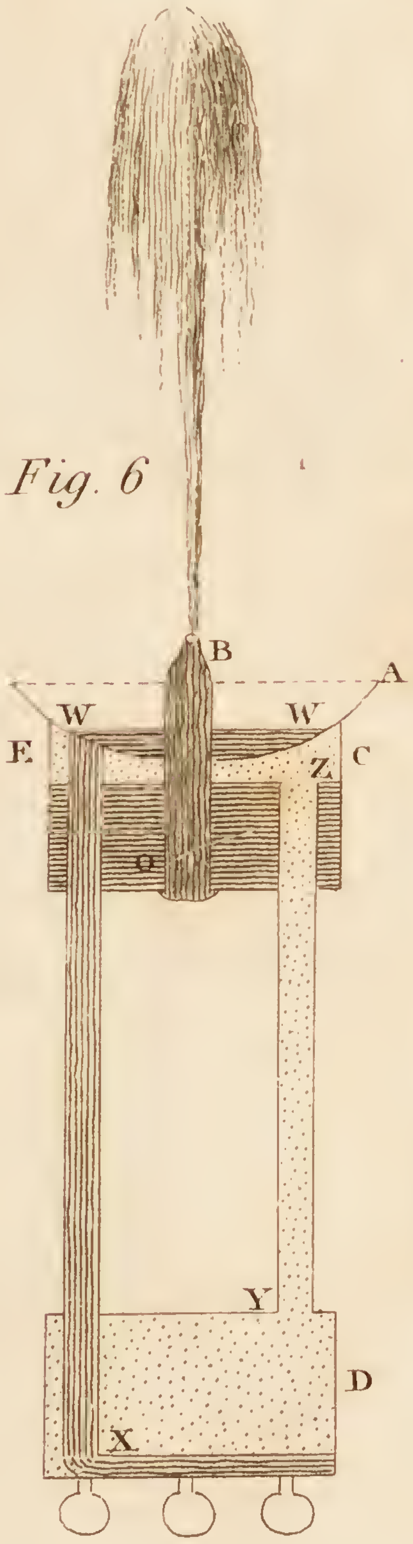


Fig. 6

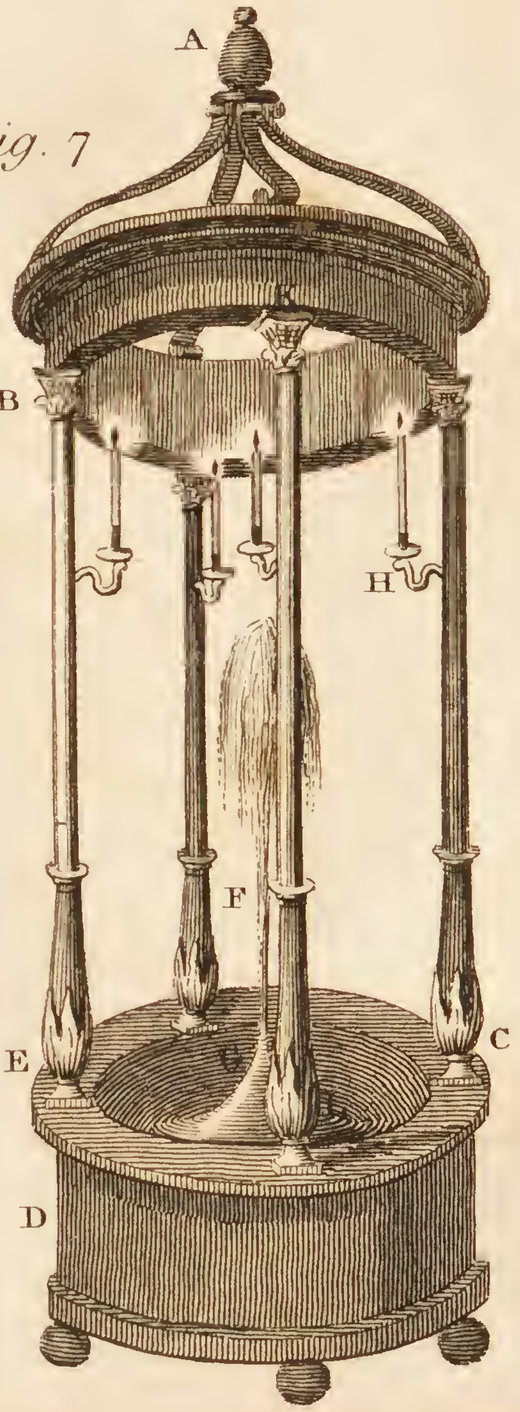


Fig. 7

Fountain Pen

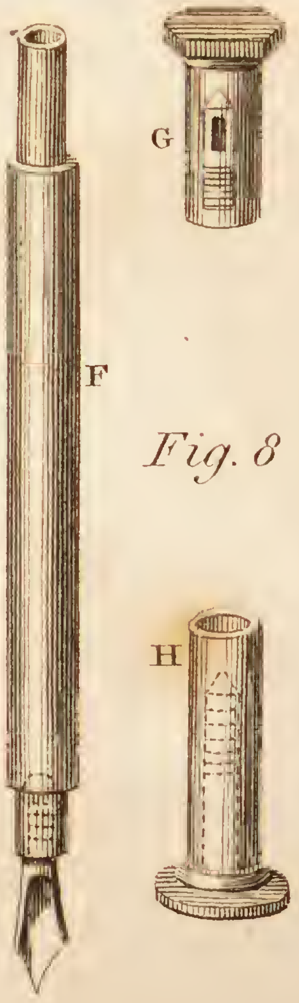


Fig. 8

Artificial Globe

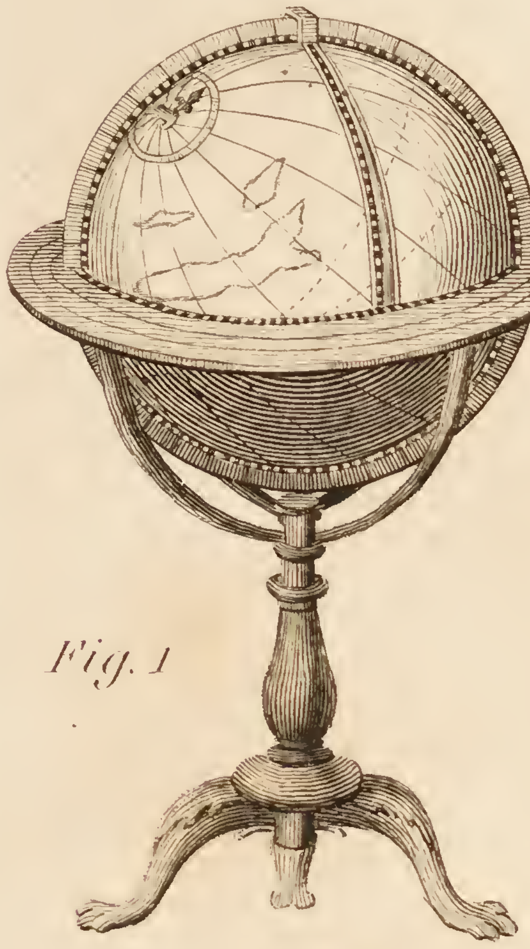
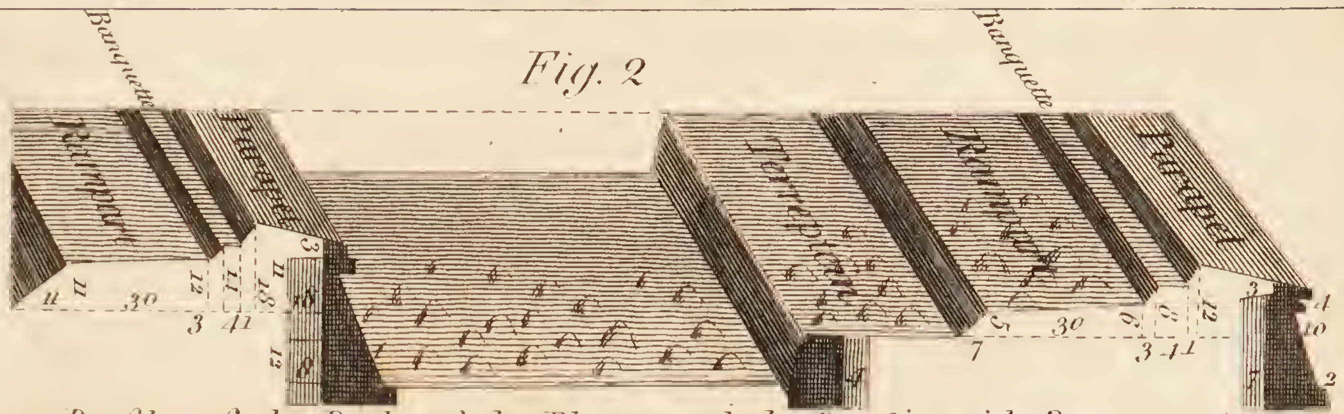
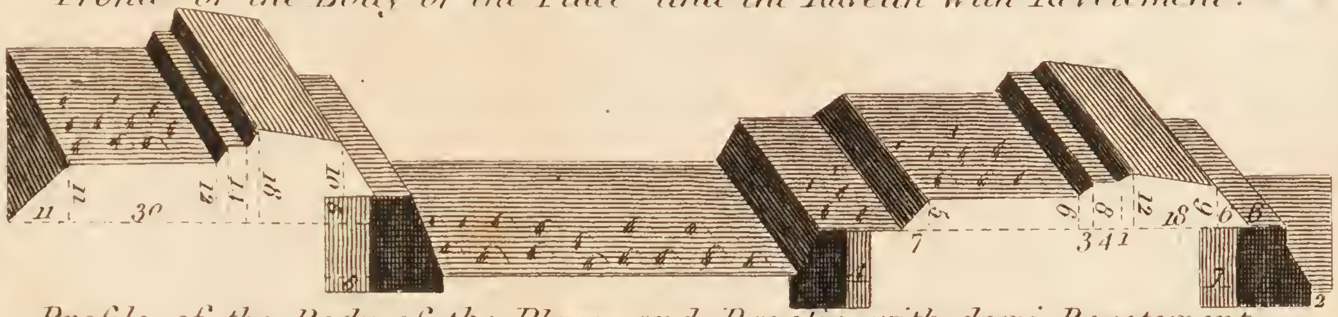


Fig. 1

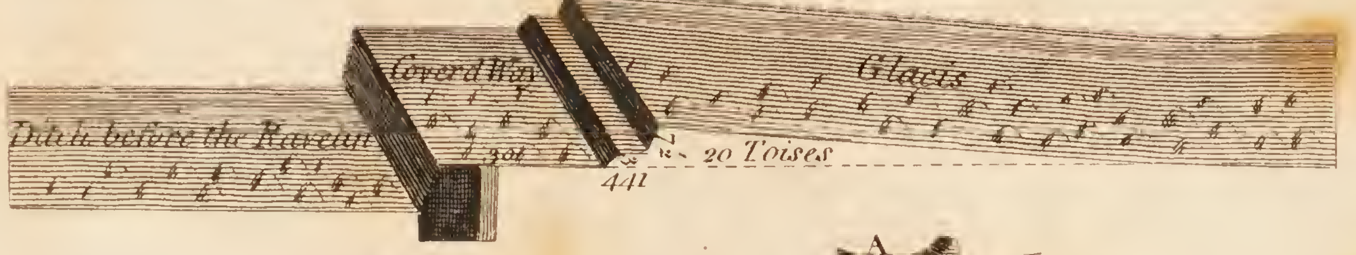
Fig. 2



Profile of the Body of the Place and the Ravelin with Revetement.



Profile of the Body of the Place and Ravelin with demi Revetement.



HYDROMANTIC

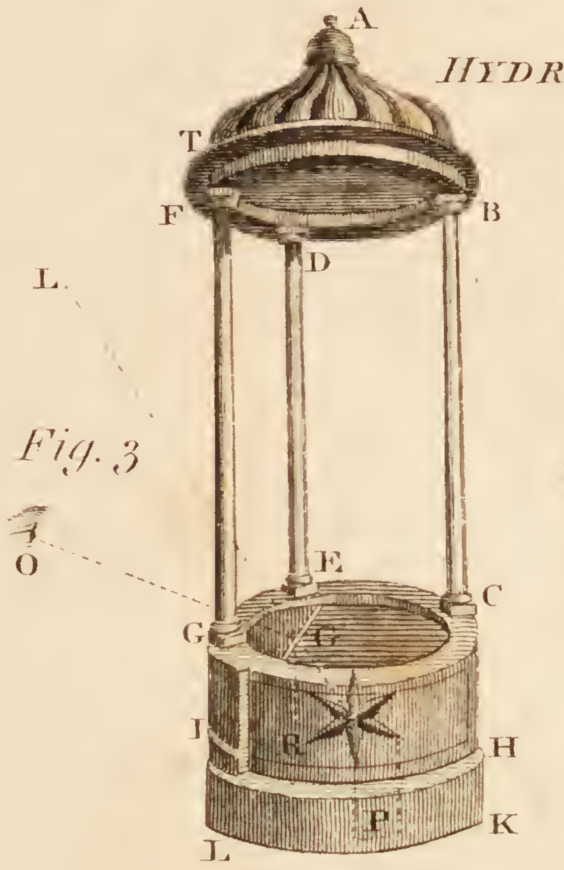
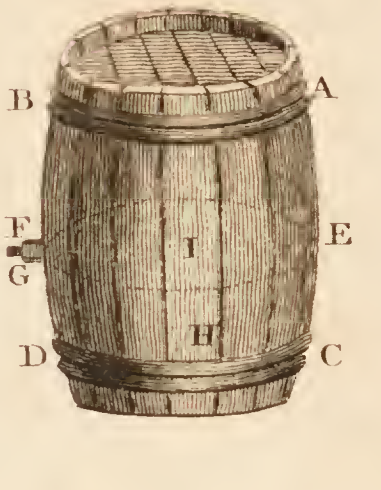


Fig. 3

Fig. 4



Hydrometer

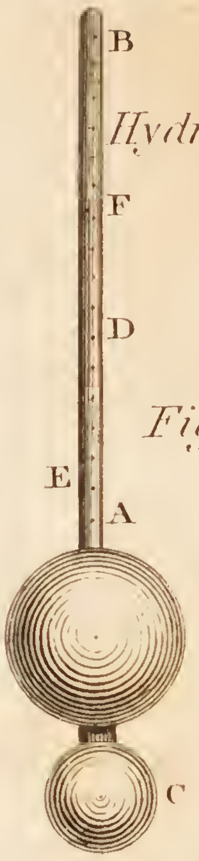


Fig. 5

JACK

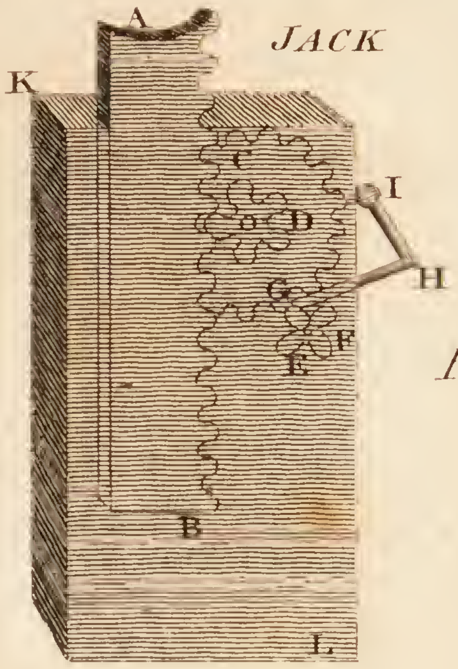


Fig. 6

Kitchen or Fly JACK

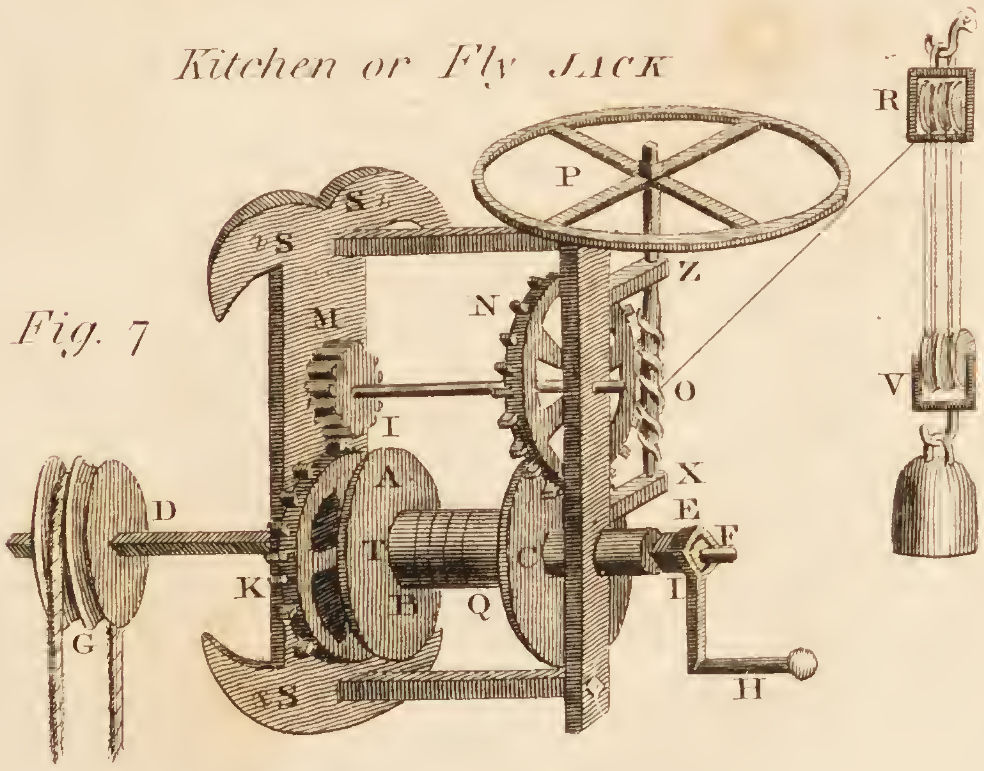


Fig. 7

Smoke JACK

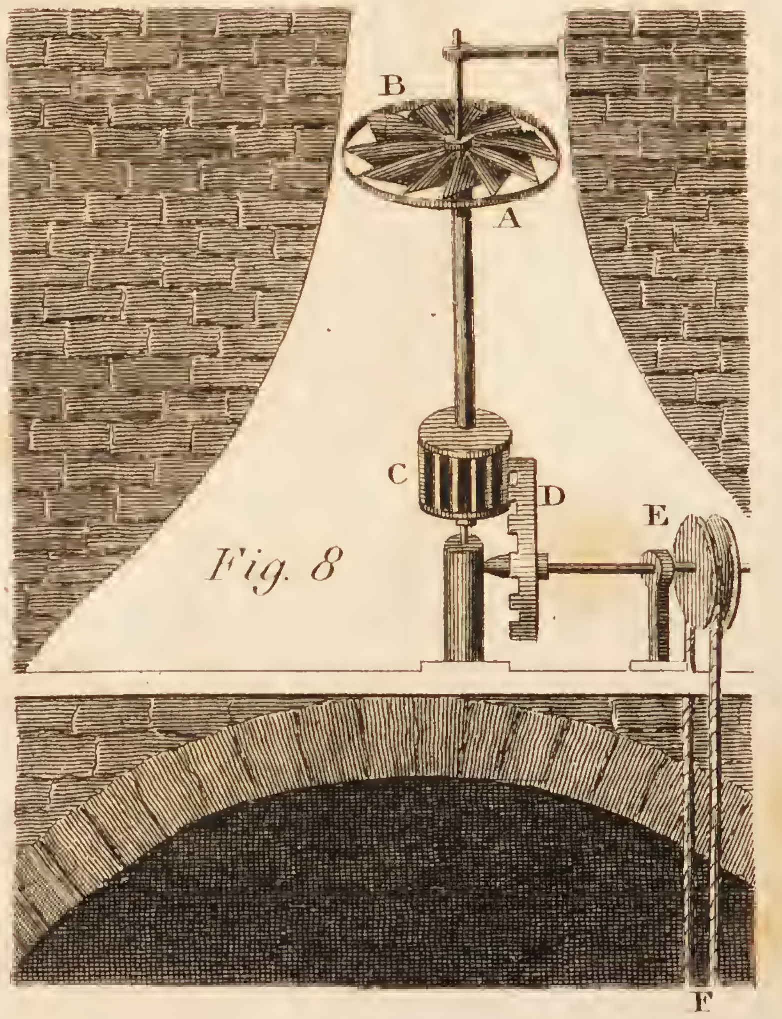


Fig. 8

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